Essais en Economie Evolutionniste

Bilge Ozturk

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Par

BILGE OZTURK

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Soutenue le 8 juillet 2005, devant le jury composé de :

Seyfettin GÜRSEL	Professeur à l'Université Galatasaray
Jean-François LASLIER	Directeur de Recherche au CNRS, *Directeur de Thèse*
Vincent MERLIN	Chargé de Recherche au CNRS, *Rapporteur*
André ORLEAN	Directeur de Recherche au CNRS, *Rapporteur*
Remzi SANVER	Professeur a l'Université Bilgi Istanbul
Bernard WALLISER	Directeur de Recherche au CNRS

Aileme
Dostlarıma
Özlem'e

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CHAPITRE 1

CHAPITRE INTRODUCTIF

1.1 Economie évolutionniste

La rationalité et l'information parfaite sont les hypothèses fondamentales sur le comportement individuel en théorie des jeux, et pour cette partie de l'économie qui se base sur la théorie des jeux pour étudier le comportement des agents économiques. L'agent dispose d'information sur les actions des autres agents, ou du moins sur la distribution de probabilité de ces actions, et maximise son gain en prenant en compte la conséquence de toutes ses actions étant donné l'action des autres agents. De cette façon, les agents peuvent collectivement se trouver en équilibre. La théorie, telle qu'elle est, présente certains défauts: le problème de la sélection d'équilibre, la question des agents hyperrationnels et l'absence d'aspect dynamique. Le recours à la théorie des jeux évolutionnistes pour traiter de ces trois difficultés est naturel car (a) le concept de stabilité évolutionniste offre une rafinement d'équilibre, (b) les critères de la rationalité ne sont pas aussi forts et (c) la théorie de l'évolution est une théorie dynamique. La théorie des jeux évolutionnistes consiste à l'origine en l'application de la théorie des jeux dans des contextes biologiques. Les trois changements relatifs à la nature de l'intelligence des agents, à l'interaction entre agents et au concept d'équilibre sont destinés à adapter l'aspect stratégique au contexte biologique.

La théorie évolutionniste fait l'étude d'une population d'agents. Un état de la population est une répartition dans l'espace des stratégies pures. Les agents ne sont plus supposés choisir leurs actions dans un espace stratégique et avoir un effet sur les actions des autres agents. La population est composée d'agents, chacun porteur d'une unique stratégie, et appariés au hasard d'une façon répété pour jouer le jeu. La fréquence des stratégies dans la population change suivant l'investigation des stratégies alternatives par apprentissage, par mimétisme ou par hasard. L'étude de la stabilité remplace le concept d'équilibre de Nash.

1.1.1 La stabilité évolutionniste

1.1.1.1 Le cas statique: le concept des stratégies évolutionnairement stables (SSE)

Le concept de stratégie évolutionnairement stable est dû à Maynard Smith et Price (1973). Une telle stratégie est définie comme une stratégie robuste aux pressions évolutionnistes. C'est une situation stationnaire du processus évolutionniste. C'est une stratégie telle que si elle est adoptée par tous les membres d'une population, aucune stratégie adoptée par une fraction suffisamment petite de la population (une stratégie mutante) ne pourra venir envahir cette population par les mécanismes de la sélection naturelle. Cette définition ne décrit pas la manière dont toute la population arrive à jouer cette stratégie mais détermine la propriété d'une telle situation. Néanmoins c'est un état stable potentiel de l'évolution et l'étude de l'evolution nécessite un tel concept d'équilibre.

1.1.1.2 Le cas dynamique: La dynamique du réplicateur

Le dynamique du réplicateur propose un contexte dynamique qui combine la sélection et la mutation. Contrairement à la notion de stratégie évolutionnairement stable (SSE) qui met l'accent sur le rôle de la mutation, la dynamique du réplicateur, formulée par Taylor and Jonker (1978), est un modèle explicite de la sélection. La mutation intervient au niveau des critères de stabilité dynamique. La dynamique du réplicateur est une formulation de la variation des parts de la population associées à une stratégie déterminée. Le taux de croissance de la part de la population qui joue une certaine stratégie dépend directement de la différence entre l'espérance de gain de cette stratégie et la moyenne des espérances de gain des autres stratégies, c'est à dire le gain moyen général. Cette formulation vérifie le principe de sélection: elle contribue à faire augmenter la proportion des individus les mieux adaptés à leur environnement.

1.1.2 Les dynamiques de sélection

1.1.2.1 Les dynamiques de sélection régulières

Les dynamiques de sélection considèrent le taux de croissance des parts de la population en attribuant cette croissance à la favorisation de certaines stratégies. Le taux de croissance satisfait certaines conditions de régularité: i. Continuité au sens de Lipschitz, qui implique l'existence et l'unicité de la solution du système d'équations différentielles associé. ii. Monotonicité, qui assure que la part de la population associée à une stratégie plus performante croît à un taux supérieur. iii. Positivité des gains, qui nécessite que l'ordre des taux de croissance soit le même que celui des gains.

La dynamique de réplicateur est une dynamique de sélection régulière. La formulation de la dynamique du réplicateur est basée sur le "réplicateur" qui caractérise un gène, un organisme, une stratégie, une croyance, une technique, une convention ou une forme institutionnelle et qui est capable de faire des copies de lui-même. A chaque période, la population de réplicateurs est renouvelée de façon à favoriser la reproduction des réplicateurs qui s'adaptent le mieux à l'environnement.

1.1.2.2 L'apprentissage

L'apprentissage peut être classé en trois catégories: la modélisation du comportement des autres joueurs et la révision des croyances à partir des informations acquises par observation (l'apprentissage bayésien), la modification du comportement compte tenu des résultats observés de ses propres actions dans le passé (l'apprentissage par renforcement), et le choix d'un comportement basé sur le gain obtenu lors de confrontations aléatoires (l'apprentissage évolutionniste).

Le mimétisme Les dynamiques de l'imitation supposent que, de temps en temps, un agent est tiré au hasard dans la population, et que cet agent compare la performance de son comportement avec le comportement d'un autre agent, lui aussi tiréau hasard dans la population, puis change son comportement si c'est nécessaire. Dans le modèle de Gale, Binmore et Samuelson (1995) l'imitation se fait si la performance est en dessous du niveau d'aspiration de l'agent. Au lieu de supposer un tirage au hasard on peut supposer que le besoin de réviser la performance suit un certain processus stochastique.

1.1.3 La stabilité stochastique

La recherche dans le domaine de la stabilité des processus évolutionnistes stochastiques est initiée par les travaux de Foster et Young (1990) et Fudenberg et Harris (1992). L'aspect stochastique des jeux évolutionnistes provient de deux sources: le processus de mutation est de toutes manières aléatoire, et le processus de sélection peut aussi être sujet à des chocs aléatoires. Le problème de la stabilité stochastique s'impose. Un état stable de la population est décrit comme un état robuste aux petites perturbations isolées. Cette définition de la stabilité exclut le cas des chocs simultanés ou des chocs enchainés qui peuvent éloigner le système du domaine d'attraction du processus de sélection. Quant aux mutations, leur indépendance et leur rareté change le caractère du processus qui peut devenir ergodique.

Dans la description de la sélection, Kandori, Mailath et Rob (1993) supposent que le changement de stratégies se fait en calculant la meilleure réponse à la distribution de stratégies de la période précédente, et analysent la répartition des stratégies d'un jeu symmétrique à deux joueurs dans une population finie. Young (1993) suppose que les agents collectent un échantillon fini des jeux passés et calculent la meilleure réponse à cette échantillon de jeux passés. Le bruit est créé par la diminution de la taille de l'échatillon. Dans ce contexte, le processus de sélection converge dans certaines classes de jeux si l'échatillon est suffisamment petit comparé à l'histoire du jeu.

Kandori, Mailath et Rob (1993) et Young (1993) montrent que l'introduction des probabilités d'erreur a un effet critique sur la convergence. Le processus de mutation est modélisé par l'introduction d'un taux de mutation exogène (déterministe ou alétoire) et ce processus doublé du processus de sélection devient une chaine de Markov finie et irréductible, qui possède distribution stationnaire unique, à laquelle tous les états de la population convergent quelle que soit la situation initiale.

Bergin et Lipman (1996) critiquent les modèles de Kandori, Mailath et Rob

(1993) ou de Young (1993) pour l'absence d'une théorie des mutations. Robson et Vega-Redondo (1996) montrent que, quand l'appariement est aléatoire et l'imitation des joueurs performants est possible, l'équilibre Pareto efficace est atteint dans le long terme. Dans le contexte de l'apprentissage, Nöldeke et Samuelson (1993) analysent des jeux en forme extensive.

Nous allons présenter maintenant les trois chapitres de la thèse en détail. Trois modèles en économie évolutionniste sont étudiés. Le premier modèle (chapitre 2) se place dans le cadre de la théorie monétaire, ou plus exactement de l'émergence de la monnaie, alors que le deuxième et le troisième modèles (chapitres 3 et 4) se placent dans le cadre de la théorie du choix social. Dans ces chapitres 3 et 4, nous allons étudier deux modèles dans le cadre des modèles spatiaaux de la compétition électorale. Les traits communs vont être soulignés.

1.2 L'émergence de la monnaie

Les modèles d'équilibre général néoclassique introduisent, explicitement ou implicitement, un planificateur central pour résoudre les problèmes de coordination et de détermination des prix d'échange quand il existe un grand nombre d'agents. Néanmoins, une telle formalisation de l'échange entre les agents n'arrive pas à modéliser la demande de monnaie. L'émergence de la monnaie, comme un moyen d'échange n'est donc pas expliquée et la demande de monnaie est plutôt rationalisée par le fait que la monnaie est un actif de bas risque et de haute liquidité. Les modèles de prospections monétaires résolvent ce problème par une modélisation explicite d'allocations des ressources et définissent un contexte où l'utilisation de la monnaie dépend du degré d'acceptation de celle-ci comme un moyen d'échange. Le processus de prospection et de recrutement introduit des frictions comme l'échange bilatéral, le défaut d'engagement et de mémoire. Dans Kiyotaki et Wright (1989)[1], de telles frictions donnent à la monnaie un rôle essentiel, sans introduire une contrainte particulière pour qu'elle soit utilisée dans les échanges.

Le modèle de Kiyotaki et Wright (1989) où la monnaie est considérée comme un bien cherche à retrouver la structure du commerce en troc dans une économie ayant les caractéristiques suivantes: Il existe trois biens indivisibles et durables. Le coût de stockage de chaque bien est différent et défini en terme de désutilité instantanée. Comme tous les biens sont indivisibles, il y a un troc un pour un des stocks quand il existe un accord mutuel pour faire un échange. Il existe un continuum d'agents, dont la durée de vie est limitée mais incertaine. Il existe trois types d'agent. Les agents sont spécialisés dans la production et la consommation et donc ils existent autant de type d'agents que de biens. L'agent i acquiert l'utilité seulement de la consommation du bien i et produit seulement le bien $i+1$ (L'agent

[1] Kiyotaki et Wright (1989) est la source d'une littérature importante. Quelques extensions du modèle incluent la monnaie fiduciaire dans Kiyotaki et Wright (1991, 1993) et dans Aiyagari et Wallace (1991,1992), la détermination endogène des biens comme des moyens d'échange dans Kehoe, Kiyotaki et Wright (1993), l'introduction de plusieurs monnaies différentes dans Matsuyama, Kiyotaki, et Matsui (1993), et de négociation bilatérale dans Trejos et Wright (1995) ou Shi (1995). Une autre extension du modèle est faite par Marimon, McGartten et Sargent (1990), qui utilisent des populations artificielles de rationalité limitée pour tester les prédictions d'équilibre de Kiyotaki et Wright (1989).

3 consomme le bien 3 et produit le bien 1). Comme les biens sont indivisibles et le stockage est coûteux, les agents ne peuvent stocker qu'un seul bien à la fois. Si un agent de type i obtient son bien de consommation par échange bilatéral, il le consomme tout de suite, gagne une unité d'utilité et produit une unité de bien $i+1$ à nouveau. Ainsi, les agents ont toujours une unité de bien en stock autre que leur bien de consommation. A la période 0, n agents de chaque type possédant une unité de bien de production entrent dans le marché. Le nombre d'agents reste constant. Pour chaque type d'agent, il existe deux stratégies: soit accepter seulement leur bien de consommation en échange quand ils rencontrent un autre agent, soit accepter les deux biens. Un agent du premier type a toujours son bien de production $i+1$ en stock. En revanche, un agent du deuxième type peut avoir les biens $i+1$ ou $i+2$. A chaque période, un agent est sélectionné de manière aléatoire. Ensuite, cet agent est supposé contacter un autre agent pour faire l'échange. La paire d'agents échange leur stock si cela est mutuellement acceptable. La paire se dissout ensuite, que le commerce aie été mutuellement accepté ou rejeté. Les agents savent le type de la personne qu'ils rencontrent mais n'ont pas d'information sur sa détention de stock, ni sur sa stratégie.

Dans ce contexte, certaines biens émergent comme un moyen d'échange, suivant à la fois les propriétés intrinsèques et les croyances extrinsèques. Ces deux cas sont nommés stratégie *fondamentale* et *spéculative* respectivement, par Kiyotaki et Wright (1989). La stratégie fondamentale est une stratégie qui consiste à faire le commerce pour le bien de consommation ou pour un bien de coût de stockage plus élevé contre un bien de coût de stockage plus faible. La stratégie spéculative nécessite de faire l'échange du bien de production contre un autre bien de coût de stockage plus élevé. Kiyotaki et Wright (1989) déterminent deux équilibres dans le contexte décrit, selon la différence des coûts de stockage des biens. Dans l'un des équilibres, tous les agents utilisent des stratégies fondamentales et dans l'autre, les agents du type 1 utilisent la stratégie spéculative et les autres agents utilisent les stratégies fondamentales. Ces résultats sont obtenus sous l'hypothèse standard de rationalité des agents. Le modèle de Kiyotaki et Wright (1989) et la plupart de ses extensions supposent que les agents rationnels sont collectivement capables de déterminer un équilibre du modèle.

1.2.1 Un modèle évolutionniste de l'émergence de la monnaie

Dans la thèse, l'approche évolutionniste est appliquée au modèle de l'émergence de monnaie. On suppose que la population initiale soit suffisamment hétérogène, de façon à refléter tous les comportements possibles et leurs utilités matérielles. La population évolue de façon que la part de la population qui a plus d'utilité croît plus vite que la part ayant moins d'utilité. Alors on arrive à identifier le point d'équilibre asymptotiquement stable de ce système dynamique de sélection.

Le point de départ est Sethi (1999) qui analyse le modèle de prospection monétaire de Kiyotaki et Wright (1989) dans un cadre évolutionniste. Dans cette version, où il existe un appariement aléatoire, on montre que les états fondamentaux et spéculatifs, ainsi que les états polymorphiques, peuvent tous être stables suivant les valeurs des coûts de stockage, et qu'il peut exister plusieurs équilibres stables.

Pour obtenir la stabilité évolutionniste de ces équilibres, Sethi (1999) utilise la procédure suivante: Etant donné une composition de comportements de la population, la dynamique des stocks est définie et les valeurs d'équilibre des stocks sont trouvées. Par la suite, les dynamiques de la sélection évolutionniste sont appliquées aux différents états de la population pour établir la stabilité par rapport aux dynamiques évolutionnistes. L'hypothèse sous-jacente est que la distribution des stocks est toujours à sa valeur d'équilibre temporaire même si la composition des comportements évolue. L'approche évolutionniste analyse la distribution des comportements dans la population (règles de décision, stratégies) qui sont soumis à des dynamiques de sélection spécifiques. Par conséquent, tous les comportements possibles sont inclus et les comportements qui vont survivre au processus de sélection dynamique sont déterminés. Etant donné les différentes distributions initiales, les points de convergence des dynamiques des stocks sont déterminés. Ainsi, la robustesse de ces distributions de stratégies est testée.

Dans le Chapitre 2 de la thèse nous visons deux objectifs: L'objectif principal est d'analyser les implications de l'introduction d'asymétries dans le processus d'appariement et d'étudier la relation de l'hypothèse d'appariement aléatoire avec l'émergence d'un moyen d'échange. Cette introduction nous permet de retrouver les conditions de stabilité pour les équilibres définis dans Kiyotaki et Wright (1989). L'objectif secondaire est d'étudier les dynamiques du déséquilibre, dans lesquelles la distribution des stocks n'est plus supposée avoir toujours sa valeur d'équilibre temporaire. Dans ce cadre, on détermine l'espace des valeurs des paramètres où les équilibres fondamentaux et spéculatifs sont stables, et la dynamique des parts de la population est analysée lorsque la distribution des stocks n'est plus supposée être à la valeur de l'équilibre temporaire.

