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We show that the introduction of Tobin taxes in agent-based models of currency markets can lead to a reduction of speculative trading and reduce the magnitude of exchange rate fluctuations at intermediate tax rates. In this regime revenues for the market maker obtained from speculators are maximal. We here focus on Minority Game models of markets, which are accessible by exact techniques from statistical mechanics. Results are supported by computer simulations. Our findings suggest that at finite systems sizes the effect is most pronounced in a critical region around the phase transition of the infinite system, but much weaker if the market is operating far from criticality and does not exhibit anomalous fluctuations.

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I. INTRODUCTION

In 1972 James Tobin proposed to “throw some sand in the wheels of our excessively efficient international money markets” [2] by imposing a tax of 0.05 to 0.5% on all international currency transactions. The Bretton Woods agreement – a system of fixed foreign exchange rates tied to the price of gold – at that time was gradually being dismantled, with the USA stepping out in 1971. This system had been introduced in the wake of World War II in order to rebuild global capitalism. Tobin feared the effects of countries exposed to freely fluctuating exchange rates and suggested, as a second best solution, the introduction of what is now called a ‘Tobin tax’ in order to suppress speculative behaviour and thus containing exchange rates volatility within tenable levels.

Since then, under floating exchange rates, the trading volume on international currency markets has grown sharply, especially after the introduction of electronic trading, reaching a level of 1.5 trillion US-Dollar per day in 2000 [4]. Most of these transactions are effected on time-scales of less than seven days, more than 40% involve round-trips within two days or less, and 90% are of a speculative nature, only one tenth are carried out within the production sector [4].

While the Tobin tax has never been implemented in reality the discussion of this issue is still lively and opinions are widely divided between proponents who claim that a Tobin tax would improve the situation of countries damaged by international currency speculation [3], and opponents who reject the proposal on various grounds. They claim that its implementation would hardly be feasible and extremely expensive, that it would mostly damage developing countries and that through a reduction of market liquidity, Tobin taxes might indeed result in more, not less volatility [5].

The question of whether a tax on financial transactions reduces volatility or not is indeed a subtle one. On the one hand excess volatility is related to trading volume, hence reducing the currency trading activity through the introduction of a tax on currency transactions can be expected to reduce volatility. On the other hand speculators provide liquidity and eliminate market inefficiencies, hence volatility might increase when a tax is levied and speculators drop out of the market due to the increasing trading costs. The latter consideration is particularly relevant given that the margin on which speculators live is extremely small and even a 0.5% tax could turn their marginal gains negative.

In order to make these considerations quantitative some authors have investigated the effects of Tobin taxes in agent-based models of financial markets [4, 6, 7]. These model are capable to reproduce typical features of price fluctuations in real markets – so-called stylized facts [8] – and are therefore well suited to shed light on the effects of introducing a Tobin tax. In the context of zero-intelligence percolation models [9], Ehrenstein and co-workers found that generally the introduction of Tobin tax brings about a reduction in volatility, as long as the tax rate is not too high to cause liquidity problems [4, 6]. Westerhoff [7] comes to similar conclusions building on the Chiarella-Hommes approach of modeling financial markets with systems of heterogeneous agents [10]. At variance with previous models, this study introduces considerations of agents’ rationality through heuristic expectation models of future returns of a chartist or fundamentalist nature.

The present paper addresses similar general questions. In contrast with previous works, however, we take the view of a financial market as an ecology of different types of agents interacting along an ‘information food chain’. In our picture speculators ‘predate’ on market inefficiencies (arbitrage opportunities) created by other investors (the so-called producers). This approach has been recently formalized in the Minority Game (MG) [11, 12, 13]. The MG captures the interplay between

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volatility and market efficiency in a vivid though admittedly simplified and stylized way. Indeed the analysis of the MG has revealed that within this model framework excess volatility and market efficiencies are identified as two sides of the same coin, both resulting as consequences of speculative trading. The MG exhibits two different regimes, one in which the market is fully efficient and another in which arbitrage opportunities are not entirely eliminated by the dynamics of the agents. These regimes are separated sharply in the parameter space of the model, and it turns out that the boundary at which the market becomes efficient coincides precisely with the locus of a phase transition in the language of statistical physics. At the same time critical fluctuations – very similar to the stylized facts observed in real market data – emerge in the vicinity of this transition but not further away [14]. Hence, at odds with previous models, the MG might offer a perspective of understanding how the introduction of transactions taxes affects the information ecology and market efficiency. It will here be important to distinguish between regimes close to and far away from the above transition between efficient and inefficient phases of the market. A further advantage of the MG over other more elaborate agent-based models lies in its analytic tractability. Despite its stylized setup the phenomenology of the MG is remarkably rich, but at at the same time the MG model can be understood fully analytically with tools of statistical physics of disordered systems [11, 12]. This analytical solution provides an understanding of the model, which goes much deeper than approaches to other models purely based on numerical simulations. In particular it is possible to derive closed analytical expressions for key observables such as the market volatility, the trading activity of the agents and the revenue for the market maker.

