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CONTROL DESIGN OF A NONLINEAR CONTROLLER TO STABILIZE THE NONLINEAR TCP MODEL

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ABSTRACT

This article presents the design of a highly efficient nonlinear0 controller which is a kind of an Active Queue Management (AQM) scheme to stabilize the nonlinear TCP model dynamics. Specific boundary conditions have been considered for stability occurrences and have been compared with other existing Active Queue Management Schemes. All analytical experiments have been carried out using MATLAB.

KEY WORDS: Active Queue Management (AQM), Transport Control Protocol (TCP)

INTRODUCTION

With greater demand for internet services due to the present industrial revolution, security of documents transferred from host to receivers should be ensured. The speed and performance of internet services should be enhanced as well. Protocols responsible for transportation in communication systems are the User Datagram Protocol (UDP) and the Transport Control Protocol (TCP). TCP is a more reliable, robust and adaptable transport protocol employed to combat the problem of proper queue utilization, busty packet drop and adaptable delay. TCP model which dynamically captures the dynamics of TCP protocol developed by Misra et al, (2000) is used in this work.

For a controller that can stabilize the TCP model, an Active Queue Management (AQM) scheme employing router controlled packet flow and systematic packet drop is used. This scheme ensures that the buffer is not overloaded. Several AQM schemes which have been developed include Random Early Detection (RED) (Hollot et al, 2001; 2002), Random Exponential Marking (REM) (Athuraliya et al, 2001), and Proportional-integral (PI) (Liebeherr and Christin, 2000). These controllers have been applied to the linearized TCP model about certain operating points but in this case the nonlinear TCP dynamics, modelled by Misra et al (2000) is used for all analysis.

Modelling

According to Misra et al (2000), a TCP dynamics was developed through the study of fluid flow and stochastic differential equation analysis. The results obtained show that the model fully captures the TCP dynamics. In this work, the TCP time out mechanism will be ignored which simplifies the model without losing its meaning. The TCP model is described by the following nonlinear differential equation.

$$\left.\begin{array}{l} \dot{W} = \dfrac{1}{R(t)} - \dfrac{W(t)W(t-R(t))}{2R(t-R(t))}p(t-R(t)) \\ \dot{q} = \left\{\begin{array}{ll} \dfrac{W(t)}{R(t)}N(t) - C, & q>0 \\ \max\left\{0, \dfrac{W(t)}{R(t)}N(t) - C\right\}, & q=0 \end{array}\right\} \end{array}\right\}, \qquad (1)$$

where $\dot{x}$ denotes the derivative of x and

$W \overset{\Delta}{=}$ expected TCP sending window size (packets);

$q \overset{\Delta}{=}$ expected queue length (packets);

$R \overset{\Delta}{=}$ round-trip $\text{time} = \dfrac{q(t)}{c} + T_p$ (seconds);

$C \overset{\Delta}{=}$ link capacity (packets/sec);

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$T_p \overset{\Delta}{=}$ Propagation delay (seconds);

$N \overset{\Delta}{=}$ number of TCP session; and

$p \overset{\Delta}{=}$ probability of packet drop.

The first part of equation 1 describes the positive bounded TCP window control dynamics $W \in [0, \overline{W}]$, where $\frac{1}{R(t)}$ describes the additive increase of packet through the window and $\frac{W(t)}{2}$ the multiplicative decrease of packets through the window to avoid congestion. The second part of equation (1) describes the positive bounded bottleneck queue length $q \in [0, \overline{q}]$ as the difference between the packet arriving rate $\frac{NW(t)}{R(t)}$ and link capacity C, with the assumption that the bottleneck does not possess internal dynamics. The probability of packet drop p, takes the values 1 and 0 i.e $p \in [0,1]$.