1.2.1.1 La technologie de transaction

Les modèles de prospection standards formalisent la circulation monétaire sous forme d'interaction stratégique et doivent alors spécifier une technologie de transaction. L'approche commune proposée par Kiyotaki et Wright (1989, 1993) est de supposer que les échanges ont lieu sur une place de marché unique avec des rencontres bilatérales aléatoires entre un grand nombre de producteurs. Deux probabilités, supposées indépendantes, sont essentielles pour ce mécanisme de rencontre: la probabilité d'être apparié et celle de rencontrer un agent d'un type particulier. La probabilité d'être apparié est proportionnelle au nombre d'agents et les probabilités de rencontrer un agent d'un type particulier dépendent de la proportion d'agents de chaque type dans le secteur de l'échange. Les extensions évolutionnistes de ces modèles retiennent l'hypothèse de l'appariement aléatoire.

Cette hypothèse est plutôt irréaliste (Clower et Howitt (1995, 1996)) parce que les agents ne choisissent pas leurs actions en se basant sur les rencontres aléatoires. Les travaux plus récents ayant recours à des mécanismes de rencontre plus complexes (choix du partenaire d'échange, intermédiation marchande)[2] ont montré que l'hypothèse de rencontres aléatoires n'est pas essentielle pour l'obtention de l'équilibre monétaire, au contraire de l'hypothèse de rencontres bilatérales. Dans ce travail, le processus d'appariement est modifié de façon à introduire des probabilités d'appariement dans le modèle de Sethi (1999). Les agents d'un certain type ne veulent pas renconter les autres types d'agents avec les mêmes probabilités. Ils préfèrent renconter un certain type d'agent plus qu'un autre. Les conditions de stabilité des états fondamentaux, spéculatifs et polymorphiques sont définis en fonction des probabilités d'appariement. L'appariement aléatoire étant un cas particulier de ce processus d'appariement, les conditions de stabilité obtenues sont une généralisation des conditions de stabilité dans Sethi (1999). Dans cette analyse, l'hypothèse que la distribution des stocks est toujours à la valeur d'équilibre stationnaire implique que les effets des changements de la distribution des stocks sont négligés.

[2] Les différents modifications de la mécanique de l'appariement incluent: Corbae, Temzelides et Wright (1999), qui autorisent les agents à choisir le type d'agents qu'ils souhaitent rencontrer, Iwaï (1988, 1996), qui propose un modèle où les rencontres sont aléatoires mais où l'échange se fait sur des places de marchés distinctes pour chaque bien où les biens s'échangent deux à deux, Matsui et Shimizu (2000), qui permettent aux agents de choisir à la fois le marché et leur spécialisation (être un acheteur ou un vendeur), Hellwig (2000) et Howitt (2000), qui proposent un modèle dans lequel les agents doivent à la fois choisir leur activité (producteur ou marchand) et, pour les producteurs, leur mode d'échange (échange direct ou par l'intermédiaire de magasins spécialisés dans les biens offerts).

1.2.1.2 Les dynamiques de déséquilibre

La dynamique de déséquilibre est analysée quand la distribution des stocks n'est pas anticipée être toujours à sa valeur d'équilibre. Pour tenir compte de l'effet des changements des stocks, nous proposons un modèle d'appariement aléatoire dans lequel la population est classifiée suivant la distribution des stocks et des comportements. Cette classification nous permet d'analyser l'évolution des parts de la population avec l'échange des stocks et la sélection évolutionniste, ces deux processus se déroulant avec des vitesses différentes.

Pour pouvoir analyser ce modèle d'un point de vue évolutionniste, nous devons caractériser la population totale des agents dans un groupe suivant deux critères: la stratégie qu'ils adoptent et le bien qu'ils ont en stock. Par conséquent, chaque groupe est composé de quatre catégories. Ensuite la dynamique de la modification des parts de la population est défini. Les parts de la population changent d'une part avec l'échange et d'autre part avec la réplication par imitation. Les agents sont supposés faire l'échange des stocks plus fréquemment qu'ils ne révisent leurs stratégies. La vitesse de la dynamique d'échange est plus grande que celle de la réplication par imitation. L'état fondamental et l'état spéculatif sont les points de convergence de cette dynamique. La stabilité de ces états est étudiée. Pour déterminer les conditions sous lesquelles ils sont stables, le système est linéarisé, les conditions pour la stabilité de l'état fondamental et de l'état spéculatif sont déterminées, en terme des paramètres. Les résultats ne sont pas en accord avec les résultats antérieurs dans la littérature. Par exemple, pour une même gamme de paramètres que dans Sethi (1999), les dynamiques de déséquilibre définies dans ce travail aboutissent à l'instabilité.

1.3 La compétition électorale

1.3.1 Position des problèmes considérés

L'interprétation politique des modèles spatiaux remonte à la fameuse discussion des duopolistes par Hotelling (1929). Hotelling avait formulé la tendance des concurrents à être exactement semblables selon le principe de différenciation minimale, et il avait suggéré que ce principe pouvait être appliqué à un large éventail de phénomènes sociaux, y compris la compétition électorale, en faisant référence aux similarités idéologiques entre les politiques proposées par les républicains et les démocrates dans les élections de 1928. Cette intuition a été formalisée par la suite dans la théorie de l'électeur médian par Black (1948). Dans ce cas, les électeurs sont caractérisés par des préférences unimodales.

Downs (1957) a élargi le modèle spatial de concurrence dans le domaine de la démocratie représentative où deux candidats se font concurrence en proposant des politiques d'un espace de politique unidimensionnel conceptualisé soit comme un espace des niveaux d'effort soit, plus classiquement, comme une dimension idéologique. Selon l'approche downsienne, les partis politiques sont organisés dans le but de gagner les élections, de ce fait les responsables politiques sont censés formuler leurs propositions politiques de manière à contenter la majorité. Dans ce sens, les concurrents sont identiques à tous les égards et la proposition de la politique d'équilibre commun est déterminée par la politique préférée de l'électeur médian.

A côté de cette vision trop simple des politiciens considérés comme des maximisateurs de vote, il y a eu une longue tradition, de Michels (1915) à Lipset (1959), selon laquelle les partis sont "idéologiques" et ont des préférences propres, définies sur l'espace de politiques. Cette idée a été formalisé par la suite par Wittman (1983), Calvert (1985) et Hansson et Stuart (1984), qui ont identifié les partis à des institutions qui représentent les groupes d'intérêt adversaires dans la société. Ainsi les candidats adversaires deviennent différenciés car chacun représente un groupe d'intérêt différent et cherche la publicité pour une plateforme idéologique différente, déterminée dans ces travaux d'une manière exogène.

Ces deux approches ont été combinées par Roemer (1999) qui a conceptualisé les partis comme composés d'une faction opportuniste, d'une faction militante et d'une faction réformiste. Les politiciens opportunistes sont ceux qui veulent maximiser leur probabilité de victoire, et les militants sont ceux qui veulent maximiser l'utilité du citoyen représenté par le parti. Quant aux réformistes, ils maximisent l'utilité prévue des constituants du parti. Les opportunistes appartiennent à la conception politique de Downs (1957), et les militants à celle de Wittman (1983). Dans ce contexte, l'équilibre de Nash d'unanimité du parti (PUNE, "Party unanimity Nash Equilibrium") est défini. Il consiste en des propositions politiques partisanes qui sont "Nash" dans le sens suivant: ni dans un parti ni dans l'autre, étant donné la politique proposée par l'autre parti, les factions (internes) ne peuvent se mettre unanimement d'accord sur une déviation de la politique proposée.

Nous adoptons l'approche précédente des partis politiques dans une démocratie représentative. Les citoyens choisissent le gouvernement qui, à son tour, choisit la politique à appliquer. Les partis s'engagent, de manière crédible, à respecter leur propositions politiques au cas où ils seraient élus. Les citoyens votent sincèrement pour les politiques proposées.

Le cycle électoral est le suivant: Etant donné les profils des partis, chaque parti annonce la politique qui est obtenue par l'aggrégation des propositions des factions. La règle de l'agrégation est la moyenne pondérée des propositions de chaque faction. L'influence de chaque faction est proportionnelle à son poids dans le parti. La proposition de la faction militante est donnée par sa préférence idéologique. La proposition de la faction opportuniste est obtenue à travers la maximisation de l'utilité des opportunistes, étant donné la proposition de politique du parti adverse. Contrairement à Roemer (1999) qui adopte une règle de décision selon laquelle chaque faction dans le parti a droit de veto sur les politiques proposées, le mécanisme de décision interne utilisé ici est une moyenne pondérée des propositions des différentes factions. Roemer (1999) montre que si chaque parti élabore un méthode de négociation interne, la proposition qui en résulte en tant que conséquence de négotiation à l'intérieur du parti est un PUNE puisque les factions d'aucun parti ne pourrait être unanimes pour dévier vers une autre politique.

Par rapport à ces travaux, nous adoptons l'approche évolutionniste pour endogénéiser les décisions des candidats opportunistes. Ces derniers révisent leur choix d'adhésion aux partis politiques dans la période de pré-élection selon les dynamiques évolutionnistes.

Il y a deux types d'états d'une population opportuniste: les états purs, où tous les opportunistes sont dans un parti ou un autre, et les états mixtes, où les opportunistes sont distribués dans les deux partis. Toutefois, les opportunistes sont censés être sujet à une certaine contrainte dans leur processus de la prise de décision. La faction opportuniste du parti gauche (droite) ne peut pas proposer un proposition plus grande (petite) que celui du parti droite (gauche). Ceci semble raisonable, par exemple, si la faction opportuniste du parti gauche propose une politique plus grande que celle du parti droite, on peut imaginer qu'ils vont perdre de leur crédibilité en tant que candidats, et subir une punition électorale (non modélisée) dans le futur.

Le résultat politique est détérminé par les élections. Nous prenons en considération deux systèmes politiques qui se distinguent par la manière dont les votes sont traduits en sièges à l'assemblée: les systèmes politiques majoritaire et proportionnel. Ces deux systèmes diffèrent par la récompense qui croît selon le partage des votes. Les utilités des candidats opportunistes sont définis suivant le système politique considéré.

1.3.2 L'opportunisme et l'évolution de la compétition électorale

Dans le chapitre 3 de la thèse, nous étudions la compétition électorale entre deux parties politiques. Les partis s'affrontent sur la base de propositions de politiques dont l'espace est unidimensionnel. Suivant Roemer (1999) on suppose que les partis sont composés de candidats hétérogènes. Les candidats diffèrent quant à leurs motivations politiques. Il y a deux types de candidats: Les candidats "opportunistes" ont pour seul objectif de remporter les élections ou de maximiser le nombre de voix pour bénéficier du prestige et de la puissance, alors que les candidats "militants" ont des préférences idéologiques. L'analyse de Roemer (1999) suppose qu'il y ait des candidats opportunistes dans chaque parti. Pourquoi un candidat opportuniste, qui ne s'intéresse qu'aux perspectives de remporter les élections, choisit-il un parti de préférence à un autre? On peut répondre à cette question en permettant aux candidats opportunistes de changer de parti politique et en endogénéisant cette décision.

Au début du cycle électoral, les candidats opportunistes réexaminent leurs choix de partis politiques pour les prochaines élections. L'approche standard serait de supposer que les candidats rationnels peuvent collectivement trouver l'équilibre du modèle et ensuite calculer l'équilibre d'un jeu à 2 périodes où les opportunistes révisent leurs décisions dans une première période et les partis font leurs propositions dans une deuxième période. Cette approche ne fourniera pas une idée sur l'évolution du comportement des candidats opportunistes à partir des état initiaux différents. Comme nous essayons d'étudier les dynamiques de déséquilibre, nous allons supposer que initialement il y a des candidats opportunistes dans chaque parti. La proportion des candidats dans chaque parti va alors se modifier, suivant des rencontres aléatoires avec les autres candidats opportunistes, et par adoption des comportements les plus performants dans l'environnement politique et économique. Les militants, par définition, ne sont pas censés réexaminer dynamiquement leurs engagements et décisions.

Dans ce cadre, on compare les systèmes politiques majoritaire et proportionnel. On étudie l'existence d'un équilibre de la compétition politique de court terme, puis, dans un contexte évolutionniste, on étudie la décision des candidats opportunistes dans le long terme. Un état pur est le cas où tous les candidats opportunistes choisissent un même parti de préférence à l'autre. Un état mixte est le cas où les candidats opportunistes sont répartis dans les deux partis. Les états purs sont des solutions des dynamiques évolutionnistes dans les deux systèmes. Dans le système politique majoritaire, il n'y a qu'un seul état mixte solution des dynamiques. Dans le système politique proportionnel, il peut y avoir un ou plusieurs états mixtes, suivant la distribution des électeurs et de la distribution des candidats. Il se peut aussi qu'il n'y ait aucun. Dans le système politique majoritaire, seuls les états purs sont stables selon les dynamiques évolutionnistes, indépendamment de la distribution des électeurs. L'état mixte n'est pas stable. Dans le système proportionnel, la stabilité des états purs et des états mixtes dépend de la distribution des électeurs et de la distribution des candidats. Seulement dans ce cas il peut y avoir des états mixtes stables.

1.3.3 Un modèle dynamique d'adhésion aux partis et aux idéologies

Les partis politiques sont caracterisés par leurs idéologies, mais celles-ci ne sont que vaguement définies. L'aspect dynamique de l'idéologie d'un parti est du à la variété des opinions et des motivations de ses candidats. Dans le chapitre 4 de la thèse, on essaye d'aborder cet aspect en s'intéressant au comportement des candidats. Les candidats peuvent être plus ou moins flexibles idéologiquement ou bien seulement avoir un intérêt pour le pouvoir. Nous analysons la compétition électorale entre deux partis politiques dont les candidats diffèrent par leur motivations politiques: les opportunistes et les militants. Les opportunistes ne sont loyaux à aucun parti. Leur décision d'adhésion aux partis est une fonction de la probabilité que le parti remporte les élections. Les militants s'intéressent seulement à représenter leur électorat en adoptant les préférences du citoyen moyen qui les soutient. La préférence exprimée par la faction militante d'un parti est donc endogène. Les idéologies des partis sont déterminés par les préférences de leur électorat. Ce processus d'endogénéisation est repris de Roemer (2001) qui a utilisé la notion de citoyen-candidat de Osborne et Slivinski (1996) et Besley et Coate (1997). Ces approches sont modifiées pour inclure les partis et l'engagement.

Au début du cycle électoral, les candidats opportunistes réexaminent leurs choix d'adhésion aux partis politiques pour les prochaines élections en se basant sur des rencontres aléatoires avec les autres candidats opportunistes et en adoptant la meilleure décision du point de vue de la probabilité de remporter les élections. Les militants réexaminent leurs décisions en se basant sur les résultats des élections passées. On étudie l'existence de l'équilibre de compétition électorale sous l'hypothèse de l'information parfaite. Cet équilibre représente une situation où, étant donné le choix de politique d'un parti, l'autre parti ne peut pas dévier en proposant une autre politique, les idéologies des partis sont données par les préférences de l'électeur moyen et les opportunistes proposent des politiques qui maximisent la probabilité de remporter les élections étant donné la politique de l'autre parti. On introduit ensuite une analyse dynamique pour étudier la stabilité de ces équilibres.

Le premier processus dynamique est la dynamique de sélection qui endogénéise les décisions des opportunistes. Le second processus est le processus d'ajustement des militants. Dans ce double cadre, on étudie le systèmes politique proportionnel. On étudie l'existence d'un équilibre de compétition politique de court terme, puis, dans un contexte évolutionniste, on étudie la décision des candidats opportunistes dans le long terme. On détermine les conditions de stabilité des états pures et mixtes. Dans le système proportionnel, la stabilité des états purs dépend de la distribution des électeurs et de la distribution des candidats et la stabilité des états mixtes dépend de la vitesse relative des deux dynamiques.

CHAPITRE 2

UN MODÈLE ÉVOLUTIONNISTE DE L'ÉMERGENCE DE LA MONNAIE

Abstract

This paper uses an evolutionary version of the commodity money model (Kiyotaki and Wright (1989)). The main objective of this paper is to study the implications of endogenising the matching process. Under the endogenous set up we find stability conditions for each kind of equilibrium (fundamental or speculative). The second objective is to analyse the disequilibrium dynamics, when the inventory distribution is not assumed to be continuously at its temporary equilibrium value. We prove that under this setting, for some values of the parameters the fundamental and speculative states are unstable.

