Optimization of Local Routing for Connected Nodes with Single Output Ports - Part I: Theory

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Abstract

The optimization of packet flows in a set of cooperative nodes with single output ports is considered. A single output port relays a packet to a single connected node at a time. The different service time distributions to distinct connected nodes are considered in terms of multiclass queuing with a single first-come first-serve queue and a single server in each node. The analytic model is applied to cases of two, three and four connected nodes with M/M/1 queues relaying packets in a chosen direction. Analytical solutions for two connected nodes are obtained. The influence of other arbitrary packet flows is considered as background traffic. Directed links are used for local connectivity within the set of cooperative nodes.

Keywords: *Local routing, cooperative node, single output port, multiple service rates, background traffic, directed links.*

Introduction

The implementation of multi-hop routing in mobile ad hoc networks (MANET 2011) relies on the existing technological constraints which restrict the number of wireless channels which can be used by a mobile user to relay information (IEEE 802.11 2007; IEEE 802.11b 1999; IEEE 802.11g 2003; IEEE 802.11n 2009; IEEE-SA 2010). Although multiple-input multiple-output (MIMO) technology is used in the physical layer of the latest wireless standards, the increased bandwidth is utilized for a directed transmission towards a single destination in most cases. Power consumption, frequency reuse, topology control, security issues, etc., are factors which limit the number of output ports in a mobile node. Also, the existing mobile units, which communicate directly only with an access point or a base station, are designed to operate with a single output port and connect to a single destination in a centralized network. The modification of the existing hardware configurations to operate in a decentralized network would use an upgrade of the existing technology and the utilization of a single output port, which can switch consecutively between different connections with different service distributions, is a realistic scenario. The packets to be relayed to different connected nodes with different service times can be considered as belonging to different classes, or different types of customers for the single server at the output port. The single queue which stores packets arriving independently from several input ports will contain several classes of packets. The packets of a given class which could be serviced faster than other packets will have to wait for the slower servicing of packets to be delivered in channels with low signal-to-noise ratios. The performance of such multi-class single-server systems for mobile communications is obviously outperformed by alternative prospective systems containing several independent output ports which could relay packets to several distinct destinations at the same time. As the advent of such advanced multi-port systems is delayed due to bandwidth limitations, it is worth studying to what extent the single output port systems can be optimized to relay packets in a particular direction (by using underutilized links with shorter service times) and reduce the average delay in small sets consisting of several cooperative nodes. Such local routing may prove beneficial in congested parts of the network (hot spots) where some local paths are utilized more frequently that others by the global routing algorithms. The decentralized networks implement best effort algorithms which result in overlapping paths and the absence of global routing control requires local optimization of packet flows. The arbitrary inter-arrival time distributions at the input ports and the multi-class service time distributions at the single output port of each local mobile node result in a multi-variable optimization in locally re-routing portions of the global packet flows among adjacent neighbors by applying a local decision making.

Analytic Model

The ideal optimization of packet flows is difficult to achieve in practice because the number of variables is a function of the local connectivity and even a node degree of just two or three links poses challenges for real-time estimation. The engineering approach is to reduce the complexity by separating the problem into smaller ones which can be treated independently from each other in first approximation in order to achieve sub-optimal performance. One can optimize the traffic in particular direction by considering all other traffic as a background traffic and partition the general problem into smaller solvable cases.

In a set of cooperative nodes, the arrival traffic in one of the detected general directions is described by the inter-arrival time distributions characterized by the average arrival rates $\lambda_{in,i}$ and the squared coefficients of time variation $c^2_{\lambda,in,i}$ of said distributions, i = 1, 2,…, N, where N is the number of local cooperative nodes.

The multi-class service time distributions are known by the average service rates $\mu_{out,i}$ and the squared coefficients of time variation $c^2_{\mu,out,i}$, i = 1, 2,…, N, of departing packets, and also the average service rates $\mu_{i,j}$ and the squared coefficients of time variation $c^2_{\mu,i,j}$, i = 1, 2,…, N, of packets circulating among the cooperative nodes.

The unknown average departure rates $\lambda_{out,i}$ and the corresponding unknown squared coefficients of time variation $c^2_{\lambda,out,i}$, i = 1, 2,…, N, of departing packet flows are to be determined by optimizing the local packet exchange with the unknown rates $\lambda_{i,j}$ and the corresponding unknown squared coefficients of time variation $c^2_{\lambda,i,j}$, i, j = 1, 2,…, N for $i \neq j$, assuming that if $\lambda_{i,j} > 0$, then $\lambda_{j,i} = 0$.

All average rates are measured in packets/sec and their reciprocal values, the average time intervals of the time distributions, are measured in seconds/packet. The range of the indexes i and j, i = 1, 2,…, N, goes without saying in the formulae that follow.

