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Children struggling to make sense of fractions: an analysis of their argumentation

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We used Toulmin's scheme for analysis of argumentation to analyse the interaction between three Grade 5 learners solving a common fractions problem. This analysis identified several issues, e.g. the ability of young children to participate in discourse characterised by argumentation, the complexity of the mathematical constructs that children have to deal with, and the nature of their discourse when they grapple with such complexities. The analysis showed that the process was driven by the classroom mathematical culture and the social and socio-mathematical norms, keeping the learners from closing their argumentation prematurely.

Introduction

Our work is informed by a socio-constructivist view of the nature of knowledge and learning — learners construct their own knowledge and this is both an individual and a social process. "The central issue is not whether students are constructing, but the nature or quality of those constructions" (Cobb, Yackel & Wood, 1992:28).

Contrary to the perception that in "constructivism" anything goes, one of the most important aspects of the mathematics teacher's role is to establish individual and social procedures to monitor and improve the nature and quality of children's constructions. This is a multi-facetted task ...

Our framework is problem-centred, in the sense that we regard problem-solving as the vehicle for learning (Murray, Olivier & Human, 1998). It is therefore important to design a sequence of activities (a hypothetical anticipated "trajectory" of learning) to provide learners with the experiences to construct their mathematical knowledge. However, no matter how well-designed such a trajectory is, the quality of learning depends on the classroom culture. It falls on the teacher to establish an enquiry classroom mathematical culture where learners are expected to develop personally meaningful solutions to problems, to explain and justify their thinking and solutions, to listen and attempt to make sense of each other's interpretations of and solutions to problems, and to ask questions and raise challenges in situations of misunderstanding or disagreement (*cf.* Yackel, 2001). To regulate the nature of this interaction, the teacher has the task of continually negotiating appropriate classroom norms.

Our framework is also learner-centred, in the sense that we plan further teaching based on diagnostic evidence of learners' present understandings. Formative assessment or diagnostic assessment is therefore an integral part of teaching and learning.

"Group work" can be seen as a monitoring procedure and a formative assessment tool — children can monitor their understandings by comparing it to the understandings of others. However, for us, "group work" is fundamental to learning: through classroom social interaction, the teacher and learners construct a consensual domain of taken-to-be-shared mathematical knowledge. In the course of their individual construction of knowledge, learners actively participate in the classroom community's negotiation and institutionalisation of mathematical knowledge and in turn the community facilitates or constrains the individual's learning. Social interaction is therefore a prerequisite for the individual construction of knowledge.

To understand and monitor the process of learning through social interaction and the iterative product of learners' mathematical constructs, our research closely analyses learners' task-orientated argumentation. The analysis serves the purpose to improve our teaching and learning: it gives us information about the quality of the functioning of specific groups as well as fine-grained formative feedback about learners' constructs. This specific information enables us to address specific aspects relating to the classroom culture, and to adapt our anticipated learning trajectory to learners' needs. Such analysis further serves the purpose to illustrate and confirm or refute our theory!

Argumentation

Argumentation is "... a social phenomenon, when co-operating individuals tried to adjust their intentions and interpretations by verbally presenting the rationale of their actions." (Krummheuer, 1995:229). The aim of argumentation is to convince oneself as well as the other participants of the validity of one's own reasoning and to win over the other participants to this special kind of "rational enterprise" (Krummheuer, 1995:247). Krummheuer explains how participants in a classroom situation constitute an argument interactively. He uses a model, proposed by Toulmin (Krummheuer, 1995:239) as a "lay-out" of an argument, showing how statements made by learners in classroom interaction construct an argument.

Toulmin (Krummheuer, 1995:240-247) describes four parts in an argument — a claim, data, a warrant, and a backing. A speaker makes a statement that s/he claims to be certain (for instance a suggestion made by a learner towards finding a solution for a problem). This is the claim of the argument and the claim has to be supported by more information or evidence explaining why it should be considered as true. The additional supportive information is the data of the argumentation, while the warrant explains why the data should be accepted as support for the claim. Further support for the warrant is provided in the form of a backing. The backing refers to global convictions and primary strategies and binds the core of an argument to collectively accepted basic assumptions (Krummheuer, 1995; Yackel 2001; Yackel, Underwood, Stephan & Rasmussen, 2001).

