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Pre-service teachers engaged in noticing aspects of learner written work

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The study reported on here focused on pre-service teachers noticing learner thinking in the context of written work. The results show how pre-service teachers engaged in noticing learner thinking and on which aspects of learner thinking they focused. These results and related discussion broaden our conceptualisation of teacher noticing learner thinking as involving both disciplinary and non-disciplinary-specific aspects and provides related pedagogical implications for those who educate teachers.

Keywords: learner thinking; mathematics education; teacher education; teacher noticing

Introduction

Globally, educational stakeholders and those who educate teachers are paying increasing attention to how pre-service teachers learn to teach and how to effectively ground teacher education in classroom instruction (Arends, Winnaar & Mosimege, 2017; Darling-Hammond, Burns, Campbell, Goodwin, Hammerness, Ling Low, McIntyre, Sato & Zeichner, 2017; Jenset, Klette & Hammerness, 2018; Moon, 2016). These kinds of practice-based approaches to teacher education suggest that pre-service teachers should be engaging in analysing learner thinking during both their practicum experiences and in university courses (Ball, 1993; Jenset et al., 2018; Kazima, Pillay & Adler, 2008; National Council of Teachers of Mathematics [NCTM], 2014; Puig Gutiérrez, Cruz-Guzmán & Rodríguez-Marín, 2019). Prior research suggests that pre-service teachers can learn to analyse learner thinking when they are provided with opportunities to practice noticing learner thinking during teacher preparation experiences (see Sun & Van Es, 2015).

In mathematics education, as teachers learn to notice, they often first direct their attention (noticing) on more general aspects of classroom settings (Van Es & Sherin, 2010). While there are many aspects that a teacher could notice in a classroom at any given moment, the most critical is to actively notice learner thinking (Grosser & Nel, 2013; Jacobs, Lamb & Philipp, 2010; Puig Gutiérrez et al., 2019). For this reason, we designed a research context that allowed pre-service mathematics teachers to focus specifically on noticing learner thinking. As we focused pre-service teachers on noticing learner thinking, we engaged the participants in a semi-structured interview to support them in noticing learner thinking in written work. We believe that this is a particularly rich area of research, as past and current research indicates the importance of pre-service teachers learning to focus on (notice) learner thinking in their teaching (e.g. Grosser & Nel, 2013; Jacobs et al., 2010; Puig Gutiérrez et al., 2019).

Literature Review and Theoretical Framework

Situating a pedagogy of teacher education focused on learner thinking

An emphasis on learner thinking is not a new focus in mathematics teacher education pedagogy. For example, 20 years ago, Cognitively Guided Instruction provided a context that successfully focused teacher learning on engaging with learner thinking (Franke & Kazemi, 2001). More recently, university mathematics method courses are beginning to include core practice-based accounts of effective teacher preparation (McDonald, Kazemi & Kavanagh, 2013). For example, the core practice of eliciting and responding to learner thinking through video analysis, written cases, modelling and rehearsals (McDonald et al., 2013) is in many ways akin to noticing learner thinking. Furthermore, focusing pre-service teachers on noticing learner thinking is a proven effective guiding conceptualisation for designing, enacting, and researching university mathematics method course pedagogy (Levin, Hammer & Coffey, 2009; Star & Strickland, 2008; Sun & Van Es, 2015). In other words, we suggest that the study of pre-service teachers noticing learner thinking is situated in current mathematics education research and pedagogy and provides its own pedagogical and empirical value. For these reasons, we used pre-service teachers noticing learner thinking to guide our study. We share the current literature on both teacher noticing and more specifically, teachers noticing learner thinking.

Teacher noticing

Teacher noticing is a theoretical construct of teacher practice that stems from the work of Sherin, Jacobs and Philipp (2011). How teachers engage in noticing is described in the literature as involving three main

components: (1) attending to, (2) interpreting, and (3) responding to particular events in an instructional setting (e.g. Colestock & Sherin, 2016; Jacobs et al., 2010; Sherin & Van Es, 2009). Learning to notice is a professional skill, and we expect pre-service teachers to notice in more diverse, less focused ways than experienced teachers do (Jacobs et al., 2010; Star & Strickland, 2008). Research investigating pre-service teacher noticing examines various aspects of this construct, from *how* they notice (e.g. focus on describing rather than making sense of learner thinking), to *what* is noticed (e.g. the classroom organisation, teacher moves, content), to *who* is noticed (e.g. whole group, individual learners, small group), and *why* something is noticed (e.g. correct/incorrect, interesting) (Levin et al., 2009; Rosaen, Lundeberg, Cooper, Fritzen & Terpstra, 2008; Simpson & Haltiwanger, 2017; Star & Strickland, 2008; Van Es, 2011).

