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THERE IS, PROBABLY, NO NEED FOR A DESIGN FRAMEWORK

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Abstract

I present my perspective on the design process in this article, arguing for a focus on student learning and "slow design" that stems from knowledge of mathematics and their support system in the learning process. I have a question about the design process academization and task design research direction. Numerous examples from my work at the Freudenthal Institute are used to illustrate this paper.

Keywords: Design Framework, Design Process, Design Research, Slow Design, Mathematics Knowledge, Task Design, Freudenthal Institute

Abstrak

Saya menyajikan perspektif pribadi saya tentang proses desain dalam artikel ini, dengan alasan untuk fokus pada pembelajaran siswa dan slow design yang berasal dari pengetahuan matematika dan sistem pendukungnya dalam proses pembelajaran. Saya punya pertanyaan untuk proses desain akademik dan arahan tugas penelitian desain. Banyak contoh dari karya saya di Institut Freudenthal digunakan untuk mengilustrasikan makalah ini.

Kata kunci: Kerangka Desain, Proses Desain, Penelitian Desain, Slow Design, Pengetahuan Matematika, Desain Tugas, Institut Freudenthal

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This article is heavenly based on a Chapter from the present author with the title, "There Is, Probably, No Need for This Presentation," presented in ICMI study 22 (de Lange, 2015). However, there is some information that has not been written in that paper. Therefore, in this article, the author completes or adds some information to improve the previous article.

It's all starting when an article about the Millennial generation was published by TIME magazine in the spring of 2013. According to the author, J. Stein, the 'Millennials' are a generation primarily composed of teenagers and twenty-somethings known for constantly holding up cameras, photographing themselves, and posting them online. In addition, they are selfish, arrogant, entitled, and slothful. Then a deep sigh follows:

> "Now, imagine being used to that technology your whole life….. and having to sit through Algebra" (Stein, 2013).

Indeed, if you are required to sit through Algebra, is there any way for designers to engage millennials in mathematics learning? Was everything we did in the recent past pointless? The reader will grasp the significance of the tile I have chosen for this presentation. It is even worse if we believe Chris Schunn when he observes:

> "The educational design community has no communal mechanisms for codifying craft knowledge" (Schunn, 2008).

Thus, not only does the designer struggle to relate to today's youth, but we, as designers, lack the tools and knowledge necessary to articulate our craft knowledge. Do you believe this is bad news? Consider this for a moment: we lack communal mechanisms for codifying craft knowledge, which has an advantage if we take Collopy seriously (which we do):

"Codifying design thinking threatens it's central value of flexibility" (Collopy, 2009).

There is a (need for a) framework for educational design. Thus, I must take the title of this article seriously; is a system necessary in educational design? Assuming we are experienced and proven designers (based on research?), how do we assist young and promising designers to improve their skills based on data that may be integrated into some framework or system in the future?

We can find some definitions of educational design in the literature. Among the most promising:

"Educational Design:

- to create and execute according to plan
- to conceive and plan out in the mind
- is very ego involving (own mark)
- to make drawing or sketch: process of design"

(Webster, 2008)

Creating and carrying out a plan according to the project may appear trivial. Even so, it's easy to interpret this as a cautionary tale: creative processes frequently continue indefinitely; there is always room for improvement. This could even be a problem, particularly in the design of educational materials. If you view design as a recursive process that must be validated through classroom experiences, declaring that the plan has been carried out without conducting research requires courage.

The second point, on the other hand, appears more reasonable. Before you begin the actual design, your mind is filled with thought experiments, dreams, and wandering thoughts. The sublimation of the study and mental processes appears to be the origin of concrete design. Indeed, it creates the illusion that you are executing a plan. At least temporarily.

One may wish to deny that design is or must be in some way ego evolving. There is, however, no reason to contest this fact. A designer is constantly attempting to create something beautiful, and it would be nice if others recognized your work as uniquely yours. Additionally, the creator can quickly be credited with several instances of innovative or challenging educational design.

Several stages of the process require drawing and sketching, or more broadly, visualization. This may include the following: designing longitudinal development over time, identifying different levels of competency, visualizing concepts in ways other than through words, and many others.

Thus, it is possible to reflect on one's practices through the use of definitions. However, numerous facets are omitted from this concise list, which may be detrimental to future designers. As a result, it's unsurprising that Schunn (2008) poses the following question:

"Is there a system to the madness of design?" (Schunn, 2008)

To compound matters, we must admit that it is not easy to reflect on one's system (assuming there is one):

> "People do not have direct access to the mental processes they follow" (Anderson, Lebiere, & Lovett, 1998)

Thus, describing your mental processes is extremely difficult, even more so when combined with Schoenfeld's observation of reality:

> "Educational designers have few incentives to codify their reflections and present them publicly" (Schoenfeld, 2009).

Given the preceding observations, we see that writing about the design process may indeed be a near-impossible task. However, the challenge is even more significant. Consider Schön's observation:

> "Design takes place in the swampy lowlands: problems are messy. The irony is that the problems of the high ground (theory) tend to be relatively unimportant to individuals or society at large, while in the swamp lie the greatest problems. The practitioner is confronted with a choice: shall he remain on the high ground where he can solve relatively unimportant problems or shall he descend to the swamp of important problems where he cannot be rigorous in any way he knows how to describe?" (Schön, 1995)

Given these observations, one must exercise caution when writing about the design process. As a result, one should not take the following statements too seriously: I am unable to access the mental processes that I am following, there may be no system to my madness, I am forced to choose between high and low ground, and the entire context is confusing. Nevertheless, we'll attempt it regardless.

The next section of this article discusses about my personal reflection on the design process based on my work experiences at Freudenthal Institute, Utrecht University. Several examples of these works are used to illustrate this paper. De Lange (2015) has been explore several reflections on the process starting with slow design until fast design, including the principles, process, meet the real world, example, and conditions. Furthermore, I will continue to explore more about integrating STEM from pre-primary to primary, pre-algebra, design as art, geometry, exponents and logarithms.

PERSONAL REFLECTIONS ON THE DESIGN PROCESS

Slow Design

De Lange (2015) stated that there are some principles for slow design. First, it's tempting, to begin with, a design process that appears to have vanished: a process founded on developmental research (nowadays frequently referred to as Design Research) and guided by high quality and demonstrated effectiveness (evidence). However, it requires considerable effort and time, requires serious planning and execution, goals (when will we achieve these goals?), and assessments, among other things. As a result, it is considered excessively expensive.

The question is whether we are deluding ourselves here. Wouldn't it be interesting to compare the costs of a genuinely innovative and evidence-based 'program' to the costs of a hastily constructed 'program' with marginal evidence that serves the purpose and objectives but will sell well due to intelligent and professional marketing? The prices of such a process may ultimately be much higher in terms of output: ineffective at achieving actual improvement, failing to adhere to any 'standard' of educational design, and necessitating the development of 'even better' materials.

Slow design leading principle is based on integrated partnerships: researcher/designer collaborations and researcher/designer collaborations with teacher/student collaborations. Alternatively, speak a different language: from the heights to the muck of the swamp, and attempting to be relevant at all levels. Following that, we know we must describe a plan. This plan is a description of a process that evolved from a wandering mind and years of experience. Of course, this is only a sketch and should not be taken too seriously; instead, it should guide through the maze that is an educational design.

Additionally, de Lange (2015) delves into the process of slow design's first phase, which consists of seven stages, namely:

1. Choose a subject (especially if you are a junior designer), a duration, and a level.
2. Create a (mental) flow diagram and educational/didactic vision.
3. Utilize your intuition.
4. Concept selection.
5. Contextualization.
6. Look for inspiration in the library's 'random' search function (associative thinking).
7. Refine your initial concept.

