Design of Tools for PEV-integration Studies

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Abstract

This paper presents and analyzes four different optimal power flow formulations designed to find the maximum number of plug-in electric vehicles that can be charging simultaneously for a given power system operating condition. The modeling approach to represent the coincident charge of PEVs is based on the modeling philosophy of homotopy methods. The analyzed models are intended to help in tasks related to both planning and operation of power systems. The developed formulations are tested in the IEEE RTS-96 benchmark system. The obtained results are discussed and suggestions on the applicability of the models are provided.

Index Terms

Optimal power flow, Homotopy methods, Simultaneous charge of PEVs

I. Introduction

The widespread usage of Plug-in Electric Vehicles (PEVs) will come true in the near future. This fact poses a huge challenge for current power systems since they will have to adapt to a great increase in the power demand. The necessary transformation will not only affect the physical components of the system but it will also entail the re-design of the manner that power systems are planned and operated.

In the technical literature, studies on the integration of PEVs are based on either estimated or given penetration levels, mostly in the context of distribution networks, where the impact of the charge of PEVs on aspects such as load profiles, grid components aging, losses, voltage profile, etc., are assessed and analyzed. See, for example, the comprehensive reviews [1]–[5], and the references therein.

The idea of this paper originates from the following question: how many PEVs could be charging simultaneously for a given power system operating condition? In an attempt to give an appropriate answer to this question, this paper explores Optimal Power Flow (OPF) formulations where the charge of PEVs is modeled in four different ways. The resulting models could serve as assistant tools in the planning and operation of future power systems. The modeling approach for the charge of PEVs is based on the modeling philosophy of both single-parameter and multi-parameter homotopy methods [6]. These methods were conceived to construct convergent series of solutions of nonlinear systems. For that, one or more parameters are introduced in the nonlinear mathematical model and system equilibrium points are found by varying such parameters. In the context of power systems, homotopy methods are well-known due to their application in voltage stability studies [7]. In the analyzed OPF formulations, homotopy parameters turn into the main optimization variables that lead to find out the maximum coincident charge of PEVs that could withstand a power system when operating at a given condition.

The remainder of the paper is organized as follows. Section II lists and defines all symbols used in the paper. Section III formulates and describes the four developed optimization problems. Section IV presents the results of applying the OPF models to the IEEE RTS-96 benchmark system. The particular characteristics of the solutions that each model attains are analyzed and possible practical applications are suggested. Finally, Section V summarizes the paper and provides the main conclusions drawn.

II. Notation

The notation used throughout the paper is stated below for quick reference.

Sets

$\mathcal{D}_\mathrm{n}$	Set of demands located at bus n.
$\mathcal{G}$	Set of generators.
$\mathcal{G}_\mathrm{n}$	Set of generators located at bus n.
$\mathcal{N}$	Set of buses.
$\mathcal{N}_\mathrm{D}$	Set of demand buses.
$\mathcal{N}_\mathrm{G}$	Set of generator buses.
Θ_n	Set of buses connected to bus n through a branch.
Ω	Set of network branches.

Indices

i	Index of demands.
j	Index of generators.
k	Index of network branches.
n, m	Indices of buses.

Functions

$I_k(\cdot)$	Current magnitude through branch k as a function of the problem variables.
$I_{nm}(\cdot)$	Current magnitude from bus n to bus m, $(n,m) \in \Omega$, as a function of the problem variables.
$P_{nm}(\cdot)$	Active power flow from bus n to bus m, $(n,m) \in \Omega$, as a function of the problem variables.
$Q_{nm}(\cdot)$	Reactive power flow from bus n to bus m, $(n,m) \in \Omega$, as a function of the problem variables.
z	Optimization function.

Variables

$P_{\mathrm{D}n}$	Total active power consumption in bus n.
$P_{\mathrm{EV}n}$	Total active power consumption due to the charge of PEVs in bus n.
$P_{\mathrm{G}j}$	Active power production of generator j.
$P_{\mathrm{G}n}$	Total active power production in bus n.
$Q_{\mathrm{D}n}$	Total reactive power consumption in bus n.
$Q_{\mathrm{G}j}$	Reactive power production of generator j.
$Q_{\mathrm{G}n}$	Total reactive power production in bus n.
V_n	Voltage magnitude at bus n.
θ_n	Voltage angle at bus n.
$\lambda^{\#}$	Uniform number of PEVs that can be simultaneously charged at each system bus.
$\lambda_n^{\#}$	Number of PEVs that can be simultaneously charged at bus n.
$\lambda^{\%}$	Uniform percentage of the total number PEVs that can be simultaneously charged at each system bus.
$\lambda_n^{\%}$	Percentage of the total number of PEVs that can be simultaneously charged at bus n.

