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Analysis of Pressure Variation of Fluid in Bounded Circular Reservoirs under the "No Flow" Outer Boundary Condition

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ABSTRACT: Predicting the pressure at the wellbore in bounded circular reservoir has been the hub in the study of Reservoir Engineering. Predicting the pressures outside the reservoir was not an easy task. The Ei function method has been the only method for determining the pressure outside the wellbore of a bounded circular reservoir. The disadvantage of the Ei function method is that it involves rigorous iterations. This study sorts to use another approach to determine the pressure within and outside the wellbore at a glance. The finite element method was used in this study. The reservoir domain was divided into smaller subdomains and analysed. The results from these subdomains were assembled to represent pressure in the entire reservoir. The results obtained where shown tabular form (dimensionless pressure and dimensionless time) for dimensionless radii of 1.5, 2, 2.5, 3, 3.5, 4, 5, 6, 7, 8, 9, and 10. It was shown that the relationship between dimensionless pressure and dimensionless time was linear. The result obtained at the wellbore was compared with the results obtained by Van Everdigen and Hurst. It was shown that there was a strong positive correlation between the results. ©JASEM

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***Keyword**s*: *Bounded circular reservoir, constant terminal rate, dimensionless variables, diffusivity equation, and Crank-Nicholson scheme.* Nomenclature

B	Formation volume factor, RB/STB
c	Compressibility, psia^{-1}
h	Thickness, ft
K	Stiffness matrix
k	Permeability, md
M	Mass matrix
n	Number of elements
P	Pressure, psi
P_D	Dimensionless pressure
$\dot{P}_D$	Dimensionless pressure rate
P_i	Initial reservoir pressure, psi
Q	Terminal flow rate
q	Volumetric flow rate, STB/D
r	Radius, ft
r_D	Dimensionless radius
r_e	External radius, ft
r_{eD}	Dimensionless external radius
r_w	Wellbore radius, ft
s	Time step, hr
t	Time, hr
t_D	Dimensionless time
w	Weight function
$\forall$	For all
Greek letters	
Δt	Time increment, hr
α	Family of approximation
ϕ	Porosity, fraction
μ	Viscosity, cp
π	Pi
ψ	Interpolation function

Understanding the trend of pressure profile in a reservoir during primary depletion is essential in reservoir studies. Especially, for those reservoirs which are susceptible to perform two phases of hydrocarbon due to pressure drop (Danesh, 1998). In this regard, any precise approach and model which include less assumption can be a reliable method. The fluid flow in reservoir or in porous medium has been a great interest of physicists, engineers and hydrologists who tried to predict the behaviours of compressible and incompressible fluids. They have designed several experiments so as to validate the implementation of their proposed correlations (Ahmed and McKinney, 2011). The basic equation for predicting pressure distribution in a reservoir is the diffusivity equation. For this equation, the reservoir temperature is supposed to be constant which is a valid assumption in most cases.

Several methods have proposed to solve the diffusivity equation including numerical and analytical approaches. The diffusivity equation has been solved in dimensionless form (Lee and Wattenbarger, 1996). Chakrabarty et al. (1993) provided a quantitative analysis of the effects of neglecting the quadratic gradient term on solving the diffusion equation governing the transient state. It should be noted that among the flow regimes in reservoir, the transient flow is the most significant state upon which such important characteristics such as permeability, reservoir capacity, and skin factor can be determined using well test analysis (Van Everdingen, 1953; Lee, 1992)

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Transient pressure response for a well producing from a finite reservoir of circular, square, and rectangular drainage shapes has been studied by Van Everdigen and Hurst (1949); Miller et al. (1950); Aziz and Flock (1963); Earlougher et al. (1968); Ramey and Cobb (1971); Kumar and Ramey (1974); Cobb and Smith (1975); and Chen and Brigham (1978) among others. Van Everdingen and Hurst presented the solution to diffusivity equation in eq. 8 in the form of infinite series of exponential terms and Bessel functions. The authors evaluated this series for several values of r_{eD} over a wide range of values for t_D. Chatas (1953) and John (1982) conveniently tabulated these solutions for the following two cases: Infinite-acting reservoir and Finite-radial reservoir.

