African Journal of Science and Technology (AJST)
Science and Engineering Series Vol. 12, No. 2, pp. 105 - 115

ANALYSIS OF THE WELD STRENGTH OF THE HIGH DENSITY POLYETHYLENE (HDPE) DAM LINER

Mbuge, D.O.[1], Gumbe, L.O.[1] and Rading G.O.[3]
[1]Department of Environmental and Biosystems Engineering
[2]Department of Mechanical and Manufacturing Engineering
University of Nairobi
P.O. Box 30917, Nairobi, Kenya.

Email: dmbuge@yahoo.com

ABSTRACT: *An analysis was carried out to determine the strength of welded joints in High Density Polyethylene (HDPE) dam liners. Samples were collected of welded joints and subjected to tensile tests and creep test. It was observed that the welded joints from field welded samples were much weaker and had a very low straining capacity than the un-welded material. The field welded specimens registered average strengths of 5 MPa as compared to 12 MPa of un-welded specimens and a maximum strain at fracture of 0.5 as compared to 3.5 of un-welded specimens. As a result of the weak welds it was found necessary to conduct further research into the best welds possible and a protocol for hot air and hot knife welding was developed. It was shown that for hot air welding while holding the width of the weld, applied pressure and dwell time of the pressure constant at 2.6cm, 0.3 MPa and 60 s, the recommended temperature of the hot air to achieve the strongest weld was 176 and for hot knife welding the recommended temperature of the knife was 400°C*

INTRODUCTION

In the world today, there is a big shortage of potable water for domestic use as well as agricultural water for irrigation and this has led to much suffering for many people in the world [1, 2]. The most important factor that undermines the availability of water is the prohibitive cost of water storage structures. In systems that depend on rainwater harvesting for example, water security and availability lies squarely on how much water one can store [3]. This is because rainfall patterns are such that it rains unexpectedly, and the total seasonal rainfall is concentrated in two to three week episodes. Also, the commencement of the rains is unpredictable. It is therefore important for water users to collect as much water as they can during these short rainy seasons and store it for use in times when there is no rainfall. This calls for large water storage structures, which most water users cannot afford due to their high costs [3].

It is in recognition of this that there is need to develop cheaper alternatives for water storage. Already a lot of work has been done in this line which has led to the development of new products such as plastic tanks. A more recent introduction to Kenya is the plastic lined water reservoir for storage of water for both domestic and agricultural use. Using this technology, it is possible to store water at about one tenth of the cost of the conventional water storage structures such as ferro-cement tanks and plastic tanks. It also offers a unique opportunity to make collapsible tanks, which are vital in refugee camps and other conditions, which involve temporary settlement [3].

One of the more popular materials used in the fabrication of these plastic lined reservoirs is High Density Polyethylene (HDPE). It is normally extruded from the factories as 50 m long and 2 m. wide sheets and needs to be welded to cover the reservoir. The welded joints were observed to be a common failure point and there was need to study them so as to find out the problems and recommend remedies.

A linear polymer, HDPE is prepared from ethylene by a catalytic process. The absence of branching results in a more closely packed structure with a higher density and somewhat higher chemical resistance than LDPE. HDPE is also harder and more opaque than LDPE. It has a melting point that ranges from 130°C to 137°C, a maximum continued use temperature of 65°C and a glass transition temperature of between -110°C to -125°C. It has a density of about 0.941 - 1.45 g/cm^3, a coefficient of thermal expansion of 100 - 220 x 10^{-6} per °C, a tensile strength of 15 - 40 MPa, a flexural modulus of 1.2GN/m^2, Young's modulus of 0.7 GN/m^2 , Brinnel hardness of 2 and an elongation of 150%-500% at failure [4, 5, 6, 7]When used for lining reservoirs, HDPE sheeting does not require soil cover unlike the other materials which are readily degraded if not covered. Installation is generally undertaken using fusion-weld joining equipment. Thickness range is 0.4 to 2.5 mm. It would be used on sites where puncturing of cheaper products cannot be avoided or where steep slopes (steeper than 2:1) preclude the use of other products [8].

HDPE is the most widely used geomembrane in the world and is used more commonly internationally due to its availability and relatively low material cost. HDPE is an excellent product for large dam applications that require UV and ozone resistance, chemical resistance or high-quality installations. The sheets are delivered in large rolls and may be heat welded in the field by trained technicians. This product has been used in landfills, wastewater treatment lagoons, animal waste lagoons, mining applications and for water storage [8].

Welding in polymers is done in order to produce seals or joints of sufficient strength to survive the work environments. When these polymer surfaces are brought into intimate contact and are partially molten, a bond is achieved. In order to achieve a realistic bond, the surfaces are pressed together for a period of time sufficient for the polymer chains to diffuse across the interface and form connecting bridges. The formation of these bridges is a function of the material being welded and at a given thickness is a function of the material composition, average molecular weight, molecular weight distribution, and thermal conductivity [9].

A weld seal is usually obtained by sealing like materials together. Weld seals are based on obtaining the strongest seal such that the material fails before the seal. The method of sealing, in particular how heat sealing is achieved, can be carried out by using different techniques like jaw-type bar sealers, rotary sealers, band rotary sealers, impulse sealers, bead sealers, hot knife, or side weld sealers. The basic process involves the welding of two polymer films when forced into intimate contact while they are in at least their semi-molten state. The two main sealing parameters that affect the heat seal quality are the interfacial temperature and sealing time. Therefore, in order to assess effectiveness of a welded joint in a polymeric material, it is very important to determine the interfacial temperature and heat sealing time that will result in a desirable seal. Many instruments have been developed, but no instrument or technique is capable of assessing these conditions yet [9].