Thus, the design process's first phase is complete. After that, it's time to leave your comfort zone and venture into the real world. The primary challenge at this point is to avoid becoming excessively defensive: take all comments seriously and avoid taking definitive positions until you have heard everything. Following that, however, the options are entirely up to you.

De Lange (2015) discusses two activities in the meet, the real-world process. First, we must have a 'real' discussion with experts of all kinds, including 'real' mathematicians, to have your design is torn apart (very desirable and uncommon). Then, finally, we write a revised version that is close to being ready for classroom experiments.

As we all know, there are numerous alternate realities, each with its own set of rules and culture. However, interaction with experts is not an accurate representation of the real world. In and around the classroom, the ultimate authorities are located. In the subsequent phase, we will meet with teachers to discuss the following: (1) discuss your design with experienced teachers and change it if you believe it is a significant improvement; (2) revise and have the teacher teach using your design (NEVER teach yourself); (3) observe classroom activities without video; (4) make revision notes; take discrepancies between the terms 'intended,' 'implemented,' and 'achieved' seriously; (5) Priority should be given to fundamental conceptual development, not to details (de Lange, 2015). Unsurprisingly, the following steps restart the cycle in a different classroom. If necessary, the next redesign emphasizes designing formative and summative assessments and indicating opportunities for excellent feedback (Black & Wiliam, 1998).

Only six conditions exist for slow design (de Lange, 2015). Firstly, some freedom is in terms of what to design and a great deal of time. Next, we must freedom of thought (no pressure from publishers, standards, etc.) and freedom to explore. Lastly, with one restriction, we design within the philosophy you have chosen; in my case, a free interpretation of realistic mathematics education and take that restriction with freedom.

Fast Design

A different set of rules applies in the highly commercialized educational design community. First, we observe a greater degree of separation:

1. The designer creates.
2. The researchers conduct classroom observations.
3. The technology component is delegated to specialists.
4. The teacher prepares and teaches.
5. The student does as the teacher directs.

The primary issue with this type of process is obvious—a lack of communication between all of the design's compartments. Coherence and convergence are being strained. The designer has no idea what happens to his or her design later in the design cycle, and the likelihood is that he or she will feel no responsibility for the end product. Design is treated as a noun rather than a process. In this mode of invention: design is a noun rather than a process.

Of course, you must deal with and adhere to standards for a variety of reasons. One can frequently argue about the quality of even the 'best means, and according to the designers, all 'standards' are the best. And while they all engage in some form of mathematical practice, the description leaves a lot to be desired from a designer's perspective.

Rapid design necessitates rapid product development and is highly market-driven. Thus, 'appearances' are critical: many books feature full-color illustrations that bear little resemblance to mathematics. Quite frequently, names are everything: the book's cover features the names of more or more minor well-known mathematicians or math educators. It's difficult for an educational designer to be delighted with the more creative and out-of-the-box thinking aspects of design. Lastly, on occasion, design can be considered an art form.

ONE FINAL OBSERVATION

From Pre-Primary to Primary: Integrating STEM (de Lange et al., 2013)

Curiosity as starting point

It seems desirable from the child's point to start more general in scientific reasoning without identifying specific school disciplines like math. Following Mechler (2015), we see that "integrating math and science concepts throughout early childhood (classrooms) promotes children's development in all domains: cognitive, social, emotional, and physical."

We should avoid limiting the children to their great potential by specializing too soon. Young children do not perceive their world as divided into separate cubbyholes such as 'Math' or 'Literacy' (Clements, 2001).

Playing in a challenging context should be part of the children's lives forever. Or, as Resnick put it: We need a lifelong kindergarten (Resnick, 2017). Slowly moving into more formal representations and defining more narrowly the disciplines offers children the possibility to use their natural curiosity longer. We show four examples of the category 'classification and seriation' as critical concepts for young children.

1. Animals

   Question with Figure 1: “Which ones ‘belong’ together?”

   This turns out to be a very successful activity. Children start to explore if they ‘know’ animals. Where they might have seen them. Whether or they are dangerous and many more observations. The simple question gets them all excited. Children will start by collecting the white ones, the flyers, the swimmers, the ones with two or four legs, the ones with feathers (excluding the bat), and numerous more ways to ‘classify’ them. They make groups by classifying them, compare the groups: more than, less than, equal, ordering the groups in size by actually making some kind of histograms. They can go on forever.

   Another question is: “Can you put them in order?”.

   The answer by children is “Yes!”. Again, different options: you can put them in order of size as they are as plastic models, or you can put them in order as they are in the real world. Exciting discussions may follow: which one is larger, the crocodile or the deer? The children know definitions to answer this question as well.

Figure 1. A Collection of Animals

2. Other objects (Figure 2)

   No animals available? No problem! Ask children to bring one shoe each to collect in front of the classroom. It is incredible how many different ways there are to classify running shoes. Classification and seriation are the main underlying concepts, but lots of ordinary curriculum content can be covered or addressed.

Figure 2. A Collection of Running Shoes

3. Shapes

   Question with Figure 3: "Which ones 'belong' together?"

   Especially for younger children (from 3 years onward) mosaic tiles are wonderful. Children and parents alike embark first on the simple (?) task to copy and make the mosaic. Can there be made others? Count the pieces. Classify them, just as the animals by shape or color. If we look at the above pictures we might give the following answers: shapes as organizing principle (triangles), next one color as principle (Red/orange), next shapes again (squares) etc.

   Prepare them for the familiar curricular shapes from any geometry curriculum. Make your own more complex collection of shapes in different colors.

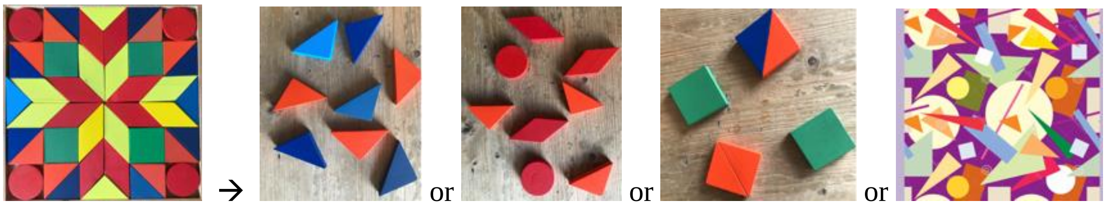


Figure 3. Mosaic Tiles

4. Numbers

   Although the picture might seem different from the mosaic ones, the question is again the same question with Figure 4: "Which ones 'belong' together?"

Again, the same question as before. It shows the abstraction process: classification and seriation can be done in many contexts. The worksheet shows 'numbers' in a variety of colors and sizes. How are the children making classifications and seriation?

Figure 4. A Collection of Numbers

The square root of 4 might seem an outlier but that is only so for math-experts. Children might argue that 4 and V4 are a group by itself.

The design of this sequence of related and connected activities is an example of slow design. We started with a large collection of animals with a wide array of classifications and seriation as result. As there were occasions where the animals were not available we realized ourselves that sport shoes could do just as well, but maybe less exciting.

From there on we went to the already available mosaic tiles: that we could still do the same questions was quite a surprise. Classification and seriation (put the animals in order of size) are seen as natural ways to organize this part of the children's world.

The next sheet with the numbers is again a surprise: we can organize them by color, by the physical size, by the size of the quantity they represent (is 4 the largest number or 123? From here on it becomes almost 'natural' to classify numbers in the mathematical way.

