Modeling and Analytical Simulation of High-Temperature Gas Filtration Combustion

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ABSTRACT: High temperature filtration in combustion and gasification processes is a highly interdisciplinary field. Thus, particle technology in general has to be supported by elements of physics, chemistry, thermodynamics and heat and mass transfer processes. Presented in this paper is the analytical method for describing high-temperature gas filtration combustion in an inert porous medium. We assume the porous media is highly permeable and both the contact time between the phases and the rate of oxidizer diffusion through the gas stream to the surface of the solid particles where the reaction occurs are not large. Also, we assume that the initial temperatures increase lengthwise. The coupled nonlinear partial differential equations describing the phenomenon have been decoupled using the parameter-expanding method and solved analytically using eigenfunctions expansion technique. The results obtained revealed that the combustion wave is propagated and oxidizer is consumed. A self-oscillating mode of gas filtration combustion was found with variation in the values of interfacial heat transfer. ©JASEM

https://dx.doi.org/10.4314/jasem.v21i5.20

*Keywords***: A**nalytical method, filtration combustion, fuel, oxidizer, porous medium, temperature.

Filtration combustion, when an oxidizer is injected into a porous medium containing fuel, has numerous applications in technology and nature (enhanced oil recovery by in situ ombustion, coal gasification, self-propagating high-temperature synthesis (SHS), smolder waves etc.). A High temperature, high pressure gas filtration is a fundamental component of several advanced coal-fired power systems (Endo Kokubun *et al.*, 2016).

Smoldering and SHS are both complicated processes involving chemistry; diffusive and convective transport of reactants, products, and heat through a porous medium; heat losses to the environment by radiation and convection (Wahle *et al.*, 2003).

Mathematical models for filtration combustion of gases and condensed materials are based on the equations of the mechanics of multiphase media supplemented with the equations of chemical kinetics (Aldushin and Merzhanov, 1988). A fairly complete review of experimental and theoretical studies of filtration gas combustion in inert porous media is presented in (AbdulMujeebu *et al.*, 2009). Numerical two-dimensional calculations of temperature and concentration fields supported experimentally have been performed in (Brenner *et al.*, 2000) for combustion of a methane–air mixture in a rectangular burner with an inert coarse-pore section. Unsteady filtration combustion of gases in an inert porous layer is considered in (Prokof'ev *et al.*, 2010) taking into account the gas pressure distribution in the pores. Due to the complexity of two-temperature models, very few results have been obtained. Wahle *et al.* (2003) employed a two-temperature model to study coflow FC in a sample open to gas flow only at the ends of the sample.

The objectives of this paper are to propose a two-temperature model for gas filtration combustion in an inert flat porous layer taking into account the gas pressure distribution in the pores, heat loss to the environment, energy accumulation, and net gas production in the reaction and to provide an approximated analytical solution to describe the phenomena.

MATERIALS AND METHODS

We consider a horizontal one-dimensional inert porous medium with an initially available concentration of a solid fuel. A combustion wave travels in the x-direction through a sample of length L. The space variable is x, $0 < x < L$ and time t, $t > 0$. Energy balances for each phase account for the heat flux through a given volume of the medium as well as heat exchange between the two phases. We assume the interphase heat transfer rate depends on the gas flux and follow Frank-Kamenetskii expression (Frank-Kamenetskii, 1969). The reaction rate depends on both the solid-fuel concentration and the oxidizer concentration in the gas at the surface of the solid particles where the reaction occurs. The kinetic function is assumed to be Arrhenius with large activation energy and the rate of oxidizer diffusion to the interface to be sufficiently large. We consider first-order solid-fuel reaction kinetic, so that $f(\eta) = 1 - \eta$. Based on the above assumptions, the two-temperature filtration gas combustion model consists of equations describing conservation of energy in each phase, gas mass, oxidizer mass, solid fuel mass and gas momentum as well as an equation of state and appropriate boundary and initial conditions as shown below:

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Conservation of energy of solid:

$$(1-\phi)\rho_s c_{ps}\left(\frac{\partial T_s}{\partial t}+u\frac{\partial T_s}{\partial x}\right)=\frac{\partial}{\partial x}\left(\lambda_s\frac{\partial T_s}{\partial x}\right)-\alpha(T_g-T_s)-h(T_s-T_0)+(1-\phi)\rho_f Qk_0 Y_{ox}e^{-\frac{E}{RT_s}}$$

(1)

Conservation of energy of gas:

$$\phi\rho_g c_{pg}\left(\frac{\partial T_g}{\partial t}+u\frac{\partial T_g}{\partial x}\right)=-\phi\rho_g c_{pg}v\frac{\partial T_g}{\partial x}+\frac{\partial}{\partial x}\left(\lambda_g\frac{\partial T_g}{\partial x}\right)+\alpha(T_g-T_s)+$$

$$\mu_g c_{pg}(T_g-T_s)(1-\phi)\rho_f k_0 Y_{ox}e^{-\frac{E}{RT_s}}$$

(2)

Conservation of gas species (Oxygen):

$$\frac{\partial}{\partial t}(\phi\rho_g Y_{ox})+u\frac{\partial}{\partial x}(\phi\rho_g Y_{ox})=-\frac{\partial}{\partial x}(\phi\rho_g vY_{ox})+\frac{\partial}{\partial x}\left(\phi\rho_g D\frac{\partial Y_{ox}}{\partial x}\right)-\mu_0(1-\phi)\rho_f k_0 Y_{ox}e^{-\frac{E}{RT_s}}$$

