Modelling Load Stochastic Jumps for Power Systems Dynamic Analysis

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Abstract

This letter proposes an approach to model power system loads as stochastic processes that incorporate both continuous and event-driven dynamics. The letter provides a brief theoretical background on the stochastic differential equations defining Ornstein-Uhlenbeck processes with jumps used for the stochastic modeling of power system voltage-dependent loads. The all-island 1479-bus Irish transmission system serve to illustrate and test the proposed jump-diffusion model.

Index Terms

Ornstein-Uhlenbeck processes, jump-diffusion processes, exponential load recovery.

I. Introduction

This letter originates from the observation that the noise of measured quantities of a power system, such as the frequency, is not distributed as a Gaussian process, but shows heavy tails. These are known to be caused by discrete, sporadic events, which, in the literature on stochastic differential equations, are called *jumps* [1]. These events, e.g., load step variations, have been scarcely studied in the literature as they are not big enough to be considered as contingencies and not small enough to be classified as noise. However, in systems with high penetration of stochastic sources, such as wind, they can contribute to reducing the overall system stability.

Stochastic processes able to combine both continuous small perturbations and event-driven random phenomena are called jump-diffusion processes. These processes are reckoned to be appropriate to model the discrete events occurring in various physical systems [2], and are largely used in financial modeling [1]. Particularly, jump-diffusion models of the Ornstein-Uhlenbeck type have shown a great modelling potential in the context of econometrics [3]. However, the application of these models in power system analysis is very limited. In [4], stochastic processes including jumps are used for modeling the perturbations due to the operation of transformer tap changers. More recently, jump-diffusions are employed in [5] for the stochastic characterization of network faults.

In the last decades, a number of studies have taken into account the load volatility to define generalized load models, to determine the system operational bounds, and to prevent instability issues by means of load sensitivity analysis [6], [7]. A number of studies develop and validate load models via measurement approach [8]–[11].

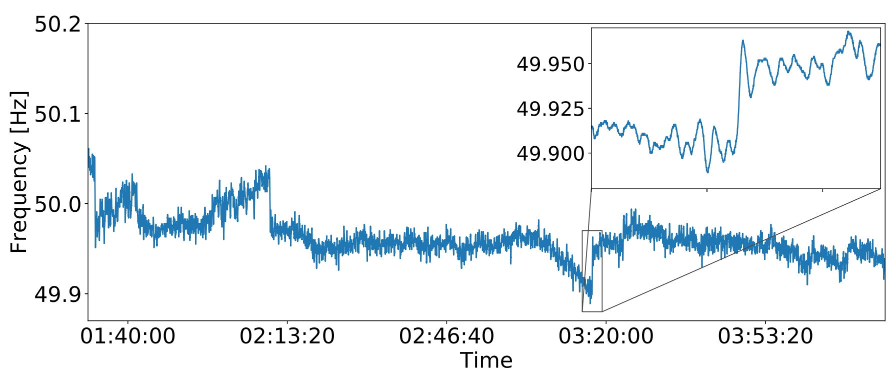

Fig. 1: Irish system frequency measured at transmission level. Excerpt of the frequency series.

The letter proposes to use SDAEs with jumps as a general model to describe continuous noise as well as random discrete events in power systems, and it addresses the impact on power system stability of the discrete events that originated the heavy-tail distribution in power system quantities.

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II. ORNSTEIN-UHLENBECK PROCESSES WITH JUMPS

A one-dimensional standard Ornstein-Uhlenbeck process, $x(t)$, is the solution of the following SDE:

$$dx(t) = a\,[\mu(t) - x(t)]\,dt + b\,dW(t)\ , \tag{1}$$

with initial value $x(t_0) = x_0$. In (1), a, μ, and b are constant parameters and $W(t)$ represents a standard Wiener process. As $t \to \infty$, the standard Ornstein-Uhlenbeck process represented by (1) tends to a stationary Gaussian distribution with mean $\mathrm{E}\,[x(t)] = \mu$, and variance $\mathrm{Var}\,[x(t)] = b^2/2a$. Moreover, in the stationary state, process $x(t)$ is exponentially autocorrelated according to $\mathrm{Aut}\,[x(t), x(t+\tau)] = e^{-a\tau}$. The mean-reversion parameter a controls both how fast the process $x(t)$ tends to the stationary mean value μ, and the memory of the process, such that the lower the value of a the slower the process autocorrelation decays. In the context of power systems, the standard Ornstein-Uhlenbeck process have been directly and/or indirectly used for the modelling of different stochastic phenomena (see, e.g., [12]–[16]).

