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Acceptance sampling plans from a truncated life test based on the power Lomax distribution with application to manufacturing

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ABSTRACT

In this research, a new acceptance sampling plan for a truncated life test is presented, assuming that the quality characteristic follows the power Lomax distribution. The operating characteristic function values are calculated for the proposed sampling plan, jointly with the optimal sample size and the producer's risk for a selection of distribution parameters. Furthermore, a comparative study with other sampling plans is introduced to demonstrate the advantages of the proposed plan. Finally, a real-life example illustrating the applicability of the proposed sampling plan in a manufacturing company is discussed.

Key words: acceptance sampling plan, operating characteristic, power Lomax distribution, industry, data analysis.

1. Introduction

Acceptance sampling plans play a very important role in the statistical quality control, especially in the lot production process to decide whether to reject or accept the lot (Al-Nasser and Al-Omari, 2013). The decision on the quality of all entire items in each lot depends on drawing a random sample of size *n* from a selected lot; after that, within a specific timeframe, testing procedure is initiated to discover the number of failure or defective items included in the sample before the pre-indicated time is terminated (Al-Nasser and Gogah, 2017; Al-Omari et al., 2016; Malathi and Muthulakshmi, 2017).

Then, the problem is to find the optimal sample size *n* that is necessary to assure a certain average life, when the life test is terminated at a pre-assigned time *t*. Such that the observed number of failures does not exceed a given acceptance number *c*. Accordingly, the decision is to reject all entire items in the lot if the number of failures

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in the sample within the timeframe is greater than the pre-assigned acceptance number (c); elsewhere the lot is accepted.

Many authors proposed truncated life test plans for different lifetime distributions. For example, Epstein (1954) was the first who considered truncated life tests in the exponential distribution. The truncated life tests in the Pareto distribution of the second kind are discussed by Baklizi (2003). The Rayleigh model is proposed by Baklizi *et al.*, (2005). The generalized Birnbaum-Saunders Distribution is discussed by Balakrishnan *et al.*, (2007). The Marshall-Olkin extended Lomax distribution is given by Rao *et al.*, (2008). The generalized exponential distribution is considered by (Rao, 2010). The new Weibull-Pareto distribution is proposed by Al-Omari *et al.*, (2016), weighted exponential distribution is discussed by Gui and Aslam (2017), exponentiated generalized inverse Rayleigh distribution is discussed by Al-Nasser *et al.*, (2017). The inverse gamma model is given by Al-Masri (2018), and Tsallis q-exponential distribution is proposed by (Al-Nasser & Obeidat, 2020).

The purpose of this article is to develop and discuss an acceptance sampling plan (ASP) for a truncated life test on the power Lomax distribution (PLxD) and illustrate the results on manufacturing data. The rest of the paper is organized as follows. Section 2 is based on summaries of PLxD and some of its properties. ASPs and operating characteristic (OC) values and the producer's risk for PLxD are analysed. The analysis and illustrative examples are presented in Section 4. A comparative study between the proposed ASP and other sampling plans based on different distributional assumptions is discussed in Section 5. A real manufacturing data analysis is given in Section 6. The work is concluded in Section 7.

2. The Power Lomax distribution

Power Lomax distribution (PLxD) originally proposed by (Rady, Hassanein, & Elhaddad, 2016) is a lifetime distribution obtained by taking the power of the Lomax distribution random variable. The PLxD distribution is very flexible due to its variable shapes of hazard rate, which accommodates both inverted bathtub and decreasing.

The probability density function (pdf) of PLxD is

$$f(x) = \alpha\beta\lambda^{\alpha}x^{\beta-1}\left(\lambda + x^{\beta}\right)^{-\alpha-1}, \quad x > 0, \qquad \alpha,\beta,\lambda > 0. \tag{1}$$

The corresponding cumulative distribution function (CDF) is

$$F(x) = 1 - \lambda^{\alpha}\left(\lambda + x^{\beta}\right)^{-\alpha} \tag{2}$$

From Figure 1 it can be easily concluded that the shapes of PLxD have a decreasing behaviour for $\beta < 1$, the distribution has an exponentially decreasing behaviour but starting from the y-axis for $\beta=1$. For $\beta > 1$ the pdf curves of the model are unimodal and symmetrical for some combinations of parameters.

