How to Use Mathematical and Physical Constants*

The technical studies utilize a large number of constants which occur in mathematics and physics. The symbolic representation of such constants is often straightforward but their numerical values should also be shown, especially when the calculations are sensitive to the number of true digits taken into consideration.

Mathematical Constants

A constant can be named in accordance with its meaning, after a person, or on the basis of historical accounts. In mathematics, most constants are denoted by lower case letters of the Latin and Greek alphabets but upper case letters and complex notations are also used. Some constants may not have a designated symbol, like the Laplace limit (Sloane 2011), 0.662 743 419…

Specified Mathematical Constants

The numbers 0 (zero) and 1 (one, or unity) which form the binary set {0, 1} can be considered as constants. The imaginary unit, $i = (-1)^{0.5}$, is also a constant.

The following typical examples illustrate the representation of specified mathematical constants which are shown with a certain number of true digits (Sloane 2011):

- Archimedes' constant, or Ludolph's number (the ratio of a circle's circumference to its diameter), $\pi = 3.141\ 592\ 653\ldots$;
- Napier's constant, or Euler's number, $e = 2.718\ 281\ 828\ldots$;
- Euler-Mascheroni constant, $\gamma = 0.577\ 215\ 664\ldots$;
- golden ratio, $\varphi = 1.618\ 033\ 988\ldots$, etc.

Unspecified Mathematical Constants

Some constants are unspecified when they appear in analytical expressions, like in the solutions of indefinite integrals which are unique up to an arbitrary constant C:

$$\int(1/x)dx = \ln(x) + C.$$

Unspecified constants (or parameters) are common in mathematical equations as opposed to variable quantities (variables). The constancy of a mathematical parameter is context-dependent because it can remain constant with respect to some variable(s) and eventually depend on other additional variable(s) as shown in the following equation:

$$f(x,y) = C(x)y,$$

where C depends on the variable x but it does not depend on the variable y.

New Mathematical Constants

Whenever a new mathematical constant is obtained and introduced for the first time, the author(s) of the paper should provide a proper definition which can be verified by other mathematicians. A survey of existing constants should be conducted before to announce the existence of the new constant. Such a constant can remain unnamed or be given a specific name by the author(s) depending on its estimated frequency of use and importance. In many cases, new constants are named and denoted later in works of other authors.

For example, Batovski (2005) compared a discrete Bernoulli distribution with a corresponding continuous linear distribution in order to determine the distinct probabilities for which the standard deviations of the two distributions are equal, $\sigma_{continuous} = \sigma_{discrete}$, and obtained the quadratic equation,

$$p_1(1 - p_1) = (1 + 2\,p_1)/6 - [(1 + p_1)/3]^2, \quad (1)$$

which has two solutions, $p_1 = 0.0669\ldots$, and $p_1 = (1 - 0.0669\ldots) = 0.9330\ldots$ The numerical values in this particular case are new constants which have not been introduced in earlier works of other mathematicians.

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Fundamental Physical Constants

The International Bureau of Weights and Measures (BIPM, in French: Bureau International des Poids et Mesures) uses the International System of Units (SI, in French: Système International d'Unités), or SI base units (meter, m; kilogram, kg; second, s; Ampère, A; Kelvin, K; mole, mol; and candela, cd) to represent the measuring units of fundamental physical constants. A list of some frequently used physical constants (CODATA 2010) is provided below with standard uncertainty shown in parentheses.

Dimensional Physical Constants

Universal physical constants:

- electric constant (permittivity of free space), $\varepsilon_0 = 8.854\ 187\ 817... \times 10^{-12}$ m^{-3} kg^{-1} s^{4} A^{2};
- magnetic constant (permeability of free space), $\mu_0 = 4\pi \times 10^{-7}$ kg m s^{-2} A^{-2};
- Newtonian constant of gravitation (universal gravitational constant), $G = 6.673\ 84(80) \times 10^{-11}$ m^{3} kg^{-1} s^{-2};
- Planck length, $l_P = 1.616\ 199(97) \times 10^{-35}$ m;
- Planck mass, $m_P = 2.176\ 51(13) \times 10^{-8}$ kg;
- Planck time, $t_P = 5.391\ 06(32) \times 10^{-44}$ s;
- Planck temperature, $T_P = 1.416\ 833(85) \times 10^{32}$ K;
- Planck constant, $h = 6.626\ 069\ 57(29) \times 10^{-34}$ kg m^{2} s^{-1};
- speed of light in vacuum, $c = 299{,}792{,}458$ m s^{-1}; etc.

Electromagnetic constants:

- Bohr magneton, $\mu_B = 927.400\ 968(20) \times 10^{-26}$ kg m^{2} s^{-2} T^{-1};
- nuclear magneton, $\mu_N = 5.050\ 783\ 53(11) \times 10^{-27}$ kg m^{2} s^{-2} T^{-1};
- elementary (positive) charge, $e = 1.602\ 176\ 565(35) \times 10^{-19}$ s A; etc.

Atomic and nuclear constants:

- Bohr radius, $\alpha_0 = 0.529\ 177\ 210\ 92(17) \times 10^{-10}$ m;
- alpha particle mass, $m_\alpha = 6.644\ 656\ 75(29) \times 10^{-27}$ kg;
- electron mass, $m_e = 9.109\ 382\ 91(40) \times 10^{-31}$ kg;
- neutron mass, $m_n = 1.674\ 927\ 351(74) \times 10^{-27}$ kg;
- proton mass, $m_p = 1.672\ 621\ 777(74) \times 10^{-27}$ kg; etc.

Physico-chemical constants:

- Avogadro's number, $N_A = 6.022\ 141\ 29(27) \times 10^{23}$ mol^{-1};
- Boltzmann constant, $k_B = R/N_A = 1.380\ 6488(13) \times 10^{-23}$ kg m^{2} s^{-2} K^{-1};
- ideal gas constant, $R = 8.314\ 4621(75)$ kg m^{2} s^{-2} K^{-1} mol^{-1}; etc.

Dimensionless (or of Dimension One) Physical Constants

Some physical constants are dimensionless, for instance:

- the fine-structure constant $\alpha = 7.297\ 352\ 5698(24) \times 10^{-3} = 1/137.035\ 999\ 074(44)$;
- the gravitational coupling constant (calculated on the basis of m_e and m_P), $\alpha_G = (m_e/m_P)^2 \approx 1.7518 \times 10^{-45}$; etc.

References

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