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FACULTAD DE CIENCIAS HUMANAS, SOCIALES Y DE LA EDUCACIÓN
GIZA, GIZARTE ET HEZKUNTZA ZIENTZIEN FAKULTATEA

Graduado o Graduada en Maestro en Educación Primaria
Lehen Hezkuntzako Irakaslean Graduatua

Trabajo Fin de Grado
Gradu Bukaerako Lana

Relational Understanding as Inclusion Tool for Children with Math Learning Disabilities

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Campo/Arloa: Matemáticas **Mayo, 2021**

Resumen

El presente proyecto resalta la importancia de enseñar las operaciones matemáticas de la suma y la resta desde el entendimiento relacional, para permitir al alumnado con dificultades de aprendizaje adquirir una comprensión matemática significativa, huyendo así de la memorización de múltiples procedimientos. Esto les permite concebir las matemáticas como una herramienta que les acerca y facilita la comprensión de la vida que les rodea. Concretamente en nuestro caso, su aplicación les permite reconocer y entender las operaciones de la suma y la resta en diferentes situaciones o contextos de su vida cotidiana.

Se propone la implantación de un taller matemático en un colegio público de Pamplona para un alumno de sexto curso desde el aula externa de PT. Este taller, basado en el juego, pretende trabajar la aritmética de las matemáticas haciendo uso de materiales manipulativos como herramienta de aprendizaje para que el alumno con dificultades posea unos recursos que le permitan desarrollar sus habilidades matemáticas y construir conocimiento, de modo que cree y desarrolle sus propios procedimientos de resolución de problemas. Por último, analizamos los beneficios de esta propuesta y comprobamos la existencia de una progresión en el aprendizaje del alumno.

Palabras clave: aritmética; dificultades de aprendizaje; entendimiento relacional; situaciones reales aritméticas.

Abstract

The following project emphasizes the importance of teaching the mathematical operations of addition and subtraction from a relational understanding in order to enable students with learning disabilities (LD) to acquire a meaningful mathematical comprehension; thereby avoiding the memorisation of multiple procedures. This allows them to conceive mathematics as a tool that brings them closer and facilitates the understanding of the life around them. Specifically in our case, they can recognise and understand the operations of addition and subtraction in different situations or contexts of their daily lives throughout the application of mathematics.

We propose the implementation of a mathematics workshop in a public school in Pamplona for a sixth-grader from the external special education (SPED) classroom. This workshop, based on play, aims to work on arithmetic using manipulative materials as learning tool. This enables the learners with difficulties to possess resources that allow them to develop their mathematical skills and build knowledge that will help them creating and developing their own problem-solving procedures. Lastly, we analyse the benefits of this proposal and verify the existence of a progression in the student's learning.

Keywords: arithmetic; learning disabilities (LD); relational understanding; actual arithmetic situations.

AGRADECIMIENTOS

Quiero agradecer primero, a la universidad por la formación impartida durante estos años; a mi familia, pareja y amigos por haber sido un apoyo fundamental de motivación para seguir adelante. Y en especial agradezco a mi tutora de TFG : Raquel García Catalán, por haberse mostrado siempre disponible, alentadora y cercana. Valoro sobre todo su guía a lo largo del proceso, su capacidad de trasmisión de tranquilidad, y el contagio por la ilusión y amor hacia las matemáticas.

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INTRODUCCIÓN

Las matemáticas han sido siempre consideradas una parte fundamental de nuestro aprendizaje, pero no sólo durante nuestros estudios, sino para facilitarnos pensar y razonar, y para ser capaces de enfrentarnos a los diferentes retos de la vida con éxito. Esta ciencia nos abre la mente y nos es útil para la vida, ya que nos ayuda en la resolución de problemas, y el éxito frente al reto nos lleva a sentir una gran satisfacción. Esto produce un empoderamiento en la persona y nos lleva a sentirnos más felices, por lo que podemos decir que las matemáticas impactan favorablemente en nuestro estado de ánimo y nos facilitan la vida que nos rodea.

Sin embargo, y cada vez más, en la escuela se ve a muchos niños que presentan cierto rechazo hacia ellas y argumentan que las matemáticas les producen frustración y enfado. El haber desligado las matemáticas de la vida real, sumado a una enseñanza puramente memorística de ellas resulta en una incapacidad por parte de los niños de ver el potencial que tienen para facilitarles la vida y ayudarles a comprenderla a través de su aplicación. Esta frustración además es más notable en niños que experimentan dificultades de aprendizaje. Si estas ciencias no se explican desde una perspectiva que incite a pensar y ayude al alumno a desarrollar sus competencias matemáticas, entonces quizás algo está fallando en el método de enseñanza y aprendizaje.

La observación de este fenómeno en los colegios como una constante que se repite, nos llevó a investigar acerca de diferentes formas de acercar las matemáticas a los niños de una manera más efectiva y personalizada, lo que nos lleva a descubrir la enseñanza basada en el entendimiento relacional, que permite al alumno construir su conocimiento y habilidades de razonamiento, en el cual el niño primero comprende el porqué antes que el cómo realizar ciertos procedimientos, para que así las consigan comprender y aplicar a su vida.

La inclinación por realizar este trabajo tiene varios orígenes. Por una parte una natural predisposición y vocación por enseñar a niños con dificultades. Por otra parte, el haber participado en un seminario del Proyecto Europeo Matemático 'ANFoMAM', donde desarrollan diversos talleres para acercar de una manera eficaz y lúdica las matemáticas a los niños con Síndrome Down, haciendo que incluso llegasen a percibir estas ciencias como algo divertido y útil. Por último, el haber realizado la mención de pedagogía terapéutica (PT) cursada durante la carrera. Todo ello nos ha llevado a plantearnos cómo podíamos hacer para acercar las matemáticas a niños con dificultades de aprendizaje dentro del entorno escolar permitiéndoles así un aprendizaje significativo. Esto nos lleva a nuestros objetivos principales que son estudiar si un método de enseñanza y aprendizaje basado en el entendimiento relacional puede ayudar a estos niños con dificultades desde el aula de PT para alcanzar el nivel de sus compañeros y así reducir el desfase académico existente entre ellos promoviendo la inclusión.

Para ello decidimos implantar unos talleres matemáticos que trabajarán los conceptos aritméticos de la suma y la resta, los cuales eran los siguientes conocimientos a trabajar con un niño de sexto de primaria desde el aula externa de PT. Esta propuesta didáctica se basa en aprender desde el juego y la propia experiencia, a través de materiales manipulativos funcionales, para que la primera toma de contacto y reflexión acerca de las matemáticas sea desde algo tangible y palpable a sus sentidos, para luego poder ir progresivamente pasando a la abstracción y simbolismo de esta ciencia exacta.

En cuanto a la estructura que este trabajo sigue, se presenta en primer lugar un marco teórico dividido en dos capítulos, en los que se abordan conceptos matemáticos, como una breve evolución de los sistemas numéricos a lo largo de la historia y su respectiva aplicación en los diferentes algoritmos de suma y resta, así como diferentes estrategias y técnicas de enseñanza relacional cuyo objetivo es conseguir una enseñanza de las matemáticas eficaz e integrada en la vida del alumnado. En segundo lugar, desarrollamos una propuesta didáctica en formato de talleres como herramienta de aprendizaje para los niños con dificultades promoviendo el aprendizaje relacional a través de los materiales manipulativos funcionales. Tras esto, analizamos los resultados obtenidos durante los meses de implementación del taller y observamos cambios significativos del alumno en cuanto a la actitud para afrontar las matemáticas y la existencia de una progresión de aprendizaje siendo ambos cambios positivos. Por último, en las conclusiones, se hará hincapié en la futura posibilidad de implantar el taller matemático dentro del aula reglada para que el beneficio de aprender las matemáticas jugando se traslade a todo el grupo, permitiendo así que sean atendidos los diferentes niveles de desarrollo del aprendizaje matemático dentro de una misma clase. Esto se haría a través de una variedad de rincones respondiendo a los diferentes niveles de abstracción matemática: manipulativo, pictórico y simbólico, para que así el alumnado pueda ir rotando de manera progresiva de uno a otro una vez completado el respectivo nivel de abstracción de un concepto. De esta manera todos los alumnos trabajan lo mismo, aprenden desde la experiencia y se benefician de un tipo de aprendizaje más significativo y lúdico creando una atmósfera positiva para afrontar las matemáticas y desarrollando unas competencias que les ayudarán a enfrentar retos que se encuentren en su vida diaria y futura.

1. THEORETICAL FRAMEWORK

1.1 Numeral systems

1.1.1 Evolution of the Representations of Natural Numbers and its application for children

Throughout history many different civilizations have developed their own methods and rules for representing natural numbers; in other words, as Ore (1988) mentions "all the various forms of human culture and human society, even the most rudimentary types, seem to require some concept of number and some process for counting" (p.1). Nowadays, we use the decimal positional system (DPS) for representing numbers. However, it should be stressed that children do not only use DPS when they are learning, but depending on their stage of learning and use of the number some other systems suit their understanding, reasoning and skills better. In other words, they use several ones at different times in their learning, and therefore, it is necessary to make a brief introduction of the most important ones in order to emphasise the contributions of each one for the learners. Being these the tally system, the additive and the multiplicative systems, and lastly, the one used by our society: the decimal positional system.

1. *Unary Numeral System or Tally System*

In the early days, humans used fingers to count "when ten fingers were not adequate, stones, pebbles, or sticks were used to indicate values" (Das & Lanjewar, 2012, p.236). Primitive societies did not have the necessity to represent very large numbers, they just needed to represent cardinals of sets surrounding them to keep track of their elements. For example, if they wanted to know the exact number of cows they had in a herd, they would draw a symbol such as a tally stick | to represent each cow in the herd; for instance, eight animals are represented as ||||||||. Hence, the only rule they had for the representation of natural numbers was drawing a tally symbol for each of the units, corresponding later the total amount of animals to the quantity of tally sticks (number of times this tally/unit was repeated).

However, when counting large numbers this type of representation unit by unit could be very tiring and confusing. Imagine reading, for example, the number 36 as ||||||||||||||||||||||||||||||||||||. At a single glance, they could not deduce the amount they were talking about, requiring them to count them one by one until you reached the total. This becomes unsustainable and naturally leads to develop a more advanced method with an additive approach, grouping the sticks in sets of a fixed quantity (ten in our case), representing then the same number 36 as |||||||||| |||||||||| |||||||||| ||||||. The reading now is easier, but this would fail again when representing very large numbers as 369.

This simple grouping tally system is very powerful when working with small numbers. Besides, it is a very intuitive method, and happens to be incredibly useful for discovering the addition and subtraction algorithms: children can observe easily that if adding up means putting things together, this traduces in the tally system as putting together all the sticks; and if subtracting means diving the group into two subgroups, taking away one of them and see what remains, just the same has to be done with the tallies. Even more, the regrouping becomes natural. Math & Learning Videos 4 Kids (2014) created different videos for kids to understand the addition and Math & Learning Videos 4 Kids (2013) for the subtraction using the tally system.

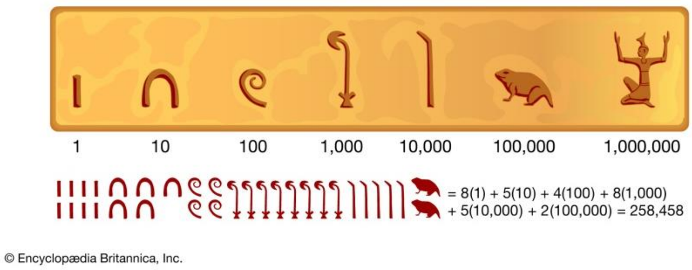
2. *Additive Multibase System.*

Because of the low efficient application of the tally system with large numbers, other numeral systems appeared such as the ancient Egyptian system. It inspired the Greek Alphabetic Numeral System and the Roman Numeral System in which they use as symbols their alphabet letters to represent different numerals.

In these systems, each symbol represents a fixed quantity. For instance, if a tally stick | represents a unit and the symbol ♦ replaces 10 tally sticks (10 units), then the number thirteen is represented as ♦|||. However, this enables the representation of the number 99 maximum because each time you have 10 things a new symbol needs to be used for each power of 10. For example, in number 145, a new symbol needs to be created for representing the 100 quantity (i.e. * symbol), then after the creating of the new symbol, the number 245 is represented as **♦♦♦♦|||||. For this reason, Egyptians invented different symbols for 10 powers to represent up to the natural number of 9.999.999 using only seven symbols as shown in figure 1.

Figure 1.

Ancient Egyptian numeral system



Note: Retrieved from *Ancient Egyptian hieroglygraphic numeral system [Photography]*, by Smith and LeVeque, 2019, Encyclopedia Britannica.

This system corresponds with the one learners work with when using manipulative materials such as the multi-base arithmetic blocks also known as Dienes blocks (block as the Thousands, flat: Hundreds, rods: Tens, little cubes: Ones). The additive along with the tally system are the ones that work better for children when they are learning for the first time the addition and subtraction algorithms (the Dienes blocks also are very helpful for understanding the multiplication and division algorithms or representing the measure units). This is the reason for the combination of both in the learning proposal when using the toothpicks.

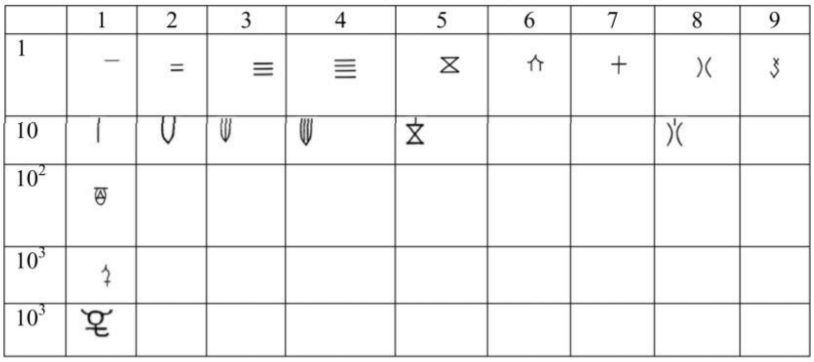
3. Multiplicative System

In this system there are two sets of symbols: one for different powers of 10 and other one for the quantities from 1 to 9. For example, taking as set of symbols for the powers of 10 {|=1,♦=10,#=100} and as set of symbols representing quantities from 1 to 9 as: {a,b,c,d,e,f,g,h,i} respectively, we would represent 35 as c♦ e| (3*10+5*1), and 268 as b# f♦ h| (2*100+6*10+8*1=200+60+8).

This method of natural numbers representations was used by the Chinese Traditional civilization having its origin with Shang Dynasty (Subedi, 2017) and developing the symbols shown in figure 2. Whereas in previous systems the addition was the operation undertaken to represent the different numbers, now in the multiplicative system, both the multiplication and addition take place.

Figure 2.

Chinese (Shang) numeral system



Note: Retrieved from *Table 5: Shang Numeral System [Photography]* (p.12), by N. Subedi, 2017, JMC Research Journal.

Children use this system when decomposing large numbers by orders. For instance, the number 1.324 as 1.000+300+20+4 representing each order all the repeated times (additive decomposition); or as 1 time 1000 + 3 times 100 + 2 times 10 + 4 times 1 (1*1000+3*100+2*10+4*1), in which the number graphically represented is to have for each order a value next to it indicating the times it is being repeated (multiplicative approach).

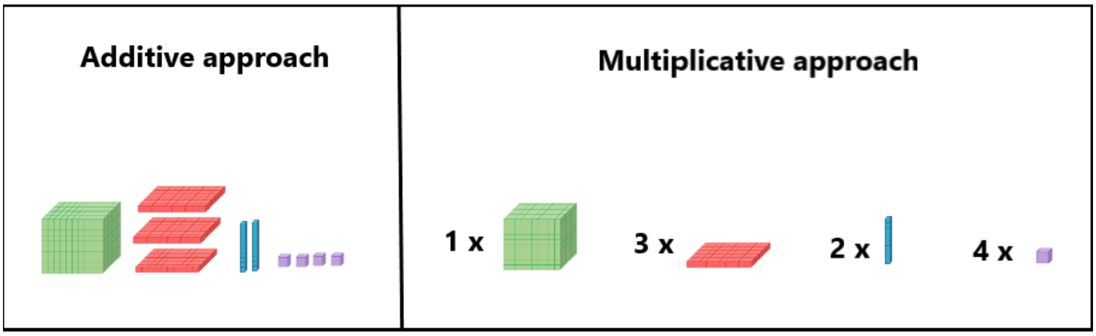
This is also the system that children use when multiplicating with the dienes blocks and all we practise whenever we pronounce a number, as all oral languages use a multiplicative system for building the words related to any number.

The transition from an additive to a multiplicative method is also useful for children, because they start representing a multiplication as a reiterated addition. For example, 5 times 2 = 2+2+2+2+2, which equals to having 5 little cubes. Gradually, they pass to understand that instead of writing repeated times a number, they can indicate with a number (values 1 to 9 in base 10), before the symbol '2', the times it is been repeated; having then, 5 times 2 as 5x2. Therefore, they have understood that a multiplication n x m is a repeated addition of m +m+...+m, n times.

In the following figure 3, both approaches (additive and multiplicative) can be seen represented pictorially to see the difference between them: the additive repeats an order symbol as many times as necessary until getting the quantity indicated, and the multiplicative uses a representative symbol of each order and writes a number (value) next to it, indicating the number of times it should be repeated.

Figure 3.

Representation of the natural number 1.324 in both approaches



4. *Positional system*

The previous system still involves a problem: for each power of ten a new symbol has to be created; nevertheless, this is solved with the positional numeral system, being the decimal positional system the one used nowadays (base 10). Instead of having a symbol for each of the powers, the concept of placing a number within a specific position comes into play. Thus, as its name indicates, a position is added as a variable of the number.

Here, we do not need two sets of symbols, but just the symbols for the values from 1 to 9, and the special symbol 0 denoting “nothing”. In addition, the real value of a symbol changes depending on the position it occupies in the writing of the number. It is not the same having the number ’23’ placing the 2 on the Tens and the 3 on the Ones, or having the same symbols in a different position: ‘32’ (3 Tens and 2 Ones). Children tend to think that an ‘8’ means eight regardless of its position; if it is in the Tens means 8 or Hundreds: 80 (Kamii & Dominick, 1998). For this reason, the position that each digit of a number takes is so important. These positions are not arbitrary, but organised by orders from right to left in an increasing order: Thousands, Hundreds, Tens, Ones. This is a problem for many of us, because we cannot begin the reading of a number until we realise which is the biggest order, and without external help this can only be done counting the positions from the last right digit. For example, try to recognize the number 234156324783219 (and now 234$_2$156.324$_1$783.219).

This system has a further advantage: it allows us to represent also fractions of a unit, and so it can be extended for writing all rational numbers (in their decimated expression).

This positional system has been developed over different societies but its origin is attributed to the Babylonians in 3000 BC. There have been found numbers carved on Babylonian cuneiform tablets which show a Sexagesimal Positional-Additional System (the positions were taken with base 60, although there were only two symbols standing for Units and Tens). However, their numerals had no sign for zero, leading to the emergence of confusions when reading the numbers. Nothing was expressing a "void or missing class; for instance, 204 is different from 24" (Ore, 1988, p.16), being this the reason for the appearance of more advanced positional systems. In our place-value system, the zero symbol acts then as a place holder; for example with the number 307, it indicates the position of the tens' place and that it is worth nothing while the "3 says three hundred because of the position it is in" (p.77). Then, the function of the zeros is to make the position it occupies clearer and indicate an absence (Haylock & Manning, 2014).

The natural numbers writing, as we use them now, is known as the Hindu-Arabic numeral system. Its etymology stems, as the studies point out, from its origins to India and the Arabs to be the responsible for its transmission to Europe.

As Ore (1988) explains, an enormous benefit of using this type of system is that the execution of operations in the place-value system in comparison with others becomes very fast and much easier (not simpler), although learners lose the manipulative and visual support that the additive or multiplicative systems provide when operating with classic algorithms.

It is very important to explain patiently to children the involvement of the place value in our system using for instance flashcards for the number formation. For example, the number 269 using the card 200, 60 and 9, they see how each of the digits of the number has a different value depending on its position. For children to be able to put numbers in order requires them to have a great understanding of the basic principles of place value (Haylock & Manning, 2014). Despite the idea that it seems obvious, many times the misunderstanding of the positional system is the origin of experimenting problems when executing additions and subtractions. This occurs because they have never worked with materials that facilitate their learning when having to understand, for example, the regrouping. By using an additive-tally mixture system based on the handling of toothpicks, the children intuitively understand those concepts and gradually acquire them in a symbolic level of abstraction.

1.1.2 Addition and subtraction algorithms worked from different numeral systems

The different numeral systems explained above interfere directly in the way of teaching the arithmetical algorithms of addition and subtraction leading to the appearance of different types of methods to solve algorithms such as the traditional, the ABN, or the relational understanding methods.

Depending on the method it is being used, children use some solving strategies established or create their own procedures, being the latter the aim nowadays for mathematics teaching. Narode, Board & Davenport (1993) believe that:

> ...by encouraging students to use only one method (algorithmic) to solve problems, they lose some of their capacity for flexible and creative thought. They become less willing to attempt problems in alternative ways, and they become afraid to take risks. Furthermore, there is a high probability that the students will lose conceptual knowledge in the process of gaining procedural knowledge. (p.260)

Teaching conventional algorithms is a regular practice widespread in primary classrooms. However, there are both advantages and disadvantages when teaching these algorithms in the early years of primary school. It can be summarised that algorithms are automatic and fast because they have a straight route to the answer; therefore, they can be instructive. This leads teachers to teach them because they are easy to manage and assess; and lastly, the primary mathematics curriculum has counted with them for many years being commonly taught worldwide (Clarke, 2005). However, remember that the fastest algorithm is: 'take the calculator, write the first number, write the operation symbol, write the second number, press the = symbol' as seen in Ramos-Alonso (2019).

Nevertheless, one should not forget that their development has taken centuries, so we must not limit ourselves to teaching only the conventional method, but rather let our learners (with the help of manipulative materials) discover the process to be followed and enable them to create their own strategies. Teachers need to remember that there is not only one method to calculate but different routes to the answer involving always an understanding of the process. Thus, Clarke (2005) claims that learner's invented algorithms should be accepted by teachers as alternatives to the conventional solving strategy as long as children are encouraged to consider whether these procedures are efficient enough to be used regularly without compromising loss of time, valid mathematically, and generalizable to be applicable to the whole range of possibilities of this type of algorithm.

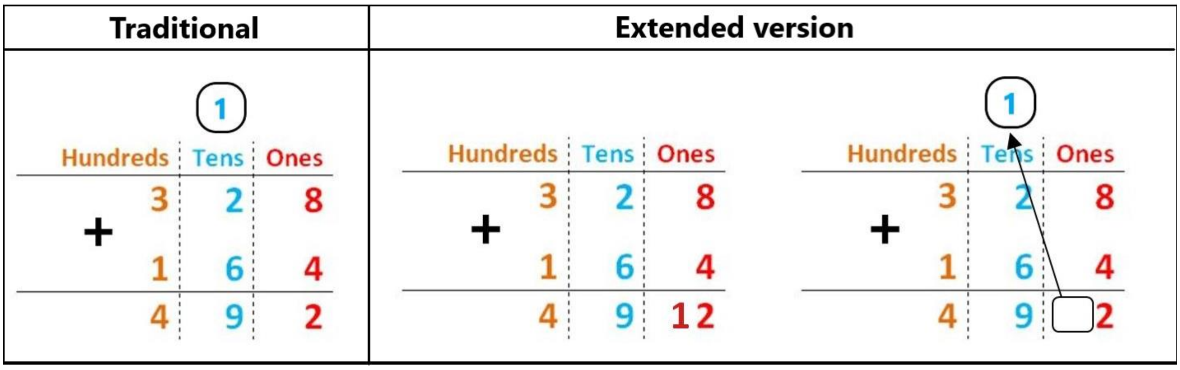
1. *Traditional Addition and Subtraction Algorithms*

The traditional method is the most common to execute the arithmetical operations of addition and subtraction. They lead to a fast performance of the operations; however, they reinforce the idea of explaining to children how to execute before having understood the reason for being performed in that particular way. This ends up with learners having memorized procedures that will be forgotten because children have not grasped why, sometimes, they have to regroup in an addition or open a group of the next upper order in the subtraction. Of course, they have been taught that they have to write a '1' each time they get 10, but they wonder about the meaning of the phenomenon 'carrying a ten' in addition, or 'lending a ten' in subtraction.

On one hand, the procedure followed to perform a traditional addition algorithm is firstly lining up both addends vertically matching the place values: having then, the Ones under the Ones, the Tens under the Tens and so on. Secondly, each order values are added up to get the total, starting always from the Ones order (moving right-to-left). It should be noted that if when executing this combination any of the order gets bigger than 10, then, we carry the number 10 and place it in the next upper order as a 1. The rule followed in here is that each time the child gets 10 in a column; he has to replace it in the next upper level. This is visible in figure 4:

Figure 4.

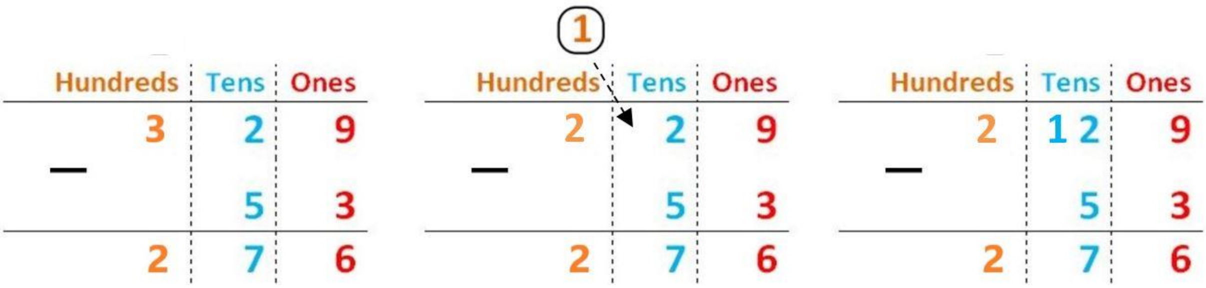
Traditional Addition Algorithm (with regrouping) vs extended version:



On the other hand, in the subtraction procedure, the first goal is determining the minuend as the total and the subtrahend as the part taken away to see what remains (the rest). Therefore, the subtrahend is placed under the minuend by matching the Ones under the Ones, Tens with Tens, etc. Secondly, subtract the subtrahend units from the minuend units following the same process with all the orders, moving from right to left. If the minuend value in a category is smaller than the one on the subtrahend, as seen in figure 5, then we ask the next order for one and place it on the order where it is needed as a 10 (remember to record it by writing a 1 on the Hundreds).

Figure 5.

Extended version of the Traditional Lending Subtraction Algorithm (with regrouping)



Children find it more natural and instinctive to execute and understand the subtraction operation with regrouping throughout the lending algorithm rather than the borrowing model that is much more difficult to be understood, as it uses more arithmetical properties. Canto-López (s.f.) argues that the procedure based on this method is absurd because children do not understand the process and limit to follow the rules and the mechanical way to solve it. As it can be seen, these algorithms tend to the mechanization of the process and when children forget one step they do not know how to continue, they block; thus, teaching conventional algorithms to primary learners has its dangers. Furthermore, Clarke (2005) agrees with Ramos-Alonso (2019) that they might no longer be the most efficient because now calculators can be used to solve these calculations and even an estimation method is more appropriate. After all, in life, most calculations require only to be estimated, which encourages the children's application number sense. Children may develop for subtraction a strategy using the understanding of place value and integer numbers. For example, "for 354 – 278, a child might use an algorithm that involves the following steps: 300 – 200 = 100; 50- 70 is -20 (take 20); 4 -8 is -4 (take 4); so the answer is 100, then take 20, then take 4, giving 76" (Clarke, 2005, p.94).

There are different reasons to say that algorithms are harmful because "they encourage children to give up their own thinking and they unteach place value, thereby preventing children from developing number sense" (Kamii & Dominick, 1998, p.135). A study showed that children with no previous teaching on algorithms got more correct answers by carrying their own thinking and reinforced their place value knowledge than those who were instructed (Kamii, 1994, as cited in Kamii & Dominick, 1998). For this reason, the procedures to execute an algorithm should be discovered by the primary learners, but always having the teacher as a guide to help them construct their own mathematical reasoning. Colburn (1912) agrees:

> The learner should never be told directly how to perform any operation in arithmetic… Nothing gives scholars so much confidence in their own powers and stimulates them so much to use their own efforts as to allow them to pursue their own methods and to encourage them in them. (p.463)

2. *Algorithms Based on Numbers (ABN)*

Traditional algorithms worked because they were designed for an economic purpose in the markets, but not for academics. Conventional algorithms impact the children making them have a negative attitude towards mathematics and experience frustration because they require them to memorize the process. This ends up being difficult for pupils because they "do not understand anything and at no time know what they do" (Canto-López, s.f., p.5).