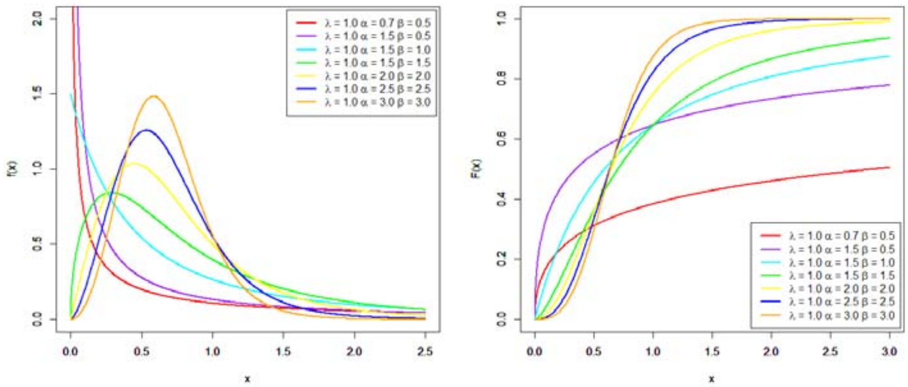

Figure 1. pdf and CDF of PLxD for some given parameter values

The PLxD has the following properties. The rth moments about origin and mean of PLxD are, respectively:

$$\mu_r' = \frac{\alpha\lambda^{\frac{r}{\beta}}\,\Gamma\left[\alpha - \frac{r}{\beta}\right]\Gamma\left[\frac{r+\beta}{\beta}\right]}{\Gamma[1+\alpha]}$$

Therefore, the mean of PLxD is given as:

$$\mu = \frac{\alpha\lambda^{\frac{1}{\beta}}\,\Gamma\left[\alpha - \frac{1}{\beta}\right]\Gamma\left[\frac{1+\beta}{\beta}\right]}{\Gamma[1+\alpha]} \tag{3}$$

The PLxD has an increasing-decreasing hazard rate function, for more details see (Rady, Hassanein, & Elhaddad, 2016).

3. The suggested sampling plans

Suppose that the lifetime of the products being tested follows the PLxD as given in (1) and the specified mean lifetime is μ_0. Now, the ASP problem is to find the optimal sample size that ensures an actual average life (μ) such that no more than c units fail to pass the test period (t). To perform the test according to this plan, a random sample of size n units is selected from a lot. If μ_o can be obtained with a pre-assigned probability, P^*, as specified by the consumer, then the lot is accepted. If not, then it is rejected.

Following Al-Nasser and Obeidat (2020), the ASP-based on truncated life tests consists of the following parameters:

1) The sample size: number of units' n to be drawn from the lot.
2) The test duration t: the maximum test duration time.
3) Acceptable number of defective (d) items: c; if $d \leq c$ remains the same until the end of the test period t_0, the lot is accepted.

4) The minimum ratio t/μ_o: where μ_0 is the quality parameter of the product life; and t is the maximum test duration.
5) The ASP parameters will be (n, c, t/μ_o).

3.1. Optimal sample size of the ASP (n, c, t/μ$_o$)

Let P^* be the confidence level where $P^* \in (0,1)$; in the sense that the possibility of rejecting a considered lot having a specified mean less than or equal to the actual mean ($\mu_0 \leq \mu$) is greater or equal to P^*. The shopper's risk, that is the probability of accepting a defective lot, is fixed and less or equal to $1-P^*$. Also, suppose that the lot size is large enough to use the binomial distribution. Then, the problem of the ASP (n, c, t/μ_o) is to find the minimum sample size n such that the number of defective units d does not exceed c, to ensure that $\mu > \mu_0$ satisfies the following inequality:

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - p^* \tag{4}$$

where

$$p = F(t;\mu_0) = 1 - \lambda^{\alpha} \left(\lambda + \left(\frac{t}{\mu_0} \frac{\alpha \lambda^{\frac{1}{\beta}} \Gamma\left[\alpha - \frac{1}{\beta}\right] \Gamma\left[\frac{1+\beta}{\beta}\right]}{\Gamma[1+\alpha]} \right)^{\beta} \right)^{-\alpha}$$

By using the binomial theory, the probability of success in (4), which is used for finding a defective item in each a lot during the test process time t, is $p = F(t;\mu_0)$; this probability in terms of distribution function is a monotonically increasing function of the ratio t/μ_o. Then, for the acceptance sampling ASP (n, c, t/μ_o) and inequality (4), we assure that $F(t;\mu) \leq F(t;\mu_0)$ with probability P^*, or alternatively $\mu_0 \leq \mu$. The results for this plan when the lifetime distribution is PLxD with $\alpha = 1;\ \beta = 2\ and\ \lambda = 1$ are given in Table 1, under the classical initial values of the ratio t/μ_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712, when P^* = 0.75, 0.9, 0.95, 0.99 and c = 0, 1, 2, … , 10 (Gupta and Groll, 1961; Kantam and Rosaiah, 2001; Baklizi, 2003; Baklizi et al., 2005; Al-Nasser et al., 2018; Al-Masri, 2018; Al-Omari et al., 2019).

3.2. Operating characteristic function of the ASP (n, c, t/μ$_o$)

Operating characteristic (OC) function is an important parameter in the ASP, it provides the exact information about the probability of acceptance of a lot. For the *ASP (n, c, t/μ$_o$)*, the OC can be computed using binomial distribution as:

$$OC(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$$

which can be computed using the incomplete beta function $B_p(a,b)$ as:

$$OC(p) = 1 - B_p(c+1, n-c)$$

where $p = F(t;\mu)$. Table 2 presents the OC function values for the *ASP (n, c, t/μ$_o$)*.

Table 1. Minimum sample size to assert that the mean life exceeds a given value μ_0 with probability P^* and acceptance number *c* based on binomial probabilities when $\alpha = 1;\ \beta = 2\ and\ \lambda = 1$

P^*		t/μ_0							
	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	3	2	1	1	1	1	1	1
	1	5	3	3	3	2	2	2	2
	2	7	5	4	4	3	3	3	3
	3	10	7	6	5	4	4	4	4
	4	12	8	7	6	6	5	5	5
	5	14	10	8	8	7	6	6	6
	6	16	11	10	9	8	7	7	7
	7	19	13	11	10	9	9	8	8
	8	21	15	12	11	10	10	9	9
	9	23	16	14	12	11	11	10	10
	10	25	18	15	14	12	12	11	11
0.9	0	4	2	2	2	1	1	1	1
	1	7	4	4	3	3	2	2	2
	2	9	6	5	4	4	4	3	3
	3	12	8	7	6	5	5	4	4
	4	14	10	8	7	6	6	6	5
	5	17	11	9	8	7	7	7	7
	6	19	13	11	10	8	8	8	8
	7	22	15	12	11	10	9	9	9
	8	24	16	14	12	11	10	10	10
	9	26	18	15	13	12	11	11	11
	10	29	20	16	15	13	12	12	12
0.95	0	5	3	2	2	2	1	1	1
	1	8	5	4	4	3	3	3	2
	2	11	7	6	5	4	4	4	4
	3	14	9	7	6	5	5	5	5
	4	16	11	9	8	7	6	6	6
	5	19	12	10	9	8	7	7	7
	6	21	14	12	10	9	8	8	8
	7	24	16	13	12	10	9	9	9
	8	26	18	14	13	11	11	10	10
	9	29	19	16	14	12	12	11	11
	10	31	21	17	15	14	13	12	12
0.99	0	7	4	3	3	2	2	2	2
	1	11	7	5	5	4	3	3	3
	2	14	9	7	6	5	4	4	4
	3	17	11	9	7	6	6	5	5
	4	20	13	10	9	7	7	6	6
	5	23	15	12	10	9	8	8	7
	6	25	17	13	12	10	9	9	8
	7	28	18	15	13	11	10	10	10
	8	30	20	16	14	12	11	11	11
	9	33	22	18	16	13	13	12	12
	10	36	24	19	17	15	14	13	13