The guiding principles of creating welded joints have been presented in part by some references [9]. They stated that when two pieces of a plastic film are heat sealed, there is an inflection point in the sealing time–temperature profiles of the materials. The inflection point occurs at a temperature below the melting point of the materials as measured by the Differential Scanning Calorimeter (DSC) technique. It was also found that pressure has limited effect on the sealing properties of the sealed films. The highest peel seal strength is achieved at a temperature near the fusion point. Seals made above the fusion point result in a weld seal that is not separable by the peel test [9].

Three main design guidelines that should be considered when designing welded or adhesive joints as [10]:

1. Maximizing Shear and minimizing peel and cleavage
2. Maximizing compression and minimizing tensile pull
3. Joint width is more important than overlap length and as a general rule it would be more feasible to increase the joint width rather than the overlap length.

Adhesives have been used successfully in joining plastics. However, the use of adhesives for joining HDPE sheets rather than welding is deemed as difficult, unsuccessful or complicated. Because of their non-polar nature, polyethylene requires an oxidation treatment using flames, plasma treatment, or chemical etching to enhance the adhesive bond). Some references have listed HDPE as one of the hard-to-bond plastics alongside LDPE, polypropylene and Teflon [10, 11]. The use of adhesives for joining HDPE was therefore not considered in this research study.

The broad objective of the study was to determine the expected lifespan of High Density Polyethylene (HDPE) and the specific objective was to determine the effect of welding HDPE on the expected lifespan.

MATERIALS AND METHODS

The specimens used in this project were obtained from HDPE plastic dam liner manufactured by A-Plus Ltd. in Nairobi. The material is available commercially in four classes with regard to the thickness ranging from 0.5 mm to 1.2 mm. The material used in the tests was 0.8mm thick, manufactured at 15–30 MPa extrusion pressure, the die temperature ranging from 131 °C to 139 °C and a rotation speed of 15 rpm. It had a melting temperature of 135 °C. The rest of the dimensions are as shown in Figure 1 as per [12] for Type II specimens.

There were three sets of samples. The first batch consisted of samples with welded joints of width 1.3 cm. Collected from 5 reservoirs made from the HDPE lining material from the same supplier of equal thickness of 8 mm, installed within one week of the test. Test specimens were then prepared from these samples such that the welded section fell in the middle of the narrow section of the dumbbell shaped test piece. These specimens were then subjected to the tensile test as well as the creep test at different expected operating temperatures. It was ascertained that all the samples had been welded using the hot air technique. The samples under this category were called 'field welded' samples.

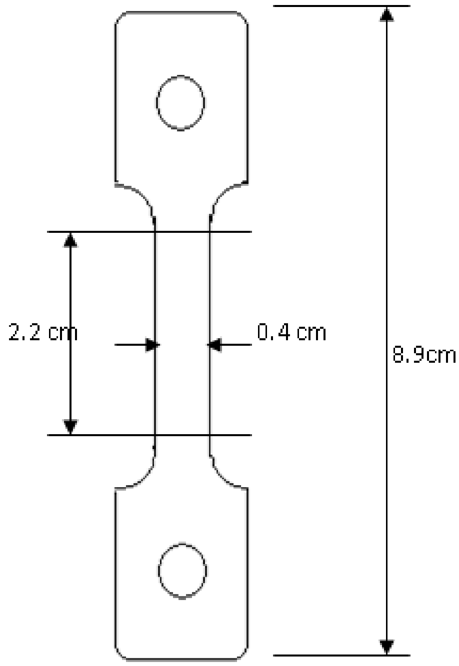

Figure 1: Specimen Geometry – Die Type II Dimensions used in the research study

The second batch of samples was obtained from un-welded pieces from the same reservoirs used to obtain the first batch of samples. Hence the punched dumbbell specimen did not have a welded joint and were called 'un-welded' samples.

The third batch of samples was obtained from the same reservoirs as in the first and second batch and did not have any welded joints, but was welded under laboratory conditions to obtain the best weld possible. They were called 'laboratory welded' samples. To obtain 'laboratory welded' specimens, the joints were first welded on large pieces and then the required specimens were punched at random locations along the welded joint. The punching was done so that the welded section fell at the centre of the narrow section of the dumbbell specimen.

The specimens from all three categories were then tested in tensile creep at temperatures of 30°C, 40°C and 50°C and stresses of 0.78MPa, 0.94MPa and 1.56MPa. The methods used in these test was similar to that described by [9]. All the welded joints were lap joints. Two popular field welding techniques namely the hot air and hot-knife techniques were used for the 'laboratory welded' specimens. The pressure applied on the welded joint and the dwell time of the pressure were held constant at 0.3MPa and 10s respectively from [9]. A variation was also made in the width of the welded joint from the 'field welded' dimension of 1.3 cm. to 2.6 cm. The tensile and creep tests were done on the specimens.

Hot air welding was simulated in the laboratory by heating the oven to the desired temperature. The specimens to be welded were prepared by covering the sections not to be melted with flat wood as insulation. The specimens were then placed in the oven for 10 s. The sections to be joined were placed together and pressure applied.

The hot knife welding was done by incubating flat bars of width 1.3 cm. and 2.6 cm. in the oven at a given temperature for 20 minutes. Two pieces of the material to be welded were placed to form a welded joint and the hot flat bar passed in the lap joint at the rate of 1cm/s for a weld piece of 10 cm. length. Pressure was then applied for 10 s. Specimens for subsequent tests were then prepared with a dumbbell cutter, with the welded section at the centre of the specimen.