At a local node, the total average arrival rate is given by

$$\lambda_i = \lambda_{in,i} + \sum_{j=1}^{N} \lambda_{j,i} , \qquad (1)$$

the utilizations of a node due to packets of distinct classes are

$$\rho_{i,out} = \frac{\lambda_{i,out}}{\mu_{i,out}} , \text{ and} \qquad (2)$$

$$\rho_{i,j} = \frac{\lambda_{i,j}}{\mu_{i,j}} , \qquad (3)$$

and the total utilization of a node is

$$\rho_i = \rho_{i,out} + \sum_{j=1}^{N} \rho_{i,j} . \qquad (4)$$

The mean service rate of a node for all packet classes (customers) is, as follows:

$$\mu_i = \frac{1}{\frac{1}{\lambda_i}\left(\rho_{i,out} + \sum_{j=1}^{N} \rho_{i,j}\right)} . \qquad (5)$$

The following constraint applies to all average rates assuming initially that there are no packet drop rates:

$$\lambda_i = \lambda_{in,i} + \sum_{j=1}^{N_{j\to i}} \lambda_{j,i} = \lambda_{out,i} + \sum_{k=1}^{N_{i\to j}} \lambda_{i,j} , \qquad (6)$$

where $N_{j\to i}$ and $N_{i\to j}$ are the numbers of active links used in each direction, $j \to i$ and $i \to j$, and the cumulative average rate for all cooperative nodes is

$$\lambda_{total} = \sum_{i=1}^{N} \lambda_i \ . \qquad (7)$$

The unknown squared coefficients of variation $c^2_{\lambda,out,i}$ and $c^2_{\lambda,i,j}$ can be obtained as described by Belch *et al*. (1998).

In the private case of M/M/1 first come-first serve (FCFS) queues (Kleinrock 1975), the number of waiting packets of a given class is estimated by the expression (Pujolle and Wu 1986; Belch *et al*. 1998):

$$\overline{Q}_{i,j,\text{M/M/1}} = \frac{\rho_{i,j}}{1-\rho_i} \, . \qquad (8)$$

The number of waiting packets for GI/G/1 queues:

$$\overline{Q}_{i,j,\text{GI/G/1}} = f(\rho_i \, , c^2_{A,i,j}, c^2_{S,i,j})\overline{Q}_{i,j,\text{M/M/1}}, \qquad (9)$$

is estimated using a chosen approximation, $f(\rho_i \, , c^2_{A,i,j}, c^2_{S,i,j})$, where $c^2_{A,i,j}$ and $c^2_{S,i,j}$ represent the squared coefficients of variation of arrival and service time distributions involved in the method of decomposition for open non-product-form networks (Allen 1990; Whitt 1983a,b, 1993, 1994; Belch *et al*. 1998; Batovski 2008).

The corresponding average delay of waiting in a queue for a given class of packets is obtained from the average number of waiting packets for the said class with the use of the Little's Law (Little 1961). The sum of average waiting time and average service time (which is reciprocal to the average service rate) gives the average delay in a node for the given class of packets.

The average delay for a set of N independent nodes is obtained from the average rate-delay product (Inthawadee and Batovski 2008; Batovski 2009), which in a general form is shown as:

$$D = \frac{1}{\lambda_{Total}} \sum_{i=1}^{N} \lambda_{out,i} D_{out,i} \ . \qquad (10)$$

The cooperation among the set of local nodes results in a modified expression:

$$D = \frac{1}{\lambda_{Total}} \sum_{i=1}^{N} \left(\lambda_{out,i} D_{out,i} + \sum_{j=1}^{N_{j \to i}} \lambda_{j,i} D_{j,i} \right), \qquad (11)$$

where

$$\lambda_{out,i} = \lambda_{in,i} + \sum_{j=1}^{N_{j \to i}} \lambda_{j,i} - \sum_{k=1}^{N_{i \to j}} \lambda_{i,j} \geq 0 \, . \qquad (12)$$

Numerical solutions for the minimization of the average delay, *D*, have been obtained for multiple independent output ports and M/M/1 queues in each port with the method of Lagrange multipliers (Inthawadee and Batovski 2008; Inthawadee 2009).

The formula of Theorem 1 by Inthawadee and Batovski (2008) for M/M/1 queues in cooperative nodes forming a distributed gateway with multiple output ports can be modified to include the background traffic in each node:

$$\lambda_{out,i} = \frac{\sqrt{\mu_i \, (1-\rho_{B,i})} \left(\lambda_{Total} - \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} \mu_j (1-\rho_{B,j}) - \sqrt{\mu_i \, (1-\rho_{B,i})} \sum_{\substack{j=1 \\ j \neq i}}^{N} \sqrt{\mu_j (1-\rho_{B,j})} \right) \right)}{\sum_{j=1}^{N} \sqrt{\mu_j (1-\rho_{B,j})}} \geq 0 \, , \qquad (13)$$

where $\rho_{B,i}$, i = 1,…, N, is the utilization of node i due to background packet flows.

The method of Lagrange multipliers results in systems of non-linear equations for nodes with single ports.