Mishra and Ramey (1987) presented a buildup derivative type curve for a well with storage and skin, and producing from the centre of a closed, circular reservoir. Their type-curve applies for large producing times. The work by Ambastha and Ramey (1988) presents drawdown and buildup pressure derivative type-curves for a well producing at a constant rate from the centre of a finite, circular reservoir. The outer boundary may be closed, or at a constant pressure. The differences between the responses for a well in a closed, circular reservoir (fully developed field), and a well in a circular reservoir with a constant-pressure outer boundary (active edge water drive system, or developed five-spot fluid-injection pattern) were discussed. Design relations were developed to estimate the time period which corresponds to infinite-acting radial flow, or to a semi-log straight line on a pressure vs. logarithm of time graph. Producing time effects on buildup responses were studied using the slope of a dimensionless buildup graph proposed in Agarwal (l980).

In all the literature reviewed so far, none has been able to predict reservoir pressure outside the well bore. To this end, this work sort to predict the reservoir pressure both within and outside the wellbore using the finite element method.

Theory: The law of conservation of mass, Darcy's law and the equation of state has been combined to obtain the following partial differential equation:

$$\frac{\partial^2 P}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}=\frac{\phi\mu c}{0.000264k}\frac{\partial P}{\partial t} \quad (1)$$

with the assumptions that compressibility, c is small and independent of pressure, P; permeability, k, is constant and isotropic; viscosity, μ, is independent of pressure; porosity, ϕ, is constant; and that certain terms in the basic differential equation (involving pressure gradients squared) are negligible. This equation is called the diffusivity equation and the term $\frac{\phi\mu c}{0.000264k}$ is the inverse of the diffusivity constant, η.

In this work, the diffusivity equation was analysed for bounded circular reservoirs, the case in which the well is assumed to be located in the centre of a cylindrical reservoir with no flow across the exterior boundary and also the case of constant external boundary.

Governing Equation

$$\frac{\partial^2 P}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}=\frac{\phi\mu c}{0.000264k}\frac{\partial P}{\partial t}$$

Initial and boundary conditions:

i. $P = P_i$ at t = 0 $\forall$ r (2)

ii. $\left(r\frac{\partial P}{\partial r}\right)_{r_w}=\frac{qB\mu}{2\pi kh}$ for t > 0 (3)

iii. $\left(\frac{\partial P}{\partial r}\right)_{r_e}=0 \ \forall$ t (4)

Dimensionless Variables: The above equations incorporate physical parameters such as permeability, and it would be futile to solve this problem for a particular combination of values for these parameters. Dimensionless variables are designed to eliminate the physical parameters that affect quantitatively, but not qualitatively, the reservoir response. The above equations are in Darcy units, and the dimensionless terms will render the system of units employed irrelevant. For this line source model, 3 dimensionless variables are required. In US Oilfield units, distance, time and pressure are replaced as follows:

Dimensionless time:

$$t_D=\frac{0.0002637kt}{\phi\mu c r_w^2} \quad (5)$$

Dimensionless distance:

$$r_D = \frac{r}{r_w} \qquad (6)$$

Dimensionless pressure:

$$P_D = \frac{kh}{141.2qB\mu}\left(P_i - P\right) \qquad (7)$$

By defining dimensionless pressure and dimensionless time in this way, it is possible to create an analytical model of the well and reservoir, or theoretical 'type-curve', that provides a 'global' description of the pressure response that is independent of the flow rate or of the actual values of the well and reservoir parameters. Eq.1 can be transformed by substituting the following dimensionless variables in Eqs. 5-7 into eq. 1 and this becomes:

$$\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D} \qquad (8)$$

and the boundary and initial conditions become:

1. Dimensionless initial condition:
   Uniform pressure in the reservoir
   $$P_D\left(r_D, t_D = 0\right) = 0 \qquad (9)$$
2. Dimensionless inner boundary condition:
   Constant rate at the well
   $$\frac{\partial P_D}{\partial r_D}\left(1, t_D\right) = -1 \qquad (10)$$
3. "No Flow" boundary:
   No flux across the reservoir
   $$\frac{\partial P_D}{\partial r_D}\left(r_{eD}, t_D\right) = 0 \qquad (11)$$

Eq. 8 can also be written in a condensed form as:

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial P_D}{\partial r_D}\right) = \frac{\partial P_D}{\partial t_D} \qquad (12)$$

Assumptions; The assumptions used in proposing a solution to the diffusivity equation is as follows:

- The well is producing at constant flow rate. The reservoir is at uniform pressure, P_i when production begins.
- The well, with a wellbore radius of r_w is centred in a cylindrical reservoir of radius r_{eD}.
- No flow across the outer boundary, i.e., at r.
  The diffusivity equation was analysed for bounded circular reservoirs.