Toulmin's lay-out for an argument can be represented diagrammatically as follows:

$$\text{DATA} \xrightarrow{\text{so}} \text{CLAIM} \xrightarrow{\text{because}} \text{WARRANT} \xrightarrow[\text{account of}]{\text{on}} \text{BACKING}$$

Background to the research

The data used for this analysis were generated during a two-year project titled "The role of articulation and argumentation in the development of children's conceptions of fractions". Members of our research team monitored one Grade 5 and one Grade 6 class twice a week and audio- and videotapes were made of children's interaction while they were working on problems.

A typical lesson would start by the teacher presenting a problem and the learners engaging with the problem collaboratively in groups. When the groups have reached consensus on a solution (or when at least most of the learners have sufficiently engaged with the problem), a whole class discussion, handled by the teacher, takes place.

This article describes analysis of the interaction of three Grade 5 children while they were working on a problem. A transcript was made of the audiotape recording with the aid of comprehensive field notes taken by the researcher who acted as facilitator during the episode. The children's interaction was then analysed according to Toulmin's model of argumentation.

The problem

The Williams family buys a Gatsby bread[1] with lots of tasty fillings. Father is the biggest so he takes a half of the bread, whilst Mother, Jonathan and Catherine share the rest equally. What fraction of the original bread do they each get?

The problem was designed to introduce the notion of a fraction of a fraction as part of a learning trajectory for learners to develop understanding and a method for multiplying fractions, as opposed to the teacher giving learners the rule to multiply the numerators and multiply the denominators, as is common practice in most traditional classrooms.

The underlying mathematical constructs contextualised in this problem are:

- equal partitioning whereby fractional units are produced
- iteration of the unit
- the naming of the fractional units
- changing of the referent unit (expressing the same fraction in terms of another unit)
- equivalence of fractions of different units.

The group

The three children in the group are of average to below average mathematical ability. They were in other groups that did not function well and landed in this group as a result of reshuffling. They seemed to work well together with no one trying to dominate the others. All three of them were willing to express their ideas and to listen to the others, admitting when they did not understand. They were also willing to admit when they were confused but seemed determined to resolve their confusion themselves, without relying on the teacher or researcher to make decisions for them.

The analysis of the interaction

Four different claims were made in the course of the group's efforts to solve the problem. These were expressed in different ways throughout the lesson.

We will discuss each of the claims with excerpts from the transcript giving information about how and when they were expressed. Line numbers indicate the chronological order in which the statements were made.

Claim 1a: Each of the mother and the children gets a third

After Simone had read the problem aloud, Jane suggested making a drawing (see Figure 1). This was her way of trying to understand the problem. The others closely followed what she was doing.

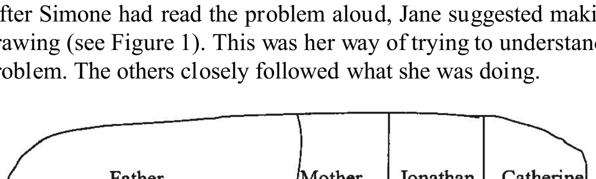

Figure 1 Diagrammatic representation of the situation

The conversation leading to the first claim went as follows:

18 Jane: Father gets half ... So, how much do ...?
19 Stacey: Is each one getting ...?
20 Simone: Must the mother and Jonathan and Catherine like get the same? Like share equally?
22 Jane: They share the other half.
23 Simone: Equally.
24 Others: Equally.
27 Simone: One, two, three, ... each one gets a third (claim 1a)
28 Stacey: The mother and the father ... the mother and the children get the same fraction
29 Others: Yes
30 Stacey: So, we have to divide it into three now.
31 Simone: A third ... (claim 1a)

Although nobody referred verbally to Jane's sketch, it served the purpose of helping to understand the problem correctly. Much of the initial talk is also about clarifying the problem. The sketch also helped them to explain what they cut and how they did it to get a third as the answer to the problem. While the learners were clarifying the problem, they said that the father must get a half (lines 9 and 18) and that the rest must be shared equally (lines 20–24). This, as well as the sketch with the inscription they made on it (indicating how they wanted to share the other half equally) serve as the data to support the claim.