(3)

Solid fuel mass:

$$\frac{\partial}{\partial t}((1-\phi)\rho_f)+u\frac{\partial}{\partial x}((1-\phi)\rho_f)=-(1-\phi)\rho_f k_0 Y_{ox}e^{-\frac{E}{RT_s}}$$

(4)

Gas mass:

$$\frac{\partial}{\partial t}(\phi\rho_g)+u\frac{\partial}{\partial x}(\phi\rho_g)=-\frac{\partial}{\partial x}(\phi\rho_g v)+\mu_g(1-\phi)\rho_f k_0 Y_{ox}e^{-\frac{E}{RT_s}}$$ (5)

Darcy's Law:

$$\frac{\partial P}{\partial x}+rv=0$$

(6)

The equation of State:

$$P=\rho_g\overline{R}T_g, \qquad \overline{R}=\frac{R}{\phi M},$$

(7)

where

$$\alpha=\alpha_0\left(1+\left(Re_0\frac{G}{G_0}\right)^{0.6}Pr^{0.3}\right)$$

where ϕ is the porosity of the sample, v is the gas velocity, u is the propagation velocity, E is activation energy, R is the universal gas constant, L is the length of sample, λ is thermal conductivity, Q is the enthalpy of reaction, c is specific heat, T is temperature, Y is the mass fraction of the species, t is time, x is the spatial coordinate along the sample, α is the heat exchange rate between the two phases, h is the coefficient of heat loss to the external environment, D is the gas mass diffusivity, r is the filtration resistance, μ_g is the net mass of gas produced ($\mu_g>0$) or consumed ($\mu_g<0$) per unit mass of fuel consumed in the reaction, μ_0 is the mass stoichiometric coefficient for oxygen, P is pressure, ρ is density, k_0 is the pre-exponential factor, M is the molecular weight of the gas, G is the gas mass flux in the direction of propagation which can be expressed as $G=-\phi\rho_g v$, α_0

is the interphase heat exchange rate when the gas is at rest with respect to the solid matrix, Pr is the Prandtl number, Re_0 is the Reynolds number corresponding to the gas flux at the inlet and the subscripts s, g, f, p, ox, gp and 0 represent solid, gas, solid-fuel, pressure, oxygen, gas product and initial respectively.

Coordinate Transformation
Here, we assume ρ_s, ρ_g, ρ_f, c_p, c_{pg}, c_{ps}, λ_s, λ_g, ϕ, D, α to be constants and $\rho_s=\rho_g=\rho$.

By means of streamline function (Olayiwola, 2015),

$$\eta(x,t)=\int_0^x\rho(s,t)ds \qquad (8)$$

The coordinate transformation becomes,

$$\frac{\partial}{\partial x}\rightarrow\frac{\partial}{\partial\eta}\frac{\partial\eta}{\partial x}=\rho\frac{\partial}{\partial\eta} \qquad (9)$$

$$\frac{\partial}{\partial t}\rightarrow\frac{\partial}{\partial\eta}\frac{\partial\eta}{\partial t}+\frac{\partial}{\partial t}=-\rho u\frac{\partial}{\partial\eta}+\frac{\partial}{\partial t}$$

(10)

We make the additional assumptions that $\rho\lambda_s$, $\rho\lambda_g$ and $\rho^2 D$ are constants. Although these assumptions could be relaxed in the future, they considerably simplify the equations.

Then, equations (1) – (7) reduce to

$$(1-\phi)\rho c_{ps}\frac{\partial T_s}{\partial t}=\rho^2\lambda_s\frac{\partial^2 T_s}{\partial\eta^2}-\alpha(T_g-T_s)-h(T_s-T_0)+(1-\phi)\rho_f Qk_0 Y_{ox}e^{-\frac{E}{RT_s}}$$

(11)

$$\phi\rho c_{pg}\frac{\partial T_g}{\partial t}=\phi\rho^3 c_{pg}\frac{1}{r}\frac{\partial P}{\partial\eta}\frac{\partial T_g}{\partial\eta}+\rho^2\lambda_g\frac{\partial^2 T_g}{\partial\eta^2}+\alpha(T_g-T_s)+$$

$$\mu_g c_{pg}(T_g-T_s)(1-\phi)\rho_f k_0 Y_{ox}e^{-\frac{E}{RT_s}}$$

$$\phi\rho\frac{\partial Y_{ox}}{\partial t}=\phi\rho^3\frac{\partial}{\partial\eta}\left(\frac{1}{r}\frac{\partial P}{\partial\eta}Y_{ox}\right)+\phi\rho^3 D\frac{\partial^2 Y_{ox}}{\partial\eta^2}-\mu_0(1-\phi)\rho_f k_0 Y_{ox}e^{-\frac{E}{RT_s}}$$

(13)

The initial and boundary conditions were formulated as follows:
Initial condition:
At $t=0$ and $\forall\ \eta$

$$T_s=\frac{RT_0^2}{E}\left(1-\frac{\eta}{L}\right)+T_0, \quad T_g=\frac{RT_0^2}{E}\left(1-\frac{\eta}{L}\right)+T_0, \quad Y_{ox}=Y_0$$

(14)

