MATHEMATICAL MODEL OF TRAINING OPTIMIZATION IN THE COMPANY AS A COMPONENT OF INFORMATION TECHNOLOGY FOR DUAL EDUCATION SYSTEM

Ivan Hrod

https://orcid.org/0000-0002-0678-1456

e-mail: igrod@ukr.net

Ternopil Volodymyr Hnatiuk National Pedagogical University, Ternopil, Ukraine

Taras Lechachenko

https://orcid.org/0000-0003-1185-6448

e-mail: taras5a@ukr.net

Ternopil Ivan Puluj National Technical University, Ukraine, Ternopil

The development of models for the tasks of human resource management (HRM) is quite relevant. One of these tasks is to optimize the cost of training a student in a company, since a student in the dual training system is also an employee of the company. Maximizing the usefulness of training employees and minimizing the cost of them in the company is one of the necessary conditions for the effective operation of the enterprise.

The purpose of the study is to develop and analyze a model for optimizing the cost of training in a company as a component of information technology of a dual form of education.

In the paper analyzed the currently relevant methods of multi-criteria analysis of decision support, and as a result of the analysis, the Trapezoidal fuzzy - AHP method was motivated to be chosen as the most effective method for researching a model for determining the usefulness of professional competencies for a company. Trapezoidal fuzzy - AHP method

was used to parameterize the objective function with subsequent optimization using linear-integer programming. To analyze the obtained mathematical model, an algorithm was developed in the Python environment and a numerical experiment was carried out.

As a result of the study, a model for optimizing the cost of training in a company was developed and a numerical experiment was carried out to demonstrate the acceptability of its application. The developed model is an integral component of the information technology of the dual form of education.

The proposed training cost optimization model can be used in practice to maximize the usefulness of training in a company with a given budget and time constraints in a dual system.

Key words: Analytic Hierarchy Process, trapezoidal fuzzy numbers, linear programing, optimization, linear-integer programing

Problem statement. Given the constant changes and innovations in the era of Industry 4.0, training in a company should be integrated with academic education even at the stage of its receipt by a student at an educational institution. One of the ways of such integration is mediated by information technology, which is at the heart of the interaction of stakeholders. Thus, the optimization of student learning in a company in a dual education system is a task of human resource management, since a student, studying in parallel in a company, is an employee performing job duties, generating productivity for the total income of the company. Taking into account the problems of collaboration [1, 2] of a company and an educational institution in the dual training system and the significant costs of training a student in a company [3], the development of a mathematical model for optimizing the cost of training a student in a company requires separate consideration.

Optimization of training in a company is a component of information technology together with such components as: information portal, online learning diary, assessment technology, LMS (learning management system), database, is an essential tool in the era of the transformational economy of Industry 4.0. The training cost optimization component of information technology of dual education is a software service for decision support.

The need is due to the problems of collaboration of stakeholders in the dual system (domination of one of the stakeholders in the learning process [1,2] and reducing costs [3] for training a student in a company while maintaining the balance of training competencies according to the educational program).

The purpose and tasks of the article. The purpose of the study is to develop and analysis a model for optimizing the cost of training in a company as a component of information technology of a dual form of education.

To achieve this purpose, it is necessary: 1. Analyze decision support methods and choose the most relevant one for prioritizing competencies; 2. Calculate the cost of training in the company; 3. Construct objective function; 4. Parameterize the constraints of the optimization model; 5. Analyze optimization methods, choose the current one and use the chosen method to find the optimal solution.

The main material presentation. The first step of the optimization algorithm provides for the prioritization of student competencies (within the framework of the educational program of the specialty). Prioritization of competencies is a definition of their usefulness for the company, ranking from the least useful to the most necessary. In this work, to take into account the uncertainty when making a decision by an expert, we will use the AHP method with trapezoidal fuzzy numbers as in work [4]. Let us apply

in this work the AHP method using fuzzy trapezoidal numbers with the scale of the relative importance of alternatives proposed in work [4].