Table 2. Operating characteristic function values for the sampling plan $(n, c = 2, t/\mu_0)$ for a given probability P^*

P^*	n	t/μ_0	μ/μ_0 2	4	6	8	10	12
0.75	7	0.628	0.85933	0.994463	0.999411	0.999888	0.99997	0.99999
	5	0.942	0.758991	0.985557	0.998274	0.999656	0.999905	0.999967
	4	1.257	0.697065	0.974337	0.996542	0.999275	0.999795	0.999928
	4	1.571	0.51865	0.933522	0.9892	0.997548	0.999276	0.999741
	3	2.356	0.536389	0.901897	0.979068	0.994523	0.998252	0.999345
	3	3.141	0.366443	0.780305	0.934349	0.979078	0.992497	0.996976
	3	3.927	0.259086	0.651114	0.864325	0.948163	0.97906	0.990869
	3	4.712	0.190563	0.536389	0.78025	0.901897	0.955662	0.979068
0.90	9	0.628	0.749567	0.987815	0.998641	0.999736	0.999928	0.999975
	6	0.942	0.639516	0.973686	0.996695	0.999329	0.999813	0.999935
	5	1.257	0.512032	0.945163	0.991985	0.998266	0.999501	0.999824
	4	1.571	0.51865	0.933522	0.9892	0.997548	0.999276	0.999741
	4	2.356	0.222004	0.743325	0.933578	0.980987	0.99364	0.99755
	4	3.141	0.098206	0.518914	0.816848	0.933606	0.974393	0.989217
	3	3.927	0.259086	0.651114	0.864325	0.948163	0.97906	0.990869
	3	4.712	0.190563	0.536389	0.78025	0.901897	0.955662	0.979068
0.95	11	0.628	0.631772	0.978039	0.997433	0.999494	0.999861	0.999952
	7	0.942	0.523698	0.958024	0.994463	0.998854	0.999677	0.999888
	6	1.257	0.35586	0.906065	0.985131	0.996681	0.99903	0.999655
	5	1.571	0.311317	0.867571	0.975906	0.994266	0.998268	0.999372
	4	2.356	0.222004	0.743325	0.933578	0.980987	0.99364	0.99755
	4	3.141	0.098206	0.518914	0.816848	0.933606	0.974393	0.989217
	4	3.927	0.047651	0.341284	0.666449	0.850637	0.933556	0.969202
	4	4.712	0.025325	0.222004	0.518826	0.743325	0.869726	0.933578
0.99	14	0.628	0.464414	0.957378	0.994663	0.99892	0.999699	0.999896
	9	0.942	0.328618	0.916115	0.987815	0.997383	0.99925	0.999736
	7	1.257	0.237226	0.858908	0.975856	0.99444	0.99835	0.999408
	6	1.571	0.174323	0.787955	0.956955	0.989273	0.996684	0.998782
	5	2.356	0.079876	0.572449	0.867672	0.958688	0.985527	0.994273
	4	3.141	0.098206	0.518914	0.816848	0.933606	0.974393	0.989217
	4	3.927	0.047651	0.341284	0.666449	0.850637	0.933556	0.969202
	4	4.712	0.025325	0.222004	0.518826	0.743325	0.869726	0.933578

Table 3. Minimum ratio of the true mean life to specified mean life for the acceptance of a lot with producer's risk of 0.05 with $\alpha = 1;\ \beta = 2\ and\ \lambda = 1$