An alternative approach has been used (Batovski 2009; Inthawadee 2009) in equalizing (whenever possible) the rate-delay products among connected nodes, so that

$$\lambda_{out,1} D_{out,1} = \lambda_{out,2} D_{out,2} = \dots . \qquad (14)$$

The level of complexity increases for nodes with a single output port, because the delays $D_{out,i}$ and $D_{j,i}$ in Eq. (11) depend on a multitude of local arrival rates which are included in ρ_i in Eq. (4).

The advantage of the method of decomposition for open networks is that the average delay of individual flows can be estimated separately from the rest of the traffic.

The analytic approach for the optimization of the average delay of the set of cooperative nodes can be summarized, as follows:

- For a given set of connected nodes, the direction of internal flows is chosen so that the underutilized nodes receive and relay more traffic in a particular direction.
- The method of decomposition for open networks allows one to estimate separately the average number of waiting packets of every packet flow.
- The Little's Law is used to estimate the average delay of every packet flow given the average number of waiting packets of said flow and the average source rate.

Internal Link Directivity

As stated by Inthawadee and Batovski (2008), the internal packet flows have chosen directions pointing to the underutilized nodes which may share the load of congested neighbors. It is assumed that a packet may perform only one hop within the set before leaving it. A set of two connected nodes forms a cooperative pair. A set of three nodes forms a triangular configuration of three pairs in a closed loop. A set of four nodes forms a tetrahedral configuration of four pairs in four interconnected triangular closed loops. However, the directivity of chosen internal links reduces the number of pairs participating in the optimization problem.

For two nodes, (N_1 and N_2), there is a single pair, so that the following directed links could be established in two different scenarios:

$N_1 \leftarrow N_2$, or $N_1 \rightarrow N_2$

For three nodes (N_1, N_2, and N_3), there is one node sending traffic to two neighbors or one node receiving traffic from two neighbors, or two pairs, so that the following directed links could be established in two different scenarios:

$N_2 \leftarrow N_1 \rightarrow N_3$, or $N_2 \rightarrow N_1 \leftarrow N_3$.

For four nodes (N_1, N_2, N_3 and N_4), there is one node sending traffic to three neighbors or receiving traffic from three neighbors, so that the following directed links could be established in two different scenarios:

$$N_2 \leftarrow N_1 \rightarrow N_3, \text{ with } N_1 \rightarrow N_4; \text{ or } N_2 \rightarrow N_1 \leftarrow N_3, \text{ with } N_4 \rightarrow N_1.$$

Other scenarios could split the triangular or tetrahedral configuration into individual pairs to be optimized independently, because (according to the two assumptions: only one internal hop is allowed, and a chosen directivity applies to the selected links) a node is not allowed to receive and send internal traffic at the same time for a given optimization.

Note that the directivity of the links (for a portion of the traffic to be optimized locally in a particular direction) is a logical concept used by the optimization algorithm and it does not apply to the background traffic related to other packet flows which may flow in both directions for a given connection.

M/M/1 Queues

A simplification of Eq. (14) is obtained for M/M/1 queues since all squared coefficients of variation are equal to 1 and the solution depends only of the average source and service rates.

Two nodes: Let the known average rates be denoted as $\lambda_{in,i}, \mu_{out,i}$, and $\mu_{j,i}$ and the corresponding unknown rates be denoted as $\lambda_{out,i}$ and $\lambda_{j,i}$, i, j =1, 2.

In the absence of arbitrary background traffic, the equalization of rate-delay products as given by Eq. (14) for the scenario $N_1 \leftarrow N_2$ for M/M/1 queues can be written, as follows:

$$\frac{\lambda_{in,1} + \lambda_{21}}{\mu_{out,1}(1 - \frac{\lambda_{in,1} + \lambda_{21}}{\mu_{out,1}})} + \frac{\lambda_{21}}{\mu_{21}(1 - \frac{\lambda_{in,2} - \lambda_{21}}{\mu_{out,2}} - \frac{\lambda_{21}}{\mu_{21}})}$$

$$= (\lambda_{in,2} - \lambda_{21}) \frac{1}{\mu_{out,2}(1 - \frac{\lambda_{in,2} - \lambda_{21}}{\mu_{out,2}} - \frac{\lambda_{21}}{\mu_{21}})}, \tag{15}$$

where:

$$\lambda_{out,1} = \lambda_{in,1} + \lambda_{21}, \tag{16}$$

$$\lambda_{out,2} = \lambda_{in,2} - \lambda_{21}, \tag{17}$$

$$\rho_1 = \frac{\lambda_{in,1} + \lambda_{21}}{\mu_{out,1}}, \text{ and} \tag{18}$$

$$\rho_2 = \frac{\lambda_{in,2} - \lambda_{21}}{\mu_{out,2}} + \frac{\lambda_{21}}{\mu_{21}}. \tag{19}$$