Unlike the traditional method, in which only one solution is possible, the open calculation method based on numbers (ABN) as Canto-López (s.f.) describes, is an "open and personalized algorithm because students can solve operations in multiple ways, each student solves operations easily, comprehensibly and [...] children acquire a real number and amount concept" (p.2).

Martínez-Montero (2001) published, in his first article, a new method that makes possible teaching mathematics in a different way. This methodology poses a new approach to operate with numbers. Later, it is introduced as a solution for teaching mathematics because it entails a method for problem-solving and mental arithmetic that "promotes real learning" (Canto-López, s.f., p.12), fostering the principles of calculation with real problem solving and calculation using estimation. This is a methodology followed and used by teachers for their teaching nowadays in their classrooms (Martínez-Montero, 2000, 2010, 2011). It adapts to the student learning pace taking into account his learning moment (when children experiment difficulties the learning is slowed down) and the learning curriculum amount (eliminating contents if needed for the children with learning disabilities 'LD'). For this reason, this methodology could prevent the struggles learners might have and solve them whenever they appear. It has been shown that because it allows children to create or use their own procedures when calculating, it creates a favourable attitude in the child towards learning mathematics (Canto-López, s.f.) and that children performing this method achieve better results than using the conventional approach widespread in Spanish schools (Cerda et al., 2018).

3. Relational Addition and Subtraction algorithms based on the Tally-Additive system

In this approach, manipulative materials with base ten are key to help children construct their mathematical knowledge and abilities, enabling the learner to understand first mathematical operations from a concrete perspective and gradually transfer it to a symbolic level of abstraction. The pupil then sets the pace of work because once he understands how to use the materials, little by little, does through the different acquisition steps.

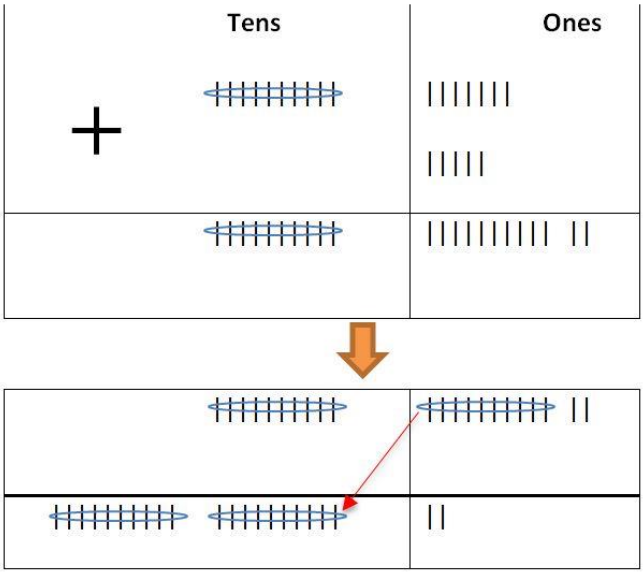
This method is based on Skemp's (2006) idea of relational understanding (also developed by Hazekamp (2011)), where the process needs to be understood before knowing the execution procedure. This enables children to be later capable of deducing and recalling the processes whenever they are needed.

The procedure, when teaching mathematics operations with this method, starts with the only rule that children need to keep in mind: whenever they have ten elements (in our case they are sticks) they have to put an elastic around them and place this mini-group in its corresponding place (the Tens order); if they are 10 mini-groups into the Hundreds (making a super-group). Soon, they identify the orders of a number and represent them making different collections of Hundreds, Tens and Ones (Fuson et al., 1997).

Once children have grasped the representation of a number with the sticks, the next step is operating with them in addition and subtraction. At this point, teachers can ask children to add two sets of Ones reminding them to follow the regrouping rule, for example, if they add the sets: ||||| + ||||||| equals a total of 12 sticks. Therefore, children get the solution both by counting all the sticks, but also by representing the quantity with one group of Tens and two Ones groups, which introduces very naturally and intuitively the regrouping concept in addition. A representation with sticks can be seen in figure 6.

Figure 6.

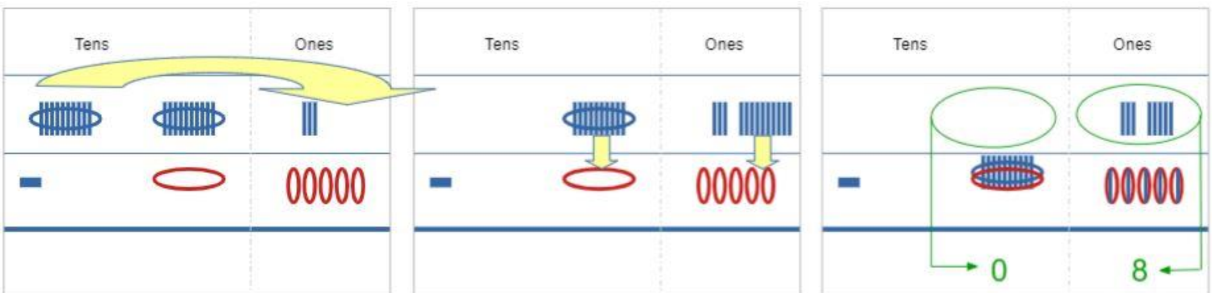
Addition with regrouping performance (17+5) using the tally-additive approach



When subtracting pupils are asked to represent with sticks the biggest number (minuend) of the subtraction algorithm and taking away to the boxes the number of sticks the subtrahend indicates to see what remains. Children soon notice that the subtrahend cannot be represented with sticks but as holes to be filled because it indicates the part to take away. However, sooner than later, they come to the step in which they do not have enough sticks to bring to the boxes as the subtrahend indicates, being then the moment to introduce the subtraction with regrouping by using the lending algorithm. The learner is asked that if he does not have enough sticks on the Ones (as seen in figure 7): where does he have more sticks? Can he use them? And then, where can he get them from? This makes the pupil realise that on the Tens he has more sticks but in groups and starts wondering if he can do something with those 2 groups. The teacher can give him a hint and remember that if in addition each time they had ten elements they grouped them with an elastic and placed them in the next upper order, perhaps he could do the opposite when subtracting: open a group of the Tens in this case and place the 10 elements on the Ones to have enough sticks to bring to the holes (symbol **O**) as the subtrahend determines.

Figure 7.

Subtraction with regrouping (23-15) using the tally-additive system approach



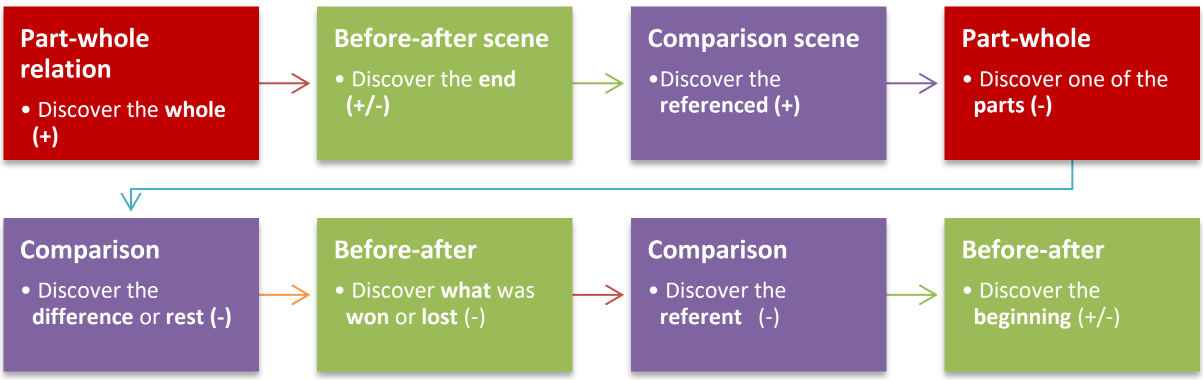
Teachers need to keep in mind that each of the operations needs to be immersed in a story or word-problem to provide meaning to the operation; being better if these contexts or situations are close to the children real-life scenarios so that they realise how and why mathematics helps them to understand their life. For example, figure 6 could be introduced as: *'if your parents give you 17 euros and your uncles 5. How much do you have in total?'*; and figure 7 as: *'if you want to buy a new suit in Fortnite that costs 23 coins and you have 15 coins. Can you buy the suit? And if not, how many coins do you need to buy the suit?'*

These situations can be categorised according to the three possible relations that can appear in addition and subtraction problems: part-whole (involves three magnitudes: two disjoint parts and their union); before-after (involves one single magnitude that changes over time creating at least three moments: before, then and after); and additive comparison relation (involves two magnitudes that are compared in absolute value).

Within these relations, different possible situations may occur. They are ordered regarding their progressive difficulty (steps children undertake) on its identification, understating and acquisition (Ban Har, 2014; Fuson et al., 2011); thus, detailed the unknown aspects on each of the situations as follows in figure 8.

Figure 8.

Steps undertaken by children when contextualising the problem in a situation



By immersing the operations in situations, teachers help children to identify problems in their daily life but also being capable of formulating them themselves.

1.2 Didactical Frame

1.2.1 Mathematics for primary learners

Mathematics is more than just a simple subject or a bunch of instruments to learn, it is an area that encourages children to develop skills and knowledge to face and connect with their reality, enabling them to understand it. This phenomena, many times, occurs when this science is taught throughout games and challenges magnifying their world´s awareness (Gil-Clemente, 2021). Thus, mathematics is more than just something used at school: it prepares children for facing challenges. Taylor (2005) defines a challenge as:

> not only an important component of the learning process but also a vital skill for life. People are confronted with challenging situations each day and need to deal with them. Fortunately, the processes in solving mathematics challenges (abstract or otherwise) involve certain types of reasoning which generalise to solving challenges encountered in everyday life. Mathematics has a vital role in the classroom not only because of the direct application of the syllabus material but because of the reasoning processes the student can develop. (p. 2)

This causes mathematics to become more important in school making its proper and adequate teaching critical. Many times we encounter children experiencing a negative attitude towards the area, even adults, and a wrong mathematics instruction, based on memorising and not making them see that they are a tool that brings us closer to life, may be one of the reasons for this coolness. Children might experience frustration and hate blocks when they feel incapable of solving a problem and it is not understood, driving them into an avoiding attitude (de Guzmán, 1991).

In her prologue, Millán-Gasca argues that it is the teacher's responsibility to make children see that mathematics is interesting and that it is part of our humanity because it is in ourselves, so if we do not know it, we are missing a part of our humanity (Gil-Clemente, 2021). To do so, UpToYou (2021) suggests that education has to be done from a personal perspective linking directly the learner with his life. For all that, the teacher has to help the child to make sense of all that the child is discovering here and now. As the example explained, it is not the same to give instructions on what to do with the numbers in addition, than to explain the value of addition, when someone thought of this addition, what needs it responded to, or what relationship addition has with his life today. By teaching according to this approach, it eases the child to find meaning in this operation and he will see that the educator cares about him and treats him as a capable person. Therefore, it is not a question of putting the additions in front of the pupil no matter how he learns them. He deserves to understand their value as well as why and what he is learning them for, encouraging that inner motivation to continue learning and improving. Faragher and Brown (2005) defend that learning mathematics is a privilege that all children deserve.

Furthermore, teaching this science from a meaningful perspective enables learners to become a participant of their own learning because they enjoy thinking mathematically when they understand it. Pound (2006) created a guide of the curriculum for teachers to foster this type of thinking on children. All this is a consequence of an effective mathematics teaching, stated as a teaching principle, which requires a teacher to understand what students know and need to learn and, consequently, challenges and supports them to learn new knowledge well (NCTM, 2000).

Teachers should remember that there is not only a single way to get to the answer, but that mathematics, in reality, consists in searching different paths to solve a problem and all are equally valid. As Pòlya (1945) presents, it is more efficient and meaningful to propose a variety of different types of problems better than repeating many times the same problem (Gil-Clemente, 2021; Godino et al., 2003).

On the other hand, pupils learn mathematics through the experiences provided by teachers; therefore, their mathematical knowledge and reasoning skills are conditioned by the teaching at school. Then, the starting point to build mathematical knowledge should be through the practical and everyday experience in order to start establishing spontaneous and intuitive mathematical relationships in their life. This mathematical reasoning is worked through the use of manipulative and concrete materials at the beginning, to gradually pass into a more essentially symbolic, abstract and formal mathematical learning (Dienes & Perner, 1999; Godino et al., 2003).

The materials used in the classes will drive to a more meaningful pupils' learning if they are prepared from things that motivate him, attracting and exercising their attention (Gil-Clemente, 2021). This motivation arises if mathematics is taught through games as it is done in the ANFoMAM project workshop as a method to rescue creative mathematics, which enables children to discover their world through their senses and movement. Millán-Gasca, in the presentation of Gil-Clemente's book (Cálamo, 2021), said that "when learning is based on play, mistakes are considered as part of the process and not as failures". Additionally, as Edouard Séguin (s.f., cited in Gil-Clemente, 2021) claimed, working geometry in mathematics with children with disabilities helped him to discover the clear steps any child undertakes when learning geometry concepts. This phenomenon can be extrapolated to all the areas that mathematics deals with, and it needs to be taken into account and applied to improve everyone´s education and the teachers´ training.

An application of this type of mathematics education is more effective, approachable and intuitive for primary learners. It can be seen in the Singapore math model teaching approach developed in the following section.

1.2.2 Strategies, techniques and models in the Singapore math approach

This 'Singapore math model' is a highly effective teaching approach which has been widely developed and adopted in different forms around the world, being progressively included and introduced in the mathematics curriculum of different countries such as USA in 1998, or Chile in 2014, and lately Spain in 2019 through SM. It instils the children and adults with a deep understanding of mathematics, helping them to conceive mathematics as an approachable and important science that helps them understand their life instead of something isolated that has no utility (Singapore Math Inc, 2020).

In other words, it is a method that compiles all the effective strategies and techniques when teaching mathematics, developed by previous mathematicians (covering the ages from early childhood to university) in the form of a series of several books that include concepts such as the Dienes *CPA (Concrete, Pictorial, Abstract)* progression, Fuson *number bonds*, Pòlya *problems solving*, Skemp *relational understanding* or Ban Har *bar modelling*. These aspects lead students to learn to think mathematically developing a relational understanding, comprehending the Hazekamp *why before how* or developing some reasoning strategies to solve problems.

Additionally, it should be highlighted that, as Skemp (2006) proposes, there are many benefits when meeting a teaching-learning model based on a relational understanding because it promotes a functional process where children understand what, and most importantly, why they are doing math as opposed to an instrumental approach understating that fosters rote memorization of tons of procedures without understanding. Simply instrumental understanding limits to a set of “rules without reasons” (p.89) which is not the idea of understanding that Skemp defends.

To determine how widespread the instrumental approach is, here follow some examples, to which the reader might have been exposed: “borrowing in subtraction” or “take it over to the other side and change the sign” (p.89) which are obvious examples of instrumental explanations in which children have just memorized and applied these rules to achieve the correct solution of the operation, but without having understood the why behind these rules.

As opposed to what teachers do when teaching mathematics based on an instrumental approach (defending a page of right answers in which they advocate principles such as an easier understanding, a fast right answer and immediate reward) there are many advantages when teaching from a relational perspective.

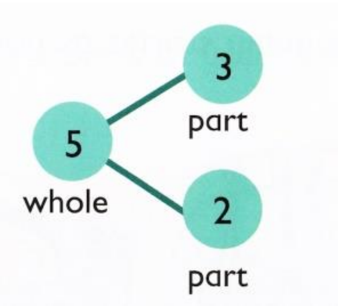
Firstly, Skemp (2006) claims that relational mathematics are more adaptable to new tasks and easier to remember: the relational knowledge can be effective as a goal itself as well as the relational schemas are organic in quality as an agent for the children’s own growth. Secondly, he believes that teachers should not limit only to instruct on the former understanding but combine both: the instrumental and relational understanding. The reason is that the latter refers to the former when giving the child the instructions of the process to follow, but only after they have understood why they use it, for what and how. Then, in that particular moment of the children learning, the explanation of the procedure appears, but not before.

Skemp perception of understanding links directly with Hazekamp (2011) idea of teaching and learning the ‘*why before the how*’ to help the child develop his learning at his own pace and build meaningful knowledge that later can be extrapolated to other situations because he has completely comprehended the concept, which enables him to deduce it in a future.

Other techniques to teach additive decompositions of numbers (a previous step for addition and subtraction) are the number bonds and the bar modelling. The number bonds (Fuson, 2012) are a pictorial technique that shows the part-whole relationship between numbers and fosters a children´s number sense. In figure 9, the adjoining circles ('3' and '2') are the parts making up the whole '5', in other words, it replaces the four mathematical equalities: 3+2=5, 2+3=5, 5-3=2 and 5-2=3.

Figure 9.

Number bonds (part-whole relationship)

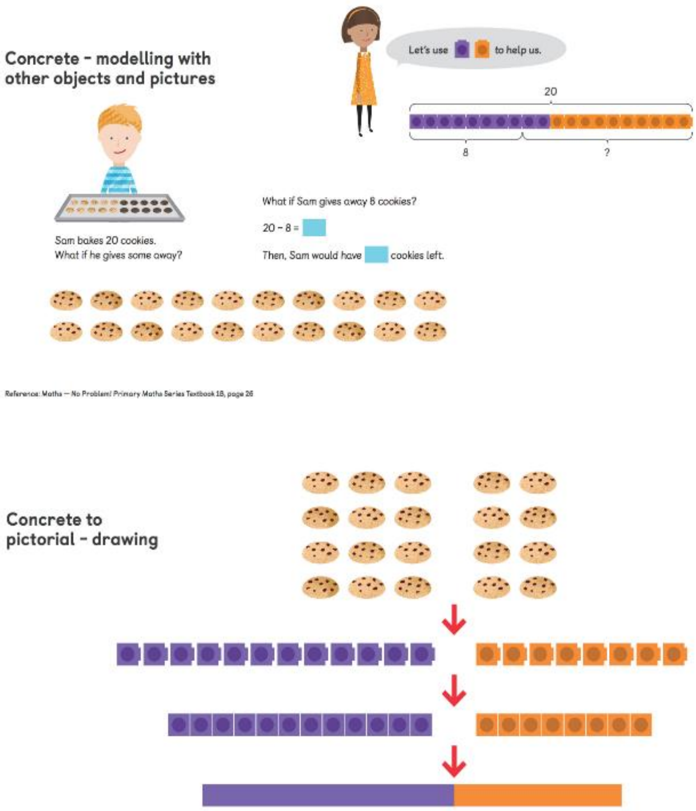


Note: By Acollins, 2016.

On the other hand, created by Ban Har (2014), the bar modelling allows pupils to picture and visualize mathematical concepts in order to solve problems. It can be used in a variety of different mathematical concepts such as fractions, percentages, ratios (Singapore Math Inc, 2020). Its functioning is similar to the number bonds, but it also can be applicable to more complex problems because it helps learners indicate the knowns and unknowns in a given situation (Singapore Math Inc, 2020; Maths-No Problem, 2021). Pupils, when solving a problem, pass from concrete to pictorial abstraction level as seen in figure 10, in which the children need to picture the situation of Sam having 20 cookies and giving 8 away.

Figure 10.

Concrete to pictorial in bar modelling



Note: by Maths-No problem, 2021

These techniques can be used when solving a problem. However, this can be worked in-depth by following the strategies mentioned by Pòlya (1945) as the steps children should perform when they approach a word-problem. The phases are:

1st. - understand the problem;

2nd. - devise a plan;

3rd. - carry out the plan;

4th. - look back and learn.

If teachers want their children to expertise problem solving following these steps will guide them through the process of developing reasoning skills. It is a key element to immerse these word-problems within real-life situations to help children recognize the application and usefulness of mathematics in their daily life.

After having worked with Down Syndrome (DS) children, Faragher and Clarke (2014) raise the question of whether teachers should approach the teaching of mathematics for its own sake (value or worth) or focus on its skills for daily life. This project believes in using mathematics as a way to understand our world and simplify it; therefore, as a science that eases our learning and life along with our existence.

All these strategies mentioned above were considered important, and; therefore, applied in our didactical proposal to help children develop a deep and meaningful understanding of mathematics. We only add one more thought due to Leo Tolstoy (1967): if you want to educate the student through knowledge, want and master your subject, and the students will like the subject and love you, and you'll influence their education; but if you don't like what you teach, you can already force the students to study that you will not exert any educational influence.

2. EMPIRICAL STUDY

2.1 Procedure and Phases of the Study

The current pandemic conditions have necessitated a case study as the mixing of different students has been prevented to comply with the new COVID-19 measure of acting on the basis of bubble classes. As a consequence of the actual situation, only one pupil has benefited from the maths workshop proposal; however, this has allowed us to have an exhaustive follow-up of the child, paying special attention to his particular mathematical knowledge and skills evolution after having worked personally with him on some very specific and precise topics of the mathematics curriculum.

The empirical study proposes a recreational mathematical workshop based on learning through play, using manipulative materials which help the child build his mathematical knowledge and abilities based on a relational understanding. During 3 months, it is implemented outside of the mainstream classroom allowing one-to-one classes and a personalized instruction based on meeting the educational needs of the pupil. All this is done to observe that even if a child has a curricular accommodation if, after providing him with some learning tools previously worked, he could be able to follow the learning pace of his peers within the regular classroom to fulfil the terms of inclusion.

The duration of the sessions started being 30 minutes long and before the playtime which, after a few classes, was perceived as a distraction for the pupil adding an extra adversity to his concentration difficulty; therefore, we noticed the necessity of rescheduling the workshop to a longer session (50 minutes) because otherwise, by the time he understood the procedure of the activity, it was recess time break. This timetable change really has helped in developing the proposal and has given both, the learner and teacher, more flexibility in working on the activities planned.

In addition to the proposal, the study is composed of more steps reflected in table 1, which describes the different phases of the study and the periods in which the present Final Degree Project has been developed.

Table 1.

Project Timeline

	December	January	February	March	April	May
Literature Review						
Election of objectives						
Objectives modifications						
Design and Selection of instruments						
Participants Selection						
Development of the proposal						
Implementation of the proposal						
Data Collection						
Data Analysis						
Final Writing of the Paper						
Editing of FDP						

2.2 Objectives of the Study

The general objectives that we pursue by means of our Final Degree Project are the successive:

- Study if the relational understanding as a mathematical tool for working with arithmetic can help children with difficulties to reach the formal classroom from the external special education (SPED) classroom.
- Reduce the academic gap between these children with LD and their peers to facilitate their academic and personal learning process encouraging the inclusion of the child and the development of his personal competencies and relationships.

The following are the specific goals that we aim to achieve once our practical proposal is implemented:

- Search and elaborate a theoretical framework to understand the power and benefits of a teaching-learning model based on a relational understanding promoting a functional process where children understand what and, most important, how they are doing math, as opposed to the instrumental understanding (Skemp, 2006), which fosters the memorization of thousands of procedures without understanding.
- Find out to which extent manipulative materials are really a learning tool and, if so, how to use them from a relational methodology, with the aim of helping the child to build and integrate mathematical knowledge and skills.
- Create and develop a Learning Proposal in the form of a mathematical workshop which will provide the children a more meaningful way to understand how to solve the arithmetical operations of addition and subtraction, and related problems in real life scenarios.
- Design and implement a recreational mathematical workshop based on learning through play, using manipulative materials which help the child build his mathematical abilities and applicable knowledge in his daily life through a relational understanding approach.
- Enable the children to understand and internalise the application of arithmetic in their daily life.
- Analyse the evolution of the case study to evaluate the learning proposal effectiveness and its possibility to be applicable to other children suffering LD in other classrooms.

2.3 Learning Proposal

2.3.1 Justification for the choice of the Learning Proposal

This project proposal arises from the criticism of manipulative materials which are not based on a relational understanding, but on an instrumental understanding, or do not facilitate the knowledge building and scaffolding. Just because they are mathematical concrete materials, it does not mean that they are useful for the pupils to learn. The reality is that not all of them are effective as it happens, for example, with the multiplication pizza (see figure 11): it helps to review the multiplication tables and make them more playful, but it does not stop us from purely memorizing the tables without understanding, which contrast with the idea of facilitating the construction of knowledge and providing the child with a tool that can be easily assembled anytime and can help him in the learning process. Another material example that might help the learner just with the execution of the arithmetical operation of addition or subtraction with regrouping is figure 12, where it is seen that the process is mechanized, but its use and the reason for the process is not understood completely, it only eases the proper execution of the algorithm.

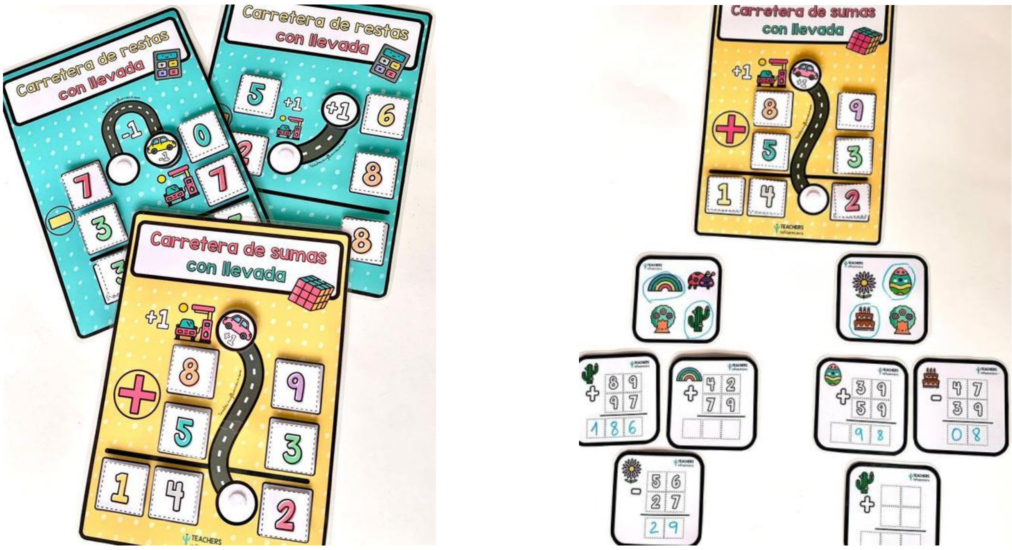
Figure 11.

Pizza with the multiplication tables

Note: Retrieved from A. López, Junio 2020, Instagram (https://www.instagram.com/p/CA51RNfCkjd/).

Figure 12.

Road of addition and subtraction with carrying



Note: Retrieved from A. López, Diciembre 2020, Instagram (https://www.instagram.com/p/CIdH7A7l49g/).

Little by little, the pupil gradually leaves these materials aside because he has not grasped the why of the process before knowing the how (Hazekamp, 2011), which leads them to assimilate a myriad of diverse procedures to solve mathematical problems (without understanding them), which are later forgotten because if there is no understanding and reasoning of the process followed the knowledge is not internalized, thus, the learner does not know how to deduce the procedures in the future and, as a consequence, these procedures are forgotten. As Benjamin Franklin said: "Tell me and I forget. Teach me and I remember. Involve me and I learn". However, if children understand the why of the process with a relational understanding, later they are capable of deducing it, extrapolating it to other situations, remembering it and using it when necessary. As teachers, we have to be aware of what we want them to carry out and ensure their correct understanding of the meaning of the two algorithms of addition and subtraction, but above all, we have to make them understand in which real-life situations they should apply each one. This enables the avoidance of mathematical mechanization because mechanizing the process results in the correct execution of the operation in the short term, but not in the understanding.

Besides, all we have experienced that in adult life, for the correct execution, we use the calculator, so it is not so important the execution or the how of the procedure, but the why of the process and knowing when to use it in real life. The study of mathematics helps us to understand the world around us and its application in it, in essence, to make our lives easier, which is achieved through the correct understanding and reasoning of the why of things surrounding us.

A final aim of this proposal is to point out some manipulative materials that will empower the children in their abilities, skills and arithmetic knowledge, so that they feel capable of successfully facing problems that were previously out of their reach.

This study is based on the ANFoMAM mathematical project carried out in a leisure environment in the SESDOWN association for Trisomy 21 or DS children. This programme is a member of the European Erasmus+ project "*Aprender de los Niños para Formar a los Maestros en el Área de Matemáticas",* which was born from different initiatives that seek the deep understanding of the basic concepts of mathematics, moving away from the transmission of mechanical and repetitive procedures.

Therefore, the combination of this ANFoMAM workshop together with the attendance to a seminar on solving arithmetical word-problems for students with cognitive diversity given by Dr G. Catalán, is the motivation to develop this proposal because it arouses the curiosity to know if this could be applied to other types of students with learning difficulties in order to discover if what works for DS children can also be useful for any other type of common difficulty during the learning process.