2.1 Introduction

The neoclassical general equilibrium models get away with the difficulty of coordination of trade in a many person economy by a fictional coordinating and price setting central authority. The weakness of modelling trade among agents by assuming a central authority results in a failure to model the demand for money. The emergence of money as a medium of exchange is not explained and the demand for money is rather rationalised on the grounds that it is an asset of low risk and high liquidity. Search theoretic models overcome this failure by an explicit modelling of resource allocation process and provide a framework where the use of money depends on the degree of acceptance of money as a medium of exchange. The process of search and recruitment introduces trade frictions such as bilateral exchange, lack of commitment and memory. Such frictions generate in Kiyotaki and Wright (1989) an essential role for money with no particular constraint that it must be used in exchange[1].

This paper uses an evolutionary version of the commodity money model (Kiyotaki and Wright (1989)) which explores the structure of barter trades in an economy with the following characteristics: A continuum of rational agents living a finite but uncertain number of periods and specialised in consumption and production meet pairwise and engage in bilateral exchange. The goods are indivisible and durable but costly to store. Since all goods are indivisible, there is one to one swap of inventories in case of mutually agreed upon trade. In this setting, a medium of exchange is a good that is accepted in trade but not desired

[1] Kiyotaki and Wright (1989) has been the source of a fruitfull literature. This literature is by now large; a few extensions include the introduction of fiat money in Kiyotaki and Wright (1991,1993), Aiyagari and Wallace (1991,1992), the endogenous determination of the commodities that serve as the media of exchange in Kehoe, Kiyotaki and Wright (1993), the introduction of different currencies in Matsuyama, Kiyotaki, and Matsui (1993), the introduction of bilateral bargaining in Trejos and Wright (1995), Shi (1995). Another extension is the use of populations of artificial, boundedly rational agents in order to test the equilibrium predictions of Kiyotaki and Wright (1989) by Marimon, McGartten and Sargent (1990), Başçı (1999) and Staudinger (1998). The predictions of Kiyotaki and Wright (1989) model of commodity money have been tested through laboratory experiments with human paid subjects (Brown (1996), Dufy and Ochs (1999) and Dufy (2000)).

for consumption purposes. The acceptance of a good as a medium of exchange depends both on intrinsic properties and extrinsic beliefs. These two situations are referred to as *fundamental strategy* and *speculative strategy* by Kiyotaki and Wright (1989). More specifically, *Fundamental strategy* is the acceptance of a non consumption good to facilitate further trade if it has a lower storage cost than the one currently held in inventory or the rejection of a non consumption good if it has a higher storage cost than the one currently held in inventory. On the other hand, *speculative strategy* requires agents to accept a good with a higher storage cost.

These results are obtained based on the standard assumption of rationality. Kiyotaki and Wright (1989) model and most of its extensions assume that the rational and optimizing agents are collectively able to locate an equilibrium of the model. However, the evolutionary approach suggests that the initial population consists of a variety of heterogeneous types reflecting all permissible behaviours with their related material rewards. The population evolves in such a manner that the population share of more highly rewarded behaviours grows relative to that of poorly rewarded behaviours. Then the asymptotically stable rest points of this dynamic selection process are identified.

The point of departure of this paper is Sethi (1999) who analyses Kiyotaki and Wright (1989) model of money within an evolutionary framework. In this version of the model with exogenous random matching, it is shown that *fundamental*, *speculative* as well as 'polymorphic' states can all be stable and there may exist a multiplicity of stable states. In order to show the evolutionary stability of these states, Sethi (1999) uses the following procedure. Given a behavioural population composition, the dynamics of inventory holdings are defined and the equilibrium values of inventories are expressed. Then, evolutionary selection dynamics are applied to various population states in order to establish their stability with respect to the evolutionary dynamics. The assumption behind this analysis is that the inventory distribution is expected to be continuously at its temporary equilibrium value even when the behavioural composition evolves.

The present paper deals with two issues. In Section 2, the relation of the random matching assumption with the emergence of media of exchange is explored. The standard search theoretic models assume that agents meet at random. This assumption is rather unrealistic since agents do not choose their actions on the basis of random encounters. Section 2 introduces matching probabilities to the model of Sethi (1999) to study their effects on the stability. The conditions of stability of fundamental, speculative and polymorphic states are defined as a function of the matching probabilities. The preferences of the agents for their trading partners affect the stability of the states even if we hold the relative costs of the goods constant. As a second issue, in Section 3, the dynamics of population shares are analysed when the inventory distribution is not assumed to be continuously at its temporary equilibrium value. The assumption that the inventory distribution is continuously at its temporary equilibrium value implies that the effect of changes in the inventory distribution in response to disequilibrium is neglected. In order to take into account this effect, we propose a model with random matching in which the population is classified according to the inventory distribution and the behavioural distribution. This classification allows us to analyse the evolution of population shares through trade and evolutionary selection affecting the process at different rates. The conditions of stability of fundamental and speculative states are defined as a function of the difference of the costs of the goods and the relative speed of the two dynamics. The two previous sections differ with respect to the evolutionary dynamics used to describe the changes in the share of the population adopting a particular strategy. In the first section the standard replicator dynamics is used while in the second section the changes are due to the replication by imitation. The reason is simply the convenience of the replication by imitation for the model in the second section since we use a different classification for the agents and rather than looking at the performance of the strategies relative to the mean performance we make a pairwise comparison and better performing strategies are imitated. When the relative frequency of trade is increased we observe that the stability conditions looks similar to those of the first model.

2.2 Endogenous matching

In this section the results in Sethi (1999) are reviewed with a different matching setup. The notation of the original article is adopted. First, the environment is described. Then, given behavioural population distribution, temporary equilibrium values of inventory holdings are calculated. The existence and the stability of those values are discussed. Given the equilibrium values of inventories, the utilities corresponding to different behavioural situations are given and the evolutionary stability of these is analysed.

2.2.1 The model

Goods: There exist three indivisible goods indexed by i. They are durable. Storing the good i entails the cost c_i (in terms of instantaneous disutility). We suppose that $c_1 < c_2 < c_3$[2]. A one for one swap of inventories occurs in case of mutually agreed upon trade.

Time: Time is discrete and indexed by $t \in N$.

Economic agents: There exists three types of agents again indexed by i. Agents specialize in consumption and production. Agent i derives utility only from consuming good i and produces only good $i+1$ modulo 3[3]. Agents can store only one good at a time since goods are indivisible and stored at

[2] This particular specification for storage costs is referred to as Model A in Kiyotaki and Wright (1989).

[3] Agent 3 consumes good 3 and produces good 1.

a cost. If an agent of type i gets through trade his consumption good i he consumes it immediately, gets one unit of utility and produces a new unit of good $i+1$. Thus agents always have in stock one unit of one good other than their consumption good.

In period 0, n agents of each type, each endowed with one unit of their production good enter the market. The number of agents stays fixed thereafter.

For each type of agent there exist two strategies: Agents of type i can accept only their consumption good i when they meet another agent. These agents are denoted by αi. On the other hand agents denoted by βi can accept both goods i and $i+2$ if they trade. An agent of type αi has always in stock his production good $i+1$. In return agents of type βi can have either the good $i+1$ or $i+2$.

Matching: Each period one agent is selected randomly (with probability $\frac{1}{3n}$). Then this agent is supposed to contact an agent to trade from the other two populations. Agents of type i are assumed to contact agents of type $i+1$ with probability π_i and with agents of type $i+2$ with probability $1-\pi_i$. If we denote by I_i the set of agents of type i and by (x, y) the pair trading at period t then the following probabilities apply:

– $\Pr(x \in I_i) = \frac{1}{3}$

– $\Pr(y \in I_i | x \in I_i) = 0$

- $\Pr(y \in I_{i+1} | x \in I_i) = \pi_i$
- $\Pr(y \in I_{i+2} | x \in I_i) = 1 - \pi_i$

Therefore, the probability that agent i and agent $i+1$ are matched is $\frac{\pi_i + 1 - \pi_{i+1}}{3}$. In order to simplify the notation, we will denote

$$a_i = \frac{(1 - \pi_i + \pi_{i+2})}{3}.$$

The pair exchanges their inventories if it is mutually agreeable. The pair dissolves in case of both mutually agreed trade or rejection.

Information: The agents know the types of people they meet but have no information on the inventory holdings and the strategies of these agents.

Definition 1 *In this setting, a medium of exchange is a good that is accepted in trade but not desired for consumption purposes.*

In order to analyse this model from an evolutionary point of view we need to allow all behaviours from the part of agents and study the ones that are robust to a dynamic selection mechanism. Thus, the total population of agents can be characterised according to two criteria: the share of agents adopting a certain behaviour and the share of agents holding a certain good. We denote the proportion of agents of type β among agents of type i by s_i. Therefore the share of α agents is $1 - s_i$. The proportion of agents of type i having in stock their production good

$i+1$ is denoted by p_i. Hence the share of agents of type βi holding $i+2$ is $1-p_i$ and the share of agents of type βi holding $i+1$ is p_i+s_i-1. The player types, their inventory holdings and population shares are represented in Table 1.

Table 1 Agent types, inventory holdings and population shares

Type	Inventory holding	Population shares among i types
αi	$i+1$	$1-s_i$
βi	$i+1$	s_i+p_i-1
	$i+2$	$1-p_i$

2.2.2 Temporary equilibrium

2.2.2.1 Inventory dynamics

Suppose the composition of behaviours $s \in [0,1]^3$ is given. We denote the set containing the values of inventory holdings by $\Delta(s)$ which is defined as follows:

$\Delta(s) = [1-s_1, 1] \times [1-s_2, 1] \times [1-s_3, 1]$

Agents of type αi have always in stock their production good $i+1$ as they accept only their consumption good. Therefore, agents of type αi do not have an effect on the inventory distribution when they trade.

Agents of type βi holding their production good $i+1$ can decrease the share p_i in their population if they exchange $i+1$ for $i+2$. This happens if an agent of type βi holding $i+1$ is selected and he in return selects an agent $\beta(i+1)$ holding $i+2$ (with probability $\frac{1}{3}(s_i+p_i-1)\pi_i p_{i+1}$) or an agent of type $\beta(i+1)$ holding $i+2$ is selected and he in return selects an agent βi holding $i+1$ (with probability

$\frac{1}{3}p_{i+1}(1-\pi_{i+1})(s_i+p_i-1))$.

On the other hand, agents of type βi holding the good $i+2$ can increase the share p_i in their population if they exchange $i+2$ for $i+1$. This happens if an agent of type βi holding $i+2$ is selected and he in return selects an agent $\beta(i+2)$ holding i (with probability $\frac{1}{3}(1-p_i)(1-\pi_i)p_{i+2}$) or an agent of type $\beta(i+2)$ holding i is selected and he in return selects an agent βi holding $i+2$ (with probability $\frac{1}{3}p_{i+2}\pi_{i+2}(1-p_i)$).

The resulting inventory dynamics is given by the following equation:

$$\dot{p}_i = (1-p_i)p_{i+2}a_i - (s_i+p_i-1)p_{i+1}a_{i+1} \tag{2.1}$$

The inventory dynamics do not cross the boundary of $\Delta(s)$. In other words if $p(0) \in \Delta(s)$, then $p(t) \in \Delta(s)$ for all t. To see this, notice the following limits:

$$\lim_{p_i \uparrow 1} \dot{p}_i = -s_i p_{i+1} a_{i+1} \leqslant 0$$
$$\lim_{p_i \downarrow 1-s_i} \dot{p}_i = s_i p_{i+2} a_i \geqslant 0$$

2.2.2.2 Temporary equilibria and their stability

For any given vector of population shares $s \in [0,1]^3$, the rest points of equation (2.1) are defined as:

$$\Phi(s) = \{p \in \Delta(s) \mid \dot{p} = 0\}$$

A state s is *monomorphic* if individuals belonging to the same population are of the same behavioural type. States which are not monomorphic are *polymorphic*. The rest points of equation (2.1) are calculated for monomorphic populations and the values are provided at Table 2.

Table 2 Equilibrium inventories for monomorphic populations

	s	$\rho(s)$
Case 1	$(0,0,0)$	$(1,1,1)$
Case 2	$(0,1,0)$	$(1,\frac{a_2}{a_3+a_2},1)$
Case 3	$(1,1,0)$	$(\rho_1^3,\rho_2^3,1)$
Case 4	$(1,1,1)$	$(\rho_1^4,\rho_2^4,\rho_3^4)$

Case 1 refers to a situation where all agents accept only their consumption good in trade. In Case 2, agents of type 1 refuse to trade their lower cost production good 2 for the more costly to store good 3, agents of type 2 always offer to trade their high storage cost good 3 for the less-costly-to store good 1 and agents of type 3 always refuse to offer to trade their low storage cost good 1 for the more costly-to-store good 2. Case 2 is referred to as the *fundamental state* since fundamental factors i.e. storage cost are taken into account in the decision process and either agents trade high storage cost goods for lower storage cost goods or refuse to trade low storage cost goods for higher storage cost goods. In Case 3, agents of type 1 trade good 2 for the more costly to store good 3, agents of type 2 always offer to trade their high storage cost good 3 for the less costly to store good 1 and agents of type 3 always refuse to offer to trade their low storage cost good 1 for the more costly-to-store good 2. In case 3 agents of type 1 use speculative trading strategies since they accept in trade a more costly to store good to increase the likelihood of trading for their desired consumption good. We will refer to the Case 3 as the *speculative state.* Case 4 refers to a situation where all agents accept a good that is not desired for consumption purposes.

Remark 2 *In case 2 Good 1 is a medium of exchange since agents of type 2 trade for good 1 but consume good 2. In case 3, there are two media of exchange Good 1 and Good 3. Again, Good 1 is a medium of exchange since agents of type 2 trade for good 1 but consume good 2. Good 3 fulfills this role as agents of type 1 trade for good 3 but consume good 1. As good 3 is the most costly-to-store good, it is dominated in rate of return.*

Notice that for $s = (1, 1, 0)$ the equilibrium inventories is given by $(\rho_1^3, \rho_2^3, 1)$ where

$$\rho_1^3 = \frac{a_1}{2a_2(a_1 + a_2)} \left(a_2 - a_3 + \sqrt{\frac{4a_2^2 a_3}{a_1} + (a_2 + a_3)^2} \right) \tag{2.2}$$

and

$$\rho_2^3 = \frac{a_1(1 - \rho_1^3)}{a_2 \rho_1^3} \tag{2.3}$$

For $s = (1, 1, 1)$ the equilibrium inventories is $(\rho_1^4, \rho_2^4, \rho_3^4)$ where

$$\rho_1^4 = \frac{a_3 \left[(\rho_3^4 - 1)a_2 + (\rho_3^4)^2 a_1 \right]}{\rho_3^4 a_1 a_2} \tag{2.4}$$

and

$$\rho_2^4 = 1 - \frac{(\rho_3^4)^2 a_1}{(1-\rho_3^4)a_2} \tag{2.5}$$

and ρ_3^4 is given by the following fourth degree equation:

$$\begin{aligned} 0 \quad = \quad & \rho_3^4(a_1a_2a_3 - 2a_2^2a_3) + (\rho_3^4)^2(-4a_1a_2a_3 + a_1^2a_2 + a_2^2a_3) \\ & +(\rho_3^4)^3(3a_1a_2a_3 - a_1^2a_2 - a_1^2a_3) + 2(\rho_3^4)^4a_1^2a_3 + a_2^2a_3 \end{aligned} \tag{2.6}$$

The set $\Phi(s)$ of rest points calculated given a behavioural population composition has at least one element for each admissible composition. The analogous of the following propositions have been proved in Sethi (1999) for the case of random matching. The proof of these are provided in the Appendix (Annexe A).

Proposition 3 *The set of rest points $\Phi(s)$ is non-empty for all values of s_i for all $i = 1, 2, 3$.*

Proposition 4 *Suppose $s \neq (1,1,1)$, the set of rest points $\Phi(s)$ contains a single element.*

Proposition 5 *Suppose $s = (1,1,1)$, $\pi_i \neq 0$ and $\pi_i \neq 1$. The set of rest points $\Phi(s)$ contains exactly two elements, exactly one of which is stable with respect to the dynamics of equation (2.1).*

Finally, we will provide a technical proposition that will be useful in the sequel.