Equation (15) is to be solved for the unknown internal rate λ_{21}, and it is a quadratic equation for λ_{21} which can be simplified, as follows:

$$(\lambda_{in,1} + \lambda_{21})\mu_{out,2}\mu_{21}(1 - \frac{\lambda_{in,2} - \lambda_{21}}{\mu_{out,2}} - \frac{\lambda_{21}}{\mu_{21}})$$
$$= ((\lambda_{in,2} - \lambda_{21})\mu_{21} - \lambda_{21}\mu_{out,2})\mu_{out,1}(1 - \frac{\lambda_{in,1} + \lambda_{21}}{\mu_{out,1}}). \tag{20}$$

The analytical solution of Eq. (15) is given by the following analytic expression:

$$\lambda_{21} = \frac{1}{4\mu_{out,2}}$$
$$\times [(\mu_{21}\mu_{out,1} - 2\lambda_{in,1}\mu_{out,2} + \mu_{21}\mu_{out,2} + \mu_{out,1}\mu_{out,2})$$
$$\pm \sqrt{\begin{array}{l} 8\mu_{out,2}(-\lambda_{in,2}\mu_{21}\mu_{out,1} + \lambda_{in,1}\mu_{21}\mu_{out,2}) \\ + (\mu_{21}\mu_{out,1} - 2\lambda_{in,1}\mu_{out,2} \\ + \mu_{21}\mu_{out,2} + \mu_{out,1}\mu_{out,2})^2 \end{array}}], \tag{21}$$

From the two possible solutions of the quadratic Eq. (15), the positive real solution is chosen, if it does exist, for:

$$8\mu_{out,2}(-\lambda_{in,2}\mu_{21}\mu_{out,1} + \lambda_{in,1}\mu_{21}\mu_{out,2})$$
$$+ (\mu_{21}\mu_{out,1} - 2\lambda_{in,1}\mu_{out,2}$$
$$+ \mu_{21}\mu_{out,2} + \mu_{out,1}\mu_{out,2})^2 \geq 0. \tag{22}$$

In the presence of arbitrary background traffic, the equalization of rate-delay products is based on the following equation:

$$\frac{\lambda_{in,1} + \lambda_{21}}{\mu_{out,1}(1 - \frac{\lambda_{in,1} + \lambda_{21}}{\mu_{out,1}} - \rho_{B,1})}$$
$$+ \frac{\lambda_{21}}{\mu_{21}(1 - \frac{\lambda_{in,2} - \lambda_{21}}{\mu_{out,2}} - \frac{\lambda_{21}}{\mu_{21}} - \rho_{B,2})}$$
$$= \frac{\lambda_{in,2} - \lambda_{21}}{\mu_{out,2}(1 - \frac{\lambda_{in,2} - \lambda_{21}}{\mu_{out,2}} - \frac{\lambda_{21}}{\mu_{21}} - \rho_{B,2})}, \tag{23}$$

where $\rho_{B,1}$ and $\rho_{B,2}$ are the average utilizations of background traffic in nodes N_1 and N_2, correspondingly.

Equation (23) is also to be solved for the unknown internal rate λ_{21} and similarly to Eq. (15) can be simplified, as follows:

$$(\lambda_{in,1} + \lambda_{21})\mu_{out,2}\mu_{21}(1 - \frac{\lambda_{in,2} - \lambda_{21}}{\mu_{out,2}} - \frac{\lambda_{21}}{\mu_{21}} - \rho_{B,2})$$
$$= ((\lambda_{in,2} - \lambda_{21})\mu_{21} - \lambda_{21}\mu_{out,2})$$
$$\times \mu_{out,1}(1 - \frac{\lambda_{in,1} + \lambda_{21}}{\mu_{out,1}} - \rho_{B,1}). \tag{24}$$

The analytical solution of Eq. (23) is given by the following analytic expression:

$$\lambda_{21} = \frac{1}{4\mu_{out,2}}$$
$$\times [(\mu_{21}\mu_{out,1} - 2\lambda_{in,1}\mu_{out,2} + \mu_{21}\mu_{out,2} + \mu_{out,1}\mu_{out,2}$$
$$- \mu_{21}\mu_{out,1}\rho_{B,1} - \mu_{out,1}\mu_{out,2}\rho_{B,1} - \mu_{21}\mu_{out,2}\rho_{B,2})$$
$$\pm \sqrt{\begin{array}{l} 8\mu_{out,2}\mu_{21}(\lambda_{in,2}\mu_{out,1}(\rho_{B,1} - 1) - \lambda_{in,1}\mu_{out,2}(\rho_{B,2} - 1)) \\ + (\mu_{21}(\mu_{out,1}(\rho_{B,1} - 1) + \mu_{out,2}(\rho_{B,2} - 1)) \\ + (\mu_{out,2}(2\lambda_{in,1} + \mu_{out,1}(\rho_{B,1} - 1)) \\ + \mu_{21}(\mu_{out,1}(\rho_{B,1} - 1) + \mu_{out,2}(\rho_{B,2} - 1)))^2 \end{array}}], \tag{25}$$