Finite Element Formulation: Weak Formulation: In the development of the weak form, we assumed a quadratic element mesh and placed it over the domain and applied the following steps:
From eq. 12, we have:

$$\frac{\partial P_D}{\partial t_D} - \frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial P_D}{\partial r_D}\right) = 0 \qquad (13)$$

Multiply eq. 13 by the weight w function and integrate the final equation over the domain.

$$\int_v w\left[\frac{\partial P_D}{\partial t_D} - \frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial P_D}{\partial r_D}\right)\right]dv = 0 \qquad (14)$$

Eq. 14 becomes,

$$\int_0^1\int_0^{2\pi}\int_{r_{DA}}^{r_{DB}} w\left[\frac{\partial P_D}{\partial t_D} - \frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial P_D}{\partial r_D}\right)\right] r_D\,dr_D\,d\theta\,dz = 0 \qquad (15)$$

Integrating eq. 15 with respect to z, then θ, over the limits, we have:

$$\int_{r_{DA}}^{r_{DB}} w\left[\frac{\partial P_D}{\partial t_D} - \frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial P_D}{\partial r_D}\right)\right] r_D\,dr_D = 0 \qquad (16)$$

Eq. 16 can be exploded into:

$$\int_{r_{DA}}^{r_{DB}} w\frac{\partial P_D}{\partial t_D} r_D\,dr_D - \int_{r_{DA}}^{r_{DB}} w\frac{\partial}{\partial r_D}\left(r_D\frac{\partial P_D}{\partial r_D}\right)dr_D = 0 \qquad (17)$$

Integrating eq. 17 by part, we have:

$$\int_{r_{DA}}^{r_{DB}} r_D\frac{\partial w}{\partial r_D}\frac{\partial P_D}{\partial r_D}dr_D - w\left[r_D\frac{\partial P_D}{\partial r_D}\right]_{r_{DA}}^{r_{DB}} + \int_{r_{DA}}^{r_{DB}} r_D w\frac{\partial P_D}{\partial t_D}dr_D = 0 \qquad (18)$$

Grouping eq. 18 into linear and bilinear components, we have:

$$\int_{r_{DA}}^{r_{DB}} r_D\frac{\partial w}{\partial r_D}\frac{\partial P_D}{\partial r_D}dr_D + \int_{r_{DA}}^{r_{DB}} r_D w\frac{\partial P_D}{\partial t_D}dr_D - w\left[r_D\frac{\partial P_D}{\partial r_D}\right]_{r_{DA}}^{r_{DB}} = 0 \qquad (19)$$

$$\int_{r_{DA}}^{r_{DB}} r_D \frac{\partial w}{\partial r_D}\frac{\partial P_D}{\partial r_D} dr_D + \int_{r_{DA}}^{r_{DB}} r_D w \frac{\partial P_D}{\partial t_D} dr_D - wQ_A - wQ_B = 0 \quad (20)$$

Where $Q = r_D \dfrac{\partial P_D}{\partial r_D}$

Interpolation Function: The weak form in eq. 20 requires that the approximation chosen for P_D should be at least quadratic in r_D so that there are no terms in eq. 20 that are identically zero. Since the primary variable is simply the function itself, the Lagrange family of interpolation functions is admissible. We proposed that P_D is the approximation over a typical finite element domain by the expression:

$$P_D(r_D, t_D) = \sum_{j=1}^{n} P_{Dj}(t_D)\psi_j^e(r_D) \quad and \quad w = \psi_i^e(r_D) \quad (21)$$

Substituting eq. 21 into eq. 20, we have:

$$\int_{r_{DA}}^{r_{DB}} r_D \frac{d\psi_i^e}{dr_D}\frac{\partial}{dr_D}\sum_{j=1}^{n} P_{Dj}(t_D)\psi_j^e(r_D) dr_D + \int_{r_{DA}}^{r_{DB}} r_D \psi_i^e \frac{d}{dt_D}\sum_{j=1}^{n} P_{Dj}(t_D)\psi_j^e(r_D) dr_D - Q_i^e = 0 \quad (22)$$