Boundary conditions:

$$\frac{\partial T_s}{\partial \eta}\bigg|_{\eta=0} = 0, \qquad \frac{\partial T_s}{\partial \eta}\bigg|_{\eta=L} = 0$$

$$T_g\big|_{\eta=0} = T_0, \qquad \frac{\partial T_g}{\partial \eta}\bigg|_{\eta=L} = 0$$

$$Y_{ox}\big|_{\eta=0} = Y_0, \qquad \frac{\partial Y_{ox}}{\partial \eta}\bigg|_{\eta=L} = 0$$

(15)

Method of Solution

In our analysis, we consider the pressure gradient to be parabolic (Olayiwola, 2012; Olayiwola *et al.*, 2013), i.e.:

$$\frac{\partial P}{\partial \eta} = f(\eta) = \frac{\eta}{L}\left(1 - \frac{\eta}{L}\right)$$

(16)

Non-dimensionalization

Here, we non-dimensionalized equations (11) – (16), using the following dimensionless variables:

$$\eta' = \frac{\eta}{L}, \qquad t' = \frac{t}{t_0}, \qquad Y = \frac{Y_{ox}}{Y_0}, \qquad \epsilon = \frac{RT_0}{E},$$

$$\theta = \frac{E}{RT_0^2}(T_s - T_0), \qquad \varphi = \frac{E}{RT_0^2}(T_g - T_0)$$

(17)

where t_0 is the initial time for ignition to occur and we obtain and we obtain

$$\frac{\partial \theta}{\partial t} = \lambda_1 \frac{\partial^2 \theta}{\partial \eta^2} - \alpha_1(\varphi - \theta) - h_1\theta + \delta Y \exp\left(\frac{\theta}{1 + \epsilon\,\theta}\right)$$

(18)

$$\frac{\partial \varphi}{\partial t} = \lambda_2 \frac{\partial^2 \varphi}{\partial \eta^2} + k\eta(1 - \eta)\frac{\partial \varphi}{\partial \eta} + \alpha_2(\varphi - \theta) + \beta Y(\varphi - \theta)\exp\left(\frac{\theta}{1 + \epsilon\,\theta}\right)$$

(19)

$$\frac{\partial Y}{\partial t} = D_1 \frac{\partial^2 Y}{\partial \eta^2} + k\eta(1 - \eta)\frac{\partial Y}{\partial \eta} + k(1 - 2\eta)Y - \sigma Y \exp\left(\frac{\theta}{1 + \epsilon\,\theta}\right)$$

(20)

Together with initial and boundary conditions:

$$\theta(\eta,0) = 1 - \eta, \qquad \frac{\partial \theta}{\partial \eta}\bigg|_{\eta=0} = 0, \qquad \frac{\partial \theta}{\partial \eta}\bigg|_{\eta=1} = 0,$$

$$\varphi(\eta,0) = 1 - \eta, \qquad \varphi(0,t) = 0, \qquad \frac{\partial \varphi}{\partial \eta}\bigg|_{\eta=1} = 0$$

$$Y(\eta,0) = 1, \qquad Y(0,t) = 1, \qquad \frac{\partial Y}{\partial \eta}\bigg|_{\eta=1} = 0$$

(21)

where

$$\lambda_1 = \frac{\rho \lambda_s t_0}{L^2(1 - \phi)c_{ps}} = \text{Solid phase scaled thermal conductivity,}$$

$$\lambda_2 = \frac{\rho \lambda_g t_0}{L^2 \phi c_{pg}} = \text{Gas phase scaled thermal conductivity,}$$

$$\alpha_1 = \frac{\alpha t_0}{(1 - \phi)\rho c_{ps}}, \quad \alpha_2 = \frac{\alpha t_0}{\phi \rho c_{pg}}, \qquad \delta = \frac{\rho_f k_0 QY_0 t_0 e^{-\frac{E}{RT_0}}}{\rho c_{ps} \in T_0} = \text{Frank-}$$

Kamenetskii parameter, $h_1 = \frac{ht_0}{(1 - \phi)\rho c_{ps}}$, $k = \frac{\rho^2 t_0}{rL}$, $D_1 = \frac{\rho^2 t_0 D}{L^2} =$

Species diffusion coefficient, $\beta = \frac{\mu_g \rho_f k_0(1 - \phi)t_0 Y_0 e^{-\frac{E}{RT_0}}}{\phi \rho}$,

$$\sigma = \frac{\mu_0 \rho_f k_0(1 - \phi)t_0 e^{-\frac{E}{RT_0}}}{\phi \rho}$$

Analytical Solution

We solve equations (18) – (21) using parameter-expanding method (where details can be found in (He, 2006)) and eigenfunctions expansion method (where details can be found in (Myint-U and Debnath, 1987)).

Ayeni (1982) has shown that $\exp\left(\frac{\theta}{1 + \epsilon\,\theta}\right)$ can be approximated as $1 + (e - 2)\theta + \theta^2$. In our analysis we are going to take an approximation of the form:

$$\exp\left(\frac{\theta}{1 + \epsilon\,\theta}\right) \approx 1 + (e - 2)\theta$$

(22)

so that equations (18) - (20) can be approximated as:

$$\frac{\partial \theta}{\partial t} = \lambda_1 \frac{\partial^2 \theta}{\partial \eta^2} - \alpha_1(\varphi - \theta) - h_1\theta + \delta Y(1 + (e-2)\theta)$$

(23)

$$\frac{\partial \varphi}{\partial t} = \lambda_2 \frac{\partial^2 \varphi}{\partial \eta^2} + k\eta(1-\eta)\frac{\partial \varphi}{\partial \eta} + \alpha_2(\varphi - \theta) + \beta Y(\varphi - \theta)(1 + (e-2)\theta)$$

