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Dynamics of a stage–structure Rosenzweig–MacArthur model with linear harvesting in prey and cannibalism in predator

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Abstract

A kind of stage-structure Rosenzweig–MacArthur model with linear harvesting in prey and cannibalism in predator is investigated in this paper. By analyzing the model, local stability of all possible equilibrium points is discussed. Moreover, the model undergoes a Hopf–bifurcation around the interior equilibrium point. Numerical simulations are carried out to illustrate our main results.

Keywords: *Stage–Structure; Rosenzweig–MacArthur Model; Linear Harvesting; Cannibalism; Stability; Hopf–Bifurcation*

1. Introduction

In the recent decade, mathematics ecology has become one of the dominant analytical scopes [1, 2]. It studies the reality of predator-prey interactions, including changes in population densities because of their interaction [3–5]. The famous modified one is a stage-structure Rosenzweig-MacArthur model defined by

$$\begin{aligned}\frac{dx_1}{dt} &= rx_2\left(1-\frac{x_1}{k}\right)-\alpha x_1-\frac{\beta x_1 x_3}{x_1+n_1}\\ \frac{dx_2}{dt} &= \alpha x_1-\delta_1 x_2\\ \frac{dx_3}{dt} &= \frac{\varphi\beta x_1 x_3}{x_1+n_1}-\delta_2 x_3\end{aligned} \tag{1}$$

where x_1, x_2 and x_3 represents the densities of immature and mature prey as well as predator population at time t, respectively. We assume that the immature prey grows logistically with constant intrinsic rate r and k is the carrying capacity of the environment. β and n_1 are maximum values which per capita reduction rate of immature prey can attain and measure the extent to which enviroment provides protection to immature prey. α and δ_1 represent the surviving rate of immaturity to reach maturity and the per capita death rate of the mature prey, respectively. We assume that the predator does not attack and eat the mature prey. A conversion rate of the consumed prey into the predator birth is φ, and next, δ_2 denotes the per capita death rate of the predator [6–8].

Now, we consider the harvesting and cannibalism processes to modify the stage-structure predator-prey model in [6]. Theoretically, these two aspects can affect the existence of any populations in the system. For consumption and commercial, the immature prey and mature prey are continually being harvested at a linear function rate by various interested parties [9, 10]. The harvesting process changes the population density in the system [11–13]. Although there are several harvesting scheme exist such as threshold harvesting [14–17] which assumes the harvesting is stopped when the population density attains a constant level; and Michaelis-Menten (or nonlinear) harvesting [18–20] which assumes the harvesting has a saturation level, we prefer to employ the linear harvesting [3, 13] which is suitable for a large number of population density. This type of harvesting also fitted for some cases of bioeconomic resources such as fisheries and plantation. On the other hand, many scientists study the vigorous behaviors of the ecosystem with cannibalism. This circumstance represents the behavior of the species which consumes each others and impacts to the decrease of the population density. [21–26]. Another impact of the cannibalism process is to help provide a source of food in the system [21, 25, 26]. The combination of harvesting and cannibalism aspects in the system (1) is very appealing to investigate. As long as we know, The stage-structure Rosenzweig-MacArthur model involving these two ecological component (linear harvesting in prey and canibalism in predator) has never been previously studied. Thus, the local dynamics of this model is the main novelty of our research.

Note that the local stability of the system (1) was investigated by Beay et al. [6]. Furthermore, dynamics of the system (1) with prey refuge was studied by Beay and Saija [7]. Next, in [8] Beay et al. consider intraspecific competition in the system. In this paper, we consider the immature prey and mature prey populations of model (1) where both populations are subjected to a constant rate of harvesting, where h_1 and h_2 represents linear harvesting rate of immature and mature preys, respectively. In addition, there is a process of cannibalism in the predator population. $\frac{\sigma {x_3}^2}{x_3+\omega}$ denotes the cannibalism of the predator, where σ is the rate of cannibalism. The model with linear harvesting in prey and cannibalism in predator is

$$\begin{aligned}\frac{dx_1}{dt} =& rx_2\left(1-\frac{x_1}{k}\right)-\alpha x_1-\frac{\beta x_1 x_3}{x_1+n_1}-h_1x_1\\ \frac{dx_2}{dt} =& \alpha x_1-\delta_1 x_2-h_2x_2\\ \frac{dx_3}{dt} =& \frac{\varphi\beta x_1x_3}{x_1+n_1}-\delta_2x_3-\frac{\sigma {x_3}^2}{x_3+\omega}\end{aligned} \tag{2}$$