Factor out $\sum_{j=1}^{n} P_{Dj}$

$$\sum_{j=1}^{n} P_{Dj}\int_{r_{DA}}^{r_{DB}} r_D \frac{d\psi_i^e}{dr_D}\frac{d\psi_j^e}{dr_D} dr_D + \sum_{j=1}^{n} \dot{P}_{Dj}\int_{r_{DA}}^{r_{DB}} r_D \psi_i^e \psi_j^e dr_D - Q_i^e = 0 \quad (23)$$

Where $\dot{P}_{Dj} = \dfrac{dP_D}{dt_D}$

In matrix form we can represent the semi-discrete finite element model thus,

$$[K_{ij}^e]\{P_D\} + [M_{ij}^e]\{\dot{P}_{Dj}\} = \{Q_i^e\} \quad (24)$$

Where

$$K_{ij}^e = \int_{r_{DA}}^{r_{DB}} r_D \frac{d\psi_i^e}{dr_D}\frac{d\psi_j^e}{dr_D} dr_D \quad (25)$$

$$M_{ij}^e = \int_{r_{DA}}^{r_{DB}} r_D \psi_i^e \psi_j^e dr_D \quad (26)$$

T*ime Approximation:* Recalling eq. 24,

Using Quadratic Lagrange Interpolation functions for a quadratic element:

$$\psi_1(r) = \frac{1}{h^2}(h + r_A - r)(h - 2r + 2r_A) \quad (27)$$

$$\psi_2(r) = \frac{4}{h^2}(r - r_A)(h + r_A - r) \quad (28)$$

$$\psi_3(r) = \frac{-1}{h^2}(r - r_A)(h - 2r + 2r_A) \quad (29)$$

The coefficient matrix can be easily derived by substituting the Lagrange interpolation functions into eq. 25 respectively. The matrices are shown below:

$$[K^e] = \frac{1}{6h}\begin{bmatrix} 3h+14r_A & -(4h+16r_A) & h+2r_A \\ -(4h+16r_A) & 16h+32r_A & -(12h+16r_A) \\ h+2r_A & -(12h+16r_A) & 11h+14r_A \end{bmatrix} \quad (30)$$

Also, the mass matrices can be easily derived by substituting the Lagrange interpolation functions into eq. 27 respectively. The matrices are shown below:

$$[M^e] = \frac{h}{60}\begin{bmatrix} h+8r_A & 4r_A & -h-2r_A \\ 4r_A & 16h+32r_A & 4h+4r_A \\ -h-2r_A & 4h+4r_A & 7h+8r_A \end{bmatrix} \quad (31)$$

Using four quadratic elements,

$$r_A = r_w + (n-1)h \quad (32)$$

In this analysis, we have withheld the computational details of the shape assembly of the finite element analysis (FEA) used. However, the authors would be glad to interact with researchers who may want to refer to the computational mathematics involved.

$$\left[K_{ij}^{e}\right]\left\{P_D\right\}+\left[M_{ij}^{e}\right]\left\{\dot{P}_{Dj}\right\}=\left\{Q_i^{e}\right\}$$

For a given time step s, eq. 24 becomes

$$\left[K_{ij}^{e}\right]\left\{P_D\right\}_s+\left[M_{ij}^{e}\right]\left\{\dot{P}_{Dj}\right\}_s=\left\{Q_i^{e}\right\}_s \quad (33)$$

For the next time step s+1, eq. 25 becomes

$$\left[K_{ij}^{e}\right]\left\{P_D\right\}_{s+1}+\left[M_{ij}^{e}\right]\left\{\dot{P}_{Dj}\right\}_{s+1}=\left\{Q_i^{e}\right\}_{s+1} \quad (34)$$

Multiply eq. 33 by $(1-\alpha)$ and eq. 34 by α , then we add the two resulting equations,

$$\left[M_{ij}^{e}\right]\left[(1-\alpha)\left\{\dot{P}_{Dj}\right\}_s+\alpha\left\{\dot{P}_{Dj}\right\}_{s+1}\right]+\left[K_{ij}^{e}\right]\left[(1-\alpha)\left\{P_{Dj}\right\}_s+\alpha\left\{P_{Dj}\right\}_{s+1}\right]=(1-\alpha)\left\{Q_i^{e}\right\}_s+\alpha\left\{Q_i^{e}\right\}_{s+1} \quad (35)$$