From the two possible solutions of the quadratic Eq. (23), the positive real solution is chosen, if it does exist, for:

$$8\mu_{out,2}\mu_{21}(\lambda_{in,2}\mu_{out,1}(\rho_{B,1} - 1) - \lambda_{in,1}\mu_{out,2}(\rho_{B,2} - 1))$$
$$+ (\mu_{21}(\mu_{out,1}(\rho_{B,1} - 1) + \mu_{out,2}(\rho_{B,2} - 1))$$
$$+ (\mu_{out,2}(2\lambda_{in,1} + \mu_{out,1}(\rho_{B,1} - 1))$$
$$+ \mu_{21}(\mu_{out,1}(\rho_{B,1} - 1) + \mu_{out,2}(\rho_{B,2} - 1)))^2 \geq 0. \tag{26}$$

Obviously, Eq. (21) can be obtained from Eq. (25) for $\rho_{B,1} = 0$ and $\rho_{B,2} = 0$.

The solution for scenario $N_1 \rightarrow N_2$ for M/M/1 queues can be written by simply exchanging the places of indexes 1 and 2 in the above equations.

Three nodes: In the presence of arbitrary background traffic, the equalization of rate-delay products as given by Eq. (14) for the scenario $N_2 \rightarrow N_1 \leftarrow N_3$ for M/M/1 queues can be written, as follows:

$$\frac{(\lambda_{in,1}+\lambda_{21}+\lambda_{31})}{\mu_{out,1}(1-\frac{\lambda_{in,1}+\lambda_{21}+\lambda_{31}}{\mu_{out,1}}-\rho_{B,1})} + \frac{\lambda_{21}}{\mu_{21}(1-\frac{\lambda_{in,2}-\lambda_{21}}{\mu_{out,2}}-\frac{\lambda_{21}}{\mu_{21}}-\rho_{B,2})} + \frac{\lambda_{31}}{\mu_{31}(1-\frac{\lambda_{in,3}-\lambda_{31}}{\mu_{out,3}}-\frac{\lambda_{31}}{\mu_{31}}-\rho_{B,3})}$$

$$= \frac{(\lambda_{in,2}-\lambda_{21})}{\mu_{out,2}(1-\frac{\lambda_{in,2}-\lambda_{21}}{\mu_{out,2}}-\frac{\lambda_{21}}{\mu_{21}}-\rho_{B,2})} = \frac{(\lambda_{in,3}-\lambda_{31})}{\mu_{out,3}(1-\frac{\lambda_{in,3}-\lambda_{31}}{\mu_{out,3}}-\frac{\lambda_{31}}{\mu_{31}}-\rho_{B,3})}, \quad (27)$$

where:

$$\lambda_{out,1} = \lambda_{in,1}+\lambda_{21}+\lambda_{31}, \quad (28)$$

$$\lambda_{out,2} = \lambda_{in,2}-\lambda_{21}, \quad (29)$$

$$\lambda_{out,3} = \lambda_{in,3}-\lambda_{31}, \quad (30)$$

$$\rho_1 = \frac{\lambda_{in,1}+\lambda_{21}+\lambda_{31}}{\mu_{out,1}} + \rho_{B,1}, \quad (31)$$

$$\rho_2 = \frac{\lambda_{in,2}-\lambda_{21}}{\mu_{out,2}} + \frac{\lambda_{21}}{\mu_{21}} + \rho_{B,2}, \text{ and} \quad (32)$$

$$\rho_3 = \frac{\lambda_{in,3}-\lambda_{31}}{\mu_{out,3}} + \frac{\lambda_{31}}{\mu_{31}} + \rho_{B,3}. \quad (33)$$

and $\rho_{B,1}$, $\rho_{B,2}$, and $\rho_{B,3}$ are the average utilizations of background traffic in nodes N_1, N_2, and N_3, correspondingly.

For the scenario, $N_2 \leftarrow N_1 \rightarrow N_3$, the equalization of rate-delay products for M/M/1 queues can be written, as follows:

$$\frac{(\lambda_{in,1}-\lambda_{12}-\lambda_{13})}{\mu_{out,1}(1-\frac{\lambda_{in,1}-\lambda_{12}-\lambda_{13}}{\mu_{out,1}}-\frac{\lambda_{12}}{\mu_{12}}-\frac{\lambda_{13}}{\mu_{13}}-\rho_{B,1})} = \frac{(\lambda_{in,2}+\lambda_{12})}{\mu_{out,2}(1-\frac{\lambda_{in,2}+\lambda_{12}}{\mu_{out,2}}-\rho_{B,2})} + \frac{\lambda_{12}}{\mu_{12}(1-\frac{\lambda_{in,1}-\lambda_{12}-\lambda_{13}}{\mu_{out,1}}-\frac{\lambda_{12}}{\mu_{12}}-\frac{\lambda_{13}}{\mu_{13}}-\rho_{B,1})}$$

$$= \frac{(\lambda_{in,3}+\lambda_{13})}{\mu_{out,3}(1-\frac{\lambda_{in,3}+\lambda_{13}}{\mu_{out,3}}-\rho_{B,3})} + \frac{\lambda_{13}}{\mu_{13}(1-\frac{\lambda_{in,1}-\lambda_{12}-\lambda_{13}}{\mu_{out,1}}-\frac{\lambda_{12}}{\mu_{12}}-\frac{\lambda_{13}}{\mu_{13}}-\rho_{B,1})}, \quad (34)$$