Teacher noticing of learner thinking

While there is a growing body of research on teacher noticing in general, fewer studies specifically examine pre-service teacher noticing of learner thinking. Much of the research on teacher noticing focuses on whether and how they notice learner thinking instinctively (Van Es & Sherin, 2010). Jacobs et al. (2010) have extended the general noticing construct by focusing directly on learner thinking and introducing the idea of *professional noticing of children's mathematical thinking*. Professional noticing of children's mathematical thinking is comprised of three interrelated skills: attending to learner's strategies, interpreting children's understanding, and deciding how to respond based on these understandings (Jacobs et al., 2010). In our study we drew on Jacobs and colleague's conceptualisation of noticing while focusing directly on the learner and referred to this as *noticing learner thinking.* We used a research context in which pre-service teachers were primed to focus on learner thinking and our interview prompts were tied specifically to *noticing learner thinking*. Prior and current research demonstrates value in narrowing down the construct of teacher noticing in this way (Luna & Selmer, 2021; Luna & Sherin, 2017; Selmer & Luna, 2018). For example, Luna and Sherin (2017) found that when teachers use point-of-view cameras to capture learner thinking and then watch and discuss these excerpts with other teachers in a professional development setting, the discussion often immediately focuses on learner thinking. This work demonstrates that by narrowing down the construct of teacher noticing to noticing learner thinking, professional development experiences can better support teachers in learning to attend to, make sense of, and respond to their learner's thinking. Literature shows that this premise holds true for pre-service teachers' educational experiences, and that pre-service teachers can develop the ability to effectively focus on learner thinking when appropriate scaffolding (e.g. explicit questioning, written case studies and video analysis) is used (Levin et al., 2009; Puig Gutiérrez et al., 2019; Sun & Van Es, 2015).

In the design of our research, we built on prior work (see Luna & Selmer, 2021) that required a practicing teacher to notice learner thinking in the context of a semi-structured interview focused on learner written work in mathematics and science. Like this study, we used explicit interview questions that required participants to engage in the noticing of learner thinking. Our focus was on pre-service teachers rather than practicing teachers and we used learner written work in mathematics only. Therefore, we built on the existing literature by examining pre-service teachers noticing learner thinking to answer the following research question: What does pre-service teacher noticing of learner thinking look like when focused on written work? In answering this question, we focused on two dimensions of their noticing: (a) *How* do pre-service teachers engage in *noticing learner thinking*, and (b) on what *aspects* of learner thinking do pre-service teachers focus?

Methods

Context

We present a study of eight pre-service mathematics teachers enrolled in an initial teacher preparation programme in a city in Southern Africa. Each of the participants were enrolled in the fourth and final year of a Bachelor of Education degree programme that allowed them to teach mathematics in primary school (Grades 4 to 7). The pre-service teachers had 9 weeks of practicum experience in their second and third years, but it is unlikely that they would have observed many instances of learner constructed calculation strategies, since written mathematics work tends to be formulaic. During their final year, the pre-service teachers were involved in a weekly mathematics club at a local primary school as part of their mathematics method course. Furthermore, during a 2-hour weekly session, theory and literature related to learner thinking, as well as examples of learner thinking, obtained from the mathematics club by the participants, were discussed.

Researcher-Provided and Participant-Chosen Examples of Learner Written Work

We used two distinct types of learner written work. The first was directly connected to a pre-service teacher's experiences in the mathematics club and the second was situated in a real learning context, but not directly drawn from the actual pre-service teachers' experiences. We used both types of learner written work in response to Land, Tyminski and Drake's

(2019) suggestion to examine both written work provided by researchers and written work directly related to pre-service teachers' classroom teaching contexts to comprehensively consider teacher preparation experiences. The participant-chosen examples of learner written work under investigation show mathematical topics including counting and cardinality, base-10 understanding, and basic operations. The participants were asked to collect written work that showed learner thinking, and to then purposefully select two of these examples in order to discuss them during two semi-structured interviews (a total of four participant-chosen examples of learner written work). Figure 1 displays a sample of three participant-chosen examples of learner written work.