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	7.512	9.181	8.607	10.757	16.132	21.507	26.888	32.263
	1	3.429	3.74	4.991	6.238	6.896	9.194	11.495	13.792
	2	2.567	3.063	3.433	4.291	4.846	6.461	8.077	9.692
	3	2.348	2.744	3.236	3.419	3.908	5.21	6.513	7.815
	4	2.099	2.32	2.744	2.911	4.366	4.47	5.588	6.705
	5	1.937	2.242	2.417	3.021	3.858	3.971	4.964	5.957
	6	1.822	2.018	2.452	2.727	3.49	3.606	4.509	5.41
	7	1.807	1.996	2.248	2.503	3.209	4.278	4.158	4.989
	8	1.732	1.975	2.086	2.325	2.986	3.98	3.877	4.652
	9	1.672	1.847	2.14	2.18	2.802	3.736	3.646	4.375
	10	1.623	1.843	2.018	2.304	2.649	3.531	3.452	4.142
0.90	0	8.684	9.181	12.251	15.311	16.132	21.507	26.888	32.263
	1	4.155	4.5	6.004	6.238	9.354	9.194	11.495	13.792
	2	2.998	3.48	4.087	4.291	6.434	8.578	8.077	9.692
	3	2.636	3.027	3.662	4.044	5.127	6.835	6.513	7.815
	4	2.324	2.767	3.096	3.429	4.366	5.82	7.276	6.705
	5	2.209	2.426	2.721	3.021	3.858	5.143	6.43	7.715
	6	2.054	2.332	2.692	3.065	3.49	4.653	5.817	6.98
	7	2.001	2.261	2.465	2.809	3.753	4.278	5.349	6.418
	8	1.905	2.093	2.468	2.607	3.486	3.98	4.976	5.971
	9	1.828	2.059	2.309	2.442	3.268	3.736	4.671	5.604
	10	1.809	2.031	2.176	2.522	3.086	3.531	4.415	5.297
0.95	0	9.715	11.268	12.251	15.311	22.961	21.507	26.888	32.263
	1	4.473	5.144	6.004	7.504	9.354	12.471	15.591	13.792
	2	3.373	3.85	4.643	5.108	6.434	8.578	10.725	12.868
	3	2.895	3.285	3.662	4.044	5.127	6.835	8.545	10.254
	4	2.529	2.964	3.408	3.869	5.142	5.82	7.276	8.731
	5	2.372	2.596	2.991	3.401	4.53	5.143	6.43	7.715
	6	2.194	2.474	2.911	3.065	4.089	4.653	5.817	6.98
	7	2.121	2.382	2.663	3.081	3.753	4.278	5.349	6.418
	8	2.011	2.309	2.468	2.857	3.486	4.648	4.976	5.971
	9	1.97	2.157	2.465	2.674	3.268	4.357	4.671	5.604
	10	1.895	2.118	2.322	2.522	3.455	4.114	4.415	5.297
0.99	0	11.503	13.025	15.036	18.792	22.961	30.612	38.272	45.922
	1	5.314	6.232	6.864	8.578	11.253	12.471	15.591	18.708
	2	3.867	4.496	5.137	5.803	7.66	8.578	10.725	12.868
	3	3.244	3.745	4.383	4.576	6.065	8.086	8.545	10.254
	4	2.895	3.321	3.692	4.259	5.142	6.855	7.276	8.731
	5	2.669	3.047	3.464	3.738	5.1	6.039	7.55	7.715
	6	2.451	2.854	3.112	3.638	4.596	5.451	6.815	6.98
	7	2.341	2.605	3.017	3.328	4.213	5.003	6.255	7.505
	8	2.209	2.506	2.793	3.084	3.909	4.648	5.81	6.972
	9	2.146	2.425	2.748	3.08	3.662	4.882	5.447	6.536
	10	2.094	2.359	2.587	2.902	3.782	4.606	5.144	6.172

3.3. Producer's risk of the *ASP (n, c, t/μ_o)*

Producer's risk (PR) is another important parameter of the acceptance plans, it measures the probability that a consumer rejects a good lot. Based on binomial theory, PR is computed as follows:

$$PR(p) = \sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

or by using the incomplete beta function:

$$PR(p) = B_p(c+1, n-c)$$

Therefore, for the *ASP (n, c, t/μ_o)* is solved as an inequality to ensure that the producer's risk is at most equal to a specific small value (i.e. say P^*) such that the ratio of the actual mean to the specified mean (i.e., μ/μ_o) is as specified by the producer. Therefore, the minimum ratio μ/μ_o is specified as a solution of the following inequality:

$$PR(p) > 1 - P^*$$

where $p = F([(t/\mu_o)\ (\mu_o/\mu)];\ \mu)$. The minimum values of the ratio μ/μ_0 for the *ASP (n, c, t/μ_o)* are given in Table 3.