Parameters

b_k	Series susceptance of element k.
$b_{\mathrm{p}k}$	Half of the shunt susceptance of element k.
g_k	Series conductance of element k.
$I_k^{\max}$	Maximum current magnitude through branch k.
$N_{\mathrm{EV}n}^{\mathrm{total}}$	total number of PEVs located at bus n.
$P_{\mathrm{D}i}^0$	Active power consumption of demand i at the considered operating condition.
$P_{\mathrm{D}n}^{\mathrm{peak}}$	Active power consumption at bus n for the annual peak load of the power system.
$P_{\mathrm{G}j}^0$	Active power production of generator j at the considered operating condition.
$P_{\mathrm{G}j}^{\max}$	Capacity (maximum power output) of generator j.
$P_{\mathrm{G}j}^{\min}$	Minimum power output of generator j.
$P_{\mathrm{EV}}^{\mathrm{single}}$	Active power consumption due to the charge of a single PEV.
$P_{\mathrm{EV}n}^{\mathrm{total}}$	Active power consumption due to the charge of the total number of PEVs located at bus n.
$Q_{\mathrm{D}i}^0$	Reactive power consumption of demand i at the considered operating condition.
$Q_{\mathrm{G}j}^0$	Reactive power production of generator j at the considered operating condition.
$Q_{\mathrm{G}j}^{\max}$	Maximum reactive power limit of generator j.
$Q_{\mathrm{G}j}^{\min}$	Minimum reactive power limit of generator j.
T_k	Tap ratio of transformer k.
V_n^0	Voltage magnitude at bus n at the considered operating condition.
$V_n^{\max}$	Maximum voltage magnitude at bus n.
$V_n^{\min}$	Minimum voltage magnitude at bus n.

III. Formulation of the Optimization Problems

A. Charge of PEVs

We assume that the charge of PEVs is carried out at unity power factor. Thus, from the transmission system point of view, this process represents a constant active power demand ($P_{\mathrm{EV}}^{\mathrm{single}}$). Taken this into account, in the optimization problems the coincident charge of PEVs is modeled in four different ways, as follows:

1) The demand attributable to PEVs is represented as

$$P_{\mathrm{EV}n} = P_{\mathrm{EV}}^{\mathrm{single}} \cdot \lambda_n^{\#}, \quad \forall n \in \mathcal{N}_{\mathrm{D}} \tag{1}$$

where $P_{\mathrm{EV}}^{\mathrm{single}}$ is the demand of a single PEV, and $\lambda_n^{\#}$ is the number of PEVs that are simultaneously charged at bus n. This model would correspond to a multi-parameter homotopy representation where $\lambda_n^{\#}$ are the homotopy parameters.

2) The demand of PEVs is defined as

$$P_{\mathrm{EV}n} = P_{\mathrm{EV}}^{\mathrm{single}} \cdot \lambda^{\#}, \quad \forall n \in \mathcal{N}_{\mathrm{D}} \tag{2}$$

where $P_{\mathrm{EV}}^{\mathrm{single}}$ is the demand of a single PEV, and $\lambda^{\#}$ is the number of PEVs that are simultaneously charged at all buses. This formulation emulates a single-parameter homotopy technique where $\lambda^{\#}$ is the homotopy parameter, common to all buses, that represent a uniform growth on the number of on-charge PEVs.

3) The demand corresponding to PEVs is modeled as

$$P_{\mathrm{EV}n} = P_{\mathrm{EV}}^{\mathrm{single}} \cdot N_{\mathrm{EV}n}^{\mathrm{total}} \cdot \frac{\lambda_n^{\%}}{100}, \quad \forall n \in \mathcal{N}_{\mathrm{D}} \tag{3}$$

where $P_{\mathrm{EV}}^{\mathrm{single}}$ is the demand of a single PEV and $N_{\mathrm{EV}n}^{\mathrm{total}}$ is the total number of PEVs at bus n. In this case, $\lambda_n^{\%}$ is the percentage of the existing number of PEVs at bus n that are simultaneously charged. As in the case of equation (1), this formulation is based on multi-parameter homotopy representations.

