Optimal Adjustments on the Market Dispatch Solution to Supply System Losses

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Abstract

This paper proposes an Optimal Power Flow (OPF) problem to obtain a balanced power system operating condition from the solution of a market dispatch. The goal of this OPF problem is to correct the system power imbalance due to transmission losses. The solution provided by the proposed OPF problem corresponds to the generation power adjustments required to achieve the system power balance in an economically optimal manner. The performance of the proposed model is tested on the IEEE One Area RTS-96 benchmark system. The obtained base-case operating condition is compared with others obtained by two variations of the proposed model.

Index Terms

Optimal power flow, Base-case operating condition, Power imbalance.

I. Introduction

Most electricity markets provide a dispatching solution only based purely on economics, in which the total generation equals the total demand. This solution does not correspond to a power balanced operating condition due to transmission losses that should be supplied. As a consequence, one generator (or a group of generators) should adjust its scheduled power to balance the system. In some electricity markets, the dispatching solution includes an estimate of transmission losses. In this case, the power imbalance corresponds to the error in the estimation of transmission losses, [1]–[3]. Once the power balance has been obtained, the resulting operating condition is the so-called base-case operating condition, which is used as the reference operating condition for the security analyses that give support to security redispatching procedures [4]–[8].

The procedure traditionally employed to solve the system power imbalance due to losses, and then obtain the base-case operating condition, is the well-known Power Flow (PF) algorithm. In this procedure, two different approaches can be used: (i) the single slack bus; and (ii) the distributed slack bus. In the former approach, one generator (the slack) is selected to compensate for the system power imbalance. The selection of the slack generator is usually arbitrary, e.g., the generator with the largest capacity or a generator of the bus with the largest number of connected lines [9]. The single slack bus could be a rough approach in current power systems, which are operated under market rules, and where the increasing penetration of fluctuating renewable generation could lead to notably different hourly generation dispatches. This situation will be probably more apparent in the future with the presence in the system of plug-in electric vehicles and/or energy storage devices, which could also participate in the compensation of the power imbalance due to losses. In the distributed slack approach, a set of generators contributes to balance the system according to pre-specified participation factors. These factors can be determined based on a variety of criteria, e.g., machine inertias, governor droop characteristics, frequency control participation factors, or economic dispatch, [2], [3], [10], [11].

Regardless of whether the single or distributed slack bus approach is adopted, the voltage magnitude at generator buses and the set point of control devices, such as FACTS devices, must be specified to solve the PF problem. These values are usually fixed to those provided by the state estimator that correspond to the operation of the system with a similar demand profile [12].

In this work, the base-case operating condition is obtained by solving an Optimal Power Flow (OPF) problem based on the one analyzed in [1] for the identification of the optimal set of slack buses. As in [1], in our model no assumption is made a priori about whether the power imbalance is supplied by a single or a distributed set of generators. However, unlike [1], the proposed formulation does not require complementary constraints. Furthermore, voltage magnitudes at generator buses are treated as variables instead of being fixed at a predefined value. These voltages are provided by the solution of the proposed OPF problem.

The remainder of the paper is organized as follows. Section II formulates and describes the proposed optimization model. Section III analyzes the performance of the proposed OPF model in the IEEE One Area RTS-96 benchmark system. The comparison of the obtained results with the results provided by two variations of the proposed approach is discussed. Finally, Section IV summarizes the paper, provides the main conclusions, and points out possible future working lines.

II. Problem Description

This subsection provides a detailed description of the objective function and all constraints used in the proposed OPF problem to identify the base-case operating condition for security analyses. The objective of the proposed OPF problem is to minimize the cost associated with the system power imbalance resulting from the market solution. We assume that the market dispatching solution does not take into account any estimation of transmission losses; thus, minimizing the cost associated with the system power imbalance directly implies minimizing the cost of generating the energy dissipated in system losses.