Proposition 6 *Suppose* $\pi_i \neq 0$ *and* $\pi_i \neq 1$. *The function* $\rho(s)$ *is continuous at all monomorphic states and at all polymorphic states of the type* $s = (x, 1, 0)$ *where* $x \in (0, 1)$.

2.2.3 Evolutionary stability

Given that inventories are continuously at their equilibrium values, we proceed with the evolution of the behavioural composition s. The evolutionary approach analyses the population distribution of behaviours (decision rules, strategies) subject to specific selection dynamics. Consequently, we will allow for all permissible behaviours on the part of agents and analyse the behaviours which will survive the dynamic selection process.

2.2.3.1 Expected payoffs

We denote the utility of agent i by u_i. According to the strategies the agents adopt, the utility of an agent of type αi is denoted by $u_{\alpha i}$ and the utility of an agent of type βi is denoted by $u_{\beta i}$. The expected payoffs to each type of player are functions of the population composition s and the corresponding equilibrium inventories $\rho(s)$. The expected payoff of an agent of type αi is:

$$u_{\alpha i}(s) = (1 - \rho_{i+1}(s))a_{i+1} + (s_{i+2} + \rho_{i+2}(s) - 1)a_i - c_{i+1} \tag{2.7}$$

where the first two terms indicate the expected payoff from consumption and the last term is the cost of storing good $i+1$. Agents of type αi will have in stock the good $i+1$ whether they trade or not.

Agents of type βi have in stock either their production good $i+1$ or the good $i+2$. Since the good held by these agents changes over time through trade, we need to define the probability of holding the good $i+1$ and the good $i+2$ in order to compute the expected payoff of agent of type βi. The probability of having $i+1$ in inventory is denoted by $\tau_i(s)$. The probability of having $i+2$ in inventory is consequently equal to $1-\tau_i(s)$. The probability of having $i+1$ in stock, $\tau_i(s)$ is given by:

$$\tau_i(s) = \frac{\rho_{i+2}(s)a_i}{\rho_{i+2}(s)a_i + \rho_{i+1}(s)a_{i+1}}$$

The expected payoff of an agent of type βi holding $i+1$ is:

$$u_{\beta i}^{i+1}(s) = (1-\rho_{i+1}(s)(1+c_{i+2}-c_{i+1}))a_{i+1} + (s_{i+2}+\rho_{i+2}(s)-1)a_i - c_{i+1} \quad (2.8)$$

The expected payoff of an agent of type βi holding $i+2$ is:

$$u_{\beta i}^{i+2}(s) = \rho_{i+2}(s)a_i(1-c_{i+1}) - (1-\rho_{i+2}(s)a_i)c_{i+2} \quad (2.9)$$

Then the expected payoff of an agent of type βi will be:

$$u_{\beta i}(s) = \tau_i(s)u_{\beta i}^{i+1}(s) + (1 - \tau_i(s))u_{\beta i}^{i+2}(s) \tag{2.10}$$

Given the payoffs to each strategy in each population, define the mean payoff in population i as:

$$\bar{u}_i(s) = (1 - s_i)u_{\alpha i}(s) + s_i u_{\beta i}(s) \tag{2.11}$$

2.2.3.2 Definition of the replicator dynamics

In the evolutionary setting, there are interactions among boundedly rational agents from each of the three populations. These agents have little or no information about the environment. In each population, we allow for all types of behaviours on the part of agents. Evolutionary pressures select better performing behaviours in the long run. The selection dynamics governing change are in continuous time and are regular selection dynamics. Given the payoffs to each of the two behavioral types in each of the three sub-populations, the evolution of the behavioural composition of the population is given by the following system of continuous-time differential equations: $\dot{s}_i = \xi_i(s)$. The function ξ is said to yield a monotonic selection dynamic if the following conditions are satisfied:

i. ξ is Lipschitz continuous

ii. $s_i = 0 \Rightarrow \xi_i(s) \geqslant 0$ and $s_i = 1 \Rightarrow \xi_i(s) \leqslant 0$

iii. $\lim\limits_{s_i \to 0} \frac{\xi_i(s)}{s_i}$ exists and is finite.

iv. $u_{\beta i}(s) > (=)\ u_{i\alpha}(s) \Rightarrow \frac{\xi_i(s)}{s_i} > (=)\ 0$

These conditions ensure that s_i remains in $[0,1]$, its growth rates are defined and continuous at all points $s \in [0,1]^3$ and the growth of the share of β types in population i is proportional to its relative payoff. Taylor and Jonker (1978) defined a special case of the class of monotonic selection dynamics as the replicator dynamics.

$$\frac{\dot{s}_i}{s_i} = \beta(u_{\beta i}(s) - \bar{u}_i(s)) \tag{2.12}$$

2.2.3.3 Asymptotic stability for the replicator dynamics

Note that all monomorphic population states are rest points of monotonic dynamics. Asymptotically stable monomorphic rest points will now be described. Notice that results analogous to the following two propositions have been proved in Sethi (1999) for the case of random matching. The proof of these propositions are provided in the Appendix (Annexe A).

Proposition 7 *Suppose $c_1 < c_2 < c_3$.*

1. *If $(c_3 - c_2) > a_1 - a_2(1 - \rho_2^2)$ there is an asymptotically stable rest point at $s = (0,1,0)$.*

2. *If $(c_3 - c_2) < a_1 - a_2(1 - \rho_2^3)$ there is an asymptotically stable rest point at $s = (1,1,0)$.*

The left hand side is the difference in costs from storing good 3 rather than good 2. The right hand side is the difference in expected utility of storing good 3 rather than good 2 (with probability $a_1\rho_1^2$ agents of type 1 trade meet agents of type 3 and trade good 3 for good 1 and with probability $a_2(1-\rho_2^2)$ agents of type 1 meet agents of type 2 and trade good 2 for good 1). The first part of the proposition requires that all players play fundamental strategies. In the second part of the proposition agents of types 2 and 3 play fundamental strategies while agents of type 1 speculate. For the Economy A, Kiyotaki and Wright (1989) show that there is at most one equilibrium in pure strategies depending on the values of storage costs and the discount rate. Either the unique equilibrium requires that all players play fundamental strategies or that agents of types 2 and 3 play fundamental strategies while agents of type 1 speculate[4]. The proposition resembles the results in Kiyotaki and Wright (1989) with the difference that s represents the population composition of behaviours and the results are obtained through the study of the evolutionary dynamics.

In Sethi (1999), the simulations are done based on the cost vector

$$c = (0.01, 0.04, 0.09)$$

[4]The equilibrium is $s = (0, 1, 0)$ if $(c_3 - c_2) > \frac{1}{3}\beta$ and $s = (1, 1, 0)$ if $(c_3 - c_2) < \frac{\sqrt{2}-1}{3}\beta$ where β is the discount factor and s_i is the strategy of an agent of type i. For $(c_3 - c_2) \in [\frac{\sqrt{2}-1}{3}\beta, \frac{1}{6}\beta]$, there is no equilibrium in pure strategies. Kehoe et al. (1993) show that there exists a symmetric equilibrium in mixed strategies for this set of parameters where agents of type 1 randomize over their two possible strategies while those of other types play fundamental strategies.

and the replicator dynamics. At this cost vector, the population of agents converge to a speculative state. Through holding c_1 and c_2 constant and raising c_3, Sethi (1999) shows that the speculative state loses stability at $c_3 = 0.18$ and the population converges to a polymorphic state where some agents of type 1 use fundamental strategies while others speculate. At $c_3 = 0.21$, there is a stable fundamental state.

In this paper, the conditions of stability are expressed in terms of matching probabilities. In order to simplify the analysis, we will suppose that agents of type 1 and 2 will randomly choose their trading partners. Then the conditions of stability for the first part of the proposition becomes:

$$3(c_3 - c_2) > \frac{2\pi_3 - 1}{2} + \frac{2}{5 - 2\pi_3}$$

and the conditions of stability for the second part of the proposition becomes the following complicated equation:

$$3(c_3 - c_2) < \frac{-(2\pi_3 - 1)(6\pi_3 - 5) + \sqrt{(73 - 2\pi_3 - 36\pi_3^2 + 8\pi_3^3)(1 + 2\pi_3)}}{4(3 - 2\pi_3)} - 1$$

Figure (2.1) illustrates the stability conditions for the fundamental and speculative states. On the x-axis we represent π_3 and on the y-axis we represent

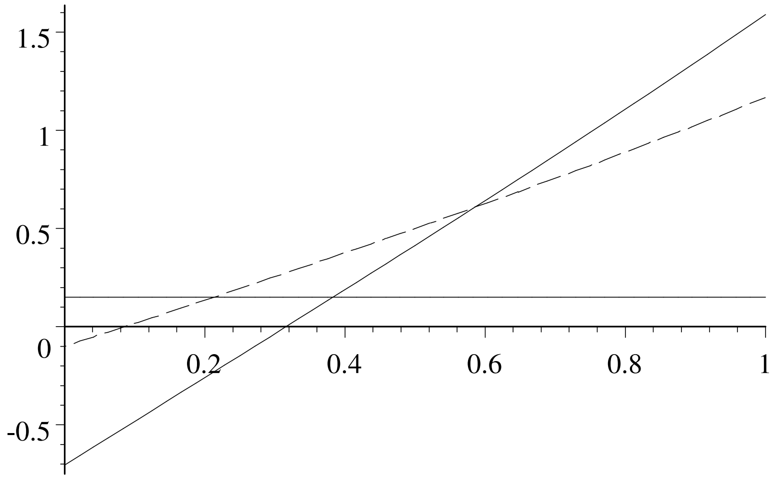

Figure 2.1: The first and second condition of stability in terms of π_3

$(c_3 - c_2)$. The dashed curve illustrates the points for which we have the equality of the right hand side and the left hand side of the above first inequality. Above the dashed curve the fundamental state is stable. The graphical analysis shows that based on the same cost vector, up to $\pi_3 = 0.21$, the fundamental state is stable (the intersection of the dashed curve and the solid horizontal line representing $3(c_3 - c_2)$ in Figure (2.1)). The solid curve represents the points for which the right hand side and the left hand side are equal in the above second inequality. Below the solid line the speculative state is stable. The graphical analysis shows that beyond $\pi_3 = 0.38$, the speculative equilibrium is stable based on the same cost vector (the intersection of the solid curve and the solid horizontal line representing $3(c_3 - c_2)$ in Figure (2.1)).

Proposition 8 *Suppose* $c_1 < c_2 < c_3$.

If $a_1 - a_2(1-\rho_2^3) < (c_3 - c_2) < a_1 - a_2(1-\rho_2^2)$ *there is an asymptotically stable rest point at* $s = (x, 1, 0)$ *where* $x \in (0, 1)$.

When we suppose that the agents of type 1 and 2 will randomly choose their trading partners, the previous condition of stability becomes:

$$\frac{-(2\pi_3 - 1)(6\pi_3 - 5) + \sqrt{(73 - 2\pi_3 - 36\pi_3^2 + 8\pi_3^3)(1 + 2\pi_3)}}{4(3 - 2\pi_3)} - 1$$
$$< \quad 3(c_3 - c_2) < \frac{2\pi_3 - 1}{2} + \frac{2}{5 - \pi_3}$$

In order to visualise this function, we can use the same graph we used for the stability of the fundamental and speculative states (Figure 2.1) since the right and left hand sides turn out to be the same equations as the previous equations for the stability of the fundamental and speculative states. In the area between the dashed curve and the solid curve the polymorphic state is stable. We can also conclude that when π_3 varies between 0.21 and 0.38, the polymorphic state is stable for the cost vector in consideration.

2.3 Disequilibrium dynamics

In the previous section, given various initial distributions of strategies, the rest points of inventory dynamics are determined. Then the robustness of these distributions of strategies is checked. The assumption that the inventory distribution is continuously at its temporary equilibrium value implies that the effect of changes in the inventory distribution in response to disequilibrium is neglected. In this section, the population shares change according to trade and evolutionary selection affecting the process at different rates. Thus the inventories are allowed to be in disequilibrium. The dynamics are studied with $\pi_i = 0.5$ for all $i = 1, 2, 3$.

2.3.1 The model

The description of the previous section will be used. In order to analyse this model from an evolutionary point of view, we consider that the agents may change their strategies. The total population of agents in a group can be characterised according to two criteria: the strategy they adopt (either α or β) and the good they have in stock (either $i+1$ or $i+2$). Consequently, each group is composed of four categories. We denote the proportion of agents of type αi having in stock their production good $i+1$ by e_i and the proportion of agents of type αi having in stock the good $i+2$ by f_i. The share of agents of type βi holding $i+1$ is denoted by g_i and the share of agents of type βi holding $i+2$ by h_i. The player types, their inventory holdings and population shares are represented in Table 3.

Table 3 Agent types, inventory holdings and population shares

Type	Inventory holding	Population shares among i types
αi	$i+1$	e_i
αi	$i+2$	f_i
βi	$i+1$	g_i
βi	$i+2$	h_i

Remark 9 *The fact that there are agents of type αi having in stock the good $i+2$ is technically necessary since as we allow agents of type agents of type βi holding $i+2$ changing their strategy and if it is better performing adopting the other strategy.*

Given the previous definition of population shares, the population of agents can be represented by the matrix $r = (\sigma_1, \sigma_2, \sigma_3)$ where σ_1, σ_2 and σ_3 are the row vectors $\sigma_1 = (e_1, f_1, g_1)$, $\sigma_2 = (e_2, f_2, g_2)$ and $\sigma_3 = (e_3, f_3, g_3)$. We define the utility of agent i according to the group they belong (for example the utility of an agent of type αi holding in stock the good $i+1$ will be denoted by $u_{e_i}(r)$). Consequently there will be four utility functions for each group.

The expected payoff to an agent of type αi holding the good $i+1$ is:

$$u_{e_i}(r) = \frac{1}{3}(f_{i+1} + h_{i+1}) + \frac{1}{3}g_{i+2} - c_{i+1} \tag{2.13}$$

The expected payoff to an agent of type αi holding the good $i+2$ is:

$$u_{f_i}(r) = \frac{1}{3}(e_{i+2} + g_{i+2})(1 - c_{i+1}) - (1 - \frac{1}{3}(e_{i+2} + g_{i+2}))c_{i+2} \tag{2.14}$$

The expected payoff to an agent of type βi holding the good $i+1$ is:

$$u_{g_i}(r) = \frac{1}{3}(f_{i+1}+h_{i+1})+\frac{1}{3}(e_{i+2}+g_{i+2})-\frac{1}{3}(e_{i+1}+g_{i+1})c_{i+2}-(1-\frac{1}{3}(e_{i+1}+g_{i+1}))c_{i+1} \tag{2.15}$$

The expected payoff of an agent of type βi holding the good $i+2$ is:

$$u_{h_i}(r) = \frac{1}{3}(e_{i+2} + g_{i+2})(1 - c_{i+1}) - (1 - \frac{1}{3}(e_{i+2} + g_{i+2}))c_{i+2} \tag{2.16}$$

2.3.2 Evolution and trade

The change in population composition results from the change in stocks due to trade and from the change in strategies which is due to the evolutionary mechanism. We denote the change due to trade by Δ^{trade} and the change due to the evolutionary mechanism by Δ^{imit}. These changes do not occur at the same rate. In this paper, it is supposed that agents have a chance to trade more often than they have a chance to revise their strategies. Accordingly, the rate at which the agents trade, v_1 is greater than the rate at which the agents revise their strategies,

v_2 ($v_1 > v_2$). Without loss of generality, let $v_2 = 1$ and $v_1 = v > 1$.

The change in e_i is given by the following equation:

$$\dot{e_i} = v\Delta_{e_i}^{trade} + \Delta_{e_i}^{imit} \tag{2.17}$$

Agents of type αi having in stock their production good $i+1$ do not affect the inventory distribution when they trade as they accept only their consumption good whenever they trade. On the other hand, agents of type αi having in stock the good $i+2$ matched with agents of type $\alpha(i+2)$ and $\beta(i+2)$ having in stock the good i will increase the share e_i and consequently decrease the share f_i. Notice that $\Delta_{e_i}^{trade} = -\Delta_{f_i}^{trade}$.

$$3\Delta_{e_i}^{trade} = f_i(e_{i+2} + g_{i+2}) \tag{2.18}$$

The evolutionary dynamics modelled in this section is replication by imitation. In the previous section the replicator dynamics allowed replication by way of biological reproduction where each agent reproduces according to his relative fitness measured in terms of the payoff for his strategy and each offspring inherits his single parent's strategy. In case of replication by imitation, agents live forever but review their pure strategies. Each reviewing agent samples another agent at random from his player population.

