

Applying Butterfly Method in the Learning of Addition and Subtraction of Fractions

Dalilah Rauqah Laidin¹, Khairul Amilin Tengah²

¹Menglait Secondary School, Ministry of Education of Brunei Darussalam, Brunei Darussalam

²Sultan Hassanal Bolkia Institute of Education, Universiti Brunei Darussalam, Brunei Darussalam

Email: khairul.amilin.tengah@gmail.com

Abstract

Several researches attempt to improve the performance in addition and subtraction of fraction by focusing on students' understanding, such as applying fraction tiles manipulatives and number line in the teaching, or using visual aids such as area model and drawings or virtual diagrams. Similarly, few studies focus on adopting simple procedures that will facilitate students in attempting the topic. Butterfly Method is an approach that not only simplifies the pedagogical approach of learning addition and subtraction of fraction, but also indirectly promotes students' conceptual understanding in the topic. Yet this approach is not commonly practiced in Brunei. This action research employed quantitative approach involving forty-one Year 9 IGCSE level students from a government secondary school in Brunei Darussalam. The study aimed to investigate the effect on applying Butterfly Method in the learning of addition and subtraction of fraction through pre-test and post-test. The study revealed a significant improvement in students' performance through Wilcoxon Signed rank test ($Z = 4.2332$, p -value < 0.05), with large effect size ($r = 0.6611$). This is also supported with the improvement of the overall mark distribution of students from pre-test to post-test, with Hake's normalised gain values indicated majority of the participants achieved high (56.1%), medium (7.32%) and low (2.44%) improvements. Item analysis also indicated overall improvement of correct attempts in questions related to addition and subtraction of fraction, most apparently on fraction problems involving denominators not multiple of each other, improper fractions and mixed numbers. Thus, applying Butterfly Method significantly improved the overall students' performance in the learning of addition and subtraction of fractions.

Keywords: Addition and Subtraction of Fraction, Butterfly Method

Abstrak

Beberapa penelitian berusaha untuk meningkatkan kinerja penjumlahan dan pengurangan pecahan dengan berfokus pada pemahaman siswa, seperti menerapkan manipulatif ubin pecahan dan garis bilangan dalam pengajaran, atau menggunakan alat bantu visual seperti model kawasan dan gambar atau diagram virtual. Demikian pula, beberapa studi fokus pada mengadopsi prosedur yang akan memfasilitasi siswa dalam mencoba topik. Metode Butterfly merupakan pendekatan yang tidak hanya menyederhanakan pendekatan pedagogis dalam pembelajaran penjumlahan dan pengurangan pecahan, tetapi juga secara tidak langsung meningkatkan pemahaman konseptual siswa pada topik tersebut. Namun pendekatan ini tidak umum dilakukan di Negara Brunei Darussalam. Penelitian ini menggunakan pendekatan kuantitatif yang melibatkan empat puluh satu siswa IGCSE kelas 9 dari sebuah sekolah menengah di Brunei Darussalam. Penelitian ini bertujuan untuk mengetahui pengaruh penerapan Metode Butterfly dalam pembelajaran penjumlahan dan pengurangan pecahan melalui pra-tes dan pasca-tes. Hasil penelitian menunjukkan peningkatan yang signifikan dalam kinerja siswa melalui uji Wilcoxon Signed rank ($Z = 4.2332$, p -value < 0.05), dengan ukuran efek besar ($r = 0.6611$). Hal ini juga didukung dengan peningkatan distribusi nilai siswa secara keseluruhan dari pra-tes hingga pasca-tes. Nilai Hake gain dinormalisasi menunjukkan kebanyakan siswa memperoleh peningkatan tinggi (56,1%), sedang (7,32%) dan rendah (2,44%). Analisis item juga menunjukkan peningkatan keseluruhan dari setiap soal penjumlahan dan pengurangan pecahan, terutama pada masalah pecahan yang melibatkan penyebut tidak kelipatan satu sama lain, pecahan yang tidak tepat dan bilangan campuran. Dengan demikian, penerapan Metode Butterfly secara signifikan meningkatkan kinerja siswa secara keseluruhan dalam pembelajaran penjumlahan dan pengurangan pecahan.