In brief, our main result is that within the picture of the MG model a small tax decreases volatility whenever a *finite* market exhibits anomalous fluctuations. Indeed the fundamental effect of the tax is to draw the market away from its critical point. At the same time, the tax introduces an information inefficiency and thus too high a tax might not be advisable. The total revenue from the tax exhibits a maximum for intermediate rates similar to what was found in earlier works on different models [4]. Furthermore, the effects of imposing a tax materialize in the market behaviour only after times which scale inversely with the tax rate. Extremely small tax rates may thus need a long time to stabilize turbulent markets. This can be quite relevant if this time scale becomes comparable to that over which market's composition changes. In particular, if speculators leave and enter the population of traders at a too fast rate a small Tobin tax may fail to stabilize the market. Large taxes in such a scenario reduce the time the population needs to co-ordinate below the rate with which new agents enter the market. Under such circumstances a transaction tax may thus prevent high volatility states and reduce the volatility significantly even in an *infinite* system.

In the following we shall first introduce the grand-canonical MG (GCMG) and re-iterate its known main features. In the main sections we then comment on how a tax on transactions can be introduced and discuss the effects on the market within the present model. We then turn to a brief discussion on how to relate these results to real markets, summarize our results and give some final concluding remarks.

II. THE GRAND-CANONICAL MINORITY GAME

A. The model

The so-called grand-canonical MG (GCMG) describes a simple market of N agents $i = 1, \ldots, N$ who at each round of the game make a binary trading decision (to buy or to sell) or who each may decide to refrain from trading. They thus each submit bids $b_i(t) \in \{-1, 0, 1\}$ in every trading period $t = 0, 1, 2, \ldots$ resulting in a total excess demand of $A(t) = \sum_{i=1}^{N} b_i(t)$.

These trading decisions are taken to be based on a stream of information available to the agents. This common information on the state of the market (or on other questions relevant to the market) is encoded in an integer variable $\mu(t)$ at time t taking values in $\{1, 2, \ldots, P\}$ [15]. Here we assume that $\mu(t)$ models an exogenous news arrival process, and that the $\{\mu(t)\}$ are drawn at random from the set $\{1, \ldots, P\}$, independently and with equal probabilities at each time [16]. The objective of each agent is to be in the minority at each time-step, i.e. to place a bid $b_i(t)$ which has the opposite sign of the total bid $A(t)$. This minority setup corresponds to contrarian behaviour and can be derived from a market mechanism taking into account the expectations of the traders on the future behaviour of the market [17]. In order to do so, each agent has a 'trading strategy' at his disposal. Trader i's strategy is labelled by $\mathbf{a}_i = (a_i^\mu)_{\mu=1,\ldots,P} \in \{-1, 1\}^P$ and provides a map from all values of the information μ onto the binary set $\{-1, 1\}$ of actions (buy/sell). Upon receiving information μ the trading strategy of agent i thus prescribes to take the trading action $a_i^\mu \in \{-1, 1\}$. These strategies are assigned at random and with no correlations at the beginning of the game, and then remain fixed [18]. Agents in the GCMG are adaptive and may decide not to trade if they do not consider their strategy adequate. More precisely, each agent keeps a score $u_i(t)$ measuring the performance of his strategy vector. He then trades at a given time-step t only if his strategy has a positive score $u_i(t) > 0$ at that time. Therefore, the bids of agents take the form $b_i(t) = n_i(t) a_i^{\mu(t)}$ with $n_i(t) = 1$ if $u_i(t) > 0$ and $n_i(t) = 0$ otherwise. Accordingly the excess demand is given by

$$A(t) = \sum_{i=1}^{N} n_i(t) a_i^{\mu(t)}. \qquad (1)$$

Agent i keeps a record of the past performance of his strategy a_i^μ by updating the score $u_i(t)$ as follows

$$u_i(t+1) = u_i(t) - a_i^{\mu(t)} A(t) - \varepsilon_i. \quad (2)$$

at each step, with constant ε_i. The first term $-a_i^{\mu(t)} A(t)$ is the Minority Game payoff, it is positive whenever the trading action $a_i^{\mu(t)}$ proposed by i's strategy vector and the aggregate bid $A(t)$ are of opposite signs, and negative whenever i joins the majority decision. The idea of Eq. (2) is that whenever the payoff $-a_i^{\mu(t)} A(t)$ is larger than ε_i the score of player i's strategy is increased, otherwise it is decreased. The constant ε_i in (2) thus captures the inclination of agent i to trade in the market. This inclination will in general be heterogeneous across the population of agents, with agents with high values of ε_i being more cautions to trade than agents with low ε_i. In our simplified model we only consider two types of agents. First we assume that there are $N_s \leq N$ speculators who trade only if their perceived market profit obtained by using their strategy exceeds a given threshold, and hence we set $\varepsilon_i = \epsilon \geq 0$ for such agents. Here ϵ can be considered as the speculative margin of gain in a single transaction.