In this model, the distribution of agents in each population does not exactly represent the distribution of strategies since the differences of stocks are taken into account. Each reviewing agent is supposed to meet at random an agent from his population and imitate the strategy of the agent he meets if it is better performing. The contribution of our model rises from the fact that the performance of the strategies depends also on the inventory holdings of the agents. If an agent of type αi having in stock $i+1$ meets an agent of type βi having in stock $i+2$ and finds out that the strategy β performs better, he will imitate this agent and adopt the strategy β but in return he will not become part of agents βi having in stock $i+2$. The imitation will not affect the share h_i.

As a result we have to define four categories and write the dynamics due to imitation in these categories.

A	$u_{f_i}(r) > u_{g_i}(r)$ and $u_{h_i}(r) > u_{e_i}(r)$
B	$u_{f_i}(r) > u_{g_i}(r)$ and $u_{h_i}(r) < u_{e_i}(r)$
C	$u_{f_i}(r) < u_{g_i}(r)$ and $u_{h_i}(r) > u_{e_i}(r)$
D	$u_{f_i}(r) < u_{g_i}(r)$ and $u_{h_i}(r) < u_{e_i}(r)$

Note two following points.

1. $u_{h_i}(r) > u_{e_i}(r)$ whenever $u_{f_i}(r) > u_{g_i}(r)$.

$$u_{h_i}(r) > u_{e_i}(r) \Longrightarrow \frac{\frac{1}{3}(f_{i+1}+h_{i+1}-e_{i+2})}{1-\frac{1}{3}(e_{i+2}+g_{i+2})} < -(c_{i+2}-c_{i+1})$$

$$u_{f_i}(r) > u_{g_i}(r) \Longrightarrow -(c_{i+2}-c_{i+1}) > \frac{\frac{1}{3}(f_{i+1}+h_{i+1})}{1-\frac{1}{3}(e_{i+2}+g_{i+2}+e_{i+1}+g_{i+1})}$$

$u_{h_i}(r) > u_{e_i}(r)$ whenever $u_{f_i}(r) > u_{g_i}(r)$ since we have

$\frac{\frac{1}{3}(f_{i+1}+h_{i+1}-e_{i+2})}{1-\frac{1}{3}(e_{i+2}+g_{i+2})} < \frac{\frac{1}{3}(f_{i+1}+h_{i+1})}{1-\frac{1}{3}(e_{i+2}+g_{i+2}+e_{i+1}+g_{i+1})}$

Consequently it is impossible to be in the B region.

2. $u_{f_i}(r) < u_{g_i}(r)$ and $u_{h_i}(r) > u_{e_i}(r)$.

$u_{f_i}(r) < u_{g_i}(r) \iff -(c_{i+2} - c_{i+1}) < \frac{\frac{1}{3}(f_{i+1}+h_{i+1})}{1-\frac{1}{3}(e_{i+2}+g_{i+2}+e_{i+1}+g_{i+1})}$

$u_{h_i}(r) > u_{e_i}(r) \iff \frac{\frac{1}{3}(f_{i+1}+h_{i+1}-e_{i+2})}{1-\frac{1}{3}(e_{i+2}+g_{i+2})} < -(c_{i+2} - c_{i+1})$

Denote by k^* $\frac{\frac{1}{3}(f_{i+1}+h_{i+1})}{1-\frac{1}{3}(e_{i+2}+g_{i+2}+e_{i+1}+g_{i+1})}$ and by k_* $\frac{\frac{1}{3}(f_{i+1}+h_{i+1}-e_{i+2})}{1-\frac{1}{3}(e_{i+2}+g_{i+2})}$. We can rewrite the categories as functions of the costs.

A	$-k_i > k^*$
C	$k_* < -k_i < k^*$
D	$-k_i < k_*$

Remark 10 *Notice that the conditions differ from the previous results since an agent of type αi compare the performance of his strategy with the performance of the strategy of an agent of type βi either having in stock the good $i+1$ or the good $i+2$ and he does not compare the performance of his strategy with the mean of the performance of the strategy of an agent of type βi.*

The dynamics of replication by imitation for e_i is given by the following equation:

$$3\Delta_{e_i}^{imit} = 2e_i g_i(u_{e_i}(r) - u_{g_i}(r)) + \begin{cases} f_i g_i(u_{f_i}(r) - u_{g_i}(r)) - e_i h_i(u_{h_i}(r) - u_{e_i}(r)) & A \\ -e_i h_i(u_{h_i}(r) - u_{e_i}(r)) & C \\ 0 & D \end{cases} \tag{2.19}$$

The change in f_i is given by the following equation:

$$\dot{f}_i = v\Delta_{f_i}^{trade} + \Delta_{f_i}^{imit} \tag{2.20}$$

Notice again that $\Delta_{e_i}^{trade} = -\Delta_{f_i}^{trade}$.

$$3\Delta_{f_i}^{trade} = -f_i(e_{i+2} + g_{i+2}) \tag{2.21}$$

The dynamics of replication by imitation for f_i is given by the following equation:

$$3\Delta_{f_i}^{imit} = 2f_i h_i(u_{f_i}(r) - u_{h_i}(r)) + \begin{cases} 0 & A \\ -f_i g_i(u_{g_i}(r) - u_{f_i}(r)) & C \\ e_i h_i(u_{e_i}(r) - u_{h_i}(r)) - f_i g_i(u_{g_i}(r) - u_{f_i}(r)) & D \end{cases} \tag{2.22}$$

The change in g_i is given by the following equation:

$$\dot{g}_i = v\Delta_{g_i}^{trade} + \Delta_{g_i}^{imit} \tag{2.23}$$

Agents of type βi holding their production good $i+1$ can decrease the share g_i, consequently increase the share h_i in their population if they exchange $i+1$ for $i+2$. This happens if an agent of type βi holding $i+1$ and an agent $\beta(i+1)$ holding $i+2$ are selected (with probability $\frac{1}{3}g_i(e_{i+1}+g_{i+1})$). If agents of type βi holding their production good $i+1$ exchange $i+1$ for $i+2$, they decrease the share g_i and consequently increase the share h_i.

On the other hand, agents of type βi holding the good $i+2$ can increase the share g_i and consequently decrease the share h_i in their population if they exchange $i+2$ for $i+1$. This happens if an agent of type βi holding $i+2$ and an agent $\beta(i+2)$ holding i are selected (with probability $\frac{1}{3}h_i(e_{i+2}+g_{i+2})$). Notice that $\Delta_{g_i}^{trade} = -\Delta_{h_i}^{trade}$.

The resulting dynamics is given by the following equation:

$$3\Delta_{g_i}^{trade} = h_i(e_{i+2}+g_{i+2}) - g_i(e_{i+1}+g_{i+1}) \tag{2.24}$$

The dynamics of replication by imitation for g_i is given by the following equation:

$$3\Delta_{g_i}^{imit} = 2e_i g_i(u_{g_i}(r) - u_{e_i}(r)) + \begin{cases} f_i g_i(u_{g_i}(r) - u_{f_i}(r)) + e_i h_i(u_{h_i}(r) - u_{e_i}(r)) & A \\ e_i h_i(u_{h_i}(r) - u_{e_i}(r)) & C \\ 0 & D \end{cases} \tag{2.25}$$

The change in h_i is given by the following equation:

$$\dot{h}_i = v\Delta_{h_i}^{trade} + \Delta_{h_i}^{imit} \tag{2.26}$$

Notice again that $\Delta_{g_i}^{trade} = -\Delta_{h_i}^{trade}$.

$$3\Delta_{h_i}^{trade} = -h_i(e_{i+2} + g_{i+2}) + g_i(e_{i+1} + g_{i+1}) \tag{2.27}$$

The dynamics of replication by imitation for h_i is given by the following equation:

$$3\Delta_{h_i}^{imit} = 2f_i h_i(u_{h_i}(r) - u_{f_i}(r)) + \begin{cases} 0 & A \\ f_i g_i(u_{g_i}(r) - u_{f_i}(r)) & C \\ e_i h_i(u_{h_i}(r) - u_{e_i}(r)) + f_i g_i(u_{g_i}(r) - u_{f_i}(r)) & D \end{cases} \tag{2.28}$$

2.3.3 The stability conditions

The fundamental and speculative states are rest points of the dynamics given by equations 2.17-2.28. At this point, we will study the stability of these states. In order to determine the conditions under which they are stable, the dynamic system is linearised around these states and the Jacobian is computed, the eigenvalues of the Jacobian are defined as explicit functions of the parameters. The stability of the system requires all the eigenvalues of the Jacobian to have negative real parts. According to this criteria the stability conditions are expressed as a function of the difference of the costs of the goods and the velocity parameter.

2.3.3.1 The fundamental state

First, we will analyse the conditions under which the fundamental state is stable. The fundamental state is represented by the population state $\sigma_1 = (1, 0, 0)$ for the agents of type 1, $\sigma_2 = (0, 0, 0.5)$ for the agents of type 2 and $\sigma_3 = (1, 0, 0)$ for the agents of type 3. The conditions for the evolutionary dynamics are computed for this population state.

A	$-k_1 > \frac{1}{3}$	$-k_2 > 0$	$-k_3 > 0$
C	$-\frac{1}{4} < -k_1 < \frac{1}{3}$	$-\frac{1}{2} < -k_2 < 0$	$0 < -k_3 < 0$
D	$-k_1 < -\frac{1}{4}$	$-k_2 < -\frac{1}{2}$	$-k_3 < 0$

Note that only A is satisfied for the agents of type 2, and only D is satisfied for the agents of type 3. For the agents of type 1, C and D are satisfied.

As a result, there are two cases to analyse:

1. $k_1 < \frac{1}{4}$

2. $k_1 > \frac{1}{4}$

We obtain the following proposition. The proof is provided in the Appendix (Annexe A).

Proposition 11 *Suppose $c_1 < c_2 < c_3$. If $v > \frac{-8k_1^2+18k_1-4}{3(8k_1-9)}$ the fundamental state is asymptotically stable.*

Figure (2.2) illustrates the stability conditions for the fundamental state. The x-axis is used for the difference of the costs of the good 3 and good 2 k_1 and the y-axis is used for the speed parameter v. The dashed curve illustrates the points for which we have the equality of the right hand side and the left hand side of the above first inequality. To the right of the dashed curve the fundamental state is stable. The fundamental state is stable when the difference between the costs of the two goods is higher for lower values of the velocity parameter i.e. when people interact in order to review their strategies as much as they trade.

2.3.3.2 The speculative state

Second, we will analyse the conditions under which the speculative state is stable. The speculative state is represented by the population state $\sigma_1 = (0, 0, \sqrt{2}/2)$

for the agents of type 1, $\sigma_2 = (0, 0, \sqrt{2}-1)$ for the agents of type 2 and $\sigma_3 = (1, 0, 0)$ for the agents of type 3. The conditions for the evolutionary dynamics are computed for this population state.

A	$-k_1 > \frac{4-\sqrt{2}}{7}$	$-k_2 > 0$	$-k_3 > \frac{5-\sqrt{2}}{23}$
C	$-\frac{\sqrt{2}-1}{2} < -k_1 < \frac{4-\sqrt{2}}{7}$	$0 < -k_2 < 0$	$\frac{3-\sqrt{2}}{14} < -k_3 < \frac{5-\sqrt{2}}{23}$
D	$-k_1 < -\frac{\sqrt{2}-1}{2}$	$-k_2 < 0$	$-k_3 < \frac{3-\sqrt{2}}{14}$

Note that only A is satisfied for the agents of type 2, and only D is satisfied for the agents of type 3. For the agents of type 1, C and D are satisfied.

As a result, there are two cases to analyse:

1. $k_1 < \frac{\sqrt{2}-1}{2}$
2. $k_1 > \frac{\sqrt{2}-1}{2}$

We obtain the following proposition. The proof is provided in the Appendix (Annexe A).

Proposition 12 *Suppose $c_1 < c_2 < c_3$. If $v < \frac{2\left(-5+4\sqrt{2}\right)\left(k_1-1-\sqrt{2}\right)\left(7k_1+4-\sqrt{2}\right)}{21(5\sqrt{2}-4-4k_1(2-\sqrt{2}))}$ the speculative state is asymptotically stable.*

Figure (2.2) illustrates the stability conditions for the speculative state. The x-axis is used for k_1 and the y-axis is used for v. The solid curve depicts the points for which we have the equality of the right hand side and the left hand side of

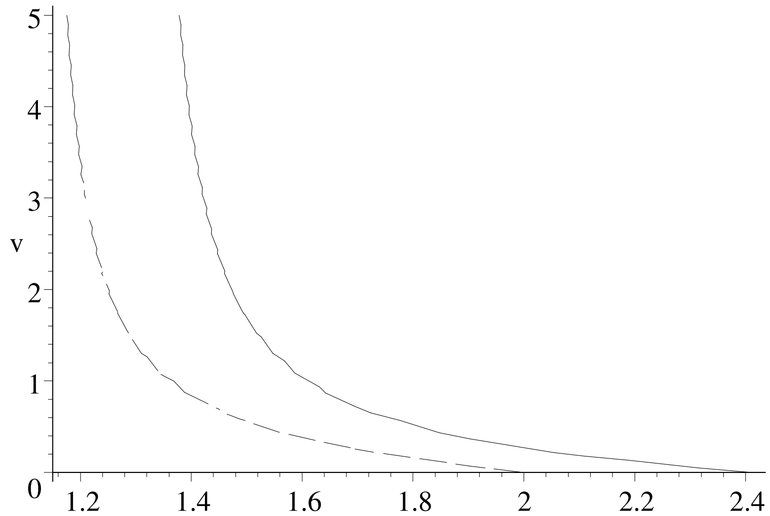

Figure 2.2: The stability of the fundamental and speculative states

the above inequality. To the left of the solid curve the speculative state is stable. The speculative state is stable when the difference between the costs of the two goods is lower for higher values of the velocity parameter i.e. when people interact in order to review their strategies more frequently than they trade or when the difference between the costs of the two goods is higher for lower values of the velocity parameter i.e. when people interact in order to review their strategies as frequently as they trade.

Remark 13 *In the Figure (2.2) as the velocity increases, the curves become steeper and for high values of the velocity, the stability of both states will be determined by the difference of the costs of the goods. Notice that there is a region where the parameter combinations allow for both states to be stable.*

2.4 Conclusion

The standard search theoretic models of the emergence of money assumes random bilateral encounters between a large number of agents. The evolutionary extensions of these models retain the random matching assumption. The first objective of this paper was to explore a version of the commodity money model in Kiyotaki and Wright (1989) with the addition of matching probabilities and to study the implications of endogenising the matching process. The addition of the matching probabilities adds asymetries to the model. The *fundamental state* in Kiyotaki and Wright (1989) in which every exchange involves either agents trading for their consumption goods or trading a higher storage cost good for a lower storage cost good, *the speculative state* requiring some individuals to trade their production good for one with a higher storage cost can be stable under this set up. Their stability is a function of the difference of the costs of the goods and the matching probabilities.

The second objective was to analyse the disequilibrium dynamics, when the inventory distribution is not expected to be continuously at its temporary equilibrium value. We found the conditions for the stability of the fundamental and speculative states. We used different evolutionary dynamics and defined the range of the parameters where the fundamental and speculative states are stable. The findings are not in accordance with the earlier results in the literature. For instance, for the same range of parameter values as in Sethi (1999), the disequilibrium dynamics defined in this paper, result in instability.

CHAPITRE 3

L'OPPORTUNISME ET L'ÉVOLUTION DE LA COMPÉTITION ÉLECTORALE

Abstract

This paper is an attempt to endogenise the party membership decisions of opportunist politicians in different political systems. We study a two-party, uni-dimensional model of political competition with two types of politicians: The office oriented politicians, referred to as "opportunists", care only about the spoils of the office. The policy oriented politicians, referred to as "militants" have ideological preferences on the policy space. We consider that opportunist politicians are, as time goes, more likely to register in a winning party. We compare a winner-take-all system, where all the spoils go to the winner, to a proportional system, where the spoils of office are split among the two parties proportionally to their share of the vote. We study the existence of short term political equilibria and then within an evolutionary setup the stability of policies and party membership decisions of opportunist candidates.