Kata kunci: Penambahan dan Pengurangan Pecahan, Metode Butterfly

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INTRODUCTION

One focus of the current education system in Brunei, SPN 21, is to increase students' achievement in Mathematics; a fundamental subject that is interrelated to almost every other curriculum subjects. Studies suggested that one could assess the students' Mathematics competency from their elementary knowledge in Fraction (Bailey, Hoard, Nugent, & Geary, 2012; Siegler, Duncan, Davis-Kean, Duckworth, Claessens, Engel, Susperreguy, & Chen, 2012). Bailey et al. (2012) claimed that students' competency in solving fraction problem may have positive impact towards subsequent mathematics achievements. This is supported by Siegler et al. (2012) who stated that students' elementary fraction knowledge could predict, and facilitate the students' performance in algebra, and thus the overall mathematics achievement in high school in United States and United Kingdom.

There are multiple definitions of fractions (McCrory, 2006). Fraction of a quantity, $\frac{x}{y}$ and $y \neq 0$, is defined as the amount obtained x by equally dividing one into y parts (Beckmann, 2003). Alternatively, fraction can also be defined as a point carrying a value within a number line with equally spaced intervals derived from the division of value of y (Parker & Baldrige, 2004).

Fraction starts being taught in primary level, yet students find it difficult despite its early introduction (Idris & Narayanan, 2011; Lortie-Forgues & Siegler, 2015). One contributing factor is the complicated and multifaceted computation of fractions (Brousseau, Brousseau, & Warfield, 2004). A similar report by Lortie-Forgues and Siegler (2015) stated opaqueness of procedures was one of the inherent difficulty encountered by students in fraction arithmetic. Hence, it is important to master these skills at the root, or at earlier stage, for benefit of future learning.

Similarly, in Brunei, fraction is the most challenging topic for students to learn (Damit, 2002; Harun, 2003). This issue is not only prevalent in primary, but also in secondary level especially on the addition and subtraction of fractions (Suffolk & Clement, 2003). Yusof and Malone (2003) suggested that poor introduction of elementary fraction, weak basic knowledge and mechanical skills in calculating operations of fractions, language barrier, and ineffective teachers' instructional activities could be the possible factors in such learning difficulty. Hence, teachers are encouraged to apply innovative pedagogies to enhance quality of mathematical education amongst student, as stated in SPN 21, specifically in fraction (Ministry of Education, 2013).

Researchers recognised the importance of the development of both conceptual and procedural knowledge in mathematics competency (Hansen, Jordan, & Rodrigues, 2017; Rittle-Johnson & Schneider, 2015; Rittle-Johnson, Siegler & Alibali, 2001). Thus, there is a need for teachers to ensure that the students grasp both conceptual and procedural knowledge to maximise students' mathematics performance (Fazio & Siegler, 2010). Fazio and Siegler (2010) defined conceptual knowledge in the topic 'Fractions' as the knowledge on the meaning of fractions, which includes the construction of fractions, the value it holds, and the reason behind each procedure in operational fractions. On the

other hand, the two researchers defined procedural knowledge as the knowledge on how to perform procedures in solving a problem. This suggests that students need to have a good foundation on fraction concept, as well procedures to progress with further fraction learning.

There are many procedures to follow when doing the four operations (addition, subtraction, multiplication and division) in fraction. Hence, one possible factor which may lead to student's difficulty is the confusion of the procedures itself (Castro, Custodio, & Escueta, 2013). In Brunei, for addition and subtraction of fraction, students are typically taught to use the procedure of determining the least common multiple (LCM) method (Khalid, 2007). Due to its long and complex sequence of actions, including finding the LCM itself, this method might not cater for all students particularly those of 'lower ability'. Consequently, this may shift the student's focus on learning and solving the LCM itself instead of the intended focus of solving fractions (Castro et. al, 2013).

LCM procedural approach in addition and subtraction of fraction (Goswami, 2018; Khalid, 2007) can be summarised in figure 1. Khalid (2007) found that the students in her research study did not understand and grasp the purpose of LCM was to determine the equivalent fractions, and they were just trying to remember the algorithm and hence, prone to make mistakes.

Step 1: Determine the LCM of the denominators.

