# Genetic Based Experimental Investigation on Finishing Characteristics of $AlSiC_p$-MMC by Abrasive Flow Machining

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## Abstract

Implementing non-conventional finishing methods in the aircraft industry by the abrasive flow machining (AFM) process depends on the production quality at optimal conditions. The optimal set of the process variables in metal-matrix-composite (MMC) for a varying reinforcement percentage removes the obstructions and errors in the AFM process. In order to achieve this objective, the resultant output functions of the overall process using every clustering level of variables in a model are configured by using genetic programming (GP). These functions forecast the data to vary the percent of silicon carbide particles (SiCp) particles without experimentation obtaining the output functions for material removing rates and surface roughness changes of Al-MMCs machined with the AFM process by using GP. The obtained genetic optimal global models are simulated and, the results show a higher degree of accuracy up to 99.97% as compared to the other modeling techniques.

**Keywords:** abrasive flow machining, mmc, polishing, genetic models, forecasting

## 1. Introduction

Limited product finishing/polishing technologies are available for achieving the excellent surface finish of composites and challenging to reach internal passages like polishing through chemicals and abrasive flow machining (AFM) [1]. AFM is a state-of-the-art finishing / polishing process for the products of intricate shapes, internal sections, the intersection of holes, slots, concave or convex edges, etc., [2]. Accelerating the abrasion medium formed by abrasive particles in a polymer to carry it into channels along with oil for maintaining the viscosity increases the material removal rate (MRR) and surface finish (Ra) [3].

AFM was introduced and successfully developed in 1960. In traditional machining, such as honing, grinding, lapping, etc., composites are hard to machine efficiently when direct tool contact leaves cutting marks and burrs on components [4-5]. Generally, the samples prepared by the electric discharge machining (EDM) process got spoiled at their surfaces and were significantly improved by AFM process [6]. AFM successfully polished composite material surfaces, which had machining the difficulties from traditional methods by producing consistent and predictable results efficiently and economically [7].

The experimental investigations into rotational magnetic field abrasion flowing (RMAFM) machining of alloys proves the improvement of their finishing efficiency by the simultaneous rotational and down movements of the flow medium [8]. Ultrasonic based on MAFM develops the vibration perpendicular to flow medium direction has improved the surface characteristics as well as operating conditions like driving force, power, applied voltage, working gap, etc., [9]. Various factors

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and their impact on the responses of AFM in alloys are studied by using the response surface (RSM) and Taguchi design (TD) methods. TD was applied to optimize the variables of AFM samples having the slots prepared by wire-EDM [10-11]. TD was also employed in detecting most influencing factors and optimizing Ra characteristics of alloys finished by MAFM involving multiple input factors [12]. RSM based experiments was employed to assess the sliding wear of an inferior vs. a superior surface finish using the "Slip line theory of plasticity" which resulted in optimum operating conditions and material properties [13]. Numerical models were generated to forecast the machining outputs as a function of operating input attributes [14]. The mathematical expressions in the turning operation of AISI1019 carbon steel were generated by using an RSM and Box-Cox transformation technique [15]. The neural network (NN) modeling technique in the AFM processes produced results could be validated with the results of the achieved optimal set via genetic algorithms [16].

Genetic programming (GP) is an automatic process run by a computer to develop the programs by the Darwinian principle of the natural selection called "Survival of the fittest". GP carefully solved the various problems having specific outputs. GP predicted the actual outputs from the extensive trial data of the inputs and outputs obtained by experimentation [17]. Fig. 1 shows the various phases of GP. The discipulus software evolves the most excellent program by mapping multiple inputs onto a single output data by a set of programs that reproduce from hundreds to thousands of iterations with each other [18]. GP is an active, productive, and fast-growing method. It is also successfully used in divergent problems. GP succeeds in outperforming the best human developed solutions [19]. GP has been successfully employed in solving problems of different fields , such as engineering methods, data modeling, time-series forecast, factor optimization, forecasting of manufacturing control processes, etc., [20].

1. Randomly generate computer programs (population) on which computerized evolution takes place.
2. The tournament is conducted by selecting randomly four programs from the population to measure best performance by 4 programs for the task chosen at developing stage of GP.
3. The tournament is repeated between two programs to select one performing the task best, which is called the winner program. From two tournaments and 4 programs, two new child (winner) programs would replace loser programs of two tournament by the crossover and mutation transformation.
4. The above steps will be repeated till it writes a program that performs the assigned task such that GP keeps developing its child programs by using the sport winning programs that are stimulated by biology.
5. The mutation (transformation) process may change an instruction of a game winner to a new child program, and crossover process may swap the instructions of the two game winning programs.
6. GP uses direct manipulation of binary machine code for evolving linear GP program which changes the best evolved models into C, Java programs etc. from machine code.

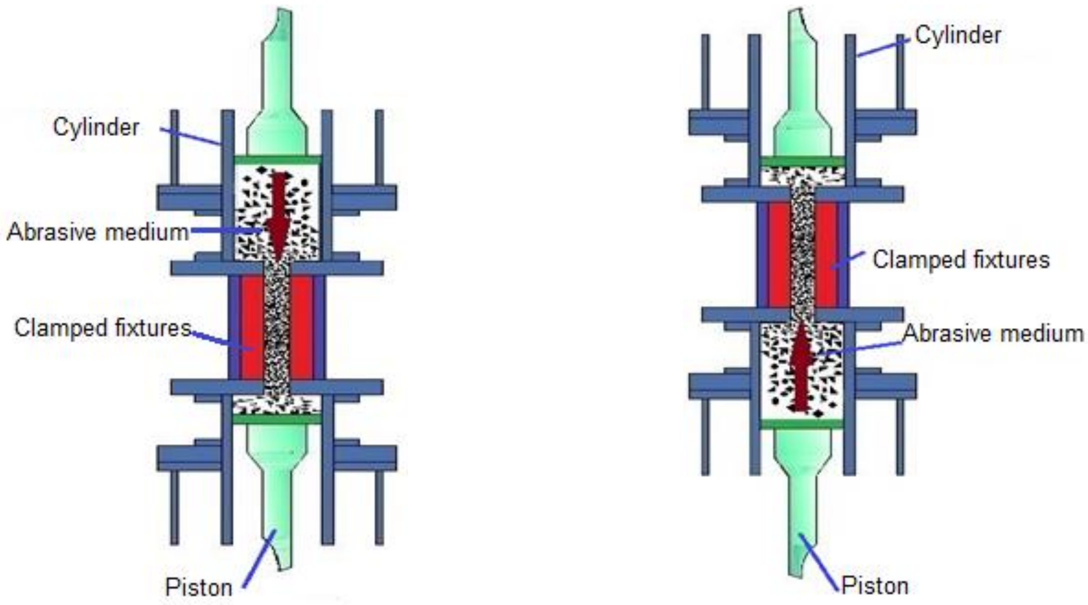
Fig. 1 Steps involved in Genetic Programming

In the recent literature survey, several researchers have worked on optimizing input attributes using TD, RSM, NN techniques, but mathematical models (MM) are only available for metals and MMCs having a fixed percent of $SiC_p$. Hard material composites polished through the AFM have a broad range of applications in the small-scale aerospace industries, which demands constant improvement. Linear Regression, ANN, ANOVA, Fuzzy logic controllers, and RSM that develops MMs produce significant errors up to three digits and omit the non-linearity portion of the experimentation process, whereas only GP can reduce error to $2^{nd}$ decimal positions.