The remaining $N_p = N - N_s$ agents – the so-called producers or institutional investors – are assumed to trade no matter what. Mathematically this is implemented by setting $\varepsilon_i = -\infty$ for this second group of agents. They have $n_i(t) = 1$ for all times t. For convenience we will order the agents such that speculators carry the indices $i = 1, \ldots, N_s$ and producers the labels $i = N_s + 1, \ldots, N$.

B. Price process, volatility and predictability

Within the MG setup the market volatility is then given by

$$\sigma^2 = \frac{\langle A(t)^2 \rangle}{N}, \quad (3)$$

where $\langle \ldots \rangle$ will stand for a time-average in the stationary state of the model from now on. The normalization to the number of agents N is here introduced to guarantee a finite value of σ^2 in the infinite-system limit, with which the statistical mechanics analysis of the model is concerned.

The information variable $\mu(t)$ allows one to quantify information-efficiency of the model market by computing the predictability

$$H = \frac{1}{PN} \sum_{\mu=1}^{P} \langle A|\mu \rangle^2 \quad (4)$$

where $\langle \ldots |\mu \rangle$ denotes an average conditional on the occurrence of information pattern $\mu(t) = \mu$. A value $H \neq 0$ indicates that for some μ the minority payoff is statistically predictable $\langle A(t)|\mu \rangle \neq 0$, whereas the market is unpredictable and fully efficient when $H = 0$.

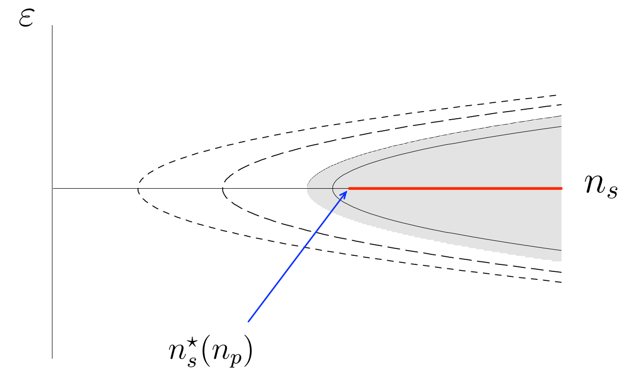

FIG. 1: Phase diagram of the GCMG in the (n_s, ε)-plane at fixed n_p. The red line segment at $\varepsilon = 0$ and $n_s \geq n_s^\star(n_p)$ marks the phase transition in the limit of infinite system size. At finite size anomalous fluctuations and stylized facts are found in a region around this critical line, as indicated by the shaded area. This so-called 'critical region' indicates regions with strong dynamical and finite size effects and is large for small systems and shrinks towards the critical line segment in the infinite-size limit.

The simplest way to relate this picture to a financial market is to postulate a simple linear impact of $A(t)$ on the (logarithmic) price (or the exchange rate), i.e to assume that

$$p(t+1) = p(t) + \frac{A(t)}{\lambda}, \quad (5)$$

where $\{p(t)\}$ denotes a price (exchange rate) process and where λ is the liquidity. In a derivation of Eq. (5) from a market clearing condition λ turns out to be inversely proportional to the number of active traders $\sum_i n_i(t)$ [17]. In the following we set the liquidity to $\lambda = \sqrt{N}$, so that σ^2 becomes the volatility of the price process, $\sigma^2 = \langle (p(t+1) - p(t))^2 \rangle$. It is found simulations that the effects of introducing a Tobin tax on the behaviour of the model are qualitatively similar for either choice of λ, so that we will stick with the technically more convenient first definition.

C. The behaviour of the GCMG

The GCMG has been studied in great detail in [14, 19] with methods well established in statistical mechanics. The analysis is here generally concerned with the stationary states of the system, i.e. the behaviour which is reached after running the learning dynamics of the agents for some sufficiently long transient equilibration time.

The statistical mechanics approach provides exact results for the model in the limit of infinite market sizes, where one takes the number of agents $N = N_s + N_p$ and the number P of possible different information states to infinity, while at the same time keeping the ratios

$n_s = N_s/P$ and $n_p = N_p/P$ fixed and finite. n_s and n_p along with ε are thus control parameters of the model. This approach [11, 12, 14] makes it possible to derive exact expressions for several quantities, including the predictability H and upon neglecting time-dependent correlations accurate approximations for the volatility σ^2 can be found [20, 21]. We will here not enter the detailed mathematics of the calculations, the resulting equations for the key quantities in the stationary states as well as a sketch of their derivation are found in the appendix. Further details regarding the statistical mechanics analysis of MGs and GCMGs can be found e.g. in [11, 12, 14] and in references therein.