The warrant can be found in the elaboration of the equal sharing, when Stacey repeated that the mother and the children must get the same fraction (line 28) and "... we have to divide it into three now" (line 30). She probably meant that they had to share "it" among three. Although they did not offer new information, they elaborated on their thinking, verbalising it differently. This may be seen as an indication of the nature of children's discourse when they are trying to make sense of a problem — they show a need to articulate the same idea in different ways.

Although the learners represented the sharing situation correctly in the diagram, they did not at this point refer to the referent unit to make their claim mathematically valid — the it (line 30), i.e. a half: each gets a third of a half! How to use the referent unit proved to be their main problem to overcome. They understood the problem correctly, they could even express it correctly verbally (e.g. see line 38 below), but it was difficult to express it mathematically. The class was introduced to the concept of a fraction as a part of a whole by giving them equal sharing problems and then giving them the relevant terminology, for example "if something is equally shared by three, each of the parts is called a third." It is possible that there was not sufficient emphasis on the referent unit at this point of their programme.

Claim 2: Each of the mother and the children gets a quarter

The conversation took a different turn when Jane expressed doubt about claim 1a:

36 Jane: Simone thinks it's a third, but ... we don't know if that's a third ... because ... should be a third ... because Father took the half.
38 Stacey: Yes we divide that, the other half is between 3 people, but should each one get a third of that?
40 Simone: What do you say, Stacey?
41 Stacey: I think it's a quarter.
42 Jane: How do you get a quarter? (hesitates) I think it's a third.

Stacey's claim of a third was immediately refuted and dismissed by Jane[2] and was not pursued further. As no support for the claim was offered, it is not clear what was behind Stacey's thinking when she made it. She made a sketch (Figure 2) on her worksheet, but because she never mentioned it, it is not certain at what point it was made.

One can only infer that she figured that if you divide something among four people, each should get a quarter but there is no clear evidence for this.

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1 A Gatsby bread is similar to a French loaf and is very popular on the Cape Flats where the participating school is situated.

2 We are here analysing only the cognitive aspects of interaction. Of course, the social aspects are important too!

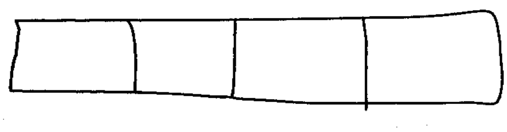

Figure 2 The bread equally shared by four people

Claim 1b: Each of the mother and the children does not get a third

In the subsequent conversation, Jane repeatedly referred to the fact that they had to find a fraction of the original loaf of bread, emphasising original. This awareness could explain the doubt she expressed about claim 1a. The conversation continued:

45 Jane: Because you have to divide it into three and the other half is divided into three , so each one ... What fraction of the original bread do they each get? So you must say how much, how much uhm do the mother and the three child ..., the two children get of the original bread.
48 Simone: A third
49 Jane: That's the problem because now the father, that's a whole thing ... is the original bread. The father eats a half, now if the father takes a half of the original bread then there's still that, the other half ... now how much is a half divided by three, that's what we have to find out now.
53 Jane: So, we don't have it, not really a third, because that's not the original bread, the whole bread ... (claim 1b)
54 Jane: So, [reads the problem again] what fraction of the original bread ... do they each ... get. Because they each get ...

Jane explicitly refuted claim 1a in line 53, which means that she put forward another claim which is a denial of the previous claim. (We call this claim 1b, because it still forms part of the argumentation linked to claim 1a.) The data for this claim is the fact that the problem said they must find out what fraction of the original bread must go to the mother and each of the children (lines 45 and 54) and as warrant she stated that that was not the original bread (line 53, referring to the half which was shared by the mother and the children).