In this study, GP is employed in $Al/SiC_p$ MMC with Al base material reinforced with varying percent of $SiC_p$ particles to generate MMs for their quality characteristics of machining responses. This work is to obtain MM at higher accuracy (with least or no error) in forecasting the output data avoiding experimentation work in future. GP approach yielded the best MM for reducing the surface roughness and increasing the MRR of the AFM process which shows 99.2 to 99.6% accuracy level [21-22].

## 2. Materials and Methods

The experimental setup is designed to accommodate the changes in the process inputs and to withstand the pump pressure up to 10 MPa in the EN8 hydraulic cylinders having an ID of 85 mm, 240 mm stroke, and lubricant oil (SAE 68). Nylon fixture is employed for supporting the work-piece because of the easy machinability to cut slots irrespective of their form and size. Converging the fixture passage gradually reduces the vibrations on a work-piece while passing the abrasive media smoothly (refer Fig. 2). After a required number of cycles, the work-piece is taken out, and is placed in the slots. Two movable plates are fixed, which can be moved up and down such that the fixture grasps firmly to prevent the outflow of the media. The work-piece material is chosen as MMC with aluminium when a base material and particles of $SiC_p$ varying in 20, 40, and 60% are as shown in Table 1.




(a) Downward stroke (b) Upward stoke

Fig. 2 The working principle of abrasive flow machine [23]

Table 1 Units and levels of control parameters

| Symbol | Parameters and their Units | Levels | | |
|---|---|---|---|---|
| | | $1^{st}$ | $2^{nd}$ | $3^{rd}$ |
| $V_0$ | Extrusion pressure (MPa) | 3.5 | 5 | 7 |
| $V_1$ | Oil in Media (%) | 10 | 12.5 | 15 |
| $V_2$ | Mesh Number | 100 | 150 | 220 |
| $V_3$ | Concentration of abrasives (weightage % of abrasives) | 50 | 55 | 60 |
| $V_4$ | Workpiece material (% of $SiC_p$ in $Al/SiC_p$) | 20 | 40 | 60 |
| $V_5$ | Number of cycles | 100 | 200 | 300 |

### *2.1. Experimental methodology*

The experiments carried out on the work piece are made of Al/SiCp MMCs having input factors namely, $V_0$ to $V_5$, to find the outputs such as MRR and ΔRa. Abrasives in liquid silicon media is used for polishing of work-pieces. Various levels of input factors have been selected based on a literature review as shown in Table 1 for the AFM process. The work-piece uses $Al/SiC_p$ MMCs with varying $SiC_p$ percent (20, 40 and 60) having aluminium as a base metal. GP requires a vast number of trial data to train, validate the output program, and are tabulated in Tables 2 and 3. The Surface roughness (Ra) is determined prior to and subsequent to the finishing operation at every trial by the profilometer (refer to Table 4). Similarly, MRR is measured by the difference of the initial and final weight of the sample, and time taken is tabulated in Table 5. MRR is calculated by:

$$MRR = \frac{initial\ weight\text{-}final\ weight}{Time} \tag{1}$$

Table 2 The experimental and GP predicted values of MRR at $R^2$ = 99.5%

| $V_0$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | Material Removal Rate | | Error | Linear regression | Error | ANOVA | Error |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Experimental | GP | | | | | |
| 3.5 | 10 | 100 | 50 | 20 | 100 | 1.91 | 1.90 | 0.01 | 1.885 | 0.025 | 157.7 | 1.39175 |
| 3.5 | 12.5 | 100 | 50 | 40 | 200 | 2.32 | 2.317 | 0.003 | 2.72425 | 0.05575 | 3.30175 | 0.90925 |
| 3.5 | 15 | 100 | 50 | 60 | 300 | 5.01 | 4.98 | 0.03 | 3.5635 | 1.4165 | 3.22925 | 1.48125 |
| 3.5 | 15 | 150 | 55 | 20 | 200 | 2.11 | 2.14 | 0.03 | 2.1125 | 0.0875 | 6.49125 | 1.49725 |
| 3.5 | 10 | 150 | 55 | 40 | 300 | 2.52 | 2.541 | 0.021 | 2.969 | 0.241 | 3.60725 | 1.15225 |
| 3.5 | 12.5 | 150 | 55 | 60 | 100 | 5.01 | 5.03 | 0.02 | 2.97125 | 1.40875 | 3.67225 | 1.25025 |
| 3.5 | 12.5 | 220 | 60 | 20 | 100 | 2.52 | 2.47 | 0.05 | 1.79885 | 0.64115 | 6.26025 | 1.23375 |
| 3.5 | 15 | 220 | 60 | 40 | 100 | 2.53 | 2.57 | 0.04 | 2.3591 | 0.5009 | 3.75375 | 1.45525 |
| 3.5 | 10 | 220 | 60 | 60 | 200 | 4.21 | 4.32 | 0.11 | 3.2156 | 1.3144 | 3.98525 | 1.13425 |
| 5 | 12.5 | 150 | 60 | 20 | 100 | 2.54 | 2.57 | 0.03 | 3.52525 | 0.83525 | 5.34425 | 1.37475 |
| 5 | 15 | 150 | 60 | 40 | 200 | 6.64 | 6.635 | 0.005 | 4.3645 | 1.2255 | 3.91475 | 1.589 |
| 5 | 10 | 150 | 60 | 60 | 300 | 7.95 | 7.93 | 0.02 | 5.221 | 0.689 | 8.229 | 1.264 |
| 5 | 10 | 220 | 50 | 20 | 200 | 3.23 | 3.31 | 0.08 | 3.8866 | 0.7366 | 9.214 | 0.753 |
| 5 | 12.5 | 220 | 50 | 40 | 300 | 7.98 | 7.95 | 0.03 | 4.72585 | 2.38415 | 3.983 | 1.80175 |
| 5 | 15 | 220 | 50 | 60 | 100 | 4.54 | 4.575 | 0.035 | 4.7281 | 0.0681 | 9.78175 | 1.49 |
| 5 | 15 | 100 | 55 | 20 | 300 | 3.43 | 3.402 | 0.028 | 4.1175 | 0.5075 | 6.03 | 1.626 |
| 5 | 10 | 100 | 55 | 40 | 100 | 6.46 | 6.55 | 0.09 | 4.137 | 2.673 | 5.056 | 0.744 |
| 5 | 12.5 | 100 | 55 | 60 | 200 | 4.51 | 4.48 | 0.03 | 4.97625 | 0.02375 | 7.204 | 1.75275 |
| 7 | 15 | 220 | 55 | 20 | 100 | 9.76 | 9.77 | 0.01 | 5.8571 | 3.4729 | 6.26275 | 1.592 |
| 7 | 10 | 220 | 55 | 40 | 200 | 3.98 | 4 | 0.02 | 6.7136 | 2.8136 | 11.352 | 1.671 |
| 7 | 12.5 | 220 | 55 | 60 | 300 | 8.68 | 8.67 | 0.01 | 7.55285 | 1.19715 | 5.651 | 1.18575 |
| 7 | 12.5 | 100 | 60 | 20 | 200 | 9.12 | 9.197 | 0.077 | 6.10525 | 3.47475 | 9.86575 | 1.47475 |
| 7 | 15 | 100 | 60 | 40 | 300 | 4.13 | 4.22 | 0.09 | 6.9445 | 2.5145 | 10.59475 | 1.808 |
| 7 | 10 | 100 | 60 | 60 | 100 | 8.18 | 8.164 | 0.016 | 6.964 | 1.396 | 5.938 | 1.274 |
| 7 | 10 | 150 | 50 | 20 | 300 | 9.01 | 9.023 | 0.013 | 6.467 | 2.713 | 9.454 | 1.312 |
| 7 | 12.5 | 150 | 50 | 40 | 100 | 4.32 | 4.38 | 0.06 | 6.46925 | 2.23925 | 10.322 | 1.35975 |
| 7 | 15 | 150 | 50 | 60 | 200 | 8.34 | 8.29 | 0.05 | 7.3085 | 0.9415 | 5.67975 | 1.705 |