The overall picture which emerges is the following [14]: at fixed n_p, the statistical behavior of the model is characterized by a critical line at $\epsilon = 0$ which extends from some critical value $n_s^\star(n_p)$ to larger values $n_s \geq n_s^\star(n_p)$ than this threshold. This is illustrated in Fig. 1. As this line (segment) is approached in parameter space the market becomes more and more efficient, i.e. $H \to 0$ as $\varepsilon \to 0$ for $n_s \geq n_s^\star$. On the critical line ($\epsilon = 0, n_s \geq n_s^\star$) itself the market is fully efficient and one finds $H = 0$ exactly in the limit of infinite system size. In addition, numerical simulations of the model at finite sizes close to the critical line reveal fluctuation properties which are similar to those observed in real markets [8]. In particular $A(t)$ has a fat tailed distribution and one observes volatility clustering. These effects become weaker as the system size is increased at constant values of n_s, n_p and ε, and similarly they disappear gradually when one moves away from the critical line at fixed system size.

The case $\epsilon = 0$ and $n_s > n_s^\star(n_p)$ is peculiar because it turns out that the stationary state is here not unique, but rather that it depends on the initial conditions from which simulations are started. In the following we will not consider the case of a strictly vanishing ε, but will assume instead that speculators have a positive profit margin $\varepsilon > 0$, even if the latter may be small. All simulations on which we report are started from zero initial conditions $u_i(t = 0) = 0$ for $i = 1, \ldots, N_s$.

We finally remark that, a detailed analysis of the transient dynamics demonstrates that for small ϵ the stationary state is reached after a characteristic equilibration time which scales as $1/\epsilon$ [22]. This long equilibration time is responsible for some relevant effects if the market composition changes in time, as discussed in Section IV.

III. TOBIN TAX IN THE GCMG

Within the model setup the introduction of a tax τ on each transaction can be accounted for by a change $\varepsilon_i \to \varepsilon_i + \tau$ for all $i = 1, \ldots, N$. Indeed by raising ε by an amount τ, an additional cost τ incurs for any given agent every time they trade and no costs for agents who refrain from trading. Note that the trading volume of any fixed (active) agent is one unit in our simple setup so that τ indeed corresponds to a transaction tax per

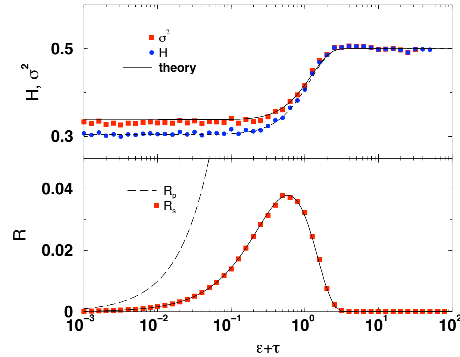

FIG. 2: Volatility, predictability (top) and revenue from speculators and producers (bottom) for $n_s = n_p = 1$. Symbols are data obtained from numerical simulations with $PN_s = 6000$, every data point is an average over at least 1000 samples, simulations are run for $1000P$ steps, with measurements taken in the second half of this interval. Lines are the corresponding predictions for infinite systems obtained from the analytical theory.

unit traded. Hence we will assume

$$\varepsilon_i = \begin{cases} \epsilon + \tau & i \leq N_s \\ -\infty & i > N_s \end{cases} \tag{6}$$

in the following. While this will discourage speculators from trading (via the reduction of their strategy score) such a tax will have no effect on the participation of producers. They will trade at every time step as before ($n_i(t) = 1 \; \forall i > N_s$ at all times t).

The total revenue from this tax τ is then given by

$$R = \frac{\tau}{N} \sum_i \langle n_i(t) \rangle = \frac{\tau}{N} \sum_{i=1}^{N_s} \langle n_i(t) \rangle + \tau \frac{n_p}{n_s + n_p} \equiv R_s + R_p, \tag{7}$$

where the first term corresponds to the revenue R_s from speculators and the second (R_p) to that obtained from the producers.

An evaluation of the effects of levying a tax τ on the GCMG then amounts to studying the behavior of the model as a function of ϵ at fixed model parameters n_s and n_p. It turns out that here one has to distinguish between two different regimes, namely close and far away from the phase transition.

Fig. 2 reports the effects of introducing a tax on markets whose parameters are far from the critical line. We here consider $n_p = 1$ and $n_s = 1 < n_s^\star(n_p = 1) \approx 4.15\ldots$ so that one operates sufficiently far to the left of the critical region depicted in Fig. 1. For such parameter values the results of numerical simulations follow the curves predicted by the analytical theory perfectly, and no anomalous fluctuations are present in the corresponding price time-series. The tax has very mild effect both on volatility and on the information efficiency, as long as $\epsilon + \tau \ll 1$.