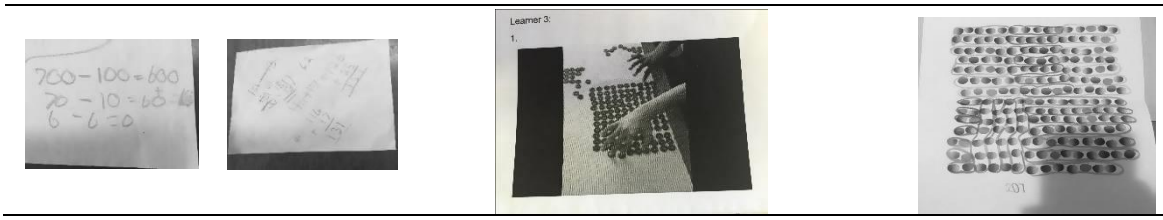

Figure 1 Participant-chosen examples of learner written work

Three researcher-provided examples of learner written work for each interview (a total of six) reflected a similar focus to the participants' current mathematical work in the local school setting. These examples were collaboratively developed by the researchers or used with permission from other studies (see Jacobs et al., 2010). The six researcher-provided examples of learner written work were meant to demonstrate a breadth of learner thinking related to direct models, problem-solving, base-10 understanding, properties of the number system, and the use of direct modelling and learner constructed strategies.

Semi-Structured Interviews

Each participant engaged in two semi-structured interviews – conducted 4 weeks apart in order to target a breadth of number concepts – in which we discussed two examples of learner written work supplied by the participants and three researcher-provided examples. The interviews followed a cyclical round of questioning on each piece of written work. The researcher first prompted the participants to share why they had chosen to bring each example of learner written mathematical work. Following that, the interviewer prompted the participants to notice learner thinking through describing, making sense of, and responding to learner thinking, (e.g. How would you describe the learner thinking? How do you make sense of the learner thinking? How would you respond to this learner?). The interview then shifted to the second participant-chosen example of mathematical learner written work, and then to the researcher-provided examples, with analogous questions.

Data Sources and Analysis

The complete data set consisted of 32 digitally scanned participant-chosen examples of learner written work, six researcher-provided examples, and 16 corresponding interview transcripts. We first segmented the 16 teacher interview transcripts, based on each separate instance of a learner's written work. We then segmented each separate example, based on when each pre-service teacher raised a new idea while discussing the written work at hand. This segmenting of the transcripts was similar to how other researchers break down data to identify a meaningful unit for analysis (e.g. Grant & Kline, 2004). One thousand two hundred segments were then analysed using a coding scheme that characterised *how* the pre-service teachers talked about learner thinking, and on what *aspects* of learner thinking they focused. The next step in our data analysis was to reduce the 1,200 data segments. Firstly, we reduced the number of segments by combining any concurrent segments for the *how* codes (N = 972). We then returned to the 1,200 segments and combined any concurrent *aspects* codes (N = 857). We then analysed the numerical data to identify tendencies. Next, we describe each coding scheme used.

How coding scheme

The *how* coding scheme (describe, evaluate, interpret) was derived from a framework by Sherin and Van Es (2009) to describe how teachers engaged in noticing classroom events, including learner thinking (see Table 1). Additionally, based on the work of Jacobs et al. (2010) we added a respond code. Thus, our final coding scheme included four codes (describe, evaluate, interpret,

and respond) in efforts to capture **how** the pre-service teachers engaged in noticing learner thinking. Since we used the interview context to engage the pre-service teachers in actively noticing learner thinking, we expected that these codes would demonstrate **how** they talked about learner thinking. We found the four *how* codes fully captured the data, and that contemplation of additional codes was not indicated for how the pre-service teachers engaged in this type of work. Therefore, the *how* coding scheme was used to verify that the pre-service teachers were engaged in a cycle of noticing learner thinking and to explicate how the pre-service teachers engaged in *noticing learner thinking*. Each segment coded as describe, evaluate, or interpret was further coded to indicate on what *aspects* of learner thinking the pre-service teachers focused. We report on what we refer to as the pre-service teachers' *responses* in a separate study.