A. Objective Function

The objective of the proposed OPF problem is to minimize the cost of generating the system losses. Since the dispatching procedure does not include an estimate of system losses, losses are equivalent to the additional active power that the generators have to supply with respect to the power assigned in the dispatching procedure. Then, the total system losses P_{loss} can be expressed as

$$P_{\mathrm{loss}} = \sum_{j \in \mathcal{G}} \Delta P_{\mathrm{G}j}^{\mathrm{loss}}, \tag{1}$$

where $\Delta P_{\mathrm{G}j}^{\mathrm{loss}}$ is the change in the power production of generator j needed to supply the system losses.

The objective function represents the total cost of the additional active power that the generators have to supply to match the system losses

$$z = \sum_{j \in \mathcal{G}} C_j \left(\Delta P_{\mathrm{G}j}^{\mathrm{loss}} \right), \tag{2}$$

where $C_j(\cdot)$ is a function representing the offer associated with the power production of generator j.

B. Power Flow Equations

The operating condition of the system is established by the active and reactive power balance at all buses:

$$P_{\mathrm{G}n} - P_{\mathrm{D}n} = \sum_{m \in \Theta_n} P_{nm}(\cdot), \quad \forall n \in \mathcal{N}, \tag{3}$$

$$Q_{\mathrm{G}n} - Q_{\mathrm{D}n} = \sum_{m \in \Theta_n} Q_{nm}(\cdot), \quad \forall n \in \mathcal{N}, \tag{4}$$

where $P_{\mathrm{G}n}$ and $Q_{\mathrm{G}n}$ are, respectively, the total active and reactive power generated at bus n, $P_{\mathrm{D}n}$ and $Q_{\mathrm{D}n}$ are, respectively, the total active and reactive power consumed at bus n, and $P_{nm}(\cdot)$ and $Q_{nm}(\cdot)$ represent, respectively, the active and reactive power flowing through the network components connected to bus n. Total generation and demand powers per bus are computed as follows:

$$P_{\mathrm{G}n} = \sum_{j \in \mathcal{G}_n} P_{\mathrm{G}j}, \quad \forall n \in \mathcal{N}, \tag{5}$$

$$P_{\mathrm{D}n} = \sum_{i \in \mathcal{D}_n} P_{\mathrm{D}i}, \quad \forall n \in \mathcal{N}, \tag{6}$$

$$Q_{\mathrm{G}n} = \sum_{j \in \mathcal{G}_n} Q_{\mathrm{G}j}, \quad \forall n \in \mathcal{N}, \tag{7}$$

$$Q_{\mathrm{D}n} = \sum_{i \in \mathcal{D}_n} Q_{\mathrm{D}i}, \quad \forall n \in \mathcal{N}, \tag{8}$$

where $P_{\mathrm{G}j}$ and $Q_{\mathrm{G}j}$ are, respectively, the active and reactive power production of the generator j connected to bus n ($j \in \mathcal{G}_n$), and $P_{\mathrm{D}i}$ and $Q_{\mathrm{D}i}$ are, respectively, the active and reactive power consumed by the demand i connected to bus n ($i \in \mathcal{D}_n$).

Demand powers are defined on the basis of the market dispatch solution as follows:

$$P_{\mathrm{D}i} = P_{\mathrm{D}i}^{\mathrm{M}}, \quad \forall i \in \mathcal{D}, \tag{9}$$

$$Q_{\mathrm{D}i} = P_{\mathrm{D}i}^{\mathrm{M}} \tan \varphi_i, \quad \forall i \in \mathcal{D}, \tag{10}$$

where φ_i is the angle corresponding to the power factor associated to the demand i.

Active generation powers are defined based on the market dispatch solution as follows:

$$P_{\mathrm{G}j} = P_{\mathrm{G}j}^{\mathrm{M}} + \Delta P_{\mathrm{G}j}^{\mathrm{loss}}, \quad \forall j \in \mathcal{G}, \tag{11}$$

and

$$\Delta P_{\mathrm{G}j}^{\mathrm{loss}} \geq 0, \quad \forall j \in \mathcal{G}. \tag{12}$$

Note that superscript "M" in (9), (10) and (11) indicates market dispatch solution.