Hence, this learning proposal has been developed for children suffering LD within the mainstream classroom in order to reduce the academic gap between them and their peers and to facilitate their academic and personal learning process, and thus, enable them to join their reference class having reduced this academic gap, encouraging the inclusion of the child and the development of his personal competences and relationships. Concretely, it has been applied to a very interesting case of a sixth-grader who experiences great difficulties with the understanding of mathematics, and lacks reading and writing skills. The description of the case is detailed in the following section.

All mentioned above is flourished in this proposal through the implementation of a mathematical workshop using the resources of manipulative material and concrete tools that aims to help the learner to develop mathematical abilities and applicable knowledge in his daily life and facilitate the built of knowledge by a functional process where he understands what, why and how he is doing math. The latter will allow him to develop a more realistic connection between mathematical concepts and real life.

Over 3 months, eleven sessions have been carried out to notice an observable evolution of his learning and test the effectiveness of the proposal. It is set out within the sessions of the Special Education teacher outside the mainstream classroom with instruction more individualized. Based on the actual relational instruction, this work has intended to help the learner to develop an actual understanding of the decimal positional system, the arithmetic operations for addition and subtraction with or without regrouping, and the importance of mathematics for his own knowledge construction; all this through the use of adequate tools to ease his mathematical learning acquisition.

2.3.2 School context and Description of the case study

The proposal has been developed at a public school in Pamplona in the multicultural and diverse neighbourhood of Mendillorri, specifically in the primary level with a pupil presenting many special educational necessities and difficulties on following the mainstream class.

The school community has, in addition to the group of teachers, management team and the head of studies, and staff specialized in working with children with special needs such as a counsellor; two speech therapists; two SPED teachers.

Since many pupils come from different backgrounds and nationalities, the school supports the participation and integration of their pupils creating a positive school atmosphere where coexistence and respect are the main pillars. It should also be highlighted that some of the students do not have the language and come from families with a mid-low socioeconomic status which impacts directly on the children learning process and the accessibility to resources. This inequality leads to the disadvantages and struggles of certain pupils in relation with the rest of their classmates, presenting learning disabilities related to their language comprehension, memory and reading and writing abilities as it happens with the learner selected for the study.

The target for this study is a singular case of a 12-year-old pupil placed in the sixth grade of primary education with a curricular adaptation (accommodation) in the Spanish language corresponding to a third-grade level and in maths to second grade, making the learner struggle along his learning process and within the educational system having already repeat the third grade of primary education.

Some relevant aspects of this personal case will help us to understand his learning difficulties. This child has been suffering from sleep epilepsy for many years, even being hospitalized, which may have caused a cognitive degeneration. He has also been diagnosed with ADHD disorder with no medication making the learner lose attention and concentration continuously. Additionally, he presents difficulties with memory, writing and reading, and mild cognitive impairment.

In addition to the previous adversities, there is very little follow-up and support from his family which pictures in a background full of chaos for the learner and intermittent truancy leading to increasing learning difficulties. This also reflects on his poor hygiene and laziness attitude because everything seems a huge effort for him, and, instead of trying harder, prefers not to face the challenge. Besides, he is not socially involved because the relationship with his peers is not good due to his lack of anger control when he gets frustrated.

The scholar intervention consists of having a weekly session with the school reading and speech therapist and five sessions with the SPED teacher inside and outside the class, but lately increasing the time outside of the mainstream class in the subjects of maths, Spanish language and science. The Educational Support Unit has determined that maths has to be taught in a manipulative, attractive and functional way, so it enables the pupil to get motivated within the activity. This intervention suits perfectly this maths workshop study that, with the accurate coordination with the special teacher, will help the child to build self-esteem and strategies to understand and reduce frustration when performing mathematics.

2.3.3 Curricular framework: contents, evaluation criteria and learning standards

To design and develop the learning proposal, the first step undertaken was the identification and choice of the specific curriculum contents, evaluation criteria and learning standards that were appropriate to work in the mathematical workshop see (table 2). They were chosen from the *Decreto Foral 60/2014*, which establishes the curriculum for elementary education in Navarre.

Table 2.

The Contents, Evaluation Criteria and Learning Standards of the Primary Education Curriculum of Navarra. Mathematics of Primary 2º Grade, Block 2: Numbers.

	Contents	Evaluation Criteria	Learning Standards
Natural numbers and numerical literacy	- Meaning and usefulness of natural numbers (counting, measuring, ordering, expressing quantities, etc.). - Decimal numbering system. Rules of number formation and place value. -Introduction to the equivalences between the elements of the decimal number system: Ones, Tens, Hundreds. -Numbers in real situations: reading, writing, ordering, comparison, decomposition,...).	1. Read and write natural numbers up to 999, using them in the interpretation and resolution of problems in real contexts.	Using numbers up to 3 digits is able to: 1.1. Count drawn objects. 1.2. Read numbers. 1.3. Write numbers with digits and letters. 1.4. Knows the Hundred and its value. 1.5. Identifies place value of numbers/digits. 1.6. Decomposes and composes numbers additively according to the place value of their digits. 1.7. Identifies the number before and the number after. 1.11. Establishes equivalences between Hundreds, Tens, and Ones.
Operations with Natural Numbers: Arithmetical Calculation	-Operations of addition (putting together or adding) and subtraction (taking apart or taking away) and their use in everyday life. -Oral and written mathematical expression of operations and the calculation of addition and subtraction.	2. Perform basic numerical calculations with the operations of addition, subtraction, using different strategies and procedures.	2.1. Identifies the terms of addition. 2.2. Understands and uses the expressions double, half ... in different contexts.
Calculation: Written mental Calculus	-Performance of non-academic algorithms of addition and subtraction, by means of numerical decompositions and other personal strategies. -Calculation of addition using the academic algorithm. -Calculation of subtraction without carried numbers using the academic algorithm. - Use of standard addition and subtraction algorithms.	3. Know, develop and use basic strategies of mental arithmetic.	3.4. Constructs ascending and descending series by 2, 3, 5, 10 and 100 3.8. Calculates the missing term in an addition or subtraction of the type: 7 + ___ = 15.
Arithmetical problems		4. Identify and solve problems of everyday life, establishing connections between reality and mathematics and valuing the usefulness of appropriate mathematical knowledge for problem-solving.	4.1. Poses and solves first-level additive-subtractive problems (one-step or one-operation). 4.3. Applies notions of numeration in solving arithmetic problems. 4.4. Determines/relates data or questions or statements or operations in a problem situation.

Note: Adapted from Gobierno de Navarra (2014, p.50).

2.3.4 Learning proposal Procedure and Introduction to the Lesson Plans

First of all, the study began by talking with the SPED teacher and the speech therapist (with whom we stayed in the SPED practicum) about the different children profiles experiencing learning difficulties which could suit well in the proposal for this thesis work. The selection of the pupils was made based on the mathematical concepts and objectives that we wanted to develop in the project, being the main target: the addition and subtraction algorithms. After deciding to which 3 learners it was going to be conducted, and because of the pandemic situation that does not allow mixing pupils from different classes to complying with the bubble class COVID measurement, the time sessions had to be specified making it only possible to work with one case from the initial ones.

This pupil became a very interesting case for the study. However, firstly, we needed to contextualize the workshop, specify his academic status and know his learning difficulties in the areas required for the development of mathematical competence being aware that the language comprehension ability takes an important place on the matter. Once known that his global achievement in the competencies of mathematics and language corresponds to a second grader performance, the concrete learning mathematical objectives for the case study were defined.

Before designing the adequate mathematical workshop for the learner, it was necessary to know at which level of mathematical abstraction, understanding and reasoning our learner was. Therefore, an initial evaluation was conducted to identify also some misconceptions the pupil might have. So, in the very first lesson, we collected some word-problems and pictorial and abstract addition and subtraction (with and without regrouping) operations. The initial test (see Annex I) was designed according to the school standards to which the child was used and completed with the rubric in Annex II, even though it was not the most appropriate one to define the child's current status in the parameters we wanted to work on.

The results showed that he experienced difficulties with the regrouping concept, especially in the subtraction algorithm, and even though he performed correctly the additions with regrouping, it could be observed that the process was mechanized leading the child to a not understanding of what he was doing and why was he following that process in which claimed that 'it was the way he was taught'. This incomprehension of the addition process makes him fail in the performance and understanding of the subtraction algorithm in all its dimensions: from a pictorial to an abstract perspective.

Additionally, it is shown that due to the mechanization of the process, he could not identify the real situations in which you chose an operation or another, based on the meaning of what you want. In other words, he could not transfer the meaning and usefulness of the operation to real life. Besides the pupil experiments frustration whenever he does not understand and struggles with a problem, feels incapable of solving it or makes him strive beyond his comfort zone driving into an avoiding attitude of the mathematical problem corresponding with de Guzmán (1991) definition of frustration and the hate blocks, experienced by pupils, towards a matter when there is a frustration because it is not understood.

Taking into account everything mentioned above, we could start designing the activities to shape our mathematics workshop to develop our didactic proposal. In the first two sessions, we intended to revise previous mathematical concepts needed by the learners to understand the reason to operate choosing the correct algorithm and integrate what they are doing and why they are following that process instead of memorizing it. Therefore, we began with different models to perform additive decomposition of numbers (i.e. ten until one hundred) which helped the teacher introduced the orders of the decimal positional number: Ones, Tens and Hundreds. The pupil revised also the names, symbols and logic order of numbers, how to move in series of 2 by 2, 5 by 5, and 10 by 10 and perform operations of addition and subtraction on the hundred chart.

Once the pupil has reviewed the previous concepts, he is ready to learn in the next lesson the importance and reason for the decimal positional value of numbers and its influence to perform operational algorithms, especially in addition and subtraction. During the core lessons of working with these operations, we have used a relational learning tool based on the CPA approach with the tally system model to achieve a complete mathematical abstraction of the learner with the support of toothpicks, number flashcards and a pen for him to write at a symbolic level; performing all this, through different activities and integrated into real-life situations to contextualize and ascribe the proper meaning to these arithmetic operations. There have been some rubrics created for the teacher to have a closer following of the case study of the learner in order to see if he is accomplishing the objectives proposed during the core lessons (see annex XIX, XX, XXI). As a closure of the approach, we have made the student create his own mathematical verbal situations throughout the proposal development, which has shown the teacher if the student has accomplished completely the understanding of these arithmetical operations when having to identify and create situations with them in real life.

The mathematical items that will be worked all along these sessions can be summarized in the following table 3:

Table 3.

Summary of the Lessons Plan

Lessons	Activities	Materials	Sessions (Date)
***0** - Mathematical initial level evaluation*	- Evaluation	- Assessment test in annex I	15/01
***1** - What makes ten?*	- Friends of Ten – Complementary numbers - Additive decomposition of number 10 and simple numbers through number bonds models	- Paper cones, Cuisenaire rods or polycubes - Annexes IV, V, VI, VII	22/01
***2** - Let´s create and move on our own Hundreds Chart*	- First contact with the Table of 100 (observation of its organization, number citation, movement by groups of 2, 5, 10) - Decoding the hidden numbers (Annex X) - 100 Chart puzzle (Annex XI)	- The 100 Chart (Annex VIII) - Little figure to move - 100 Chart Window tool (annex IX) - Worksheets in Annexes X, XI	29/01
	- Cross Numbers (Annex XII) - Moving through the 100 Chart performing addition and subtraction operations (symbolic language) - Bingo Dictation Game (Annex XIII)	- The 100 Chart (Annex VIII) - 100 Chart Window tool (annex IX) - Worksheets in Annexes XII, XIII	5/02
***3** - Why is the place value of a number so important?*	- Explanation of the decimal tool functioning - Numbers representations (concrete and symbolic level) - Identification and understanding of the orders (Hundreds, Tens, Ones) - Discovering the meaning of place value of a number - Mathematical code activity (Annex XIV)	-Decimal Positional Table (Annex XVII) -Number flashcards (Annex XVIII) -Toothpicks - Elastics - Place Value Rubric (Annex XIX)	3/03
***4** - Why and How do we perform additions?*	- Numbers representations (concrete and symbolic level) - Learn the parts of the addition - Performance of Addition without regrouping - Performance of Addition with regrouping (understand why it has carried numbers in a concrete and symbolic level) - Identify and Formulate real life contexts in which addition is used - Numbers Detective Game	-Decimal Positional Table (Annex XVII) -Number flashcards (Annex XVIII) -Toothpicks with elastics - Situations Cards of Addition (Annex XXV) - Addition Rubric (Annex XX) - Numbers Detective Game (Annex XV)	5/03 9/03
***5** - Why and How do we perform subtractions?*	*Subtraction with regrouping* - Numbers representations (concrete and symbolic level) - Learn the parts of the subtraction - Understand that the subtrahend are holes to be filled - Performance of subtraction without regrouping integrated within the domino game - Identify and Formulate real life contexts in which subtraction is used	-Decimal Positional Table (Annex XVII) -Number flashcards (Annex XVIII) -Toothpicks with elastics - Boxes - Situations Cards of Subtraction (Annex XXVI) - Subtraction Rubric (Annex XXI) - Addition and Subtraction Domino Game (Annex XVI)	18/03 24/03
	Subtraction without regrouping - Performance of subtraction with regrouping integrated within the domino game - Understand the necessity of opening an upper order group to perform subtraction with carrying in a concrete and symbolic level - Identify and Formulate real life contexts in which subtraction is used	Same materials as in the previous session - Blank Situation Cards (Annex XXVII)	25/03
Extension lesson *Arithmetical Word Problems*	- Revision of the identification and creation of addition and subtraction word problems emphasising relations of before-after and additive comparison.	- Blank Situation Cards (Annex XXVII) - Situations Cards of Addition (Annex XXV) - Situations Cards of Subtraction (Annex XXVI)	
Final Lesson Assessment and Analysis of the evolution case study and results	- Revision of the materials used across the workshop and its functioning - Performance of the final mathematical level assessment/evaluation test with the help of the materials to see if there has been a progression at the end of the mathematical workshop and if the pupil has experienced an evolution	-Decimal Positional Table (Annex XVII) -Number flashcards (Annex XVIII) -Toothpicks with elastics - Boxes - Assessment test in Annex I - Assessment test in Annex III	26/03 - Assessment of Annex I 13/04 - Assessment of Annex III

Another aspect to be aware of is that the materials for the workshop placed in the annexes are designed in Spanish because the maths workshop is conducted in this vehicular language as it is established in the school methodology protocol and voted by the school faculty.

2.3.5 Action Proposal (Lesson Plans)

Let us now turn to a more comprehensive and detailed account of the meetings.

Lesson 1 – What makes 10?

Our decimal positional numeral system is based in the necessary grouping of ten elements to form the next unit. This was widely explained in chapter 1, section 1.1.1 (4.positional system), and is the base on which our arithmetic is built. The deep understanding of our decimal

We expose the first lesson as a revision of the different mathematical concepts such as the complementary numbers of ten and some simple additive splitting with the purpose of applying later that knowledge into a simple algorithm of subtraction and addition with a maximum of ten, so he can understand that the action of subtracting is discovering the remaining part of the total from a given part, and the action of adding is putting two parts together to make the total of ten, revealing then, the friends of 10 (complementary numbers). Making groups of ten with numbers helps the teacher introduce two of the different orders of the decimal positional system: Ones and Tens, and the reason why 10 is written as 10: 1 Ten and 0 Ones, without a special symbol for this value.

Another objective of the activity is to make him think about the importance of mathematics in his surrounding atmosphere making him wonder what a number is, what they are for and how they help us in real life.

The goals above link with the curriculum contents of "*introducing the equivalences between the elements of the decimal number system: Ones, Tens, Hundreds, discovering the meaning and usefulness of natural numbers (counting, measuring, ordering, expressing quantities, etc.)"* and the learning standards of "*decomposing and composing numbers additively according to the place value of their digits, the establishment of equivalences between Tens and Ones, the calculation of the missing term in an addition or subtraction of the type: 7 + ___ = 15 and the execution of operations of addition (putting together or adding) and subtraction (taking apart or taking away")'.*

The lesson is set to work through four different activities, but before starting with them, we try to activate the previous knowledge and discover some misconceptions the child might have. Therefore, he is asked how he could explain, with his own words, what a number is, when or in which situations he uses them, and what are they useful for. The pupil answered that "a number is a number that helps to count" and that "they are used in the school". To which we exposed some examples of different situations they could be used for when buying in a store or things on *Fornite*, checking the time to know the hour or how much time has passed since a particular moment or measuring lengths, weights or heights; basically letting him know that he lives surrounded by numbers and that they are relevant for his life even though he does not notice that he uses them every day.

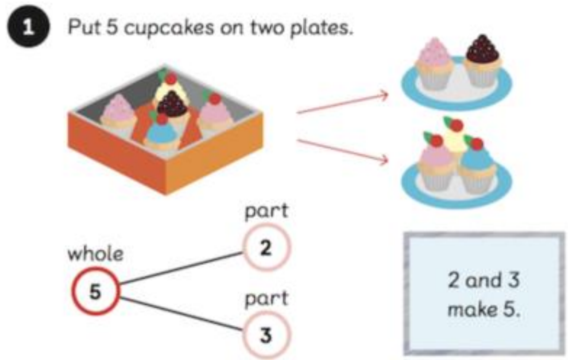
After activating previous ideas, we start with the *first activity* of the lesson which consists of *discovering the friends of ten* using his hands and, in our case two different coloured sets of paper cones which will be placed on the fingers, (this activity was designed by Dr. G. Catalán for DS children and is related in Gil-Clemente (2021) in pages 225-227), but it can also be done with poly-cubes or Cuisenaire rods. The goal is to discover from a manipulative perspective all the possible additive decompositions of the number ten, for which we start placing 1 blue cone on one finger and fill the rest with red cones, realizing that 1 blue cone + 9 red cones make ten, or in other words, that 1+9 is an additive decomposition of ten; then the learner will place 2 blue cones and notice that 8 have to be red to fill all his fingers, and so on, until the pupil fills all the fingers with one colour (blue) having recorded all additive possibilities of the complementary numbers of ten in the friends of 10 sheet (see annex IV). This activity will be reinforced with the one detailed in annex V which will help us introduce how the addition or action or putting together 10 things makes ten which is equivalent to 1 Ten, or 20 things equals 2 Tens. This second activity was not very useful, but I have learned that it is redundant because it works the same as the previous one and it is more interesting to move on to the others; however, if the learner needs to reinforce the idea of additive splitting this is a great practise to do.

The *second activity* consists in seeing if the pupil has understood *how complementary numbers work* and with no help has to find the respective pairs of numbers in the bubble (annex VI) and write down the additive expressions of 10 that he can find and discover the missing ones.

Now, that the learner has integrated this mathematical concept of decompositions with the number ten, we transfer this aspect of additive expression to other simple numbers until the order of Tens, for example, having a total of 5 in how many additive different ways the number can be decomposed, and if the teacher gives the pupil one of the numbers of the decomposition observe if he can calculate the missing gap by using either addition or subtraction. To work this we have the *number bonds activity* defined by Fuson (2012) and found in annex VII widely used in the Singapore maths strategy, which reflects the part-part-whole relationship of numbers. The pupil first starts in a concrete level to see the parts and whole with toothpicks, the paper cones or poly-cubes by asking him, for instance, to put seven poly-cubes (whole) on two different plates to see how many poly-cubes can be found on each of the plates (parts) as it can be seen on the example of figure 13.

Figure 13.

Example of the part-part-whole Singapore model in a concrete and abstract level



Note: Retrieved from *Maths - No problem! Primary Maths Series Textbook 1A* (p.26), by Y. Ban Har et al., 2014, Maths - No problem.

Once we see that after a few examples he understands the strategy to get the whole in the case of having both 'parts' or calculate the remaining part in the case of having one *part* and the whole, we pass to the next levels of abstraction having the whole written according to the symbolic level and one of the parts on pictorial, making firstly the learners to give its symbolic representation. Now, having one part and a whole, he has to give the missing part both in the visual and abstract representation. If the learner has the two parts, he will have to discover the whole realizing that he has to perform the operation of addition which consists in putting the parts together.

A symbolic example of the Fuson number bonds model is figure 14 where the parts can be seen as parts in orange 7 and 3, making the whole in green 10 (abstract step: 7+3=10). It replaces the four mathematical equalities: 7+3=10, 3+7=10, 10-3=7, 10-7=3. The other two examples (figure 15) are for the scholar to discover the unknown whole in the former (5+2=?) and the remaining unknown part in the latter (3+? =9). These examples are designed in a slightly different way, so that the learner gets used to more than one model, but also can be adapted to work at the same time the pictorial level by making him draw dots as a representation of the symbolic number being this the activity worked in class.

Figure 14.

Part-whole relationship of numbers (symbolic level)

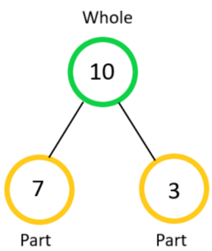
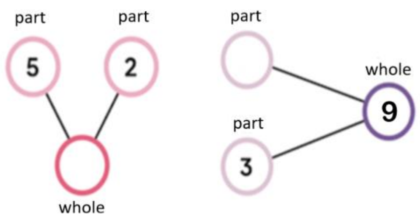


Figure 15.

Situation of unknown whole and situation of unknown remaining part



After having described the activities worked in the lesson, we proceed to show up in the following results scheme whether the activity objectives have been accomplished (see table 4).

Table 4.

Results scheme lesson 1 – Additive decompositions

Objectives evaluation	Achieved
1. Discover the complementary numbers of 10.	Yes
2. Perform additive decompositions of the number 10	Yes
3. Perform additive decompositions of simple natural numbers until the order of Tens	No (lack of time)
4. Introduce orders of Ones and Tens of the decimal positional system and understand the equivalence of 1 Ten to 10 Ones	Yes
5. Comprehend the meaning of operating in additions as putting together and in subtractions as taking away or apart	Not completely, just acquire the addition (lack of time)
6. Create curiosity about the importance of mathematics in the world around us and real-life situations.	Yes

As we can see, the learner accomplishes almost all the objectives, except the additive decomposition of simple numbers (apart of the complementary numbers of ten) due to the lack of time in the session, therefore, we would take that into account and break the lesson into two sessions taking more time for the understanding of the decomposition using the number-bonds model in a concrete and pictorial level before starting with the symbolic one. Later, we would have skipped the reinforcing activity of annex V because after having conducted it, we realised that it was a little redundant with the previous one. The aspect showing that he has accomplished the goals is seeing the child being able to identify and record the decompositions of the number 10 in a mathematical notation using the addition operation as a strategy of putting two parts together. However, because of the lack of time, we cannot conclude that the pupil has acquired the understanding of the subtraction operation when having to calculate the remaining part of the number-bonds model.

Lesson 2 – Let´s create and move on our own Hundreds Chart

The objectives of this second lesson are to reinforce the abilities of writing and reading the natural numbers until 100 correctly when having to name them aloud and write them down, review the order of natural numbers and apply it on the hundreds chart which will help the child later when having to decide which number is the subtrahend and the minuend on the operational performance of the subtraction algorithm. Also, it is reinforced the equivalence between ones and tens worked previously and introduced the decimal positional order of hundreds and its equivalences with tens and ones by working with the hundreds chart, which enables the child to learn how to perform non-traditional simple operations of addition and subtraction when moving on the table by different quantities (2 by 2, 5 by 5, 10 by 10).

To achieve the goals, it is necessary to specify them based on the curriculum contents of "*discovering the meaning and usefulness of natural numbers (counting, measuring, ordering, expressing quantities, etc.) and the performance non-academic algorithms of addition and subtraction, using numerical decompositions and other personal strategies and the evaluation criteria of reading and writing natural numbers up to 999, using them in the interpretation and resolution of problems in real contexts"* to which are linked the learning standards of "*reading and writing of numbers with digits and letters, knowing the Hundred and its value, identifying the number before and the number after and, lastly, constructing ascending and descending series by 2, 3, 5, 10 and 10".*

The lesson is set out to be undertaken in two different sessions developing in the first one the first three activities, including the puzzle exercise, and in the next session the rest of the lesson. Usually, this lesson would have been started with the practice of movement on the number line, but because of the pupil study age, we go directly to practise the movement and integration of the operations into the hundreds chart enabling also the work of the hundreds decimal positional order. The practice begins with the *initial contact* with the hundreds table (see annex VIII) asking the child to observe how numbers are organized and why it is done in that particular way, and what numbers were comprised the table. To what he answered that it seemed that in each of the rows it goes the tens, the twenties, the thirties, and so on, until the number one-hundred. Then, we emphasised that its organization was done according to the different families of ten taking place on each row.

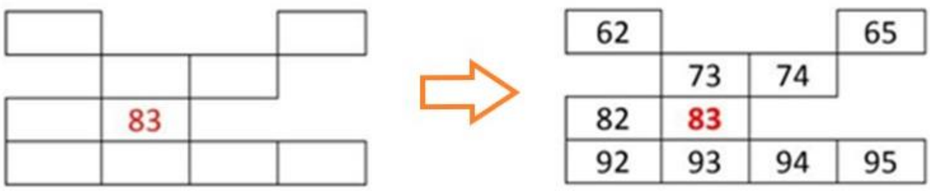
Once discovered the organization of the table, we ask him to place the figure on different numbers (i.e. 75, 80, 10, 26) to see if he identifies each number name, and later we switch roles and the teacher places the figure, which makes the child say aloud the numbers empathising on the sixties and seventies that lead him to the confusion many times. Later, the teacher tells him to move up and down by groups of 2, 5 and 10, and right after, a certain amount of places from a particular number with few examples, for instance, '*starting on the 31 let´s move forwards 11 places, to which* *number have we arrived?. And if we go backwards now 5 places, at which number station we are?'.* This helps the child realize how to move on the table with the vocabulary of moving forwards (adding) and backwards (subtracting) by counting number places that are traduced on the mathematical expressions of 31+11=42 and 42-5=38, which is worked later in the activity: *Let´s use math language to move on the 100 Table*. This activity helps him to begin establishing connections between the vocabulary keywords and the maths expressions.

The *second activity* consists in *deciphering the hidden numbers of the 100 chart* found in annex X, in which the child needs the reasoning and understanding of the ordering of numbers to accomplish the activity and decode the missing numbers. On the third activity, the learner builds his own table by assembling the different pieces of the 100 Chart puzzle placed in annex XI, and discover the four missing numbers.

The following exercise consists in completing the cross numbers by filling the number gaps as shown in figure 16 (the activity on annex XII), where the child has to first localize the part of the 100 chart that corresponds with the cross number and fill the rest numbers using the strategies of knowing that, for instance, the previous number (moving to the left) for 83 is 82 and the following after (moving to the right) is 84, but also that the number on the row right under (moving down) is calculated by adding a Ten= ten numbers, and the one right above (moving up) is one Ten less. He can use as helping tools the window tool (see annex IX) and the 100 Chart in the beginning and gradually remove them.

Figure 16.

Example of a cross number and its solution



If the child presents many learning disabilities and has problems understanding the established Hundreds Chart, then we would work with the table constructed from the bottom upwards, indicating that the higher we are in the table, the higher the numbers are.

The *last activity* to be done deals with *moving through symbolic mathematical expressions* around the 100 Chart. Therefore, the teacher explains with an example (17+42 which means start on the 17 and go 42 places forwards). Then, the kid starts counting each place, but later he has to use the strategies of knowing that moving one row up/down means 1 ten less/more and one place right/left means adding/subtracting a unit, which directly links to the additive decompositions of numbers worked on the previous lesson. The teacher later proposes different operations and the kid has to tell us what he is doing and its result, as it can be seen in the following examples:

- 12 + 5: means start in 12 and count 5 units to the right making 17
- 37 – 9: means moving backwards 9 places making 28
- 20 + 34: move 3 rows down at once (3 tens) and right 4 places making 54

An *extension activity* to do in pairs (see annex XIII) is the *Bingo dictation game* which consists in creating a figure using the numbers of the 100 Chart. Once coloured it, the child strategy is to ask in turns his partner different numbers to see if those are part of the partner drawing to guess it as if they were playing Battleship, but instead of discovering the ships, they discover the drawings making the learners work the naming of numbers.

Once the description of activities has been done, we proceed to reveal if the lesson objectives have been achieved in the following results scheme (see table 5).

Table 5.