Table 1 *How* coding scheme

Code	Description of code
Describe	statement that characterises what was noticed in the written work, or recounts events that occurred in the production of the written work
Evaluate	statement that involves any kind of judgement about what was noticed (e.g. comment on what is good/bad, correct/incorrect, etc.)
Interpret	statement that involves making an inference about what was noticed
Respond	statement that involves a teacher action connected to what was noticed

Aspects coding scheme

Drawing on existing research, we found parallels between our study and a previous study by Luna and Sherin (2017), in that both studies explicitly focused teachers on learner thinking. While the related study context used point-of-view cameras, we used learner written work. In both studies research questions focused on what *aspects* of learner thinking teachers noticed. Drawing from existing literature, we included (modified for our specific research context) the following codes: learner characteristic, source, and content.

We used the learner characteristic code for statements in which the pre-service teachers noticed a learner emotion or trait (e.g. the learner is a sad child, the learner is smart, the learner is lazy). The source code was used for statements in which the pre-service teachers were attending to or making sense of where a learner's idea and/or thinking came from (past lesson, home experience, etc.). The content code was used for statements in which the pre-service teachers talked about a learner's mathematical understanding, removed from the specific example of learner written work. For example, if a pre-service teacher was making sense of how a learner determined the sum of two three-digit numbers, referring to the specific work, it was coded as written work. However, if the pre-service teacher was making sense of a learner's more general mathematical understanding related to a chosen strategy (e.g. a strategy shows understanding of properties of the number system), it would be coded as content.

Thus, during initial data analysis, we recognised the emergence of three new *aspects* of learner thinking codes – written work, general, and pre-service teacher. The written work code was used to capture when pre-service teachers specifically noticed both the learner's written work and the related classroom experience (connected to the written work). For example, if the participant described dialogue between the teacher and the learner when the written work was created, it was coded as describe (how) and written work (aspect). The general code was used whenever a pre-service teacher shifted from talking about learner thinking related to the written work, to more general noticing (e.g. about the whole class). The teacher code captured when pre-service teachers explicitly talked about themselves. These codes aligned to similar codes (artefact, general, and teacher) found in a concurrent study conducted by Selmer and Luna (2018) in the context of an expert, practicing teacher noticing learner thinking.

Thus, our final coding scheme included six codes: written work (WW), content (C), general (G), pre-service teacher (PT), learner characteristic (LC), and source (S). Once the final coding scheme had been developed and tested, we independently coded each transcript segment. Differences were discussed, and consensus was reached.

Results

In answering our research question (What does pre-service teacher noticing of learner thinking look like when examining learner written work?), we first focus on ***how*** the pre-service teachers engaged in noticing learner mathematical thinking. We then share results related to what *aspects* of learner mathematical thinking the pre-service teachers focused on.

How the Participants Engaged in Noticing Learner Thinking

As the pre-service teachers engaged in noticing they would describe, interpret, evaluate, and respond to learner thinking. Table 2 shows the frequency for this data.

Table 2 How the pre-service teachers noticed learner thinking

Code	Number of instances	Percentage of total
Describe	325	33%
Evaluate	108	11%
Interpret	357	37%
Respond	182	19%
Total	972	100%

The data show that the participants were engaged in a cycle of *noticing of learner thinking*. The overall percentages vary slightly across with more describing and interpreting (33% and 37%) than evaluating and responding (11% and 19%). These results did not surprise us, because of the designed research context, but they affirmed that participants were engaged in *noticing of learner thinking* and allowed us to further explore what *aspects* of learner thinking the pre-service teachers noticed. These findings hold important pedagogical value for teacher education, as they show that learning experiences designed around written work are effective to engage pre-service teachers in noticing learner thinking. We examine this finding further in the discussion section.

Aspects of Learner Thinking that Pre-Service Teachers Noticed

As the participants described, evaluated, and interpreted learner thinking, we were interested in what *aspects* of learner thinking they noticed. Table 3 displays the frequency of each aspect of learner thinking that the participants noticed.