4) The demand of PEVs is formulated as

$$P_{\mathrm{EV}n} = P_{\mathrm{EV}}^{\mathrm{single}} \cdot N_{\mathrm{EV}n}^{\mathrm{total}} \cdot \frac{\lambda^{\%}}{100}, \quad \forall n \in \mathcal{N}_{\mathrm{D}} \tag{4}$$

where $P_{\mathrm{EV}}^{\mathrm{single}}$ is the demand of a single PEV, and $N_{\mathrm{EV}n}^{\mathrm{total}}$ is the total number of PEVs at bus n. In this case, $\lambda^{\%}$ is the percentage of the existing number of PEVs at all buses that are simultaneously charged. As in (2), this formulation would correspond to a single-parameter homotopy where $\lambda^{\%}$ is a common value for all buses.

B. Network model

The transmission network is modeled by the well-known AC power flow equations that represent the active and reactive power balance at network buses.

$$P_{\mathrm{G}n} - P_{\mathrm{D}n} = \sum_{m \in \Theta_n} P_{nm}(\cdot), \quad \forall n \in \mathcal{N} \tag{5}$$

$$Q_{\mathrm{G}n} - Q_{\mathrm{D}n} = \sum_{m \in \Theta_n} Q_{nm}(\cdot), \quad \forall n \in \mathcal{N} \tag{6}$$

The powers on the left-hand side of each equation are defined as

$$P_{\mathrm{G}n} = \sum_{j \in \mathcal{G}_n} \left(P_{\mathrm{G}j}^0 + P_{\mathrm{G}j}\right), \quad \forall n \in \mathcal{N} \tag{7}$$

$$P_{\mathrm{D}n} = \sum_{i \in \mathcal{D}_n} \left(P_{\mathrm{D}i}^0\right) + P_{\mathrm{EV}n}, \quad \forall n \in \mathcal{N} \tag{8}$$

$$Q_{\mathrm{G}n} = \sum_{j \in \mathcal{G}_n} \left(Q_{\mathrm{G}j}^0 + Q_{\mathrm{G}j}\right), \quad \forall n \in \mathcal{N} \tag{9}$$

$$Q_{\mathrm{D}n} = \sum_{i \in \mathcal{D}_n} \left(Q_{\mathrm{D}i}^0\right), \quad \forall n \in \mathcal{N} \tag{10}$$

The functions on the right-hand side of (5) and (6) are the power flow equations and depend on the devices connecting buses n and m. For simplicity, we only consider transmission lines and transformers. Thus, the active and reactive power flows through branch k from bus n to bus m are, respectively,

$$\begin{aligned} P_{nm}(\cdot) = & \frac{1}{T_k^2} V_n^2 g_k - \frac{1}{T_k} V_n V_m (g_k \cos(\theta_n - \theta_m) \\ & + b_k \sin(\theta_n - \theta_m)) \end{aligned} \tag{11}$$

$$\begin{aligned} Q_{nm}(\cdot) = & -\frac{1}{T_k^2} V_n^2 (b_k + b_{\mathrm{p}k}) - \frac{1}{T_k} V_n V_m (g_k \sin(\theta_n - \theta_m) \\ & - b_k \cos(\theta_n - \theta_m)) \end{aligned} \tag{12}$$

and the active and reactive power flows through branch k from bus m to bus n are, respectively,

$$\begin{aligned} P_{mn}(\cdot) = & V_m^2 g_k - \frac{1}{T_k} V_m V_n (g_k \cos(\theta_n - \theta_m) \\ & - b_k \sin(\theta_n - \theta_m)) \end{aligned} \tag{13}$$

$$\begin{aligned} Q_{mn}(\cdot) = & - V_m^2 (b_k + b_{\mathrm{pk}}) + \frac{1}{T_k} V_m V_n (g_k \sin(\theta_n - \theta_m) \\ & + b_k \cos(\theta_n - \theta_m)) \end{aligned} \tag{14}$$

In (11)-(14), g_k is the series conductance, b_k is the series susceptance, and b_{pk} is the half of the shunt susceptance of the component k. If component k is a transmission line, $T_k = 1.0$, while if component k is a transformer, parameter T_k takes the value corresponding to the power system operation condition to be analyzed.