Table 3 The experimental and GP predicted values of ΔRa at $R^2$ = 99.5%

| $V_0$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | Surface Roughness $R_a$ | | Error | Linear Regression | Error | ANOVA | Error |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Experimental | GP | | | | | |
| 3.5 | 10 | 100 | 50 | 20 | 100 | 0.4 | 0.401 | 0.001 | 0.45885 | 0.05885 | 0.61953 | 0.21953 |
| 3.5 | 12.5 | 100 | 50 | 40 | 200 | 0.75 | 0.748 | 0.002 | 0.70815 | 0.04185 | 0.8111475 | 0.0611475 |
| 3.5 | 15 | 100 | 50 | 60 | 300 | 0.9 | 0.9012 | 0.0012 | 0.95745 | 0.05745 | 0.850265 | 0.049735 |
| 3.5 | 15 | 150 | 55 | 20 | 200 | 0.7 | 0.6912 | 0.00 | 0.53105 | 0.16895 | 0.9603825 | 0.2603825 |
| 3.5 | 10 | 150 | 55 | 40 | 300 | 0.75 | 0.7611 | 0.0111 | 0.79235 | 0.04235 | 0.8892875 | 0.1392875 |
| 3.5 | 12.5 | 150 | 55 | 60 | 100 | 0.58 | 0.5711 | 0.0111 | 0.74195 | 0.16195 | 0.478155 | 0.101845 |
| 3.5 | 12.5 | 220 | 60 | 20 | 100 | 0.6 | 0.621 | 0.021 | 0.41379 | 0.18621 | 0.8411225 | 0.2411225 |
| 3.5 | 15 | 220 | 60 | 40 | 100 | 0.5 | 0.51 | 0.01 | 0.56319 | 0.06319 | 0.57644 | 0.07644 |
| 3.5 | 10 | 220 | 60 | 60 | 200 | 0.8 | 0.7912 | 0.0088 | 0.82449 | 0.02449 | 0.758045 | 0.041955 |
| 5 | 12.5 | 150 | 60 | 20 | 100 | 0.63 | 0.6324 | 0.0024 | 0.72275 | 0.09275 | 1.026425 | 0.396425 |
| 5 | 15 | 150 | 60 | 40 | 200 | 0.98 | 0.971 | 0.009 | 0.97205 | 0.00795 | 1.20695 | 0.22695 |
| 5 | 10 | 150 | 60 | 60 | 300 | 1.13 | 1.1243 | 0.0057 | 1.23335 | 0.10335 | 1.1956 | 0.0656 |
| 5 | 10 | 220 | 50 | 20 | 200 | 0.92 | 0.9112 | 0.0088 | 0.85194 | 0.06806 | 1.26105 | 0.34105 |
| 5 | 12.5 | 220 | 50 | 40 | 300 | 1.1 | 1.123 | 0.023 | 1.10124 | 0.00124 | 1.306025 | 0.206025 |
| 5 | 15 | 220 | 50 | 60 | 100 | 1.13 | 1.142 | 0.012 | 1.05084 | 0.07916 | 0.9645 | 0.1655 |
| 5 | 15 | 100 | 55 | 20 | 300 | 0.98 | 0.9625 | 0.0175 | 0.93825 | 0.04175 | 1.464725 | 0.484725 |
| 5 | 10 | 100 | 55 | 40 | 100 | 0.9 | 0.887 | 0.013 | 0.89985 | 0.00015 | 1.009875 | 0.109875 |
| 5 | 12.5 | 100 | 55 | 60 | 200 | 1.18 | 1.1921 | 0.0121 | 1.14915 | 0.03085 | 1.16565 | 0.01435 |
| 7 | 15 | 220 | 55 | 20 | 100 | 0.9 | 0.912 | 0.088 | 1.13269 | 0.23269 | 1.410415 | 0.510415 |
| 7 | 10 | 220 | 55 | 40 | 200 | 1.43 | 1.421 | 0.009 | 1.39399 | 0.03601 | 1.741425 | 0.311425 |
| 7 | 12.5 | 220 | 55 | 60 | 300 | 1.7 | 1.756 | 0.056 | 1.64329 | 0.05671 | 1.73761 | 0.03761 |
| 7 | 12.5 | 100 | 60 | 20 | 200 | 1.15 | 1.1612 | 0.0088 | 1.231 | 0.081 | 1.843645 | 0.693645 |

Table 3 The experimental and GP predicted values of ΔRa at $R^2$ = 99.5% (continued)

| $V_0$ | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | Surface Roughness $R_a$ | | Error | Linear Regression | Error | ANOVA | Error |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Experimental | GP | | | | | |
| 7 | 15 | 100 | 60 | 40 | 300 | 1.55 | 1.521 | 0.029 | 1.4803 | 0.0697 | 2.02013 | 0.47013 |
| 7 | 10 | 100 | 60 | 60 | 100 | 1.6 | 1.621 | 0.021 | 1.4419 | 0.1581 | 1.55894 | 0.04106 |
| 7 | 10 | 150 | 50 | 20 | 300 | 1.3 | 1.31 | 0.01 | 1.36185 | 0.06185 | 1.95241 | 0.65241 |
| 7 | 12.5 | 150 | 50 | 40 | 100 | 1.4 | 1.413 | 0.013 | 1.31145 | 0.08855 | 1.534645 | 0.134645 |
| 7 | 15 | 150 | 50 | 60 | 200 | 1.52 | 1.534 | 0.014 | 1.56075 | 0.04075 | 1.43538 | 0.08462 |