Model Approximations and Control Design

According to Gosiewskj and Olbrot (1980), conventional approach for evaluation of systems with delay is by approximation with certain assumptions. In the case of a very large link capacity where the queue delay is far smaller than the propagation delay, approximation is very good. Operating with the assumption that the TCP session N(t) and link capacity C(t) are time invariant N and C respectively, the equilibrium points of equation (1) are $W_0, q_{0,} p_0$. They are defined by equation (2).

$$\left.\begin{aligned} W_0^2 p_0 &= 2 \\ W_0 &= \frac{R_0 C}{N}. \end{aligned}\right\} \tag{2}$$

Equation (1) can be re-written as

$$\left.\begin{aligned} \dot{q} &= \frac{W(t)}{R(t)} N(t) - C \\ \dot{W} &= \frac{1}{R} - \frac{W(t)W\big(t-R(t)\big)p\big(t-R(t)\big)}{2R\big(t-R(t)\big)}. \end{aligned}\right\} \tag{3}$$

A nonlinear control design is considered for application in this case, so that when there is any change in network condition, the system is able to produce the desired result. This should give a robust performance. Equation (1) shows that the model is in a strict triangular feedback form. Therefore the control design to be applied is back stepping which was discovered by Kokotovic in Kristic et al (1995). Although back stepping is believed to provide robust performances, it has its own limitations. Back stepping does not ignore the effect of the nonlinearity in the system which may result in a very complex system. Again, it is possible to have more than one Lyapunov function that can stabilize the system.

Case I: Approximation without Delay

$$\left.\begin{aligned} \dot{q} &= \frac{W(t)}{R(t)} N - C \\ \dot{W} &= \frac{1}{R(t)} - \frac{W(t)^2}{2R(t)} p(t). \end{aligned}\right\} \tag{4}$$

Let

$$a = \frac{1}{R(t)}, \quad x_1 = q, \quad x_2 = W \quad \text{and} \quad u(t) = p\big(t - R(t)\big). \tag{5}$$

The control law is denoted as $u(t)$ and the states are x_1 and x_2. Therefore, equation (4) becomes,

$$\left.\begin{aligned} \dot{x}_1 &= -C + aN \\ \dot{x}_2 &= a - 0.5ax_2^2u(t). \end{aligned}\right\} \quad (6)$$

From the application of integrator back stepping to equation (6) with a stabilizing function ϕ, where

$$\phi = -\frac{1}{aN}(x_1 - C), \quad (7)$$

the Lyapunov function candidate chosen is

$$V = \frac{1}{2}x_1^2 + \frac{1}{2}z^2 > 0. \quad (8)$$

A control law

$$u(t) = \frac{1}{0.5ax_2^2}\left(aNx_1 + a - \frac{\delta\phi}{\delta x_1}(-c + aNx_2) + \left(x_2 + \frac{1}{aN}(x_1 - C)\right)\right) \quad (9)$$

is derived for the case of a nonlinear delay free marking controller by taking the derivative of equation (9) to yield equation (10)

$$\dot{V} = -x_1^2 - z^2 \quad < \quad 0. \quad (10)$$

Case II: Nonlinear Delayed Marking Controller

This case considers a delay on the control input only and the link capacity is made to be very large to accommodate the delay within a region locally. The TCP model in equation (3) is now written as

$$\left.\begin{aligned} \dot{q} &= -C + \frac{N}{R(t)}W(t) \\ \dot{W} &= \frac{1}{R(t)} - \frac{W(t)^2}{2R(t)}p(t - R(t)). \end{aligned}\right\} \quad (11)$$

Substitution of equation (5) into equation (11), yields equation (12)

$$\left.\begin{aligned} \dot{x}_1 &= -C + aNx_2 \\ \dot{x}_2 &= a - 0.5ax_2^2u(t - R(t)). \end{aligned}\right\} \quad (12)$$

Applying Integrator back stepping to equation (12) and choosing a stabilizing function ϕ as in Case I, we have

$$\phi = -\frac{1}{aN}(x_1 - C).$$

Because of the delayed term, Lyapunov-Krasovskii function is used and is defined as

$$V = \frac{1}{2}x_1^2 + \frac{1}{2}z^2 + \frac{1}{2}b\int_{-R}^{0} ds > 0 \quad (13)$$