$\frac{1}{4} + \frac{1}{6}$	2	4, 6
	2	2, 3
	3	1, 3
		1, 1

LCM = 2 x 2 x 3 = 12

Step 2: Rewrite the unlike fractions to their equivalent fraction with the LCM as the denominator.

$$\frac{1 \times 3}{4 \times 3} = \frac{3}{12} \quad \frac{1 \times 2}{6 \times 2} = \frac{2}{12}$$

$$\frac{3}{12} + \frac{2}{12} = \frac{5}{12}$$

Figure 1. An example of addition and subtraction of fractions using LCM

Goswami (2018) shared the same view of the lack of the procedural knowledge of the students using LCM method. For instance $\frac{1}{4} + \frac{2}{4}$, with LCM of 4, was solved by adding 'the quotient of denominator after divided by LCM (in this case, 1) to the numerators instead of multiplying them ($\frac{1+1+2+1}{4} = \frac{5}{4}$). Despite this, it was noted that LCM approach was beneficial to student when the fraction involved a large denominator value, where students made less careless mistakes in

multiplication of large numbers (Castro et al., 2013). In addition, the use of LCM approach saved time when the problem involved addition or subtraction of more than three fractions.

Few strategies have been introduced to aid students' learning in fractions such as a number line model, fraction tiles, region model, area model, and discrete model (Bruce, Chang, Flynn, & Yearley, 2013; Orton & Frobisher, 1996). However, there is no conclusive evidence on which of these methods is best suited or/and most effective. Therefore, this study aimed to investigate the effect of the Butterfly Method in helping students to learn the addition and subtraction of fractions. The study was guided by the research question: *What is the effect on students' performance when applying the Butterfly Method in the addition and subtraction of fraction?*

Butterfly Method

The Butterfly Method can be summarised as a mathematical mnemonic strategy through visual approach, modelling the structure of a butterfly, that applies the cross-multiplication concept and avoids the LCM algorithm. Miller and Obara (2017) outlined the procedure to follow in addition and subtraction of unlike fractions:

1. Draw the wings: Two loops across the numerators and denominator of the other fraction. Then, draw the antenna and bottom of the butterfly body.
2. As shown in figure 2, we start with base multiplication. Multiply both of the denominator and write the product in the bottom of the butterfly body. This will be the new common denominator.
3. Multiply the numbers in each loop (wing). Write the product under the antenna.
4. Add or subtract the numbers under the antenna, depending on the operation of the question.

$$\frac{1}{4} + \frac{1}{6}$$

$$= \frac{6+4}{24} = \frac{10}{24} = \frac{5}{12}$$

Figure 2. An application of Butterfly Method in addition of unlike fractions

In this 21st century learning, many students are visual learner and hence incorporating such approach could effectively support learning and performance in mathematics (Boaler, Chen, Williams, & Cordero, 2016). Boaler et al. (2016) indicated when an individual is working on mathematical problem, the visual pathway in our brain is involved. Part of the pathway, developed over the course of childhood, helps the visualization of number in the brain, enhancing the mathematical skills (Battista, Evans, Ngoon, Chen, Chen, Kochalka, & Menon, 2018).

Low, Shahrill, & Zakir (2018) conducted a study on combining Bar Model and Butterfly Method in Brunei for fraction learning development, and the result indicated significant increase in students' performance after the intervention. She also found that the students had difficulty on applying Butterfly Method on subtraction rather than addition of fractions, particularly involving mixed numbers. Some students missed the first step of converting mixed numbers to improper fractions. Few students in the study also expressed their confusion on the method, as it was newly introduced and lack of practice during the intervention.

METHODS

This action research applied quantitative methods of convenient sampling of two classes, class A and class B within the same government secondary school in Brunei, using comparison between pre-test and post-test performance in the topic. Forty-one students were involved in this study, twenty-three males and eighteen females, between 14 and 15 years old, and were of diverse academic background from low to average abilities.

The pre-test consisted of ten questions covering addition and subtraction of fractions, with each question allocated 1 mark (table 1). Any incorrect answer or unattempted question was awarded zero. The questions were a mixture of proper fractions, improper fractions and mixed numbers, and arranged in increasing difficulty. The time allocated for test was fifty minutes, without the use of calculator. The post-test was designed identical to the format of pre-test, with the same number of questions and marking system. However, there was a slight change in the sequence of the question, i.e. question 1 in pre-test becomes question 2 in the post-test (1 → 2), (2→1), (3→4) and etc. The order of difficulty of the questions was maintained. This rearrangement of sequence of questions was to avoid memory effect, since the pre-test and post-test were conducted within one week.