RESULTS AND DISCUSSIONS

Tensile Test Results

It was necessary to compare the strength parameters of field welded specimens and un-welded specimens. The tensile tests carried out on the various types of specimens yielded the results presented in Table 1. All the experiments were carried out at a room temperature of 23ºC and relative humidity of 50% on specimens subjected to hot air welding. The tensile strength and the maximum strain at fracture were the main parameters for comparing the welded specimens and specimens without welded joints.

Table 1: Results for tensile test

Specimen Type	Tensile strength (MPa)	Maximum Strain (mm/mm)
Un-welded specimen	12	3.5
Field Welded with 1.3 cm welded joint	5	1.2
Field Welded with 2.6 cm welded joint	11	0.5

* Each result is the average of 5 specimens

By this test it was confirmed that indeed the welded joint is a weak point in the dam liner assembly not just by the figures but by the fact that all the failures occurred either on the welded joint or near it. Placing the un-welded joint as the standard, it can be seen from Table 1 that all the welded joints fell far short of this standard in both the strength and the maximum strain at fracture, both of which are important parameters in the operation of dam liners and flexible tanks. From the results, it was interesting to note that increasing the width of the weld more than doubled the tensile strength from 5 MPa. to 11 MPa. However, the effect on the maximum strain at fracture was the opposite, decreasing from 1.2 to 0.5. It may thus be concluded that in general, increasing the welding width of this material increases the strength of the welded joint but reduces its elasticity. It is possible that during welding, which involves a rapid temperature increase in the prescence of oxygen, certain reactions occur to modify the material on and around the welded joint. Hence a lager welded section is stronger but has a larger section rendered inelastic, with reduced straining capacity. In practical terms dam liners and other applications using this product should be designed on the asis of strength and strain values.

Creep Data for Un-welded Specimens

Curves in Figure 2(a) through Figure 2(c) are the creep curves obtained for un-welded specimens (material without a welded joint) tested under three different stresses at three different temperatures of 30ºC, 40ºC and 50ºC. From these results, the set of master curves presented in Figure 2 (d) was developed according to the principles of time-temperature superposition presented by many authors such as [13, 14, 15]. These corves are the benchmarks against which the creep curves of the field welded specimens, presented in Figure 3 (a) through Figure 3 (c) would be compared.

Creep Curves for Field Welded Specimens

Tests were done to determine the behaviour of test pieces with machine welded joints in tensile creep over time. In all instances the specimens obtained from the field failed at the welded joint. The failure was observed to result from shear between the welded surfaces. The curves of strain versus time for welded specimens under tensile creep are presented in Figure 3 (a - c).