We take the derivative of equation (13) with respect to t and apply the Leibniz integrator rule to obtain

$$\dot{V} = -x_1^2(t) - z^2(t) - \frac{b}{2}z^2(t - R(t)) < 0. \quad (14)$$

The control law is then derived as

$$u(t - R(t)) = \frac{1}{0.5ax_2^2}\left(aNx_1 + a - \frac{\delta\phi}{\delta x_1}(-C + aNx_2) + \right.$$
$$\left. + \frac{b}{2}\left(x_2 + \frac{1}{aN}(x_1 - C)\right) + x_2 + \frac{1}{aN}(x_1 - C). \right. \quad (15)$$

Stability Analysis

In systems with delay, analysis is more complicated and a scalable analysis methodology is difficult. In this work, we consider an arbitrary network topology using Lyapunov candidate functional to tackle the delay problem. Taking advantage of the symmetrical nature of the system, stability of the system is proven. The conditions for stability of the nonlinear system in equation (1) are a pointer showing the tuning parameters. The tuning parameters are the TCP session N and the link capacity C.

Case I: Nonlinear delay free marking controller.

The equilibrium point (W_0, q_0, p_0) of equation (1) is asymptotically stable for all positive initial conditions such as $a \geq 0$. In order to get a more realistic communication link, the queue size and window size must operate under a boundary of $0 < q < \bar{q}$ and $0 < W < \overline{W}$ respectively. The positive definite Lyapunov function candidate V shows that equation (8) yields a strictly negative definite derivative of Lyapunov function $\dot{V}$ in equation (10). This proves that the system is asymptotically stable. Therefore, $\dot{V} \to 0$ as $t \to \infty$ and the states $\dot{x}_1 \to 0$ and $\dot{z} \to 0$, indicating that the states $x_1(t)$, $x_2(t)$ and the control law u(t) are bounded. Note that, for the system to be within the stable region, the roundtrip time must not be equal to zero (meaning, if $R(t) = 0$, the control output will be estimated as infinity because $a = \frac{1}{R(t)}$).

Case II: Nonlinear delayed marking controller

To ensure the stability of a system with delay, a Lyapunov argument is most suitable. Most text prefers to use Lyapunov Razumikhim function for stability analysis of nonlinear delay system, because of its simplicity and ease for manual construction. Despite this advantage, Lyapunov-Razumikhim function is limited because it tries to assess the stability of infinite dimension argument with a finite dimensional argument. Therefore, Lyapunov-Krasovskii functional is used because it sees the system as an evolving function of time.

Here, Lyapunov-Krasovskii functional is constructed to scale with the system size. The identity $\frac{1}{2} b \int_{-R}^{0} z^2 ds$ is used to formularize the delay differential equation in equation

(13), which accounts for the delayed terms in the system. By applying the Leibniz integration rule, the derivative $\dot{V}$ shows that the system is asymptotically stable with a negative definite solution in equation (14). The conditions for the stability are:

- The delay is considerably low i.e. $0 < R < \overline{R}$
- The states x_1 and x_2 are bounded for $0 < q < \bar{q}$ and $0 < W < \overline{W}$
- The control output $u(t)$ is bounded for $t - R \leq t \leq t_0$ i.e. the control input described by equation (15) ensures that the close-loop system is asymptotically stable.

Performance Evaluation and Discussion

The above analysis is verified using MATLAB. Our major consideration is the control objective which is to achieve full bandwidth utilization without congestion by regulating the tuning parameter.

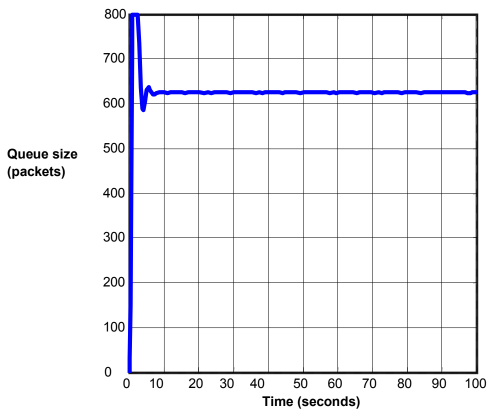
A case of a single router running different protocols at infinite duration with the greedy ftp and http flows to give a more realistic traffic scenario is used. All simulation is carried out using the nonlinear TCP model.