The α family of interpolation for time consideration is given as:

$$(1-\alpha)\left\{\dot{P}_{Dj}\right\}_s+\alpha\left\{\dot{P}_{Dj}\right\}_{s+1}=\frac{\left\{P_{Dj}\right\}_{s+1}-\left\{P_{Dj}\right\}_s}{\Delta t_{s+1}} \quad (36)$$

Substitute eq. 36 into eq. 35 and using the Crank-Nicholson Scheme where $\alpha = \frac{1}{2}$,

$$\left[\left[M_{ij}^{e}\right]+\frac{\Delta t_{s+1}}{2}\left[K_{ij}^{e}\right]\right]\left\{P_{Dj}\right\}_{s+1}=\left[\left[M_{ij}^{e}\right]-\frac{\Delta t_{s+1}}{2}\left[K_{ij}^{e}\right]\right]\left\{P_{Dj}\right\}_s+\frac{\Delta t_{s+1}}{2}\left[\left\{Q_i^{e}\right\}_s+\left\{Q_i^{e}\right\}_{s+1}\right] \quad (37)$$

From the initial condition given in eq. 9 for a constant terminal rate case, it implies that when $s = 0$, i.e., initial time, all dimensionless pressure in the reservoir will be zero. Also, the flow rate was constant all through operation. This means that $\left\{Q_i^{e}\right\}_s=\left\{Q_i^{e}\right\}_{s+1}$ .Hence, eq. 38 becomes:

$$\left[\left[M_{ij}^{e}\right]+\frac{\Delta t_1}{2}\left[K_{ij}^{e}\right]\right]\left\{P_{Dj}\right\}_1=\left[\left[M_{ij}^{e}\right]-\frac{\Delta t_1}{2}\left[K_{ij}^{e}\right]\right]\left\{P_{Dj}\right\}_0+\Delta t_1\left\{\bar{Q}_i^{e}\right\} \quad (38)$$

Where $\bar{Q}_i^{e}=\frac{1}{2}\left(Q_i^{1}\right)_{s+1}+\frac{1}{2}\left(Q_i^{1}\right)_s$

$$\left\{P_{Dj}\right\}_1=\left[\left[M_{ij}^{e}\right]+\frac{\Delta t_1}{2}\left[K_{ij}^{e}\right]\right]^{-1}\left[\left[\left[M_{ij}^{e}\right]-\frac{\Delta t_1}{2}\left[K_{ij}^{e}\right]\right]\left\{P_{Dj}\right\}_0+\Delta t_1\left\{\bar{Q}_i^{e}\right\}\right] \quad (39)$$

RESULTS AND DISCUSSION

This condition is applicable to bounded reservoirs. This is to say that the reservoir has been producing for a sufficient period of time so that the effect of the pressure disturbance has been felt in the outer boundary. In that case, the influence of the reservoir boundaries or the shape of the drainage area has effect on the rate at which the pressure disturbance spreads in the reservoir. It is therefore considered that the well acts as if it is surrounded at its outer boundary, by a solid "brick wall" which prevents the flow of fluids into the radial cell of the reservoir. This “brick wall” can either be in the form of a fault bringing about variation in the permeability of the walls of the reservoir or a high degree of anisotropy.

Furthermore, if the well is producing at a constant flow rate then the cell pressure will decline in such a way that the change in pressure with time will be approximately costant for all radius and time.

Eq. 39 was analysed to determine the various results obtained in the course of this study. The results obtained from this analysis have been presented in the form of tables of dimensionless pressure against dimensionless time. This is shown in Table 1. This table comprises of different radial values of dimensionless radius. From the Table 1, it was observed that the graph did not start from the point where the dimensionless time was zero. The reason was that when the reservoir is opened, the reservoir will act as if it was infinite in size. At these points, the behaviour of the reservoir is different from its behaviour when it was finite in size. Therefore, these regions were not captured in this part of the analysis. When the pressure disturbance in now felt by the external boundary of the formation, the reservoir at this point behaves as if there is no flow of fluid into it. As this happens, the pressure in the reservoir begins to drop. These results presented in Table 1 are the dimensionless pressure at the well bore of the reservoir.