(24)

$$\frac{\partial Y}{\partial t} = D_1 \frac{\partial^2 Y}{\partial \eta^2} + k\eta(1-\eta)\frac{\partial Y}{\partial \eta} + k(1 - 2\eta)Y - \sigma Y(1 + (e-2)\theta)$$

(25)

We let

$$\alpha_1 = r\epsilon, \qquad k = p\epsilon, \qquad \beta = q\epsilon, \qquad \delta = a\epsilon, \qquad \sigma = b\epsilon$$

Suppose that the solution of equations (23) – (25) can be expressed as:

$$\theta(\eta,t) = \theta_0(\eta,t) + \epsilon\,\theta_1(\eta,t) + \ldots$$
$$\varphi(\eta,t) = \varphi_0(\eta,t) + \epsilon\,\varphi_1(\eta,t) + \ldots$$
$$Y(\eta,t) = Y_0(\eta,t) + \epsilon\,Y_1(\eta,t) + \ldots$$

(26)

Substituting (26) into (23) – (25) and processing, we obtain

$$\frac{\partial \theta_0}{\partial t} = \lambda_1 \frac{\partial^2 \theta_0}{\partial \eta^2} - h_1\theta_0 \qquad (27)$$

$$\theta_0(\eta,0) = 1 - \eta, \qquad \theta_{0\eta}(0,t) = 0, \qquad \theta_{0\eta}(1,t) = 0$$

$$\frac{\partial \varphi_0}{\partial t} = \lambda_2 \frac{\partial^2 \varphi_0}{\partial \eta^2} + \alpha_2(\varphi_0 - \theta_0) \qquad (28)$$

$$\varphi_0(\eta,0) = 1 - \eta, \qquad \varphi_0(0,t) = 0, \qquad \varphi_{0\eta}(1,t) = 0$$

$$\frac{\partial Y_0}{\partial t} = D_1 \frac{\partial^2 Y_0}{\partial \eta^2} \qquad (29)$$

$$Y_0(\eta,0) = 0, \qquad Y_0(0,t) = 1, \qquad Y_{0\eta}(1,t) = 0$$

$$\frac{\partial \theta_1}{\partial t} = \lambda_1 \frac{\partial^2 \theta_1}{\partial \eta^2} - r(\varphi_0 - \theta_0) - h_1\theta_1 + aY_0(1 + (e-2)\theta_0)$$

(30)

$$\theta_1(\eta,0) = 0, \qquad \theta_{1\eta}(0,t) = 0, \qquad \theta_{1\eta}(1,t) = 0$$

$$\frac{\partial \varphi_1}{\partial t} = \lambda_2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + p\eta(1-\eta)\frac{\partial \varphi_0}{\partial \eta} + \alpha_2(\varphi_1 - \theta_1) + qY_0(\varphi_0 - \theta_0)(1 + (e-2)\theta_0)$$

(31)

$$\varphi_1(\eta, 0) = 0, \qquad \varphi_1(0, t) = 0, \qquad \varphi_{1\eta}(1, t) = 0$$

$$\frac{\partial Y_1}{\partial t} = D_1 \frac{\partial^2 Y_1}{\partial \eta^2} + p\eta(1-\eta)\frac{\partial Y_0}{\partial \eta} + p(1-2\eta)Y_0 - bY_0(1 + (e-2)\theta_0)$$

(32)

$$Y_1(\eta, 0) = 0, \qquad Y_1(0, t) = 0, \qquad Y_{1\eta}(1, t) = 0$$

Using eigenfunctions expansion method, we obtain the solution of equations (27) - (32) as

$$\theta_0(\eta, t) = e^{-h_1 t} + \sum_{n=1}^{\infty} Ae^{-ct} \cos n\pi\eta \tag{33}$$

$$\varphi_0(\eta, t) = \sum_{n=1}^{\infty}\left(B_1\left(e^{-h_1 t} - e^{dt}\right) + \sum_{n=1}^{\infty} B_2\left(e^{-ct} - e^{dt}\right) + Be^{dt}\right)\sin\left(\frac{2n-1}{2}\right)\pi\eta \tag{34}$$

$$Y_0(\eta, t) = 1 + \sum_{n=1}^{\infty} A_1 e^{-mt} \sin\left(\frac{2n-1}{2}\right)\pi\eta \tag{35}$$

$$\theta_1(\eta, t) = \theta_{10}(t) + \sum_{n=1}^{\infty}\theta_{1n}(t)\cos n\pi\eta \tag{36}$$

$$\varphi_1(\eta, t) = \sum_{n=1}^{\infty} s(t)\sin\left(\frac{2n-1}{2}\right)\pi\eta \tag{37}$$

$$Y_1(\eta, t) = \sum_{n=1}^{\infty} Q(t)\sin\left(\frac{2n-1}{2}\right)\pi\eta \tag{38}$$

where

$$\theta_{10}(t) = \frac{2a}{h_1}\left(1 - e^{-h_1 t}\right) + 2a(e-2)te^{-h_1 t} + \sum_{n=1}^{\infty} p_1\left(e^{-mt} - e^{-h_1 t}\right) - (e-2)\sum_{n=1}^{\infty} p_1\left(e^{-(h_1+m)t} - e^{-h_1 t}\right) -$$