Results scheme lesson 2 – Hundreds Chart

Objectives evaluation	Achieved
1. Write, read and name the natural numbers until 100 correctly	Yes
2. Order logically the first hundred natural numbers properly	No
3. Reinforce orders of Ones and Tens of the decimal positional system and the equivalence of 1 Ten to 10 Ones.	Yes
4. Introduce the order of Hundreds of the decimal positional system and understand the equivalence of 1 Hundred to 10 Tens and to 100 Ones.	Yes
5. Move forwards and backwards on the hundreds table: 2 by 2, 5 by 5, 10 by 10	Yes
6. Perform simple operations of addition and subtraction when moving on the hundreds chart	No

The results scheme shows that the child has accomplished the objectives that were developed during the first session because on the second session he did not want to work at all making it only possible to prepare the transition from the hundreds chart to the introduction of our decimal positional system using the tally system from a concrete abstraction level at first. If the child has understood that moving a number 2 Tens is moving 2 rows of ten or 20 number places, he can relate that idea with quantity expression making 2 Tens equal to 20 toothpicks in our tally system. In the first session when having to begin the motion through the table, we realized that it took him time to understand how to move by specific groups (by 2, by 10); therefore, we thought that in order to facilitate the movement, on the next session, when performing operations in a non-traditional way, it could be used the hundreds chart window tool (see annex IX). The numbers order worked by the cross numbers and puzzle could not be performed because the child lost motivation and decided not to work as it happened with the non-traditional algorithms of addition and subtraction integrated into the table.

Lesson 3 – Why the place value of a number so important?

As we were creating and designing the proposal, we realised that there had to be a previous session to the algorithm operations, which made sure that the learner understood completely the decimal positional system and its arrangement into groups of 10. Actually this lesson became a very important part of the proposal.

The goals for this third lesson are: understand the value of a digit depending on its position in the number (place value), identify the different order of the positional system (Hundreds, Tens, and Ones), and lastly, be able to represent a specific quantity within the decimal positional system at each of the levels of mathematical abstraction. All this is intended to be endeavoured during a single session.

These objectives are contextualised within the framework of the curricular contents of decimal numbering system: *"rules of number formation and place value"* and the curricular learning standards of "*counting drawn objects, identify the place value of digits and reinforce the writing of numbers with digits and letters, and equivalences between Hundreds, Tens, and Ones"*.

At the *beginning of the lesson*, the teacher explains how our decimal positional system works giving the child *only a rule*: each time you have 10 of something you group them with an elastic (10 toothpicks '10 Ones' = 1 group of 10 toothpicks '1 Ten'; or 10 groups of groups of toothpicks '10 Tens'= 1 super-group '1 Hundred'). For this activity, the materials needed are the decimal positional system table (annex XVII), toothpicks, elastics and the number flashcards (annex XVIII) pictured in figure 17.

Figure 17.

Decimal positional table tool with sticks and flashcards

After having explained the rule, the teacher gives him firstly 8 single toothpicks. Then he is asked if he can group them attending the 'rule'. This exercise is made secondly with 15 toothpicks. The learner realized that the 15 could be grouped in a group of 10 (making a Ten) and leaving 5 single ones. In this process, the child has to identify the number flashcards that (together) represent the amount of toothpicks and name it. Next, it is necessary to see if the child knows how to go from number to sticks (concrete representation), for which we give him a number trace on the cards and he represents it with sticks. This enables us to show him the symbol 1 in the different positional orders and ask him about the difference between having a 1 on the Hundreds, on the Tens or on the Ones. He answers by using the concrete representation and saying that a 1 on the Hundreds is a super-group which equals 10 mini-groups and 100 toothpicks, therefore, he understands that even though the symbol itself is the same in all of them, the position in which it is placed makes a difference, which links with the theoretical idea that the value of a digit depends on both its own value and its position in the number.

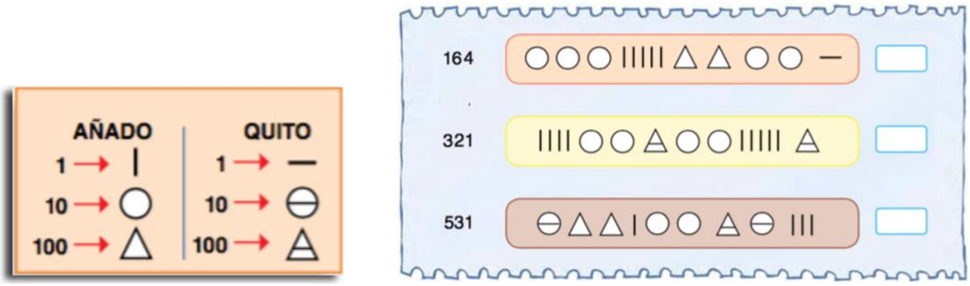
Afterwards, we want to stress the importance of the position of a number by asking him the next questions followed by his answers:

- What is the use of what we have done? '*Because it is easier to see 7 groups of 10 and 4 single sticks than having to count 74 things one by one to know the total*'
- Why is it important? '*Because otherwise we had to be counting every time everything and it would be very tiring'*
- What other civilisations took into account the grouping of numbers and used their own way of grouping? Give an example: *'Mmmm.. I do not know'* '
- Did Romans have the same numbers as we use? How did they group them? *'Oh yes, they are different to ours, they used letters like I, V, X'.* To what the teacher adds: *'Yes that is it, it was their way of grouping: each time they had a V meant 5, or an X equalled 10'.*

Later, we show him how the Roman numerical system works to help the child establish connections with our decimal positional system. After having mentioned mathematical codes among other civilizations it is the turn of the learner to work and create his own numerical system using the legend of the exercise throughout the activity of *'Let´s create our own mathematical code'* which implies the decoding of different encrypted numbers made up of symbols based on a legend as it is shown on figure 18, and an additive system rule (the final value of the number is the sum of all the values of the symbols used). For example, as seen in figure 18, the starting number is 164 to which we have to add 3 circles at first that according to the legend would be 30, later 5 sticks=5, 2 triangles=200, 2 more circles=20, and taking 10 away as it is shown by the hyphen getting a total of 418 (164+3*10+5*1+2*100+2*10-1*1=418). The child has to discover these numbers and read the numbers aloud. Later, he decides the number he wants to start with and the encrypted additive code to discover the final number or total (see annex XIV).

Figure 18.

Mathematical code activity and legend



Note: Modified from https://www.actiludis.com/2017/10/23/iniciacion-la-abstraccion-metodo-abn/

In this way, he sees and works with another way of numerical representations leading to a better understanding of our own.

Now, we proceed to show up whether the lesson goals have been achieved in the following results scheme (see table 6).

Table 6.

Results scheme lesson 3 – Place Value

Objectives evaluation	Achieved
1. Understand the importance of the position of a number within the positional system	Yes
2. Identify the different orders of the positional system (Hundreds, Tens, Ones)	Yes
3. Understand and assimilate the meaning of each digit in the writing of a number in a specific position (numerical place value)	Yes
4. Be able to represent a specific quantity within the positional system at each of the levels of mathematical abstraction	Yes

As the table shows, the learner achieves all the objectives worked on the lesson which implies that he has understood the functioning of the decimal positional system. This can be seen when he gave the teacher the following example: 66 is 6 Ones (alone sticks) and 6 Tens (groups of 10 sticks each one or 60 alone sticks).

However, in order to be able to evaluate if the child is making a progression as the materials are useful, or if the child is experimenting difficulties and then maybe the materials need to be adapted to it, it was necessary to design some rubrics for the different core lessons. In particular, for the place value practice, we have the rubric in annex XIX. It includes aspects from the objectives of the activity, the motivation and motor skills of the child at the beginning and during the activity to the ability to convert a number into its representation with sticks and vice versa, and finally, as an example, whether the child understands the number value within the place value system. Needs to be highlighted that we have created similar rubrics for the addition and subtraction lessons (found in annex XX and XXI) because more than a powerful tool helping the teacher to evaluate the child evolution, it also enables an auto evaluation of the practice itself, detecting if the proposal needs modifications or adaptations to suit the child needs.

The following table in figure 19 (see annex XXII for more detail) shows the recording of results obtained during the practice based on a rubric designed according to the different items mentioned previously which enables a closer following of the case study showing either an evolution or detecting some learning difficulties suffered by the child during the lesson. In this case, we can see that child lacked motivation at the beginning of the activity, but along the session, he understood the tally system mechanism, which implied a raising motivation.

Figure 19.

Results of the Place Value Lesson Rubric

Objetivos de la actividad	Evaluación final actividad: ¿Objetivos completados?		Observaciones
1. Entender la importancia de la posición de un número dentro del sistema posicional	SI / NO	**Motivo**: ❑ Falta de Tiempo ❑ Falta de comprensión de la actividad ❑ Entiende que la posición cambia el valor de la cifra, pero no entiende la razón. ❑ Más sesiones necesarias ❑ Poca/mucha motivación de la actividad ❑ Otro:	
2. Identificar los diferentes órdenes del sistema posicional (centenas, decenas, unidades)	SI / NO	**Motivo**: ❑ Falta de Tiempo ❑ Falta de comprensión de la actividad ❑ Más sesiones necesarias ❑ Poca/mucha motivación de la actividad ❑ Otro:	
3. Comprender y asimilar el significado que toma cada cifra en la escritura de un número en una posición concreta (valor numérico posicional) *Cuando tenemos 10 unidades se agrupan en 1 decena en el sistema numérico decimal*	SI / NO	**Motivo**: ❑ Falta de Tiempo ❑ Falta de comprensión de la actividad ❑ Entiende que la posición cambia el valor de la cifra, pero no entiende la razón. ❑ Más sesiones necesarias ❑ Poca/mucha motivación de la actividad ❑ Otro:	Me da el ejemplo del número 66 que se compone de : - 6 decenas = 6 grupos de 10 palillos cada uno = 60 palillos sueltos - 6 unidades (palillos sueltos)
4. Ser capaz de representar una cantidad concreta dentro del sistema posicional en cada uno de los niveles de abstracción matemática e identificar sus órdenes concretos.	SI / NO	**Motivo**: ❑ Falta de Tiempo ❑ Sabe hacerlo pero no explicarlo ❑ Sabe representar de manera concreta ❑ Sabe representar de manera simbólica, pero no grafica o pictórica ❑ Poca/mucha motivación de la actividad ❑ Otro:	

Evaluación durante la actividad				Observaciones
¿Cómo es la actitud del niño?	Emocionalmente	Antes de empezar la actividad su disposición es buena	SI - NO	Viene dormido, pocas ganas de trabajar. Luego, se mete en actividad.
		Durante la realización de la misma, ¿le motiva la actividad?	SI - NO	Metido en actividad se ve capaz
¿Maneja bien los palillos?	Motricidad	¿Dificultad encontrada?	SI - NO	Se le resbalan los palillos un poco, pero cuando pilla el truco buen manejo
¿Comprende el valor numérico en el sistema posicional?	Comprende que no es lo mismo un 1 en las unidades que en las decenas, lo cual que implica que es un grupo de 10		SI - NO	
¿Es capaz de pasar correctamente de ... ? *(Número o símbolo = grafía)*	Número → Palillos	Representa el número correctamente usando los palillos	SI - NO	Dificultad entre sesenta y setenta, hacer posterior hincapié y comprobar en suma y resta
		Verbaliza correctamente el número	SI - NO	
	Palillos → Número	Elige o escribe la grafía correcta que representa la cantidad de palillos dados	SI - NO	
		Verbaliza correctamente el número	SI - NO	Dificultad entre sesenta y setenta
		Escribe el nombre del número correctamente	SI - NO	No trabajado : trabajar en suma y resta
En el manejo de palillos ¿entiende que con los números se pueden realizar diferentes operaciones de agrupación, partición, multiplicación o diferencia?			SI - NO	
Agrupación de unidades al orden inmediato superior - ¿Agrupa de manera espontánea en grupos de 10? *Regla: Cada 10 agrupo (goma)*	SI			Entiende regla perfectamente
	NO	*Frecuencia* - ¿Cuántas veces le ayudo hasta que él lo integra? Primero un ejemplo de prueba para enseñarle y luego él lo integra perfectamente.		
¿Qué me muestra que entiende que es capaz de realizar la actividad?	- Junta en grupos de 10 antes de situar los palillos dependiendo el tipo de grupo sabe que va en una columna u otra (órdenes identificados) - De 10 en 10 → creación de grupos - Me verbaliza que no es lo mismo tener un 6 en unidades (6 palillos) que un 6 en decenas (6 grupos de 10=60 palillos) - Importancia de la agrupación de números porque se hace más sencilla y rápida la representación de la cantidad usando los diferentes órdenes (decena, unidad) frente a tener todos los palillos sueltos.			A la hora de hacer código matemático está cansado y entonces hay dificultad para nombrar y descubrir los números sin las tarjetas. Le cuesta mucho ese cálculo mental.

In general, the results show that the pupil has understood the tally system and the place value importance. However, when it comes to mental calculation on the mathematical code lesson, the child is unable to add mentally 10 or 100 to a given number. He needed to use the decimal system table to write down the sums. This could be because of his tiredness at the end of the session.

Lesson 4 – Why and How do we perform additions?

Once the child has understood how the material works, it is time to start operating using it. One of the most important aspects when teaching children arithmetical algorithms is to give them meaning, in other words, contextualise the operation with situations that children can find every day in their real life, so they can understand the real meaning of adding, subtracting, multiplying or dividing. In this way, the pupil is learning and building a relational understanding of the operations instead of memorizing tons of procedures that in a near future will be forgotten. Therefore, during this lesson and the following practices, the teacher always immerses the operation in a story or word-problem, which will provide meaning to the operation.

This lesson is set out to be undertaken in one session because once he has understood the functioning of our tool the only difference between the performance of addition without or with regrouping is replacing the group of 10 formed to the column of Tens. Firstly, it aims to reinforce the understanding of the positional system and the positional numeral value that each digit in a number has when it is written. The other targets are to understand that adding (in the natural numbers context) means putting together two parts (which measures are the addends) to get the whole (which measure is the sum); to be able to perform the addition in the way that makes more sense to the learner; explain what and why he is doing it; and lastly, recognize the addition in different situations in his life.

All mentioned above is linked with the curriculum contents of "*oral and written mathematical expression of operations and the calculation of addition; use of standard addition algorithms; identify and use numbers in real situations: reading, writing, ordering, decomposition, comparison …); and reinforce the idea that adding is putting together*". On one hand, regarding evaluation criteria, we have "*the performance of basic numerical calculations with the operations of addition using different strategies and procedures*", and ultimately, the learning standards of "*identifying the terms of addition and reinforcement of count drawn objects, writing of numbers with letters and digits, and a calculation of the missing term in an addition or subtraction of the type: 7+? =15*".

On the other hand, we also develop the ability to create and solve arithmetical problems for integrating the addition and subtraction operations within a real-life situation, providing meaning to the operations. This appears in the evaluation criteria of "*the identification and resolution of everyday life problems, establishing connections between reality and mathematics"*. We also value the use of appropriate mathematical knowledge for solving problems; and, the learning standards of "*posing and solving first-step additive-subtractive problems (one-operation), the application of numeration notions in solving arithmetic problems",* and finally*, "the determination and relation of data, questions, statement or operations in a problem situation"*.

The lesson starts with the refreshing of the rule: each time the child has 10 items of something he has to group them (10 sticks → 1 elastic), and identify the decimal positional orders with their equivalences. Later, as it happened in the Place Value lesson the teacher gives the child *two decks of different amounts of sticks that do not require regrouping*. Firstly, the child is asked what he can do with both decks for adding them, to which he answered that he puts them together to get the result. So he discovers that adding is putting the sticks together. After discovering that there are different parts in an addition, we name them (both addends and the total). Later, he is asked to give the number for the specific representations and execute the operation both in the concrete and symbolic abstraction level to see if they can be regrouped or not. In this case, he realises that with those numbers, as ten is not reached, he does not have to regroup.

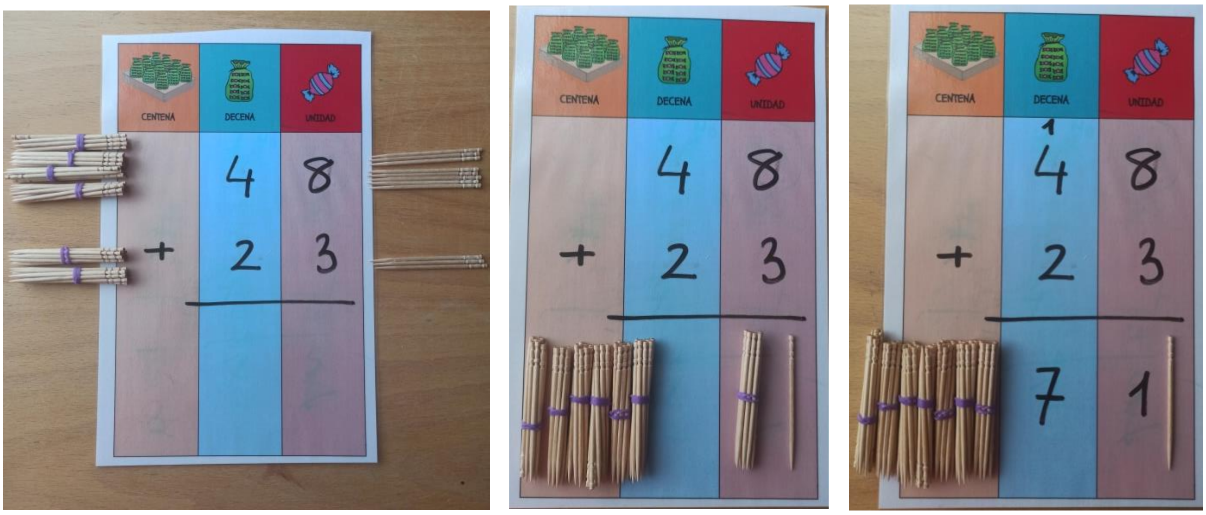
Then, based on the Singapore model for the addition algorithm execution, the teacher gives the pupil different examples of *two decks of sticks which addition requires regrouping*. For instance, if he has 15 sticks (1 Ten and 5 Ones) and 9 sticks, he gets 24 Ones, which gives another Ten and 4 Ones. In the end, he has 2 Tens and 4 Ones left after the regrouping. This process is done with different examples in both directions passing from the representation to the number and vice versa, asking the pupil also to write some of the numbers with letters.

When performing with sticks there is not a specific division between adding with or without regrouping because the only change experienced is that each time the learner gets ten in a specific order, he groups it and relocates it with its equals. Therefore, it is performed in a very instinctive way as you can see the process (shown in figure 20).

Figure 20.

Process of addition with regrouping using toothpicks and table tool

1º 2º 3º



It should be highlighted that the teacher always has to provide *meaning to the numbers* involved integrating them into a real-life context. For example, with the following before-after situation: *Ash had on Monday 4 Pokémon and, the next day, and he got another 3 Pokémon. How many has he had after the second day?*

In this lesson, the situations are worked both verbally and written, but always with the support of cards that picture the situation simply and graphically along with some vocabulary in the back. This helps the pupil to develop his own stories using the operation requested (see annex XXV for all the possible scenarios). Some card examples of situations, for the stories or word-problems in which you immerse the operation of addition, are shown in figure 21 (a part-whole scene in which the total is unknown) and figure 22 (a before-after scene, being the latter image original from Soy Visual).

Figure 21.

The front and back of the card 'part-whole relation' in a Pokemon scenario

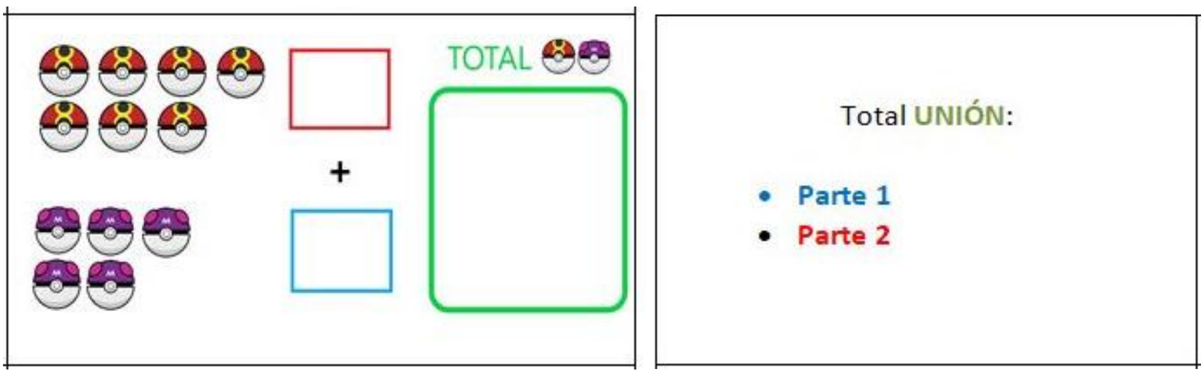
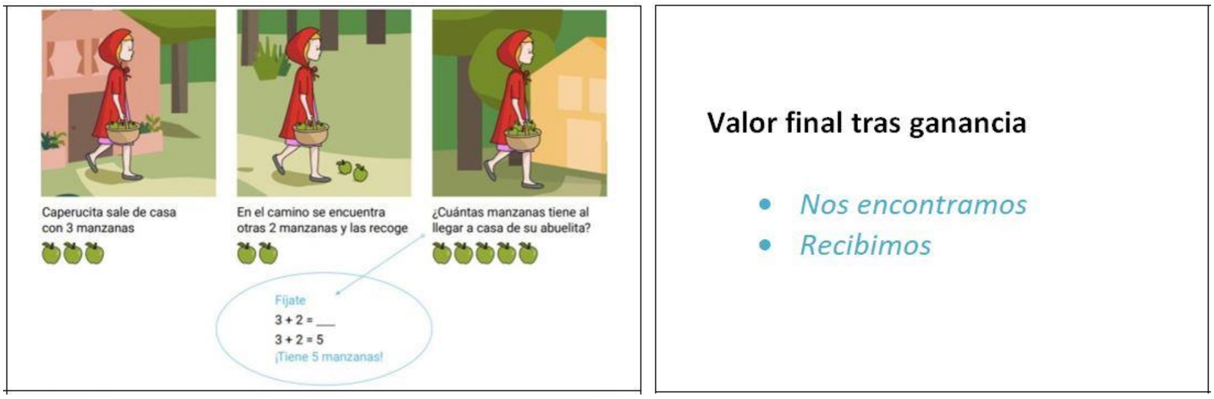


Figure 22.

The front and back of the card 'before-after relation' of a win-win scene



In this way, it can be seen the development of the lesson and its activities. Once having them described, we proceed to show up in the following results scheme (see table 7) whether the activity objectives have been acquired according to the rubric found in annex XX.

Table 7.

Results scheme lesson 4 – Addition

Objectives evaluation	Achieved
1. Strengthen the understanding of the positional system and the meaning that each digit takes in writing a number in a specific position (positional numerical value) through the exercise of the addition	Yes
2. Understanding that adding is grouping or putting together both parts of the addition (addends) to get the whole (sum or total)	Yes
3. Be able to perform the sum in the way that the learner considers most logical and explain what and why he does it.	Yes
4. Recognize the addition in different situations in his life	No

As we can see, the learner accomplishes almost all the objectives, except the recognition of all the different sample situations. The child only identifies the situations of parts making up the whole and a win-win scene in a before-after relationship but lacks the identification of contexts: the initial value after a loss scenario and the calculation of the referenced knowing how much greater than the referent (number of units) it is. Then, it can be seen that he is making a natural evolution and following the steps that all children undertake (discover the whole → in win/loss scene: discover the end → comparison scene: discover the referenced → parts-whole scene: discover the parts → comparison: discover the difference or rest → win/loss situation: discover what was won or lost → comparison: discover the referent → win/loss: discover the beginning). Thus, he has obtained two of the steps in a single session. He showed that the identification of the situations was better developed than the formulation which is directly related to the progression of the acquisition of these mathematical skills and knowledge. Figure 23 pictures the addition rubric which shows the child results according to the identification and formulation of contexts for the addition operation, which is an aspect that differs from the previous rubric of the place value results. To see the results recorded during the addition lesson go to annex XXIII.

Figure 23.

Rubric part of the identification and formulation of addition situations

Identifica la suma en distintas situaciones de su vida (docente da significado al algoritmo y contextualiza): a) Como el cálculo del total de la unión de las partes b) Como la magnitud (referido) que es unidades mayor que otra menor (referente) c) Como el valor final tras una ganancia d) Como el valor inicial tras una pérdida	a) SI –NO b) SI –NO c) SI – NO d) SI – NO
Formula o relata situaciones posibles que den sentido a la suma: a) Como el cálculo del total de la unión de las partes b) Como la magnitud (referido) que es unidades mayor que otra menor (referente) c) Como el valor final tras una ganancia d) Como el valor inicial tras una pérdida	a) SI –NO b) SI –NO c) SI – NO d) SI – NO

Due to the little predisposition of the pupil to work along and the lack of time in the session, he could not integrate all the addition contexts in real life, therefore, we would break the lesson into two sessions taking more time for the understanding of the addition performance with and without regrouping in a concrete and symbolic abstraction level and make the child pose the operation within contexts so it helps him formulate the scenes. We thought about interesting and close situations to the child where he could add up as things to but in the videogame “Fortnite”.

During the session the child did not grasp completely the addition with regrouping and formulate stories with it, thus, we decide to work on it in a second session; however, the child was very little predisposed to work and it was impossible to get to work with the situations cards, so we did the *Detective Number game* to motivate him towards the rest of the class. This activity (see annex XV) consists of discovering the chosen number after following the clues provided by the detective and that the number has to verify: for instance, being even or odd, number of digits, the sum of digits and divisibility of the number. This provides a revision of previous concepts. We discovered that using the addition for measuring objects in the class motivates him.

Lesson 5 – Why and How do we perform subtractions?

This lesson is set out to be undertaken in two sessions: one for the performance of subtraction without regrouping and the other one with regrouping. It aims to reinforce the understanding of the value of each digit in a number through the subtraction execution. The other targets are to understand that subtracting is taking a part away from the minuend and see how much remains; be able to perform the subtraction in the way that the learner considers most logical; explain what and why he does it; and lastly, recognize the subtraction in different contexts in his life.

The goals above link with the curricular contents: *"the oral and written mathematical expression of operations and the calculation of subtraction, calculation of subtraction without carried numbers using the academic algorithm, use of standard subtraction algorithms, identify and use numbers in real situations: reading, writing, ordering, decomposition, comparison, …) and reinforce the idea that subtracting is taking a part away from the total"*.

Regarding evaluation criteria, we have "*the performance of basic numerical calculations with the operations of subtraction using different strategies and procedures",* and ultimately, the learning standards of *"understanding and using of expressions double, half … in different contexts; reinforcement of count drawn objects; writing of numbers with letters and digits; and a calculation of the missing term in a subtraction of the type: 7+? =15".*

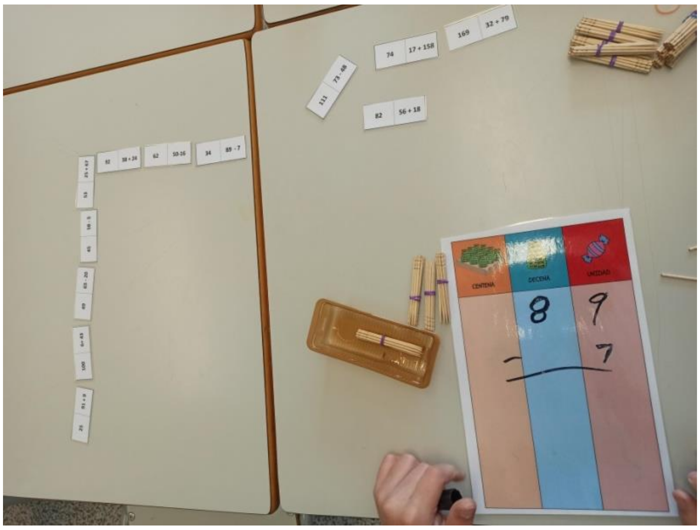
Besides, during the practice we go on practising the arithmetical contents and abilities developed in the previous lesson of addition using the subtraction operations immerse in real-life contexts, for which the teacher gives the child the situations cards.

The lesson development is similar to the addition one. Therefore, we are not going into further detail on the functioning of *the tally system with the sticks* in the execution of the operations; however, it is explained below how it is done with the pictures support, worked through play and integrated within real contexts close to the learner reality, so he can identify the subtraction in real life and create his own word problems, which later will ease him the process of arithmetical problem-solving. Firstly the teacher asks the child if he can recall a situation of his life in which he had to undertake a subtraction operation. The pupil answered: "to know how much money is left after buying a suit in *Fortnite* or knowing if I have enough to buy it and if not how much more I need".

For this lesson, the teacher has thought that once understood how carrying works in arithmetic operations, it is the moment to integrate the operations of subtraction within a game to work through the play, which is developed through a *mathematical domino* made of additions and subtractions (example in figure 24 and for the complete activity go to annex XVI), which allows the learner to see if the operation's solution is correct: if he can find it in one of the pieces which marks the following piece to solve. This enables a review of previous concepts learned in the addition, helps the child to realize that what works for addition also does for subtraction. For instance the regrouping: "each time you have ten put an elastic around it and place it in the correct new order". To perform the operations, the pupil uses the decimal table tool with the marker, the sticks, and the boxes that act as holes to be filled in the subtraction.