Using the following transformation

$$(a,b,c,t)\rightarrow\left(\frac{x_1}{k},\frac{x_2}{k},\frac{\beta x_3}{rk},rt\right)$$

the model in system (2) can be simplified as

$$\begin{aligned}\frac{da}{dt} =& b\left(1-a\right)-\phi a-\frac{ac}{a+m}\\ \frac{db}{dt} =& \theta a-\delta b\\ \frac{dc}{dt} =& \frac{\xi ac}{a+m}-\eta c-\frac{\mu c^2}{c+\psi}\end{aligned} \tag{3}$$

where $\phi=\frac{\alpha+h_1}{r}$, $m=\frac{n_1}{k}$, $\theta=\frac{\alpha}{r}$, $\delta=\frac{\delta_1+h_2}{r}$, $\xi=\frac{\beta\varphi}{r}$, $\eta=\frac{\delta_2}{r}$, $\mu=\frac{\sigma}{r}$, and $\psi=\frac{\omega\beta}{rk}$.

The paper is arranged as follows. In the next section, we analyze the existence and local stability of the all equilibrium points of system (3). In Section 3, we explore the existence of Hopf–bifurcation. Numerical simulations are performed in Section 4. We end this work with a conclusion.

2. Existence and stability analysis of equilibrium points

It is easy to show that system (3) has three non-negative equilibrium points as follows.

- The extinction equilibrium $E_0=(0,0,0)$, which there is no population in the habitat.
- The predator–free equilibrium $E_1=(a_1,b_1,0)$, which exists if

$$\theta>\phi\delta, \tag{4}$$

  where $a_1=\frac{\theta-\phi\delta}{\theta}$ and $b_1=\frac{\theta-\phi\delta}{\delta}$.
- The interior equilibrium $E^*=(a^*,b^*,c^*)$, i.e. all of species coexist, where a^* in the equilibrium point E^* is the positive solution of the cubic equation

$$A_1{a^*}^3+A_2{a^*}^2+A_3a^*+A_4=0,$$

  where

$$\begin{aligned}A_1 =& \theta(\eta+\mu-\xi),\\ A_2 =& (\delta\phi-\theta)(\eta+\mu-\xi)+m\theta(2\eta+2\mu-\xi),\\ A_3 =& m(\delta\phi-\theta)(2\eta+2\mu-\xi)+m^2\theta(\eta+\mu)+\delta\psi(\xi-\eta),\\ A_4 =& m^2(\delta\phi-\theta)(\eta+\mu)-m\eta\delta\psi.\end{aligned}$$

The existence of positive root a^* of a cubic equation can be easily derived by Cardano's criteria. The detail of Cardano's criteria can be seen, for example, in [27] and is not discussed here. Furthermore, the values of b^* and c^* are respectively given by

$$b^* = \frac{\theta}{\delta}a^* \quad \text{and} \quad c^* = \frac{1}{\delta}\left[\theta(a^*+m) - (m\delta\phi + a^*(\theta a^* + \delta\phi + m\theta))\right].$$

Theorem 1. *For system (3), we have the following stability properties of its equilibrium points:*

(i) The equilibrium point E_0 is locally asymptotically stable if $\theta < \phi\delta$.
(ii) The equilibrium point E_1 is locally asymptotically stable if $\eta < \xi + \varpi$.
(iii) The coexistence equilibrium point E^* is locally asymptotically stable if $\tau_1 > 0, \tau_3 > 0$ and $\tau_1\tau_2 > \tau_3$ where τ_1, τ_2 and τ_3 are defined as in the proof.

proof. Now to study the local stability of these equilibrium points, the Jacobian matrix from system (3) is determined as

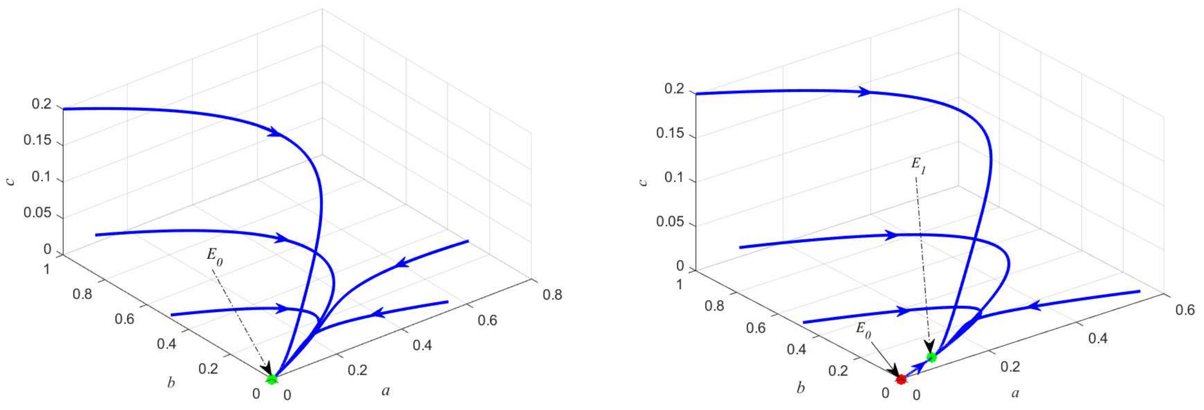
$$J = \begin{pmatrix} -b-\phi-\frac{ac-a(a+m)}{(a+m)^2} & 1-a & -\frac{a}{a+m} \\ \theta & -\delta & 0 \\ \frac{\xi cm}{(a+m)^2} & 0 & \frac{\xi a}{a+m} - \mu + \frac{\mu c^2 - 2\mu c(c+\psi)}{(c+\psi)^2} \end{pmatrix} \tag{5}$$