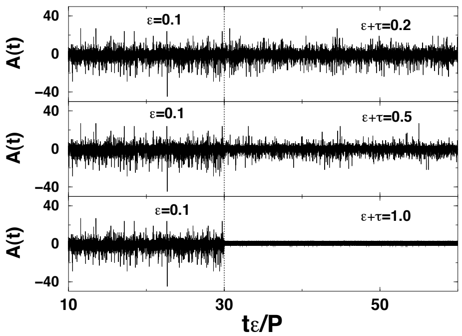

FIG. 3: Effect of introducing a tax on the exchange rate fluctuations $A(t)$. Each panel corresponds to a single run with parameters $n_s = 20$, $n_p = 1$, $PN_s = 16000$, $\varepsilon = 0.1$. In the initial period up to $t = 30P/\varepsilon$ no tax is imposed ($\tau = 0$). At $t\varepsilon/P = 30$ a tax rate ($\tau > 0$ is introduced and then kept fixed for the rest of the simulation. The tax rate increases from top ($\varepsilon + \tau = 0.2$) to bottom ($\varepsilon + \tau = 1$). .

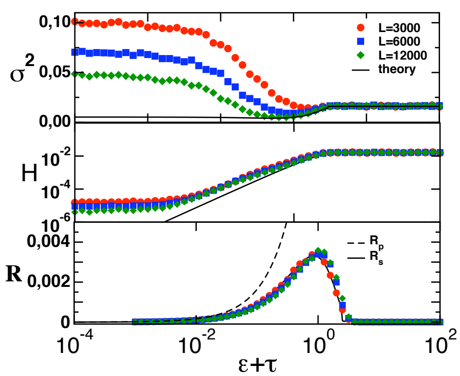

FIG. 4: Volatility (top), predictability (middle) and revenue (bottom) from speculators and producers for $n_s = 60$ and $n_p = 1$. Markers are data obtained from numerical simulations of systems with different (effective) size $L = PN_s$ with circles, squares and diamonds corresponding to $L =$ 3000, 6000, 12000 respectively. Every data point represents an average over at least 1000 different strategy assignments. Simulations are run for $400P + 20Ns/(\varepsilon+\tau)$ steps, with measurements taken in the second half of this interval. Lines are the predictions of the analytical theory for the infinite system.

Fig. 2 also shows that the contribution of speculators to the tax revenue has a peak at intermediate tax rates, but that at the same time the revenue R_s obtained from speculators is smaller than that from institutional investors.

Fig. 3 on the other hand illustrates the response of the market at the other extreme where $n_s \gg n_s^\star(n_p)$. For such values of the parameters one is within the critical region (for small enough $\varepsilon + \tau$), and the model exhibits strong anomalous price fluctuations for market sizes of a few thousand agents. The deviations from the analytical theory (which is valid only in the limit of infinite systems) mark strong finite-size effects in the critical region. As illustrated in the lower panel of Fig. 3 imposing

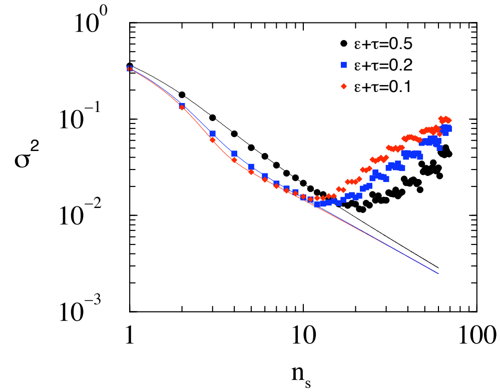

FIG. 5: Volatility as a function of n_s at fixed $n_p = 1$ for different tax rates ($\epsilon + \tau = 0.1,\ 0.2$ and 0.5). Markers represent data obtained from numerical simulations with $PN_s = 3000$, run for $200P + 200P/(\varepsilon + \tau)$ steps (with measurements in the second half of this interval). An average over 3000 samples is taken. Lines are the corresponding predictions obtained from the analytical theory.

a sufficiently large tax may in such markets have a pronounced effect on the volatility, whereas smaller transaction fees may influence the time-series of the market only marginally. Fig. 4 presents a systematic account of these effects and shows the dependence of the volatility, the predictability and the revenue from the tax on the system size and the tax rate τ at $n_s \gg n_s^\star(n_p)$. In particular, a significant reduction of the market volatility can be obtained, while still keeping the market relatively information efficient. Furthermore, the contribution to the tax revenue of speculators largely outweighs that of producers and it is peaked at a value close to that where the volatility is minimal. The effect of a tax, as shown in Fig. 4, also depends on the size of the market. The volatility at low $\varepsilon + \tau$ indeed decreases with the size of the system and approaches the theoretical line, making a tax more effective in small than in large markets.