Ornstein-Uhlenbeck processes represented by (1) are only able to model continuous stochastic phenomena. Discrete stochastic events can be modelled by introducing in (1) a *jump* term, as follows:

$$dx(t) = a\,[\mu(t) - x(t)]\,dt + b\,dW(t) + c\,dC(t)\ , \tag{2}$$

with initial value $x(t_0) = x_0$. In the jump term, c is a constant parameter and $C(t)$ is a compound Poisson process. A compound Poisson process $C(t), t \in [0, \infty)$, with initial value $C(t_0) = C_0 = 0$, is defined as

$$C(t) = \sum_{k=1}^{N(t)} \zeta_k\ , \tag{3}$$

where $N(t)$ is a Poisson process with finite intensity $\lambda > 0$, and $\zeta_1, \zeta_2, ...$ are independent identically distributed random variables, known as *marks*, which are independent from $N(t)$. A compound Poisson process generates a sequence of pairs (t_k, ζ_k), with $k \in \mathbb{N}$, of Poisson distributed jump times t_k and marks ζ_k defined on the mark set $\mathcal{E} = \mathbb{R}\backslash\{0\}$, that can have an arbitrary distribution.

The solution of the SDE (2) is a Ornstein-Uhlenbeck process with jumps, which belongs to a wider class of processes so-called jump-diffusion processes [2]. The presence of the jump term does not modify the autocorrelation of the resulting process, which also follows an exponential decaying law similar to the autocorrelation of the process (1). However, the distribution of the process (2) generally shows heavier tails and higher peaks than the standard Gaussian distribution.

Although $W(t)$ and $C(t)$ are not differentiable, two types of white noise processes $\xi_W(t)$ and $\xi_C(t)$, namely *Gaussian white noise* and *Poisson white noise*, respectively, can be formally defined as follows [17]:

$$\xi_W(t) = \frac{dW(t)}{dt}, \quad \xi_C(t) = \frac{dC(t)}{dt}\ . \tag{4}$$

In view of (4) and dropping the time dependency of $\xi_W(t)$ and $\xi_C(t)$, the SDE (2) is reformulated as follows

$$\dot{x}(t) = a[\mu(t) - x(t)] + b\,\xi_W(t) + c\,\xi_C(t)\ . \tag{5}$$

Finally, the solution of SDEs with jumps is generally carried out by numerical integration methods. A wide collection of them is described in [2].

Expression (5) allows us to model the resulting stochastic voltage dependent load as:

$$\begin{aligned}
p_L(t) &= p_{L0}(t) + x_p(t) \\
q_L(t) &= q_{L0}(t) + x_q(t) \\
\dot{x}_p(t) &= a_p[\mu_p(t) - x_p(t)] + b_p\xi_{Wp} + c_p\xi_{Cp} \\
\dot{x}_q(t) &= a_q[\mu_q(t) - x_q(t)] + b_q\xi_{Wq} + c_q\xi_{Cq}\ ,
\end{aligned} \tag{6}$$

and $x_p(t_0) = x_q(t_0) = 0$, where $p_{L0}(t)$ and $q_{L0}(t)$ are the transient active and reactive power consumptions, which are time dependent as they can depend on the voltage magnitude at the bus where the load is connected. Interestingly, the deterministic part of (6) is formally equivalent to the dynamic load model proposed in [8], and later validated in [9], if p_{L0}, q_{L0}, μ_p and μ_q are assumed to be voltage dependent.