$$(e-2)\sum_{n=1}^{\infty} A_1\sum_{n=1}^{\infty} p_2\left(e^{-(c+m)t} - e^{-h_1 t}\right) + 2rte^{-h_1 t} +$$

$$r\sum_{n=1}^{\infty} A_1\left(\begin{array}{c}B_1\left(te^{-h_1 t} - p_3\left(e^{dt} - e^{-h_1 t}\right)\right) + p_4\left(e^{dt} - e^{-h_1 t}\right) -\\ \sum B_2\left(p_5\left(e^{-ct} - e^{-h_1 t}\right) - p_3\left(e^{dt} - e^{-h_1 t}\right)\right)\end{array}\right)$$

$$\theta_{1n}(t) = a(e-2)\sum_{n=1}^{\infty} Ate^{-ct} - \sum_{n=1}^{\infty} p_6\left(e^{-mt} - e^{-ct}\right) - (e-2)\sum_{n=1}^{\infty} p_7\left(e^{-(h_1+m)t} - e^{-ct}\right) -$$

$$(e-2)\sum_{n=1}^{\infty} A_1\sum_{n=1}^{\infty} p_8\left(e^{-(c+m)t} - e^{-ct}\right) + r\sum_{n=1}^{\infty} Ate^{-ct} -$$

$$r\sum_{n=1}^{\infty} A_2\left(\begin{array}{c}B_1\left(p_9\left(e^{-h_1 t} - e^{-ct}\right) - p_{10}\left(e^{dt} - e^{-ct}\right)\right) + p_{11}\left(e^{dt} - e^{-ct}\right) -\\ \sum B_2\left(te^{-ct} - p_{10}\left(e^{-dt} - e^{-h_1 t}\right)\right)\end{array}\right)$$

$$Q(t) = p\sum_{n=1}^{\infty} q_1 te^{-mt} + p\left(q_2\left(1 - e^{-mt}\right) - \sum_{n=1}^{\infty} q_3 te^{-mt}\right) - b\left(A_1\left(1 - e^{-mt}\right) + \sum_{n=1}^{\infty} A_1 te^{-mt}\right) -$$

$$b(e-2)\left(q_4\left(e^{-h_1 t} - e^{-mt}\right) + \sum_{n=1}^{\infty} q_5\left(e^{-ct} - e^{-mt}\right) - \sum_{n=1}^{\infty} q_6\left(e^{-(h_1+m)t} - e^{-mt}\right)\right)$$

$$s(t) = e(t) + e_1(t) + e_2(t) - e_3(t) - e_4(t) - e_5(t) - e_6(t)$$

$$e(t) = q\sum_{n=1}^{\infty}\left(B_1\left(p_3\left(e^{-h_1 t} - e^{dt}\right) - te^{dt}\right) + Bte^{dt} - \sum_{n=1}^{\infty} B_2\left(-p_{10}\left(e^{-h_1 t} - e^{dt}\right) - te^{dt}\right)\right)$$

$$e_1(t) = q\sum_{n=1}^{\infty} A_1 A_3\sum_{n=1}^{\infty}\left(\begin{array}{c}B_1\left(-q_7\left(e^{-(m+h_1)t} - e^{dt}\right) + \frac{1}{m}\left(e^{(d-m)t} - e^{dt}\right)\right) - \frac{B}{m}\left(e^{(d-m)t} - e^{dt}\right) -\\ \sum B_2\left(-q_8\left(e^{-(m+c)t} - e^{dt}\right) + \frac{1}{m}\left(e^{(d-m)t} - e^{dt}\right)\right)\end{array}\right)$$

$$e_2(t) = q(e-2)(r_1(t) + r_2(t) + r_3(t))$$

$$r_1(t) = \sum_{n=1}^{\infty}\left(\begin{array}{c}B_1\left(-q_9\left(e^{-(2h_1)t} - e^{dt}\right) + \frac{1}{h_1}\left(e^{(d-h_1)t} - e^{dt}\right)\right) - \frac{B}{h_1}\left(e^{(d-h_1)t} - e^{dt}\right) -\\ \sum B_2\left(-q_{10}\left(e^{-(h_1+c)t} - e^{dt}\right) + \frac{1}{h_1}\left(e^{(d-h_1)t} - e^{dt}\right)\right)\end{array}\right)$$

RESULTS AND DISCUSSION

We solve the systems of coupled nonlinear partial differential equations describing high-temperature gas filtration combustion in an inert porous medium analytically. We decouple the equations using parameter-expanding method and solve the resulting equations using eigenfunctions expansion technique. Using (33) – (38) in (26), analytical solutions of equations (18) - (21) are computed for the values of $\delta = 0.4, \lambda_1 = 0.1, \lambda_2 = 0.1, D_1 = 0.1, \in = 0.01, k = 1, \sigma = 0.2, \beta = 0.1, h_1 = 4, \alpha_1 = 0.00025, \alpha_2 = 0.125$. The following figs. explain the temperatures and species mass fraction distribution against different dimensionless parameters.

Fig. 1 shows the effect of Frank-Kamenetskii number (δ) on the solid phase temperature. It is observed that the solid phase temperature decreases as time increases and decrease along distance but increase with increase in Frank-Kamenetskii number.
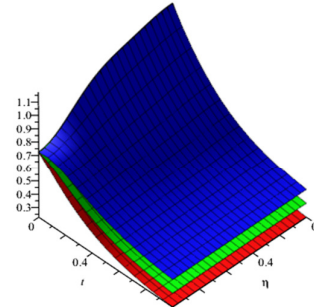

Fig. 1: Variation of solid phase temperature $\theta(\eta, t)$ with Frank-Kamenetskii number δ

Fig. 2 depicts the effect of oxygen mass diffusivity (D_1) on the oxidizer mass. It is observed that the oxidizer mass increases as time increases and behave sinusoidally along distance but decrease with increase in oxygen mass diffusivity.