Case I: Experiment with Nonlinear Delay Free Marking Controller.

Case I above is simulated considering 800 TCP sessions N with a maximum buffer size of 500 packets. The propagation delay of 0.2 is used bearing in mind that the queue delay must be very small and the link capacity C = 3750 packets/sec (i.e, $\text{bandwith} \approx 2$ Mbyte/sec). The maximum queue size $\bar{q}$ is set at 800 packets and a constraint is applied to the window size and queue size since they cannot be less than 0.

Figures 1, 2 and 3 are the responses of the queue size, window size and round-trip time respectively.

From Figures 1, 2 and 3, the system is stable at N = 800 and the queue size is properly utilized



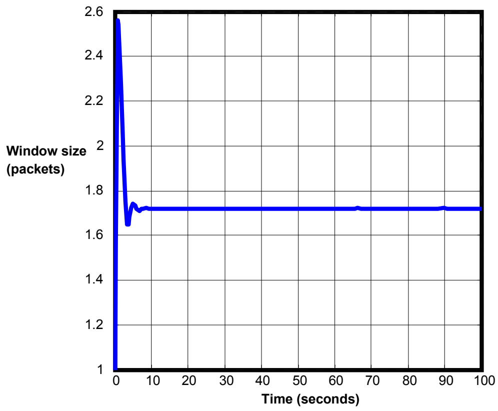


Fig. 1: Queue size responses for nonlinear delay free controller; Fig. 2: Window size response of nonlinear delay free controller

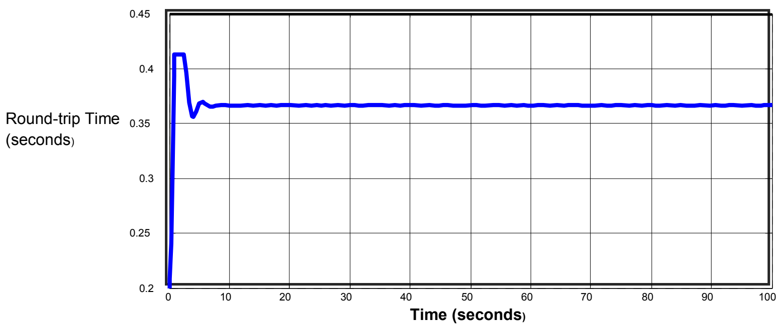


Fig. 3: Response showing the delay in system (Round-trip time) using the nonlinear delay free marking controller

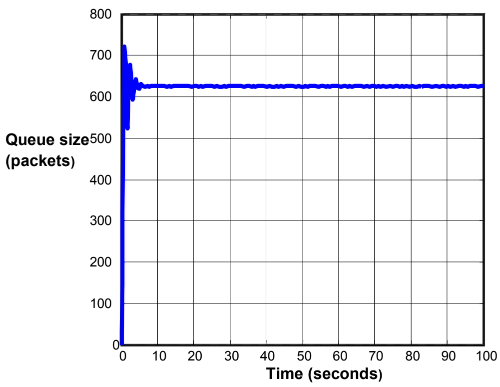
as shown in Figure 1. The queue size and window size in Figure 1 and 2 respectively show that they are bounded. Figure 3 shows a delay in the system within the range of 220-400ms based on the roundtrip time response.

When $N > 800$, the system experiences jittering which shows that the bandwidth is not fully utilized. This means there is incessant packet drop and window size adjustment based on available bandwidth. The variations in queue size invariably cause great variation in the roundtrip time, showing that the system is unstable.

Case II: Experiment with Nonlinear Delayed Marking Controller.

In this second case, we consider a queue of 800 TCP session with average packet size of 500 bytes, link capacity of 3750 packets/sec (about 2.1 Mbytes/sec), constant $b = 1$, maximum window size $\overline{W} = 500$ packets and maximum queue size $\overline{q} = 800$ packets. Note that the window size and queue size cannot be less than zero.