We will divide the competencies into three groups based on the similarity of program results and the class of competencies (general and professional). Let us determine the relative usefulness of the educational program competencies for the company using the Trapeziodal fuzzy AHP method according to the formula [5]:

$$U_i = \sum_{j=1}^{m} M_{g_i}^{j} \square \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{i} \right]^{-1} \tag{1}$$

where *i* – ranking object, *m* -aims.

In this paper, we propose to determine the usefulness of the educational program competencies as follows: Initially, the company's expert (instructor) divides professional competencies into groups based on the similarity of program results and a class of competencies. Using formula (1) and the ranking scale of fuzzy trapezoidal numbers presented in [4], the company expert and expert from educational institution determines the relative usefulness of competencies using pairwise comparison, thus forming the assessment matrix. Having received two matrices of pairwise comparison from the instructor and the expert from the educational institution, it is necessary to take into account the assigned grades from the educational institution. Taking into account the opposite goals of experts from the educational institution and the company and, accordingly, taking into account the polarity of the assigned assessments, the method of group expert assessment with averaging of values will biasedly reflect the weights of the pairwise comparison AHP (because the averaged arithmetic value takes into account the equal influence of the parties on the average result). As the training will be conducted in the company, the assessments of the prioritization of competencies by the educational institution should be considered as advisory and not on a par with the assessments of the

instructor. Grades assigned by the educational institution will reflect the recommendations for the company on preparation of competencies for the student in accordance with the educational program and the amount of workload in the educational institution. Using the following formula, we determine the weighted estimates received from the company and the educational institution (advisory):

$$
\begin{cases}
U_i = \sum_{j=1}^{m} M_{g_i}^{j} \cdot \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{i} - (\sum_{j=1}^{m} E_{q_i}^{j} - \sum_{j=1}^{m} M_{g_i}^{j}) \right]^{-1}, & \sum_{j=1}^{m} E_{q_i}^{j} > \sum_{j=1}^{m} M_{g_i}^{h} \\[4mm]
U_i = \sum_{j=1}^{m} M_{g_i}^{j} \cdot \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{i} + (\sum_{j=1}^{m} M_{g_i}^{j} - \sum_{j=1}^{m} E_{q_i}^{j}) \right]^{-1}, & \sum_{j=1}^{m} E_{q_i}^{j} < \sum_{j=1}^{m} M_{g_i}^{h}
\end{cases}
\tag{2}
$$

where $\sum_{j=1}^{m} M_{g_i}^{h}$ - fuzzy trapezoidal numbers assigned by the instructor

of the company;

$\sum_{j=1}^{m} E_{q_i}^{j}$ fuzzy trapezoidal numbers assigned by the expert of the

educational institution.

The next stage of the algorithm is to determine the components of the costs of a student's training. Each competency of a profession program is characterized by a different training cost.

Education in an educational institution is measured according to ECTS (European Community Course Credit Transfer System) a uniform learning assessment procedure. One ECTS credit equals 30 academic hours. Thus, the cost of preparation will be carried out on the basis of one hour of time.

The components of the objective function will reflect the competencies of the specialty program curriculum in which the student is studying in the dual form of education. We represent the objective function in the form:

$$
x_1 U_1 + x_2 U_2 + ... + x_{n-1} U_{n-1} + x_n U_n \to max
$$

(3)

where *x* is the amount of time allocated for training n competence, *U* is the utility of n competence

The optimization task is to maximize the usefulness of the set of competencies for the employer. The total utility of training a student-employee for a company is the product of the time allocated for training at the firm by the usefulness of each individual competency. The presented objective function reflects the combination of training time for a student-employee in a company in order to maximize its utility.