C. Technical Limits

The power production is limited by the capacity of the generators.

$$P_{\mathrm{G}j}^{\min} \leq P_{\mathrm{G}j}^0 + P_{\mathrm{G}j} \leq P_{\mathrm{G}j}^{\max}, \quad \forall j \in \mathcal{G} \tag{15}$$

$$Q_{\mathrm{G}j}^{\min} \leq Q_{\mathrm{G}j}^0 + Q_{\mathrm{G}j} \leq Q_{\mathrm{G}j}^{\max}, \quad \forall j \in \mathcal{G} \tag{16}$$

Voltages magnitudes throughout the system should be within operating limits,

$$V_n^{\min} \leq V_n \leq V_n^{\max}, \quad \forall n \in \mathcal{N} \tag{17}$$

Bus voltage magnitudes controlled by generators are fixed to their values at the considered operating condition,

$$V_n = V_n^0, \quad \forall n \in \mathcal{N}_{\mathrm{G}} \tag{18}$$

The current flow through all branches of the network must be below thermal limits,

$$I_k(\cdot) \leq I_k^{\max}, \quad \forall k = (n, m) \in \Omega \tag{19}$$

where the functions $I_k(\cdot)$ depend on the device k connecting buses n and m. The current flow through branch k from bus n to bus m is

$$\begin{aligned} I_{nm}(\cdot) = \Big(& \big(\frac{1}{T_k^2} V_n (g_k \cos\theta_n - (b_k + b_{\mathrm{pk}}) \sin\theta_n) \\ & - \frac{1}{T_k} V_m (g_k \cos\theta_m - b_k \sin\theta_m)\big)^2 + \\ & \big(\frac{1}{T_k^2} V_n (g_k \sin\theta_n + (b_k + b_{\mathrm{pk}}) \cos\theta_n) \\ & - \frac{1}{T_k} V_m (g_k \sin\theta_m + b_k \cos\theta_m)\big)^2 \Big)^{1/2} \end{aligned} \tag{20}$$

and the current flow through branch k from bus m to bus n is

$$\begin{aligned} I_{mn}(\cdot) = \Big(& \big(V_m (g_k \cos\theta_m - (b_k + b_{\mathrm{pk}}) \sin\theta_m) \\ & - \frac{1}{T_k} V_n (g_k \cos\theta_n - b_k \sin\theta_n)\big)^2 + \\ & \big(V_m (g_k \sin\theta_m + (b_k + b_{\mathrm{pk}}) \cos\theta_m) \\ & - \frac{1}{T_k} V_n (g_k \sin\theta_n + b_k \cos\theta_n)\big)^2 \Big)^{1/2} \end{aligned} \tag{21}$$

Considerations on T_k stated for the power flow equations also apply for current flow equations (20) and (21).

D. OPF Models

With the goal of finding out the maximum number of PEVs that can be charged simultaneously in a power system, four OPF models are formulated. Each model includes a different PEV demand representation and, consequently, a different objective function. As introduced in Section III-A, models I and III are based on the approach of multi-parameter homotopy methods, whereas single-parameter homotopy methods inspire models II and IV.

1) Model I: This model searches the objective by the independent maximization of the number of on-charge PEVs per bus. The OPF model is as follows:

$$\underset{\Theta}{\text{Maximize}} \quad z = \sum_{n \in \mathcal{N}_{\mathrm{D}}} \lambda_n^{\#}$$

subject to:

- PEV demand (1)
- Network model (5)-(14)
- Technical limits (15)-(21)

2) Model II: This model is based on maximizing the total number of on-charge PEVs considering an equal number of them simultaneously charging at each bus. The formulation of the OPF problem is as follows:

$$\underset{\Theta}{\text{Maximize}} \quad z = \lambda^{\#}$$

subject to:

- PEV demand (2)
- Network model (5)-(14)
- Technical limits (15)-(21)

3) Model III: This model maximizes independently the percentage of existing PEVs at each bus that can be synchronously charging. The formulation is as follows:

$$\underset{\Theta}{\text{Maximize}} \quad z = \sum_{n \in \mathcal{N}_{\mathrm{D}}} \lambda_n^{\%}$$

subject to:

- PEV demand (3)
- Network model (5)-(14)
- Technical limits (15)-(21)

4) Model IV: This model is based on the maximization of an equal percentage of the existing PEVs at system buses. The OPF formulation is as follows:

$$\underset{\Theta}{\text{Maximize}} \quad z = \lambda^{\%}$$

subject to:

- PEV demand (4)
- Network model (5)-(14)
- Technical limits (15)-(21)

Models I-IV are non-linear and non-convex optimization problems. In the next section, these models are solved by using CONOPT [8] under GAMS [9].