As a summary, we started with real toys which can be used in many different ways. Young children will try to find the names of the animals, compare their 'skin', cluster them by color (When can we say an animal is white?), How they move: flying, swimming, running. Look at their size; whether in reality or as a model toy. By speed: Catlike animals are fast. Or by having 2 or 4 legs. As long as the students have a good argument it's a good classification. Seriation is another way of looking at this set of animals. First you should be prepared to see that we can do this in at least two ways: as animal toys or as real animals. Avery interesting discussion might take place when we enter the discussion about how to measure: we observed the discussion between the size of a deer and a crocodile presented in Figure 5.

Figure 5. Looking at the Size of the 2 Animals: Students Did Choose the Deer

Popular choices: Polar animals; Flying animals, Swimming Animals, 2 legged or 4 legged, White animals, Black animals, Running Birds, Flying Birds and Bats, meat eaters and Green eaters, Zoo Animals, Home Animals, Wild Animals, etc.

Creativity and Curiosity

Young children are curious and frequently inventive. Curiosity is a characteristic associated with inquisitive thinking, such as exploration, investigation, and learning, evident in humans and other animals (Berlyne, 1954). Curiosity plays a significant role in all aspects of human development, including the learning process and the desire to acquire knowledge and skills. This is an innate quality.

> "Curiosity and imagination have been neglected in epistemology. We argue that the role of curiosity and imagination is central to the way we think, regardless of whether it is thinking about problems of ethics or problems of science. In our ever more materialistic society, curiosity and reason are either discouraged or narrowly channeled". (Loewy, 1998).

Children can stay curious or even develop their curiosity by designing and offering new activities that are not common in curricula. "Cultivating Creativity through (Projects, Passion, Peers and) Play" offers some insight into creativity in relation to curiosity (Resnick, 2017). He asserts that there is no doubt that babies are born curious. According to some, the best way to foster children's creativity is to stay out of their way: you should not teach creativity; instead, you should stand back and allow children's natural curiosity to take over.

That is too easy. Creative thinking indicates a creating, new, uncommon phenomenon. Children are excellent in it, based on their curiosity. But they need to be nurtured.

Creativity and Curiosity Broadens: The Interest and Concepts

As indicated before you can also change the 'Animals' into 'Running Shoes, and do the same activities. The concept of similarity (if not already done with their animals' cab introduced here easily: especially through asking the questions about dissimilarity. And we can discuss how the shoes are similar, but very importantly" how are hey different?"

We can discuss how solving the problem with the animal toys is different from the shoe problem. For parents and kids there are numerous problems that are the same from a conceptual point of view. Take you visit to a shop or a Supermarket and observe how the goods are 'categorized' in a shop. A typical supermarket may look like in Figure 6.

Figure 6. Supermarket

Let the students discuss their supermarket: what can you find where? Well known difficulties in Supermarket can be: where to find the Sugar and where to find Eggs?

If you go to another NEW supermarket you might be surprised that there is no real logic. The following article will not come as a surprise:

Where's the sugar? Supermarket robot creates product maps as it takes stock

Another beautiful subject for an activity like the previous ones: Taking stock of your books; for instance, in a library seen in Figure 7.

Figure 7. Sorting Books

Left book collection: by color; right collection: the best 100 books, hard cover. The earlier shown number example is easy to organize in different ways, for different levels of students. By color: black, blue, red, green/ by size from large to small: 4, sqr4, 1,33333, 0.66, -5, 0,34, 7/8, -1/22, sqr2, -3/4, 34, 123, and 22.22, as shown in Figure 8.

Figure 8. Sorting Numbers

On A Higher Level: What Kind of Number

Starting with this exploration can lead to an introduction to number systems: for instance, by putting them on the number-line: also ordering from small to large but not by size in the geometrical sense.

-26, -5, -11/22, -1/22, 0,34, 1/2, 7/8, 1,3333, sqrt2, sqrt4, 4, 22,22, 34, 123

From the animals to number-systems is an example of slow design.

Pre-Algebra (van Reeuwijk, 1995)

The students are presented the following problem presented in Figure 9, such as 2 T shirts plus 2 drinks cost 44 dollars; 1 T shirt plus 3 drinks cost you 39 dollars. The problem is what is the price of a T shirt? And of a drink?

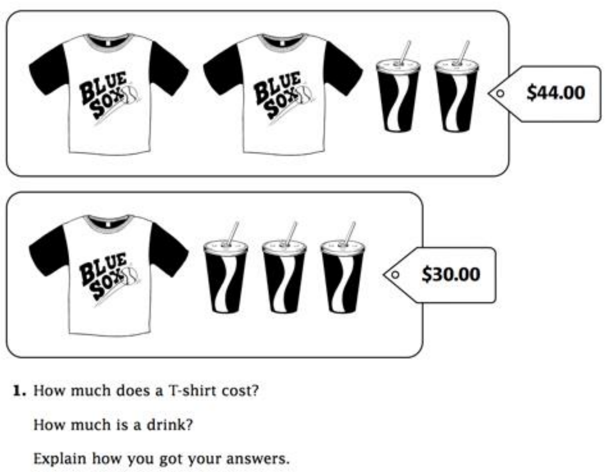


Figure 9. Solving an Algebra Problem

The students in the classroom are ranging from 11 to 14 years of age. We have collected some solutions to see how they reflect students thinking presented in Figure 10.

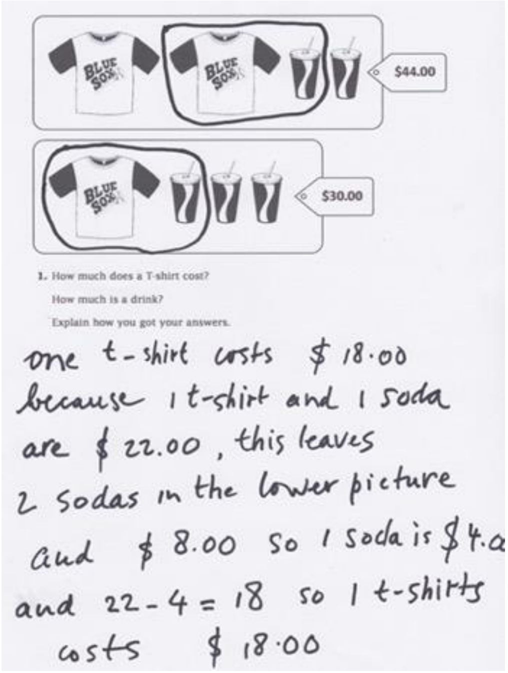


Figure 10. The First Student's Solution of problem

The student's solution goes as follows: 1 t-shirt and 1 drink cost 22 dollars (first sentence). In the second sentence we see again the 1 T shirt and 1 drink. This leaves 8 dollars for 2 Drinks. Or: 1 drink costs 4 dollars, 1 T shirt 18 dollars.

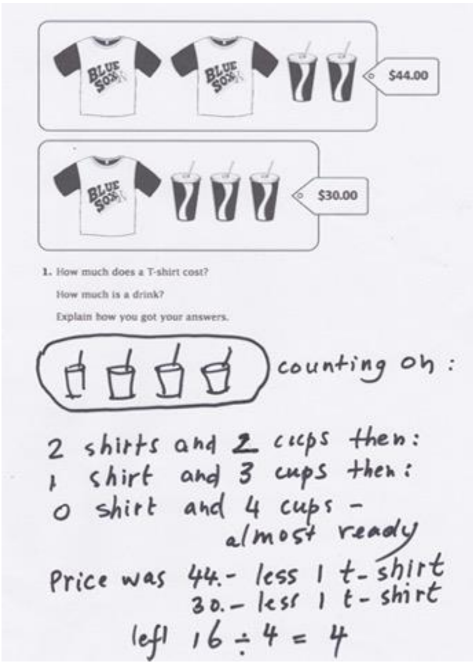


Figure 11. The Second Student's Solution of Problem

Figure 11 shows that the student's solution here differs quite a lot from the first student: The student counts on as follows 2 shirts and 2 cups for 44; 1 shirt and 3 cups for 30; 0 shirt and 4 cups must be 16; and 1 t-shirt is 4 dollars.