At this point the children were unable to answer the question Jane raised in line 46, "now how much is a half divided by three?" It could have been a semantic obstacle, in that they did not have the language to give a name to "this part" in terms of the whole, original loaf of bread, or they lacked the mathematical knowledge about how fractional parts are named. Her struggle with this concept is indicated by her repeated reference to "what part of the original bread", and her clumsy way of communicating her thoughts (e.g. lines 49, 50 and 53). Two claims were now "hanging in the air", the one a refutation of the other one. The learners did not reach closure on any of these claims, because they were searching for a backing for what was claimed. Their persistence with the search indicates how well the classroom mathematical culture was established, respecting the socio-mathematical norm of what counts as a good explanation.

The search for a backing led to yet another line of thinking, which can be seen as another claim, related to the first argument:

Claim 1c: Each gets a third, with or without father

87 Simone: Are they now talking about the whole bread like the whole bread without father?
88 Jane: The what? I think without father.
90 Simone: I think it's with father, but if it's without father it would've been easier
91 Stacey: ... of the original bread
92 Jane: I think it's with father, but if it's with father or without father it's the same answer.
93 Stacey: But then we must
94 Researcher: Repeat again Jane, I didn't hear?
95 Jane: If it's with the father or without father you will still get the same answer.
96 Stacey: Exactly.
97 Jane: It will both be ... (hesitates)... a third
98 Simone: A third
99 Stacey: Then maybe a third is the answer

This part of the conversation highlights the complexity of the mathematical constructs involved and we found its analysis problematic. It is obvious that the learners were not only trying to make sense of the problem, they were also trying to make sense of the mathematics involved. All three of them suggested "a third" as the answer (with or without father). We have to try to follow their thinking — if the answer is a third "without father" they were possibly seeing the situation like that shown in Figure 3, ignoring the father's share:

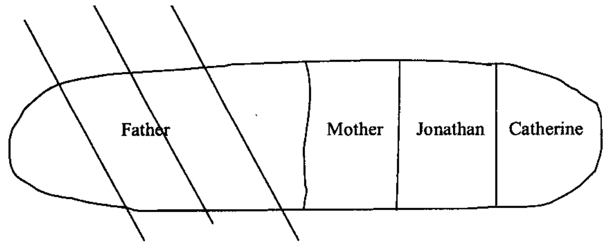

Figure 3 The answer is the same (a third) without father

That means Jonathan, Catherine and Mother would each get a third of a new unit, which is mathematically correct. However, the problem required that they express the answer in terms of the original loaf of bread.

The representation of the other possibility (Figure 4), with father, would probably be their first sketch showing father's share:

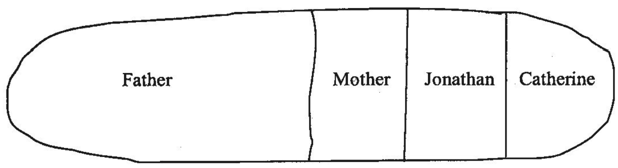

Figure 4 The answer is the same (a third) with father

Their reasoning could be the result of either a semantic or mathematical problem (as referred to earlier) — they only had the scheme of a third available to describe the part concerned.

Although they seemed to reach consensus on an answer, they still did not reach closure. Their search for a backing (or their obligation to give a good explanation) was clear in the following:

100 Jane: But we must work now, ... to get to where we get to that answer
101 Stacey: If you say for instance, like, something divided by three the answer
102 Jane: We say a chocolate bar divided by ... the father gets a half and the others get a third — it will still be a third
103 Simone: But they want to know how we get the answer
104 Jane: I think it's a third but I'm not sure

In terms of Toulmin's model they were not able "to indicate why the warrant should be accepted as having authority" (Yackel, 2000:15) and that was what Jane was looking for (line 100), supported by Simone (line 103). Jane tried to make a connection with mathematical models (sharing a chocolate bar), which they had used in previous lessons, to make her explanation clear. Her recurrent doubt about claim 1 (line 104) may be due to the fact that they failed to find a backing.

At this point the researcher intervened, trying to get them to realise that they were dealing with different units. She asked them to show a third of the whole bread. They made another sketch of a Gatsby bread shared into three pieces (see Figure 5).