The functions on the right-hand side of (3) and (4) are the power flow equations and depend on the device connected between buses n and m. For simplicity, we only consider transmission lines and transformers. Thus, the active and reactive power flows through branch k from bus n to bus m are, respectively,

$$\begin{aligned} P_{nm}(\cdot) =& \frac{1}{T_k^2} V_n^2 g_k - \frac{1}{T_k} V_n V_m (g_k \cos(\theta_n - \theta_m) \\ &+ b_k \sin(\theta_n - \theta_m)) \end{aligned} \tag{13}$$

$$\begin{aligned} Q_{nm}(\cdot) =& -\frac{1}{T_k^2} V_n^2 (b_k + b_{\mathrm{p}k}) - \frac{1}{T_k} V_n V_m (g_k \sin(\theta_n - \theta_m) \\ &- b_k \cos(\theta_n - \theta_m)) \end{aligned} \tag{14}$$

and the active and reactive power flows through branch k from bus m to bus n are, respectively,

$$\begin{aligned} P_{mn}(\cdot) =& V_m^2 g_k - \frac{1}{T_k} V_m V_n (g_k \cos(\theta_n - \theta_m) \\ &- b_k \sin(\theta_n - \theta_m)) \end{aligned} \tag{15}$$

$$\begin{aligned} Q_{mn}(\cdot) =& -V_m^2 (b_k + b_{\mathrm{p}k}) + \frac{1}{T_k} V_m V_n (g_k \sin(\theta_n - \theta_m) \\ &+ b_k \cos(\theta_n - \theta_m)) \end{aligned} \tag{16}$$

In (13)-(16), V_n (V_m) is the magnitude of the voltage at bus n (m), θ_n (θ_m) is the phase angle of the voltage at bus n (m), T_k is the tap ratio, g_k is the series conductance, b_k is the series susceptance, and $b_{\mathrm{p}k}$ is the half of the shunt susceptance of the component k. Accordingly, if component k is a transmission line, $T_k = 1.0$, while if component k is a transformer, parameter T_k takes the value corresponding to the power system operation condition to be analyzed.

C. Technical Limits

The power production is limited by the capacity of the generators.

$$P_{\mathrm{G}j}^{\min} \leq P_{\mathrm{G}j} \leq P_{\mathrm{G}j}^{\max}, \quad \forall j \in \mathcal{G}, \tag{17}$$

$$Q_{\mathrm{G}j}^{\min} \leq Q_{\mathrm{G}j} \leq Q_{\mathrm{G}j}^{\max}, \quad \forall j \in \mathcal{G}. \tag{18}$$

Voltage magnitudes at generator buses should be within operating limits,

$$V_n^{\min} \leq V_n \leq V_n^{\max}, \quad \forall n \in \mathcal{N}_{\mathrm{G}}. \tag{19}$$

Other technical limits, such as the voltage magnitude limits at load buses or the current flow limits of the elements of the network are not included to obtain the base-case solution. These limits, along with stability considerations, should be incorporated into subsequent security redispatching procedures.

D. OPF Formulation

The formulation of the OPF problem is summarized below:

$$\text{Minimize} \quad (2)$$

subject to

- Power flow equations (3)-(4).
- Technical limits (17)-(19).

The variables of the optimization problem are:

- Magnitudes and phase angle at bus voltages: V_n, θ_n.
- Reactive power production of generators: $Q_{\mathrm{G}j}$.
- Adjustment on active power production of generators to satisfy system losses: $\Delta P_{\mathrm{G}j}^{\mathrm{loss}}$.

III. Case Study

The performance of the proposed procedure is tested on the IEEE One Area RTS-96 system reported in [13]. The starting point of our procedure is a market dispatch solution. We assume that the demands corresponding to this solution are the values of the peak demand reported in [13]. With respect to the dispatch of generators, we assume that the market solution is as shown in Table I. These values are essentially the ones reported in [13] for the peak demand case, where the powers of the generators at bus 13 have been adapted to balance the assumed scheduled demand.

Table I also provides the offering costs of the generators for this case study. Observe that, for simplicity, linear offering costs are considered, i.e, offering cost functions are expressed as follows:

$$C_j\left(\Delta P_{\mathrm{G}j}^{\mathrm{loss}}\right) = c_j \Delta P_{\mathrm{G}j}^{\mathrm{loss}}, \quad \forall j \in \mathcal{G} \tag{20}$$

where linear cost coefficients c_j are constant parameters.