3.1 Introduction

In this paper we present a model of representative democracy with endogenous membership decisions of opportunist politicians. The citizens choose the government in the elections which in turn chooses the policy to be implemented. The parties compete by offering policies and make credible commitments to implement these policies in case they are elected. The preferences of voters and the political competition together determine the collective outcome and the membership decisions of opportunist politicians are driven by the prospects of being elected. A main feature of political parties is that they are composed of factions who differ in their political motivations: one faction is an office seeker while the other faction cares about the ideological platform of the party. The former will be referred to as the opportunist faction and the latter will be referred to as the militant faction following Roemer (1999). The previous formal analysis about political competition suppose that there are opportunist politicians in each party competing in the elections but we need to answer why the opportunist politicians who care only about the prospects of winning the elections and holding an office choose to be in one party rather than the other or whether there should be opportunist politicians in each party. This paper attempts to answer these questions by introducing an evolutionary setup. While the membership decisions of opportunist politicians is analysed within an evolutionary setup, the political outcome is identically to previous formal analysis given by the equilibrium of the simultaneous move electoral competition.

3.1.1 *Related literature*

The political interpretation of spatial models of competition dates back to the famous discussion of duopolists by Hotelling (1929). Hotelling formulated the tendency of competitors to be exactly alike under the principle of minimum differentiation and suggested that this principle can be applied to a wide range of social phenomena including the political competition referring to the ideological similarities between the Republican and the Democratic platforms in the elections of 1928. This intuition was later formulated under the median voter theorem by Black (1948) in the case voters were characterised by single peaked preferences.

Downs (1957) extended the spatial model of competition to representative democracy where two candidates competed by offering policies from a unidimensional policy space conceptualised either as a space of effort levels or as an ideological dimension. Under the Downsian approach the political parties are considered to be organised for the purpose of winning the elections, therefore the policy makers are supposed to shape their policy proposals in order to please the majority. In this sense the competitors are identical in all respects and the common equilibrium policy proposal is found at the preferred policy of the median citizen.

Along with this oversimplified view of the politicians as vote maximisers, there has been a long tradition from Michels (1915) to Lipset (1959) in which parties are ideological and they have policy preferences. This idea has later been formalised by Wittman (1983), Calvert (1985) and Hansson and Stuart (1984) who characterised parties as institutions that represent contesting interest groups in the society. Thus the competing candidates become differentiated as each represents an interest group in the society and seeks publicity for a different ideological platform. The politicians are supposed to have preferences over the policy space and to propose policies accordingly with the essential feature that parties and their ideologies are exogenously given.

These two approaches have been combined by Roemer (1999) who conceptualised parties as consisting of an opportunist faction, a militant faction and a reformist faction. Opportunists are those who wish to maximise the probability of victory, and militants are those who want to maximise the utility of the citizen the party is representing. On the other hand, the reformists maximise the expected utility of its constituents. The opportunists belong to Downs's (1957) conception of politics and the militants to Wittman's (1983) conception of politics.

3.1.2 *Results of the present paper*

This paper adopts the previous approach in order to endogenise the membership decisions of opportunist politicians. The opportunist politicians are allowed to review their membership decisions in the pre-election period according to evolutionary dynamics. Unlike Roemer (1999) which adopted the decision rule, the party-unanimity Nash equilibrium (PUNE) which consists of policies which are Nash in the following sense: in neither party can the (internal) factions agree on a deviation from the proposed policy, given the other party's policy proposal and where each faction in the party has veto power over the policy proposals, the internal decision mechanism is taken to be a weighted average of proposals of different factions[1]. There are two kinds of states of the opportunist population: the *pure states* where all the opportunists are in one party or the other and the *mixed states* where the opportunists are distributed in both parties.

We analyse the behaviour of the opportunist politicians according to two political systems: the proportional system and the winner-take-all system. The pure states are rest points of evolutionary dynamics in both systems. In the winner-take-all system, there is only one mixed state as a rest point of the dynamics. In the proportional system, there may be one or more mixed states as rest points of the dynamics depending on the distribution of voters and the distribution of politicians, there may also be none. In the winner-take-all system, only the pure states are stable according to the evolutionary dynamics regardless of the distribution of the voters. The mixed state is not stable. In the proportional system, the stability of the pure and mixed states depend on the distribution of voters and the distribution of politicians. Only in this case can there be mixed stable states.

[1] In Roemer (1999), it has been also shown that when each party works out a method of inner-party bargaining, the policy proposal that they reach as a consequence of inner-party bargaining is a PUNE since at that proposal, no parties' factions would agree to deviate to another policy.

3.1.3 *An outline of the model*

We consider the following electoral cycle:

1. At the beginning of the cycle, the opportunist candidates review their membership decisions to the political parties, party L and party R, for the following elections. The standard approach is to assume that the rational and optimising politicians can collectively locate the equilibrium of the model and then to compute the equilibria of the two stage game where the opportunists review their membership decisions at the first stage and the parties make their policy proposals at the second stage[2]. This approach will not provide any insight about the evolution of the behaviour of opportunist candidates from any initial state. As we attempt to study the disequilibrium dynamics, we will suppose that initially there are opportunist politicians in each party. The proportion of candidates in each party will then evolve based upon random encounters with the other opportunists candidates and the adoption of better performing behaviours in the political economic environment considered. The militants by definition are not supposed to review their membership decisions.

2. Given the party membership profiles each party announces the party platform which is obtained by the aggregation of the policy proposals of the factions. The aggregation rule is the weighted average of the policy propos-

[2] The formal results are provided in the Appendix (Annexe B).

als of each faction. The influence of each faction is proportional to its weight in the party. The proposal of the militant faction is given exogenously. The proposal of the opportunist faction is obtained through the maximisation of their utilities given the policy proposal of the competing party. However, the opportunists are supposed to be subject to a certain constraint in their decision making processes. The opportunist faction of the party L (R) can not propose a platform greater (less) than the platform of the party R (L). For instance, if the opportunist faction of the party L propose a platform greater than the platform of the party R, they will lose their credibility as politicians and there would be some (unmodelled) electoral punishments inflicted in the future.

3. The political outcome is determined by the elections. We consider two political systems differing in the way that votes are translated into seats in an assembly: the proportional system and the winner-take-all system. These two systems differ by the rewards that accrue to vote shares. In the winner-take-all system, all the spoils of office go to the winner. In the proportional system, the spoils of office are split among the candidates proportionally to their share of the vote. The spoils of office represent the benefits for a party of being able to implement its policy, and the rents from power. The utilities of the opportunist candidates are defined following the political system considered.

3.1.4 *An outline of the paper*

The paper is organised as follows. In the next section the political economic environment is formulated. Section 3 describes the aggregation of policy proposals of different factions within each party. Section 4 introduces the evolutionary process used to analyse the behaviour of opportunist politicians and gives the stability results concerning the evolutionary dynamics. Section 5 concludes with a discussion of the results.

3.2 The model

3.2.1 The voters

A society has to decide collectively on a policy t such as a redistributive income tax levied by the government in order to finance a public good that is equally valued by all citizens. The policy space is the unit interval. Each citizen evaluates the policies according to their utility and uses the only one vote he has for the policy he likes best. Each voter has well-defined single-peaked political preferences given by an ideological position. The voters are distributed according to their ideal policies on the unit interval by a cumulative distribution function F, so that $F(t) = \Pr(T \leq t)$ where T is a random variable describing the voters' ideal policies. The average ideal policy is

$$\bar{t} = \int_0^1 t dF(t) \tag{3.1}$$

and the median voter is given by

$$t^* = F^{-1}(\frac{1}{2}) \tag{3.2}$$

The utility for a voter of a given policy t is given by minus the square of his actual distance with the policy. Then, the utility of a voter whose ideal policy is t_i is given by the following equation:

$$u(t) = -(t - t_i)^2 \tag{3.3}$$

This utility definition implies that all voters will prefer the policy which is closer to their ideal policy.

3.2.2 The parties

We consider a representative democracy where the citizens choose the government which in turn chooses the policy to be implemented. There are two political parties: party L and party R. These parties compete by offering the policies t_L and t_R. They make a credible commitment to implement these policies when they are elected. The citizens vote for the candidates according to these proposals. Thus they indirectly choose the policy. In other words, the voters' preferences and the parties' proposals together determine the policy to be implemented.

A majority (or Condorcet) winning tax policy is the policy t^c that is preferred by some majority of individuals to any other policy $t \in [0, 1]$. In this setting, we define by $\pi(t_L, t_R)$ the probability that the party L wins when party L propose t_L and party R propose t_R. Consequently, the probability that party R wins will be $1 - \pi(t_L, t_R)$. If the majority of the population prefers t_L to t_R then $\pi(t_L, t_R) = 1$. If the majority of the population prefers t_R to t_L then $\pi(t_L, t_R) = 0$. If the same number of people vote for t_L and t_R, we have $\pi(t_L, t_R) = \frac{1}{2}$; in this case each party is elected with probability $\frac{1}{2}$.

Since the preferences of the voters are single-peaked, the majority of the votes will depend on the preferences of the median voter. Thus, the probability that the party L wins when party L proposes t_L and party R proposes t_R is given by the following equation:

$$\pi(t_L, t_R) = \begin{cases} 1 & \text{if } \frac{t_L + t_R}{2} > t^* \\ \frac{1}{2} & \text{if } \frac{t_L + t_R}{2} = t^* \\ 0 & \text{if } \frac{t_L + t_R}{2} < t^* \end{cases} \tag{3.4}$$

Each party consists of two competing factions, the ‘militants’ and the ‘opportunists’. The opportunists care only about wining the elections and coming to office. Consequently, they are only willing to maximise the probability of their party’s victory. On the other hand, the militants have no interest in winning the elections. They care about the party’s ideal policy. They always propose the party’s ideal point. Since the members of the factions have divergent interests, the policy proposals of each party is based on the weighted average of the proposals of the factions. The electoral platforms are determined according to the aggregation rule of the parties and the strategic behaviour of the opportunist faction. More specifically, the opportunist faction maximises its expected utility and gives a policy proposal accordingly. The party platform is chosen by the aggregation of the preferences of the members.

3.3 The policy proposals with given party membership

Political parties can choose among the same set of feasible policies. The electoral competition can be represented in a game theoretical way. The decision within the parties for a policy proposal is based on a weighted average of the policy proposals. The weight is determined by the proportion of the members of each faction in the party. The proportion of militants in party i is α_i where $i = L, R$. We denote by t_i^j the policy proposals of the militants and the opportunists in party i where j will stand for O, M. We suppose that the policy of the party i is as follows:

$$t_i = \alpha_i t_i^M + (1 - \alpha_i) t_i^O \tag{3.5}$$

The militants derive utility from belonging to their preferred party and expressing its ideology. They do not derive utility from the party platform or the final outcome. Consequently, the militant members propose always the party ideology. The ideology of the party L and the party R are 0 and 1 respectively.

When we define the utility of the opportunist politicians we have to consider the political systems since the opportunist politicians are seeking for the benefits from implementing their policies and the rents from power. We consider two political systems: the proportional system and the winner-take-all system. In the former, the rewards that accrue to votes are split proportionally to the share of votes of each party and the utility of an opportunist candidate is the share of the votes per candidate. In the latter the winning party has all the benefits and the utility of an opportunist candidate is the probability of winning divided by the number of candidates. This captures the idea that the benefits from the elections have to be equally shared by all the politicians or the politicians have and equal opportunity to get the benefits.

The party L opportunists maximise $\pi(t_L, t_R)$ or $F(\frac{t_L+t_R}{2})$ per candidate depending on the political system subject to the constraint $t_L^O \leq t_R$. As the opportunists want to win the elections in order to hold office, they have after election considerations as it has been mentioned earlier. In case they violate this constraint, they will lose their credibility as politicians and lose the elections. Consequently, they will propose the highest possible value, namely

$$t_L^O = t_R. \tag{3.6}$$

The party R opportunists maximise $1 - \pi(t_L, t_R)$ or $1 - F(\frac{t_L+t_R}{2})$ per candidate depending on the political system subject to the constraint $t_R^O \geq t_L$ by the same reason as before. Consequently, they will propose the smallest possible value, namely

$$t_R^O = t_L. \tag{3.7}$$

Solving for $t_L^O = t_R$ and $t_R^O = t_L$ in equations (3.5) will result in a Nash equilibrium of the game which is a pair of policies (t_L^N, t_R^N) such that:

$$t_L^N = \frac{(1-\alpha_L)\alpha_R}{1-(1-\alpha_L)(1-\alpha_R)} \tag{3.8}$$

$$t_R^N = \frac{\alpha_R}{1-(1-\alpha_L)(1-\alpha_R)} \tag{3.9}$$

In order to analyse the probability of victory, we calculate the midpoint of the pair of the policies $t^N = \frac{t_L^N + t_R^N}{2}$:

$$t^N = \frac{\alpha_R(2-\alpha_L)}{2(1-(1-\alpha_L)(1-\alpha_R))} \tag{3.10}$$

Notice $0 \leq t^N \leq 1$.

3.3.1 The membership decisions

The electoral cycle is as follows. At the beginning of the electoral cycle, the politicians review their membership decisions for the following elections. The political parties choose their policies given the new membership profiles. The parties announce their policy proposals simultaneously. The citizens vote for the proposals according to their preferences. Then one of the policy proposals, receiving the most votes is chosen in the election. The winning party comes to power and implements the policy. In the event of ties, a fair coin decides which party wins the election.

By definition, the militants do not update their membership decisions since they are supposed to derive satisfaction only by being a member of their party and struggling for the ideology of the party. On the other hand, the opportunists are supposed to update their membership decisions between the elections. We allow the opportunist candidates to have two alternatives. They can either choose party L or party R between the elections.

Consequently, the set of politicians has three subsets: the militants of party L, the militants of party R and the opportunists. We denote by l the number of militants in party L and by r the number of militants in party R. The number of the opportunists in the total politician population is n. The proportion of opportunists in party L is denoted by s. Thus, the proportion of opportunists in party R is denoted by $1 - s$. The number of opportunists in party L is ns. The number of opportunists in party R is $n(1 - s)$. Without loss of generality, we normalise the number of politicians to unity ($l + r + n = 1$). We can express the previous values and results in terms of the new parameters as follows.

The proportion of militants in party L as a function of the share of the opportunists in party L is:

$$\alpha_L(s) = \frac{l}{l + ns} \tag{3.11}$$

The proportion of militants in party R as a function of the share of the opportunists in party L is:

$$\alpha_R(s) = \frac{r}{r + n(1 - s)} \tag{3.12}$$

Consequently, the average of the equilibrium pair of policies is given by the following equation:

$$t^N(s) = \frac{2nsr + lr}{2(lr + n(l(1 - s) + sr))} \tag{3.13}$$

Remark 14 *Notice that* $t^N(0) = \frac{r}{2(r+n)} \leq \frac{1}{2}$ *and* $\frac{1}{2} \leq t^N(1) = \frac{2n+l}{2(l+n)} \leq 1$. *The first and second derivatives of* $t^N(s)$ *with respect to* s *are* $\frac{\partial t^N(s)}{\partial s} = \frac{nrl(r+2n+l)}{2(lr+n(l(1-s)+sr))^2} \geq 0$ *and* $\frac{\partial^2 t^N(s)}{\partial s^2} = \frac{n^2 r(l-r)l(r+2n+l)}{(lr+n(l(1-s)+sr))^3} \geq 0$ *if* $l \geq r$ *and* $\frac{\partial^2 t^N(s)}{\partial s^2} = \frac{n^2 r(l-r)l(r+2n+l)}{(lr+n(l(1-s)+sr))^3} \leq 0$ *if* $l \leq r$ *so that we have the graph (3.1) for* $t^N(s)$ *when* $l \geq r$ *and* $s \in [0, 1]$. *Notice also that* $t^N(s)$ *has its minimal value* $\frac{r}{2(r+n)}$ *when* $s = 0$ *i.e. when all the opportunists are in party* R *and* $t^N(s)$ *has its maximal value* $\frac{2n+l}{2(l+n)}$ *when* $s = 1$ *i.e. when all the opportunists are in party* L.

Remark 15 *In the winner-take-all system, there will be always one winner and one loser in the elections.*

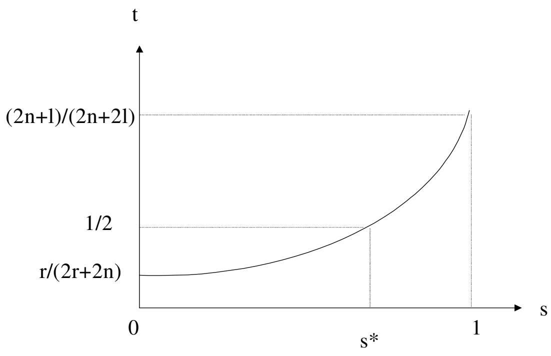

Figure 3.1: The average of the equilibrium pair of policies

3.4 The evolutionary stability

3.4.1 The replicator dynamics

The rationalistic approach to game theory assumes that players are perfectly rational, the game is played once and the game and the equilibrium are common knowledge. On the other hand, the evolutionary approach assumes that boundedly rational players who are randomly drawn from large populations and who have little or no information about the game, play the game repeatedly. Thus, the evolutionary approach allows us to analyse a game theoretic situation when we relax the perfect information and unbounded rationality assumptions. The main difference between these approaches is that the rationalistic approach analyses the individual behaviour while the evolutionary approach analyses the population distribution of behaviours (strategies). The analysis of population dynamics is done by two processes: *the selection process* favoring better performing strategies and *the mutation process* introducing varieties.