Table 1. Questions in the pre-test and post-test on addition and subtraction of fractions

Number	Pre-test	Post-test	Difficulty Level
1	$\frac{1}{4} + \frac{1}{6}$	$\frac{1}{4} - \frac{1}{6}$	Easy
2	$\frac{1}{4} - \frac{1}{6}$	$\frac{1}{4} + \frac{1}{6}$	Easy

Number	Pre-test	Post-test	Difficulty Level
3	$\frac{4}{5} + \frac{2}{15}$	$\frac{4}{5} - \frac{2}{15}$	Easy
4	$\frac{4}{5} - \frac{2}{15}$	$\frac{4}{5} + \frac{2}{15}$	Easy
5	$\frac{3}{4} + \frac{5}{7}$	$\frac{3}{4} - \frac{5}{7}$	Medium
6	$\frac{3}{4} - \frac{5}{7}$	$\frac{3}{4} + \frac{5}{7}$	Medium
7	$\frac{5}{3} + \frac{9}{8}$	$\frac{5}{3} - \frac{9}{8}$	Medium
8	$\frac{5}{3} - \frac{9}{8}$	$\frac{5}{3} + \frac{9}{8}$	Medium
9	$1\frac{1}{4} + 1\frac{1}{6}$	$1\frac{1}{4} - 1\frac{1}{6}$	Difficult
10	$1\frac{1}{4} - 1\frac{1}{6}$	$1\frac{1}{4} + 1\frac{1}{6}$	Difficult

Both pre-test and post-test were validated by two experienced teachers teaching the level of topic. For the internal consistency reliability of the instrument, the pre-test was conducted to other twenty-three Year 9 IGCSE students of the same school, who were not involved in the main study. Since the scores for each item in the pre-test were either 0 (incorrect response) or 1 (correct response), then Cronbach's alpha could be used to assess the internal consistency reliability (Brown, 2002; Huck, Cormier, & Bounds, 2012). A Cronbach's alpha of 0.74 was obtained indicating an acceptable reliability (Tavakol & Dennick, 2011). Since the same questions were used in the post-test, with only slight arrangement of order, it was also assumed to have an acceptable reliability for the post-tests items.

The intervention lessons took three normal lessons, each lasting 50 minutes. The first intervention lesson was on the introduction of the butterfly structure, including recap of conversion of improper fractions to mixed numbers and vice versa, solve some questions on addition and subtraction of fraction using individual's preferred method, followed with introduction and process of the Butterfly Method with several simple examples. The lesson then concluded with students' individual attempts of some simple questions. The second intervention lesson started with a recap of the previous lesson, followed with more examples with added complexity involving mixed numbers. The increase in the difficulty of the material was to encourage students to apply deeper mathematical thinking, and at the same time enhance their understanding in the topic. The final intervention lesson

consisted of a mix questions involving proper fraction, improper fraction and mixed numbers for students to attempt, to reinforce the method and further familiarise the students with the Butterfly Method.

The mean of pre-test and post-test scores was initially to be compared via paired sample t-test, however since one of the underlying assumption was not fulfilled (difference between pre-test and post-test score of the students were not normal), Wilcoxon-signed rank test was carried out instead (Rockinson-Szapkiw, 2013). Further analysis included comparing the trend of overall marks distribution between pre-test and post-test, and determining the normalised gain value of the participants (Hake, 1998). Item analysis of the pre-test and post-test questions was also included to investigate which questions were mostly affected when using Butterfly Method as an intervention.

RESULTS AND DISCUSSION

Comparison of Pre-test and Post-test

There was a mean increase in the post-test results from the pre-test results for overall participants, with a value of 3.24399 (table 2). The mean shifted from an average range (5.9268) to higher range (9.1707). Contrarily, the standard deviation value showed a decrease of 2.3059 (from 3.9012 in the pre-test to 1.5953 in the post test). This showed that the students' mark became less spread out from the mean value in the post-test as compared to the pre-test. From the result, it showed that the Butterfly Method had a positive impact in students' performance in addition and subtraction of fractions.