Table 4 L27 orthogonal array, ΔRa after each experimental factors

| Exp. No. | A | B | C | D | E | F | $\Delta Ra_1$ µm | $\Delta Ra_2$ µm | $\Delta Ra_3$ µm | Mean ΔRa |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.5 | 10 | 100 | 50 | 20 | 100 | 0.30 | 0.30 | 0.60 | 0.4 |
| 2 | 3.5 | 12.5 | 100 | 50 | 40 | 200 | 0.80 | 0.90 | 0.60 | 0.75 |
| 3 | 3.5 | 15 | 100 | 50 | 60 | 300 | 0.68 | 1.20 | 0.82 | 0.9 |
| 4 | 3.5 | 15 | 150 | 55 | 20 | 200 | 0.64 | 0.90 | 0.56 | 0.7 |
| 5 | 3.5 | 10 | 150 | 55 | 40 | 300 | 0.69 | 0.86 | 0.70 | 0.75 |
| 6 | 3.5 | 12.5 | 150 | 55 | 60 | 100 | 0.72 | 0.42 | 0.60 | 0.58 |
| 7 | 3.5 | 12.5 | 220 | 60 | 20 | 100 | 0.50 | 0.77 | 0.53 | 0.6 |
| 8 | 3.5 | 15 | 220 | 60 | 40 | 100 | 0.44 | 0.58 | 0.48 | 0.5 |
| 9 | 3.5 | 10 | 220 | 60 | 60 | 200 | 0.82 | 0.91 | 0.67 | 0.8 |
| 10 | 5 | 12.5 | 150 | 60 | 20 | 100 | 0.80 | 0.38 | 0.71 | 0.63 |
| 11 | 5 | 15 | 150 | 60 | 40 | 200 | 1.04 | 0.70 | 1.20 | 0.98 |
| 12 | 5 | 10 | 150 | 60 | 60 | 300 | 1.20 | 1.05 | 1.14 | 1.13 |
| 13 | 5 | 10 | 220 | 50 | 20 | 200 | 1.22 | 0.54 | 1.00 | 0.92 |
| 14 | 5 | 12.5 | 220 | 50 | 40 | 300 | 1.45 | 0.25 | 1.50 | 1.1 |
| 15 | 5 | 15 | 220 | 50 | 60 | 100 | 1.35 | 0.74 | 1.30 | 1.13 |
| 16 | 5 | 15 | 100 | 55 | 20 | 300 | 0.97 | 0.67 | 1.25 | 0.98 |
| 17 | 5 | 10 | 100 | 55 | 40 | 100 | 0.90 | 0.64 | 1.16 | 0.9 |
| 18 | 5 | 12.5 | 100 | 55 | 60 | 200 | 1.55 | 0.54 | 1.45 | 1.18 |
| 19 | 7 | 15 | 220 | 55 | 20 | 100 | 1.17 | 0.33 | 1.20 | 0.9 |
| 20 | 7 | 10 | 220 | 55 | 40 | 200 | 1.45 | 1.14 | 1.70 | 1.43 |
| 21 | 7 | 12.5 | 220 | 55 | 60 | 300 | 1.90 | 1.00 | 2.20 | 1.7 |
| 22 | 7 | 12.5 | 100 | 60 | 20 | 200 | 1.3 | 0.70 | 1.45 | 1.15 |
| 23 | 7 | 15 | 100 | 60 | 40 | 300 | 1.25 | 2.00 | 1.40 | 1.55 |
| 24 | 7 | 10 | 100 | 60 | 60 | 100 | 1.66 | 0.69 | 1.87 | 1.6 |
| 25 | 7 | 10 | 150 | 50 | 20 | 300 | 1.50 | 0.69 | 1.71 | 1.3 |
| 26 | 7 | 12.5 | 150 | 50 | 40 | 100 | 1.06 | 1.93 | 1.21 | 1.4 |
| 27 | 7 | 15 | 150 | 50 | 60 | 200 | 1.70 | 1.04 | 1.82 | 1.52 |

Table 5 L27 orthogonal array, MRR after each experimental factors

| Exp. No. | A | B | C | D | E | F | $MRR^1 10^{-6}$g/s | $MRR^2 10^{-6}$g/s | $MRR^3 10^{-6}$g/s | Mean |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2.20 | 1.90 | 1.63 | 1.91 |
| 2 | 1 | 2 | 1 | 1 | 2 | 2 | 2.32 | 2.17 | 2.45 | 2.32 |
| 3 | 1 | 3 | 1 | 1 | 3 | 3 | 5.01 | 4.78 | 5.25 | 5.01 |
| 4 | 1 | 3 | 2 | 2 | 1 | 2 | 2.11 | 1.84 | 2.38 | 2.11 |
| 5 | 1 | 1 | 2 | 2 | 2 | 3 | 2.52 | 4.01 | 3.09 | 2.52 |
| 6 | 1 | 2 | 2 | 2 | 3 | 1 | 5.01 | 4.10 | 5.95 | 5.01 |
| 7 | 1 | 2 | 3 | 3 | 1 | 1 | 2.52 | 2.3 | 2.75 | 2.52 |
| 8 | 1 | 3 | 3 | 2 | 1 | 1 | 2.53 | 2.33 | 2.81 | 2.53 |
| 9 | 1 | 1 | 3 | 3 | 3 | 2 | 4.21 | 4.2 | 4.26 | 4.21 |
| 10 | 2 | 2 | 2 | 3 | 1 | 1 | 2.54 | 2.52 | 2.57 | 2.54 |
| 11 | 2 | 3 | 2 | 3 | 2 | 2 | 6.64 | 6.26 | 7.00 | 6.64 |
| 12 | 2 | 1 | 2 | 3 | 3 | 3 | 7.95 | 8.43 | 7.48 | 7.95 |
| 13 | 2 | 1 | 3 | 1 | 1 | 2 | 3.23 | 3.38 | 3.18 | 3.23 |
| 14 | 2 | 2 | 3 | 1 | 2 | 3 | 7.98 | 7.68 | 8.26 | 7.98 |
| 15 | 2 | 3 | 3 | 1 | 3 | 1 | 4.24 | 4.58 | 4.80 | 4.54 |
| 16 | 2 | 3 | 1 | 2 | 1 | 3 | 3.4 | 3.6 | 3.3 | 3.43 |
| 17 | 2 | 1 | 1 | 2 | 2 | 1 | 6.46 | 6.22 | 6.7 | 6.46 |
| 18 | 2 | 1 | 1 | 2 | 3 | 2 | 4.51 | 4.48 | 4.54 | 4.51 |
| 19 | 3 | 3 | 3 | 2 | 1 | 1 | 9.86 | 9.78 | 9.64 | 9.76 |