Table 3 Aspects of learner thinking that pre-service teachers noticed

Code	Instances	Percentage
Written work	369	43%
Content	170	19%
General	39	5%
Pre-service teacher	111	13%
Learner characteristic	138	16%
Source	30	4%
Total	857	100%

The data reveal that pre-service teacher *noticing learner mathematical thinking* involves both disciplinary aspects (written work and content) and non-disciplinary aspects (general, pre-service teacher, learner characteristic, and source). Firstly, the data reveal that the pre-service teachers focused on *aspects* related to disciplinary thinking, either directly evident in written work (43%) or the related content (19%) codes, 62% of the time. Secondly, the remaining 38% of the instances showed that pre-service teachers focused on non-disciplinary aspects of learner thinking including pre-service teacher (13%), learner characteristics (16%), general (5%), and source (4%). We found it encouraging that the pre-service teachers only engaged in noticing general aspects of learner mathematical thinking in 5% of the instances, suggesting that the use of learner mathematical written work and explicit questioning hold pedagogical promise in its use within teacher education.

In the following section, we share two illustrative examples of participants noticing learner thinking (as evidenced by the **how** codes) and engaging in noticing learner thinking focused on discipline and non-discipline-specific aspects of learner thinking. These results align with the concurrent study conducted by Selmer and Luna (2018) that showed that expert teachers similarly focused on both disciplinary and non-disciplinary aspects of learner thinking, although data sources and frequency results varied. Two examples are presented, the first involves a participant-chosen example of learner written work, the second involves an example of learner work provided by the researcher.

Illustrative example 1

In this first example, the pre-service teacher, Simone, discusses the written work generated in a classroom experience, in which her learners were asked to create three equations using any operation that would result in 140 (for instance 70 + 70 = 140). The task was meant to explore learner understanding of operations and of base-10 concepts, as they could use any operation and any form of composition/decomposition. The following transcript excerpts, related descriptions, and codes share a sequence of Simone noticing learner thinking:

> Segment 1: *She's in Grade 6 and she's also a Xhosa learner*. [Describe: Learner characteristic]
> Segment 2: *She wrote down 50 + 50 and 2 + 2 and + 0 and that equals 140*. [Describe: Written work]
> Segment 3: *I had a feeling she could get to the answer, she just needed a little probing*. [Describe: Pre-service teacher]
> Segment 4: *The learner shared that she got 140 and asked if she could use the column method to verify the solution*. As the learner used the column method [to add the 50 + 50 and the 2 + 2 = and the 0], Simone continued, *She wasn't even half-way, she wrote down the first step, and was like, 'Oh, this must be 20 and 20!'* [Describe: Written work]
> Segment 5: *Through many of the things that she does, when she works things out mentally, or the way that she's used to, she gets the answers right. But as soon as you let her do it your way or tell her to do it a different way, or when she has to write it down, she gets the answer wrong.* [Interpret: content]
> Segment 6: *I don't know, I think learners like her can very easily fall through the cracks because a teacher doesn't have the time necessary to sit with them and work on a sum until they are able to have that, 'Aha-I-realise-what-I-did-wrong' moment.* [Interpret: general]
> Segment 7: *Because it's not that she's stupid.* [Describe: Learner characteristic] (Participant interview #4)

This example illustrates how Simone engaged in describing and interpreting learner thinking while focusing on varied aspects, including *learner characteristics*, the *written work*, herself as a *pre-service teacher*, the mathematical *content*, and *general* aspects of learners.

The next example illustrates a different pre-service teacher, Rochelle, engaged in noticing learner thinking in a researcher-provided example of learner written work.

Illustrative example 2

The researcher-provided example of learner written work was used with permission (see Jacobs et al., 2010 M&M problem) after a slight modification.