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$$r_2(t) = \sum_{n=1}^{\infty} A_1 A_3 \sum_{n=1}^{\infty} \left(\begin{array}{c} B_1\left(-q_{11}\left(e^{-(m+2h_1)t} - e^{dt}\right) + \frac{1}{m+h_1}\left(e^{(d-m-h_1)t} - e^{dt}\right)\right) - \\ \frac{B}{m+h_1}\left(e^{(d-m-h_1)t} - e^{dt}\right) - \sum_{n=1}^{\infty} B_2 \left(\begin{array}{c} -q_{12}\left(e^{-(m+h_1+c)t} - e^{dt}\right) + \\ \frac{1}{m+h_1}\left(e^{(d-m-h_1)t} - e^{dt}\right) \end{array}\right) \end{array}\right)$$

$$r_3(t) = \sum_{n=1}^{\infty} A_1 \sum_{n=1}^{\infty} A \sum_{n=1}^{\infty} B_4 \left(\begin{array}{c} B_1\left(-q_{12}\left(e^{-(m+c+h_1)t} - e^{dt}\right) + \frac{1}{m+c}\left(e^{(d-m-c)t} - e^{dt}\right)\right) - \\ \frac{B}{m+c}\left(e^{(d-m-c)t} - e^{dt}\right) - \sum_{n=1}^{\infty} B_2 \left(\begin{array}{c} -q_{13}\left(e^{-(m+2c)t} - e^{dt}\right) + \\ \frac{1}{m+c}\left(e^{(d-m-c)t} - e^{dt}\right) \end{array}\right) \end{array}\right)$$

$$e_3(t) = q\left(-q_{14}\left(e^{-h_1 t} - e^{dt}\right) + \sum_{n=1}^{\infty} q_{15}\left(e^{-ct} - e^{dt}\right) - \sum_{n=1}^{\infty} q_{16}\left(e^{-(m+h_1)t} - e^{dt}\right)\right)$$

$$e_4(t) = q(e-2)\left(\begin{array}{c} -q_{17}\left(e^{-2h_1 t} - e^{dt}\right) + 2\sum_{n=1}^{\infty} q_{18}\left(e^{-(c+h_1)t} - e^{dt}\right) + \sum_{n=1}^{\infty} A \sum_{n=1}^{\infty} q_{19}\left(e^{-2ct} - e^{dt}\right) - \\ \sum_{n=1}^{\infty} q_{20}\left(e^{-(m+2h_1)t} - e^{dt}\right) - -\frac{1}{2}\sum_{n=1}^{\infty} A_1 \sum_{n=1}^{\infty} A \sum_{n=1}^{\infty} q_{21}\left(e^{-(m+2c)t} - e^{dt}\right) \end{array}\right)$$

$$e_5(t) = p\sum_{n=1}^{\infty} A_4\left(B_1\left(-q_{22}\left(e^{-h_1 t} - e^{dt}\right) - te^{dt}\right) + Bte^{dt} - \sum_{n=1}^{\infty} B_2\left(-q_{23}\left(e^{-ct} - e^{dt}\right) - te^{dt}\right)\right)$$

$$e_6(t) = \alpha_2\left(A_1\left(r_4(t) + r_5(t) + r_6(t) - r_7(t) - r_8(t) + r_9(t) + r_{10}(t) - r_{11}(t)\right)\right)$$

$$r_4(t) = \frac{2a}{h_1}\left(-\frac{1}{d}\left(1 - e^{dt}\right) + \frac{1}{d+h_1}\left(e^{-h_1 t} - e^{dt}\right)\right)$$

$$r_5(t) = 2a(e-2)\left(\frac{1}{(d+h_1)^2}\left(e^{-h_1 t} - e^{dt}\right) - \frac{1}{d+h_1}te^{-h_1 t}\right)$$

$$r_6(t) = \sum_{n=1}^{\infty} q_{24}\left(\frac{1}{(d+m)}\left(e^{-mt} - e^{dt}\right) - \frac{1}{d+h_1}\left(e^{-h_1 t} - e^{dt}\right)\right)$$

$$r_7(t) = (e-2)\sum_{n=1}^{\infty} q_{25}\left(-\frac{1}{(d+m+h_1)}\left(e^{-(h_1+m)t} - e^{dt}\right) + \frac{1}{d+h_1}\left(e^{-h_1 t} - e^{dt}\right)\right)$$

$$r_8(t) = (e-2)\sum_{n=1}^{\infty} A_1 \sum_{n=1}^{\infty} q_{26}\left(-\frac{1}{(d+m+c)}\left(e^{-(c+m)t} - e^{dt}\right) + \frac{1}{d+h_1}\left(e^{-h_1 t} - e^{dt}\right)\right)$$

$$r_9(t) = 2r\left(\frac{1}{(d+h_1)^2}\left(e^{-h_1 t} - e^{dt}\right) - \frac{1}{d+h_1}te^{-h_1 t}\right)$$