Table 5 L27 orthogonal array, MRR after each experimental factors (continued)

| Exp. No. | A | B | C | D | E | F | $MRR^1 10^{-6}$g/s | $MRR^2 10^{-6}$g/s | $MRR^3 10^{-6}$g/s | Mean |
|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 3 | 1 | 3 | 2 | 2 | 2 | 3.98 | 4.36 | 3.6 | 3.98 |
| 21 | 3 | 2 | 3 | 2 | 3 | 3 | 8.68 | 8.67 | 8.69 | 8.68 |
| 22 | 3 | 2 | 1 | 3 | 1 | 2 | 9.12 | 8.97 | 9.27 | 9.12 |
| 23 | 3 | 3 | 1 | 3 | 2 | 3 | 4.13 | 4.01 | 4.25 | 4.13 |
| 24 | 3 | 1 | 1 | 3 | 3 | 1 | 8.18 | 7.64 | 8.71 | 8.18 |
| 25 | 3 | 1 | 2 | 1 | 1 | 3 | 9.49 | 8.23 | 9.31 | 9.01 |
| 26 | 3 | 2 | 2 | 1 | 2 | 1 | 4.32 | 4.16 | 4.48 | 4.32 |
| 27 | 3 | 3 | 2 | 1 | 3 | 2 | 8.34 | 8.23 | 8.45 | 8.34 |

### *2.2. Genetic models*

Discipulus is a manifold-run GP system, which is designed to perform several runs prudently by adapting its parameters of the problem at supply smartly. Multi-run GP is a statistical algorithm that produces a wide range of results by running it over-and-over with the same input variables ranging from very bad to very good. The $R^2$ value denotes the program output quality ranging from 0.05 to 0.99. For generating mathematical models based on GP for MRR and Ra of the AFM process, GP employs a statistics from a large number of experiments including optimal set of input values given in Tables 1-4. The collected experimental data are haphazardly arranged (refer to Tables 2-3) by using a pre-processor called Notitia data preparation software. Then, it is sent to the discipulus GP software in three sets of the data group, viz., validation, training, and applied [24].

A large number of experimental runs are sometimes required more than 100 for the best solution. The discipulus software develops program sequences. They discover useful parameters forming an optimum output in the least potential duration. Initially, the developed programs are run at the arbitrarily available program parameters settings. These are the population size/number of computer programs, number of team size (generations), crossover, transformation (Mutation) and imitation rates, functions, termination sets, number of trials, the extent of subset size, etc. They are then altered to locate optimal sets. The modified factors involve achieving the precise mathematical equation that fulfills the results detailed in Table 6. The parameters set expectation to accomplish the conclusive model providing an empirical relation model that meets the defined conditions [25].

Table 6 The factor setting for GP

| | Value Assigned |
|---|---|
| Parameters | Number of players size (G) = 800 |
| | Number of Team size = 1000 |
| | Depth of subset tree = 6 |
| | Maximum teams / match = 50 |
| | Number of trials = 100 |
| Sets | Function:* (multiplication), + (addition), - (subtraction), / (division) etc. |
| | Termination: “$V_0$, $V_1$, $V_2$, $V_3$, $V_4$, $V_5$, -10, 10”<br>N = {G, arbitrarily chosen constants} |
| Rate of GP | 0.14 (Mutation) |
| | Crossover (70% and 30% non-homologous and homologous respectively) Reproduction: 0.08 |
| Fitness scheme, $r^2$ | $r^2 = \sqrt{(\text{error1})^2 + (\text{error2})^2 + \cdots + (\text{errorn})^2}$<br>where $\text{error} = (\text{program's output} - \text{observed data})$ |

### *2.3. Regression analysis, fitness and correlation coefficient (r/R) estimation*

A regression study is a hypothetical process involving allegorical regression and discovers both the target (operational model of response) function. Target function fixed coefficients of inputs uses an approximation of an error measurement using linear or square fit. The fitness analysis shows the closeness of the response value forecast using GP with the trial results. The correlation coefficient attributes the potential and direction of a direct correlation between response A and inputs B. The squaring of the correlation coefficients gives the relationship between the variation of the predicted output from the inputs. Also, it helps determine the accuracy level in making predictions from a defined model. $r^2/R^2$ is evaluated as the ratio of to the total deviations in the range, $0 < r^2/R^2 < 1$. Additionally, it implies the linear relationship's stability between A and B. It characterizes the percentage of the results confined to the group of best-fitting [26]. For r/R = 0.95 or $r^2/R^2 = 0.9025$, 90.254% of the overall deviation of B (existing between A and B) is clarified with the help of the relationship and, the remaining (9.746%) is in the unsolved/unclear portion of B.

### 2.4. GP modelling parameters

The parameters of GP modeling are presented in Table 6 which demonstrates the group of the fixed factors required to obtain the mathematical equation meeting the conditions of the quantity. The present investigation confidently proposes that the specific GP allocation of trial results obtained by using multiple runs of GP. Mostly, by the standard of the best solutions, not the one which is replicated by other training algorithms. A machine-code GP approach is superior to other methods for its swift actions compared to different available approaches. Their capability to achieve various runs in reasonable time margins on a computer system increases the capability of searching the programs in the high-quality standard of the population.

## 3. Results and Discussion

GP programs hierarchically accommodate the ranked programs which are dynamically accomplished by adjusting the size and configuration. They are developed throughout the simulation by using the convenient function set *F* (arithmetic operators: $+, -, *$ and $/$) and terminal genes set T of symbolic regression. The initial population is produced to accommodate the environment of the solution. Further program structures are continuously modified by using fitness judgment function based on adaptability guidelines and by predicting the output through GP program to match the experimental results. The genetic changes in program structures begin adjusting by following the fitness judgment applied to an initial population to make the repetitive process called evolution.

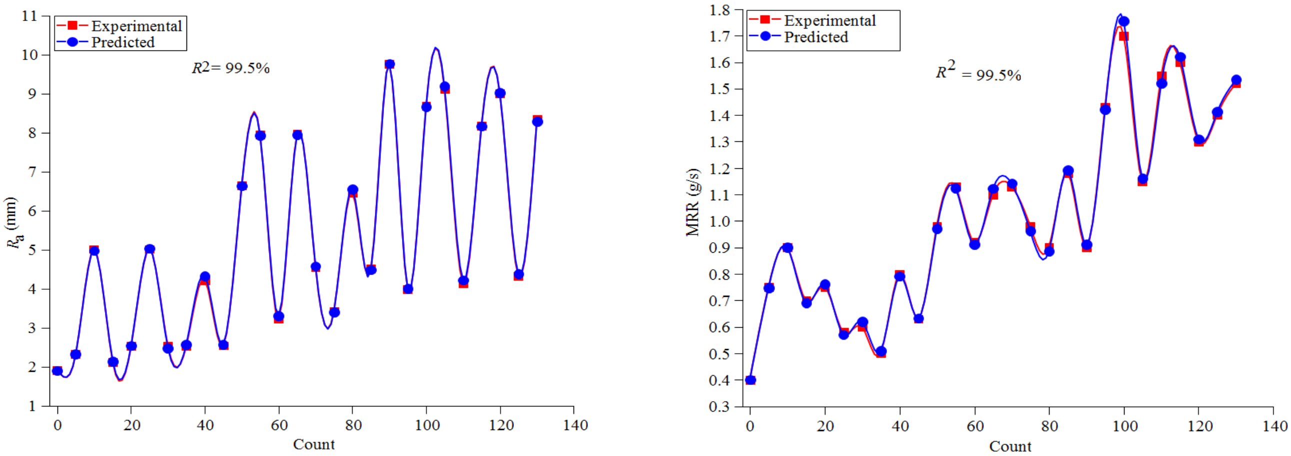
The evolution process stops when the termination standards are met, which produces many solution models of the result sets received from the various experimental runs of finishing quality characteristics of the AFM process. The MRR, Ra are the outputs for defining the finishing quality. The parameters A to F are the input variables. The data units are randomly arranged by using Notitia software through inputting the collected data set into it. The program splits them into three data-set files, namely, training, validation, and applied [27]. The discipulus software treats the first few and last columns as the inputs and an output. The crossover rate uses homologous as a transformation to imitate natural evolution more accurately. It uses non-homologous when instruction programs are to exchange between two successful growing programs without referring to their size and location.