The general form of the task of optimizing training in the company will be as follows:

$$\begin{cases} x_1 U_1 + x_2 U_2 + ... + x_{n-1} U_{n-1} + x_n U_n \to max \\ C_1 x_1 + C_2 x_2 + C_3 x_3 ... C_{n-1} x_{n-1} + C_n x_n \leq F \\ 0.25kE \leq \sum_{i=1}^{n} x_i \leq 0.60kE \\ 1E < x_i \leq 9E, \quad i = \overline{1,n} \quad x_i \geq 0 \end{cases} \qquad (4)$$

where *C* is the cost of one hour training of *n* competence, *F-* is a company fund dedicated for training, *E* – credit ECTS, *k* - the number of study credits allocated for a year according to the Law of Ukraine. In the third line the limitations of the minimum and maximum duration of training the competence of a student-employee in a company in the planning period presented. In the fourth line presented restriction sets the minimum requirements for student training in each competency while not exceeding the specified time threshold.

The determination of the usefulness of the competencies was carried out by the instructors of the company in the field of cybersecurity Cyberoo of the educational program of the specialty 125 "Cybersecurity" of the dual form of education of the Ternopil Ivan Puluj National Technical University. The instructors of the company divided 9 competencies of the educational program into three classes according to the importance and educational

content of the competencies. In table 1 presented instructor's assessments adjusted using formula (2).

As can be seen from Table1, the results of adjusting the instructor's assessments compared to the group mean method (when the arithmetic mean of the instructor's and the expert's from the educational institution is taken) are more weighted with less variance in values from the assessments assigned only by the instructor, which are taken as the basis.

According to the results presented in the table, the order of objects ranking will be as follows: $\Phi K4 > 3K2 > \Phi K2 > \Phi K5 > 3K4 > \Phi K8 > \Phi K10 > \Phi K6 > 3K5$ - the results of ranking by the instructor. $3K2 > \Phi K2 > \Phi K4 > \Phi K5 > 3K4 > \Phi K10 > \Phi K8 > \Phi K6 > 3K5$ - the results of ranking by the instructor, taking into account the adjustment of the expert from the educational institution. $3K2 > \Phi K2 > \Phi K4 > \Phi K10 > \Phi K8 > \Phi K6 > 3K4 > \Phi K5 > 3K5$ - the results were obtained by the method of the weighted group average.

Table 1.

Comparison of the instructor's assessments adjusted using formula (2) with the group mean method

Group 1 (PC - professional competencies)					Average values
PC 2	0,22	0,32	0,64	0,92	0,52
Mean PC 2	0,23	0,32	0,59	0,80	0,48
PC 5	0,12	0,18	0,38	0,57	0,32
Mean PC 5	0,12	0,16	0,31	0,44	0,26
PC 10	0,15	0,20	0,32	0,45	0,28
Mean PC 10	0,17	0,24	0,45	0,62	0,37
Group 2 (PC - professional competencies)					
PC 4	0,27	0,34	0,52	0,65	0,45
Mean PC 4	0,26	0,33	0,51	0,64	0,43
PC 6	0,11	0,13	0,19	0,24	0,17

Mean PC 6	0,15	0,20	0,35	0,47	0,30
PC 8	0,13	0,19	0,34	0,45	0,28
Mean PC 8	0,18	0,24	0,40	0,52	0,33
Group 3 (OK - general competences)					
GC 2	0,39	0,51	0,87	1,05	0,70
Mean GC 2	0,38	0,49	0,78	0,95	0,65
GC 4	0,17	0,22	0,36	0,44	0,30
Mean GC 4	0,15	0,20	0,33	0,42	0,27
GC 5	0,08	0,09	0,13	0,16	0,12
Mean GC 5	0,08	0,09	0,13	0,17	0,12

After the instructor of the company determined the relative weights of the pairwise comparison by the AHP method of the groups into which the competencies were divided, the estimates taking into account the weights of the first, second and third groups, respectively, 0.45; 0.41; 0.12 will be:

$$ФК2 > ФК4 > ФК5 > ФК10 > ФК8 > ЗК2 > ФК6 > ЗК4 > ЗК5$$

In Table 2 present the average values of the usefulness of competencies for an employer.