Table 1: Dimensionless time against dimensionless pressures and different dimensionless radii

reD=1.5		reD=2		reD=2.5		reD=3		reD=3.5		reD=4		reD=4.5	
tD	PD	tD	PD	tD	PD	tD	PD	tD	PD	tD	PD	tD	PD
0.06	0.2500	0.22	0.4430	0.40	0.5650	0.52	0.6260	1.00	0.8020	1.5	0.9270	2.0	1.0220
0.08	0.2880	0.24	0.4600	0.42	0.5760	0.54	0.6360	1.10	0.8310	1.6	0.9480	2.1	1.0390
0.10	0.3220	0.26	0.4760	0.44	0.5870	0.56	0.6450	1.20	0.8570	1.7	0.9680	2.2	1.0550
0.12	0.3550	0.28	0.4920	0.46	0.5980	0.60	0.6620	1.30	0.8830	1.8	0.9880	2.3	1.0710
0.14	0.3870	0.30	0.5070	0.48	0.6080	0.65	0.6830	1.40	0.9070	1.9	1.0060	2.4	1.0860
0.16	0.4200	0.32	0.5220	0.50	0.6180	0.70	0.7030	1.50	0.9300	2.0	1.0240	2.5	1.1010
0.18	0.4520	0.34	0.5370	0.52	0.6280	0.75	0.7220	1.60	0.9520	2.2	1.0590	2.6	1.1160
0.20	0.4840	0.36	0.5510	0.54	0.6380	0.80	0.7400	1.70	0.9740	2.4	1.0920	2.7	1.1300
0.22	0.5160	0.38	0.5650	0.56	0.6470	0.85	0.7570	1.80	0.9940	2.6	1.1230	2.8	1.1440
0.24	0.5480	0.40	0.5790	0.58	0.6570	0.90	0.7740	1.90	1.0150	2.8	1.1540	2.9	1.1570
0.26	0.5800	0.42	0.5930	0.60	0.6660	0.95	0.7900	2.00	1.0350	3.0	1.1830	3.0	1.1700
0.28	0.6120	0.44	0.6070	0.65	0.6880	1.00	0.8050	2.25	1.0830	3.5	1.2550	3.2	1.1960
0.30	0.6440	0.46	0.6210	0.70	0.7100	1.20	0.8650	2.50	1.1300	4.0	1.3240	3.4	1.2210
0.35	0.7240	0.48	0.6340	0.75	0.7310	1.40	0.9200	2.75	1.1770	4.5	1.3920	3.6	1.2450
0.40	0.8040	0.50	0.6480	0.80	0.7520	1.60	0.9730	3.00	1.2220	5.0	1.4590	3.8	1.2690
0.45	0.8840	0.60	0.7150	0.85	0.7720	2.00	1.0760	4.00	1.4010	5.5	1.5260	4.0	1.2920
0.50	0.9640	0.70	0.7820	0.90	0.7920	3.00	1.3280	5.00	1.5800	6.0	1.5930	4.5	1.3480
0.55	1.0440	0.80	0.8490	0.95	0.8120	5.00	1.8280	6.00	1.7570	6.5	1.6600	5.0	1.4030

Table 1: Contd

reD=5		reD=6		reD=7		reD=8		reD=9		reD=10	
tD	PD	tD	PD	tD	PD	tD	PD	tD	PD	tD	PD
3.0	1.1660	4.0	1.2730	6.0	1.4330	8.0	1.5500	10.0	1.6430	12.0	1.7190
3.1	1.1790	4.5	1.3200	6.5	1.4660	8.5	1.5770	10.5	1.6640	12.5	1.7370
3.2	1.1910	5.0	1.3620	7.0	1.4980	9.0	1.6010	11.0	1.6850	13.0	1.7550
3.3	1.2030	5.5	1.4012	7.5	1.5280	9.5	1.6250	11.5	1.7050	13.5	1.7720
3.4	1.2140	6.0	1.4390	8.0	1.5560	10.0	1.6480	12.0	1.7240	14.0	1.7880
3.5	1.2260	6.5	1.4750	8.5	1.5830	10.5	1.6700	12.5	1.7420	14.5	1.8040
3.6	1.2370	7.0	1.5090	9.0	1.6090	11.0	1.6910	13.0	1.7600	15.0	1.8200
3.7	1.2480	7.5	1.5420	9.5	1.6350	11.5	1.7120	13.5	1.7778	15.5	1.8350
3.8	1.2590	8.0	1.5740	10.0	1.6590	12.0	1.7320	14.0	1.7950	16.0	1.8500
3.9	1.2690	8.5	1.6050	11.0	1.7070	12.5	1.7510	14.5	1.8110	17.0	1.8780