The models of the experimental results in GP discipulus software are obtained in the form of C or C++ or java format. They are simplified in the structure of mathematical equations containing input and output parameters. For the two output characteristics, separate simulations are carried out. The mathematical equations generated for outputs are genetic models involving independent input variables and a dependent output variable. They use the training dataset, which is authenticated by a validation dataset, and checked by an applied dataset [21, 28]. The obtained fittest genetic models are presented in the form of equations 2 and 3. Appendices A and B provide the coefficients used in the empirical equations of the MRR and ΔRa, respectively.

When these Genetic model results are compared with Regression-based mathematical models, which are obtained from ANOVA response surface equations, the error amount is very high. In non-Genetic models up to 3-digit numbers, no error or a deficient error (second or third decimal levels) from the Genetic models is obtained. Other simulation methods need only high valued digit numbers even without decimal level numbers for outputs conforming to input values to minimize error. These techniques accommodate few inputs and generate incorrect empirical models. The above non-GP techniques use the data required to fill design matrix tables. The empirical relations are acquired by using optimal input values instead of every set of data.

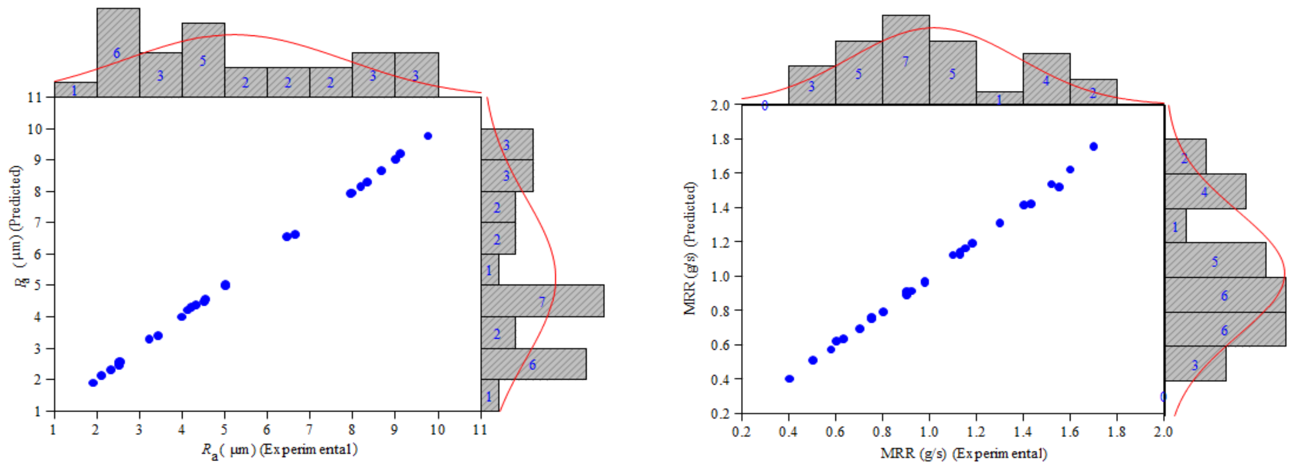
From the results of Tables 2 and 3, the optimal level input parameters produce the equations at the 84% precision level if the outputs are limited to two-digit whole numbers. Thus, the easily optimal combination of the parameters to derive the empirical correlations is acquired by using the regression method. Therefore, the genetic model solutions sustain higher accuracy exceeding statistical examinations for any number of outputs and input values.

A regression fitting curve of GP program involving a number of generations called counts for two output quality characteristics of the AFM process is shown in Fig. 3. The plots comparing the results of the experimental runs and GP predicted for MRR and ΔRa by using the normal distribution chart are shown in Fig. 4. The genetic models score the good accuracy over the experimental data and exhibit higher accuracy of prediction regarding the experimental results.




(a) $R_a$ (b) MRR

Fig. 3 The regression fit for the percentage of finishing process parameters of the AFM




(a) $R_a$ (b) MRR

Fig. 4 The experimental-predicted relationship

The genetic models carry out the analysis of AFM finishing output quality features under various levels of input parameters to generate vast data of Al-MMCs at specific competences. The genetic models produce the experimental results without a skilled operator or adequate familiarity of the process or experimental setup. The genetic models study alone explains the role of input variables range and each input level effect on other levels to associate with finishing quality featureseven it covers all levels of the work-piece to extend their applications.

- Surface roughness

$$\Delta Ra=\left\{1.53\frac{(8(1+2D))^2}{V2}-0.528*D\right\}^2 \tag{2}$$

- Material removal rate

$$MRR=\left\{\frac{3.2\left[\frac{(K+V4)^2}{(\frac{J}{K}+K)}\right]}{V2}+\frac{3.2V4}{V2}+\frac{\frac{1.58(\frac{J}{K}+K)}{K+V4}}{V2}\right\}V0 \tag{3}$$

### 3.1. Comparison with ANOVA models

To understand the accuracy of GP results and their level, these models have to be compared with other standard statistical models of a higher order. Researchers widely accept ANOVA to produce regression models (mathematical models) and declare each factor's significant level. Using Minitab 19 software for the L27 array experimental plan is carried out, and the following equations (4) and (5) are obtained at a 95% confidence interval level. The comparisons between GP and ANOVA results presented in terms of an error obtained among experimental, GP, and ANOVA, are shown in Tables 2 and 3.