Pointing at one of the thirds in this sketch, Jane expressed uncertainty about claim 1a: "That's why we think it's a third, because, but now that we've done, we're not sure because the father is also involved here". She continued:

153 Jane: So we can't say a half of the ..., we're not sure if we can say a half of this uhm bread. ... Then with this third. I'm not sure about that. Because what if we can't just take half of the bread and say that's a third. (Refutation of claim 1)

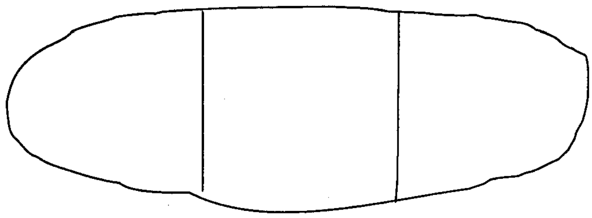

Figure 5 A Gatsby cut into thirds

In this part of the conversation Jane was toying with two ideas — on the one hand she wanted to call the piece a third (thereby accepting claim 1a), but on the other hand she realised that what was shared by the mother, Jonathan and Catherine, was not the whole (original) bread (accepting claim 1b).

The researcher then reminded them that they had previously said they had to find out what the piece was called. This lead to the final claim:

Claim 3: Each of the children gets a sixth

183 Jane: Oh, I understand now! If you cut the father in half and it'll be six. If you cut that part into into If you cut, uhm if you put into into a three it'll be six.

185 Stacey: Uhm

186 Jane: And if you uhm, they ask here what fraction of the original bread do you each give them.

187 Stacey: Also

188 Jane: Because this doesn't actually make sense when the father gets half and you made it three

189 Simone: How much OK how much fraction do they get

190 Jane: Then it makes more sense

191 Simone: I'm confused now ...

192 Jane: So I don't think they get a third anymore, I think they get like, a sixth because if you divide the father in then the whole thing is divided and then you get a sixth.

Jane had difficulty in verbalising her claim. By "If you cut the father in half and it'll be six" (line 183) and "if you put it into three it'll be six" (line 184) she probably meant that if the father's half had also been cut into three equal pieces, the whole bread would be cut into six equal pieces. That explained why the mother and each of the children would get one sixth of the original bread (claim 3) and therefore serve as data in this emerging argument. The ideas were all suggested, albeit not clearly articulated. The warrant is provided in line 192 when she explicitly formulates that "the whole thing is divided and you get a sixth."

The rest of the group was confused and did not follow Jane's explanation, but she assumed the responsibility to keep on trying to improve her explanation until the others accepted it. Her effort to make her explanation understandable according to the socio-mathematical norm what counts as a good explanation can be interpreted in terms of Toulmin's model as a search for a warrant and a backing. She had to provide evidence to explain why her data supported the claim and why her whole argument had authority.

While Jane was trying to establish the "whole bread" as the referent unit, Stacey challenged the claim:

205 Stacey: I don't know if we can cut this up,

206 Jane: Because the father must get a half, like they say here.

207 Stacey: But then how can you cut the father's up if the answer is half for the father

208 Everybody is trying something, drawing.

209 Simone: He must get a half and they must share the other half equally

210 Stacey: But the children ... father's not gonna get, ... we must share equally the father must get a half

211 Jane: We can still cut it up into six pieces, then the father can still get a half, we can just give him three, three thirds ... that's still a half

A crucial step in constructing knowledge about how to express a fraction of a fraction as a fraction of the whole is understanding the notion of equivalence of fractions (when two fractions with the same value are expressed in different ways). Jane's response to Stacey's challenge was an explanation of how equivalence works (finding a name for the new piece by cutting the whole bread into equal sized parts), thereby succeeding in giving a backing for the claim. However, the group did not accept Jane's explanation at this point, so she tried again:

233 Jane: Pointing to the relevant parts of the figure:
Because, look here, that can still be the father's half, but that's the father's half, right? And now that we have to divide into three. But the father still wants his half. He doesn't want to lose his half, so we can still cut that into three pieces. So the father can still get his half out of the deal. He still gets his half, he doesn't lose anything. That's why I think of a sixth. Because he doesn't lose anything on that. If we cut his up. We still give his amount. We just give by making it easier for us to see what fraction they are, what fraction they get.