III. CASE STUDY

The parameters of the stochastic load model (6) should be computed on the basis of the statistical analysis of high-voltage load measurements. In absence of such data, we have roughly conjectured the stochastic behaviour of the load from the analysis of frequency measurements of the All-Island Irish Transmission System (AIITS). For simplicity, we assume that deviations from the nominal frequency are only consequence of changes in the system load.

Frequency data are pre-processed as follows: frequency changes above a certain threshold are considered jumps (see Fig. 1). A new set of data with the value of these changes is created, whose statistical properties are used to set the parameters

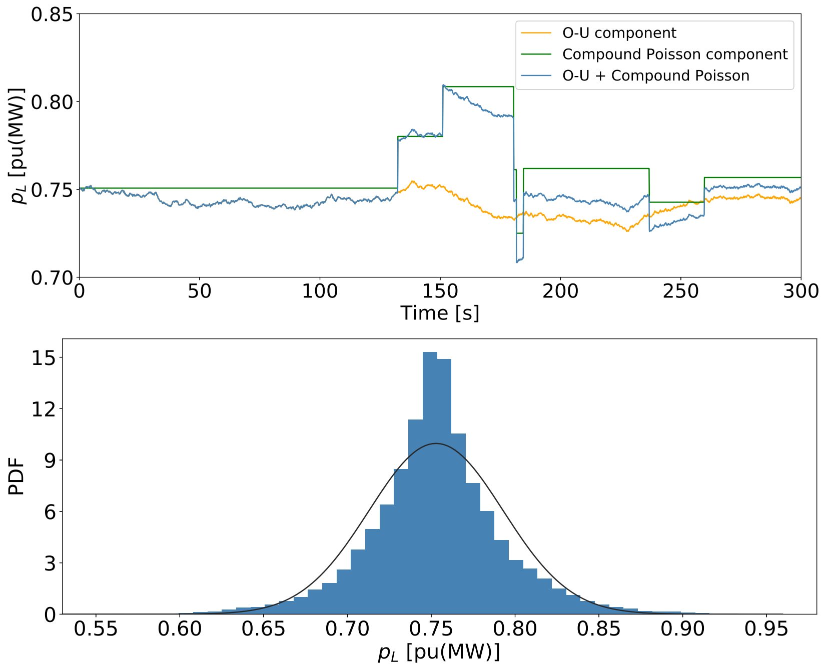

Fig. 2: Active power of the stochastic load. Upper panel: dynamic trajectory; lower panel: Histogram of the active power and normal PDF fit.

of the jump term of the model. The statistical analysis of the rest of the frequency series is used to set the parameters of the drift and diffusion terms of the Ornstein-Uhlenbeck process following the procedure shown in [13], [14], [16]. Quantities in terms of load are obtained based on the inertia and frequency control droop characteristics of the system.

The statistical analyses previously described draw the following results:

- An average of 6 jumps every 300 s is found.
- The distribution of the load jump sizes can be approximated by a normal distribution with $\mu = 0$ and $\sigma = 0.046$.
- The distribution of the load volatility can be approximated by a normal distribution with $\mu = 0$ and $\sigma = 0.008$.
- The autocorrelation exponent of the load volatility is set to 0.0125.

According to these estimations, the parameters of the stochastic part of the active load model in (6) are set as follows:

- $a_p = 0.0125$.
- $\mu_p(t) = \mu_p = 0$.
- $b_p = \sqrt{2a_p}\sigma_p = 0.0001$.
- $\lambda = 0.02$ jumps per second. This value, together with the simulation time, defines the Poisson process that determines the jump times t_k of the compound Poisson process.
- Marks ζ_k of the compound Poisson process are random values taken from a $\mathcal{N}(0, 0.046^2)$.
- $c_p = 1$.

For simplicity, the stochastic part of the reactive load model in (6) is modelled according to $x_q = x_p \tan\varphi$, where φ is the angle corresponding to the power factor of the considered load.