By analyzing the eigenvalues of the Jacobian matrix (5) at each equilibrium point, we have the following stability properties.

(i) The Jacobian matrix of the system (3) at E_0 has eigenvalues $\lambda_1 = -\eta$ and $\lambda_{2,3} = -\frac{1}{2}B_1 \pm \frac{1}{2}\sqrt{B_1^2 - 4(\phi\delta - \theta)}$, where $B_1 = \delta + \phi > 0$. If
$$\theta < \phi\delta, \tag{6}$$
then $\lambda_{2,3} < 0$. This causes the equilibrium point E_0 to be locally asymptotically stable.

(ii) The Jacobian matrix of the system (3) at E_1 has eigenvalues $\lambda_1 = \frac{\delta\phi[(\eta-\xi)-\varpi]}{(1+m)\left(\theta - \frac{\delta\phi}{1+m}\right)}$, and $\lambda_{2,3} = -\frac{1}{2\delta}B_1 \pm \frac{1}{2\delta}\sqrt{B_1^2 - 4\delta^2(\theta - \phi\delta)}$, where $B_1 = \delta^2 + \theta$. If
$$\eta < \xi + \varpi, \tag{7}$$
then $\lambda_1 < 0$, where $\varpi = \frac{\theta(\eta(1+m)+\xi)}{\delta\phi}$. This causes the equilibrium point E_1 to be locally asymptotically stable.

(iii) The characteristic equation of the Jacobian matrix of the system (3) at E^* is given by the following cubic equation
$$\lambda^3 + \tau_1\lambda^2 + \tau_2\lambda + \tau_3 = 0, \tag{8}$$
where
$$\begin{aligned}
\tau_1 &= \delta - (\vartheta_1 + \vartheta_5), \\
\tau_2 &= \vartheta_1\vartheta_5 - \left[\delta(\vartheta_1 + \vartheta_5) + \theta\vartheta_2 + \vartheta_3\vartheta_4\right], \\
\tau_3 &= \vartheta_5(\delta\vartheta_1 + \theta\vartheta_2) - \delta\vartheta_3\vartheta_4, \\
\vartheta_1 &= -b^* - \phi - \frac{a^*c^* - a^*(a^*+m)}{(a^*+m)^2}, \\
\vartheta_2 &= 1 - a^*, \\
\vartheta_3 &= -\frac{a^*}{a^*+m}, \\
\vartheta_4 &= \frac{\xi c^* m}{(a^*+m)^2}, \\
\vartheta_5 &= \frac{\xi a^*}{a^*+m} - \mu + \frac{\mu c^{*2} - 2\mu c^*(c^*+\psi)}{(c^*+\psi)^2}.
\end{aligned}$$



(a) For $\theta = 0.3$ **(b)** For $\theta = 0.4$

Figure 1. Phase-portraits of the system (3) with parameter values: $\phi = 0.6, m = 0.9, \delta = 0.6, \xi = 0.5, \eta = 0.2, \mu = 0.1, \psi = 0.09$. The red and green circles represent unstable and stable equilibrium point, respectively.