238 Simone: I know, but ... then this, ... piece is gonna go to the father, and that's gonna be a third, like, and each one's gonna get a third.

240 Jane: No. Look here, we're gonna divide the father into three, right? Then the father's still gonna be ... we're still gonna give the father his half. And we're still gonna get ours. But we then we say it's a sixth. It's not a third. Because they ask ask of the original slab, of the original uhm

243 Stacey: Gatsby

245 Stacey: So you're saying that they must cut the father's also

246 Jane: Yes, but the father, you can cut it up but then the father still gets a half. He doesn't lose anything. It can be a sixth

248 Jane: I think it's a sixth, but I don't know about you. What do you say? A sixth? And you? You mustn't just say it's right because I say so.

250 Simone: I know you said it's a sixth but if he, if he gets his half then, then it means the two children and the mother also gets a deal and he gets ...

252 Jane: No, no, uh-uh. No. He gets his half what he had in the first place. And then the mother and then the mother and the children must still get the half that they had in the first place, but instead of a third they get a sixth because when we divided the whole original bread, then we divide

Jane's explanation undoubtedly contributed to deepening her own understanding of the construct of equivalence. She understood that the "cutting up" of the father's half was "... to make it easier for us to see what fraction" (line 237) and that taking away the half meant that dividing the leftover half into three equal parts gave thirds of a new unit (a half), but they could not be called thirds of the original bread (lines 252–254).

Then Simone challenged the naming of the part as a sixth:

255 Simone: How can they get a sixth because...when father gets his half, né, then this is mos gonna be gone, né and then

there's only gonna be like, then they've got to divide into thirds

258 Jane: But they don't say a half of the bread, they say the original bread, the whole bread. See.

Jane explained again, explicitly stating that six sixths made a whole (line 266 and 268) and that three sixths made a half (line 275), thus giving her argument authority by putting it in more general terms (providing a backing according to Toulmin).

In response to Simone's request to explain again, Jane now reverted to an iconic explanation rather than just a numerical one —"frame-switching" according to the Toulmin model (Krummheuer, 1995:251). She made two sketches (Figure 6); one showing the Gatsby bread divided into 6 parts, with "father" written across three of the parts and in the second one she wrote = across father's half and also across the other three parts.

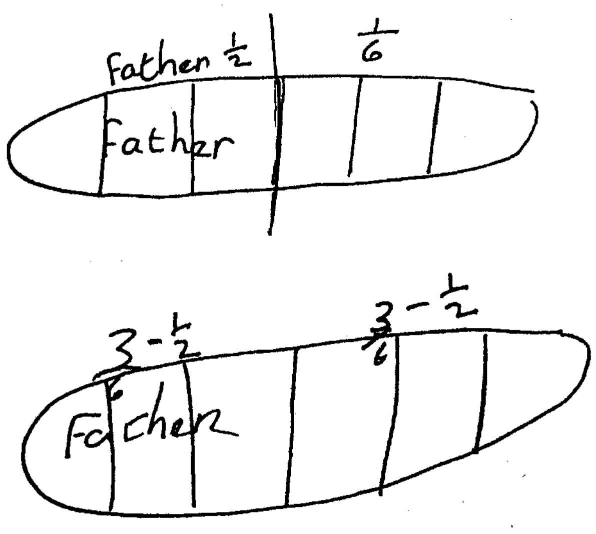

Figure 6 The bread cut into sixths

She used these sketches in her explanation:

282 Jane: This is the father's. That's the father's. But, we have to divide this into three for the, for the mother and the two children. But, that's now the father's. Say. So, we don't know, we can't now just take the half away and leave that and say that's a third. That's three thirds. We can't. Because this is a whole bread (pointing to her sketch). So we cut that also up into three. So we say one, two, three, four, five, six. That's six blocks. Cut into six. So sixes (meaning sixths) Six sixes

283 Simone: Six sixes making it a whole

284 Jane: Yes. And three sixes, three sixes will give you one, a half. And then that's also three sixes.

The iconic backing for claim 3 made it possible for Simone and Stacey to accept her explanation and consensus was finally reached on the solution.