The stochastic load model is tested by means of time-domain simulations carried out with DOME [18]. The upper panel in Fig. 2 shows the time evolution of a given load with $p_{L0}(t) = p_{L0} = 0.75$ pu. This is an example of the kinds of trajectories generated by the proposed model, whose stochastic behaviour can be viewed as the composition of two stochastic contributions: the orange line is p_{L0} plus the drift-diffusion component of $x_p(t)$, the green line represents p_{L0} plus the pure jump component of $x_p(t)$, and the blue line corresponds to the actual $p_L(t)$. The lower panel of Fig. 2 depicts the histogram of $p_L(t)$ along with the Probability Density Function (PDF) of the normal distribution that best fit the data. The deviations from the normal distribution are apparent. This is an expected phenomenon which results from the incorporation of a jump component to the standard Ornstein-Uhlenbeck process.

The proposed model could be used to study the robustness of power systems with respect to event-driven phenomena. A simple example is provided here where the robustness of the AIITS is analyzed with respect to the variation of the parameters characterizing the compound Poisson part of the stochastic load model. The study considers two different levels of wind penetration, 25% and 55%. A set of 2000 one-hour-long simulations is run for both wind scenarios in each analyzed case. In all simulations, the parameters of the drift and diffusion part of the load model remain the same as previously described. An intensity of $\lambda = 0.02$ jumps per second and $\mathcal{N}(0, \sigma_{p_P}^2)$, with $\sigma_{p_P} = 0.33$, as the distribution of the marks for the compound Poisson process are the base values. Different cases are defined by increasing λ and σ_{p_P}.

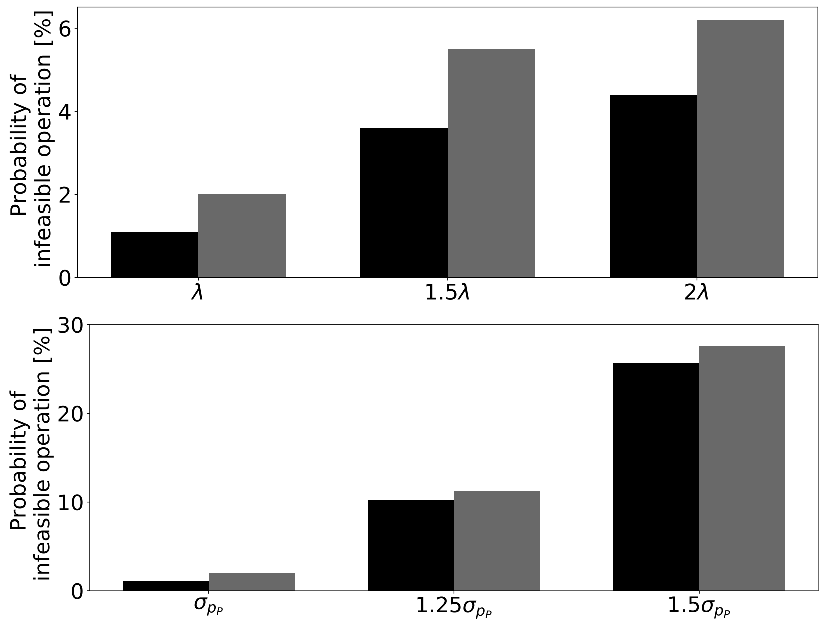

Fig. 3: Probability of system infeasible operation. Wind generation: 25% (black), and 55% (grey). Upper panel: as function of λ; lower panel: as function of σ_{p_P}.

The upper panel in Fig. 3 shows the probability of infeasible operating conditions the system might experience with respect to an increasing number of jumps (increasing intensity of compound Poisson process), in case of 25% (black bars) and 55% (grey bars) of wind penetration. The average number of jumps per second ranges from the base value of λ to 2λ with a step of 0.5λ. The chart shows that increasing the average number of events in the load dynamics does not considerably destabilize the system, albeit this fact results in introducing more randomness into the grid. The lower panel in Fig. 3 shows the same probability percentage as the upper panel but now with respect to the increasing amplitude of the jumps, that ranges from the base value of σ_{p_P} to $1.5\sigma_{p_P}$ with an increase of 25%. As expected, the bigger σ_{p_P}, the higher the chance to experience an infeasible system condition due to a sudden and relatively high load variation. This trend is particularly non-linear and amplified by high wind power production.

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