Figure 12 shows the third student's argumentation as follows subtract the first and second sentence; results in 2 Cups cost 30 minus 22 dollars; or 4 a piece and 18 for 1 T shirt.

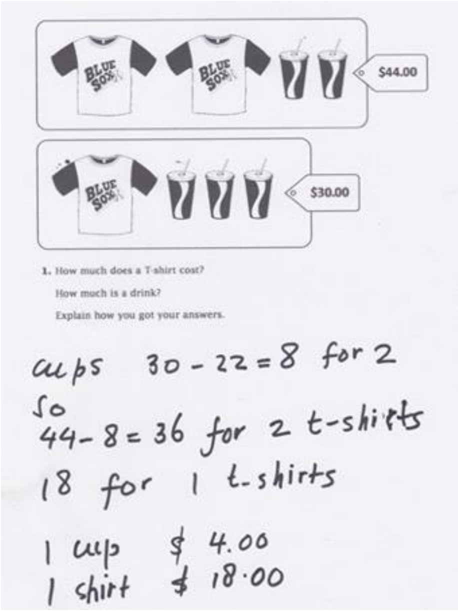


Figure 12. The Third Student's Solution of Problem

It is clear from these three solution methods that we can elaborate this part of the learning trajectory in the following way represented in Figure 13.

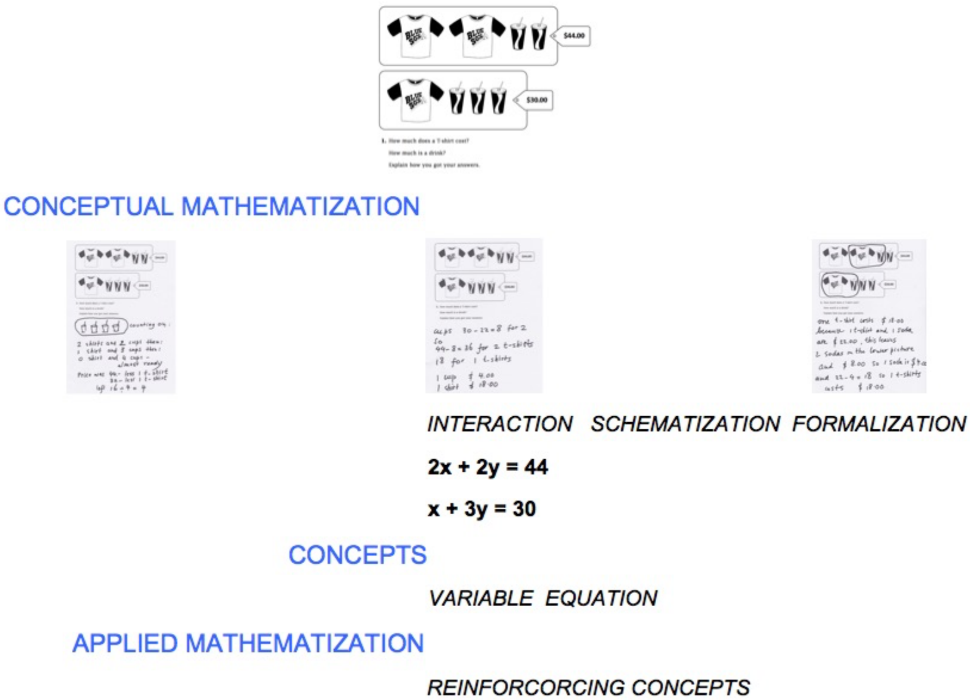


Figure 13. Schema of Mathematization

Design as An Art

Another example, which was tested in middle school and with graduate students of mathematics, will shed light on assessment possibilities and motivate students to develop into mathematical reasoners. The issue begins with a short strip detailing the process by which we created in Figure 14.

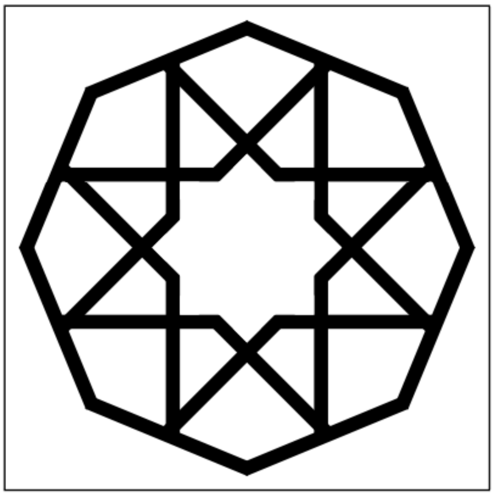


Figure 14. The Moorish Star

Figure 15 illustrates the progression toward the Moorish Star. The strip's first figure is a "cross." The second figure is formed by extending the horizontal and vertical segments. The second figure is rotated 22.5 degrees around its center, and four new elements are drawn to create the third figure. Thus, the third figure's angle is 22.5 degrees.

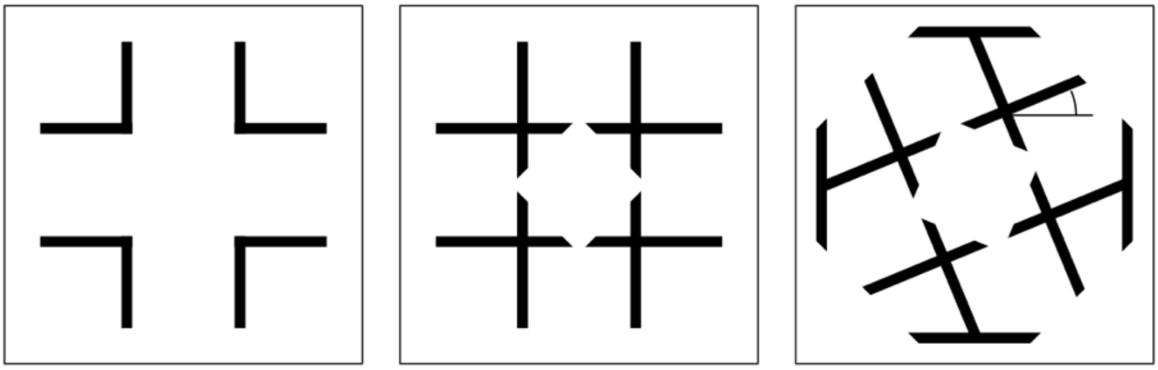


Figure 15. Progressive development towards Moorish Star

The objective is to create a Moorish star pattern using transparencies that depict the shape. A sample of this type of Moorish star pattern is defined at the top of the following page (Figure 16). After a while, the majority of students are capable of becoming a star. The real question is as follows: given that a critical angle is 22.5 degrees, what are the other curves in the star pattern measured in degrees?

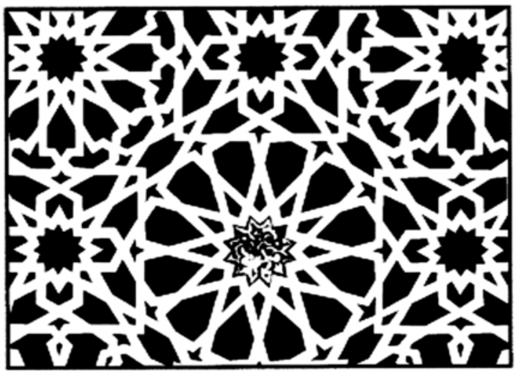


Figure 16. Moorish Art

And how did we end up with the following star using duplicate transparencies? What do you know about the number of star points and the total number of possible angles?