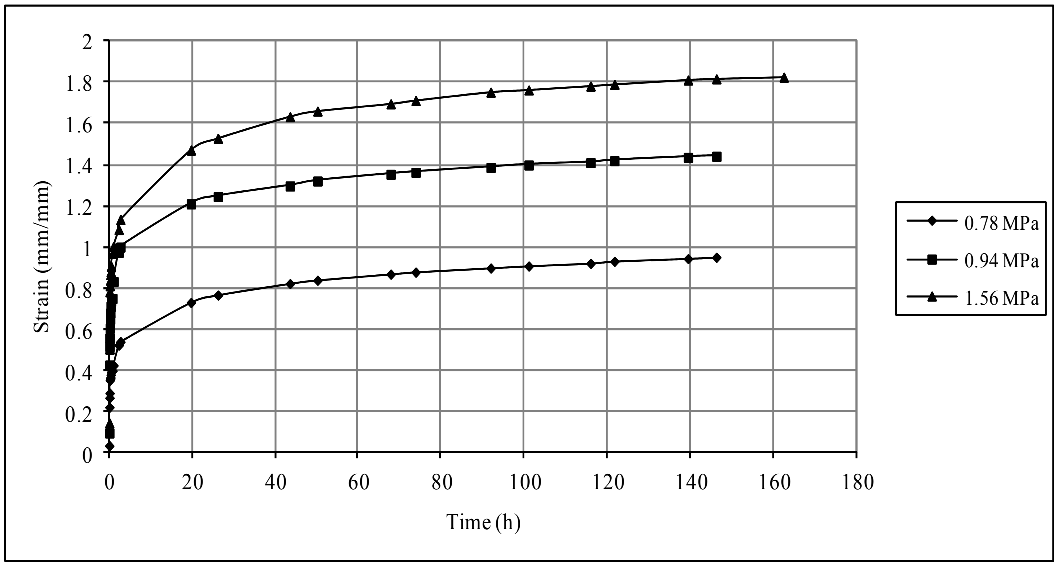

Figure 2 (a): Constant Temperature Creep Curves at 30°C

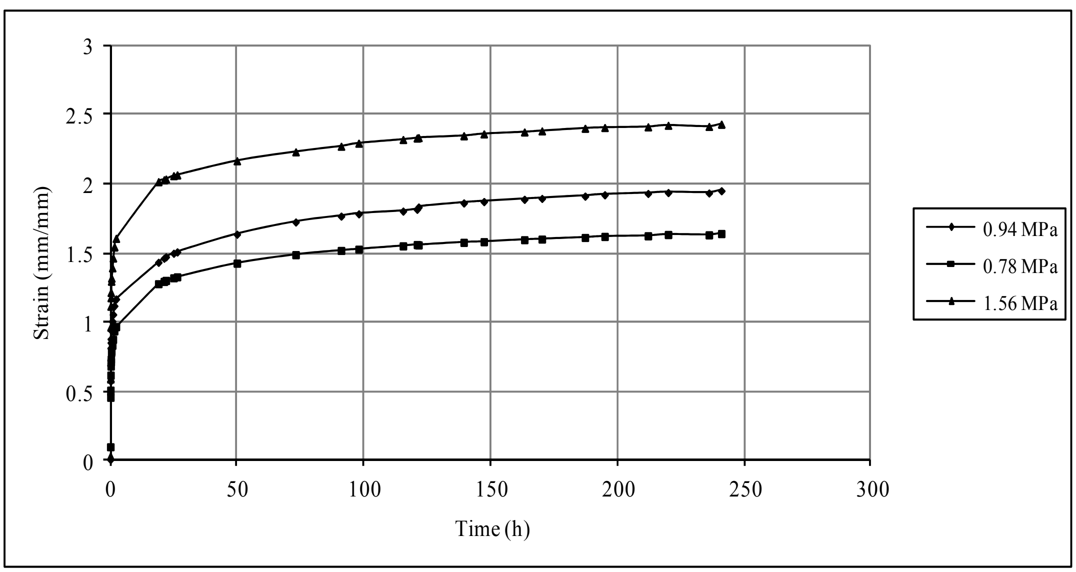

Figure 2 (b): Constant Temperature Creep Curves at 40°C

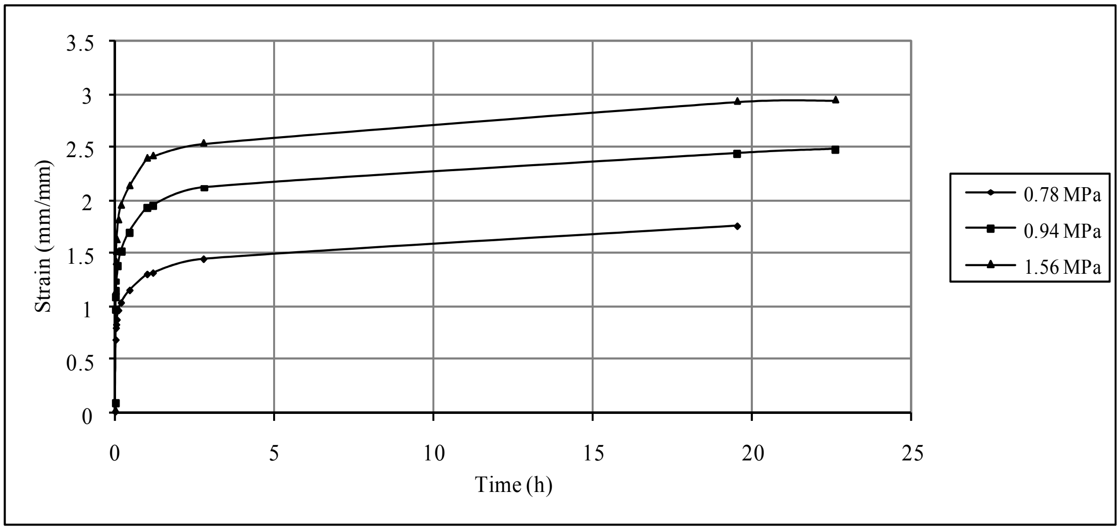

Figure 2 (c): Constant Temperature Creep Curves at 50°C

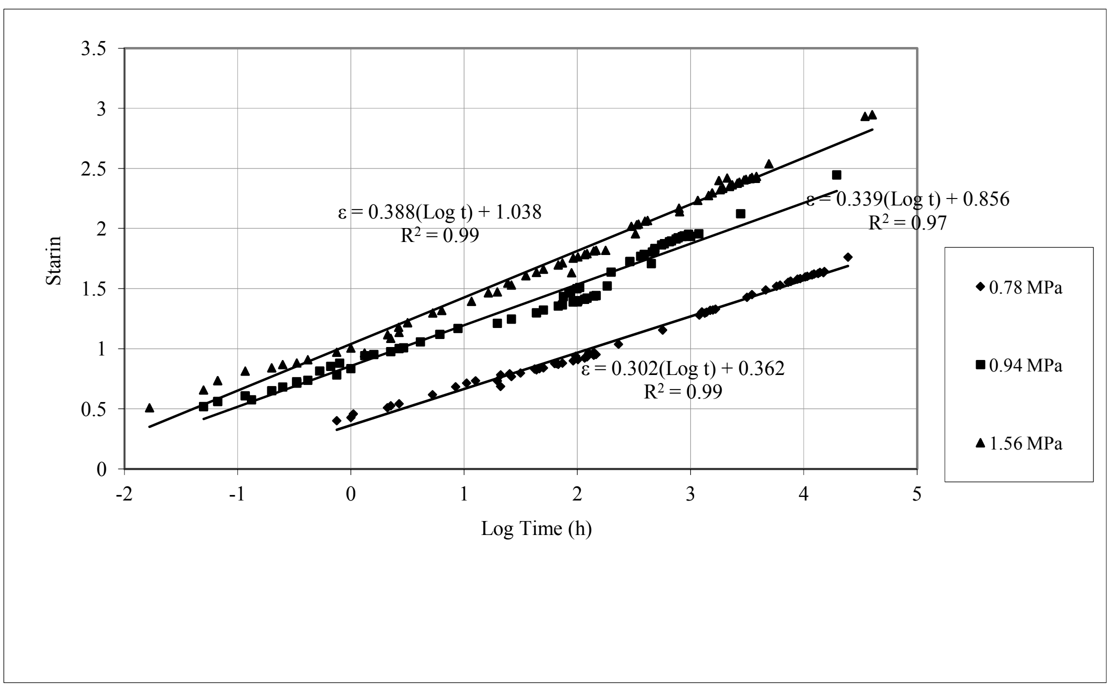

Figure 2 (d): Consolidated graph of master curves at 0.78 MPa, 0.94 MPa and 1.56 MPa.