4.0	1.2800	9.0	1.6360	12.0	1.7530	13.0	1.7710	15.0	1.8270	18.0	1.9050
4.2	1.3000	9.5	1.6660	13.0	1.7980	13.5	1.7890	16.0	1.8590	19.0	1.9310
4.4	1.3200	10.0	1.6960	14.0	1.8412	14.0	1.8080	17.0	1.8890	20.0	1.9570
4.6	1.3390	11.0	1.7550	15.0	1.8850	14.5	1.8260	18.0	1.9180	22.0	2.0050
4.8	1.3590	12.0	1.8130	16.0	1.9280	15.0	1.8440	19.0	1.9470	24.0	2.0520
5.0	1.3770	13.0	1.8710	17.0	1.9700	17.0	1.9130	20.0	1.9750	26.0	2.0960
5.5	1.4230	14.0	1.9290	18.0	2.0120	19.0	1.9800	22.0	2.0290	28.0	2.1400
6.0	1.4680	15.0	1.9860	19.0	2.0550	21.0	2.0460	24.0	2.0820	30.0	2.1830

The results obtained from this analysis were seen to agree with those already existing in literature. To test for the degree of accuracy, a percentage error between the FEM solutions and the Van Everdigen and Hurst solutions was conducted. The result shows a strong positive correlation between the two results.

As stated earlier, the results presented in Table 1 are the dimensionless pressure at the well bore at different dimensionless time. But when a pressure disturbance is created in a reservoir from the well bore, it is not only felt at the well bore but it travels through the entire reservoir to the external boundary. The dimensionless pressure at the external boundary tells us how well the reservoir is been recharged and how the reservoir pressure is been dropped. Therefore, it is important to know how this pressure disturbance affects other points of the reservoir at the same time. Thus, this analysis also presents change in dimensionless pressure with dimensionless time at different points within the reservoir at the same time. These are presented in Tables 2 to 3. It was observed that the dimensionless pressure decreases from the well bore to the external boundary of the reservoir. What this means in actual sense is that the actual pressure in the reservoir increases from the well bore to external boundary.

Table 2: Dimensionless Pressure Distribution for $r_{eD} = 1.5$, $n = 4$ and $\Delta t = 0.005$ under the "No Flow" outer boundary condition

rD / tD	1.0000	1.0625	1.1250	1.1875	1.2500	1.3125	1.3750	1.4375	1.5000
0.06	0.250	0.194	0.147	0.111	0.082	0.062	0.048	0.040	0.037
0.08	0.288	0.231	0.183	0.145	0.115	0.092	0.077	0.068	0.065
0.10	0.322	0.265	0.217	0.178	0.147	0.124	0.108	0.098	0.095
0.12	0.355	0.297	0.249	0.210	0.179	0.156	0.139	0.130	0.127
0.14	0.387	0.330	0.282	0.242	0.211	0.187	0.171	0.161	0.158
0.16	0.420	0.362	0.314	0.274	0.243	0.219	0.203	0.193	0.190
0.18	0.452	0.394	0.346	0.306	0.275	0.251	0.235	0.225	0.222
0.20	0.484	0.426	0.378	0.338	0.307	0.283	0.267	0.257	0.254
0.22	0.516	0.458	0.410	0.370	0.339	0.315	0.299	0.289	0.286
0.24	0.548	0.490	0.442	0.402	0.371	0.347	0.331	0.321	0.318
0.26	0.580	0.522	0.474	0.434	0.403	0.379	0.363	0.353	0.350
0.28	0.612	0.554	0.506	0.466	0.435	0.411	0.395	0.385	0.382
0.30	0.644	0.586	0.538	0.498	0.467	0.443	0.427	0.417	0.414
0.35	0.724	0.666	0.618	0.578	0.547	0.523	0.507	0.497	0.494
0.40	0.804	0.746	0.698	0.658	0.627	0.603	0.587	0.577	0.574
0.45	0.884	0.826	0.778	0.738	0.707	0.683	0.667	0.657	0.654
0.50	0.964	0.906	0.858	0.818	0.787	0.763	0.747	0.737	0.734
0.55	1.044	0.986	0.938	0.898	0.867	0.843	0.827	0.817	0.814
0.60	1.124	1.066	1.018	0.978	0.947	0.923	0.907	0.897	0.894
0.65	1.204	1.146	1.098	1.058	1.027	1.003	0.987	0.977	0.974
0.70	1.284	1.226	1.178	1.138	1.107	1.083	1.067	1.057	1.054
0.75	1.364	1.306	1.258	1.218	1.187	1.163	1.147	1.137	1.134
0.80	1.444	1.386	1.338	1.298	1.267	1.243	1.227	1.217	1.214