$$r_{10}(t) = r\sum_{n=1}^{\infty} A_1 \left(\begin{array}{c} B_1 \left(\begin{array}{c} \left(\frac{1}{(d+h_1)^2}\left(e^{-h_1 t} - e^{dt}\right) - \frac{1}{d+h_1}te^{-h_1 t}\right) - \\ \frac{1}{d+h_1}\left(te^{dt} + \frac{1}{(d+h_1)}\left(e^{-h_1 t} - e^{dt}\right)\right) \end{array}\right) + \\ \frac{B}{d+h_1}\left(te^{dt} + \frac{1}{(d+h_1)}\left(e^{-h_1 t} - e^{dt}\right)\right) - \\ \sum_{n=1}^{\infty} B_2 \left(\begin{array}{c} \frac{1}{h_1 - c}\left(-\frac{1}{d+c}\left(e^{-ct} - e^{dt}\right) + \frac{1}{d+h_1}\left(e^{-h_1 t} - e^{dt}\right)\right) - \\ \frac{1}{d+h_1}\left(te^{dt} + \frac{1}{(d+h_1)}\left(e^{-h_1 t} - e^{dt}\right)\right) \end{array}\right) \end{array}\right)$$

$$r_{11}(t) = \sum_{n=1}^{\infty} A_2\left(r_{12}(t) - r_{13}(t) - r_{14}(t) - r_{15}(t) + r_{16}(t) - r_{17}(t)\right)$$

$$r_{12}(t) = a(e-2)\sum_{n=1}^{\infty} A\left(\frac{1}{(d+c)^2}\left(e^{-ct} - e^{dt}\right) - \frac{1}{d+c}te^{-ct}\right)$$

$$r_{13}(t) = \sum_{n=1}^{\infty} q_{27}\left(-\frac{1}{(d+m)}\left(e^{-mt} - e^{dt}\right) + \frac{1}{d+c}\left(e^{-ct} - e^{dt}\right)\right)$$

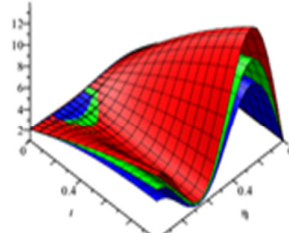
Fig. 2: Variation of oxidizer mass $T(\eta, t)$ with oxygen mass diffusivity D.

Fig. 3 presents the effect of filtration resistance (k) on the oxidizer mass. It is observed that the oxidizer mass increases as time increases and behave sinusoidally along distance but increase with increase in filtration resistance.
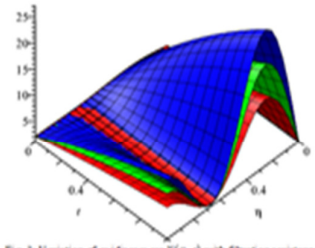

Fig. 3: Variation of oxidizer mass $T(\eta, t)$ with filtration resistance parameter k

Fig. 4 displays the effect of heat loss parameter (h_1) on the solid phase temperature. It is observed that the solid phase temperature decreases as time increases and decrease along distance but decrease with increase in heat loss to the environment.
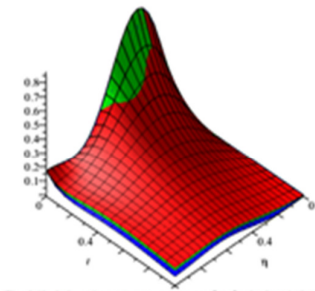

Fig. 5: Variation of gas phase temperature $\varphi(\eta, t)$ with interfacial heat transfer coefficient α_2

Fig. 5 shows the effect of interfacial heat transfer coefficient (α_2) on the gas phase temperature. It is observed that the gas phase temperature decreases as time increases and decrease along

$$r_{14}(t) = (e-2)\sum_{n=1}^{\infty} q_{28}\left(-\frac{1}{(d+m+h_1)}\left(e^{-(m+h_1)t} - e^{dt}\right) + \frac{1}{d+c}\left(e^{-ct} - e^{dt}\right)\right)$$

$$r_{15}(t) = (e-2)\sum_{n=1}^{\infty} A_1 \sum_{n=1}^{\infty} q_{29}\left(-\frac{1}{(d+m+c)}\left(e^{-(m+c)t} - e^{dt}\right) + \frac{1}{d+c}\left(e^{-ct} - e^{dt}\right)\right)$$

$$r_{16}(t) = r\sum_{n=1}^{\infty} A\left(-\frac{1}{(d+c)^2}\left(e^{-ct} - e^{dt}\right) - \frac{1}{d+c}te^{-ct}\right)$$

$$r_{17}(t) = r\sum_{n=1}^{\infty} A_2\left\{ B_1\left[\begin{array}{l}\frac{1}{c-h_1}\left(\frac{1}{(d+h_1)}\left(e^{-h_1 t} - e^{dt}\right) + \frac{1}{(d+c)}\left(e^{-ct} - e^{dt}\right)\right) - \\ \frac{1}{(d+c)}\left(te^{dt} + \frac{1}{(d+c)}\left(e^{-ct} - e^{dt}\right)\right)\end{array}\right] + \frac{B}{(d+c)}\left(te^{dt} + \frac{1}{(d+c)}\left(e^{-ct} - e^{dt}\right)\right) - \sum_{n=1}^{\infty} B_2\left[\begin{array}{l}\left(\frac{1}{(d+c)^2}\left(e^{-ct} - e^{dt}\right) - \frac{1}{(d+c)}te^{-ct}\right) - \\ \frac{1}{(d+c)}\left(te^{dt} + \frac{1}{(d+c)}\left(e^{-ct} - e^{dt}\right)\right)\end{array}\right]\right\}$$