Table 2.

The estimates obtained taking into account the weights of the competency groups.

PC 2	23,84	PC 10	12,88	PC 6	7,06
PC 4	18,56	PC 8	11,49	GC 4	3,62
PC 5	14,41	GC 2	8,51	GC 5	1,44

Having received the value of the relative usefulness of the competencies for the company and, accordingly, calculating the cost of training each of them in table. 3, we will construct optimization problem.

Table 3.

The Cost of competencies training (UAH / hour) for the company

PC 2	170	PC 10	170	PC 6	160
PC 4	165	PC 8	155	GC 4	150
PC 5	145	GC 2	160	GC 5	140

To simplify the presentation of the optimization problem, we represent $x_2=x_1$, $x_4=x_2$, $x_5=x_3$, $x_{10}=x_4$, $x_8=x_5$, $x_{2.2}=x_6$, $x_6=x_7$, $x_{4.4}=x_8$, $x_{5.5}=x_9$.

This mathematical model belongs to the class of linear integer programming problems, taking into account the specifics of the constraints and conditions for the model implementation, an additional analysis of linear optimization methods was carried out. Since curricula are formed in accordance with the allocation of a certain number of academic credits for each discipline, the multiplicity of which is 0.5 and 15 hours, respectively. The solution to this problem will be integer values that are multiples of 15 hours. Taking this feature into account, an algorithm of a partially integer linear programming method in the Python software environment was compiled to solve the problem.

The objective function and limitations of the training cost optimization problem will be as follows:

$$\begin{cases} x_1 23,84 + x_2 18,56 + x_3 14,41 + x_4 12,88 + x_5 11,49 + \\ +x_6 8,56 + x_7 7,06 + x_8 3,62 + x_9 1,44 \to max \\ x_1 85 + x_2 82 + x_3 73 + x_4 85 + x_5 77 + \\ +x_6 80 + x_7 80 + x_8 75 + x_9 70 \leq 59000 \\ 450 \leq \sum_{i=1}^{n} x_i \leq 1080 \\ 30 < x_i \leq 270, \quad i = \overline{1,n} \quad x_i \geq 0 \end{cases}$$

where i- the competence number

The task of optimization is to maximize the total usefulness of competencies and minimize the total cost of training the competencies of educational program. Limitations of the optimization task are not to exceed the total duration of the training in the limits 1080 hours and compliance

with the total minimum training time of 450 hours. In addition, the training time of one competency must also be within not less than 30 and not more than 270 hours.

As a result of applying the optimization algorithm in the Python software environment, we will get the following result: Cyberoo will spend 58 830 UAH and 720 hours for the annual training of one student worker, of which for each competency: x_2=270 h, x_4=240 h, x_5=30 h, x_{10}=30 h, x_8=30 h, $x_{2.2}$=30 h, x_6=30 h , $x_{4.4}$=30 h , $x_{5.5}$=30 h; and reaches a maximum utility effect of 12,675 utility.

Thus, using the constructed model for optimizing the cost of training, Cyberoo maximizes the total utility from a variety of competencies within the established training budget and time constraints.

Conclusion. In the paper a model is built and analyzed for the problem of optimizing the cost of training in a company in a dual system with parallel training and work of a student-employee in a company and an educational institution.

The practical value of the built model is its use by companies involved in dual education to minimize their costs and maximize the benefits fom use of many combinations of competencies. This model has been tested in practice at Cyberoo, the results of which indicate its effectiveness. Thus, the model can be used as part of the information technology of the dual form of education in the form of a web portal application of dual education as a tool to influence the company in the process of organizing training, minimizing its cost and maximizing utility.

The prospect for further research is planned to separately investigate the model for optimizing the cost of training while minimizing the cost and maximizing its utility.

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