IV. Case Study

The performance of the four models formulated in the previous section is tested on the IEEE One Area RTS-96 system reported in [10]. The supply capacity of the system is improved by considering that buses 7, 13, 21, and 23 are interconnecting buses with external power systems. Therefore, the resulting system could be view as a portion of the IEEE Three Area RTS-96 system, also reported in [10]. These interconnections are modeled as generators with a power rating equivalent to the continuous MVA rating of the interconnecting lines, existing in the 3-area system, and with a limiting power factor of 0.8.

The conventional hourly demand of the system is set to the values that correspond to each hour of the Tuesday of the week number 51, as reported in [10]. Therefore, 24 situations, one per hour, are analyzed. The peak demand of the system occurs between 5pm and 7pm of this day. Technical limits of generators, transmission lines and transformers are set according to the continuous ratings reported in [10].

We consider that the existing demands in the system are representative of population centers where PEVs can be charged. The data needed to represent the demand corresponding to the charge of PEVs in the different optimization models are estimated as follows:

- Models I and II: These models are based on the demand of a single PEV. This demand depends on the charge mode of the car. For simplicity, but without loss of generality, in this paper we only consider the single-phase (slow) charge mode, in which each PEV demands 3.7 kW ($P_{\mathrm{EV}}^{\mathrm{single}} = 3.7 \times 10^{-5}$ p.u.)

- Models III and IV: Along with the demand of a single PEV, these models are based on the total number of PEVs that exits in each system bus ($N_{\mathrm{EV}n}^{\mathrm{total}}$). The total number of PEVs per bus considered in this case study is shown in Table I. These values have been computed on the basis of 61900 PEVs/p.u.MW of peak demand. This ratio has been estimated from statistical data of Spain in 2016 [11]–[13], assuming the complete electrification of the private car fleet.

To get a picture on the figures that a complete electrification of the private car fleet would involve, Table I also includes the theoretical demand that the coincident charge of the existing PEVs per bus would suppose at the residential level for the given charge mode. Observe that even considering the least demanding charge mode, the simultaneous charge of PEVs could lead to a 200% demand increase.

TABLE I
TOTAL NUMBER AND EQUIVALENT LOAD OF PEVS PER BUS

Bus #	$P_{\mathrm{D}n}^{\mathrm{peak}}$ [p.u.]	$N_{\mathrm{EV}n}^{\mathrm{total}}$ #	$P_{\mathrm{EV}n}^{\mathrm{total}}$ [p.u.]
n1	1.08	66852	2.47
n2	0.97	60043	2.22
n3	1.80	111420	4.12
n4	0.74	45806	1.69
n5	0.71	43949	1.63
n6	1.36	84184	3.11
n7	1.25	77375	2.86
n8	1.71	105849	3.92
n9	1.75	108325	4.01
n10	1.95	120705	4.47
n13	2.65	164035	6.07
n14	1.94	120086	4.44
n15	3.17	196223	7.26
n16	1.00	61900	2.29
n18	3.33	206127	7.63
n19	1.81	112039	4.15
n20	1.28	79232	2.93

To analyze the performance of the four optimization models, they are solved to obtain the maximum number of PEVs that could be simultaneously charged in the system for each considered load level. The parameters of the models that represent the studied operating conditions are obtained from the solution of the power flow problem for each one of the 24 considered load profiles. For a given hour, the active power loads are as reported in [10], whereas the reactive power loads are set proportionally to the power factors computed for the peak load profile.

Figure 1 depicts the solution of the four models for each hour of the considered day together with the load profile. As expected, the hours with low conventional load are the hours where the greatest number of PEVs can be simultaneously charged. This situation corresponds to the 4th and 5th hour of the studied day for all cases. Models I and III, i.e., the models that independently maximize the number or percentage of on-charge PEVs per bus, allow the simultaneous charge of a greater number of them. Since these two models are basically equivalent, there should not be differences in the obtained solutions. However, Model I is able to accommodate the charge of a slightly higher number of PEVs than Model III in all hours. To gain more insight into this result we analyze the solutions of the optimization models for the fourth hour.