$$A = -\frac{2(-1)^n}{n^2\pi^2}, \quad c = h_1 + \lambda_1 n^2\pi^2, \quad d = \alpha_2 - \lambda_2 n^2\pi^2, \quad B = \frac{4(2n\pi - \pi + 2(-1)^n)}{(2n-1)^2\pi^2},$$

$$A_1 = \frac{4}{(2n-1)\pi}, \quad m = \frac{1}{4}D_1(2n-1)^2\pi^2, \quad A_2 = \frac{4(2n-1)}{(4n-1)\pi}, \quad B_3 = \frac{4(1-4n-4n^2)}{24n^3 - 4n^2 - 6n + 1},$$

$$A_3 = \frac{8}{3(2n-1)\pi}, \quad B_4 = \frac{4(48n^3 - 72n^2 + 36n - 6)}{(128n^3 - 176n^2 + 72n - 9)\pi}, \quad B_1 = \frac{4\alpha_2}{(2n-1)\pi(d+h_1)},$$

$$B_2 = \frac{4A\alpha_2(2n-1)}{(4n-1)\pi(d+c)}, \quad A_4 = \frac{2(2n-1)}{(8n^3 - 12n^2 + 6n - 1)\pi^2}$$

$$p_1 = \frac{A_1^2}{(h_1-m)}, \quad p_2 = \frac{AA_2}{(h_1-m-c)}, \quad p_3 = \frac{1}{(h_1+d)}, \quad p_4 = \frac{B}{(h_1+d)}, \quad p_5 = \frac{1}{(h_1-c)},$$

$$p_6 = \frac{A_1A_2}{(c-m)}, \quad p_7 = \frac{A_1A_2}{(c-h_1-m)}, \quad p_8 = \frac{AB_3}{m}, \quad p_9 = \frac{1}{(c-h_1)}, \quad p_{10} = \frac{1}{(c+d)}, \quad p_{11} = \frac{B}{(c+d)}$$

$$q_1 = \frac{2\pi A_1}{(2n-1)^2}, \quad q_2 = \frac{4((2n-1)\pi + 4(-1)^n)}{(2n-1)^2\pi^2 m}, \quad q_3 = \frac{4A_1}{(2n-1)^2\pi^2}, \quad q_4 = \frac{A_1}{(m-h_1)},$$

$$q_5 = \frac{AA_2}{(m-c)}, \quad q_6 = \frac{A_1}{h_1}, \quad q_7 = \frac{1}{(d+m+h_1)}, \quad q_8 = \frac{1}{(d+m+c)}, \quad q_9 = \frac{1}{(d+2h_1)},$$

$$q_{10} = \frac{1}{(c+d+h_1)}, \quad q_{11} = \frac{1}{(m+d+2h_1)}, \quad q_{12} = \frac{1}{(m+d+c+h_1)}, \quad q_{13} = \frac{1}{(m+d+2c)},$$

$$q_{14} = \frac{A_1}{(d+h_1)}, \quad q_{15} = \frac{AA_2}{(d+c)}, \quad q_{16} = \frac{A_1}{(d+m+h_1)}, \quad q_{17} = \frac{A_1}{(d+2h_1)}, \quad q_{18} = \frac{AA_2}{(d+h_1+c)},$$

$$q_{19} = \frac{AB_3}{d+2c}, \quad q_{20} = \frac{A_1}{(d+m+2h_1)}, \quad q_{21} = \frac{A}{(d+m+2c)}, \quad q_{22} = \frac{1}{(d+h_1)}, \quad q_{23} = \frac{1}{(c+d)},$$

$$q_{24} = \frac{A_1^2}{(h_1-m)}, \quad q_{25} = \frac{A_1^2}{m}, \quad q_{26} = \frac{AA_2}{(h_1-m-c)}, \quad q_{27} = \frac{A_1A_2}{(c-m)}, \quad q_{28} = \frac{A_1A_2}{(c-m-h_1)}, \quad q_{29} = \frac{AB_3}{m}$$

Fig. 4: Variation of solid phase temperature $\theta(\eta, t)$ with heat loss parameter h_1

The computations were done using computer symbolic algebraic package MAPLE
The results of an analytical investigation of the effect of Frank-Kamenetskii number, oxygen mass diffusivity, filtration resistance, heat loss to the environment and interfacial heat transfer coefficient physically means that the combustion wave is propagated and oxidizer is consumed. A self-oscillating mode of gas filtration combustion was found with variation in the values of interfacial heat transfer parameter.

Conclusio*n:* We have formulated and solved analytically a mathematical model of filtration combustion of gases in an inert porous medium with an initially available concentration of a solid fuel to determine the concentration and temperature distributions. We decoupled the equations usingparameter expanding method and solved the resulting equations using Eigen functions expansion technique. Finally, we have provided the graphical summaries of the system responses.

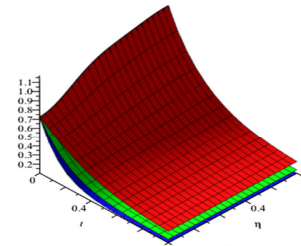*IBRAHIM, M; OLAYIWOLA, RO; COLE, AT; MD, SHEHU; MUHAMMAD, KM*

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