In this article, we analyse the distribution of behaviours of opportunist politicians. They have two alternatives: being a candidate of party L or being a candidate of party R. The utility of being a candidate of the party L is $u_L(t_L(s), t_R(s))$. The utility of being a candidate of the party R is given by $u_R(t_L(s), t_R(s))$. The average utility of the opportunist candidates is $\bar{u}(t_L(s), t_R(s))$.

$$\bar{u}(t_L(s), t_R(s)) = su_L(t_L(s), t_R(s)) + (1-s)u_R(t_L(s), t_R(s)) \tag{3.14}$$

The selection process determines how population shares corresponding to different pure strategies evolve over time. This process is based on the survival of the fittest. In other words, the share of the population playing relatively better performing strategies increases. The selection dynamics governing change are in continuous time and are regular selection dynamics[3]. Taylor and Jonker (1978) defined a special case of the class of monotonic selection dynamics as the replicator

[3] The evolution of the composition of the population is given by a system of continuous-time differential equations: $\dot{s} = \xi(s)$. The function ξ is said to yield a monotonic selection dynamic if the following conditions are satisfied:
i. ξ is Lipschitz continuous
ii. $s = 0 \Rightarrow \xi(s) \geqslant 0$ and $s = 1 \Rightarrow \xi(s) \leqslant 0$
iii. $\lim_{s \to 0} \frac{\xi(s)}{s}$ exists and is finite.
iv. $u_L(t_L(s), t_R(s)) > (=)\ u_R(t_L(s), t_R(s)) \Rightarrow \frac{\xi(s)}{s} > (=)\ 0$
These conditions ensure that s remains in $[0, 1]$, its growth rates are defined and continuous at all points $s \in [0, 1]$ and the growth of the share of the opportunists is proportional to its relative payoff.

dynamics which provides an explicit model of a selection process.

$$\dot{s} = (u_L(t_L(s), t_R(s)) - \bar{u}(t_L(s), t_R(s)))s \tag{3.15}$$

It is clear that better performing strategies have a higher growth rate which does not necessarily imply that the average payoff grows. The reason is that even if a player is replaced by a player playing a better performing strategy, this new distribution of players may reduce the payoffs of some other players.

3.4.2 The evolutionary stability

The replicator dynamics describes how the population shares of candidates playing different strategies change over time. The next step will be to determine the rest points of the replicator dynamics and analyse their stability under the assumption that the parties continuously play their instantaneous equilibrium strategies. The utilities of the opportunist candidates are defined following the political system considered. There are two political systems differing in the way that votes are translated into seats in an assembly: the proportional system and the winner-take-all system.

3.4.2.1 The proportional system

In the proportional system, the rents from power are split among the candidates proportionally to their share of the vote. Each party gets seats in the parliament equal to its vote share. The utility of an opportunist candidate is the share of the votes per candidate of his party.

Formally, the utility of an opportunist candidate of the party L is $u_L(t_L(s), t_R(s))$

$$u_L(t_L(s), t_R(s)) = \frac{1}{l+ns} F(\frac{t_L(s)+t_R(s)}{2}) \tag{3.16}$$

and the utility of an opportunist candidate of the party R is given by $u_R(t_L(s), t_R(s))$.

$$u_R(t_L(s), t_R(s)) = \frac{1}{r+n(1-s)} (1 - F(\frac{t_L(s)+t_R(s)}{2})) \tag{3.17}$$

As we study the rest points of the replicator dynamics when the parties play their equilibrium strategies, the replicator dynamics will be given by the following equation:

$$\dot{s} = (\frac{F(t^N(s))}{l+ns} - \frac{1-F(t^N(s))}{r+n(1-s)}) s(1-s) \tag{3.18}$$

The replicator dynamics has the trivial rest points at $s = 0$ and $s = 1$. The other rest points of the replicator dynamics are given by the following equation:

$$F(t^N(s)) = l + ns \tag{3.19}$$

which is the case where the number of votes of the party L is equal to the share of left party candidates in total candidate population.

Proposition 16 *In the proportional system the state where all opportunists are* *in party L ($s = 1$) is stable if $F(t^N(1)) > l + n$. The state where all opportunists are in party R ($s = 0$) is stable if $F(t^N(0)) < l$.*

The proof is provided in the Appendix (Annexe B).

Example 17 *The distribution of voters is given by $F(x) = x$. This is the case where the population is distributed uniformly along the unit line.*

The nontrivial rest points of the replicator dynamics are given by the following condition:

$$F(t^N(s)) = \frac{2nsr + lr}{2\,(lr + n(l(1-s) + sr))} = l + ns \tag{3.20}$$

When the number of militants of party L and R are equal ($l = r$), this equation has the solution $s = \frac{1}{2}$ with no additional assumptions about the weight of the opportunists in the candidate population. The right hand side and the left hand side of the equation (3.20) have been depicted by the graph (3.2) for the case $n = 0.3$. The right hand side is depicted by the dashed line and the left hand side is depicted by the solid line. We have $F(t^N(0)) < l$. In this case, since all the opportunist candidates are in party R and the utility of being a candidate of party L is less than the utility of being a candidate of party R, $s = 0$ is stable. Notice that $F(t^N(1)) > l + n$. As all the opportunists are in party L and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, $s = 1$ is stable. From the previous results, we conclude that $s = \frac{1}{2}$ is not stable.

When the number of militants of the party L and R are different ($l \neq r$), the equation (3.20) has at most two solutions. We have also the constraint $s \in [0,1]$ and the condition $r+l < 1$ to be satisfied. $F(t^N(0)) < l$ if and only if $r < 2l\,(1-l)$. In this case, since all the opportunist candidates are in party R and the utility of being a candidate of party L is less than the utility of being a candidate of party R, $s = 0$ is stable. We have $F(t^N(1)) > l + n$ under $l < 2r(1-r)$. As all the opportunists are in party L and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, $s = 1$ is stable. Notice that if we satisfy both conditions there will be only one solution to the equation (3.20) since the distribution function is always increasing in the interval $[0,1]$ and this solution will not be stable. The case where $s = 0$ is the only stable stable i.e. $F(t^N(0)) < l$ and $F(t^N(1)) < l + n$ (or $r < 2l\,(1-l)$ and $l > 2r(1-r)$) there will be no solution to the equation (3.20). The only stable outcome is the case where all the opportunists are in party R. The case where only $s = 1$ is stable i.e. $F(t^N(0)) > l$ and $F(t^N(1)) > l + n$ (or $r > 2l\,(1-l)$ and $l < 2r(1-r)$) there will be no solution to the equation (3.20). The only stable outcome is the case where all the opportunists are in party L.

Example 18 *The distribution of voters is given by $F(x) = x(2-x)$. This is the case where the population is distributed nonuniformly along the unit line.*

Then the nontrivial rest points of the replicator dynamics are given by the

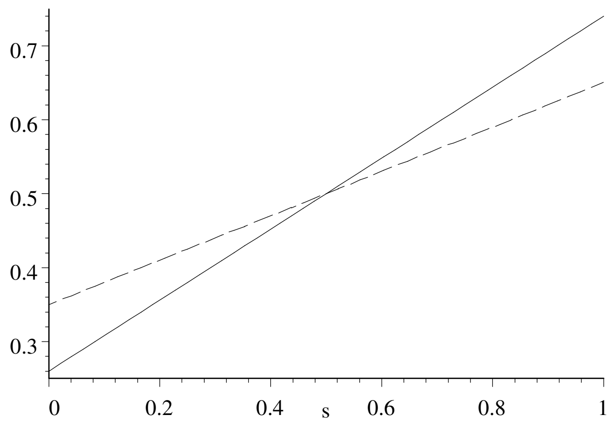

Figure 3.2: Uniform distribution of voters

following condition:

$$F(t^N) = \frac{2nsr + lr}{2\left(lr + n(l(1-s) + sr)\right)}\left(2 - \frac{2nsr + lr}{2\left(lr + n(l(1-s) + sr)\right)}\right) = l + ns \quad (3.21)$$

There are at most three solutions to this equation. When the number of militants of party L and R are equal ($l = r$), this equation has no solutions with no additional assumptions about the weight of the opportunists in the candidate population. We have $F(t^N(0)) > l$. In this case, since all the opportunist candidates are in party R and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, $s = 0$ is not stable. Notice that $F(t^N(1)) > l + n$. As all the opportunists are in party L and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, $s = 1$ is stable. In case there are equal numbers of militants in each party, only the situation where the opportunists are all in party L is stable. This result is due to the fact that the median is closer to the left party ideology. A symmetric result will apply when we have a nonuniform distribution with the median closer to the right party ideology. The right hand side and the left hand side of the equation (3.21) have been depicted by the graph (3.3) for the case $n = 0.3$. The right hand side is depicted by the dashed line and the left hand side is depicted by the solid line.

When the number of militants of the party L and R are different $(l \neq r)$, $F(t^N(0)) < l$ if and only if $r(4(1-l)-r) < 4l(1-l)^2$ and in this case since all the opportunist candidates are in party R and the utility of being a candidate of party L is less than the utility of being a candidate of party R, this point is stable. $F(t^N(1)) > l + n$ if and only if $4r(1-r)^2 > l^2$ the opportunists are in party L and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, this point is stable. The existence and the stability of the other rest points are determined accordingly.

The previous examples are provided to illustrate the relationship between the distribution of voters, the distribution of politicians and the stability of the *pure states*. In politics, the *mixed states* seem to be more common than the pure states. Next we provide an example where the *mixed state* as a rest point of the replicator dynamics is the only stable outcome.

Example 19 *The distribution of voters is given by a beta distribution* $F(x; v, w) =$

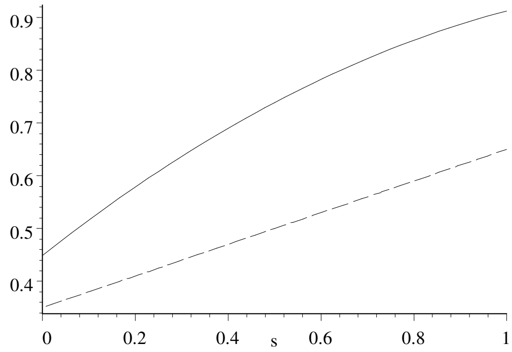

Figure 3.3: Non-uniform distribution of voters

$\frac{1}{B(v,w)} \int_0^x u^{v-1} (1-u)^{w-1} du$. *The beta function with parameters* v, w *is defined by the integral* $B(v,w) = \int_0^1 u^{v-1} (1-u)^{w-1} du$.

The nontrivial rest points of the replicator dynamics are given by the following condition:

$$F\left(t^N(s); v, w\right) = \frac{1}{\int_0^1 u^{v-1} (1-u)^{w-1} du} \int_0^{\frac{2nsr+lr}{2(lr+n(l(1-s)+sr))}} u^{v-1} (1-u)^{w-1} du = l+ns \tag{3.22}$$

We will study the case where $v = w = \frac{1}{2}$. When the number of militants of party L and R are equal ($l = r$), this equation has the solution $s = \frac{1}{2}$ with no additional assumptions about the weight of the opportunists in the candidate population. We have $F(t^N(0)) > l$ when $r < 0.32$. In this case, since all the opportunist candidates are in party R and the utility of being a candidate of party L is greater than the utility of being a candidate of party R, $s = 0$ is not stable. Notice that $F(t^N(1)) < l + n$ when $r < 0.32$. As all the opportunists are in party L and the utility of being a candidate of party L is less than the utility of being a candidate of party R, $s = 1$ is not stable. From the previous results, we conclude that $s = \frac{1}{2}$ is stable.

3.4.2.2 The winner-take-all system

In the winner-take-all system, all the rents from power go to the winner. As the winning party has all the benefits, the utility of an opportunist candidate is the probability of winning divided by the number of candidates having in mind that the benefits from the elections have to be equally shared by all the politicians or the politicians have and equal opportunity to get the benefits.

The utility of being a candidate of the party L is $u_L(t_L(s), t_R(s))$.

$$u_L(t_L(s), t_R(s)) = \frac{1}{l + ns} \pi(t_L(s), t_R(s)) \tag{3.23}$$

The utility of being a candidate of the party R is given by $u_R(t_L(s), t_R(s))$.

$$u_R(t_L(s), t_R(s)) = \frac{1}{r + n(1 - s)} (1 - \pi(t_L(s), t_R(s))) \tag{3.24}$$

As we study the rest points of the replicator dynamics when the parties play their equilibrium strategies, the replicator dynamics will be given by the following equation:

$$\dot{s} = \left(\frac{\pi(t_L^N(s), t_R^N(s))}{l + ns} - \frac{\pi(t_L^N(s), t_R^N(s))}{r + n(1-s)}\right)s(1-s) \tag{3.25}$$

The replicator dynamics has the trivial rest points at $s = 0$ and $s = 1$. The other rest points of the replicator dynamics are given by the following equation:

$$\pi(t_L^N(s), t_R^N(s)) = l + ns \tag{3.26}$$

where the left hand side is the probability of victory and the right hand side is the weight of the politicians of the party L in the total population of politicians. When the parties play their equilibrium strategies the probability of victory becomes:

$$\pi(t_L^N(s), t_R^N(s)) = \begin{cases} 1 & if \quad t^N(s) > t^* \\ \frac{1}{2} & if \quad t^N(s) = t^* \\ 0 & if \quad t^N(s) < t^* \end{cases} \tag{3.27}$$

The graph of the probability of victory as we have defined is not continuous. From the definition of the probability of victory we can conclude that there may be at most one solution to the equation (3.26). There are three cases to analyse:

1. If $\frac{r}{2(1-l)} \leq t^* \leq \frac{1-r+l}{2(1-r)}$, the case where all the opportunists are in the party L ($s = 0$) and the case where all the opportunists are in the party R ($s = 1$) will both be stable. Let s^M be the solution to the equation (3.26) then s^M must satisfy the following conditions: $t^N(s^M) = t^*$ and $l + ns^M = \frac{1}{2}$. From

the previous results, s^M will not be stable.

2. If $t^* < \frac{r}{2(1-l)}$, t^* is always less than $t^N(s)$ and $\pi(t_L^N(s), t_R^N(s)) = 1$. The case where all the opportunists are in the party L ($s = 0$) is the only stable state.

3. If $t^* > \frac{1-r+l}{2(1-r)}$, t^* is always greater than $t^N(s)$ and $\pi(t_L^N(s), t_R^N(s)) = 0$. The case where all the opportunists are in the party R ($s = 1$) is the only stable state.

3.5 Conclusion

This paper is an attempt to endogenise the party membership decisions of opportunist politicians in different political systems. When all the benefits of office are shared proportionally to the share of the votes, a situation where the opportunist politicians are in both parties can be a stable outcome. When all the benefits go to the winner of the elections, there two possible outcomes: all the opportunists are in the party L or in the party R.

The model may be extended in two ways. The behaviour of policy oriented politicians is based on exogenously given ideologies. The model as such is not complete. The ideologies of the parties may be endogenised. Another extension of the model is about the behavioural assumption about the opportunist candidates. The opportunist politicians are supposed to decide about their membership to the party based on random encounters with the other opportunist in the politician population. However we also suppose that they can perfectly calculate the optimal level of policy to maximise the probability of winning the elections. This assumption may be relaxed and an adjustment process may be coupled with the membership decision.

CHAPITRE 4

UN MODÈLE DYNAMIQUE D'ADHÉSION AUX PARTIS ET IDÉOLOGIES

Abstract

We analyse the spatial electoral competition between two parties when the ideology of each party is endogenously determined. The parties are composed of two factions: the "opportunist" faction and the "militant" faction. The ideology of the parties are endogenously determined by the preferences of the average supporters of the parties. Under the proportional system, where the spoils of office are split among the two parties proportionally to their share of the vote, we study the existence of short term political equilibria and then we introduce a dynamic setup to analyse the stability of these equilibria.