The structure of the claim 3 argument is shown in Figure 7.

Discussion

How children learn when allowed to make sense of the mathematics

Nature of the discourse

Any analysis of learners' interaction is valuable as a tool for formative assessment, yielding information about the nature and quality of the discourse and the state of learners' mathematical constructs. In particular, when the children have difficulty in finding warrants and backings, this can inform the teacher about mathematical concepts that are not yet fully mastered and will be valuable in designing follow-up tasks. Young children have difficulty in expressing and verbalising their ideas. It is therefore often necessary to have a closer look at what they were saying to find out exactly what they were thinking and to become aware of their struggles and how they can be supported to persist in their sensemaking.

Need to articulate

This analysis shows the complexity of the mathematical constructs that children are to make sense of. Their struggle between claims 1a and 1b illustrates the difficulty learners have with flexibly changing the referent unit. This is one of the examples that Sowder (2000:3) uses when she argues that "... research shows that the difficulties associated with the transition from the study of number in the early grades to the study of number in the middle grades has been vastly underestimated". Claim 1c can be seen as an effort to clarify what the actual unit was.

Jane was able to make a conceptual gain, understanding the naming of fractions and equivalence because she was allowed time to grapple with a difficult construct and the ongoing discourse facilitated optimal reflection. While her initial insight might have been an intuitive idea, her repeated explanations in response to challenges by the others in the group, lead to an explicit explanation of the important concept of equivalence. When she managed to supply two backings for claim 3 it became clear that effective learning had taken place.

The group had reached a certain level of understanding only when they succeeded in finding a backing. Finding a backing, i.e. explaining the claim in terms of shared knowledge, is crucial for the group's negotiation and institutionalisation of mathematical knowledge.

Different roles of participating children

Jane made the largest mathematical contribution in this particular episode, but all three of the girls participated and each played an important role in the discourse, illustrating that children are dependent on each other when learning a group. Jane's deeper reflection on the issues involving equivalence was the result of challenges by the other two members of the group. One example was the issue of equivalence (the "cutting up" of father's half into three equal parts) and the other the naming of those smaller parts. Both these issues were implicitly challenged — Stacey questioned the cutting of father's half (line 205) and Simone felt uneasy with calling the part a sixth (line 255). Jane's explanations in response to the challenges addressed both the issues explicitly and provided the two backings for this claim.

Although there is no evidence to the contrary in this episode, we remark that the converse is true — if children do not participate they do not learn.

Classroom mathematical culture

The three learners were able to participate in discourse characterised by argumentation, although they themselves had no idea what an argument and a valid argument consisted of. They demonstrated a willingness to engage with the mathematics and grapple with constructs. The only way they were "trained" for this type of discourse was through the efforts of the teacher and the research team to establish the social and socio-mathematical norms (e.g. explaining your thoughts, justify what you claim to be true and an explanation must be understood by everyone).

Analysis according to Toulmin's model shows to what extent these norms have been established. According to the social norms characteristic of inquiry mathematics learners are obliged to explain their ideas, i.e. whatever claims they may offer must be supported by data. Explanations must be justified and that becomes evident if warrants are supplied to explain why the data supports the claim. The socio-mathematical norm what counts as an acceptable mathematical explanation requires that an explanation should be backed by reasons that show why it should be accepted as having authority. The backing in Toulmin's model has the function of making the claim generally acceptable to the whole group.