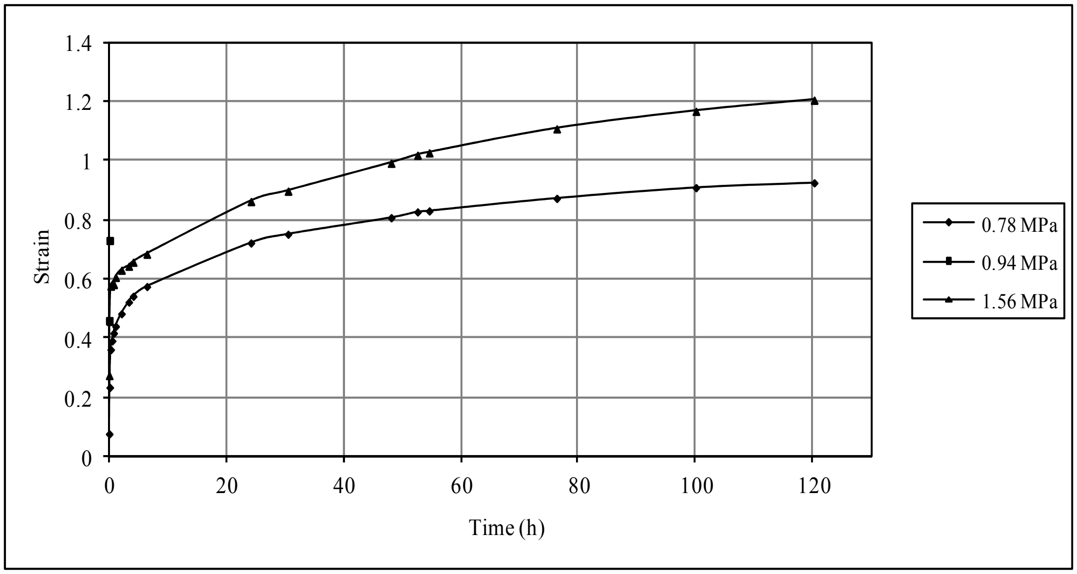

Figure 3 (a): Constant temperature hot-air machine welded specimen creep curves at 30ºc

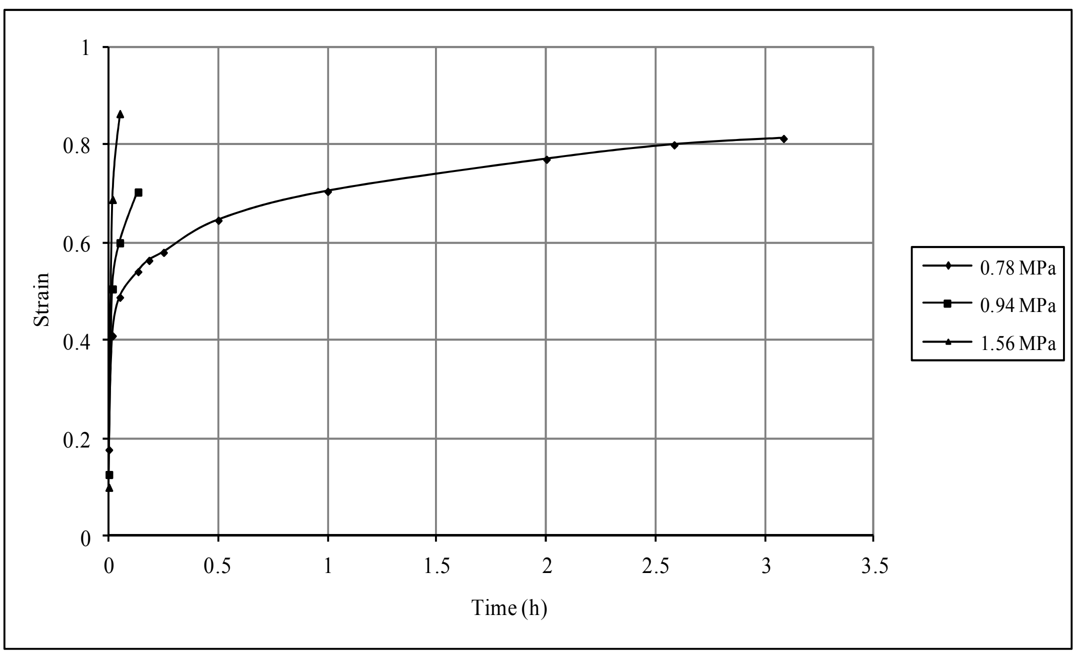

Figure 3 (b): Constant temperature hot-air machine welded specimen creep curves at 40ºC