Figure 24.

Arithmetical domino of additions and subtractions

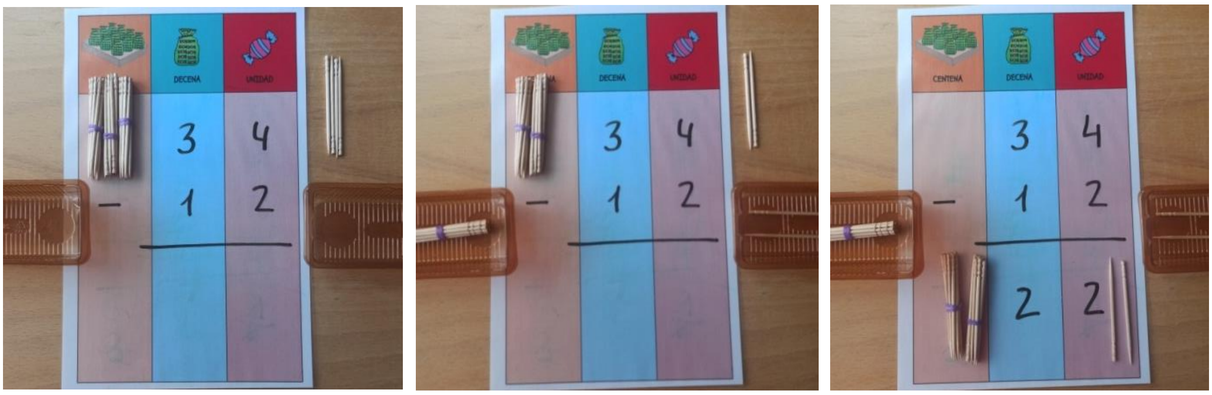


To operate with the subtraction during the lesson, we use the tools previously mentioned using toothpicks for the minuend, and boxes for the subtrahend, which act as the holes to be filled to get the rest or difference: the remaining part after having taken away the number of sticks determined in the subtrahend. The previous idea explains the execution of the algorithm without carried numbers because there are enough sticks to be taken apart as it is indicated in the subtrahend; an example of it is figure 25 which shows it in concrete and symbolic abstraction level.

Figure 25.

Process of subtraction without regrouping executed with our tally system

1º 2º 3º



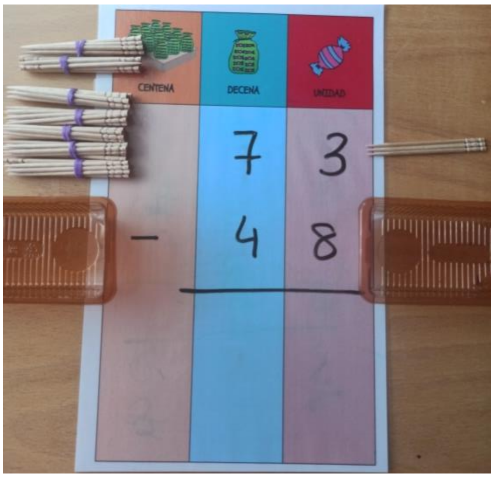
We believe that for children it is more natural and instinctive to execute and understand the subtraction operation with regrouping throughout the lending algorithm rather than the borrowing model that is much more difficult to be understood, as it uses more arithmetical properties. The lending model is supported by the Singapore methodology, which develops a mathematical relational approach and understanding to this particular arithmetical operation. Besides, this model is the one that suits perfectly with the functioning of our decimal tally system.

During the second session, it is delved the *algorithm that implies regrouping* and needs to open groups from the next upper order, in our case, it is the Tens order, which in the concrete level corresponds to the mini-group of sticks. Therefore, whenever in the minuend there are not enough sticks to be taken away to the boxes as the subtrahend indicates, a Tens group goes to the Ones and is opened. Afterwards, the execution is as non-regrouping subtraction. Then, the child has to remember that he has moved a Ten to the Ones when he operates on a symbolic level; however, when using the toothpicks, this step has already been done when needed to open a group to fill the holes with the proper number, making this process very instinctive (see figure 26). In this example, it can be seen that using this tool enables the child to benefit from the concrete level to understand the process, but also helps him to traduce the process into the symbolic language and gradually integrate it leaving aside the tools.

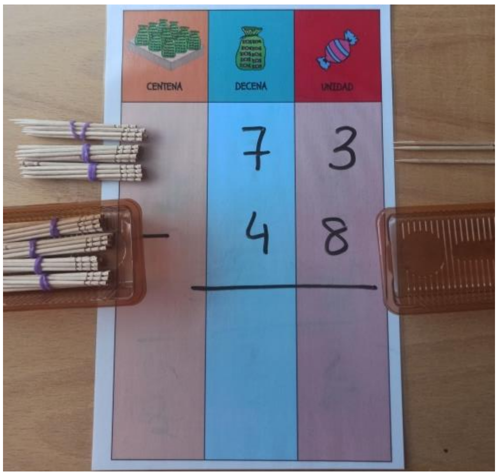
Figure 26.

Process of subtraction with regrouping executed with our tally system

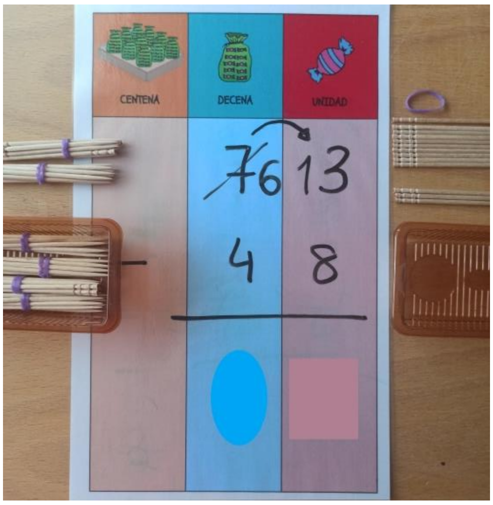
1º



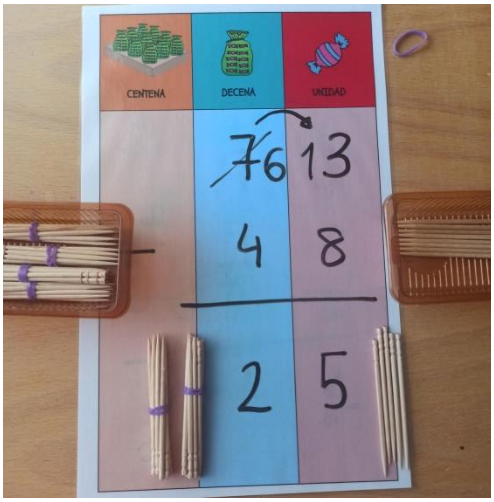
2º



3º



4º



As it was done in the addition, the teacher presents all these operations of the domino and examples models of the subtraction algorithm always immersing them *into a story* which gives meaning to the operation as the before-after example collected in the final level evaluation, in which the first part is an addition: *Ash had on Monday 4 Pokémon and, the next day, he gets another 3 Pokémon. How many Pokemons has he got the second day?;* the second part is a subtraction: *On Wednesday, 2 of the Pokemon left. How many ones stayed on Wednesday with him?* This helps the child to identify the algorithms in an interesting context for him. Then, we use the scenes cards as the examples given in figure 27: calculation of the height difference of 2 Pokemon (additive comparison relation), figure 28: final value of a before-after situation after a loss (Red riding hood image original from Soy Visual), and figure 29: calculation of an unknown part knowing the whole and the other part. The rest of the possible situations in which subtraction is performed can be found in annex XXVI.

Figure 27.

The front and back of the card 'additive comparison of unknown length difference

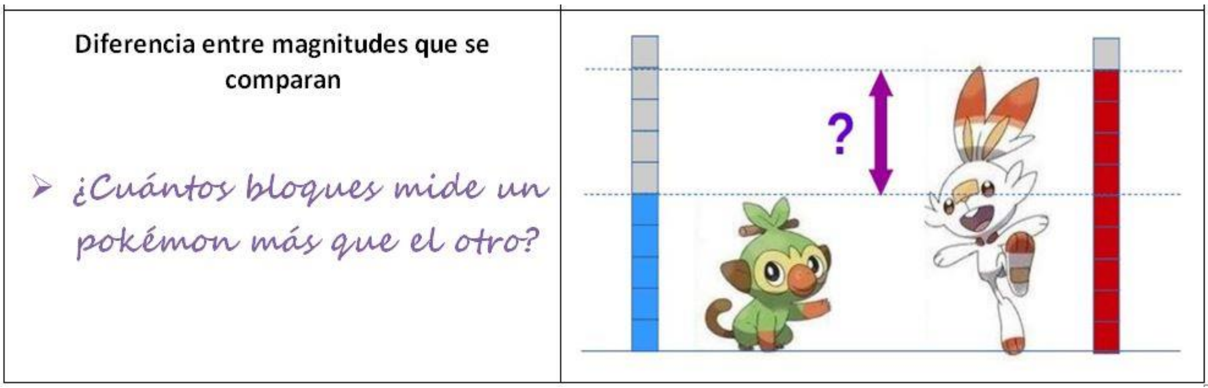
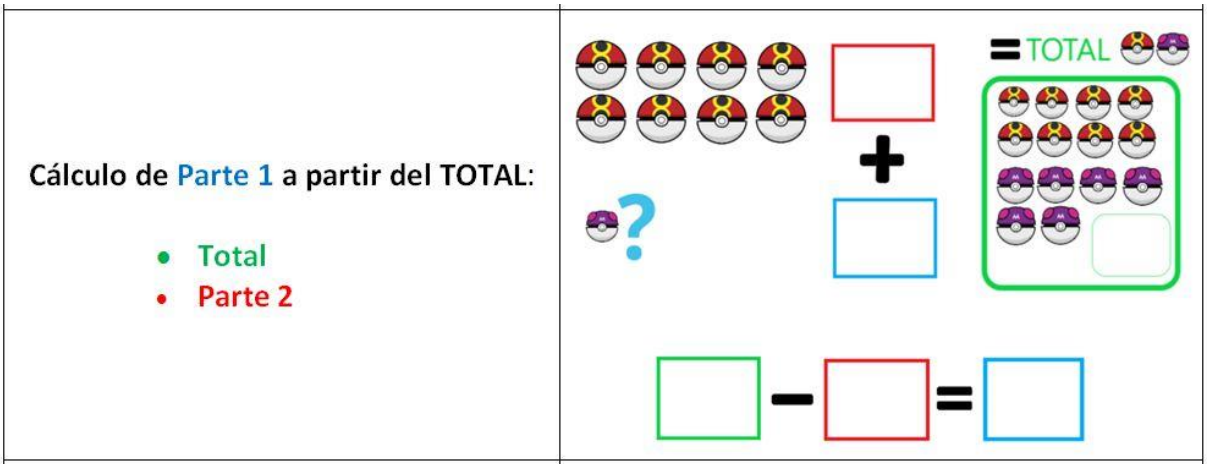


Figure 28.

The front and back of the card 'before-after relation' of a loss scene

Figure 29.

The front and back of the card in a Pokemon scenario (unknown part)



After having explained the development of the lesson, we go on to reveal in the following table 8 whether the objectives have been achieved according to the rubric found in annex XXI.

Table 8.

Results scheme lesson 5 – Subtraction

Objectives evaluation	Achieved
1. Strengthen the understanding of the positional system and the meaning that each digit takes in writing a number in a specific position (positional numerical value) through the exercise of the subtraction	Yes
2. Understand that subtracting is discovering what remains of the minuend when the subtrahend is taken away from it (subtrahend and rest are the two disjoint parts into which the minuend is separated)	No, more sessions are needed
3. Be able to carry out subtraction in the way that makes the most sense to the learner and explain what and why they are doing it	Yes
4. Recognize the subtraction in different situations in his life	Not all of them

As it can be seen half of the objectives are achieved, but due to the difficulty of the understanding of the subtraction concept and its recognition in real-life model situations, more sessions were needed to work on these particular objectives. Working with addition with regrouping (in the previous lesson it was impossible to be undertaken because of the pupil attitude) was also undone. If the child does not understand what grouping in ten and relocating means, he will not understand that in the subtraction these mini-groups sometimes need to be opened and relocated into the Ones to have enough sticks and be able to take away the proper amount of them by filling the holes in the boxes (subtrahend representation).

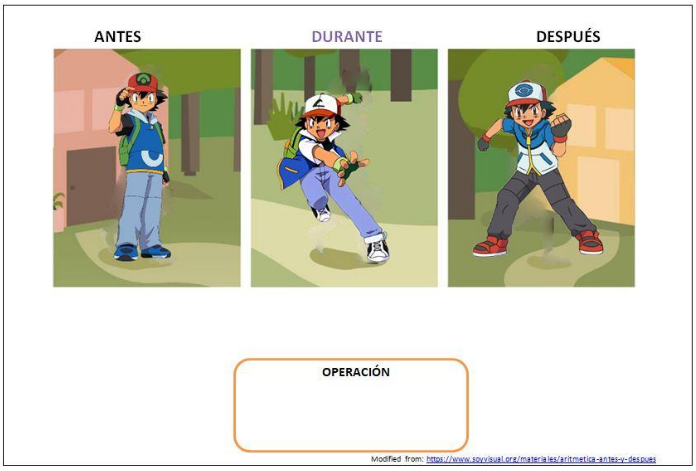
However, with the help of the manipulatives, he executes the operation perfectly but is not capable of passing to a symbolic level, and without the materials he does not perform well because he has mechanised the algorithm execution with no understanding and this leads him to making mistakes. Therefore he needs continuous help and supervision but, in this case, not only as a strategy to avoid attention lapses, but because he does not quite understand that when he puts a '1' next to the number '3' in the Minuend Ones and makes it '13', he does not interiorize that 1 comes from the Tens order and that later, it should be remembered to be taken away from them.

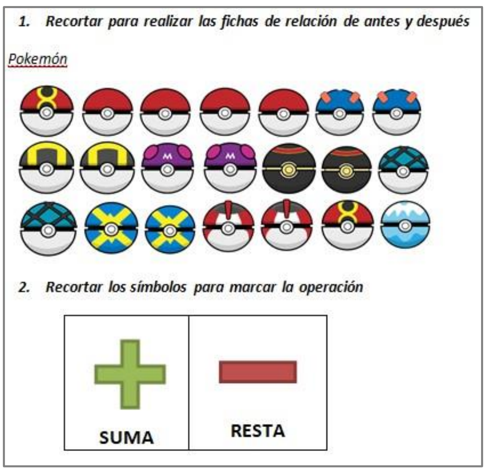
Because it requires so much effort, the teacher was aware that the subtraction causes him frustration leading him to reject it; nevertheless, he sees that the support of the tool helps him and makes him feel capable of doing it correctly and without rejection. Therefore, to release some of the pressure and frustration the teacher changed the strategy and, instead of making the child do it autonomously, she many times made the moves with the tools and the child only had to verbalize the steps to take.

Along with the frustration, the subtraction process misunderstanding and the mechanization of the algorithm made impossible to detect situations and comprehend the different meanings that subtraction takes in his life. Thus, this lesson should be extended at least with 2 more sessions to understand its applications and meanings emphasising on the continuation of the word problems work with the subtraction and using the blank cards (found in annex XXVII) as a revision of the arithmetical operations worked over the course of the mathematical workshop. Here, it should be highlighted that these blank cards pretend to work the creation of word problems with the situations that cause the pupil more difficulty such as the additive comparison and the before-after relation, which seek motivation by having modified the latter from the original of Soy Visual towards a Pokemon scenario (see figure 30). After having understood the operations situations, we intended to deepen, to see if the learner identifies in which ones addition or subtraction is performed; all this done, by mixing all the addition, subtraction and black cards.

Figure 30.

Before-after blank card in a Pokemon scenario and its icons





Furthermore, the child attitude changed when working through the play which reduced his frustration towards the subtraction execution along with the tool support. All these results explained above are recorded in the rubric in annex XXIV.

Final Lesson - Assessment Evaluation

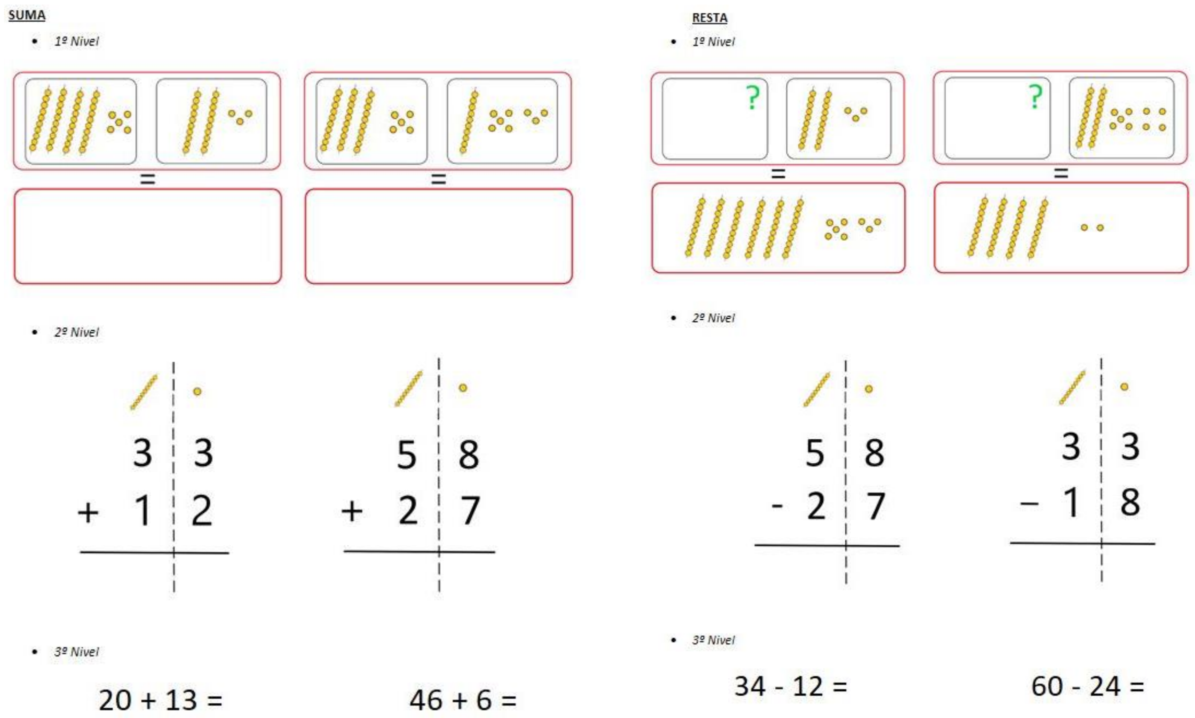
After having implemented the learning proposal, there has to be undertaken an evaluation of the effectiveness and development of the mathematical workshop as well as a study analysis of the evolution of the mathematical level of the learner regarding his competencies, knowledge and understanding. At the beginning of the proposal, the pupil took a test that allowed the teacher to observe his initial mathematical level to know the starting point of the lessons and to adapt the materials to be precise and suitable for the learner. Now, to observe and evaluate his evolution and progression, a second test will be performed at the end of the mathematical workshop with the help of the materials. The taken test is the same one, at the beginning and at the end, to allow us to see if there has actually been a progression on the mathematical level of the child comparing its results in both moments.

In this last session, the *final mathematical level assessment*, found in Annex I, was performed by the pupil. Even though it was designed according to the school standards to which the child was accustomed and complete with the rubric in Annex II, we realized that it was not the most appropriate one to define the child's current status in the parameters we wanted to work on. Therefore, we adjusted and adapted the test so that it was more in accordance with what we had been working on during the workshop. In the initial assessment, there were too many operations to solve which was unnecessary and distanced us from the point of analysis of whether they know how to do the algorithm. So, we chose just a few and developed them from different abstraction level as it is explained below. Additionally, the problems chosen in the first test were a bit confusing because the drawings lead to more distraction than aid because they were not supporting the problem situation at all and, in any case, they should be visual representations of the problem story. For example, having in the first one: two birdcages with 3 and 4 birds, instead of the Toy Story drawing). Thus, in the new test design, we created our own problem stories and left a space for the child to represent the story told in the problem instead of asking them to list the problem data and put just the operation (against problem-solving approach developed by Pòlya(1945)).

The new assessment (see Annex III) is designed according to three different levels corresponding to the abstraction levels of the CPA approach (concrete, pictorial and abstract or symbolic). As it can be seen, in figure 31, in the first level we find the addition and subtraction operations pictured with the visual representation of the decimal tally system worked during the proposal (toothpicks and elastics). In a second level, we go to a more abstract performance of the arithmetical operations, but the numbers are placed under their respective order. Lastly, in a third level, the algorithms are found in a linear form, so that the pupil has to decide how to operate them, which allows the teacher to observe the strategy chosen for its resolution and observe if he knows how to place the number digits under their respective orders. When the additions do not require regrouping, each digit is smaller than 5 to enable the child to use his hands when operating with them.

Figure 31.

Addition and Subtraction Assessment sheet of the Final mathematical level



Additionally, at the end of the test, there is an arithmetical problem that includes both operations of addition and subtraction because it is a design based on a variation over time (before-after) in three different moments (Monday, Tuesday and Wednesday). This part also counts with a space where the child can represent the problem situation with a drawing using (if he wants to) the operations contexts cards worked over the project as a support. Working the resolution of word-problems in this way allows applying the problem-solving strategies developed by Pòlya.

2.3.6 Revision of the Didactic Proposal and Analysis of the case study progression results

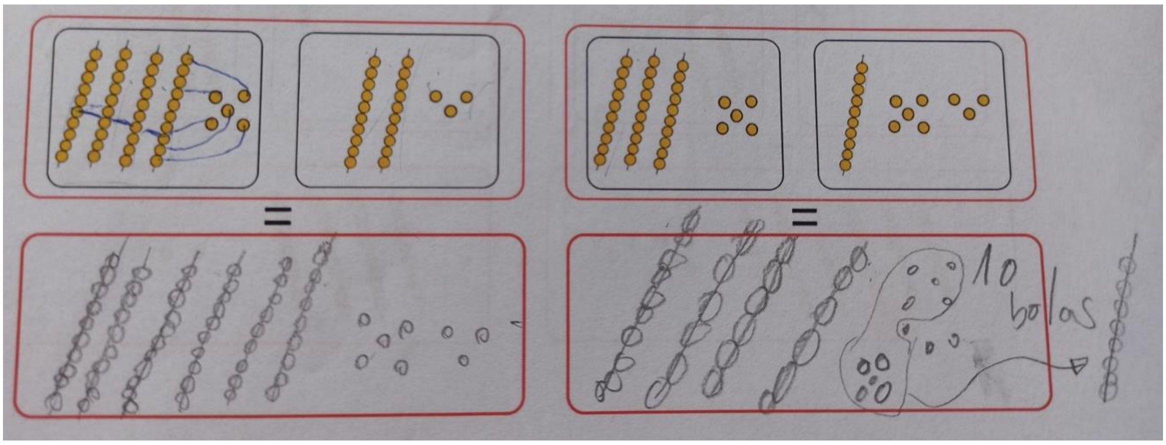
The results of the initial evaluation revealed that he struggled with the regrouping concept, especially in the subtraction algorithm, because even though he completed the additions with carrying correctly, the procedure was mechanized, contributing to the child's lack of comprehension of what he was doing and why he was pursuing that process, claiming that "it was the way he was taught." Because of this addition mechanism incomprehension, he failed in the execution and comprehension of the subtraction algorithm in all of its dimensions, from a pictorial to an abstract interpretation. He also confuses the numbers 60 and 70 when having to name them aloud and has difficulty sometimes understanding and integrating the positional value of each digit placed in a specific position (positional order).

Furthermore, due to the mechanization of the process, he could not identify real contexts situations and transfer the meaning of the operation to his real life. Besides the pupil experienced frustration whenever he did not understand and struggled with a problem, which made him strive beyond his comfort zone driving into an avoiding attitude of the mathematical problem.

After having developed and implemented the workshop for three months, the final assessment was passed to the learner. Its results showed that he had understood completely the addition operation in each of the abstraction levels and only forgot in the pictorial one to gather together the ten marbles obtained in a marble stick as a result of summing both parts (see figure 32). It should also be highlighted that when performing the addition in the 3rd level, he did it symbolically on the decimal tool using the traditional algorithm collocation.

Figure 32.

Results of the addition with and without regrouping in the 1st level



Regarding the subtraction, even though he still rejected its performance, with the help of the toothpicks, boxes and decimal tool (manipulative materials worked with), he felt capable of operating it and an instant rewarding satisfaction when solving it correctly. The subtraction with regrouping still needs to be worked longer because he only executed it correctly some of the times when he was concentrating all his attention on it and, after having done one, he just did not want to continue because it required him a lot of effort. However, he saw the material like his fingers as a learning tool that helps him when having to open a Tens group in the Ones to have enough toothpicks to take away to the boxes (lending algorithm) and just by counting what was left outside of the boxes in each of the orders he got the right result and understood the process, but did not integrate it completely to be able to perform it perfectly in a purely abstract level yet.

Due to the lack of time, the problem-solving part was unfinished, but we could ask him the problem verbally and he understood that at the beginning he had to perform addition because the character was adding more Pokemon to the previous ones when having found them in his way. Therefore, it can be observed that he had understood at least the before-after relation in addition.

When contrasting the mathematical knowledge and strategies used in the end as opposed to ones at the beginning, we see a progression and evolution especially regarding the performance and understanding of the arithmetical operations. The only aspect that requires longer work is the subtraction with regrouping, but even in this algorithm, he has experienced a progression because with the support of the materials he could solve it at a concrete level and traduced the process and solution to symbols. He has also integrated completely the importance of a number´s place value and understood that when getting a number 10 or bigger in one order he gathers ten of those order units and replaced them in their correct order.

On the other hand, the results showed that even though he identifies some of the real-life situations and gives them the proper meaning, he experiments difficulties when formulating his stories for a specific operation. However, with more deepening and slower further work with the arithmetical worksheets, he will be able to identify all the situations for addition and subtraction algorithms and formulate stories that give meaning to the operation.

Additionally, it should be noticed that only with the initial and final study of the proposal was not enough to have an exhaustive observation of the case study child; thus, it was necessary to create some observational instruments for the core lessons (place value, addition and subtraction rubrics) to be used during the implementation of the workshop to help us have a closer follow-up of the learner and to notice if the proposal needed to be modified regarding some improvement aspects adapting it better to the pace of the child's work.

According to the proposal results, during the development of each lesson, its results were recorded to examine and discover the lesson aspects and activities that needed to be improved and to detect if there was an evolution in the learner. In this way, this method and setting allowed us to go deeper on each of the sessions helping the contextualization. In other words, we believed that this was the most appropriate model for seeing the evolution of a case study. In addition, it facilitates the improvement of proposals as they are developed so that later if they are applied in other contexts or classrooms, teachers can clearly see which aspects can help them to tailor it better to their students.

To answer what we would change, in general for the proposal, would be maybe spending more previous time preparing it and promote greater integration of games because the child shows when playing that it is the time in which he really enters within the activities and performs the operations without being the main attention centre, but work with them throughout the play. Having a whole course to develop the proposal would help to go in-depth in all the aspects that could not be worked due to the lack of time or attitude of the child.

The workshop has helped him to work on mathematics from a more motivating and dynamic approach because it is really one of the subjects that requires him a big effort, which leads to a visceral rejection and a little attraction towards it. So, in this way, we have brought him closer to maths and enables him to, at least, see some applications in his daily life, as well as to enjoy playing with it.

CONCLUSIONES Y CUESTIONES ABIERTAS

Tras analizar los resultados del estudio de caso después de implantar los talleres matemáticos, respecto a nuestro primer objetivo, podemos concluir que realmente la enseñanza basada en una comprensión relacional tiene un impacto positivo en la progresión del alumno, porque le permite construir el conocimiento matemático y desarrollar sus propias habilidades a su propio ritmo y de manera significativa. Además, los materiales manipulativos diseñados realmente han actuado como herramienta de aprendizaje para construir conocimiento a través de su experiencia y sentidos lo que ha llevado al niño con dificultades de aprendizaje a verse capaz de interactuar de manera eficaz con las matemáticas.

Creemos firmemente que el modelo de aprendizaje y entendimiento que propone Skemp es una buena forma de conseguir una enseñanza eficaz donde el alumno pueda concebir las matemáticas como una herramienta que le acerca y ayuda a entender su vida diaria. Por eso, es importante que sea el propio alumno el que vaya descubriendo y entendiendo la importancia de las matemáticas como facilitadora en su vida.

Es labor del profesor acercar las matemáticas al alumno desde una perspectiva significativa y personal, donde se plantean cuestiones como el valor del concepto matemático en cuestión, por qué surgió en su día, en qué situaciones lo podemos aplicar hoy en día y para que le sirve al niño en su vida; para que así él sea capaz de crear una relación de afecto con las matemáticas. Esto permite desarrollar un aprendizaje integrador y revelador al que fácilmente podrá recurrir en un futuro.