The problem was as follows: If there are six boxes, each containing 43 items, how many items are there in total? The depicted attempt at a solution provides evidence of base-10 understanding through decomposing and composing several quantities. There is an error in the last recombination, omitting the 6 from the last 86 to the total. The following transcript excerpts, related descriptions, and codes share a sequence of Rochelle noticing learner thinking:

> Segment 1: *I can see he goes 40 + 40 is 80, in the first two boxes, then 40 + 40 is 80 in the second two boxes, and 40 + 40 is 80 in the third and fourth boxes and then he does that with the 10s and units. He leaves out the fifth and the sixth box, so he doesn't add in that 80, so he leaves that on its own as if he discards it.* [Interpret: Written work]
>
> Segment 2: *Maybe he doesn't understand the concept of multiplication and division*. [Interpret: Content]
>
> Segment 3: *I think the learner is not comfortable doing this problem or he ran out of time*. [Interpret: Learner characteristic]
>
> Segment 4: *I think, why this solution is difficult for me is because I don't understand where he's taking all of the numbers from*. [Interpret: Pre-service teacher] (Participant interview #14)

This example illustrates how Rochelle engaged in describing and interpreting learner thinking, while focusing on varied aspects, including *written work*, *learner characteristic*, *content*, and herself as a *pre-service teacher*.

These two examples show that, as these pre-service teachers engaged in noticing learner thinking, they focused on varied aspects of learner thinking that involved both disciplinary and non-disciplinary-related aspects.

Discussion

It has been suggested that improved education systems and learner learning outcomes in South Africa can lead to positive economic outcomes (Statistics South Africa, 2017). We believe that improved learner learning outcomes can begin to be realised through improvements in teaching practices. In our work we examined one such teaching practice, *noticing learner thinking*, in the context of using written work and prompting questions. Based on our results and related findings, we suggest that the use of learner written work and questions prompting a pre-service teacher to notice learner thinking hold empirical and pedagogical implications that can inform the field.

Pre-Service Teachers are Noticing Learner Thinking

Firstly, our results show that the pre-service teachers were engaged in describing, interpreting, evaluating, and responding to learner thinking across all the analysed written work. In this research context, we knew that the participants were clearly engaged in noticing learner thinking. We found it pedagogically exciting that the pre-service teachers only shifted to more general noticing in 5% of the instances. This provides evidence that explicit questioning allowed pre-service teachers to engage in describing, evaluating, interpreting, and responding about learner thinking without shifting to noticing more general aspects. This finding is promising for improving university mathematics method classroom experiences.

Our results also show that as pre-service teachers noticed learner thinking, they focused on several aspects of learner thinking (i.e. general, pre-service teacher, written work, learner characteristic, content, and source) as they analysed written work. This finding broadens our understanding of pre-service teacher noticing of learner thinking skills to involve both disciplinary (written work and content) and non-disciplinary aspects (general, pre-service teacher, learner characteristic, and source). We first focus on the disciplinary-related aspects of pre-service teacher noticing and then turn to the non-disciplinary aspects of noticing.

Disciplinary focus

We know that the participant pre-service teachers noticed learner mathematical thinking. In our research context, this noticing focus was captured as the pre-service teachers noticed learner thinking captured in the space in which the written work was created (Code: Written work) and when they moved beyond the written work to analyse the related mathematical content (Code: Content). Therefore, our findings indicate that not only were the pre-service teachers clearly noticing various aspects of learner thinking, they were often focused directly on the mathematical thinking shown in the written work (43%) or noticing the related mathematical content (19%). We know that teachers' ability to attend to and make sense of learner thinking specific to disciplinary understanding is supported by mathematics classroom practice reform efforts (NCTM, 2014).

Non-disciplinary focus

The result also shows that as pre-service teachers

analysed learner written work, various non-disciplinary specific aspects (general, pre-service teacher, learner characteristic, and source) were often part of a pre-service teacher noticing learner thinking, and that this focus may help him or her understand learner meaning. South Africa is a multi-national, multi-cultural, and multilingual country with huge variations in socio-economic status and it makes sense that these aspects of individual learners become part of a pre-service teacher noticing their thinking. For example, a pre-service teacher considering where a learner lives and connecting his/her thinking to this home context (i.e. a learner who lives in a city versus a rural setting and/or speaks a different home language, might express different ideas than one who does not). Prior work has examined different aspects of learner thinking noticed by a practicing teacher and highlighted that this practicing teacher used non-disciplinary and disciplinary aspects of learner thinking when engaged in *noticing learner thinking* (Selmer & Luna, 2018). This study adds to these findings by showing that pre-service teachers also used disciplinary and non-disciplinary aspects to noticing learning thinking when scaffolded to do so through the use of explicit questions.