The stability of E^* can be determined by the Routh-Hurwitz criterion, i.e. E^* is locally asymptotically stable if $\tau_i > 0, i = 1,3$ and

$$\tau_1\tau_2 - \tau_3 = (\vartheta_1 + \vartheta_5)\left[\vartheta_3\vartheta_4 + \delta(\vartheta_1 + \vartheta_5 - \delta^2)\right] + \theta\vartheta_2(\vartheta_1 - \delta) - (\vartheta_1^2\vartheta_5 + \vartheta_1\vartheta_1^2) > 0.$$

■

3. Existence of Hopf-bifurcation

In this part, we study the Hopf-bifurcation around the interior equilibrium point $E^* = (a^*, b^*, c^*)$ of the system (3). We consider ξ, η, and μ as the bifurcation parameters. $\xi = \frac{\beta\varphi}{r}$, $\eta = \frac{\delta_2}{r}$, and $\mu = \frac{\sigma}{r}$ are chosen as the bifurcation parameters because r is strongly related to the growth of immature prey, which controls energy input in the predator-prey system. Furthermore β, φ, σ, and δ_2 are important parameters governing the exchange of energy from prey to predator as well as towards the extinction of predator.

Theorem 2. *System (3) undergoes a Hopf-bifurcation around coexistence equilibrium $E^* = (a^*, b^*, c^*)$ when parameter ξ passes through ξ^*, where ξ^* satisfies $\tau_4(\xi^*) = \tau_1(\xi^*)\tau_2(\xi^*) - \tau_3(\xi^*) = 0$ provided that $\delta > (\vartheta_1 + \vartheta_5)$ and $\vartheta_1\vartheta_5 > [\delta(\vartheta_1 + \vartheta_5) + \theta\vartheta_2 + \vartheta_3\vartheta_4]$.*

proof. For $\xi = \xi^*$, by the condition $\tau_4 = 0$, the characteristic equation (8) from Theorem 1 can be written as

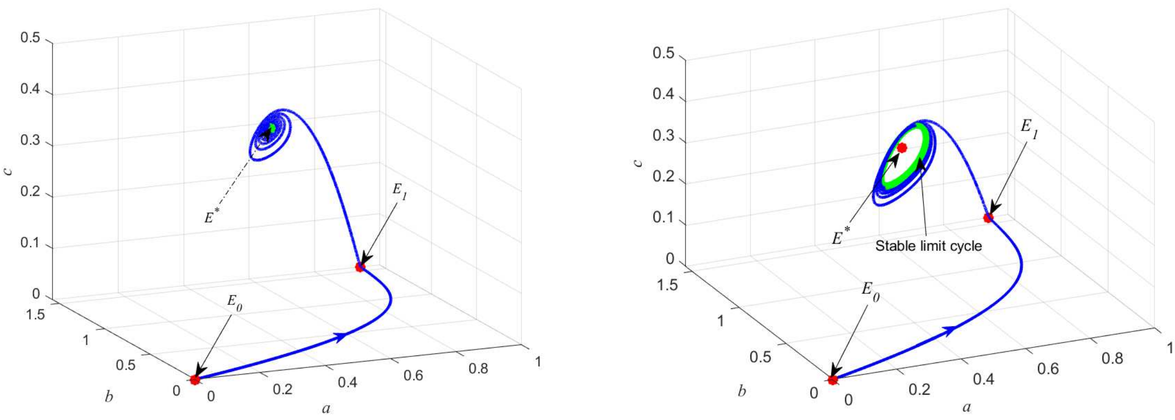
$$(\lambda^2 + \tau_2)(\lambda + \tau_1) = 0. \tag{9}$$

If $\delta > (\vartheta_1 + \vartheta_5)$ and $\vartheta_1\vartheta_5 > [\delta(\vartheta_1 + \vartheta_5) + \theta\vartheta_2 + \vartheta_3\vartheta_4]$, then from the proof of Theorem 1, we have that $\tau_1 > 0$ and $\tau_2 > 0$. The roots of equation (9) are $\lambda_1 = -\tau_1$, and $\lambda_{2,3} = i\sqrt{\tau_2}$. For any ξ, the characteristic roots are $\lambda_1(\xi) = -\tau_1(\xi)$ and $\lambda_{2,3}(\xi) = \kappa(\xi) \pm i\chi(\xi)$. Substituting $\lambda(\xi) = \kappa(\xi) \pm i\chi(\xi)$ into equation (9) and calculating the derivative, we get

$$\begin{aligned} M_1\kappa' - M_2\chi' + M_3 &= 0, \\ M_2\kappa' + M_2\chi' + M_4 &= 0. \end{aligned} \tag{10}$$