$$\begin{aligned}MRR=&157.7+25.4V_0+37.7V_1-1.634V_2-11.22V_3-0.309V_4-1.014V_5+1.505V_0^2\\&+0.441V_1^2+0.00410V_2^2+0.2146V_3^2+0.00264V_4^2+0.000812V_5^2-0.027V_0V_1\\&+0.0730V_0V_2-0.923V_0V_3-0.0678V_0V_4-0.00007V_0V_5-0.0394V_1V_2\\&-0.674V_1V_3+0.1586V_1V_4-0.0399V_1V_5-0.00702V_2V_4+0.00322V_2V_5\\&+0.01576V_3V_5-0.00411V_4V_5\end{aligned} \tag{4}$$

$$\begin{aligned}SR=&-5.15-1.158V_0-2.96V_1+0.1130V_2+0.622V_3+0.0477V_4+0.0652V_5\\&-0.11227V_0^2-0.0698V_1^2-0.0397V_2^2-0.01388V_3^2-0.000035V_4^2\\&-0.000097V_5^2-0.0903V_0V_1-0.00544V_0V_2+0.0659V_0V_3+0.002093V_0V_4\\&-0.000166V_0V_5+0.00733V_1V_2+0.048V_1V_3-0.01196V_1V_4+0.00556V_1V_5\\&+0.000483V_2V_4-0.000289V_2V_5-0.001046V_3V_5+0.00038V_4V_5\end{aligned} \tag{5}$$

## 4. Conclusions

In this GP modeling work, the empirical equations of the MRR and Ra of Al/SiCp MMCs with various percent SiCp samples undergoing a finishing by the developed AFM process and following inferences were drawn. The finishing quality characteristics of aluminum-based MMCs are developed by using discipulus software and C, C++ program. GP was based on the empirical equations, and it forecasted the actual results of the finishing parameters instantly for the given input parameters combination without experimentation.

GP based equations produced more accurate output values with an error of 0.008. They can be acknowledged by the advanced methods for predicting the finishing features at a higher degree of accuracy. Mainly, they are impossible to be acquired by other approaches based on algorithms and regression analysis.

A related issue is when the experimental methods are expensive, difficult to conduct, time-consuming, or the non-availability of mathematical models is involved. The precision of the obtained GP solutions depends on the relationship evolved factors, experimental runs, and an accuracy level of the readings. A large number of experimental readings and more levels of the input factors for improving the construction and composition of the empirical equation were achieved during GP evolution. The improves in terms of that the reliability is up to 99.35 to 99.86% predicted the experimental outputs. The GP-based modeling was thus demonstrated to be a highly proficient and beneficial technique for producing the relationship models existing in data in the absence of the theoretical methods. They overcame the level of the error produced by other methods.

The GP solutions to the present problem may produce erroneous results if the number of parameters is varied, and fewer trial results are provided to the computer program. The GP solutions to the present problem may vary if the number of parameters is varied, and fewer input data to the computer program may produce erroneous results. In the future, GP based models using other mathematical functions (trigonometric, hyperbolic, and exponential) can be generated to compare with the present GP models. This may reduce the size of equations, processing time, and that even one equation can describe the relation between multi-outputs for a given set of parameters. The suggested GP model proficiency will be evaluated with supplementary evolution-based algorithms. They are multiobjective and simulated annealing particle swarm optimization and ant colony optimization for forthcoming explorations. Besides, multiple constraints can also be contemplated for further minimizing the error to a large extent.

## Nomenclature

| | |
|---|---|
| AFM | Abrasive flow machining |
| GP | Genetic programming |
| MRR | Material removal rate |
| MMC | Metal matrix composites |
| EDM | Electrical discharge machining |
| $\Delta R_a$ | Change in Surface roughness |
| $SiC_p$ | Silicon carbide particulates |
| ANOVA | Analysis of variance |
| ID | Inner diameter |
| F | arithmetic operators |
| T | terminal genes set |
| SS | Stainless steel |
| RSM | Response surface methodology |
| Al | Aluminium |
| ANN | Artificial neural network |
| $AlSiC_p$ | Aluminium-silicon carbide particles |
| TD | Taguchi Design |
| SAE | Society of Automotive Engineers |
| $R^2$ | Coefficient determination |
| r/R | Correlation Coefficient |
| N | Number of generations |

## Conflicts of Interest

The authors declare no conflict of interest.

## References

[1] V. K. Jain, Micro manufacturing processes, 1st ed. Florida: Boca Raton, 2012.

[2] R. K. Jain, V. K. Jain, and P. M. Dixit, “Modeling of material removal and surface roughness in the abrasive flow machining process,” International Journal of Machine Tools and Manufacture, vol. 39, no. 12, pp. 1903-1923, August 1999.

[3] W. Zhenlong, G. Xuesong, C. Guanxin, and W. Yukui, "Surface integrity associated with SiCp/Al particulate composite by micro-wire electrical discharge machining," Materials and Manufacturing Processes, vol. 29, no. 5, pp. 532-539, April 2014.

[4] A. C. Wang and S. H. Weng, "Developing the polymer abrasive gels in AFM process," Journal of Materials Processing Technology, vol. 192, pp. 486-490, April 2007.

[5] V. K. Jain, "Magnetic field assisted abrasive based micro-/nano-finishing," Journal of Materials Processing Technology, vol. 209, no. 20, pp. 6022-6038, August 2009.

[6] J. Kenda, F. Pusavec, G. Kermouche, and J. Kopac "Surface integrity in abrasive flow machining of hardened tool steel AISI D2," Procedia Engineering, vol. 19, pp. 172-177, November 2011.

[7] K. B. Judal, V. Yadava, and D. Pathak, "Experimental investigation of vibration assisted cylindrical–magnetic abrasive finishing of aluminum workpiece," Materials and Manufacturing Processes, vol. 28, no. 11, pp. 1196-1202, November 2013.

[8] M. Das, V. K. Jain, and P. S. Ghoshdastidar, "The out-of-roundness of the internal surfaces of stainless steel tubes finished by the rotational–magnetorheological abrasive flow finishing process," Materials and Manufacturing Processes, vol. 26, no. 8, pp. 1073-1084, August 2011.

[9] R. S. Mulik and P. M. Pandey, "Experimental investigations and modeling of finishing force and torque in ultrasonic assisted magnetic abrasive finishing," Journal of Manufacturing Science and Engineering, vol. 134, no. 5, article 051008, September 2012.

[10] V. K. Jain and S. G. Adsul, "Experimental investigations into abrasive flow machining," International Journal of Machine Tools and Manufacture, vol. 40, no. 7, pp. 1003-1021, March 2000.

[11] H .J. Tzeng, B. H. Yan , R. T. Hsu, and H. M. Chow "Finishing effect of abrasive flow machining on micro slit fabricated by wire-EDM," The International Journal of Advanced Manufacturing Technology, vol. 34, no. 7-8, pp. 649-656, August 2007.

[12] S. S. Sidhu, A. Batish, and S. Kumar, "Study of surface properties in particulate-reinforced metal matrix composites using powder-mixed electrical discharge machining," Materials and Manufacturing Processes, vol. 29, no. 1, pp. 46-52, February 2014.

[13] X. Yin and K. Komvopoulos, "A slip-line plasticity analysis of abrasive wear of a smooth and soft surface sliding against a rough (fractal) and hard surface," International Journal of Solids and Structures, vol. 49, no. 1, pp. 121-131, September 2012.