Table 2. Descriptive statistics of pre-test and post-test result

Class	N	Mean		Standard Deviation	
		Pre-test	Post-test	Pre-test	Post-test
Overall sample	41	5.9268	9.1707	3.9012	1.5953

The null hypothesis for the Wilcoxon Signed rank test is that there is no significant difference in the median of the pre-test and post-test scores. The result revealed a statistical difference in students' performance in the pre-test and post-test, with p-value of 0.0002 which is < 0.05 (Table 3), hence the null hypothesis was rejected. The median of the pre-test score is 7.000 with IQR value of 8.000. While in the post-test, the median score is 10.000 and the IQR value is 1.000.

Table 3. Summary of Wilcoxon Signed Rank test and Effect size for overall sample

	Wilcoxon-Signed Rank test			Pre-test		Post-test	
	p-value	Z value	Effect size	Median	IQR	Median	IQR
Overall	0.0002	4.2332	0.6611	7.00	8.00	10.00	1.00

Note: Md= Median, IQR = Interquartile range

The effect size was also calculated using the r formula (Cohen, 1988; Rosenthal, 1991) as shown below;

$$r = \frac{z}{\sqrt{n}} \quad , \quad z = Z \text{ value}, \quad n = \text{number of observations}$$

The effect size measures the magnitude of the effect of the intervention in students' performance in the pre-test and post-test. According to Cohen (1988) and Coolican (2017), the r value of 0.1 to 0.3 is considered as small, 0.3 to 0.5 as moderate and 0.5 and above as large effect size. As shown in table 4, the effect size of the study was >0.5 indicating a large effect size. This meant that the Butterfly Method gave a large significant impact in students' performance in addition and subtraction of fraction. With this combination of significant difference and large effect size, it is concluded that in this study, the Butterfly Method resulted in large significant improvement on students' performance in addition and subtraction of fractions.

Mark Distribution of Pre-test and Post-test Results

The distribution of the pre-test score indicated shift to more negatively skewed in the post-test (figure 1). Most of the students' marks in post-test concentrated towards 7 to 10 marks, which is of high range. This observation further supported the decrease in the standard deviation value in post-test where there was less variation in students' marks. Furthermore, there was a noticeable huge drop in students who achieved zero marks and a big increase in the number of students who achieved full marks. Thus, the line graph indicated an improvement in the distribution of students' performance in the post-test. The post-test line graphs also showed that student achieving less than pass mark of 5 dropped while the number of students achieving 6-10 marks increased (only 1 failed in post-test while the rest of students passed).

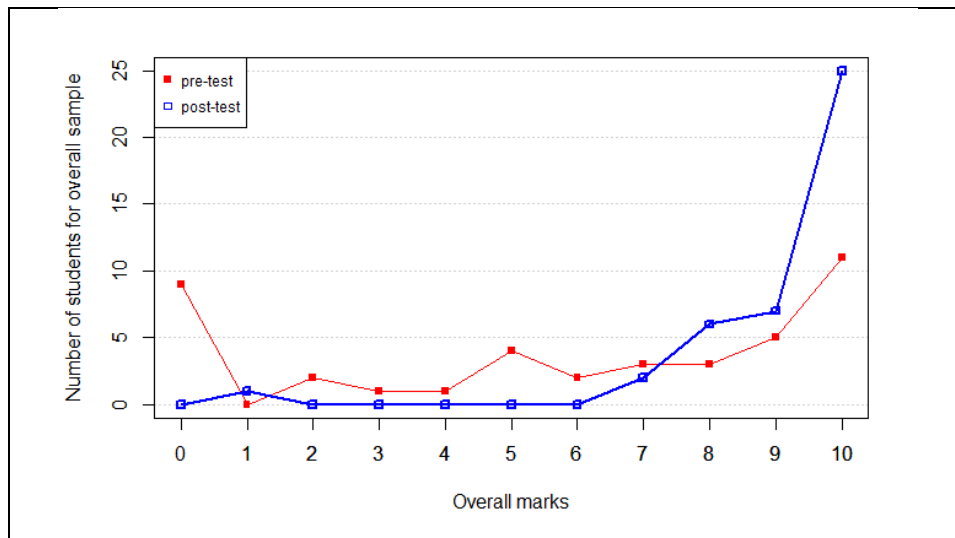


Figure 3. The pre-test and post-test mark distribution for overall samples

Further analysis using Normalised Gain Theory (Table 4), despite the post-test resulted in almost 20% of the students maintained their original pre-test marks, more than 65% had positive normalized gains, with majority of these students achieved high gain value (56.1% overall). Those who maintained their marks consisted of students that scored high marks in their pre-test, with seven students scoring full marks of 10 and one scoring 8 mark. Despite these outcomes, there was also 14.63% of mark drop amongst the participants. For normalise gain value, it was calculated using the formula:

$$\text{Normalisation gain} = \frac{(\text{post} - \text{test score}) - (\text{pre} - \text{test score})}{(\text{ideal score}) - (\text{pre} - \text{test score})}$$

Table 4. Result of Normalised Gain value of the participants in this study

Normalized Gain Value (GT)	Interpretation	Number of students	Percentage
$-1.00 \leq GT < 0.00$	Mark Drop	6	14.63%
$GT = 0.00$	Maintained mark	8	19.51%
$0.00 \leq GT < 0.30$	Low Gain	1	2.44%
$0.30 \leq GT < 0.70$	Medium Gain	2	7.32%
$0.70 \leq GT < 1.00$	High Gain	23	56.10%

Item Analysis of Pre-test and Post-test

The students' performance for each item in the pre-test were then compared to post-test focusing on the number of students who answered each item correctly, incorrectly and unattempted. Since the items used in the pre-test was the same as in the post-test, with the only difference in the sequence (i.e Pre-test item 1 = Post-test item 2), the exact items were matched for comparison as

shown in Figure 4. From the bar graph, it showed that there was increase in the number of correct responses for every items. The largest increase was for items pre-test 7 and pre-test 9 which involved addition of improper fractions, and mixed numbers, with an increase of seventeen students. On the other hand, there were noticeable decrease in the number of incorrect responses in all items. There were no unattempt items in the post-test, suggesting that the Butterfly Method encouraged the students to attempt all the items.

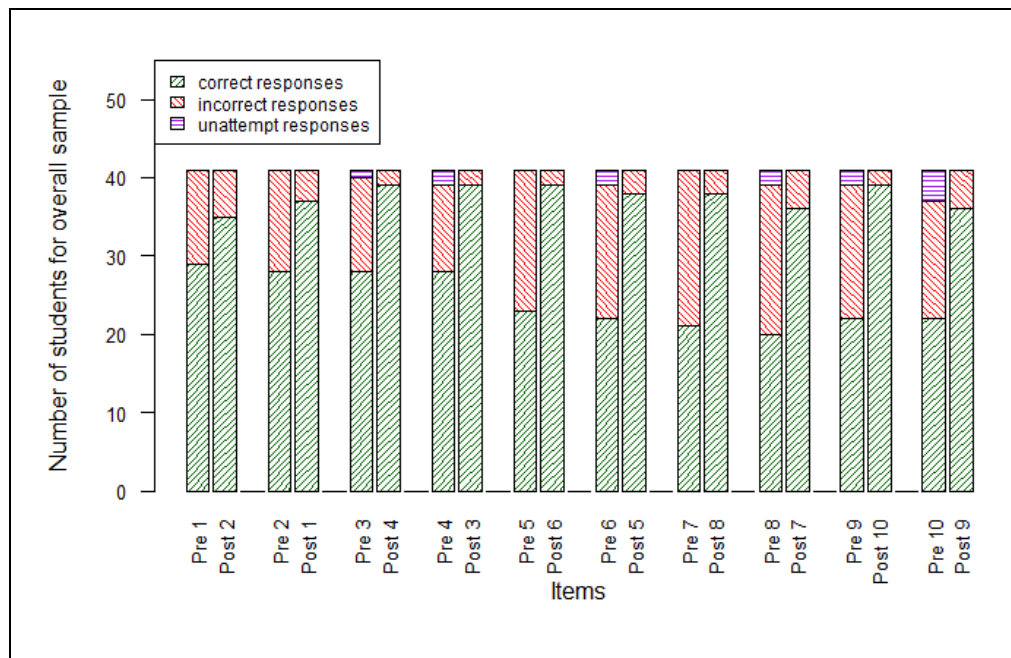


Figure 4. Bar graph of the number of students with correct, incorrect and unattempt responses for each items in pre-test and post-test