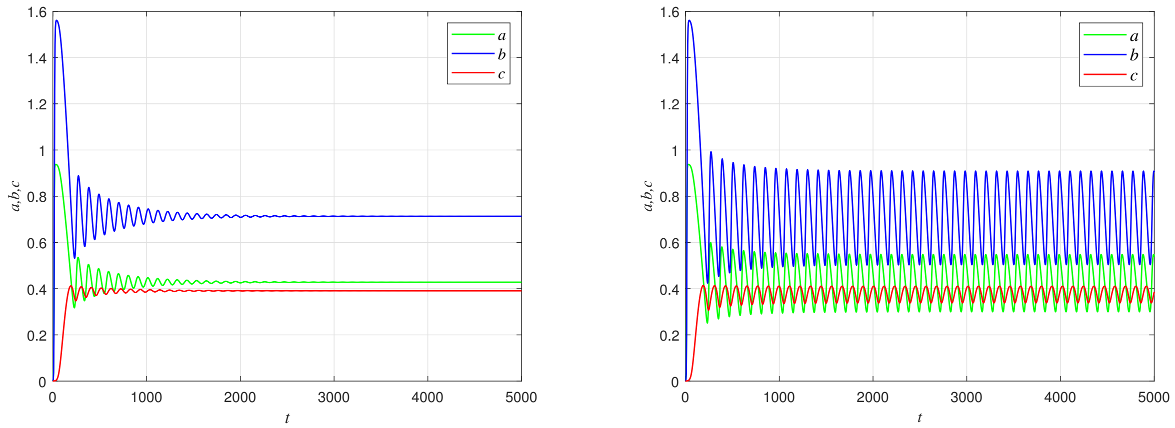
where

$$\begin{aligned} M_1 &= 3(\kappa^2 - \chi^2) + 2\tau_1\kappa + \tau_2, \\ M_2 &= 6\kappa\chi + 2\tau_1\chi, \\ M_3 &= \tau_1'(\kappa^2 - \chi^2) + \tau_2'\kappa + \tau_3', \\ M_4 &= 2\tau_1'\kappa\chi + \tau_2'\chi. \end{aligned}$$



(a) For $\xi = 0.301$ **(b)** For $\xi = 0.3015$

Figure 2. Phase-portraits of the system (3) with parameter values: $\phi = 0.6, m = 0.9, \delta = 0.6, \xi = 0.5, \eta = 0.2, \mu = 0.1, \psi = 0.09$. The red and green circles represent unstable and stable equilibrium point, respectively.



(a) For $\xi = 0.301$ **(b)** For $\xi = 0.3015$

Figure 3. Time series of solutions of the system (3) with parameter values: $\phi = 0.6, m = 0.9, \delta = 0.6, \xi = 0.5, \eta = 0.2, \mu = 0.1, \psi = 0.09$, and initial values: $(0.001, 0.001, 0.001)$.

By solving system (10) and using the fact that $M_2M_4 + M_1M_3 \neq 0$, we get

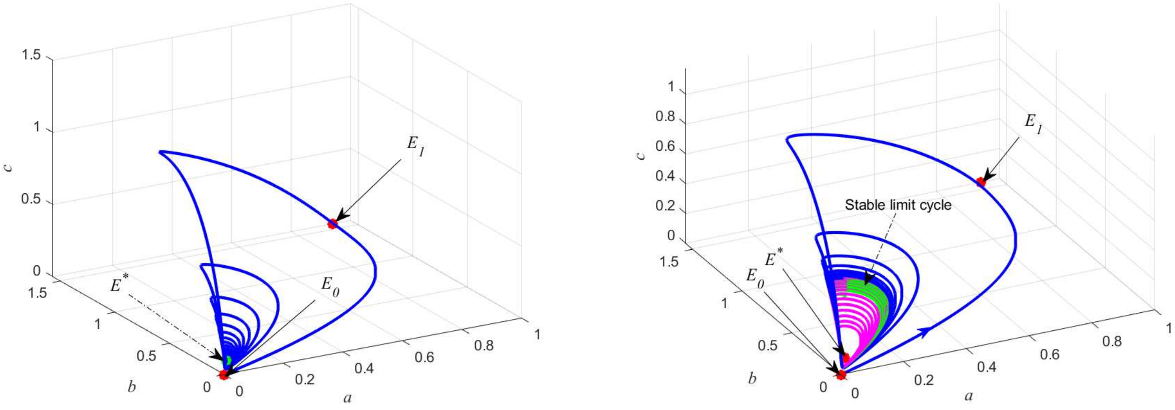
$$\left(\frac{dRe(\lambda)}{d\xi}\right)\Bigg|_{\xi=\xi^*} = \kappa'\Big|_{\xi=\xi^*} = -\left(\frac{M_2M_4 + M_1M_3}{M_1^2 + M_2^2}\right) \neq 0. \tag{11}$$

Thus, the transversality condition be in force, and Hopf-bifurcation come to pass at $\xi = \xi^*$. ■

According to Theorem 2, there exists a Hopf-bifurcation in the stage-structure predator-prey model (3) where the Hopf bifurcation is controlled by ξ. In fact, using the same argument as in the proof of Theorem 2, we can show that the Hopf-bifurcation can also be controlled by parameters η and μ. The possibility of the Hopf-bifurcation occurance is stated in the following theorems.