Fig. 5 relates these two extremes and discusses the dependence of the volatility on n_s at intermediate number of speculators for fixed $n_p = 1$. We here fix the (effective) system size by keeping $L = PN_s$ constant. For small values of n_s one is then well outside the critical region and the numerical results follow the analytical predictions (solid lines in Fig. 5). As discussed in Ref. [14] the simulations then deviate systematically from the theory at large n_s when the system has entered the zone near the phase transition line. More precisely, one finds a threshold value $\overline{n}_s(L) > n_s^\star$ so that numerical simulations agree with the theoretical lines for $n_s < \overline{n}_s(L)$ but deviations and anomalous fluctuations are observed for $n_s > \overline{n}_s(L)$. As L is increased $\overline{n}_s(L)$ is found to grow as well in simulations (not shown here), and in particular one has $\lim_{L\to\infty} \overline{n}_s(L) = \infty$ (at $\varepsilon + \tau \neq 0$) so that the critical region vanishes in the limit of infinite systems.

IV. MARKETS WITH EVOLVING COMPOSITION OF AGENTS

In the previous sections we have assumed that all traders stay in the market for an infinite amount of of time and that their trading strategies remain fixed forever. Individual agents have the option to abstain from trading at intermediate times and to join the market again at a later stage, but no agent in the setup considered so far can actually modify his strategy vector $\{a_i^\mu\}$. Thus the composition of the population of traders does not change over time. In real-market situations however it would appear more sensible to expect some fluctuation in the population of traders and to assume that the market composition will evolve and/or that strategies get replaced after some time. In the latter case, one would expect poorly performing strategies to be removed from the market and replaced by new ones.

In this section we consider the simplest case of an evolving composition of the market, namely a situation in which agents (or equivalently their strategies) are replaced randomly, irrespective of their performance. More precisely, at each time step each speculator is removed with a probability $1/(\theta N_s P)$ and replaced by a new one with randomly drawn strategy and zero initial score. Here θ is a constant, independent of the system size. This choice $\theta = \mathcal{O}(L^0)$ guarantees that the expected survival time of any individual agent scales as $N_s P$ so that one exit/entry event occurs in the entire population on average over a period of θP transaction time steps. Relaxation times in Minority Games are known to be of the order of P so that the above scaling of θ results in the composition of the market changing slowly on times comparable with those on which the system relaxes. Indeed, extensive numerical simulations show that the behaviour of the volatility on θ as well as that of other quantities characterizing the collective behaviour of the system is independent of the system size (see Fig. 6). This is in sharp contrast with the strong finite size effects observed for $n_s \gg n_s^\star$ at a fixed composition of the population of agents (Fig. 4).

The main feature of the MG market with an evolving population of agents is a pronounced minimum of the volatility as a function of $\varepsilon + \tau$ in the crowded regime $n_s > n_s^\star$. In particular the volatility increases as $\varepsilon + \tau$ is decreased, even in the limit of large system sizes (in which a corresponding system with fixed agent population would equilibrate to the flat theoretical line as shown in Fig. 4). This behaviour of the system with changing agent structure can be related to the fact that relaxation time of the GCMG scales as $1/(\epsilon + \tau)$ [22]. Thus, when $\epsilon + \tau$ is very small, the time it would take a fixed population of agents to equilibrate collectively can be much larger than the time scale θP over which the market composition changes. In this case, the market remains in a high volatility state indefinitely because the agents do not have sufficient time to 'coordinate' and to adjust their respective behaviour as the strategy pool represented in the evolving population of agents changes too quickly. Fig. 6 demonstrates that introducing a tax in such markets with dynamically evolving trader structure can reduce the volatility considerably.

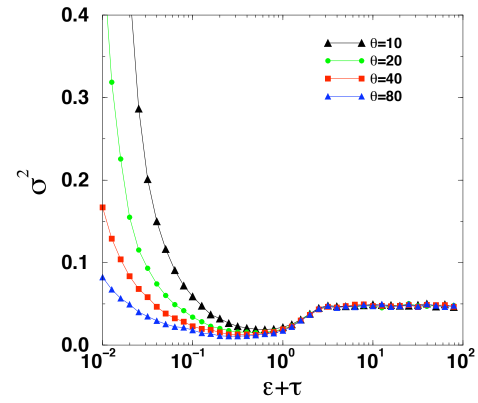

FIG. 6: Volatility as a function of $\epsilon + \tau$ in markets with slowly changing composition. The parameter θ indicates the rate at which agents are replaced; on average one replacement event occurs in the entire population every θP time-steps (see main text for further details). Simulations are at $n_p = 1, n_s = 20$ and $L = PN_s = 5000$ with an equilibration time $T = 400P/(\epsilon + \tau)$ and different values of θ as indicated.

V. CONCLUSIONS

We have shown how the theoretical picture derived for GCMG [11, 12, 14] can be used to fully characterize the impact of a Tobin tax on this toy model of a currency market. The main results of our study (see Fig. 4) is that within the GCMG the introduction of a Tobin Tax reduces the market volatility whenever the market is operating close to the critical line at $\epsilon = 0$, $n_s > n_s^\star$ which marks perfect efficiency. In this region close to criticality the efficiency scales as $H \sim \varepsilon^\alpha$ (with a theoretically predicted exponent of $\alpha = 2$) and the market is close to full efficiency $H \approx 0$. The model is known to exhibit stylized facts such as broad non-Gaussian return distributions and volatility clustering for such values of the model parameters. Thus the reduction of the volatility by an additional tax is most pronounced when the market is operating close to efficiency and exhibits anomalous fluctuations. Within the GCMG the effect of a trading tax is due to a significant amount of excess volatility close to a critical line. The tax τ draws the market away from criticality in parameter space and thus reduces volatility.