where:

$$\lambda_{out,1} = \lambda_{in,1}-\lambda_{12}-\lambda_{13}, \quad (35)$$

$$\lambda_{out,2} = \lambda_{in,2}+\lambda_{12}, \quad (36)$$

$$\lambda_{out,3} = \lambda_{in,3}+\lambda_{13}, \quad (37)$$

$$\rho_1 = \frac{\lambda_{in,1}-\lambda_{12}-\lambda_{13}}{\mu_{out,1}} + \frac{\lambda_{12}}{\mu_{12}} + \frac{\lambda_{13}}{\mu_{13}} + \rho_{B,1}, \quad (38)$$

$$\rho_2 = \frac{\lambda_{in,2}+\lambda_{12}}{\mu_{out,2}} + \rho_{B,2}, \text{ and} \quad (39)$$

$$\rho_3 = \frac{\lambda_{in,3}+\lambda_{13}}{\mu_{out,3}} + \rho_{B,3}. \quad (40)$$

and $\rho_{B,1}$, $\rho_{B,2}$, and $\rho_{B,3}$ are the average utilizations of background traffic in nodes N_1, N_2, and N_3, correspondingly.

Four nodes: In the presence of arbitrary background traffic, the equalization of rate-delay products as given by Eq. (14) for the scenario

$$\begin{array}{c} N_2 \rightarrow N_1 \leftarrow N_3 \\ \uparrow \\ N_4 \end{array}$$

for M/M/1 queues can be written, as follows:

$$\frac{\lambda_{in,1}+\lambda_{21}+\lambda_{31}+\lambda_{41}}{\mu_{out,1}(1-\frac{\lambda_{in,1}+\lambda_{21}+\lambda_{31}+\lambda_{41}}{\mu_{out,1}}-\rho_{B,1})}+\frac{\lambda_{21}}{\mu_{21}(1-\frac{\lambda_{in,2}-\lambda_{21}}{\mu_{out,2}}-\frac{\lambda_{21}}{\mu_{21}}-\rho_{B,2})}+\frac{\lambda_{31}}{\mu_{31}(1-\frac{\lambda_{in,3}-\lambda_{31}}{\mu_{out,3}}-\frac{\lambda_{31}}{\mu_{31}}-\rho_{B,3})}+\frac{\lambda_{41}}{\mu_{41}(1-\frac{\lambda_{in,4}-\lambda_{41}}{\mu_{out,4}}-\frac{\lambda_{41}}{\mu_{41}}-\rho_{B,4})}$$

$$=\frac{\lambda_{in,2}-\lambda_{21}}{\mu_{out,2}(1-\frac{\lambda_{in,2}-\lambda_{21}}{\mu_{out,2}}-\frac{\lambda_{21}}{\mu_{21}}-\rho_{B,2})}=\frac{\lambda_{in,3}-\lambda_{31}}{\mu_{out,3}(1-\frac{\lambda_{in,3}-\lambda_{31}}{\mu_{out,3}}-\frac{\lambda_{31}}{\mu_{31}}-\rho_{B,3})}=\frac{\lambda_{in,4}-\lambda_{41}}{\mu_{out,4}(1-\frac{\lambda_{in,4}-\lambda_{41}}{\mu_{out,4}}-\frac{\lambda_{41}}{\mu_{41}}-\rho_{B,4})},\quad (41)$$

where:

$$\lambda_{out,1}=\lambda_{in,1}+\lambda_{21}+\lambda_{31}+\lambda_{41}, \quad (42)$$

$$\lambda_{out,2}=\lambda_{in,2}-\lambda_{21}, \quad (43)$$

$$\lambda_{out,3}=\lambda_{in,3}-\lambda_{31}, \quad (44)$$

$$\lambda_{out,4}=\lambda_{in,4}-\lambda_{41}, \quad (45)$$

$$\rho_1=\frac{\lambda_{in,1}+\lambda_{21}+\lambda_{31}+\lambda_{41}}{\mu_{out,1}}+\rho_{B,1}, \quad (46)$$

$$\rho_2=\frac{\lambda_{in,2}-\lambda_{21}}{\mu_{out,2}}+\frac{\lambda_{21}}{\mu_{21}}+\rho_{B,2}, \quad (47)$$

$$\rho_3=\frac{\lambda_{in,3}-\lambda_{31}}{\mu_{out,3}}+\frac{\lambda_{31}}{\mu_{31}}+\rho_{B,3}, \text{ and} \quad (48)$$

$$\rho_4=\frac{\lambda_{in,4}-\lambda_{41}}{\mu_{out,4}}+\frac{\lambda_{41}}{\mu_{41}}+\rho_{B,4}, \quad (49)$$

and $\rho_{B,1}$, $\rho_{B,2}$, $\rho_{B,3}$, and $\rho_{B,4}$ are the average utilizations of background traffic in nodes N_1, N_2, N_3, and N_4, correspondingly.