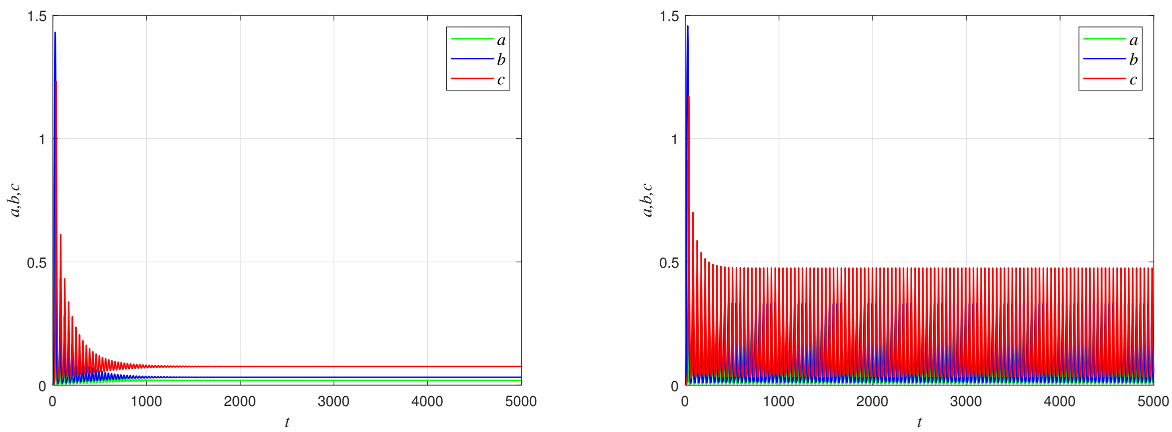
Theorem 3. *System (3) undergoes a Hopf-bifurcation around coexistence equilibrium $E^* = (a^*, b^*, c^*)$ when parameter η passes through η^*, where η^* satisfies $\tau_4(\eta^*) = \tau_1(\eta^*)\tau_2(\eta^*) - \tau_3(\eta^*) = 0$ provided that $\delta > (\vartheta_1 + \vartheta_5)$ and $\vartheta_1\vartheta_5 > [\delta(\vartheta_1 + \vartheta_5) + \theta\vartheta_2 + \vartheta_3\vartheta_4]$.*

Theorem 4. *System (3) undergoes a Hopf-bifurcation around coexistence equilibrium $E^* = (a^*, b^*, c^*)$ when parameter μ passes through μ^*, where μ^* satisfies $\tau_4(\mu^*) = \tau_1(\mu^*)\tau_2(\mu^*) - \tau_3(\mu^*) = 0$ provided that $\delta > (\vartheta_1 + \vartheta_5)$ and $\vartheta_1\vartheta_5 > [\delta(\vartheta_1 + \vartheta_5) + \theta\vartheta_2 + \vartheta_3\vartheta_4]$.*



(a) For $\eta = 0.15$ **(b)** For $\eta = 0.17$

Figure 4. Phase-portraits of the system (3) with parameter values: $\phi = 0.1, m = 0.03, \theta = 0.5, \delta = 0.3, \xi = 0.5 \mu = 0.1, \psi = 0.09$. The red and green circles represent unstable and stable equilibrium point, respectively.



(a) For $\eta = 0.15$ **(b)** For $\eta = 0.17$

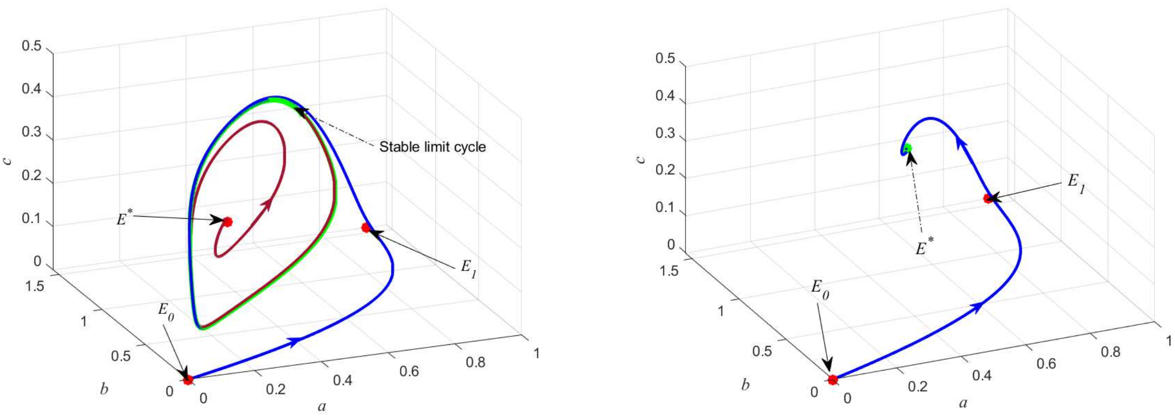
Figure 5. Time series of solutions of the system (3) with parameter values: $\phi = 0.1, m = 0.03, \theta = 0.5, \delta = 0.3, \xi = 0.5 \mu = 0.1, \psi = 0.09$, and initial values: $(0.001, 0.001, 0.001)$.