Table 3: Dimensionless Pressure Distribution for $r_{eD} = 10$, $n = 4$ and $\Delta t = 0.005$ under the "No Flow" outer boundary condition

rD / tD	1.0000	2.1250	3.2500	4.3750	5.5000	6.6250	7.7500	8.8750	10.0000
12.0	1.719	0.999	0.616	0.387	0.241	0.150	0.095	0.066	0.058
12.5	1.737	1.017	0.632	0.401	0.253	0.159	0.103	0.073	0.064
13.0	1.755	1.034	0.648	0.414	0.264	0.169	0.111	0.081	0.071
13.5	1.772	1.050	0.663	0.428	0.276	0.178	0.119	0.088	0.078
14.0	1.788	1.066	0.677	0.441	0.287	0.188	0.128	0.096	0.086
14.5	1.804	1.082	0.692	0.454	0.298	0.198	0.136	0.103	0.093
15.0	1.820	1.097	0.706	0.466	0.309	0.208	0.145	0.111	0.101
15.5	1.835	1.111	0.720	0.479	0.320	0.218	0.154	0.119	0.109
16.0	1.849	1.126	0.733	0.491	0.332	0.227	0.163	0.128	0.117
17.0	1.878	1.153	0.759	0.515	0.353	0.247	0.181	0.145	0.133
18.0	1.905	1.180	0.785	0.539	0.375	0.267	0.199	0.162	0.151
19.0	1.931	1.206	0.809	0.562	0.396	0.287	0.217	0.180	0.168
20.0	1.957	1.231	0.833	0.585	0.418	0.307	0.236	0.198	0.186
22.0	2.005	1.279	0.880	0.629	0.460	0.347	0.275	0.235	0.223
24.0	2.052	1.325	0.924	0.672	0.501	0.387	0.313	0.273	0.261
26.0	2.096	1.369	0.968	0.715	0.543	0.427	0.353	0.312	0.299
28.0	2.140	1.412	1.011	0.757	0.584	0.467	0.392	0.351	0.338
30.0	2.183	1.455	1.053	0.798	0.625	0.507	0.432	0.390	0.377
32.0	2.225	1.497	1.095	0.839	0.665	0.548	0.472	0.430	0.417
34.0	2.267	1.539	1.136	0.880	0.706	0.588	0.512	0.470	0.457
36.0	2.308	1.580	1.177	0.921	0.747	0.628	0.552	0.510	0.497
38.0	2.349	1.621	1.218	0.962	0.787	0.669	0.592	0.550	0.537
40.0	2.390	1.662	1.259	1.003	0.828	0.709	0.633	0.590	0.577
50.0	2.593	1.865	1.462	1.205	1.030	0.911	0.834	0.792	0.779
60.0	2.795	2.067	1.664	1.407	1.232	1.113	1.036	0.994	0.981
70.0	2.997	2.269	1.866	1.609	1.434	1.315	1.238	1.196	1.183
80.0	3.199	2.471	2.068	1.811	1.636	1.517	1.440	1.398	1.385
90.0	3.401	2.673	2.270	2.013	1.838	1.719	1.642	1.600	1.587
100.0	3.603	2.875	2.472	2.215	2.040	1.921	1.844	1.802	1.789

Conclusion: The analysis showed the pressure distribution across a bounded circular reservoir for the constant terminal rate case. It was shown from the "No Flow" boundary condition (Figs. 1 to 12) that the dimensionless pressure was seen to increase uniformly with dimensionless time. The results obtained from this analysis showed that there was a strong correlation with the results obtained from the Van Everdigen and Hurst. The Van Everdigen and Hurst solutions only state the pressure at the wellbore at a particular time but this work predicts the pressure variation in the entire reservoir from the wellbore to the external boundary at the same time. Therefore the Finite element method can be used to approximate the values of well pressures in the entire bounded circular reservoirs.

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