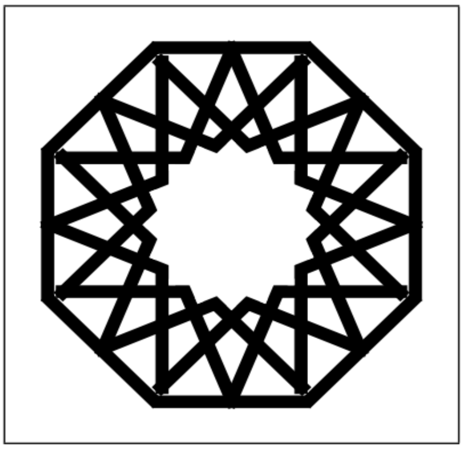
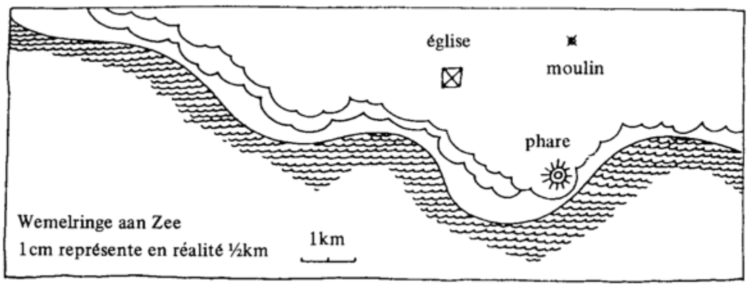


Figure 17. Complex Star

Both the 14-year-olds and 24-year-olds were, without a doubt, challenged and surprised. Above all, they encountered the unexpected beauty of mathematics, as illustrated in Figure 17. Mathematics as art, design as art.

Geometry: Orientation

A boat is sailing along the coastline. The map of that coast looks as shown in Figure 18.



church tower windmill light tower

Figure 18. Coastline Map

There are three clearly visible landmarks: a church tower, a windmill and a light tower. The captain makes a couple of pictures when sailing along the coastline as presented in Figure 19. The captain did not remember in which order the photos were taken. The question is “In which order were the pictures taken?”

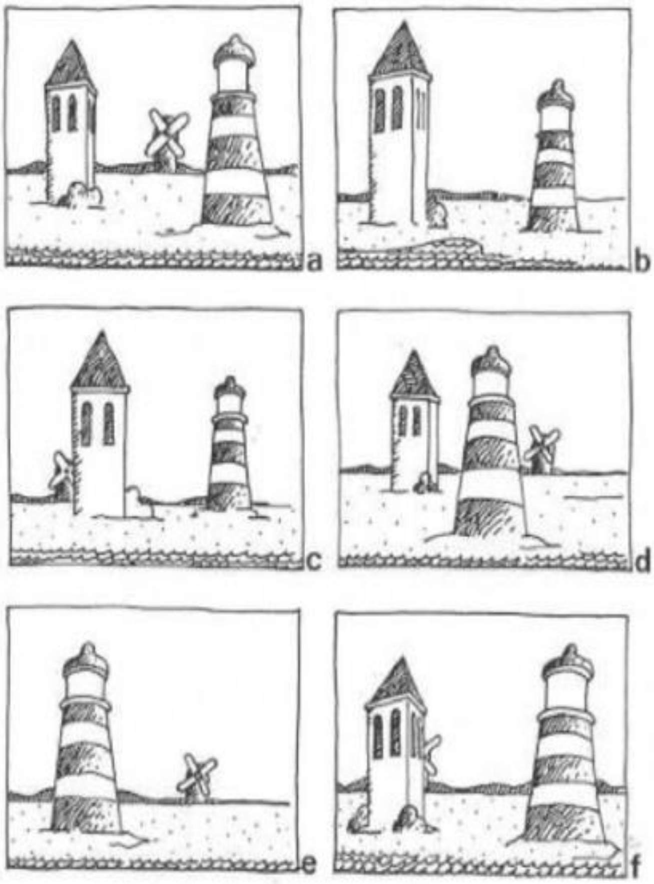


Figure 19. The six photos (Freudenthal et al., 1976)

DESIGN EXAMPLES

Central Concept Design

This type of design is guided by a central mathematical concept or subject as implied by the title. The right triangle serves as the focal point of the entire unit. At first, it's pretty informal. ("Can you see the Colorado River from the Grand Canyon's rim, and explain why?"). Thus, not only is the concept of vision lines explored, but it is also more or less formalized. Additionally, vision lines are central to the concepts of blind angle, blind area, and shadows.

Ladders have critical angles or a certain degree of steepness. Thus, the shift to this context is entirely natural, even more so when combined with increased shadows. The right triangle, the object that connects all the different contexts, reappears as the glide ratio, a more formal term than the tangent. Figure 20 illustrates the various contexts in which the same concept is used: tangent and ratio.

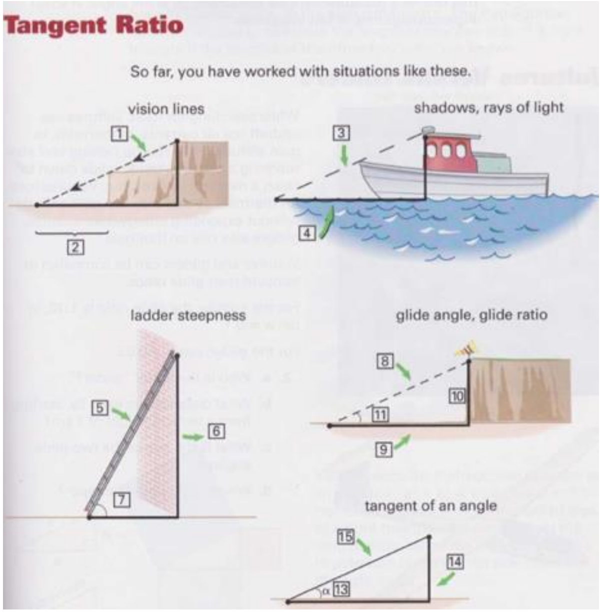


Figure 20. Example Central Concept (Feijs et al., 2006. Copyright Encyclopedia Britannica 2006)

Central Context Design

A design format that can be pretty appealing uses the same context with various mathematical concepts that fit more or less authentically in this context for a slightly more extended period. And sometimes, it's simply a matter of 'intuition' as to how far you can go in this direction. Or, as is also possible, too far. In the late 1970s, I received a request from a teacher to 'do something' with trigonometric ratios in the upper middle school region. She was certainly capable of upsetting a designer, as he was also a pilot. As a result, a two-week unit was designed around the concept of flying through trigonometry in a relatively short time. A little later in this section, you'll see an early draft of a unit, initially written in pencil and piloted in this format. The glide ratio was a central concept in the flying context, as illustrated in Figure 21.