4. Numerical Simulation

Since the field data are not available, the simulations are performed by using some hypothetical parameter values. We first consider the following parameter values: $\phi = 0.6, m = 0.9, \theta = 0.3, \delta = 0.6, \xi = 0.5, \eta = 0.2, \mu = 0.1, \psi = 0.09$. Because $\phi\delta = \left(\frac{\alpha+h_1}{r}\right)\left(\frac{\delta_1+h_2}{r}\right) > \theta = \frac{\alpha}{r}$, Theorem 1 says that E_0 is locally asymptotically stable. The behavior of this case is depicted in Figure 1a. Next, consistently using the same parameters, except $\theta = 0.4$, the simulation is done. In addition to E_0 unstable, the system (3) also has $E_1 = (0.1, 0.07, 0)$. Since $\varpi = 0.98$ and $\eta - \xi = -0.3$, therefore condition (7) holds, so E_1 is locally asymptotically stable. Behavior of this situation is plotted in Figure 1b.

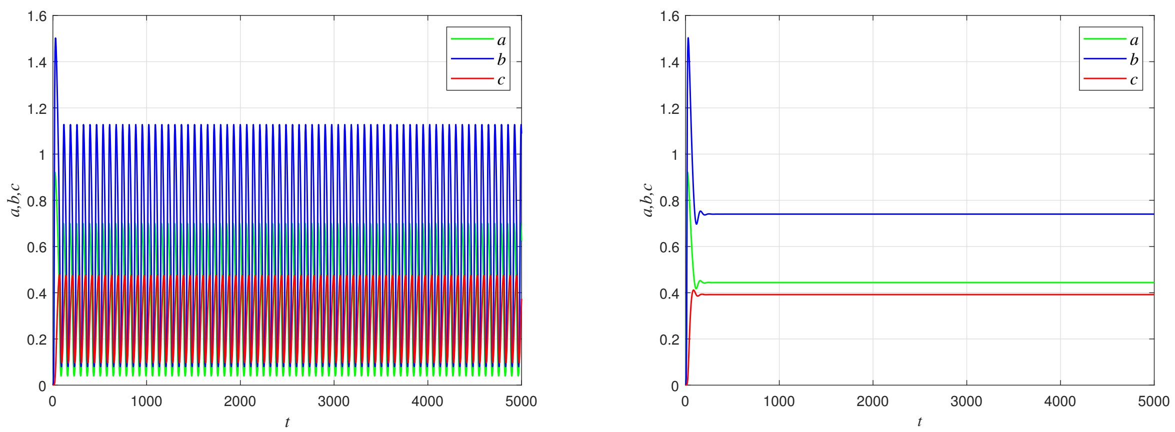
To see a Hopf–bifurcation of the system, we now choose parameter values: $\phi = 0.1, m = 0.03, \theta = 0.5, \delta = 0.3, \eta = 0.2, \mu = 0.1, \psi = 0.09$. According to Theorem 2, the system (3) undergoes a Hopf bifurcation around E^* where the bifurcation point is at $\xi^* = 0.3012$. For $\xi < \xi^*$, the solution of system is convergent to point E^*. It resulted that the point E^* is local asymptotically stable, while the solution of system is convergent to a limit cycle for $\xi > \xi^*$. Behavior of this situation is plotted in Figure 2, while the time series is plotted in Figure 3.

Based on Theorem 3, it is known that the parameter η also can control the occurrence of Hopf-bifurcation. To simulation of this case, we choose $\phi = 0.1, m = 0.03, \theta = 0.5, \delta = 0.3, \xi = 0.5, \mu = 0.1, \psi = 0.09$. We obtain that $\eta^* = 0.16234$. If $\eta < \eta^*$, we obtain that E^* is locally asymptotically stable. Different results are shown when



(a) For $\mu = 0.31$ **(b)** For $\mu = 0.33$

Figure 6. Phase-portraits of the system (3) with parameter values: $\phi = 0.1, m = 0.03, \theta = 0.5, \delta = 0.3, \xi = 0.5, \eta = 0.2, \psi = 0.09$. The red and green circles represent unstable and stable equilibrium point, respectively.