Table II provides the solution of the four models in terms of the number of PEVs that can be simultaneously charged per bus of the system, whereas Table III shows an equivalent information, but now in terms of the percentage of the existing PEVs that can be simultaneously charged per bus of the system. Both tables also include total system-wide quantities. Note that, although Models I and II are based on maximizing the number of PEVs, the results provided in Table III for these models are expressed in terms of percentages with respect to the maximum number of PEVs per bus. This way eases the quick comparison of the results. For the same reason, the results of Models III and IV have been adapted to the format of Table II. Note that Tables II and III only display quantities for the buses where the possibility to charge PEVs is considered, i.e., buses where population centers are supplied. For the rest of system buses neither conventional nor PEVs related demand is connected.

Models I and III maximize the simultaneous charge of PEVs by means of the free distribution of this charge throughout the system, only limited by technical considerations. This leads to solutions in which the charge of PEVs concentrates at the

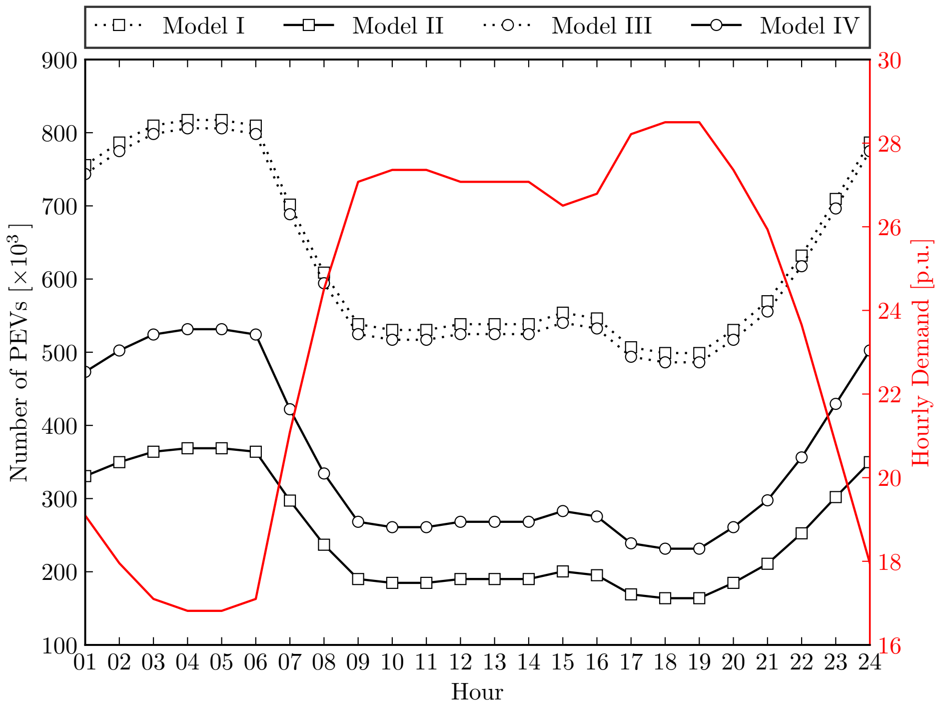

Fig. 1. Number of PEVs that could be simultaneously charged.

buses with the greatest supply capacities, while the charge in other buses is not allocated. For example, it can be observed in Table III that Model I assigns almost double of the charge needs to bus 20 while no charge is considered for buses 3-6, 8, 9, 16, and 19. Similar qualitative results are observed in the case of Model III. This type of information is of limited interest for the short-term operation of the power system, but it could be useful, for instance, to identify the strongest points of the network in planning studies related to the allocation of commercial centers that incorporate large PEV-charging stations.

It can be also observed that, although Models I and III are essentially equivalent, they distribute the charge of PEVs among the system buses in a different manner. The reasoning behind this result is the non-linear and non-convex nature of the optimization problems handled. As such, we have to assume that the results the models provide correspond to local optima. Therefore, although the results in terms of variables $P_{\mathrm{EV}n}$ of both models are quite different, in the case of Model III, the solver finds a local optimum for a total number of PEVs lower than, but similar to, the total number of PEVs of the local optimum reached for Model I. This fact is verified by fixing the values of $P_{\mathrm{EV}n}$ corresponding to the solution of Model I in the formulation of Model III. Doing so both solutions coincide.