With respect to technical data, the minimum power output for all generators are set to zero, whereas the maximum capacity of generators, the limits on the voltage magnitude at generation buses, along with the parameters of transmission lines and transformers, are set according to [13].

TABLE I
Market solution and offering costs for the generators of the IEEE One Area RTS-96 system.

Gen. #	Bus #	$P_{\mathrm{G}j}^{\mathrm{M}}$ [p.u.]	c_j [$\frac{\$}{\mathrm{MWh}}$]	Gen. #	Bus #	$P_{\mathrm{G}j}^{\mathrm{M}}$ [p.u.]	c_j [$\frac{\$}{\mathrm{MWh}}$]
1	1	0.1000	26.01	18	15	0.1200	22.02
2	1	0.1000	26.00	19	15	0.1200	22.01
3	1	0.7600	12.01	20	15	0.1200	22.00
4	1	0.7600	12.00	21	15	1.5500	11.00
5	2	0.1000	26.01	22	16	1.5500	11.00
6	2	0.1000	26.00	23	18	4.0000	7.00
7	2	0.7600	12.01	24	21	4.0000	7.00
8	2	0.7600	12.00	25	22	0.5000	3.00
9	7	0.8000	19.00	26	22	0.5000	3.01
10	7	0.8000	19.01	27	22	0.5000	3.02
11	7	0.8000	19.02	28	22	0.5000	3.03
12	13	0.4543	20.00	29	22	0.5000	3.04
13	13	0.4543	20.01	30	22	0.5000	3.05
14	13	0.4543	20.02	31	23	1.5500	11.00
15	14	0	0	32	23	1.5500	11.01
16	15	0.1200	22.04	33	23	3.5000	11.02
17	15	0.1200	22.03				

From the given market dispatch solution, the base-case operating condition to be used in security analysis procedures is obtained by three different approaches:

- Case 1) The proposed OPF problem, described in Subsection II-D, is solved, i.e., the base-case operating condition is set on the basis of the minimization of the cost of system losses.
- Case 2) The same OPF problem as in Case 1 is solved, but setting all linear cost coefficients c_j in equation (20) to 1.0. This way, the base-case operating condition is obtained based on the minimization of system losses.
- Case 3) The same OPF problem as in Case 2 is solved, but the set of eligible generators for balancing the system losses is limited to the ones connected to bus 13, i.e., the slack bus of the considered system, as described in [13]. This manner, the base-case operating condition is obtained by means of an OPF problem essentially equivalent to a standard single-slack power flow algorithm, except for the fact that voltage magnitudes at generation buses are variables instead of being fixed values.

Table II provides the results attained by means of the three approaches previously described in terms of the generation power adjustments, system losses, and cost of system losses. The power adjustments of the system generators that are not shown in Table II are zero for the three cases.

It can be observed that the three approaches provide both quantitative and qualitative different solutions. If the criterion is the minimization of system losses (Case 1), only generators connected to bus 7 are adjusted, while if the the criterion is the minimization of system losses (Case 2), the adjustments affect the generators connected to buses 7 and 13. For the standard power flow approach (Case 3), the power adjustments are restricted to the generators located at bus 13, i.e., the only eligible generators since bus 13 is the slack bus.

It is worth noting the differences between minimizing losses and minimizing their cost. For the given system and the considered offering costs, the minimization of the cost of system losses (Case 1) leads to the most economical operating condition, but at the same time, this operating condition implies the largest losses. Therefore, from a purely economical point of view, this would be the best solution, although it would not be the most efficient solution. Moreover, the minimization of system losses (Case 2) leads to a more expensive operating condition but with lower network losses. Roughly speaking, minimizing the cost of system losses implies the use of more energy resources, but these resources are more economical.