CONCLUSION

The study was conducted to two classes on one secondary school in Brunei employing the quantitative approach to investigate the effect in applying the Butterfly Method in the learning of addition and subtraction of fraction. The students involved in the study were of ‘medium ability’ on the topic. The results showed a significant increase in the median score of the pre-test and post-test when assessed using Wilcoxon Signed Rank test. For those students that showed improvement, Hake’s normalised gain values indicated that majority showed high improvement (56.1%), followed with 7.32% medium improvement and 2.44% low improvement. In addition, the item analysis showed increase in correct responses in every item, and decrease in unattempt responses. In conclusion, the study found significant and large improvement in students’ learning on the addition and subtraction of fraction through the use of Butterfly Method as an intervention. The improvement was most apparent on fraction problems involving denominator not multiple of each other, improper fractions and mixed numbers. The positive findings of this study on Butterfly Method is hoped to provide teachers in Brunei an alternative approach in teaching addition and subtraction of fractions,

and gives the opportunity to broaden and enhance their pedagogy skills in realising the mission of SPN21. The visual aid nature in Butterfly Method could be helpful in guiding cross-multiplication regardless of the type and level of learners, as most students in the study showed an improvement and have positive experience with the method. Therefore, the method could assist teachers in maximizing students' learning by catering students of different needs and preference. As previously stated, researches have emphasised on the importance of a solid conceptual knowledge of fraction in addition to the procedural knowledge. The Butterfly Method could be used to enhance the conceptual knowledge of the topic the method highlights the significance and importance of having the same denominators in equivalent fraction to the students' focal awareness as opposed to just blindly following the procedure in LCM method. Due to the limitation of the study, there are certain areas which could be considered for future researches. The study was only focusing of IGCSE students with 'average abilities' and hence, future researchers could conduct the study to more students of wider range of mathematical abilities on the main topic, for instance, students of 'lower abilities'. Since the study only involved a very small sample size, future researcher could consider a study on a larger scale of samples so that generalization to the whole population in Brunei could be made.

REFERENCES

- Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology*, *113*(3), 447-455. <http://doi.org/10.1016/j.jecp.2012.06.004>.
- Battista, C., Evans, T. M., Ngoon, T. J., Chen, T., Chen, L., Kochalka, J., & Menon, V. (2018). Mechanisms of interactive specialization and emergence of functional brain circuits supporting cognitive development in children. *NPJ Science of Learning*, *3*(1), 1-11. <https://doi.org/10.1038/s41539-017-0017-2>.
- Beckmann, S. (2003). *Mathematics for elementary teachers: Numbers and Operations (Vol. 1)*. Boston: Addison-Wesley.
- Boaler, J., Chen, L., Williams, C., & Cordero, M. (2016). Seeing as understanding: The importance of visual mathematics for our brain and learning. *Journal of Applied & Computational Mathematics*, *5*(5), 1-6. <https://doi.org/10.4172/2168-9679.1000325>.
- Brousseau, G., Brousseau, N., & Warfield, V. (2004). Rationals and decimals as required in the school curriculum: Part 1: Rationals as measurements. *The Journal of Mathematical Behavior*, *23*(1), 1-20. <https://doi.org/10.1016/j.jmathb.2003.12.001>.
- Bruce, C., Chang, D., Flynn, T., & Yearley, S. (2013). Foundations to learning and teaching fractions: Addition and subtraction. Retrieved from <http://www.edugains.ca>.
- Brown, J. D. (2002). The Cronbach alpha reliability estimate. *JALT Testing & Evaluation SIG Newsletter*, *6*(1), 17-18.
- Castro, M. R., Custodio, A., & Escueta, M. G. (2013). Improving teacher instruction of addition of dissimilar fractions that integrated lesson study. Retrieved from <http://www.academia.edu/>.