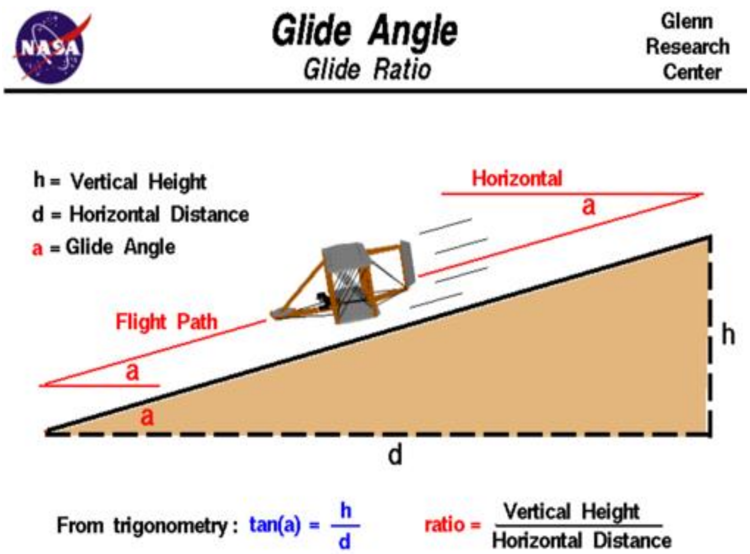
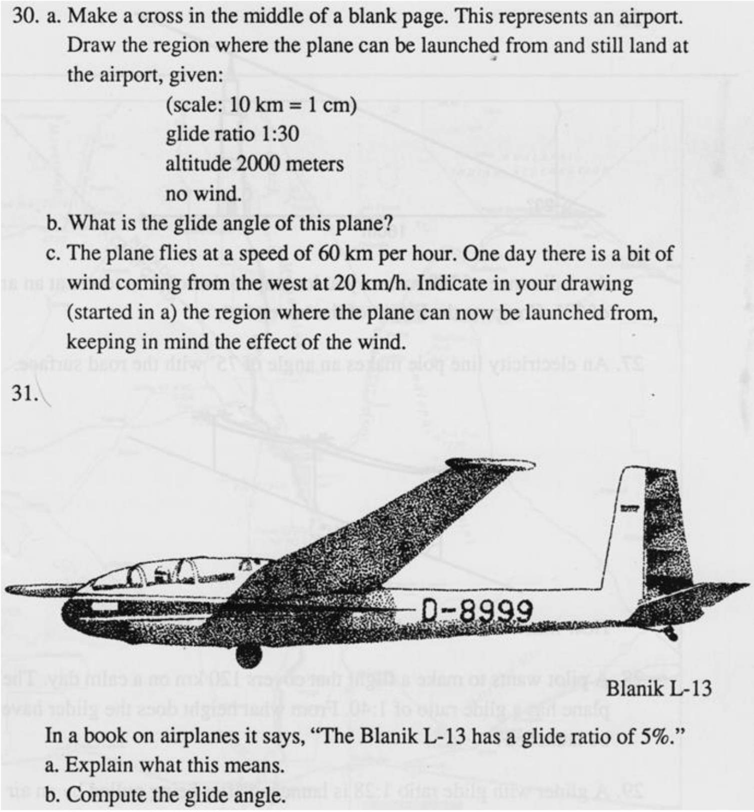


Figure 21. Glide Ratio

The classroom pilot was deemed a success, and a slightly improved format was used to continue the process, as illustrated in Figure 22.



30. a. Make a cross in the middle of a blank page. This represents an airport. Draw the region where the plane can be launched from and still land at the airport, given:
(scale: 10 km = 1 cm)
glide ratio 1:30
altitude 2000 meters
no wind.
b. What is the glide angle of this plane?
c. The plane flies at a speed of 60 km per hour. One day there is a bit of wind coming from the west at 20 km/h. Indicate in your drawing (started in a) the region where the plane can now be launched from, keeping in mind the effect of the wind.

31.

D-8999

Blanik L-13

In a book on airplanes it says, “The Blanik L-13 has a glide ratio of 5%.”
a. Explain what this means.
b. Compute the glide angle.

Figure 22. Gliding (de Lange, 1978)

However, the teacher was not satisfied: we should improve our performance and incorporate another mathematical concept into this context. As a result, a new design featured vectors as a central concept: an easy-to-understand concept for young children but frequently overlooked in curricula. Finally, we ended up with a three-lesson series on vectors (see Figure 23, 24, and 25), additional information on triangles in the context of navigation, and definitions for tan, sin, and cos. Flying as a vector context (de Lange, 1978).

Figure 23. Page Municipal Airport Landing at Runway 33 (going into direction 330, almost North (explain)

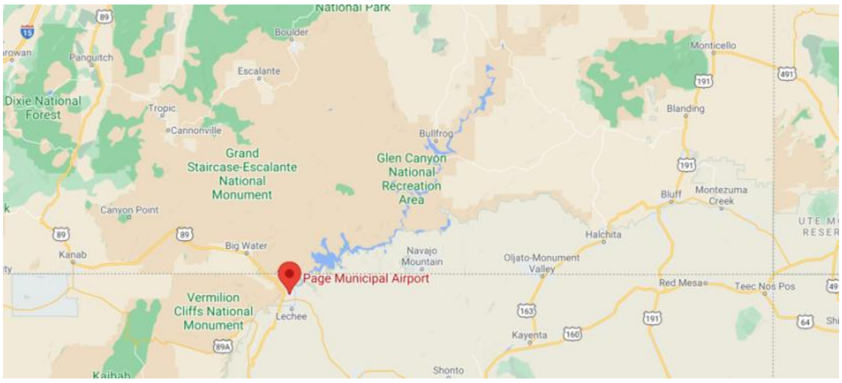
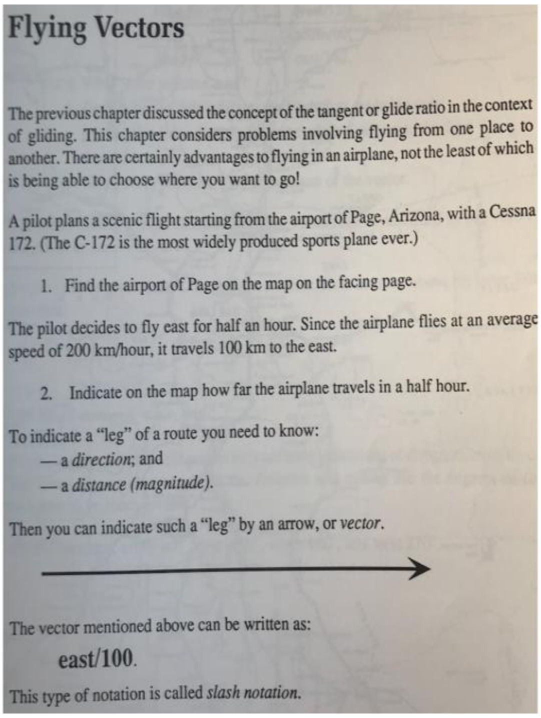


Figure 24. Map of Page Surroundings

Figure 25. A Cessna Landing at the Small Airport of Kedah in Sumatera (Indonesia)

The number of the Runway is 13 presented in Figure 26. What does that tell you about the direction?



Flying Vectors

The previous chapter discussed the concept of the tangent or glide ratio in the context of gliding. This chapter considers problems involving flying from one place to another. There are certainly advantages to flying in an airplane, not the least of which is being able to choose where you want to go!

A pilot plans a scenic flight starting from the airport of Page, Arizona, with a Cessna 172. (The C-172 is the most widely produced sports plane ever.)

1. Find the airport of Page on the map on the facing page.

The pilot decides to fly east for half an hour. Since the airplane flies at an average speed of 200 km/hour, it travels 100 km to the east.

2. Indicate on the map how far the airplane travels in a half hour.

To indicate a "leg" of a route you need to know:
— a *direction*; and
— a *distance (magnitude)*.

Then you can indicate such a "leg" by an arrow, or *vector*.

The vector mentioned above can be written as:

east/100.

This type of notation is called *slash notation*.

Figure 26. Vectors

Another instance is in the field of archaeology as shown in Figure 27. A unit was developed around this extremely intriguing context as part of an NSF-funded Middle School Project (a collaboration between Freudenthal Institute and the University of Wisconsin). The company is called 'Digging Numbers.' And while it began with the Mayan number system, it evolved into something much more profound.