Este proyecto nos ha concedido la oportunidad de formarnos en este otro tipo de enseñanza y permitido implantar unos talleres matemáticos en los que esta ciencia se trabaja desde lo lúdico y manipulativo para ver los verdaderos efectos y beneficios de esta metodología con niños con dificultades de aprendizaje. Esto ha resultado en una actitud positiva del niño ante las matemáticas pudiendo huir así de ese previo rechazo, donde el alumno veía las matemáticas como algo aislado, sin aplicación y que requería mucha memorización porque no las entendía. Ahora, tras la actuación, el alumno es capaz de ver el porqué y cómo son útiles estos conceptos y habilidades para su propia vida.

Además, nos ha posibilitado la oportunidad de descubrir si esta idea de talleres lúdicos, ya previamente implantados con niños con Síndrome Down en Zaragoza, podía ser aplicada en el contexto de un colegio desde el aula de PT para niños con dificultades de aprendizaje, como una metodología que acerque al alumno a esta ciencia desde un entendimiento integro de ella, para así poder reducir el desfase académico entre el alumno y sus compañeros. Además, hemos observado que muchas veces es necesario volver atrás en conocimientos para empezar la enseñanza a partir de lo que ya entiendan, creando así un andamiaje firme en su aprendizaje. Por ejemplo, afianzar conceptos como el de la suma que nos permita evolucionar hacia conceptos más complejos como la multiplicación o la proporcionalidad.

Tras aplicar este taller lúdico-matemático y ver que realmente hemos conseguido efectos significativos en el aprendizaje del niño y un cambio positivo de actitud a la hora de abordar esta materia, nos hubiese gustado ver si, extendiendo su implementación en el tiempo (1 año) y contando con más niños (lo cual permite actividades más cooperativas, de grupo, incluso más dinámicas), los cambios y efectos hubiesen sido más tangibles, significativos y observables. Es algo que valoramos como muy probable. Este año con las restricciones por la pandemia COVID-19 ha sido imposible aumentar el número de niños en las actuaciones, pero sería uno de los cambios que realizaría en un futuro porque pensamos que tendría muchos beneficios hacerlo, ya que incluso los propios alumnos se pueden nutrir unos de otros y motivar mutuamente para seguir intentando resolver los problemas o actividades hasta llegar a descubrir un camino de resolución apropiado.

Por ello, proponemos para futuras investigaciones la aplicación del taller dentro de la propia aula reglada a través de la asignatura de 'taller matemático' para que así se beneficien todos los alumnos del curso y donde podríamos crear diferentes grupos según el nivel matemático en el que se encuentre cada niño. La disposición a nivel logístico serían varias estaciones teniendo todas en común que trabajan un mismo conocimiento matemático desde niveles de abstracción diferentes: uno manipulativo, otro pictórico y, una vez adquiridos ambos, tendríamos la estación de manejo ya de números a nivel simbólico. Esto permite a cada alumno ir evolucionando a su propio ritmo, pero siempre desde la inclusión y sin diferenciar, porque todos están trabajando el mismo concepto. La única diferencia es que cada grupo lo haría desde un nivel diferente con fichas adecuadas a una diversidad de niveles de dificultad (ej. número de dígitos de la suma, operaciones con llevada o sin llevada, entender que es sumar o restar, etc.).

Para concluir, ratificamos la idea de que creemos firmemente que esto sería posible y que beneficiaría a todo el alumnado creando una atmosfera de entretenimiento lúdico, permitiendo al alumnado considerar las matemáticas como un aliado para su vida, donde ven su aplicación, en vez de ser una frustración constante.

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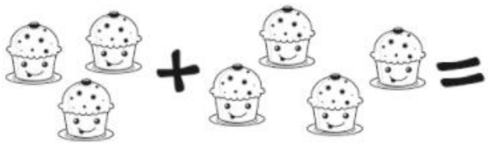
ANNEXES

Annex I: Mathematical initial level evaluation

Nombre:



$$\begin{array}{r} 5 \\ +\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 45 \\ +\ 23 \\ \hline \end{array}$$

$$\begin{array}{r} 37 \\ +\ 17 \\ \hline \end{array} \qquad \begin{array}{r} 78 \\ +\ 23 \\ \hline \end{array} \qquad \begin{array}{r} 123 \\ +\ 96 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ -\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 32 \\ -\ 12 \\ \hline \end{array} \qquad \begin{array}{r} 90 \\ -\ 58 \\ \hline \end{array} \qquad \begin{array}{r} 435 \\ -223 \\ \hline \end{array}$$

En una jaula hay tres pájaros.
Y en otra jaula hay cuatro pájaros.
¿Cuántos pájaros hay en las dos jaulas?.

- Dibuja en el recuadro lo que nos cuenta el problema

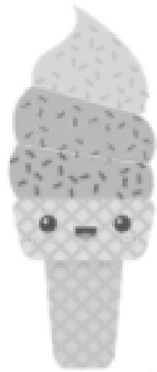
- ¿ Cuál crees que será el resultado final ?

COLOREA LA RESPUESTA

- ¿ Qué nos pide que hagamos ?: juntar quitar
- Entonces tenemos que sumar restar

1. Andrés tiene en la nevera 15 helados. Esta semana se ha comido 7. ¿Cuántos helados le quedan?

datos operaciones



solución:

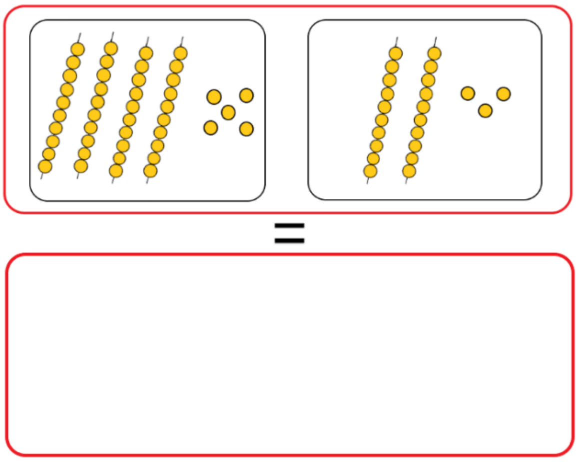
Annex II: Rubric for the mathematical level evaluation

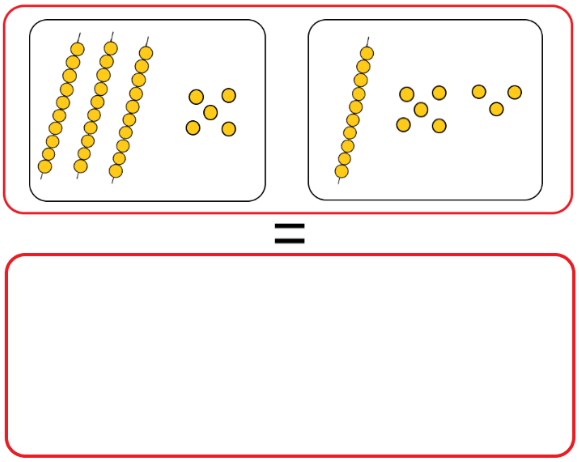
	Items to be evaluated – Learning Standards		Achieved?
Decimal Positional Value	Names the numbers correctly		
	Understands and identify the different orders of the decimal positional system	Ones	
		Tens	
		Hundreds	
	Place the digits of the numbers correctly		
	Understands and integrates the positional value of each digit placed in a specific position (positional order)		
	Represent a concrete numerical quantity in each of the levels of mathematical abstraction	Concrete	
		Pictorial	
		Abstract	
Addition	Comprehension & Understanding of the operation	Yes: *putting two parts together*	
		No	
	The mechanisation of the process of the mathematical operation	Yes	
		No	
	Correct performance of the addition operation at different levels:	Pictorial	
		Abstract	
		Without regrouping	
		With regrouping	
Subtraction	Comprehension & Understanding of the operation	Yes: *discovering the remaining part of the total from a given part*	
		No	
	The mechanisation of the process of the mathematical operation	Yes	
		No	
	Correct performance of the subtraction operation at different levels:	Pictorial	
		Abstract	
		Without regrouping	
		With regrouping	
Word-problems	Capable of identifying the correct operation from a real-life problem context	Addition	
		Subtraction	

Annex III: Mathematical final level evaluation

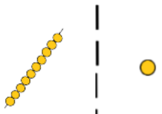
<u>SUMA</u>

- *1º Nivel*

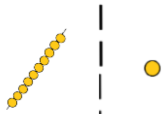




- *2º Nivel*



$$\begin{array}{r|l} 3 & 3 \\ +\ 1 & 2 \\ \hline & \end{array}$$



$$\begin{array}{r|l} 5 & 8 \\ +\ 2 & 7 \\ \hline & \end{array}$$

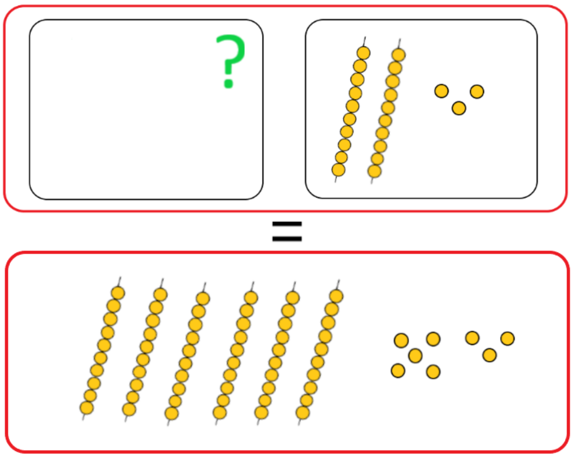
- *3º Nivel*

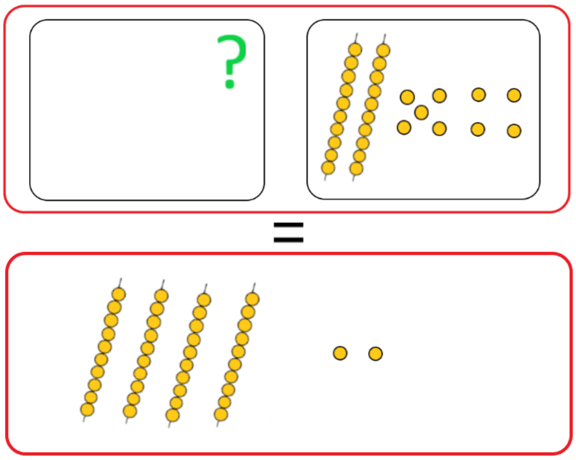
$$20 + 13 =$$

$$46 + 6 =$$

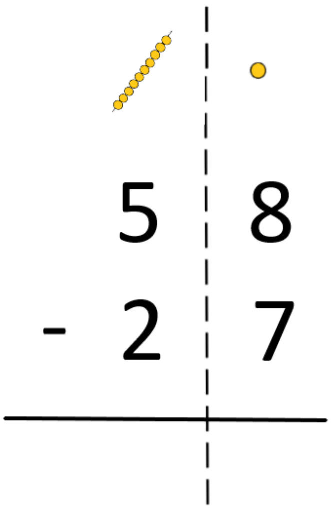
RESTA

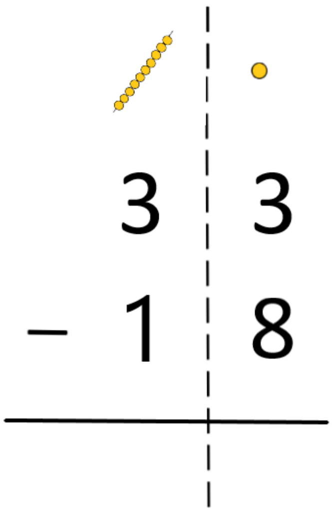
- *1º Nivel*





- *2º Nivel*





- *3º Nivel*

34 - 12 =

60 - 24 =

PROBLEMA ARITMÉTICO

Ash el lunes tenía 4 pokemons y, al día siguiente, consigue otros 3 pokemons. ¿Cuántos pokemons tiene Ash al final del martes?

- Dibuja en el recuadro lo que nos cuenta el problema

COLOREA LA RESPUESTA

- ¿ Qué nos pide que hagamos ?: juntar quitar
- Entonces tenemos que sumar restar

- ¿ Cuál crees que será el resultado final ? ______

El miércoles, 2 de los pokemons que Ash tenía, se van. ¿Cuántos se quedan al final del miércoles con Ash?

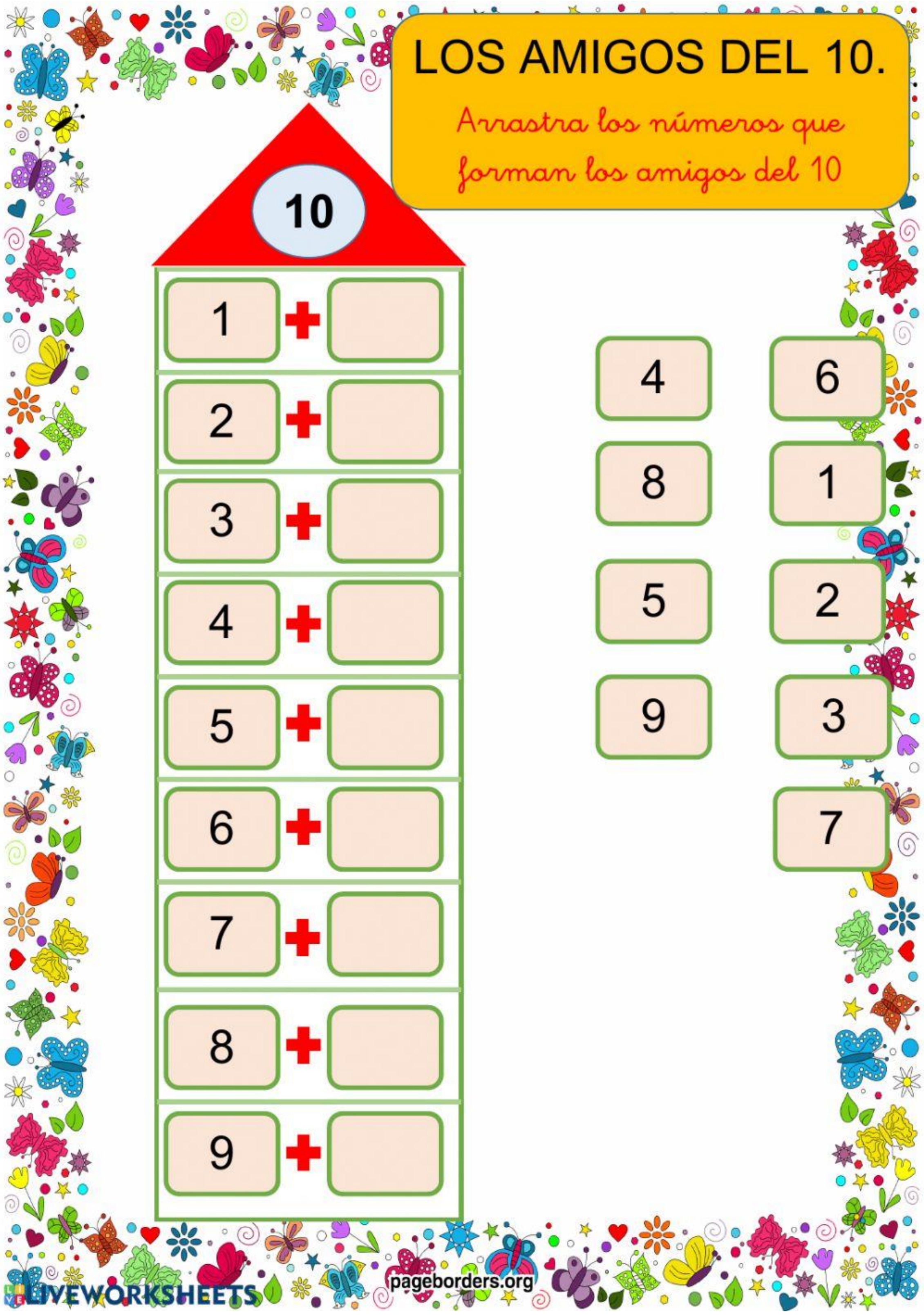
- Dibuja en el recuadro lo que nos cuenta el problema

COLOREA LA RESPUESTA

- ¿ Qué nos pide que hagamos ?: juntar quitar
- Entonces tenemos que sumar restar

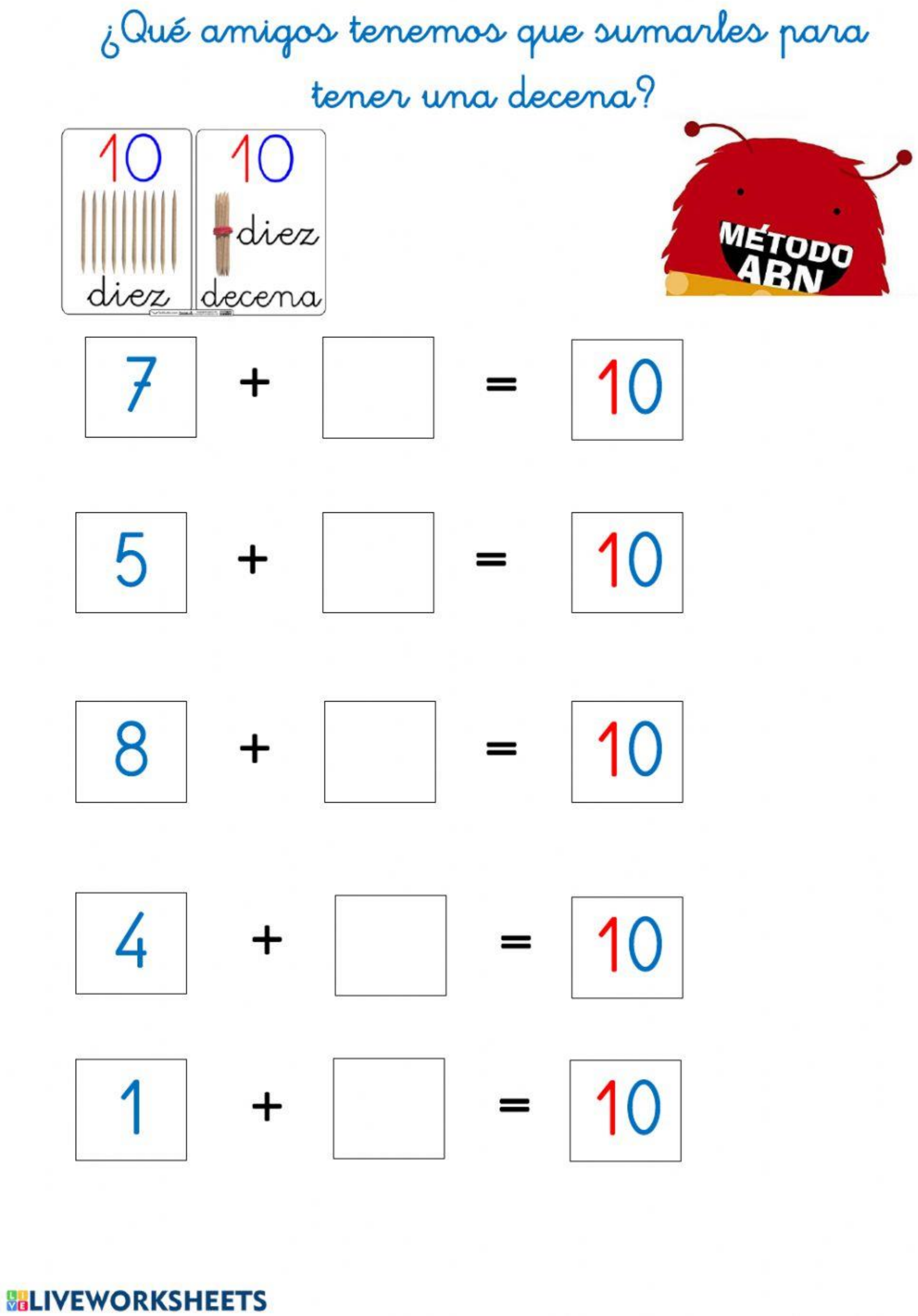
- ¿ Cuál crees que será el resultado final ? ______

Annex IV: Friends of 10 (Additive Decompositions of the number 10)



Retrieved from:
https://es.liveworksheets.com/worksheets/es/Educaci%C3%B3n_Infantil/Los_n%C3%BAmeros/Amigos_del_10_me344719ok

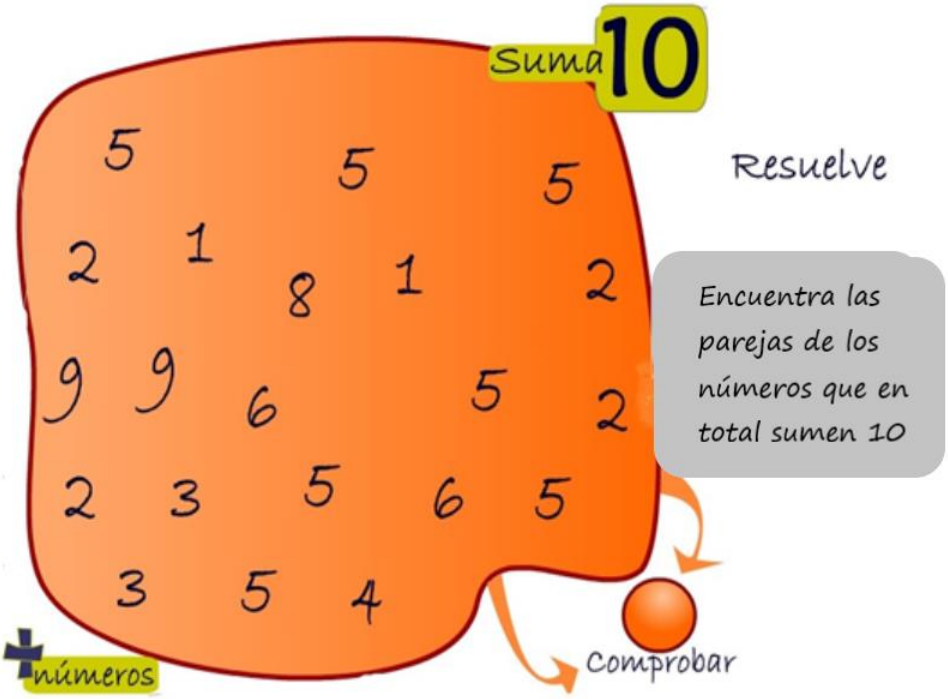
Annex V: Making a ten (Additive decompositions of 10)



Retrieved from: https://es.liveworksheets.com/worksheets/es/Educaci%C3%B3n_Infantil/L%C3%B3gico-matem%C3%A1ticas/Los_amigos_del_10_es423571bp

Annex VI: Let´s pair complementary numbers!

Encontremos las parejas de números complementarios de 10:



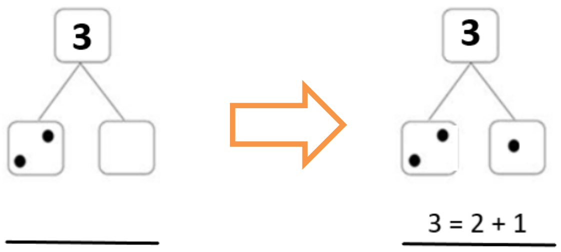
Escribe todas las opciones te encuentres de parejas que sumen 10:

-
-
-
-
-
-
-
-
-
-
-
-

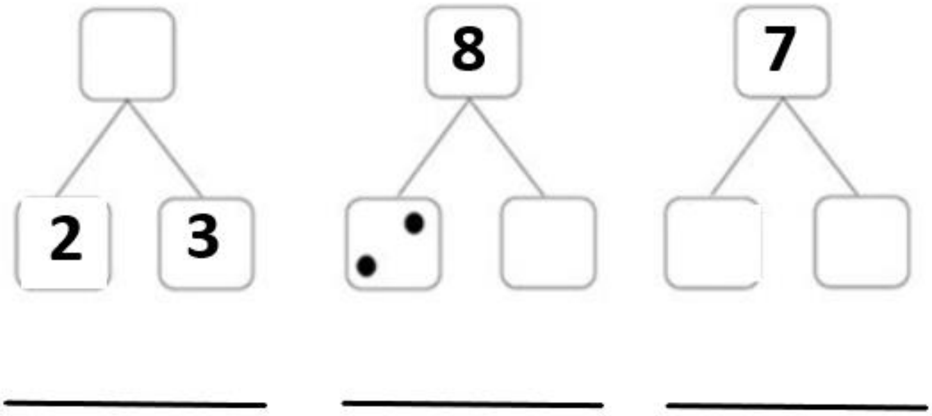
Annex VII: Number bonds - Simple number decompositions until 10 (pictorial + abstract)

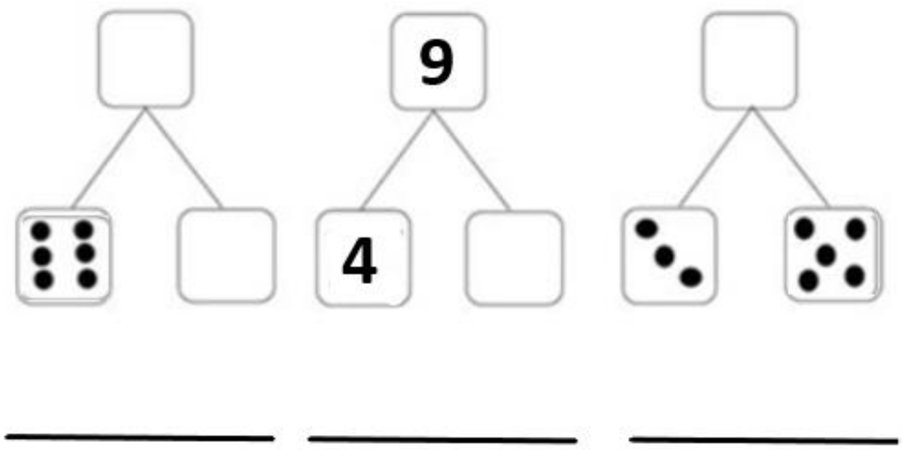
Con la ayuda de los policubos o palillos probemos a resolver las descomposiciones de los siguientes números, para ello completa los recuadros con dibujos o números según te pidan. Además, hay que poner debajo la notación matemática de la descomposición.