The approach of Models II and IV directly incorporates uniformity criteria since they maximize a variable that is common to all buses. Consequently, they maximize the presence of PEVs by distributing their charge among all the eligible buses in such a way that all of them are assigned to have charge capacity. Unlike the other two models, local optima have not been observed. Model II finds solutions where equal number of PEVs per bus are simultaneously charging. This could lead to situations in which the number of allowed coincident on-charge PEVs exceeds the existing charge infrastructure in some system buses whereas in other buses this number could fall short of the charging needs. Therefore, Model II seems to be of little interest for planning or operation studies at the system level. However, its modeling philosophy could find application, for example, in the management of charging stations to prevent unbalanced operation.

Model IV seems to be the most interesting approach from the point of view of the planning and operation studies at the whole system level, because it provides solutions characterized by the proportional sharing of the PEVs charge. For instance, it could be an assistant tool to identify optimal network expansion measurements to procure the 100% charging capability at system buses for a given period. In the case of timeframes close to real-time operation, this model could be used, for example, to find a fare limitation of the PEVs charge at system buses when the system is subjected to contingencies and/or operating in a emergency situation. In this context, Model II would be a tool to help the coordination between transmission and distribution system operators.

V. Summary and Conclusions

This paper has presented four different OPF models that could be assistant tools in planning and operation tasks of power systems where the presence of plug-in electric vehicles is generalized. Essentially, the four models maximize the simultaneous

TABLE II
NUMBER OF PEVS PER BUS FOR HOUR 4

Bus #	Model I #	Model II #	Model III #	Model IV #
n1	14750	21692	0	20140
n2	16241	21692	0	18088
n3	0	21692	0	33566
n4	0	21692	63242	13799
n5	0	21692	80842	13240
n6	0	21692	0	25361
n7	75460	21692	71931	23310
n8	0	21692	0	31888
n9	0	21692	0	32634
n10	1281	21692	0	36364
n13	199379	21692	183609	49417
n14	60798	21692	0	36177
n15	127479	21692	4470	59115
n16	0	21692	124421	18648
n18	169594	21692	91329	62098
n19	0	21692	0	33753
n20	152344	21692	186122	23869
Total	817326	368764	805966	531467

TABLE III
PERCENTAGE OF PEVS PER BUS FOR HOUR 4

Bus #	Model I %	Model II %	Model III %	Model IV %
n1	22.06	32.45	0.00	30.13
n2	27.05	36.13	0.00	30.13
n3	0.00	19.47	0.00	30.13
n4	0.00	47.36	138.07	30.13
n5	0.00	49.36	183.95	30.13
n6	0.00	25.77	0.00	30.13
n7	97.53	28.04	92.97	30.13
n8	0.00	20.49	0.00	30.13
n9	0.00	20.03	0.00	30.13
n10	1.06	17.97	0.00	30.13
n13	121.55	13.22	111.93	30.13
n14	50.63	18.06	0.00	30.13
n15	64.97	11.06	2.28	30.13
n16	0.00	35.04	201.00	30.13
n18	82.28	10.52	44.31	30.13
n19	0.00	19.36	0.00	30.13
n20	192.28	27.38	234.91	30.13
Total	46.33	20.90	45.69	30.13

charge of PEVs but they differ in the way this load is represented. The charge of the PEVs is modeled according to the approach of multi-parameter homotopy methods (Models I and III) as well as single-parameter homotopy methods (Models

II and IV). Consequently, the quantitative and qualitative characteristics of the results that each model provides also differ. It is found that Models I and III could be adequate for planning studies where the strongest buses of the network in terms of supply capacity need to be identified. Model IV seems to have application in both planning and operation tasks. More restricted applications are envisioned for Model II. It is also observed that Models I and III are basically equivalent and they are more exposed to local optima than the other models. The reason behind this fact could be the greater number of degrees of freedom that these models present. Independently of the application, the structure of the models is appropriate for sensitivity studies [14] and, therefore, the main factors that limit a grater penetration of PEVs can be identified.

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