The picture is different when a tax is introduced to a market which operates far from criticality. For a market with few speculators ($n_s < n_s^\star$) the predictability H attains a finite limit when $\varepsilon + \tau \to 0$. In this case the tax has a small effect on volatility.

In both regimes we find that the revenue for the market maker from the tax attains a maximal value at intermediate tax rates. In the case of a market near criticality this occurs approximately at a tax rate at which the reduction in volatility is largest. In both shown examples the revenue for the market maker appears to weigh more on institutional investors than on speculators. Measuring the revenues from producers and speculators respectively at other values of n_s and n_p confirms this observation and we find $R_p > R_s$ (not shown here). Only when the number of producers is extremely small or when $\varepsilon < 0$ can one observe instances in which the revenue from speculators is higher than that from producers. Both an extremely small number of producers and/or a negative value of the profit margin ε seem somewhat unrealistic in the present context so that we do not report further details here.

Thirdly our findings demonstrate that imposing a tax can also reduce the market volatility in cases where the composition of the population of traders changes slowly in time. In this case, the tax allows the agents to reach coordinated state faster so that the market can reach a stationary state of relatively low volatility.

Given the stylized nature of the MG, it is hard to make a connection between the model parameter τ and an actual tax rate in a real-world market. At any rate it seems reasonable to assume that a realistic tax rate should be of the order or smaller than the margin of profit of speculators, which is gauged by ϵ in the GCMG. The optimal tax rate τ might be unrealistically large compared to ϵ. E.g. in Fig. 4 if $\epsilon = 0.01$ volatility can be substantially reduced only for tax rates τ which are more than ten times larger.

The GCMG can at best be seen a minimalistic simplified version of a real market. Hence our conclusions on the behaviour of the MG are at most suggestive with respect to what might happen in the real world. Still, the Minority Game is able to capture the interplay between stochastic fluctuations and information efficiency in a system of adaptive agents in a simplified way. At the same time it is analytically solvable with the tools of statistical mechanics so that quantitative predictions for example of the volatility can be made based on an exact theory. In this sense, the analysis of MGs goes far beyond the results of zero-intelligence models. Most importantly in the present context our study provides a coherent theoretical picture of the effects of ‘throwing some sand in the wheels’ of markets operating close to information efficiency. The picture developed here can be extended in a number of directions toward more realistic market modes, but without giving up analytical tractability. One of the most interesting future directions might be to endow agents with individual wealth variables which evolve according to their relative success when trading in the model market.

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Appendix

We here sketch the theoretical analysis of the model with $\tau = 0$ and general values of ε. The introduction of a tax τ can be accounted for by replacing $\varepsilon \to \varepsilon + \tau$ in all equations below. The starting point of the statistical mechanics approach is the function

$$H_\varepsilon[\{\phi_i\}] = \frac{1}{P}\sum_{\mu=1}^{P}\left[\sum_{i=1}^{N} a_i^\mu \phi_i\right]^2 + \frac{2\varepsilon}{P}\sum_{i=1}^{N_s}\phi_i \qquad (8)$$

of the mean activities $\{\phi_i = \langle n_i(t)\rangle\}$ of the speculators $i = 1, \ldots, N_s$. The ϕ_i, $i = 1, \ldots, N_s$ are continuous variables within the interval $[0, 1]$, recall that producers are always active and have $\phi_i = 1$, $i = N_s + 1, \ldots, N = N_s + N_p$. Note that this function depends explicitly on the strategy assignments $\{a_i^\mu\}$ so that H_ε is a stochastic quantity. The strategy vectors which are fixed at the beginning of the game correspond to what is known as 'quenched disorder' in statistical mechanics. It turns out that the learning dynamics (2) minimizes the function H_ε in terms of the $\{\phi_i\}$ for any fixed choice of the strategy vectors. Computing the stationary states of the model thus reduces to identifying the minima of H_ε. It is here possible to characterize these minima using the so-called replica method of statistical physics [23]. A different statistical mechanics approach is based on so-called generating functionals and deals directly with the update dynamics (2), see [12]. Both methods ultimately lead to the same equations describing the stationary states of the model, so that we here restrict the discussion to the former approach.