- Cohen, J. (1988). *Statistical power analysis for the behavioral sciences (2nd Ed.)*. Hillsdale: Lawrence Erlbaum Associates.
- Coolican, H. (2017). *Testing for differences between two samples. Research methods and statistics in psychology (6th ed., pp.438-486)*. London: Psychology Press.
- Damit D. H. S. P. H. (2002). Fraction concepts and skills of some Primary Six pupils in Brunei Darussalam. *Unpublished Thesis*. Gadong: Universiti Brunei Darussalam.
- Fazio, L., & Siegler, R. S. (2011). *Teaching fractions*. Brussels: International Academy of Education.
- Goswami, R. (2018). Misconceptions in fraction. *At Right Anglesa*, 7(1), 48-51.
- Hake, R. R. (1998) Interactive-engagement vs traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. *American Journal of Physics*, 66, 64-74. <https://doi.org/10.1119/1.18809>.
- Hansen, N., Jordan, N. C., & Rodrigues, J. (2017). Identifying learning difficulties with fractions: A longitudinal study of student growth from third through sixth grade. *Contemporary Educational Psychology*, 50, 45-59. <https://doi.org/10.1016/j.cedpsych.2015.11.002>.
- Harun, H., Z., H. (2003). The development of fractions and decimals concepts among Form 4 N-level students at a girls' secondary school in Brunei Darussalam. *Unpublished Thesis*. Gadong: Universiti Brunei Darussalam.
- Huck, S. W., Cormier, W. H., & Bounds, W. G. (2012). *Reading statistics and research (6th ed., Vol. 566, pp. 225-227)*. Boston: Pearson Education.
- Idris, N., & Narayanan, L. M. (2011). Error patterns in addition and subtraction of fractions among form two students. *Journal of Mathematics Education*, 4(2), 35-54.
- Khalid, M. (2007). Incorporating mathematical thinking in addition and subtraction of fraction: real issues and challenges. *Progress report of the APEC-Khoen Kean international symposium: collaborative studies on innovations for teaching and learning mathematics in different cultures (II)-Lesson study focusing on mathematical thinking*.
- Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Why is learning fraction and decimal arithmetic so difficult?. *Developmental Review*, 38, 201-221. <http://doi.org/10.1016/j.dr.2015.07.008>.
- Low, J., Shahrill, M. & Zakir, N. (2020). Solving fractions by applying the bar model concept with the butterfly method. *Jurnal Pendidikan Matematika*, 14(2), 101-116. <https://doi.org/10.22342/jpm.14.2.11261.101-116>.
- McCrary, R. (2006). Mathematicians and mathematics textbooks for prospective elementary teachers. *Notices of the American Mathematical Society (AMS)*, 53(1), 20-29.
- Miller, G., & Obara, S. (2017). Finding meaning in mathematical mnemonics. *Australian Mathematics Teacher*, 73(3), 13-18.
- Ministry of Education. (2013). *The national education system for the 21st century: SPN21*. Berakas: Ministry of Education
- Orton, A., & Frobisher, L. (1996). *Introduction to education: Insights into teaching mathematics (1st ed., Vol. 1)*. London: Cassell.
- Parker, T. H., & Baldrige, S. J. (2004). *Elementary Mathematics for Teachers (Vol. 1)*. Okemos: Sefton-Ash Publishing.

- Rittle-Johnson, B., & Schneider, M. (2015). Developing conceptual and procedural knowledge of mathematics. *Oxford Handbook of Numerical Cognition*, 1118-1134.
- Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, 93(2), 346. <http://doi.org/10.1037//0022-0663.93.2.346>.
- Rockinson-Szapkiw, A. (2013). *Statistics guide*. Retrieved from <http://amandaszapkiw.com/>.
- Rosenthal, R. (1991). *Meta-analytic procedures for social research*. Thousand Oaks: Sage Publications.
- Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, 23(7), 691-697. <http://doi.org/10.1177/0956797612440101>
- Suffolk, J., & Clements, M. A. (2003). *Fractions concepts and skills of form 1 and form 2 students in Brunei Darussalam*. In H. S. Dhindsa, S. B. Lim, P. Achleitner, & M. A. Clements (Eds.), *Studies in science, mathematics and technical education* (pp. 145– 154). Universiti Brunei Darussalam.
- Tavakol, M., & Dennick, R. (2011). Making sense of Cronbach's alpha. *International Journal of Medical Education*, 2, 53–55. <https://dx.doi.org/10.5116%2Fijme.4dfb.8dfd>.
- Yusof, J., & Malone, J. (Eds.). (2003). *Mathematical errors in fractions: A case of Bruneian primary 5 pupils*. *Proceeding of the 26th annual conference of the mathematics education research group of Australasia*. Mathematics Education Research Group of Australasia Incorporated.