Our analysis of this episode shows that the children were not only interested in finding a solution to the problem, but that most of their

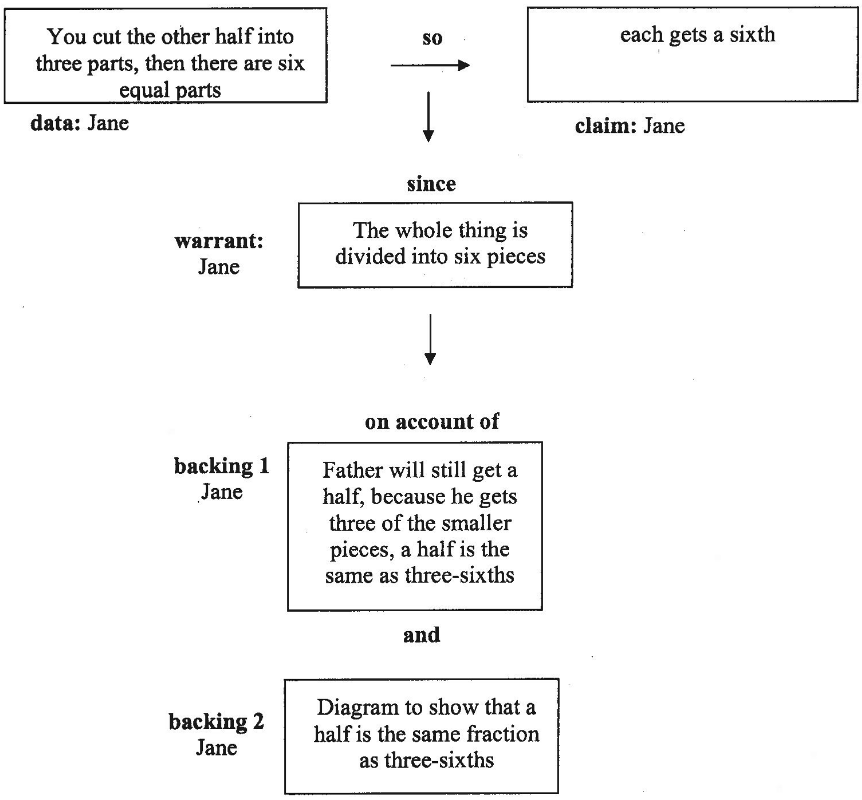

Figure 7 Diagrammatic representation of the argument for claim 3

discourse was about finding data, warrants and backings for their claims, i.e. finding reasons to support their thinking.

Towards the end of the episode, Jane was repeatedly concerned about Simone's understanding of her (Jane's) explanation, showing that it is possible for learners to monitor the nature and quality of one another's constructions. As Yackel and Cobb (1995:269) say: "When students begin to consider the adequacy of an explanation for others rather than simply for themselves, the explanation itself becomes the explicit object of discourse." Jane explained again and again in response to the socio-mathematical norm for a good explanation. In the process of explaining, her own understanding of the mathematics concepts deepened.

It seems reasonable to conclude that their participation in the discourse largely contributed to the learning experienced by all three learners and that it was their acceptance of the norms that the classroom mathematical culture required, that sustained the discourse and drove the whole episode.

The importance of learners accepting the obligation to challenge any explanation that they do not understand, is underlined by this incident. It was significant that although they seemed to agree on most of the claims that were made, they continued the discussions as if they were searching for an acceptable backing. They did not want to reach consensus if an explanation was not supported by information that was collectively acceptable.

Children's perception of mathematics

Although it might seem that the children progressed rather slowly in their construction of new knowledge, the way they displayed a rational attitude towards mathematics should be respected. Their view of mathematics is that it is a rational activity and not simply the application of rules or procedures. From the way in which they tackled the problem, immediately using the strategy of drawing a sketch to understand the problem, it is clear that they wanted to make sense and expected to make sense of the problem. They never asked the teacher what to do, but rather tried to figure out for themselves what the problem required. Their focus remained on the problem. Again, without evidence to the contrary from this episode, we emphasise that children's beliefs about mathematics influence if and what they learn! Children with a different belief system would respond quite differently.

Implications for everyday teaching in the classroom

Important information about learners' understanding of mathematical concepts becomes available if teachers are willing to listen to what they say and analyse their interaction. If learners have difficulty in supplying data, warrants and backings for their claims the teacher is able to assess to what extent they have mastered a concept and this knowledge can inform the design of follow-up tasks.

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