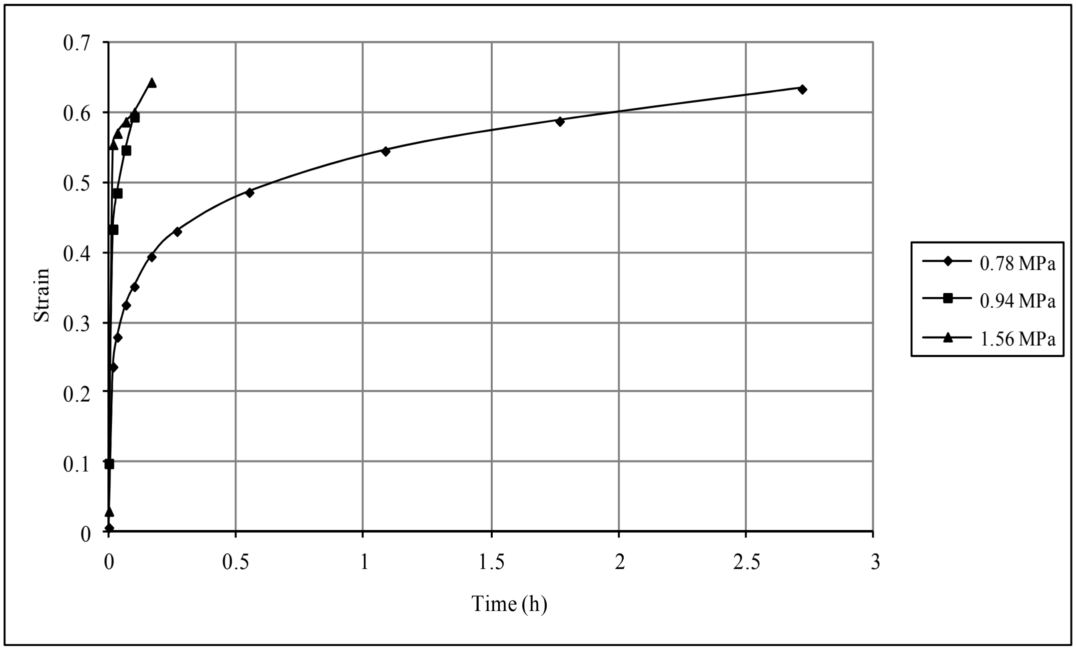

Figure 3 (c): Constant temperature hot-air machine welded specimen creep curves at 50ºC

Table 2: Summary of welded joint maximum strains at fracture, time to failure and point of failure

Temperature (ºC)	Stress applied (MPa)	Maximum strain obtained	Time to failure (h)	Point of failure
30	0.78	0.9	120	Welded joint
	1.56	1.2	120	Welded joint
40	0.78	0.8	3.1	Welded joint
	0.94	0.7	0.2	Welded joint
	1.56	0.9	0.1	Welded joint
50	0.78	0.6	2.7	Welded joint
	0.94	0.6	0.1	Welded joint
	1.56	0.6	0.2	Welded joint

The figures 3 (a – c) represent creep curves for field welded (1.3 cm width of weld) specimens subjected to the creep test at 30*℃*, 40*℃* and fresh specimens at 50*℃* respectively, all of which had been welded in the field. Table 2 gives a summary of the results obtained from figures 3 (a – c) as well as visual observations made. It was observed in all cases considered (Table 3) that the welded joint is generally a weak point. In all the experiments done, it was observed that the specimens failed on the welded joint and in much shorter times than in other experiments with clear specimens, with the highest time recorded being 120 hours and many of the specimens failing below 3 hours. The failure in all cases was in shear at the welded joint. This was an indication that in most of the specimens, complete fusion had not been attained during the initial welding.

In all the curves, failure in welded specimens was seen likely to occur as the applied stress was increased. In allthe curves, those specimens that were stressed at 0.78MPa persisted much longer than those loaded at higher stresses. The highest strain level attained for any welded specimen was 1.2. When this strain level (see dotted line in Figure 2 (d)) was superimposed on the master curves it was found that the material would fail at a maximum of 1 year (10,000 hours) when loaded at 0.78 MPa and a minimum of less than one hour when loaded at 1.56 MPa. It is clear that the welded joint is definitely a weak point in the installed dam liner exposed to tensile stresses.

With such high failure rates in welded joints, an alternative joining method was sought. But from literature it was seen that it is even more difficult to join HDPE by adhesives than by welding due to the non-polar nature of polyethylene [9]. Polyethylene is in fact listed in many sources as one of several plastics that cannot easily be joined by adhesives.

Literature points to strengths of up to 80% in the welded joints, it seems that the reason for such low strains in samples obtained from the field is poor workmanship that resulted in weak welded joints [9]. The feasible remedy is therefore factory welding of the dam liner before transportation to the field for installation. Alternatively, field welding may be done under controlled conditions particularly with regard to the welding temperature to ensure that the material is in a semi-molten state during the operation, the time and magnitude of pressure applied. It was also noted that the width of the welded joints obtained from the field was relatively small, about 1.3 cm. This had been listed in literature as another constraining factor on the strength of the welded joint apart from welding temperature, applied pressure and pressure dwell time [9].

Further tests were done in an attempt to find the conditions under which the strongest weld may be attained in HDPE. In the tests, the following parameters, earlier investigated by other researchers and found not to be significant beyond certain reasonable levels were held constant:

- Rate of heat application at 1cm/s for a width of 2.6 cm. and length of 10 cm. for hot knife welding and 10 s. of heating of the surfaces to be welded in the oven for hot air welding.
- Magnitude of pressure applied at 0.3 MPa. Pressure higher than 0.3 MPa was observed to deform the specimen.
- Dwell time of the applied pressure at 60 seconds followed by natural cooling to room temperature.

The results for hot air welding while holding the width of the weld, applied pressure and dwell time of the pressure constant at 2.6cm, 0.3 MPa and 60 s. respectively are presented in Table 3.