4. Explaining the ASP (n, c, t/μ_o) results

In this article, the parameters of the proposed *ASP (n, c, t/μ_o)*, the smallest sample size, operating characteristic values and the minimum ratio of the true mean life to specified mean life, respectively, based on the PLxD, are given in Table 1 - Table 3.

For example, assume that the researcher aims to ensure that the product's mean lifetime is at least 1000 hours, with probability $P^* = 0.90$ when $c = 2$ such that the experiment will be terminated at t = 942 hours; that is, $\frac{t}{\mu_0} = 0.942$. Then, from Table 1, the optimal sample size for this plan is 6, accordingly, the appropriated *ASP (n, c, t/μ_o)* = (6, 2, 0.942). Moreover, from Table 2, the $OC(p)$ for the ASP (6, 2, 0.942) are given in Table 4:

Table 4. OC and PR for the ASP (6, 2, 0.942)

μ/μ_0	2	4	6	8	10	12
OC(p)	0.639516	0.973686	0.996695	0.999329	0.999813	0.999935
PR	0.360484	0.026314	0.003305	0.000671	0.000187	0.000065

The plan indicates that the lot is accepted if out of 6 items less than or equal to 2 items fail before the time t. Now, if the true mean is four times as the specified mean $\mu/\mu_0 = 4$ then we are assured that the lot will be accepted under this ASP with probability equal to 0.973686 and the producer's risk is about 0.026314. The probability of accepting a lot under the ASP (6, 2, 0.942) will be more than 0.97 if and only if the true mean is four times or more than the specified mean.

Furthermore, the results given in Table 3 indicated that when the consumer's risk is 10% ($P^* = 0.90$) and by using the ASP (*6, 2,* 0.942); then, the minimum ratio $\mu/\mu_0 =$ 3.48 when the producer risk is equal to 0.05. It implies that a lot with 6 items when c=2 will be rejected with probability less than or equal to 0.05.

5. Comparative Study

The advantages of the proposed ASP based on LPxD are compared with other ASP under various types of distributions assuming that the actual mean is four times of the specified mean and the acceptable number of defectives is equal to two. The comparison criterion will be the cost of inspection based on the sample size of the ASP and the producer's risk (PR). We said that a sampling plan with a smaller sample size is more efficient in reducing the cost of inspection compared to other ASP; at the sometime, we are seeking minimum value of the PR. The proposed ASP is compared with several ASPs that were proposed by Balakrishnan *et al.* (2007) for the generalized Birnbaum–Saunders distribution (GBSD); Sampath and Lalitha (2016) for the hybrid exponential distribution (HED); Al-Nasser & Obeidat (2020) for q-Exponential distribution (QED) and Rao et al., (2008) for Marshall-Olkin extended Lomax distribution (MOELD).

The comparative results are given in Table 5, which indicated that the proposed ASP based on PLxD gave equivalents or smaller sample sizes with smaller PR than the sample sizes and PR that were obtained by all other plans. These encouraging results means the proposed ASP is more efficient than the ASP that considered in these comparisons and it is worth to be used by the decision makers.

Table 5. Comparative ASP $\left(n, c = 2, t / \mu_0\right)$ when μ / μ_0 =4 and $P^* = 0.95$.

t / μ_0	PLxD		QED		GBSD		MOLED		HED	
	n	PR	n	PR	n	PR	n	PR	n	PR
0.628	11	0.0220	10	0.2762	17	0.0351	12	0.2464	16	0.4170
0.942	7	0.0420	7	0.2665	11	0.0657	9	0.2842	11	0.4152
1.257	6	0.0939	6	0.3104	9	0.1159	7	0.2789	9	0.4543
1.571	5	0.1324	5	0.2980	7	0.1289	6	0.2929	7	0.4097
2.356	4	0.2567	5	0.5221	6	0.2699	5	0.3752	6	0.5486
3.141	4	0.4811	4	0.4846	5	0.3332	5	0.5420	5	0.5821
3.927	4	0.6587	4	0.6123	5	0.4896	4	0.4650	4	0.5194
4.712	4	0.7780	4	0.7113	4	0.4106	4	0.5662	4	0.6378