In the following we give a brief sketch of the so-called replica analysis of the model, which allows to compute the minima of the random function H_ε. To this end one first introduces the partition function

$$Z_\varepsilon(\beta) = \int_0^1 d\phi_1 \cdots \int_0^1 d\phi_{N_s} \exp\left(-\beta H_\varepsilon[\phi_1, \ldots, \phi_{N_s}]\right) \qquad (9)$$

at an 'annealing temperature' $T = 1/\beta$. In the limit $\beta \to \infty$ these integrals are dominated by configurations $\{\phi_1, \ldots, \phi_{N_s}\}$ which minimize H_ε so that the evaluation of $\lim_{\beta\to\infty} Z_\varepsilon(\beta)$ allows one to characterize the minima of H_ε.

This procedure is in general not feasible for individual realizations of the random strategy assignments as the dependence of H_ε on the $\{a_i^\mu\}$ is quite intricate. Instead we will compute 'typical' quantities in the limit of infinite systems, i.e averages over the space of all strategy assignments.

The key quantity to compute here is the free energy density

$$f_\varepsilon(\beta) = -\lim_{N\to\infty}\frac{1}{\beta N}\ln Z_\varepsilon(\beta). \qquad (10)$$

The limit $N \to \infty$ is here taken at fixed ratios $n_s = N_s/P$ and $n_p = N_p/P$ (so that N_s, N_p, P are taken to infinity as well). All relevant properties of the typical minima of H_ε can be read off from the disorder-average of $\lim_{\beta\to\infty} f_\varepsilon(\beta)$. Using the identity $\ln Z = \lim_{n\to 0}\frac{Z^n - 1}{n}$ this problem can be reduced to computing averages of Z_ε^n, corresponding to an n-fold replicated systems with no interactions between the individual copies. This is referred to as the replica method in statistical physics and is a standard tool for the analysis of problems involving quenched disorder [11, 12, 23]. The averaging procedure leads to an effective interaction between the replicas, and requires an assumption regarding the symmetry of the solution with respect to permutations of the replicas. In principle this symmetry may be broken, as different replica copies may end up in different minima of H_ε. In the non-efficient phase of this model,instead, the so-called 'replica symmetric' ansatz is exact, simplifying the analysis considerably. We will here not report the detailed intermediate steps of the calculation, but will only quote the final outcome, namely a set of closed equations for the variables characterizing the minima of H_ε (and hence the stationary states of the GCMG). Further details of the replica analysis are found in [11, 12, 14]

The minima of H_ε turn out to be described by two independent variables K and ζ, uniquely determined from the following two relations:

$$\begin{aligned} \zeta &= \frac{1}{\sqrt{n_s(Q(\zeta,K) + n_p/n_s)}} \\ K &= \varepsilon\left[1 - \frac{n_s}{2}\left(\mathrm{erf}[(1+K)\zeta/\sqrt{2}] - \mathrm{erf}(K\zeta/\sqrt{2})\right)\right]^{-1} \end{aligned} \qquad (11)$$

with

$$\begin{aligned} Q(\zeta, K) &= \frac{1}{2}\,\mathrm{erfc}[(1+K)\zeta/\sqrt{2}] \\ &\quad + \frac{1}{\zeta\sqrt{2\pi}}\left[(K-1)e^{-(1+K)^2\zeta^2/2} - Ke^{-K^2\zeta^2/2}\right] \\ &\quad + \frac{1}{2}\left(K^2 + \frac{1}{\zeta^2}\right)\left(\mathrm{erf}[(1+K)\zeta/\sqrt{2} - \mathrm{erf}(K\zeta/\sqrt{2})\right) \end{aligned}$$

These equations are easily solved numerically and one obtains K and ζ as functions of the model parameters $\{n_s, n_p, \varepsilon\}$. The disorder-average of quantities such as the predictability H or the mean activity of the speculators $\phi = \lim_{N\to\infty} N_s^{-1}\sum_{i=1}^{N_s}\phi_i$ can then be expressed in terms of K and ζ. One finds:

$$\begin{aligned} H &= \epsilon^2\frac{n_p + n_s Q(\zeta,K)}{(n_s+n_p)K^2} \\ \phi &= \frac{1}{2}\mathrm{erfc}[(1+K)\zeta/\sqrt{2}] \\ &\quad + \frac{1}{\zeta\sqrt{2\pi}}\left(e^{-K^2\zeta^2/2} - e^{-\zeta^2(1+K)^2/2}\right) \\ &\quad + \frac{K}{2}(\mathrm{erf}(K\zeta/\sqrt{2}) - \mathrm{erf}[(1+K)\zeta/\sqrt{2}]) \end{aligned} \qquad (12)$$

. These results are fully exact in the thermodynamic limit, with no approximations (except for the replica-symmetric ansatz) made at any stage. Finally, neglecting certain dynamical correlations between agents the volatility can be approximated as

$$\sigma^2 = \varepsilon^2 \frac{n_p + n_s Q(\zeta, K)}{(n_s + n_p) K^2} + n_s \frac{\phi - Q(\zeta, K)}{n_s + n_p}. \qquad (13)$$

As shown in the main text of the paper this approximation is in excellent agreement with numerical simulations (up to finite-size and equilibration effects).

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