Table 3: Effect of temperature on the strength of hot air welded joints

Temperature (ºC)	**Peel test result (Failure strength and description)**	**Conclusion**
135	0.5 MPa - Readily failed at the welded lap joint in shear	Despite being the welding point, a weak weld is formed with incomplete fusion. There is need to increase temperature.
150	1 MPa - Readily failed at the welded lap joint in shear	Weak weld with incomplete fusion
160	1 MPa - Readily failed at the welded lap joint in shear	Weak weld with incomplete fusion, slightly stronger than previous welds
165	2 MPa - Readily failed at the welded lap joint in shear	Weak weld with incomplete fusion, still stronger than previous welds
170	5 MPa - Failed at the welded lap joint in shear	Strong weld but still with almost complete fusion
175	9 MPa - Failed at the welded joint in shear	Strong weld but still with almost complete fusion. There was now need to increase the temperature by units of 1 ºC
176	11 MPa - Failure outside the welded joint, while the welded joint remained intact for five specimens tested	Complete fusion was attained at this temperature
177	The exposed section of the material melted very fast and was easily distorted. No readings were taken from the damaged specimen.	It was difficult to obtain a weld at this temperature
178	The exposed section of the material melted very fast and was easily distorted	Any further increase in temperature would only cause distortion in the material structure due to excess heat and the tests were stopped

Table 4: Effect of temperature on the strength of hot knife welded joints

Temperature (ºC)	Peel test result (MPa)	Important observations	Conclusion
135(mp)- 350	1.5 (Average)	Readily failed at the welded joint in shear. Almost no melting of the material. The knife tended to stick to the material and caused distortion.	While the knife has been heated to the material melting point, a lot of the heat is lost to raise the temperature of the material from room temperature to its melting point. The knife must be significantly hotter than the melting point of the material.
360	2	Failed at the welded lap joint in shear Stickiness of the knife still experienced	Weak weld with incomplete fusion
370	2	Failed at the welded lap joint in shear Stickiness of the knife still experienced	Weak weld with incomplete fusion
380	5	Failed at the welded lap joint in shear Stickiness of the knife still experienced	Weak weld with incomplete fusion
390	8	Failed at the welded lap joint in shear slight stickiness experienced	Strong weld with almost complete fusion. There was now need to increase the temperature by units of 1 ºC
400	11	Failed outside the lap joint. No stickiness of the knife experienced	Failure outside the welded joint indicates a strong weld. This was the best result that was obtained.
401	-	Beyond 400°C the material began to burn and the surface was too damaged to be welded	It was not possible to obtain a results beyond 400C

In hot air welding, particular attention was given to the width of the weld, which was doubled to 2.6 cm. It was also necessary to find the temperature of air that would cause the material to melt. Temperature was progressively increased from 135 °C (the average melting point of HDPE) upwards with very weak welds obtained up to 176 °C. At 176 °C, it was found that there formed enough welded material to form a strong joint within at the constant conditions specified. This was higher than the melting point of the plastic, (135 °C) to supply latent heat required for melting in a relatively short time of 10 s. and due to the inefficiency of convection heat transfer from the hot air to the plastic. Any temperature below 176°C was not capable of melting enough material to obtain the required fusion at the specified constant conditions. The simple peel test suggested by Aithani et al (2006) was used to determine whether the joint formed was strong enough. Failure (peeling) at the welded joint suggested a weak joint and failure away from the welded joint denoted a strong joint where complete fusion had occurred. In all the tested specimens prepared by hot air fusion at 176°C, failure occurred outside the welded joint.

The hot knife welding technique was also tested. The detailed results for the tests on the hot air welding technique are presented in Table 4.7. A flat bar of width 2.6 cm was heated in an oven to a constant temperature and used to weld two flaps of the lap joint together by sliding the flat bat between them at high temperature. The flat bar was incubated in an oven to acquire the required working temperature before being used for welding. It was found that at temperatures below 350°C of the flat bar, it was not possible to obtain a well fused joint at the constant conditions, since the degree of melt and quantity of molten material yielded was not adequate to create a strong joint as determined by the peel test.

Another problem at temperatures below 350°C was the fact that the hot bar tended to stick to the material, making it difficult to obtain a joint with even dimensions. This indicated that the heat energy stored by the hot knife was not adequate to maintain the material in the molten state long enough. The material therefore solidified and adhered to the knife. Temperatures between 350°C and 400°C may be described as transitory since there was still stickiness of the knife and the joints were not completely fused but the joints formed were much stronger than those formed below 350°C. However, when the bar was heated at 400°C, the sticking of the bar was no longer experienced instead the material melted readily and produced adequate melt to sustain a strong joint. Heating the flat bar beyond 400°C, resulted in the material burning and puncturing as the flat bar was slid through the joint. This indicates a need for good temperature control during welding.

The results of tensile tests on specimens made by the hot knife welding method are presented in Table 2 alongside other tensile test results and the creep test in Figure 4.

The strains in Figure 4 and Table 1 for welded joints are relatively lower than other results in the same table as a result of the increased thickness of the welded specimen (almost twice) the as a result of the lap joint. The shape of the creep curve at the beginning of the creep test in Figure 4 is also not as steep as the other curves in Figure 2 (a – c) due to the increased specimen thickness. It was concluded from these results that to obtain a strong joint, the temperature that will cause adequate melting by conduction or convection must be met by the agent causing melting. It was also observed that stronger joints are obtained with larger widths of the welded joint.