In discussing these findings, we consider that pre-service teachers position themselves in particular ways (e.g. mathematical novice or expert, novice or expert teacher, past or present learner, etc.) (Van Langenhove & Harré, 1999), as they are *noticing learner thinking*, and that perhaps, levels of expertise might affect the different ways in which teachers position themselves. For example, we wonder if our finding that pre-service teachers shifted to focusing on themselves (13% of the instances) is an attribute of a novice teacher. Rochelle (see previous illustrative examples) shifted to focusing on herself as she noticed learner thinking, sharing why she was having difficulty analysing the learners thinking because she was not understanding the mathematical solution.

> Segment 4: *I think, why this solution is difficult for me is because I don't understand where he's taking all of the numbers from*. [Interpret: Pre-service teacher]

We wonder if an expert teacher would not have had difficulty interpreting the leaner's mathematical solution, and thus would not have shifted to focusing on themselves, making this code potentially more prevalent with novice teachers.

We also found that at times the participants positioned themselves as the teacher and analysed non-mathematical-related learner characteristics (16%). There were instances of the participants focusing on learner characteristics in our previous illustrative examples. Firstly, Simone shared that she was working with a Xhosa learner.

> Segment 1: *She's in Grade 6 and she's also a Xhosa learner*. [Describe: Learner characteristic]

Secondly, Rochelle focused on learner characteristics as she noticed learner thinking.

> Segment 3: *I think the learner is not comfortable doing this problem or he ran out of time*. [Interpret: Learner characteristic]

In Simone's example, the fact that she was working with a Xhosa learner, indicated that English would be a second (or third) language for this learner, thus impacting, and having implications for teaching. We suspect that this is something important for all teachers (novice to expert) to notice and consider when working with learners. Rochelle's interpretation of a learner's work focused on whether the learner was comfortable or had run out of time working on the solution. Interestingly, Rochelle did not actually know the learner (this was a researcher-provided example of learner written work). It is thought-provoking that pre-service teachers, at times, inferred traits, attitudes, interests, and values (learner characteristics) related to an imagined learner who had created researcher-provided written work. We suspect that the tendency to judge learners characteristically capable or incapable, is perhaps a characteristic of a novice teacher. Overall, this result highlights that for pre-service teachers *noticing learner thinking* and positioning themselves in different ways, is an important element of their sense-making.

Overall, our findings show that noticing does not happen in isolation of educational contexts. In other words, learner thinking, as noticed by a pre-service teacher, is often couched in the related educational context, including non-disciplinary-focused characteristics of both the learner and the pre-service teacher. This suggests that those who educate teachers need to be aware of, ask explicit questions related to, and appropriately scaffold method classroom experiences based on how particular pre-service teachers position themselves as they notice learner thinking. Furthermore, we suggest the need for research exploring how expert teachers engage in this work compared to pre-service teachers, allowing those who educate teachers to better understand the role of education context as a valuable sense-making tool by expert and novice teachers versus a sign of a novice teacher, perhaps struggling to notice a learner's mathematical work.

Conclusion

In summary, learner written work and explicit questioning can be used to engage pre-service teachers in *noticing learner thinking*. As the pre-service teachers engage in *noticing learner thinking*, our results indicate that they will focus on both disciplinary and non-disciplinary aspects of this thinking. A pre-service teacher noticing mathematics content in learner thinking is important and well supported in the literature, the participants in this study took notice of content and

disciplinary knowledge in roughly two thirds of the noticing acts across all exemplars of learner written work. The non-disciplinary aspects of learner thinking are also clearly part of pre-service teacher noticing learner thinking. Further, we found that, at times, pre-service teachers positioned themselves in various ways. Practically, we suggest purposefully planned university mathematics method classroom experiences, including both types of learner written work, but with consideration being given to the pre-service teacher positioning as a teacher or learner, focusing on disciplinary and non-disciplinary aspects of learner thinking, and to the authenticity of learning contexts in connection to the pre-service teachers' school placements. Our results and related discussion suggest further research that engages both prospective and practicing teachers in noticing learner thinking in ways that allow us to understand what this practice looks like in development of novice to expert teachers.

Authors' Contributions

Sarah Selmer and Erna Lampen conducted the interviews and analysed the data. Sarah Selmer, Erna Lampen and Denise L. Lindstrom wrote the draft manuscript and related edits and revisions. All authors reviewed the final manuscript.

Notes

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