Ejemplo:



Completa:





Annex VIII: Our hundreds chart – Table of 100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

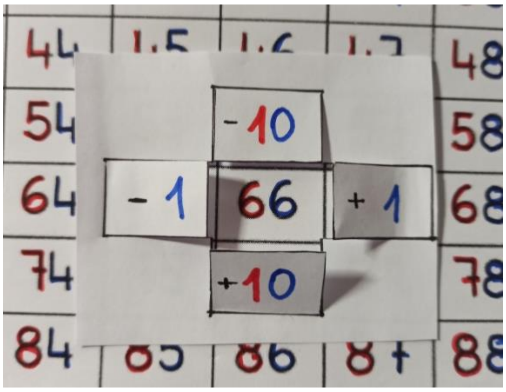
Annex IX: Hundreds Chart Window Tools

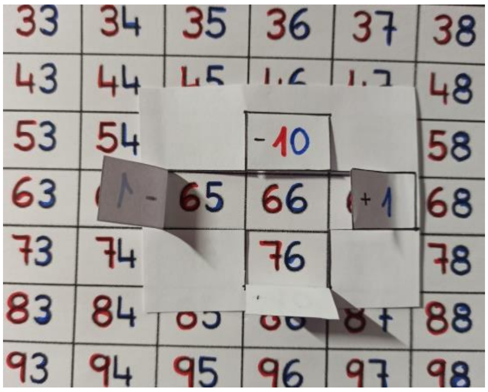
(The windows have to fit perfectly the size of the numbers on the hundreds chart)

- Model*:*

	- 10	
- 1	*CUT*	**+ 1**
	+ 10	

- Examples:





Annex X: Decoding the hidden numbers in the 100´s Chart


abn Tabla del 100

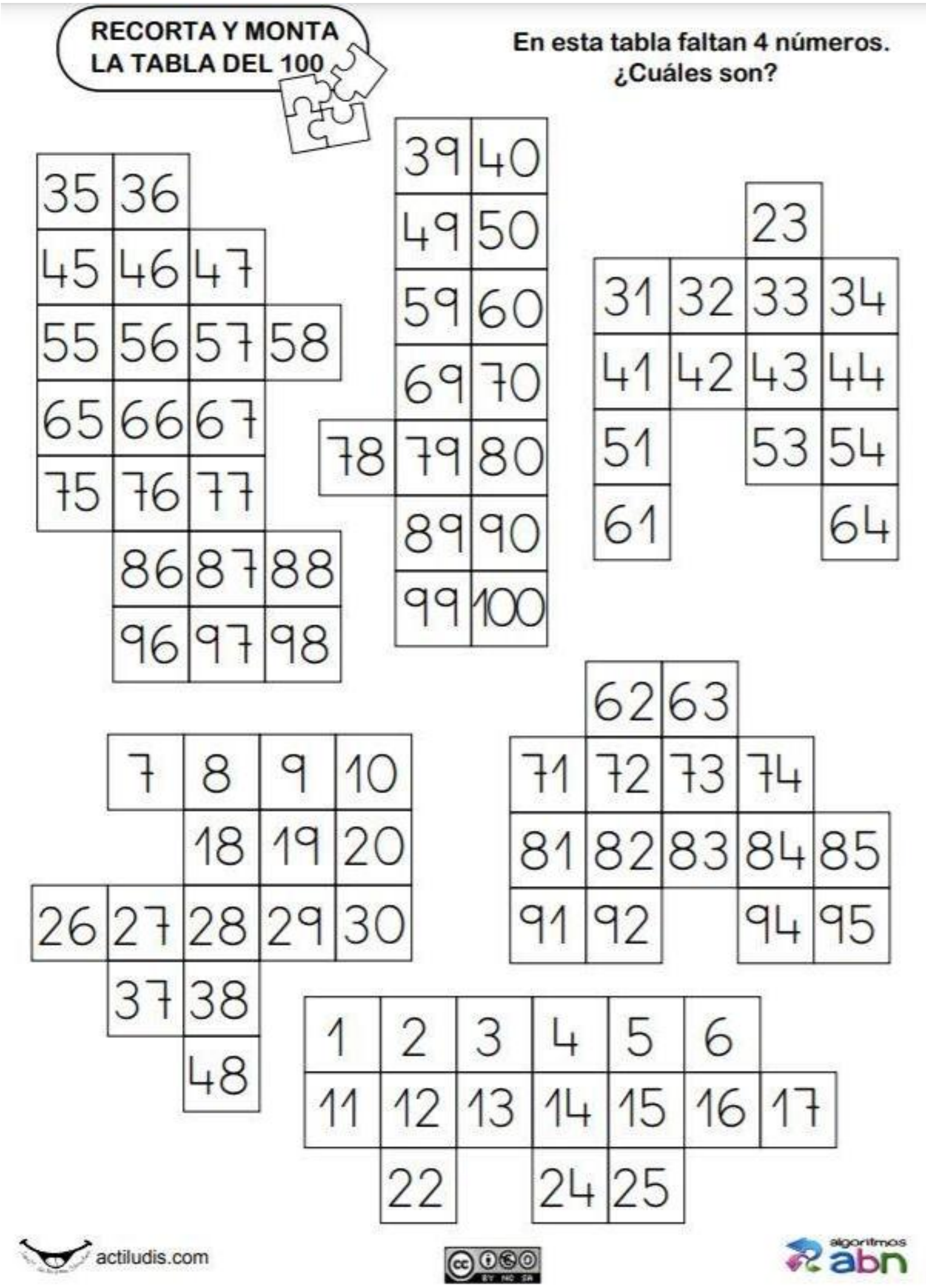
1	2		4	5	6	7	8	9	10
11		13	14	15	16	17	18	19	20
21	22	23	24	25	26	27		29	30
31	32	33	34	35		37	38	39	40
41	42	43	44	45	46	47	48	49	
51	52	53	54		56	57	58	59	60
61	62	63	64	65	66		68	69	70
	72	73	74	75	76	77	78	79	80
81	82	83		85	86	87	88	89	90
91	92	93	94	95	96	97	98		100

=
=
=
=
=
=
=
=
=
=

Retrieved from: https://www.recursosep.com/2017/04/21/metodo-abn-tabla-del-100-y-repartos-uniformes-actividades-de-primavera/

Annex XI: Let´s discover the hidden numbers in the Table 100 puzzle

(Cut the different pieces and build the table of 100 to discover which numbers have disappeared).



Retrieved from: https://www.actiludis.com/2011/06/21/puzzles-con-la-tabla-del-100/

Annex XII: Cross numbers

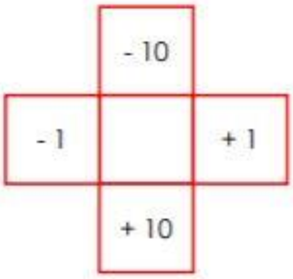
- Example:

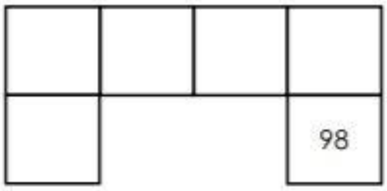
	83		

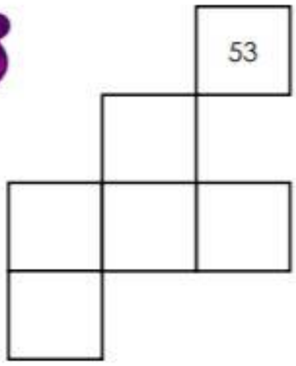
- Solution:

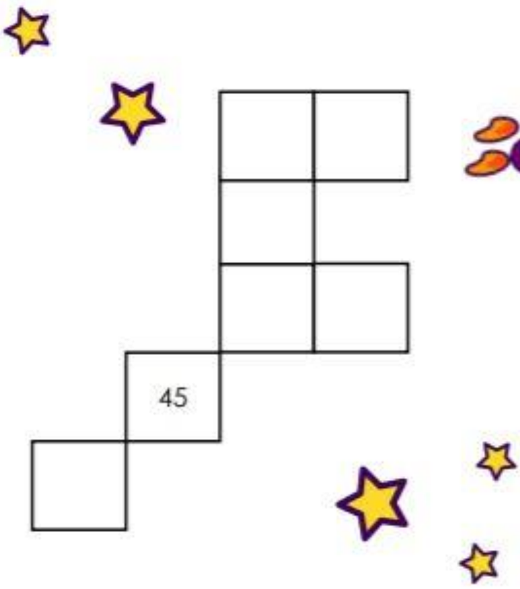
62			65
	73	74	
82	**83**		
92	93	94	95

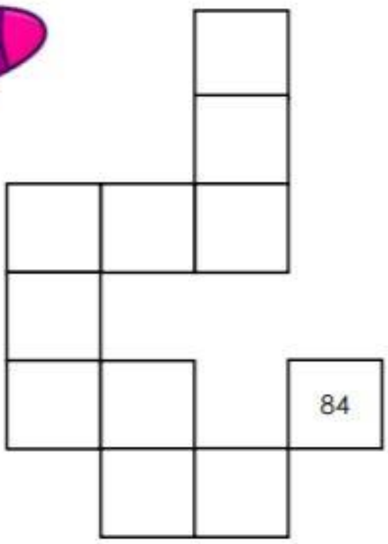
Ahora, ¡nos toca a nosotros! / Now, it´s our turn, let´s try!











Adapted from: https://www.recursosep.com/2020/09/21/nueva-actividad-de-crucinumeros/

Annex XIII: Bingo Dictation Game

BINGO DICTADO

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

BINGO DICTADO

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Retrieved from: https://www.actiludis.com/2014/04/04/bingo-dictado/

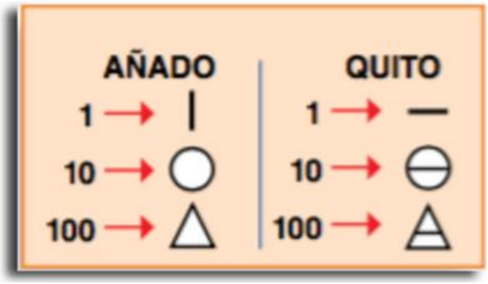
Annex XIV: Creation of Mathematical Codes (*Performed in Spanish)*

Instructions*: Decoding and creating the following mathematical codes*

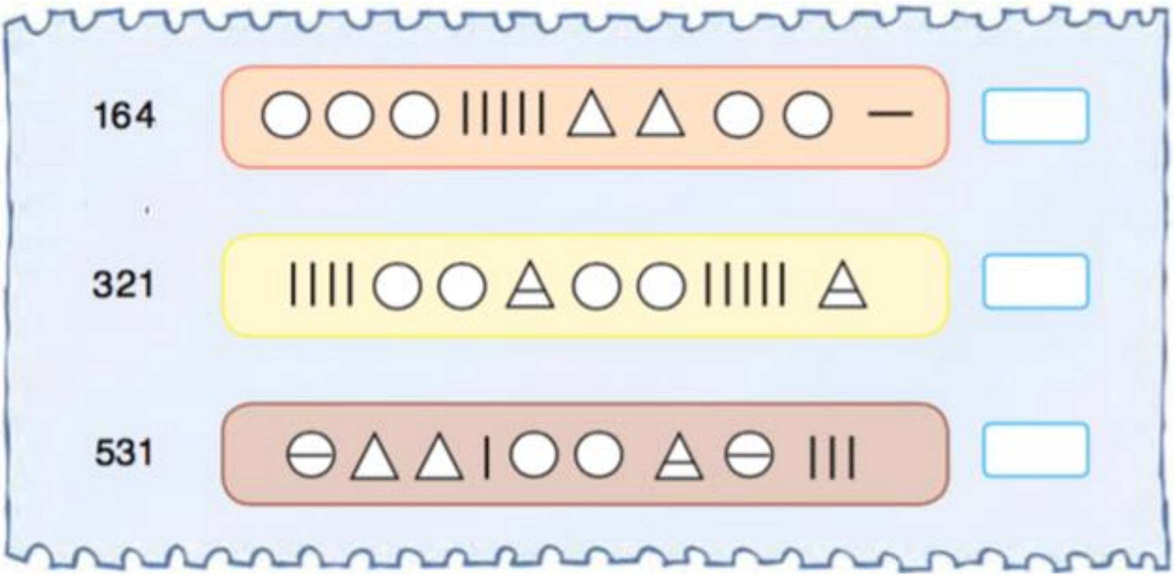
Creemos nuestro propio código matemático

Descubre los números misteriosos finales a través del siguiente código matemático.

Leyenda/Code legend:



Descubre/Discover:



Crea el tuyo propio/Create your own:

_____ _____

Adapted from: https://www.actiludis.com/2017/10/23/iniciacion-la-abstraccion-metodo-abn/

Annex XV: Number Detective *(Performed in Spanish, the original English version can be found at the link)*

Instructions*: Let´s discover the mystery number from the list below following the different clues the detective has.*

El Detective de números

¡Llamando a todos los detectives! Necesitarás pensar de manera creativa, usar las habilidades de razonamiento y tus estrategias de resolución de problemas para encontrar el número misterioso de la siguiente lista.

- El número tiene dos dígitos.
- Ambos dígitos son pares.
- El dígito de las decenas es mayor que el dígito situado en las unidades.
- El dígito de las unidades no pertenece a la tabla de multiplicar del tres.
- El dígito de las decenas no es el doble que el dígito de las unidades.
- La suma de ambas cifras/dígitos es un múltiplo de cinco.

18	86
120	42
46	64
80	8
22	83

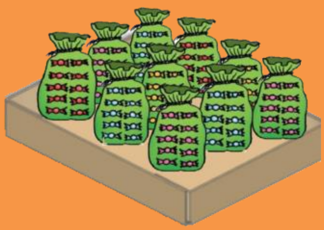
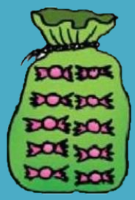
Adapted from: https://nrich.maths.org/204

Annex XVI: Domino of addition and subtraction operations

(Cut along the continuous lines)

74	18 + 148	166	32 + 79
111	73 - 48	25	91 + 9
100	6 + 43	49	65 - 20
45	58 - 5	53	25 + 67
92	38 + 24	62	50-16
34	89 - 7	82	56 + 18

Annex XVII: Table tool for decimal positional system

 CENTENA	 DECENA	 UNIDAD

Annex XVIII: Number flashcards (Hundreds, Tens and Ones)

- **Hundreds**

9	0	0
8	0	0
7	0	0
6	0	0
5	0	0
4	0	0
3	0	0
2	0	0
1	0	0

- **Tens** - **Ones**

9	0	9
8	0	8
7	0	7
6	0	6
5	0	5
4	0	4
3	0	3
2	0	2
1	0	1
		0

Annex XIX: Evaluation table of the comprehension of the decimal positional value

Objetivos de la actividad	**Evaluación final actividad:** ¿Objetivos completados?		**Observaciones**
1. Entender la importancia de la posición de un número dentro del sistema posicional	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Falta de comprensión de la actividad ❏ Entiende que la posición cambia el valor de la cifra, pero no entiende la razón. ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
2. Identificar los diferentes órdenes del sistema posicional (centenas, decenas, unidades)	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Falta de comprensión de la actividad ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
3. Comprender y asimilar el significado que toma cada cifra en la escritura de un número en una posición concreta (valor numérico posicional) *Cuando tenemos 10 unidades se agrupan en 1 decena en el sistema numérico decimal*	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Falta de comprensión de la actividad ❏ Entiende que la posición cambia el valor de la cifra, pero no entiende la razón. ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
4. Ser capaz de representar una cantidad concreta dentro del sistema posicional en cada uno de los niveles de abstracción matemática e identificar sus órdenes concretos.	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Sabe hacerlo pero no explicarlo ❏ Sabe representar de manera concreta ❏ Sabe representar de manera simbólica, pero no grafica o pictórica ❏ Poca/mucha motivación de la actividad ❏ Otro:	

Evaluación durante la actividad			**Observaciones**	
¿Cómo es la actitud del niño?	Emocionalmente	Antes de empezar la actividad su disposición es buena	SI - NO	
		Durante la realización de la misma, ¿le motiva la actividad?	SI - NO	
¿Maneja bien los palillos?	Motricidad	¿Dificultad encontrada?	SI - NO	
¿Comprende el valor numérico en el sistema posicional?	Comprende que no es lo mismo un 1 en las unidades que en las decenas, lo cual que implica que es un grupo de 10		SI - NO	
¿Es capaz de pasar correctamente de … ? *(Número o símbolo = grafía)*	Número → Palillos	Representa el número correctamente usando los palillos	SI - NO	
		Verbaliza correctamente el número	SI - NO	
	Palillos → Número	Elige o escribe la grafía correcta que representa la cantidad de palillos dados	SI - NO	
		Verbaliza correctamente el número	SI - NO	
		Escribe el nombre del número correctamente	SI - NO	
En el manejo de palillos ¿entiende que con los números se pueden realizar diferentes operaciones de agrupación, partición, multiplicación o diferencia?			SI - NO	
Agrupación de unidades al orden inmediato superior - ¿Agrupa de manera espontánea en grupos de 10? *Regla: Cada 10 agrupo (goma)*	SI			
	NO	*Frecuencia* - ¿Cuántas veces le ayudo hasta que él lo integra?		
¿Qué me muestra que entiende que es capaz de realizar la actividad?				

Annex XX: Evaluation table of the addition algorithm

Objetivos de la actividad	**Evaluación final actividad:** ¿Objetivos completados?		**Observaciones**
1. Reforzar el entendimiento del sistema posicional y el significado que toma cada cifra en la escritura de un número en una posición concreta (valor numérico posicional) a través del ejercicio de la suma	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Falta de comprensión de la actividad ❏ Entiende que la posición cambia el valor de la cifra, pero no entiende la razón. ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
2. Entender que sumar es agrupar o juntar ambas partes de la suma (sumandos) para conseguir el todo (suma o total)	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Falta de comprensión de la actividad ❏ Propuesta ambiciosa para la duración de la sesión ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
3. Ser capaz de realizar la suma de la manera que considere más lógica el alumno y explicar qué hace y por qué lo hace.	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ No sabe hacerlo por sí solo ❏ Sabe hacerlo pero no explicarlo ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
4. Reconocer la suma en distintas situaciones de su vida	SI / NO	**Motivo**: ❏ Ve solo la situación de partes formando el todo ❏ Ve solo la situación de ganancia ❏ Ve las anteriores, pero ninguna situación más ❏ Poca/mucha motivación de la actividad ❏ Otro:	

Evaluación durante la actividad			**Observaciones**	
¿Cómo es la actitud del niño?	Emocionalmente	Antes de empezar la actividad su disposición es buena	SI - NO	
		Durante la realización de la misma, ¿le motiva la actividad?	SI - NO	
¿Maneja bien los palillos?	Motricidad	¿Dificultad encontrada?	SI - NO	
¿Comprende el valor numérico en el sistema posicional?	Comprende que no es lo mismo un 1 en las unidades que en las decenas, lo cual que implica que es un grupo de 10		SI - NO	
A través del trabajo del algoritmo de la suma ¿se ha propiciado en nuestro alumno la mejora del aprendizaje en cuanto al proceso de paso de…? *(Número = grafía)*	Número → Palillos	Representa el número correctamente usando los palillos	Mejora - Se mantiene	
		Verbaliza correctamente el número	Mejora - Se mantiene	
	Palillos → Número	Elige o escribe la grafía correcta que representa la cantidad de palillos dados	Mejora - Se mantiene	
		Verbaliza correctamente el número	Mejora - Se mantiene	
		Escribe el nombre del número correctamente	Mejora - Se mantiene	
Identifica la suma en distintas situaciones de su vida (docente da significado al algoritmo y contextualiza): a) Como el cálculo del total de la unión de las partes b) Como la magnitud (referido) que es unidades mayor que otra menor (referente) c) Como el valor final tras una ganancia d) Como el valor inicial tras una pérdida			a) SI –NO b) SI –NO c) SI – NO d) SI – NO	

<table>
<tr><td colspan="3">Formula o relata situaciones posibles que den sentido a la suma:
a) Como el cálculo del total de la unión de las partes
b) Como la magnitud (referido) que es unidades mayor que otra menor (referente)
c) Como el valor final tras una ganancia
d) Como el valor inicial tras una pérdida</td><td>a) SI –NO
b) SI –NO
c) SI – NO
d) SI – NO</td><td></td></tr>
<tr><td colspan="3">En el manejo de palillos ¿entiende que sumar es juntar y que el total se adquiere al juntar ambas partes formando el todo?</td><td>SI - NO</td><td></td></tr>
<tr><td rowspan="2">Suma con llevada- ¿Agrupa de manera espontánea en grupos de 10?

Regla: Cada 10 agrupo (goma)</td><td colspan="3">SI</td><td></td></tr>
<tr><td>NO</td><td colspan="2">Frecuencia - ¿Cuántas veces le ayudo hasta que él lo integra?</td><td></td></tr>
<tr><td>¿Qué me muestra que entiende que es capaz de realizar la actividad?</td><td colspan="3"></td><td></td></tr>
</table>

Annex XXI: Evaluation table of the Subtraction Algorithm

Objetivos de la actividad	**Evaluación final actividad:** ¿Objetivos completados?		**Observaciones**
1. Reforzar el entendimiento del sistema posicional y el significado que toma cada cifra en la escritura de un número en una posición concreta (valor numérico posicional) a través del ejercicio de la resta	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Falta de comprensión de la actividad ❏ Entiende que la posición cambia el valor de la cifra, pero no entiende la razón. ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
2. Entender que restar es descubrir lo que queda del minuendo cuando se aparta de él al sustraendo (sustraendo y resta son las dos partes disjuntas en las que se separa el minuendo)	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Falta de comprensión de la actividad ❏ Propuesta ambiciosa para la duración de la sesión ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
3. Ser capaz de realizar la resta de la manera que considere más lógica el alumno y explicar qué hace y por qué lo hace.	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ No sabe hacerlo por sí solo ❏ Sabe hacerlo pero no explicarlo ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
4. Reconocer la resta en distintas situaciones de su vida	SI / NO	**Motivo**: ❏ Ve solo la situación de perdida ❏ Ve la pérdida y una parte de un todo ❏ Ve las anteriores y sabe comparar dos magnitudes, pero ninguna situación más ❏ Poca/mucha motivación de la actividad ❏ Otro:	

<table>
<tr><th colspan="3">Evaluación durante la actividad</th><th colspan="2">Observaciones</th></tr>
<tr><td rowspan="2">¿Cómo es la actitud del niño?</td><td rowspan="2">Emocionalmente</td><td>Antes de empezar la actividad su disposición es buena</td><td>SI - NO</td><td></td></tr>
<tr><td>Durante la realización de la misma, ¿le motiva la actividad?</td><td>SI - NO</td><td></td></tr>
<tr><td>¿Maneja bien los palillos?</td><td>Motricidad</td><td>¿Dificultad encontrada?</td><td>SI - NO</td><td></td></tr>
<tr><td>¿Comprende el valor numérico en el sistema posicional?</td><td colspan="2">Comprende que no es lo mismo un 1 en las unidades que en las decenas, lo cual que implica que es un grupo de 10</td><td>SI - NO</td><td></td></tr>
<tr><td rowspan="5">A través del trabajo del algoritmo de la resta ¿se ha propiciado en nuestro alumno la mejora del aprendizaje en cuanto al proceso de paso de …?

(Número = grafía)</td><td rowspan="2">Número → Palillos</td><td>Representa el número correctamente usando los palillos</td><td>Mejora - Se mantiene</td><td></td></tr>
<tr><td>Verbaliza correctamente el número</td><td>Mejora - Se mantiene</td><td></td></tr>
<tr><td rowspan="3">Palillos → Número</td><td>Elige o escribe la grafía correcta que representa la cantidad de palillos dados</td><td>Mejora - Se mantiene</td><td></td></tr>
<tr><td>Verbaliza correctamente el número</td><td>Mejora - Se mantiene</td><td></td></tr>
<tr><td>Escribe el nombre del número correctamente</td><td>Mejora - Se mantiene</td><td></td></tr>
<tr><td colspan="3">Identifica la resta en distintas situaciones de su vida (docente da significado al algoritmo y contextualiza):
a) Como el cálculo de una parte del total
b) Como la diferencia entre dos magnitudes que se comparan
c) Como la magnitud (referido) que es unidades menor que otra mayor (referente)
d) Como el valor final tras una pérdida
e) Como el valor inicial tras una ganancia.</td><td>a) SI –NO
b) SI –NO
c) SI – NO
d) SI – NO
e) SI – NO</td><td></td></tr>
</table>

<table>
<tr><td colspan="3">Formula o relata situaciones posibles que den sentido a la resta:
a) Como el cálculo de una parte del total
b) Como la diferencia entre dos magnitudes que se comparan
c) Como la magnitud (referido) que es unidades menor que otra mayor (referente)
d) Como el valor final tras una pérdida
e) Como el valor inicial tras una ganancia.</td><td>a) SI –NO
b) SI –NO
c) SI – NO
d) SI – NO
e) SI – NO</td><td></td></tr>
<tr><td colspan="3">En el manejo de palillos, entiende la diferencia con la suma: ahora el minuendo es el “todo” y por tanto el sustraendo no se representa con palillos, sino con huecos a rellenar.</td><td>SI-NO</td><td></td></tr>
<tr><td colspan="3">Resta sin llevada - ¿Entiende que restar es lo que queda del minuendo cuando se aparta de él al sustraendo (sustraendo y resta son las dos partes disjuntas en las que se separa el minuendo)?</td><td>SI - NO</td><td></td></tr>
<tr><td rowspan="2">Resta con llevada- ¿Abre un grupo del orden inmediato superior de manera espontánea cuando no hay suficientes palillos para llenar los huecos del sustraendo?

Regla: Cada 10 agrupo (goma)</td><td colspan="3">SI</td><td></td></tr>
<tr><td>NO</td><td colspan="2">Frecuencia - ¿Cuántas veces le ayudo hasta que él lo integra?</td><td></td></tr>
<tr><td>¿Qué me muestra que entiende que es capaz de realizar la actividad?</td><td colspan="3"></td><td></td></tr>
</table>

Annex XXII: Results of the Place Value Lesson → *(Anotaciones de resultados en azul)*

Objetivos de la actividad	Evaluación final actividad: ¿Objetivos completados?		Observaciones
1. Entender la importancia de la posición de un número dentro del sistema posicional	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Falta de comprensión de la actividad ❏ Entiende que la posición cambia el valor de la cifra, pero no entiende la razón. ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
2. Identificar los diferentes órdenes del sistema posicional (centenas, decenas, unidades)	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Falta de comprensión de la actividad ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	
3. Comprender y asimilar el significado que toma cada cifra en la escritura de un número en una posición concreta (valor numérico posicional) *Cuando tenemos 10 unidades se agrupan en 1 decena en el sistema numérico decimal*	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Falta de comprensión de la actividad ❏ Entiende que la posición cambia el valor de la cifra, pero no entiende la razón. ❏ Más sesiones necesarias ❏ Poca/mucha motivación de la actividad ❏ Otro:	Me da el ejemplo del número 66 que se compone de: - 6 decenas = 6 grupos de 10 palillos cada uno = 60 palillos sueltos - 6 unidades (palillos sueltos)
4. Ser capaz de representar una cantidad concreta dentro del sistema posicional en cada uno de los niveles de abstracción matemática e identificar sus órdenes concretos.	SI / NO	**Motivo**: ❏ Falta de Tiempo ❏ Sabe hacerlo pero no explicarlo ❏ Sabe representar de manera concreta ❏ Sabe representar de manera simbólica, pero no grafica o pictórica ❏ Poca/mucha motivación de la actividad ❏ Otro:	

Evaluación durante la actividad			**Observaciones**	
¿Cómo es la actitud del niño?	Emocionalmente	Antes de empezar la actividad su disposición es buena	SI - NO	Viene dormido, pocas ganas de trabajar. Luego, se mete en actividad.
		Durante la realización de la misma, ¿le motiva la actividad?	SI - NO	Metido en actividad se ve capaz
¿Maneja bien los palillos?	Motricidad	¿Dificultad encontrada?	SI - NO	Se le resbalan los palillos un poco, pero cuando pilla el truco buen manejo
¿Comprende el valor numérico en el sistema posicional?	Comprende que no es lo mismo un 1 en las unidades que en las decenas, lo cual que implica que es un grupo de 10		SI - NO	
¿Es capaz de pasar correctamente de … ? *(Número o símbolo = grafía)*	Número → Palillos	Representa el número correctamente usando los palillos	SI - NO	Dificultad entre sesenta y setenta, hacer posterior hincapié y comprobar en suma y resta
		Verbaliza correctamente el número	SI - NO	
	Palillos → Número	Elige o escribe la grafía correcta que representa la cantidad de palillos dados	SI - NO	
		Verbaliza correctamente el número	SI - NO	Dificultad entre sesenta y setenta
		Escribe el nombre del número correctamente	SI - NO	No trabajado : trabajar en suma y resta
En el manejo de palillos ¿entiende que con los números se pueden realizar diferentes operaciones de agrupación, partición, multiplicación o diferencia?			SI - NO	A la hora de hacer código matemático está cansado y entonces hay dificultad para nombrar número sin tarjetas
Agrupación de unidades al orden inmediato superior - ¿Agrupa de manera espontánea en grupos de 10? *Regla: Cada 10 agrupo (goma)*	SI			Entiende regla perfectamente
	NO	*Frecuencia* - ¿Cuántas veces le ayudo hasta que él lo integra? Primero un ejemplo de prueba para enseñarle y luego él lo integra perfectamente.		

¿Qué me muestra que entiende que es capaz de realizar la actividad?	- Junta en grupos de 10 antes de situar los palillos dependiendo el tipo de grupo sabe que va en una columna u otra (órdenes identificados) - De 10 en 10 →creación de grupos - Me verbaliza que no es lo mismo tener un 6 en unidades (6 palillos) que un 6 en decenas (6 grupos de 10=60 palillos) - Importancia de la agrupación de números porque se hace más sencilla y rápida la representación de la cantidad usando los diferentes órdenes (decena, unidad) frente a tener todos los palillos sueltos.	A la hora de hacer código matemático está cansado y entonces hay dificultad para nombrar y descubrir los números sin las tarjetas. Le cuesta mucho ese cálculo mental.