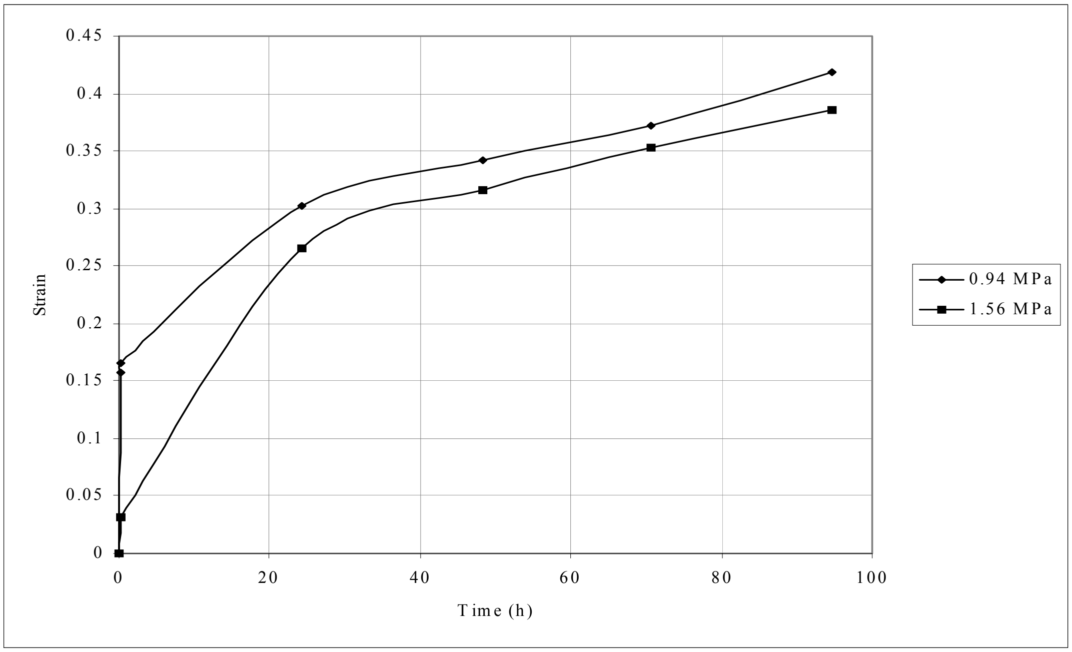

Figure 4: Laboratory welded fresh specimen creep curves at room temperature

CONCLUSIONS

It was concluded that the welded joint of samples collected from the field is the weakest point in the liner. The strength of the welded joint may be improved by correct selection of welding parameters (width of welded joint, joining pressure, dwell time of the pressure and temperature).

It is recommended for more research to be conducted with regard to developing welding devices that can satisfy the optimum conditions demonstrated in the laboratory during field welding, particularly with regard to temperature and pressure control.

REFERENCES

1. UNEP, Water – 2 Billon People are Dying for it, Nairobi (2003).

2. M.K. Tolba, O.A. El -Kholy, E.EL-Hinnawi, M.W. Holdgate, D.F. McMichael and R.E Munn, The World Environment 1972-1992. UNEP & Chapman & Hall, London (1992).

3. D.M Onyango, Women at the Source of Life: The Experience of Kenya Rainwater Association, FAO Dimitra Newsletter, 8, Brussels, Belgium (2003).

4. Sigmaaldrich, Aldrich Polymer Products Application & Reference Information, (2008).

5. Netzsch Applications Laboratory Newsletter, Characterization of Polyethylene with Differential Scanning Calorimetry (2005).

6. R.J. Crawford, Plastics Engineering, Elsevier, Butterworth Heinemann, UK (1998).

7. J.D. Idol and R.L. Lehman, "Polymers," in The CRC Handbook of Mechanical Engineering, Chapter 12, F. Kreith and Y. Goswami, Eds., CRC Press, London (2004).

8. Colorado Lining, Geomembranes / HDPE, Product Manual, (2007).

9. D. Aithani, H. Lockhart, R. Auras and K. Tanprasert, Predicting the Strongest Peelable Seal for ‘Easy-Open’ Packaging Applications, Journal of Plastic Film & Sheeting, Vol. 22, Pp 247-263, October 2006.

10. Illinois Tool Works, 2007. Plexus Guide to Bonding Plastics, Composites and Metals, www.itwplexus.com (Accessed in May 2008)

11. Salerni, C., 2007. Adhesive Technologies For the Assembly of Hard-to-Bond Plastics, Technical Briefs - Issue No.4, www.akd-tools.gr (Accessed in May 2008)

12. ASTM D638-2003 Standard Test Method for Tensile Properties of Plastics, ASTM International, West Conshohocken, PA, www.astm.org.

13. Kwan, M. K., T. H.C., Lin, and S. L.Y. Woo, 1993, On the Viscoelastic Properties of the Anteromedial Bundle of the Anterior Cruciate Ligament, Journal of Biomechanics, Vol. 26, pp. 447-452.

14. Pioletti, D. P., L. R. Rakotomanana, J. F. Benvenuti, P. F. Leyvraz, 1998. Viscoelastic Constitutive Law in Large Deformations: Application to Human Knee Ligaments and Tendons. J Biomech, 31(8):753-757.

15. Johnson, G. A., G. A. Livesay, S. L-Y. Woo and K. R. Rajagopal, 1996. A Single Integral Finite Strain Viscoelastic Model of Ligaments and Tendons, ASME Journal of Biomechanical Engineering, Vol. 118, pp. 221-226.