For the scenario,

$$\begin{array}{c} N_2 \leftarrow N_1 \rightarrow N_3, \\ \downarrow \\ N_4 \end{array}$$

the equalization of rate-delay products for M/M/1 queues can be written, as follows:

$$\frac{\lambda_{in,1}-\lambda_{12}-\lambda_{13}-\lambda_{14}}{\mu_{out,1}\left(1-\frac{\lambda_{in,1}-\sum_{j=2}^{4}\lambda_{1j}}{\mu_{out,1}}-\sum_{j=2}^{4}\frac{\lambda_{1j}}{\mu_{1j}}-\rho_{B,1}\right)}$$

$$=\frac{\lambda_{in,2}+\lambda_{12}}{\mu_{out,2}(1-\frac{\lambda_{in,2}+\lambda_{12}}{\mu_{out,2}}-\rho_{B,2})}+\frac{\lambda_{12}}{\mu_{12}\left(1-\frac{\lambda_{in,1}-\sum_{j=2}^{4}\lambda_{1j}}{\mu_{out,1}}-\sum_{j=2}^{4}\frac{\lambda_{1j}}{\mu_{1j}}-\rho_{B,1}\right)}$$

$$=\frac{\lambda_{in,3}+\lambda_{13}}{\mu_{out,3}(1-\frac{\lambda_{in,3}+\lambda_{13}}{\mu_{out,3}}-\rho_{B,3})}+\frac{\lambda_{13}}{\mu_{13}\left(1-\frac{\lambda_{in,1}-\sum_{j=2}^{4}\lambda_{1j}}{\mu_{out,1}}-\sum_{j=2}^{4}\frac{\lambda_{1j}}{\mu_{1j}}-\rho_{B,1}\right)}$$

$$=\frac{\lambda_{in,4}+\lambda_{14}}{\mu_{out,4}(1-\frac{\lambda_{in,4}+\lambda_{14}}{\mu_{out,4}}-\rho_{B,4})}+\frac{\lambda_{14}}{\mu_{14}\left(1-\frac{\lambda_{in,1}-\sum_{j=2}^{4}\lambda_{1j}}{\mu_{out,1}}-\sum_{j=2}^{4}\frac{\lambda_{1j}}{\mu_{1j}}-\rho_{B,1}\right)},\quad (50)$$

where:

$$\lambda_{out,1}=\lambda_{in,1}-\lambda_{12}-\lambda_{13}-\lambda_{14}, \quad (51)$$

$$\lambda_{out,2}=\lambda_{in,2}+\lambda_{12}, \quad (52)$$

$$\lambda_{out,3} = \lambda_{in,3} + \lambda_{13}, \quad (53)$$

$$\lambda_{out,4} = \lambda_{in,4} + \lambda_{14}, \quad (54)$$

$$\rho_1 = \frac{\lambda_{in,1} - \sum_{j=2}^{4} \lambda_{1j}}{\mu_{out,1}} + \sum_{j=2}^{4} \frac{\lambda_{1j}}{\mu_{1j}} + \rho_{B,1}, \quad (55)$$

$$\rho_2 = \frac{\lambda_{in,2} + \lambda_{12}}{\mu_{out,2}} + \rho_{B,2}, \quad (56)$$

$$\rho_3 = \frac{\lambda_{in,3} + \lambda_{13}}{\mu_{out,3}} + \rho_{B,3}, \text{ and} \quad (57)$$

$$\rho_4 = \frac{\lambda_{in,4} + \lambda_{14}}{\mu_{out,4}} + \rho_{B,4}, \quad (58)$$

and $\rho_{B,1}$, $\rho_{B,2}$, $\rho_{B,3}$, and $\rho_{B,4}$ are the average utilizations of background traffic in nodes N_1, N_2, N_3, and N_4, correspondingly.

Discussion

The equations derived for three and four nodes can be solved using standard numerical methods. Rate-delay equalization usually can take place in cases when the mean values and the coefficients of variation of the service time distributions of links between the participating nodes do not differ significantly. If there is a node with increased service times, its capability in sharing traffic is rather limited and there is no guarantee that a solution for rate-delay equalization with other adjacent neighbors does exist. Therefore, the selection of participating nodes depends on the channel conditions in each node.

The extension of the analytic model for GI/G/1 queues can be made using the method of decomposition for open non-product-form networks (Pujolle and Wu 1986; Belch *et al.* 1998; Batovski 2008) to estimate the delay with Eq. 9. A summary of the said method of merging-flow-splitting is provided below.