The control design in Case II shows that the delay terms have been estimated. Therefore the control law has been delayed before application to the system. Figures 4, 5 and 6 are the responses of the queue size, window size and roundtrip time respectively. The responses show that the system is stable.



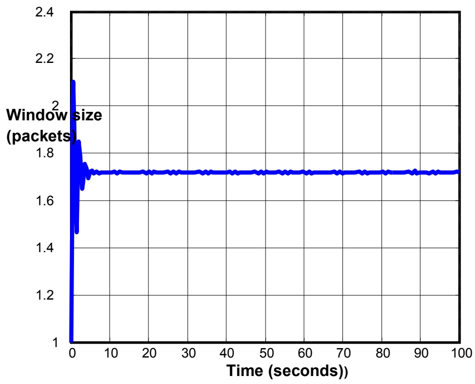


Fig. 4: Queue size responses for nonlinear delayed controller; **Fig. 5:** Window size response of nonlinear delayed controller

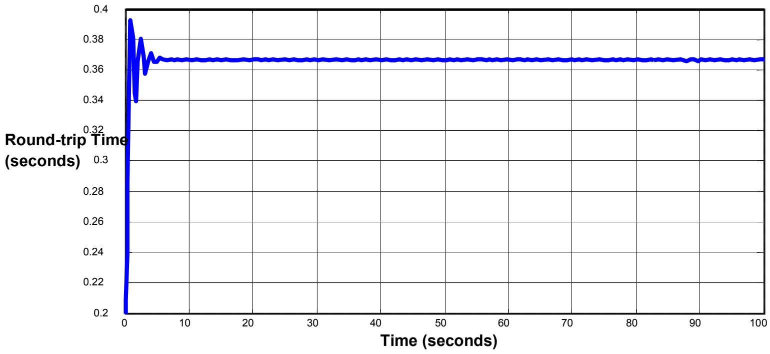


Figure 6: Response showing the delay in system (Round-trip time) using the nonlinear delayed marking controller

Comparison of Simulation Result

It is apparent that the use of nonlinear controller gives a better queue size result than the RED (Hollot et al, 2001; 2002). The queue size of the two nonlinear controllers performs better because the nonlinearities in the system have been accounted for.

The difference between the delayed free NLDF and the delayed NLD is that in the NLDF, the control law is estimated and fed into a delay block while in the NLD, the estimate of the delayed control law is obtained directly. This explains why the NLD response is better and smoother than NLDF. In several systems, delay in a system causes problems such as oscillation, instability and even bad performance of the system. The oscillatory effect seen at the beginning of the responses shown in Figures 4, 5, and 6, manifests the effects of improper delay estimate. The controller in this case is robust enough to keep the system stable despite the effect of delay.

CONCLUSIONS

In this work, two nonlinear control strategies have been considered to address the stability in the nonlinear TCP system modelled with input delay. The queue management strategies existing already produce unstable queue responses, which result in low queue utilization and in most cases cause jittering at high delay. Back stepping design method has been adopted to design a nonlinear delay free controller NLDF and nonlinear delayed controller NLD. These two nonlinear controllers have been implemented on the nonlinear TCP model as well.

Analysis and guidelines for choosing control parameters that can stabilize the system have been presented bearing in mind the AQM performance objectives, which are proper queue utilization and low queuing delay. Although certain details of TCP such as slow start, fast retransmit and fast recovery are ignored using an approximation of window size, the system's real characteristics are still retained.

NLDF operates with the control law $u(t)$ fed into a delay block while the NLD operates with an estimate of the delayed control law $u(t-R)$. From extensive simulation and comprehensive comparison of the performance of the nonlinear controller to the previous work by Hollot et al (2001, 2002), it has been discovered that the nonlinear controllers show better control responses at high TCP session of 800. However, the responses also show that the NLD experiences more oscillation than NLDF because the estimated delayed control law $u(t-R(t))$ is applied at present time (t) and the estimate of delay given by Lyapunov- Krasovskii functional candidate is an approximation. The nonlinear controllers maintain high queue utilization.

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