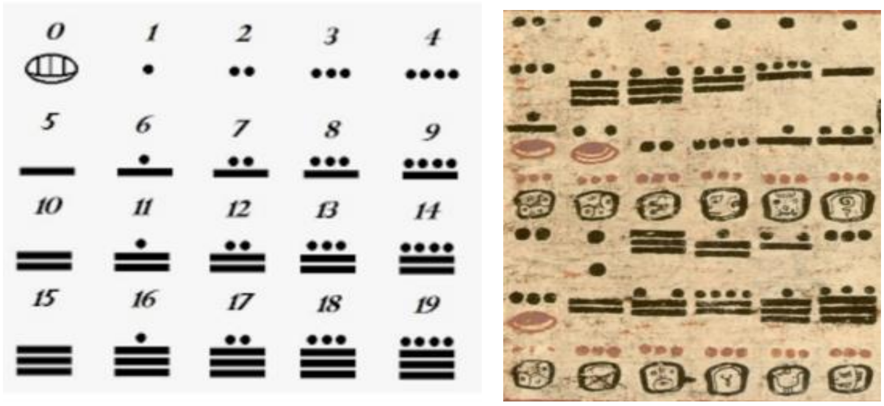
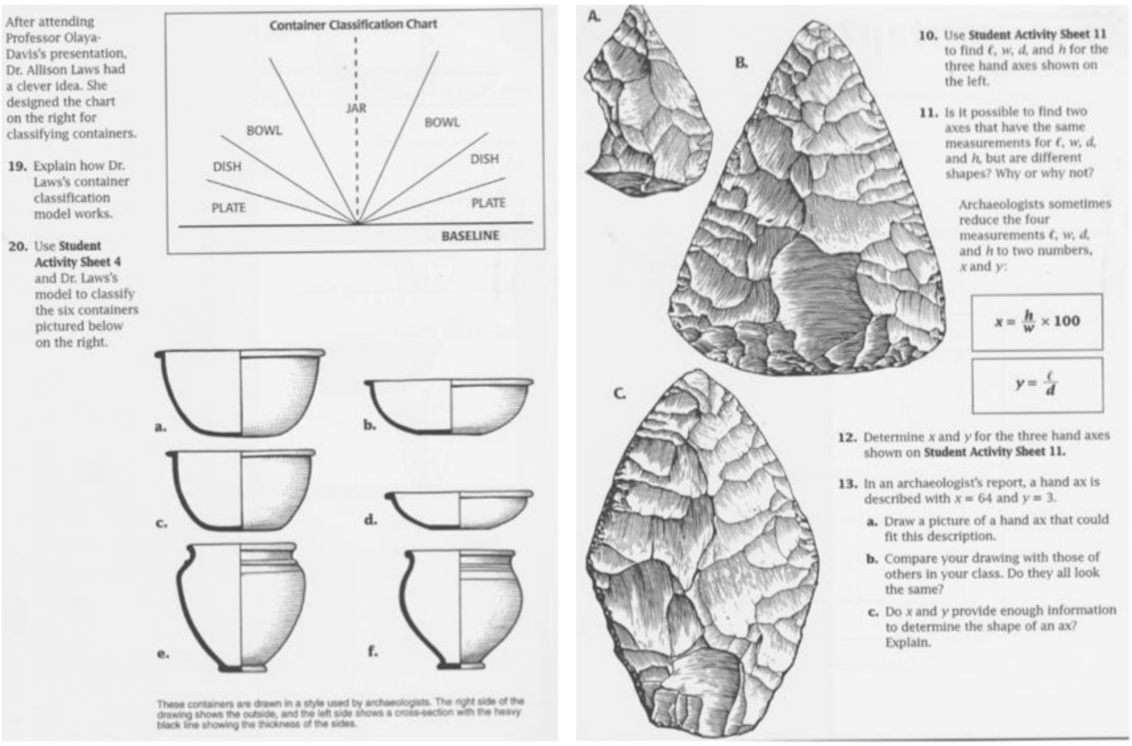


Figure 27. Maya Numbers and Maya numerals in the Dresden Codex (Maya paper book)

Classification and seriation of archaeological objects are central concepts that are treated with a high degree of authenticity. The next two examples presented in Figure 28 give the reader a slight impression: On the left: how to classify container by using the angle of steepness as indicated on the chart and digging even deeper; Right: how to classify hand-axes by using ratios.



Context: Archeology; Classification wit angles

Content: Application of Geometry

Context: Archeology; Classification of Axes

Content: Application of Algebra

Figure 28. Vases and Axes (de Lange et al., 2003)

Slow Design and Design Over Time

It is highly beneficial if the designer can reinvent himself or herself continuously. Even if a design is quite successful, the difficulty is in continuous development. Flying in the broadest sense is an excellent subject because it allows for easy extension beyond (sail) planes: flying squirrels and birds appeal to a diverse range of audiences and help students broaden their perspective on the real world (Figure 29). Is it true that a sailplane soars higher than an albatross?

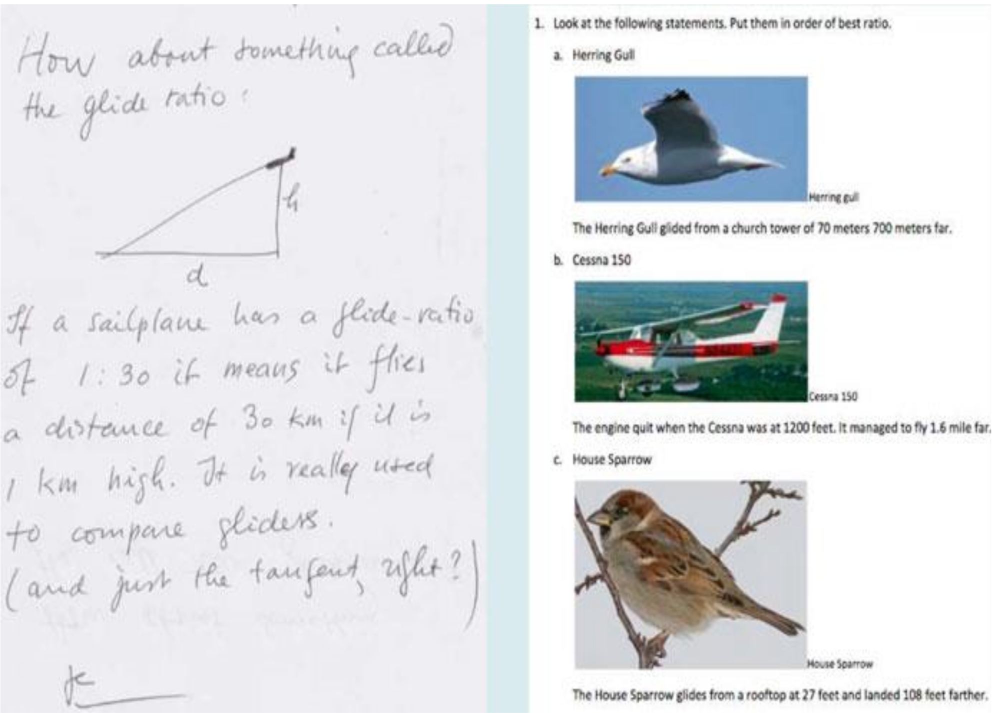


Figure 29. Slow Design (Left: de Lange, 1977; Right: de Lange, 2013)

But not only broadening the context of flying is possible. Additionally, a designer can broaden the scope of the content covered, sometimes in unexpected ways. As previously indicated, the sine can be introduced into the flying context as a ratio in a triangle (via glide ratio). However, the sine function graph can also be submitted through the movement of a propeller tip, as illustrated in the following sequence of illustrations, as shown in Figure 30.