(a) For $\mu = 0.31$ **(b)** For $\mu = 0.33$

Figure 7. Time series of solutions of the system (3) with parameter values: $\phi = 0.1, m = 0.03, \theta = 0.5, \delta = 0.3, \xi = 0.5, \eta = 0.2, \psi = 0.09$, and initial values: $(0.001, 0.001, 0.001)$.

$\eta > \eta^*$, E^* is unstable and convergent to a limit cycle. The phase-portraits and time series of this case is depicted in Figure 4 – 5.

As previously stated in Theorem 4, the Hopf-bifurcation can also be controlled by parameter μ. The following are the parameter values used to display this behavior: $\phi = 0.1, m = 0.03, \theta = 0.5, \delta = 0.3, \xi = 0.5, \eta = 0.2, \psi = 0.09$. In this case, we get $\mu^* = 0.32435$. For $\mu < \mu^*$, the limit cycle is stable and E^* is unstable. However, when $\mu > \mu^*$, the solution of system is convergent to E^* and it is asymptotically stable. Figure 6 and Figure 7 illustrates the behavior and time series of this situation, respectively.

5. Conclusion

We have considered a stage–structure Rosenzweig–MacArthur model with linear harvesting in prey and cannibalism in predator. By analyzing the eigenvalues and characteristic equations, the local stability of the equilibrium is investigated. E_1 exist if condition (4) holds. It causes condition (6) in Theorem 1 do not apply, so that E_0 becomes unstable. From the conditions (4) and (6), we can see that $\phi = \frac{\alpha+h_1}{r}$ and $\delta = \frac{\delta_1+h_2}{r}$ influence the existence of E_1 and also the stability of E_0. Hence, linear harvesting rate of immature prey (h_1) and also mature prey (h_2) are crucial to the existence or extinction of all species in the system.

The model exhibits that Hopf-bifurcation occurs by selecting the appropriate parameters. These bifurcations are ecologically important to illustrate the fluctuations of population. From our results, the rate of cannibalism in predator (σ) plays an important role in the fluctuation processes of all species in the system. This can be seen in $\mu = \sigma/r$, which is a parameter that can lead to the occurrence of Hopf-bifurcation. Numerical simulations are actualized to support our results.

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Conflict of interest

The author declares that there is no conflict of interest in publishing this paper.

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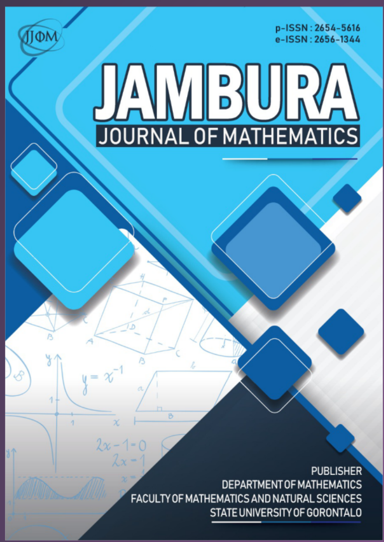

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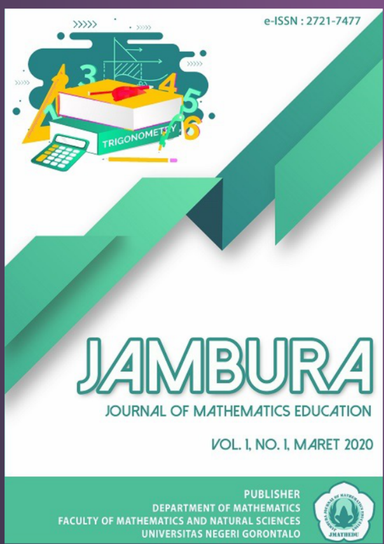

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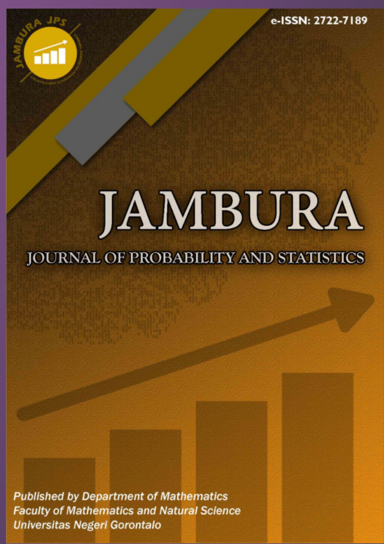

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