[14] J. N. Muguthu and D. Gao, "Profile fractal dimension and dimensional accuracy analysis in machining metal matrix composites," Materials and Manufacturing Processes, vol. 28, no. 10, pp. 1102-1109, 2013.

[15] M. Yunus and M. S. Alsoufi, "Application of response surface methodology for the optimization of the control factors of abrasive flow machining of multiple holes in Zinc and Al/SiCp MMC wires," Journal of Engineering Science and Technology, vol. 15, no. 1, pp. 655-674, February 2020.

[16] M. R. Sankar, V. K. Jain, J. Ramkumar, and Y. M. Joshi, "Rheological characterization of styrene-butadiene based medium and its finishing performance using rotational abrasive flow finishing process," International Journal of Machine Tools and Manufacture, vol. 51, no. 12, pp. 947-957, August 2011.

[17] A. Kumar, V. Kumar, and J. Kumar, "Multi-response optimization of process parameters based on response surface methodology for pure titanium using WEDM process," International Journal of Advanced Manufacturing Technology, vol. 68, no. 9-12, pp. 2645-2668, March 2013.

[18] J. R. Koza, "Genetic programming as a means for programming computers by natural selection," Statistics and Computing, vol. 4, no. 2, pp. 87-112, June 1994.

[19] S. Singh, P. Kumar, and H. S. Shan, "Quality optimization of surface finishing by magnetic field assisted abrasive flow machining through Taguchi technique," International Journal of Computing Application Technology, vol. 27, no. 1, pp. 31-37, September 2006.

[20] M. Gostimirovic, V. Pucovsky, M. Sekulic, D. Rodic, and V. Pejic, "Evolutionary optimization of jet lag in the abrasive water jet machining," International Journal of Advanced Manufacturing Technology, vol. 101, no. 9-12, pp. 3131-3141, December 2018.

[21] S. Singh and H. S. Shan, "Development of magneto abrasive flow machining process," International Journal of Machine Tools and Manufacture, vol. 42, no. 8, pp. 953-959, March 2002.

[22] M. R. Sankar, J. Ramkumar, and V. K. Jain, "Experimental investigation and mechanism of material removal in nano finishing of MMCs using abrasive flow finishing (AFF) process," Wear, vol. 266, no. 7-8, pp. 688-698, October 2008.

[23] J. Li, L. Yang, W. Liu, X. Zhang, and F. Sun, “Experimental research into technology of abrasive flow machining nonlinear tube runner,” Advances in Mechanical Engineering, vol. 6, article 752353, February 2015.

[24] M. Yunus and M. S. Alsoufi, “Mathematical modeling of multiple quality characteristics of a laser micro drilling process used in Al7075/SiCp metal matrix composite using genetic programming,” Modelling and Simulation in Engineering, vol. 2019, pp.1-15, January 2019.

[25] M. Brezocnik, M. Kovacic, and M. Ficko, “Prediction of surface roughness with genetic programming,” Journal of Materials Processing Technology, vol.157–158, pp. 28-36, December 2004.

[26] B. Jabbaripour, M. H. Sadeghi, Sh. Faridvand, and M. R. Shabgard “Investigating the effects of EDM parameters on surface integrity, MRR and TWR in machining of Ti–6Al–4V,” Machining Science and Technology, vol. 16, no. 3, pp. 419-444, August 2012.

[27] J. R. Koza, Genetic programming: on the programming of computers by natural selection, Cambridge, Massachusetts: MIT Press, 1992.

[28] M. Yunus and M. S. Alsoufi, “Mathematical modelling of a friction stir welding process to predict the joint strength of two dissimilar aluminium alloys using experimental data and genetic programming,” Modelling and Simulation in Engineering, vol. 2018, article 4183816, April 2018.



## Appendix A (Coefficients of equation of Material Removal Rate, MRR)

$$A=\left\{0.14-\frac{0.00154}{V_4}+\frac{0.14}{V_3}+0.00208\left(\frac{V_4}{V_3}\right)^2\right\} \tag{A1}$$

$$B=\left\{\frac{0.0011V_4^{\,2}}{V_3}-0.14+\frac{0.00154}{V_4}-\frac{0.14}{V_3}+\frac{0.00208V_4^{\,2}}{V_3}\right\} \tag{A2}$$

$$C=2A+\frac{16384V_0}{A^3}\left(\frac{B}{V_5}\right)^8 \tag{A3}$$

$$D=A+\frac{4096}{A^3}\left(\frac{B}{V_5}\right)^8-\frac{16\left\{\frac{8(C^2+1.25)}{V_5}+A+\frac{4096}{A^3}\left(\frac{B}{V_5}\right)^8\right\}}{\frac{4096V_0B^8}{A^3(V_5)^4}} \tag{A4}$$

## Appendix B (Coefficients of equation of Surface Roughness, ∆Ra)

$$A=\left(\frac{0.26}{V_5}-1.91\right)^2 \tag{A5}$$

$$B=\left\{\left(\frac{0.033A+V3}{V1}+1.643\right)-A\right\}^2-V0 \tag{A6}$$

$$C=\frac{1}{\left\{A+(B+0.67)V_0\right\}^2}-A \tag{A7}$$

$$\mathrm{D}=\frac{\left[\frac{\{\mathrm{A}+1.06\}^{2}+\mathrm{V}_{1}}{\mathrm{CV}_{0}}+\mathrm{C}\right]}{\mathrm{V}_{4}} \tag{A8}$$

$$\mathrm{E}=\frac{\left[2\mathrm{V}_{4}-\frac{(C+D^{2})}{V_{0}}-10.23\right]}{\frac{(C+D^{2})}{V_{0}}+12.93} \tag{A9}$$

$$F=\frac{-3.82E}{\frac{\left(C+D^{2}\right)}{V_{0}}+12.93} \tag{A10}$$

$$G=\frac{\left(C+D^{2}\right)}{V_{0}}+12.93-\frac{3.82E}{\frac{\left(C+D^{2}\right)}{V_{0}}+12.93} \tag{A11}$$

$$H=\frac{\frac{G}{(F+V_{0})}+\frac{4}{(F+V_{0})G^{3}V_{1}}-V_{2}}{\frac{4}{(F+V0)G^{3}V_{1}}-V_{2}} \tag{A12}$$

$$I=4\left[V_{0}^{\;2}+\frac{(\frac{3.34+1.928H}{V_{5}})^{2}}{V_{2}}\right]-V_{2}-1.78 \tag{A13}$$

$$J=\left\{2.928H+3.34+1.38I-(\frac{1.38IV}{V_{5}}+V_{0})^{2}\right\}*\left(\frac{(\frac{1.38IV}{V_{5}}+V_{0})^{4}}{V_{2}}-0.22\right) \tag{A14}$$

$$K=2V_{0}V_{1}\frac{\left(\frac{(\frac{1.38V_{0}}{V_{5}}+V_{0})^{4}}{V_{2}}-0.22\right)^{2}}{V_{1}}-V_{2} \tag{A15}$$