6. Real Data Application

Lifetime data measured in months of 20 small electric carts used by the manufacturing company for internal transportation and delivery services in a large manufacturing facility are used to illustrate the proposed ASP. The data are given as follows (Zimmer et al., 1998; Lio et al., 2010; Al-Omari et al., 2018): 0.9, 1.5, 2.3, 3.2, 3.9, 5.0, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15.0, 16.3, 19.3, 22.6, 24.8, 31.5, 38.1 and 53.0.

First, we need to test whether PLxD can be used or not. Several goodness of fit criteria were used to test if the data fit the model, including the minimum value of the function -log (likelihood) (-2MLL), Cramér-von Mises (CvM), Akaike information criteria (AIC), Bayesian information criteria (BIC), consistent Akaike information criteria (CAIC), Hannan-Quinn information criteria (HQIC) and two distribution tests; K-S and Anderson-Darling (A-D). The goodness of fit results were acceptable. The results indicate an excellent fit with the K-S distance value between the empirical and the theoretical PLxD equal to 0.1579606 with P-value equal to 0.6440496.

Table 6. Information measures and goodness of fit test for the small electric carts

AIC	BIC	W	AD	-log(Likelihood)	KS (P-Value)
158.030	161.017	0.039061	0.261307	76.01396	0.157961 (0.64405)

Moreover, the maximum likelihood estimation (MLE) method is used to estimate the PLxD unknown parameters. The results are given in Table 7:

Table 7. MLE estimates based on the small electric carts data

Estimator	Value	Stdev	95% C.I	
			Lower	Upper
$\hat{\alpha}$	0.7790995	0.5357288	-0.2709095	1.829109
$\hat{\beta}$	1.3513955	0.4189634	0.5302422	2.172549
$\hat{\lambda}$	10.2523672	5.7100538	-0.9391325	21.443867

Therefore, the mean life can be estimated as:

$$\mu = \frac{\alpha\lambda^{\frac{1}{\beta}}\,\Gamma\left[\alpha - \frac{1}{\beta}\right]\Gamma\left[\frac{1+\beta}{\beta}\right]}{\Gamma[1+\alpha]} = \frac{4.360922\ (25.02012)(0.9168207)}{0.9260023} = 108.03$$

assumed T = 100 months. Therefore,

$$\frac{T}{\mu_0} = \frac{100}{108.03} = 0.925$$

based on the estimated values given in Table 7; and for $\frac{T}{\mu_0} = 0.925$; we re-evaluated the minimum sample sizes as given in Table 8:

Table 8. Minimum sample sizes for the small electric carts

c		0	1	2	3	4	5	6	7	8	9	10
P^*	*0.75*	1	2	3	4	5	7	8	9	10	11	12
	0.90	1	2	4	5	6	7	8	9	10	11	13
	0.95	1	3	4	5	6	7	9	10	11	12	13
	0.99	2	3	5	6	7	8	9	10	12	13	14

From the new results, for example, corresponds to $P^* = 0.99$ and $\frac{T}{\mu_0} = 0.925$, we obtained $n = 14$ when c = 10, therefore, the optimal acceptance sampling plan will be ASP (*14*, *10*, 0.925). Based on the given data, this means that the manufacturing company can buy only 14 small electric carts in order to complete the manufacturing process within 100 hours, even if 10 out of these 14 electric carts have a mechanical failure within the manufacturing process time; with probability equal to 0.99.

7. Conclusion

In this article, we introduce the lifetime truncated acceptance-sampling plan for the power Lomax distribution. We present the table for the smallest sample size necessary to ensure a certain mean life of the test items. The operating characteristic function values and the associated manufacturer's risks are also discussed. The comparisons results with some other lifetime distribution showed that the proposed sampling plans based on PLxD are better and more efficient to be used when it applies. Therefore, the proposed plans can be used conveniently.

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