The following fit for the function $f(\rho_i, c^2_{A,i,j}, c^2_{S,i,j})$ in Eq. (9) covers a wide range of squared coefficients of variation (Whitt 1993):

$$f(\rho_i, c^2_{A,i,j}, c^2_{S,i,j}) = \phi \frac{c^2_{A,i,j} + c^2_{S,i,j}}{2}, \quad (59)$$

with (here the indexes i and j go without saying) (Whitt 1993):

$$\phi = \begin{cases} \dfrac{4(c_A^2 - c_S^2)}{4c_A^2 - 3c_S^2}\phi_1 + \dfrac{c_S^2}{4c_A^2 - 3c_S^2}\Psi; c_A^2 \geq c_S^2 \\ \dfrac{c_S^2 - c_A^2}{2(c_A^2 + c_S^2)}\phi_3 + \dfrac{c_S^2 + 3c_A^2}{2(c_A^2 + c_S^2)}\Psi; c_A^2 \leq c_S^2 \end{cases}, \quad (60)$$

$$\Psi = \begin{cases} 1; & c^2 \geq 1 \\ \phi_4^{2(1-c^2)}; & 0 \leq c^2 \leq 1 \end{cases} \text{ where } c^2 = \frac{c_A^2 + c_S^2}{2}, \quad (61)$$

$$\phi_1 = 1 + \gamma, \quad (62)$$

$$\phi_2 = 1 - 4\gamma, \quad (63)$$

$$\phi_3 = \phi_2 \exp\left(-\frac{2(1-\rho)}{3\rho}\right), \quad (64)$$

$$\phi_4 = \min\left\{1, \frac{\phi_1 + \phi_3}{2}\right\}, \quad (65)$$

$$\gamma = \min\left\{0.24, \frac{(1-\rho)(m-1)(\sqrt{4+5m}-2)}{16m\rho}\right\}, \quad (66)$$

where: ρ is the overall node utilization, c_A^2 is the squared coefficient of variation (scv) of the inter-arrival time distribution of a given packet flow; and c_S^2 is the scv of the service time distribution of said packet flow. For the case of a single server considered in this contribution, $m = 1$.

The merging process determines the overall scv coefficient of a node (Belch *et al.* 1998):

$$c^2_{A,i} = \frac{1}{\lambda_i}\left[\left(\sum_{j=1}^{N_{i\to j}} c^2_{A,ij}\lambda_{ij}\right) + c^2_{B,A,i}\lambda_{B,i}\right], \quad (67)$$

where the additional terms $c^2_{B,A,i}$ and $\lambda_{B,i}$ are the arrival scv and the average arrival rate of background traffic.

The overall scv coefficient of the service time of a node is obtained from:

$$c^2_{S,i} = -1 + \sum_{j-1}^{N_{i\to j}} \frac{\lambda_{ij}}{\lambda_i}\left(\frac{\mu_i}{\mu_{ij}}\right)^2 (c^2_{S,ij} + 1)$$

$$+ \frac{\lambda_{B,i}}{\lambda_i}\left(\frac{\mu_i}{\mu_{B,i}}\right)^2 (c^2_{B,S,i} + 1), \quad (68)$$

where the additional terms $c^2_{B,S,i}$ and $\mu_{B,i}$ are the service scv and the average service rate of background traffic.

Knowing $c^2_{A,i}$ and $c^2_{S,i}$, one can estimate the scv of the flow process (Whitt 1983a,b):

$$c_{D,i}^2 = 1 + \frac{\rho_i^2 (c_{S,i}^2 - 1)}{\sqrt{m_i}} + (1 - \rho_i^2)(c_{A,i}^2 - 1)\,, (69)$$

where the number of servers $m = 1$. Alternative estimations of the flow process can be used instead (Belch *et al*. 1998).

The splitting process determines the scv coefficient of a departing process (Belch *et al*. 1998):

$$c_{ij}^2 = 1 + \frac{\lambda_{ij}}{\lambda_i} \left(\sum_{j=1}^{N_{i \to j}} c_{D,i}^2 - 1 \right), \qquad (70)$$

where λ_i is the overall average arrival rate in node *i* including the background traffic.

The rate-delay equalization for nodes with for GI/G/1 queues is quite complex due to the non-linear analytical expressions involved in the estimation of the scv coefficients. The scv coefficients depend on the unknown rates λ_{ij} of traffic sharing among the nodes. The simpler Allen-Cunneen approximation (Allen 1990) can also be used instead of Eq. (59):

$$f(c_{A,i,j}^2, c_{S,i,j}^2) = \frac{c_{A,i,j}^2 + c_{S,i,j}^2}{2}. \qquad (71)$$

Computational experiments with the use of the method of decomposition for cooperative nodes consisting of two, three and four nodes with GI/G/1 queues and different levels of background traffic are included in the second part of this contribution.

Conclusion

The local routing depends on a significant number of statistical parameters and the level of complexity increases rapidly with the inclusion of more cooperative nodes. The obtained analytical solutions for two connected nodes with M/M/1 queues can be used as initial approximations in obtaining solutions for the more complex problems with three and four nodes and GI/G/1 queues. Statistically, pairs of only two nodes appear more frequently in arbitrary topological configurations. The sets of three and four nodes have an increased capability to share traffic whenever the local topology has such nodes in a close proximity to each other.

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