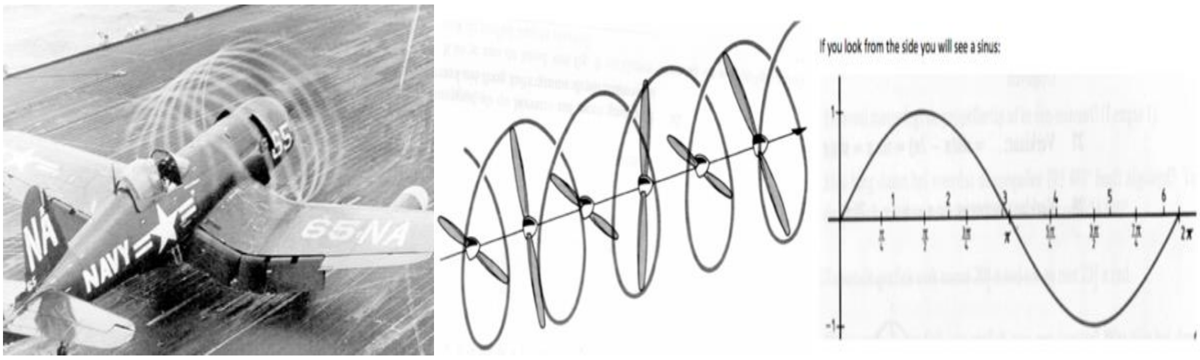


Figure 30. Helix through moving propellor-tip, Helix modelled, and Helix sideview (de Lange, 1978, 1980, 1984; de Lange & Kindt, 1984)

HINTS FOR YOUNG DESIGNERS IN MATH EDUCATION

I will summarize this article in terms of advice for young designers in mathematics education. Eleven pieces of advice are available to them. To begin, inquire of yourself. Did you teach according to the book, or did you 'design' the actual lesson if you have a teaching background? If you fall into the latter category, your chances of becoming a designer are significantly improved. Second, if you are asked to design, carefully choose your first subject: Choose an obscure concept: functions of two variables for 16-year-olds or so; have had an experience that shed new light on this subject, such as a visit to the Grand Canyon. These field experiences are near-required. After several visits to the Grand Canyon, the idea to introduce contour lines using the horizontally structured layers was practically inevitable as shown in Figure 31.

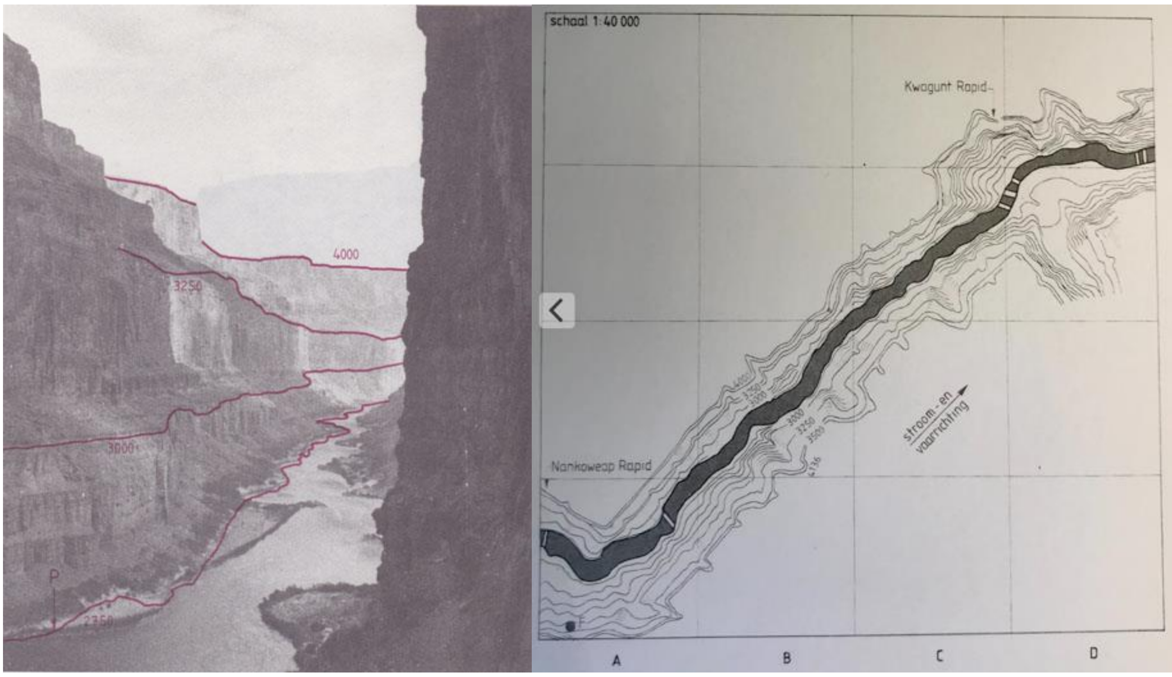


Figure 31. Grand Canyon and Contour Line Map (de Lange, 1977)

Additionally, if the first two pieces of advice were successful (if not, you should look for a more research-oriented job), choose your next subject, a less popular subject in high school (like logarithms problem presented in Figure 32). Develop a passion for comprehending the concept; if you do, you may actually design something useful.

Fourthly, whenever possible, insist on slow design. Make the case that you are conducting developmental design research, incorporating the most recent cognitive science and developmental brain research findings. Fifthly, make liberal use of your design intuition: design is also an art form. However, refrain from mentioning it excessively until you are very old. Continue teaching in a classroom for two reasons: to give you a true sense of what is possible and to keep your feet firmly planted in reality (the swamp).

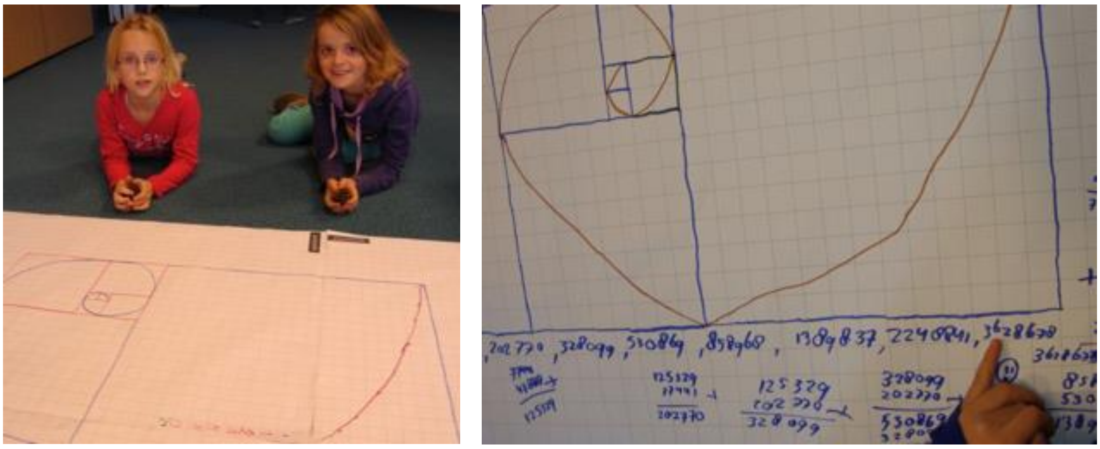


Figure 32. The Logarithmic Spiral: A Joy for All Ages (de Lange, 2013)

Next, avoid focusing exclusively on writing articles for refereed journals; instead, consider the students in your classrooms. Accept Standards if necessary, but seek out and utilize degrees of freedom. Take the initiative for Free Design: it may pay off. Especially beneficial for students. Attempt to reflect on your practices: it may prove helpful for everyone involved. I fell short in that regard. This is my very first reflection. And lastly, my feeling is there is, probably, no need for a design framework.

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