TABLE II
ALLOCATION OF POWER LOSSES $\Delta P_{\mathrm{G}j}^{\mathrm{loss}}$

Gen. #	Bus #	Case 1 [p.u.]	Case 2 [p.u.]	Case 3 [p.u.]
9	7	0.2	0.012864	0.0
10	7	0.2	0.012248	0.0
11	7	0.080524	0.012446	0.0
12	13	0.0	0.336183	0.147308
13	13	0.0	0.049486	0.162731
14	13	0.0	0.049486	0.162731
Total losses [p.u.]		0.480524	0.472713	0.472769
Cost of losses [$]		913.36	941.86	946.03

Table III lists the voltage magnitude and the total power production at system buses for the base-case operating conditions obtained in the three cases analyzed. It is observed that, although small differences exists, voltage profiles are quite similar. A common pattern for the three profiles is that the controlled voltage magnitudes, i.e., the voltage at generation buses are set to the maximum allowed values. This is a natural consequence of the fact that the objective functions of the optimization problems solved in the three cases are based on minimizing power losses. However, these kinds of voltage profiles could not be appropriate from a technical point of view.

IV. Summary, Conclusions, and Future Work

This paper proposes an OPF model to deal with a practical problem that arises after solving electricity market sessions, namely, establishing a base-case operating condition to be used in the security analyses that give support to security redispatching procedures. The proposed model is intended to correct the power imbalance existing in the market dispatching solution due to system losses. For that, the proposed OPF model computes the minimum cost adjustments in the scheduled generation powers that have to be applied in order to balance the system. The base-case operating condition obtained by the proposed model has been compared with the ones obtained by other two possible approaches, namely, (i) minimize the system losses; and (ii) consider the typical single-slack bus.

It has been observed that the proposed approach attains solutions with the largest system losses, but these losses are assigned to generators in such a way that the cost of generating the system losses is the lowest. Therefore, the proposed model leads to the use of more energy resources, but these resources are less costly than the other analyzed approaches. Comparing the voltage profiles of the obtained base-case operating conditions, none significant differences have been observed.

The results of the proposed model could be very sensitive to the values of input data (e.g. offering cost of generators). Therefore, we consider primordial to test the model in a real-world system. Furthermore, the adequacy of the attained voltage profiles needs further analysis.

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TABLE III
POWER PRODUCTION AND VOLTAGE MAGNITUDES AT BUSES FOR THE BASE CASE OPERATING CONDITIONS

	Case 1		Case 2		Case 3	
Bus #	V_n [p.u.]	$P_{\mathrm{G}n}$ [p.u.]	V_n [p.u.]	$P_{\mathrm{G}n}$ [p.u.]	V_n [p.u.]	$P_{\mathrm{G}n}$ [p.u.]
1	1.05	1.72	1.05	1.72	1.05	1.72
2	1.0127	0.0	1.0129	0.0	1.0129	0.0
3	1.0216	0.0	1.0223	0.0	1.0223	0.0
4	1.0393	0.0	1.04	0.0	1.04	0.0
5	1.0369	0.0	1.038	0.0	1.0381	0.0
6	1.05	2.8805	1.05	2.4376	1.05	2.4
7	1.0189	0.0	1.0204	0.0	1.0205	0.0
8	1.0312	0.0	1.0325	0.0	1.0325	0.0
9	1.0548	0.0	1.0562	0.0	1.0563	0.0
10	1.0336	0.0	1.0344	0.0	1.0345	0.0
11	1.0253	0.0	1.0261	0.0	1.0262	0.0
12	1.0434	1.363	1.0466	1.7982	1.0469	1.8358
13	1.05	0.0	1.05	0.0	1.05	0.0
14	1.0426	2.15	1.0425	2.15	1.0425	2.15
15	1.0449	1.55	1.0448	1.55	1.0448	1.55
16	1.0471	0.0	1.0471	0.0	1.0471	0.0
17	1.05	4.0	1.05	4.0	1.05	4.0
18	1.0392	0.0	1.0392	0.0	1.0392	0.0
19	1.044	0.0	1.044	0.0	1.044	0.0
20	1.05	4.0	1.05	4.0	1.05	4.0
21	1.05	3.0	1.05	3.0	1.05	3.0
22	1.05	6.6	1.05	6.6	1.05	6.6
23	1.0094	0.0	1.009	0.0	1.009	0.0

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