Annex XXIII: Results of the Addition algorithm Lesson → *(Anotaciones de resultados en azul)*

Objetivos de la actividad	**Evaluación final actividad:** ¿Objetivos completados?		**Observaciones**
1. Reforzar el entendimiento del sistema posicional y el significado que toma cada cifra en la escritura de un número en una posición concreta (valor numérico posicional) a través del ejercicio de la suma	SI / NO	**Motivo**: ❑ Falta de Tiempo ❑ Falta de comprensión de la actividad ❑ Entiende que la posición cambia el valor de la cifra, pero no entiende la razón. ❑ Más sesiones necesarias ❑ Poca/mucha motivación de la actividad ❑ Otro:	
2. Entender que sumar es agrupar o juntar ambas partes de la suma (sumandos) para conseguir el todo (suma o total)	SI / NO	**Motivo**: ❑ Falta de Tiempo ❑ Falta de comprensión de la actividad ❑ Propuesta ambiciosa para la duración de la sesión ❑ Más sesiones necesarias ❑ Poca/mucha motivación de la actividad ❑ Otro:	
3. Ser capaz de realizar la suma de la manera que considere más lógica el alumno y explicar qué hace y por qué lo hace.	SI / NO	**Motivo**: ❑ Falta de Tiempo ❑ No sabe hacerlo por sí solo ❑ Sabe hacerlo pero no explicarlo ❑ Más sesiones necesarias ❑ Poca/mucha motivación de la actividad X Otro: **Mecanización de la suma**	Algoritmo mecanizado, sobretodo, en suma con llevada porque no entiende el 1 que pasa al grupo de las decenas desde el de las unidades a raíz de haber conseguido un grupo de 10 unidades, que es lo mismo que 1 grupo en decenas
4. Reconocer la suma en distintas situaciones de su vida	SI / NO	**Motivo**: ❑ Ve solo la situación de partes formando el todo ❑ Ve solo la situación de ganancia X Ve las anteriores, pero ninguna situación más ❑ Poca/mucha motivación de la actividad ❑ Otro:	Lo cual se debe a la mecanización tan integrada que tiene de la ejecución de la operación, lo cual no le requiere entendimiento, pero hace que pierda el significado en si mismo.

Evaluación durante la actividad			**Observaciones**	
¿Cómo es la actitud del niño?	Emocionalmente	Antes de empezar la actividad su disposición es buena	SI - NO	1º sesión → SÍ **2º sesión → NO**
		Durante la realización de la misma, ¿le motiva la actividad?	SI - NO	En la segunda sesión no quiere trabajar
¿Maneja bien los palillos?	Motricidad	¿Dificultad encontrada?	SI - NO	
¿Comprende el valor numérico en el sistema posicional?	Comprende que no es lo mismo un 1 en las unidades que en las decenas, lo cual que implica que es un grupo de 10		SI - NO	
A través del trabajo del algoritmo de la suma ¿se ha propiciado en nuestro alumno la mejora del aprendizaje en cuanto al proceso de paso de…? *(Número = grafía)*	Número → Palillos	Representa el número correctamente usando los palillos	Mejora - Se mantiene	
		Verbaliza correctamente el número	Mejora - Se mantiene	
	Palillos → Número	Elige o escribe la grafía correcta que representa la cantidad de palillos dados	Mejora - Se mantiene	
		Verbaliza correctamente el número	Mejora - Se mantiene	
		Escribe el nombre del número correctamente	Mejora - Se mantiene	
Identifica la suma en distintas situaciones de su vida (docente da significado al algoritmo y contextualiza): a) Como el cálculo del total de la unión de las partes b) Como la magnitud (referido) que es unidades mayor que otra menor (referente) c) Como el valor final tras una ganancia d) Como el valor inicial tras una pérdida			a) SI –NO b) SI –NO c) SI – NO d) SI – NO	

<table>
<tr><td colspan="3">Formula o relata situaciones posibles que den sentido a la suma:
a) Como el cálculo del total de la unión de las partes
b) Como la magnitud (referido) que es unidades mayor que otra menor (referente)
c) Como el valor final tras una ganancia
d) Como el valor inicial tras una pérdida</td><td>a) SI –NO
b) SI –NO
c) SI – NO
d) SI – NO</td><td></td></tr>
<tr><td colspan="3">En el manejo de palillos ¿entiende que sumar es juntar y que el total se adquiere al juntar ambas partes formando el todo?</td><td>SI - NO</td><td></td></tr>
<tr><td rowspan="2">Suma con llevada- ¿Agrupa de manera espontánea en grupos de 10?

Regla: Cada 10 agrupo (goma)</td><td colspan="3">SI</td><td></td></tr>
<tr><td>NO</td><td colspan="2">Frecuencia - ¿Cuántas veces le ayudo hasta que él lo integra?
Todas →entiende que cada 10 palillos (unidades) se forma 1 grupo con goma y va a su orden correspondiente de las decenas, pero necesita guía y atención continua por parte de la profesora porque si no desconecta.</td><td>Es capaz de realizarla (ejecución), pero de manera mecanizada.</td></tr>
<tr><td>¿Qué me muestra que entiende que es capaz de realizar la actividad?</td><td colspan="3">-“Sumamos ambas partes y tenemos todo el resultado” (Relación de : Part-whole)

- “No puede haber dos cifras ‘13’ en unidades entonces el 1 va como llevada arriba al otro lado”</td><td></td></tr>
</table>

Annex XXIV: Results of the Subtraction Algorithm Lesson → *(Anotaciones de resultados en azul)*

Objetivos de la actividad	**Evaluación final actividad:** ¿Objetivos completados?		**Observaciones**
1. Reforzar el entendimiento del sistema posicional y el significado que toma cada cifra en la escritura de un número en una posición concreta (valor numérico posicional) a través del ejercicio de la resta	SI / NO	**Motivo**: ❑ Falta de Tiempo ❑ Falta de comprensión de la actividad ❑ Entiende que la posición cambia el valor de la cifra, pero no entiende la razón. ❑ Más sesiones necesarias ❑ Poca/mucha motivación de la actividad ❑ Otro:	
2. Entender que restar es descubrir lo que queda del minuendo cuando se aparta de él al sustraendo (sustraendo y resta son las dos partes disjuntas en las que se separa el minuendo)	SI / NO	**Motivo**: ❑ Falta de Tiempo ❑ Falta de comprensión de la actividad ❑ Propuesta ambiciosa para la duración de la sesión X Más sesiones necesarias ❑ Poca/mucha motivación de la actividad ❑ Otro:	- Mínimo 2 sesiones para entender la resta con y sin llevada, aunque para llevada necesarias más sesiones porque al requerirle tanto esfuerzo, le causa rechazo. - Enfatizar que sustraendo son agujeros a rellenar (son las cajitas)
3. Ser capaz de realizar la resta de la manera que considere más lógica el alumno y explicar qué hace y por qué lo hace.	SI / NO	**Motivo**: ❑ Falta de Tiempo ❑ No sabe hacerlo por sí solo ❑ Sabe hacerlo pero no explicarlo ❑ Más sesiones necesarias ❑ Poca/mucha motivación de la actividad ❑ Otro:	Algoritmo mecanizado
4. Reconocer la resta en distintas situaciones de su vida	SI / NO	**Motivo**: X Ve solo la situación de perdida X Ve la pérdida y una parte de un todo ❑ Ve las anteriores y sabe comparar dos magnitudes, pero ninguna situación más ❑ Poca/mucha motivación de la actividad ❑ Otro:	Al tener el algoritmo mecanizado no entiende su aplicación y los diferentes significados que tiene en contextos de su vida

Evaluación durante la actividad			**Observaciones**	
¿Cómo es la actitud del niño?	Emocionalmente	Antes de empezar la actividad su disposición es buena	SI - NO	Mayor dificultad en sesión de resta con llevada, porque se cierra en banda al requerirle esfuerzo el pensar y no verse capaz de realizarla bien
		Durante la realización de la misma, ¿le motiva la actividad?	SI - NO	Integrada en dominó (juego)
¿Maneja bien los palillos?	Motricidad	¿Dificultad encontrada?	SI - NO	
¿Comprende el valor numérico en el sistema posicional?	Comprende que no es lo mismo un 1 en las unidades que en las decenas, lo cual que implica que es un grupo de 10		SI - NO	
A través del trabajo del algoritmo de la resta ¿se ha propiciado en nuestro alumno la mejora del aprendizaje en cuanto al proceso de paso de …? *(Número = grafía)*	Número → Palillos	Representa el número correctamente usando los palillos	Mejora - Se mantiene	
		Verbaliza correctamente el número	Mejora - Se mantiene	
	Palillos → Número	Elige o escribe la grafía correcta que representa la cantidad de palillos dados	Mejora - Se mantiene	
		Verbaliza correctamente el número	Mejora - Se mantiene	
		Escribe el nombre del número correctamente	Mejora - Se mantiene	
Identifica la resta en distintas situaciones de su vida (docente da significado al algoritmo y contextualiza): a) Como el cálculo de una parte del total b) Como la diferencia entre dos magnitudes que se comparan c) Como la magnitud (referido) que es unidades menor que otra mayor (referente) d) Como el valor final tras una pérdida e) Como el valor inicial tras una ganancia.			a) SI –NO b) SI –NO c) SI – NO d) SI – NO e) SI – NO	

<table>
<tr>
<td colspan="3">Formula o relata situaciones posibles que den sentido a la resta:
a) Como el cálculo de una parte del total
b) Como la diferencia entre dos magnitudes que se comparan
c) Como la magnitud (referido) que es unidades menor que otra mayor (referente)
d) Como el valor final tras una pérdida
e) Como el valor inicial tras una ganancia.</td>
<td>a) SI –NO
b) SI –NO
c) SI – NO
d) SI – NO
e) SI – NO</td>
<td>La formulación le cuesta mucho más que la identificación</td>
</tr>
<tr>
<td colspan="3">En el manejo de palillos, entiende la diferencia con la suma: ahora el minuendo es el “todo” y por tanto el sustraendo no se representa con palillos, sino con huecos a rellenar.</td>
<td>SI - NO</td>
<td></td>
</tr>
<tr>
<td colspan="3">Resta sin llevada - ¿Entiende que restar es lo que queda del minuendo cuando se aparta de él al sustraendo (sustraendo y resta son las dos partes disjuntas en las que se separa el minuendo)?</td>
<td>SI - NO</td>
<td></td>
</tr>
<tr>
<td rowspan="2">Resta con llevada- ¿Abre un grupo del orden inmediato superior de manera espontánea cuando no hay suficientes palillos para llenar los huecos del sustraendo?

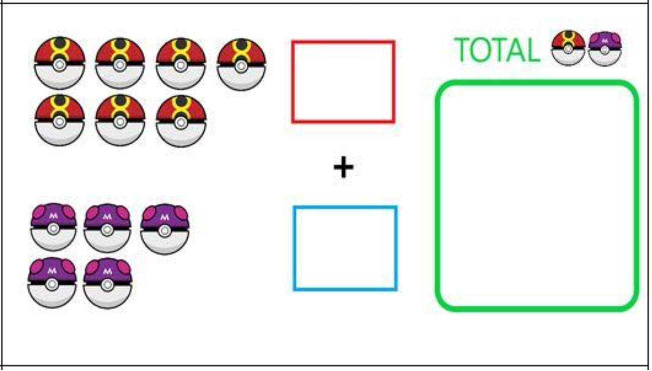
Regla: Cada 10 agrupo (goma)</td>
<td colspan="3">SI</td>
<td></td>
</tr>
<tr>
<td>NO</td>
<td colspan="2">Frecuencia - ¿Cuántas veces le ayudo hasta que él lo integra?
Todas, pero en este caso ya no es sólo como estrategia para que no se despiste, sino porque verdaderamente no entiende que cuando le pone un 1 al lado del número de las unidades ‘3’ y lo hace ‘13’, no entiende que ese 1 viene del orden de las decenas y que luego hay que quitárselo a este.</td>
<td></td>
</tr>
<tr>
<td>¿Qué me muestra que entiende que es capaz de realizar la actividad?</td>
<td colspan="3">A través del uso de los palillos y las cajitas como agujeros (representación sustraendo), es capaz de realizar la resta correctamente y sin rechazo, porque ve que esa herramienta le ayuda a hacerlo</td>
<td></td>
</tr>
</table>

Annex XXV: Addition situations flashcards - *Back and Front*

- **Part-Whole relation**

Total **UNIÓN**:

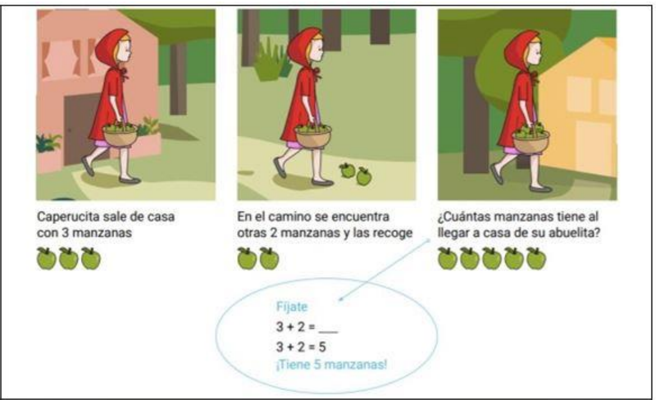
- Parte 1
- Parte 2



- **Before-After relation**

Valor final tras ganancia

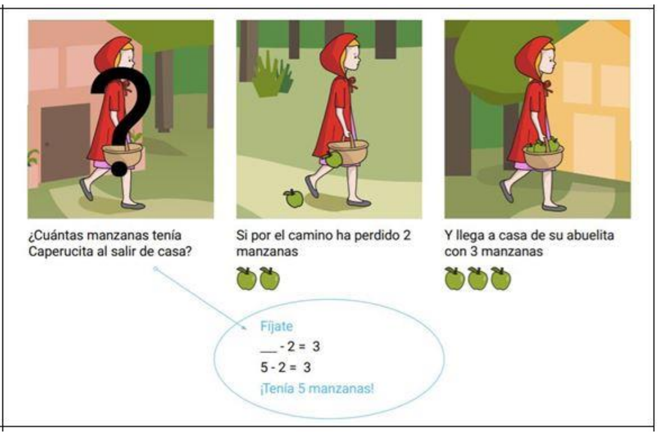
- *Nos encontramos*
- *Recibimos*



Valor inicial tras pérdida

¿Cuánto teníamos al principio tras:

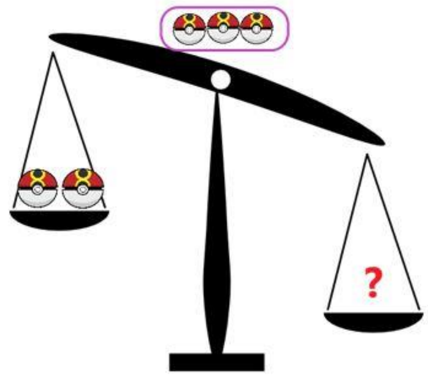
- *Dar…?*
- *Regalar…?*
- *Gastar…?*
- *Perder…?*



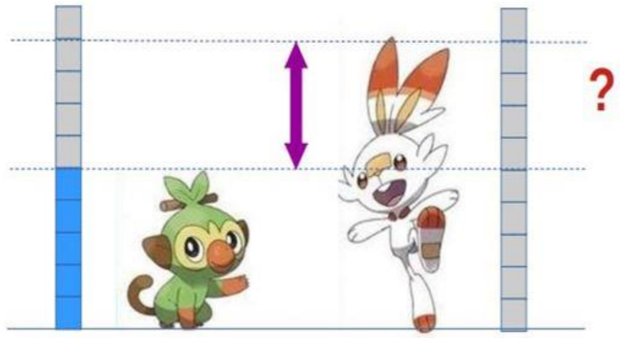
Red Riding Hood images original from: Aritmética. Antes y después: Matemáticas #Soyvisual

- **Additive comparison relation**

<u>Weight</u>

Cálculo **magnitud mayor** teniendo: • la diferencia • la magnitud menor ➢ "¿Cuántas pokeballs pesa (o hay en) el lado rojo de la balanza?"	

<u>Heights/Lenghts</u>

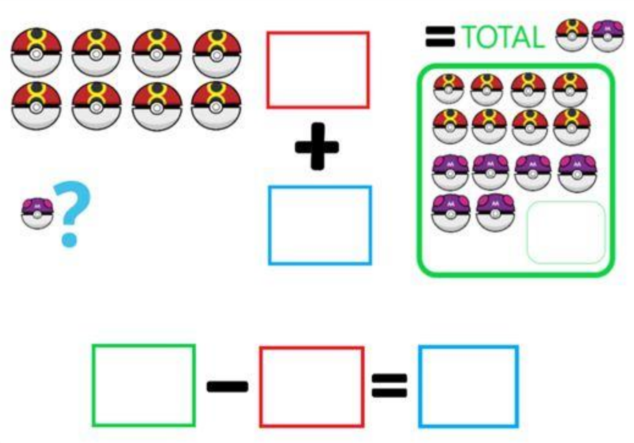
Cálculo **magnitud mayor** teniendo: • la diferencia • la magnitud menor ➢ "¿Cuántos bloques mide el pokemon rojo?"	

Annex XXVI: Subtraction Situations flashcards - *Back and Front*

- **Part-Whole relation**

Cálculo de Parte 1 a partir del TOTAL:

- Total
- Parte 2

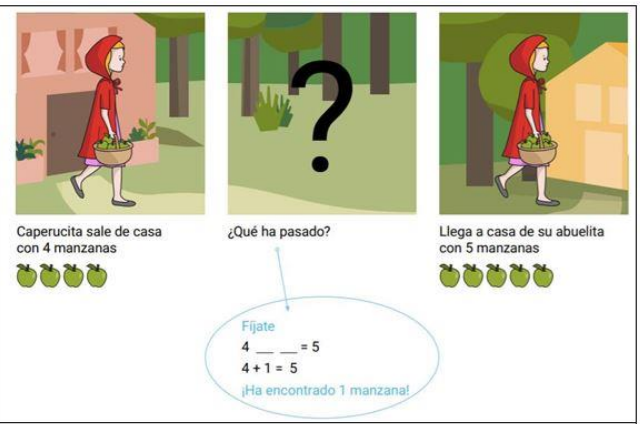


- **Before-after relation**

Red Riding Hood images original from: [Aritmética. Antes y después: Matemáticas #Soyvisual]

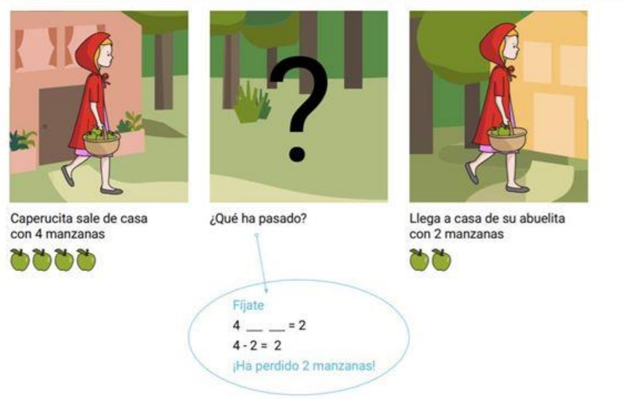
Ganancia encontrada

- *Nos encontramos*
- *Recibimos*



Valor pérdida durante

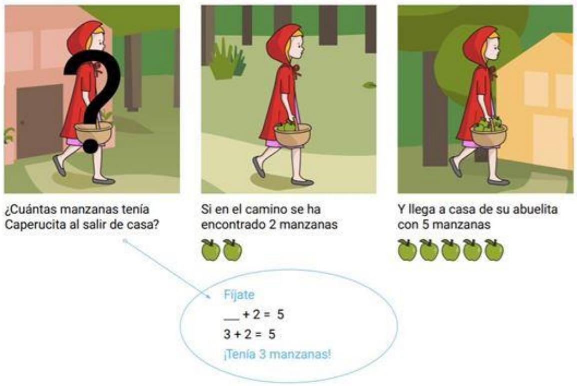
- *Perdemos*
- *Gastamos*
- *Damos*
- *Regalar*



Valor inicial tras ganancia

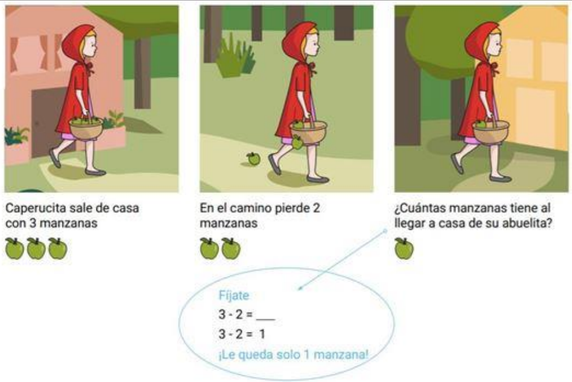
¿Cuánto teníamos al principio antes de:

- *ganar …?*
- *recibir… ?*
- *conseguir…?*



Valor final tras pérdida

- *Perdemos*
- *Gastamos*
- *Damos*
- *Regalar*



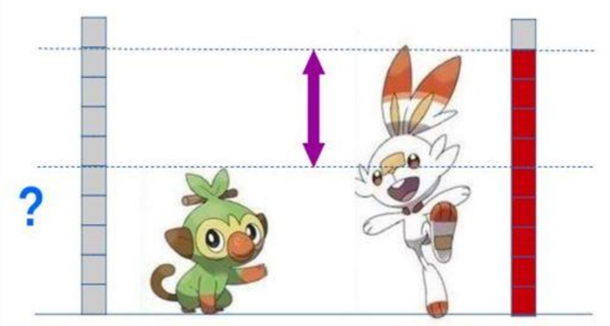
- **Additive comparison relation**

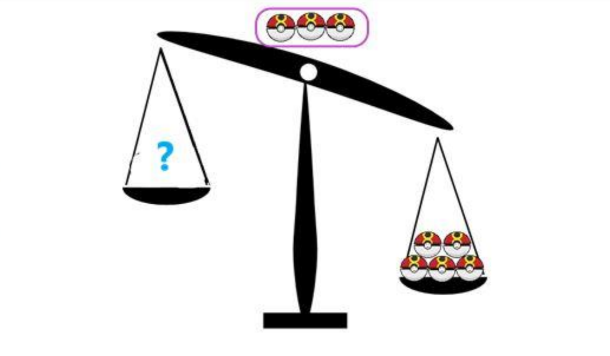
Diferencia entre magnitudes que se comparan

> *"¿Cuántas pokeballs pesa un lado de la balanza más que el otro?"*

Diferencia entre magnitudes que se comparan

> *¿Cuántos bloques mide un pokémon más que el otro?*

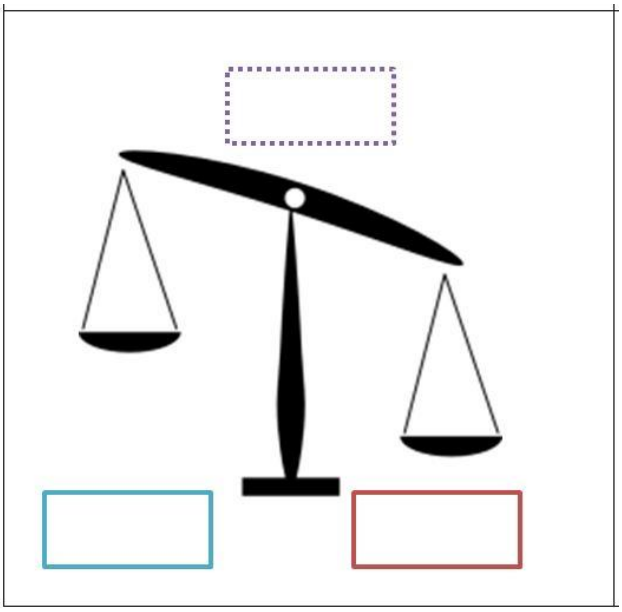
Cálculo **magnitud menor** teniendo: • la diferencia • la magnitud mayor ➢ *¿Cuántos bloques mide el pokémon azul?*	

Cálculo **magnitud menor** teniendo: • la diferencia • la magnitud mayor ➢ *¿Cuántas pokeballs pesa (o hay en) el lado azul de la balanza?*	

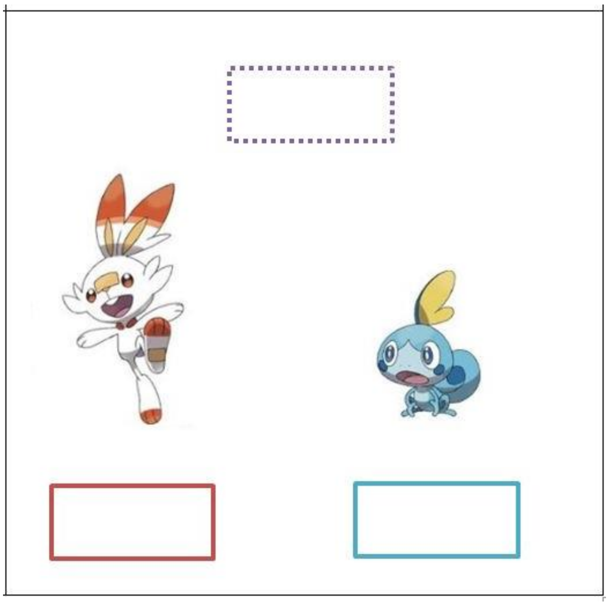
Annex XXVII: Blank Scene models for stories

- **Additive comparison relation**

<u>Weight</u>

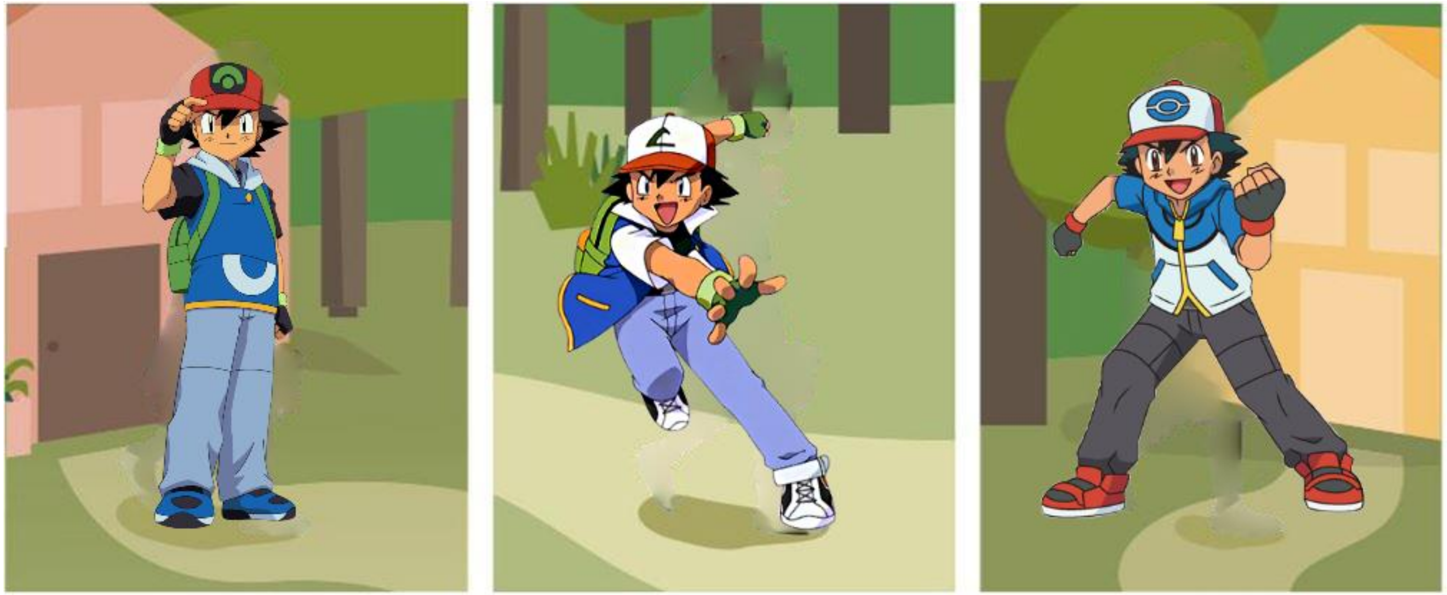


<u>Height/Lenghts</u>



- **Before-after relation**

ANTES **DURANTE** **DESPUÉS**



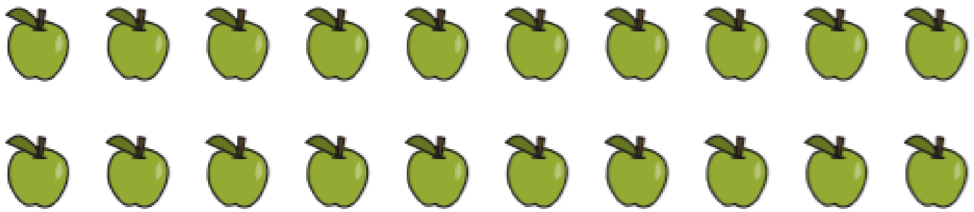
OPERACIÓN

Modified from: https://www.soyvisual.org/materiales/aritmetica-antes-y-despues

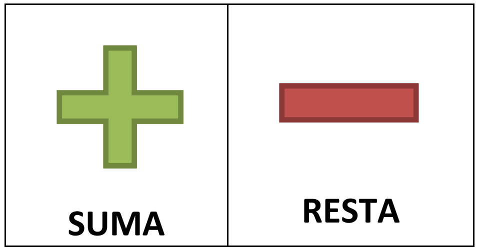
1. Recortar para realizar las fichas de relación de antes y después

Pokemón

Caperucita Roja



2. Recortar los símbolos para marcar la operación



ANTES **DURANTE** **DESPUÉS**

OPERACIÓN

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