4.1 Introduction

Political parties are supposed to be characterised by their ideologies. However deviations from the "true path" are common. Parties are loosely formed around broad political ideologies. Within parties, there are people who hold a variety of opinions and have a variety of motivations. Behaviourally we can consider that politicians are either inflexible ideologically or selfishly interested in winning office. We attempt to model these aspects of the political process through an electoral game, in which two parties simultaneously and independently announce their policies. There are different types of candidates in both parties and the policy proposals are determined through the aggregation of different political motivations. The paper attempts to explain the dynamic patterns of the policy proposals of the parties through the behaviours of the candidates. The candidates motivated by winning an office do not have any loyalty to the party since the party is a means to achieve the goal. They can change their parties if they see that another party can offer a better chance to achieve their goals. The candidates motivated by representing their electors can not accurately calculate the preferences of their electors before the elections results. They have only past elections results to decide about their proposals. Given the previous results, they adjust their ideology to better represent their electorate. The decision rules of these candidates create a coupled dynamics.

Spatial models of electoral competition have employed a variety of assumptions concerning the objectives of the party elites. The first is due to Downs (1957) where the candidates are office or vote oriented and the second is given by Wittman (1973) where the candidates are policy oriented with the ideological preferences of the parties taken to be exogenously determined. The two approaches about the political motivations are combined by Roemer (1999) who argues that parties are composed of three factions: the opportunist faction (inherited from Downs), the reformist faction (inherited from Wittman) and the militant faction focused upon the publicity of the ideal policy of the party. We use the latter view of the political parties but we limit the analysis to two factions, the opportunist faction and the militant faction. The militant faction instead of promoting an exogenously defined ideology determines the ideology of the party endogenously by the preferences of the average citizen supporting the party. The endogeneisation procedure is due to Roemer (2001) who adapted the notion of the citizen-candidate introduced by Osborne and Slivinski (1996) and Besley and Coate (1997). These approaches are modified to include parties and commitment.

The paper is organised as follows: In Section 2, we look for a policy equilibrium during a single election period i.e. a set of party platforms such that no party can improve its position by changing its policies, given the policies of its rivals. The computation of the policy equilibrium involves the inner-party determination of policy proposals and the intra-party electoral competition. In the party the opportunist faction decide strategically and maximise the probability of winning an office and the militant faction computes the preferences of the average citizen voting for the party. We have two important hypothesis about the behaviour of both types of candidates: First the opportunist faction of the party L (R) can not propose a platform greater (less) than the platform of the party R (L). For instance, if the opportunist faction of the party L propose a platform greater than the platform of the party R they lose their credibility as politicians and there would be some (unmodelled) electoral punishments inflicted in the future. Second, the militant faction have the information about the policy preferences of the average voter that supports the parties. Thus the ideology of the party is endogenised and the ideology of the parties come into being from the preferences of the voters. The inner-party proposal is determined by the simple weighted average rule where each candidate has an equal weight. The electoral outcome is given by the Nash equilibrium of the electoral game.

In Section 3, we introduce a dynamic analysis of the previous political environment in order to study the stability of the electoral equilibrium. The first dynamics concerns the membership decisions of the opportunist candidates. The opportunist candidates are allowed to change the party they belong if the other party will give a better chance to achieve their goal. The opportunist review their membership decisions by comparing their probability of success with the average probability of success. The second dynamics are the adjustment dynamics for the militant faction since the previous assumption about the decisions of the militant faction is not very realistic. In reality the politicians are not able to have a perfect information about the average voter supporting the party. The militant factions have the previous election results and based on this results they adjusts their proposals. The stability analysis show that the stability of the states where all the opportunists have chosen to be only in one party is conditional not only on the success of the party but also on the adjustment dynamics.

4.2 The model

The political competition model presented here have three aspects: the decision of individual voters, the decision of individual candidates and a model of party behaviour.

We consider a society consisting of a continuum of heterogeneous individuals. They have to decide collectively on a policy x such as a price for a regulated monopolist's services or a redistributive income tax. The voters have preferences over policies $x \in [0,1]$, a single dimensional policy space. A voter i has a utility function $u_i(x)$ over the relevant policy space. We suppose that the preferences of the voters are single-peaked around their well-defined ideal policies i.e. $x_i = \arg\max u_i(x)$ where x_i is an ideological position. The voters are distributed continuously according to their ideal policies along the relevant policy dimension by a cumulative distribution function F, so that $F(x) = P(X \leq x)$ where X is a random variable describing the voters' ideal policies. The average ideal policy is

$$\bar{x} = \int_0^1 x dF(x) \tag{4.1}$$

and the median voter is given by

$$x^* = F^{-1}(\frac{1}{2}) \tag{4.2}$$

There are two political parties: party L and party R. These parties compete by offering the policies x_L and x_R. They make a credible commitment to implement these policies when they are elected. The citizens vote for the candidates according to these proposals and they use their only one vote for the policy they like best.

4.2.1 Inter-party competition

We consider the proportional system where the rewards that accrue to votes are split proportionally to the share of votes of each party. In this setting, we define by $F(\frac{x_L+x_R}{2})$ the share of the votes of the party L when party L proposes x_L and party R proposes x_R. Consequently, the share of the votes of the party R will be $1 - F(\frac{x_L+x_R}{2})$.

4.2.2 Intra-party preference formation

Each party consists of two competing factions, the 'militants' and the 'opportunists'. The opportunists are purely office motivated. Consequently, they are only willing to maximise the probability of their party's victory or the share of votes of the party. These politicians do not act in any sense as a representative of a coalition of citizens. On the other hand, the militants have any direct interest in winning the elections. They care about the preferences of the coalition of citizens supporting the party. The utility of these politicians is the average of the utility of the citizens supporting the party.

Since the members of the factions have divergent interests, the electoral platforms are determined according to the strategic behaviour of the opportunist faction, the preference formation of the militant faction and the aggregation rule of the parties. More specifically, the opportunist faction maximises its expected utility taking the proposal of the other party as given and gives a policy proposal accordingly and the militant faction determines his policy given by the average of the utilities of the supporting coalition of citizens. The party platform is chosen by the aggregation of the preferences of the members. In this paper the aggregation rule is taken to be a weighted average of the proposals of two factions and the weight is given by the proportion of each faction in the party.

4.2.3 Policy proposals with given party membership

The electoral cycle is as follows. At the beginning of the electoral cycle, the politicians review their membership decisions for the following elections. The political parties choose their policies given the new membership profiles. The policy proposals of the parties and the vote for the proposals are simultaneous. Then given the political system (the proportional system or the winner-take-all system) the political outcome is determined.

The decision within each party for a policy proposal is based on a weighted average of the policy proposals of the factions. The weight of each faction is the proportion of the members of the faction in the party. The proportion of militants in party i is denoted by α_i where $i = L, R$. We denote by x_i^j the policy proposals of the militants and the opportunists in party i where j will stand for O, M. We suppose that the policy of the party i is as follows:

$$x_i = \alpha_i x_i^M + (1 - \alpha_i) x_i^O \tag{4.3}$$

The militants derive utility from expressing the ideology of the average citizen who votes for the party. To do so, we have to suppose that parties represent a coalition of citizens who will vote for the party in equilibrium. This is the endogenous party ideology. If we denote by (x_L^N, x_R^N), the pair of policies which constitutes the equilibrium outcome of the inter-party competition, the midpoint of the equilibrium policy proposals is $x^N = \frac{x_L^N + x_R^N}{2}$. The proposal of the left militants is

$$x_L^M = \frac{\int_0^{x^N} x dF(x)}{F(x^N)} \tag{4.4}$$

and the proposal of the right militants is:

$$x_R^M = \frac{\int_{x^N}^1 x dF(x)}{1 - F(x^N)} \tag{4.5}$$

When we define the utility of the opportunist politicians we have to consider the political systems since the opportunist politicians are seeking for the benefits from implementing their policies and the rents from power. We consider the proportional system where the rewards that accrue to votes are split proportionally to the share of votes of each party and the utility of an opportunist candidate is the share of the votes per candidate.

The party L opportunists maximise $F(\frac{x_L+x_R}{2})$ per candidate subject to the constraint $x_L^O \leq x_R$. This constraint is due to the unmodelled after-election considerations of the opportunists as the opportunists want to win the elections and hold office. In case they violate this constraint, they will lose their credibility as politicians and lose the elections. Consequently, they will propose the highest possible value, proposing $x_L^O = x_R$. The party R opportunists maximise $1 - F(\frac{x_L+x_R}{2})$ per candidate subject to the constraint $x_R^O \geq x_L$ by the same reason as before. They will propose the smallest possible value, proposing $x_R^O = x_L$.

Definition 20 *An Endogenous Party Nash Equilibrium of the political competition is a pair of policies (x_L^N, x_R^N) satisfying the following conditions:*

1. $x_L^O = x_R$ and $x_R^O = x_L$
2. x_L^M satisfies equation (4.4) and x_R^M satisfies equation (4.5)
3. x_L^N and x_R^N satisfy the equations (4.3)

The equilibrium results may be written more explicitly as follows:

$$x_L^N = \frac{\alpha_L \int_0^{x^N} xdF(x)}{(\alpha_R + \alpha_L(1-\alpha_R))F(x^N)} + \frac{(1-\alpha_L)\alpha_R \int_{x^N}^1 xdF(x)}{(\alpha_R + \alpha_L(1-\alpha_R))(1-F(x^N))} \tag{4.6}$$

$$x_R^N = \frac{\alpha_L(1-\alpha_R)\int_0^{x^N} xdF(x)}{(\alpha_R+\alpha_L(1-\alpha_R))F(x^N)} + \frac{\alpha_R\int_{x^N}^1 xdF(x)}{(\alpha_R+\alpha_L(1-\alpha_R))(1-F(x^N))} \tag{4.7}$$

In order to analyse the probability of victory, we calculate the middle point of the pair of the policies x^N:

$$x^N = \frac{\alpha_L(2-\alpha_R)\int_0^{x^N} xdF(x)}{2(\alpha_R+\alpha_L(1-\alpha_R))F(x^N)} + \frac{(2-\alpha_L)\alpha_R\int_{x^N}^1 xdF(x)}{2(\alpha_R+\alpha_L(1-\alpha_R))(1-F(x^N))} \tag{4.8}$$

4.2.4 The membership decisions and ideology of the parties

The militants derive utility only by representing the average citizen voting for the party and being a member of the party. They will not update their membership decisions. On the other hand, the opportunists have career interests and they are supposed to update their membership decisions between the elections. They can either choose party L or party R. Consequently, the set of politicians has three subsets: the militants of party L, the militants of party R and the opportunists. We denote by l the number of militants in party L and by r the number of militants in party R. The number of the opportunists in the total politician population is n. The proportion of opportunists in party L is denoted by s. Thus, the proportion of opportunists in party R is denoted by $1-s$. The number of opportunists in party L is ns. The number of opportunists in party R is $n(1-s)$. Without loss of generality, we normalise the number of politicians to unity ($l+r+n=1$). We can express the previous values and results in terms of the new parameters as follows.

The proportion of militants in party L as a function of the share of the opportunists in party L is:

$$\alpha_L(s) = \frac{l}{l+ns} \tag{4.9}$$

The proportion of militants in party R as a function of the share of the opportunists in party L is:

$$\alpha_R(s) = \frac{r}{r+n(1-s)} \tag{4.10}$$

Consequently, the ideology of party L as a function of the share of opportunists is

$$x_L^N(s) = \frac{l(r+n(1-s))\int\limits_0^{x^N(s)} xdF(x)}{(rl+rns+ln-lns)F(x^N(s))} + \frac{nsr\int\limits_{x^N(s)}^1 xdF(x)}{(rl+rns+ln-lns)(1-F(x^N(s)))} \tag{4.11}$$

and the ideology of party R as a function of the share of opportunists is:

$$x_R^N(s) = \frac{nl(1-s)\int\limits_0^{x^N(s)} xdF(x)}{(rl+rns+ln-lns)F(x^N(s))} + \frac{r(l+ns)\int\limits_{x^N(s)}^1 xdF(x)}{(rl+rns+ln-lns)(1-F(x^N(s)))} \tag{4.12}$$

The midpoint of the equilibrium pair of policies is given by the following expression:

$$x^N(s) = \frac{l(r+2n(1-s))\int\limits_0^{x^N(s)} xdF(x)}{2(rl+rns+ln-lns)F(x^N(s))} + \frac{r(l+2ns)\int\limits_{x^N(s)}^1 xdF(x)}{2(rl+rns+ln-lns)(1-F(x^N(s)))} \tag{4.13}$$

This result may be reformulated as a weighted average of the equilibrium ideologies of the parties $x_L^{MN}(s)$ and $x_R^{MN}(s)$ by the following equation:

$$x^N(s) = (1 - \frac{\alpha(s)}{2})x_L^{MN}(s) + \frac{\alpha(s)}{2}x_R^{MN}(s) \tag{4.14}$$

where $\alpha(s) = \frac{r(2s(1-l-r)+l)}{(l(1-l)(1-s)+rs(1-r))}$.

Remark 21 $x^N(s) = \frac{\alpha(s)}{2}$ *when the ideology of the parties is taken to be exogenous and symmetric with respect to the midpoint of the unit interval. This is a special case of the result above (equation (4.14)) where* $x_L^{MN}(s)$ *takes the value* 0 *and* $x_R^{MN}(s)$ *takes the value* 1*. The current setup allows us to endogenise the ideologies of the parties.*

In order to illustrate the results of the inter-party competition and the ideology of the parties we will provide some examples.

Example 22 *The distribution of voters is given by the cumulative distribution* $F(x, m) = mx^2 + (1-m)x$ *where the parameter* m *satisfies the condition* $-1 \leq m \leq 1$*. Asymmetries can be captured by this distribution. When* $-1 \leq m < 0$ *the median is on the left of the midpoint of the unit interval and more than half of the population will prefer policies on the left. When* $0 < m \leq 1$ *the median is on the right of the midpoint of the unit interval and more than half of the population will prefer policies on the right of the midpoint.* $m = 0$ *is the case where the population is distributed uniformly along the unit interval. We can study the changes in the*

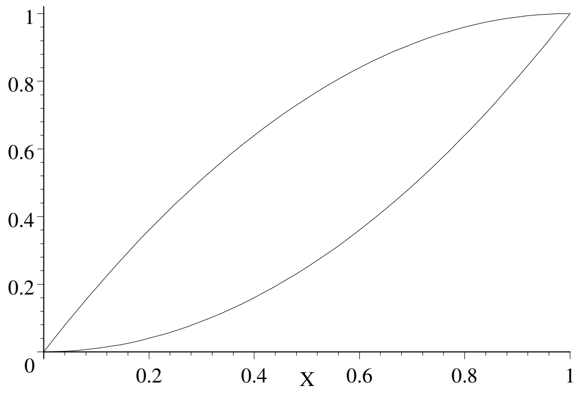

Figure 4.1: The distribution function $F(x,m) = mx^2 + (1-m)x$

equilibrium strategies as a function of m *characterising these asymmetries. This distribution function can be visualised by the graph (4.1) where the cumulative distribution functions for* $m = -1$ *and for* $m = 1$ *are depicted. All the other functions are between those two curves.*

In this case, by solving the equation (4.14), we end up at the following third degree equation:

$$4m^2(x^N(s))^3+2m(5-3m)(x^N(s))^2+2(3(1-m)-2\alpha m)x^N(s)+\alpha(m+3)(m-1)=0$$

Remark 23 *That this polynomial has at least one root in the unit interval follows from the intermediate value theorem. Let* $f(x^N(s)) = 4m^2(x^N(s))^3 + 2m(5-3m)(x^N(s))^2 + 2(3(1-m) - 2\alpha m)x^N(s) + \alpha(m+3)(m-1)$*. Since at* $x^N(s) = 0$ *this function has the value* $\alpha(m+3)(m-1)$ *which is negative and at* $x^N(s) = 1$ *this function has the value* $(m+1)(m-3)(-2+\alpha)$ *which is positive. We know*

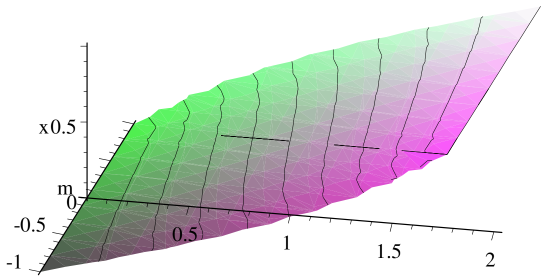

Figure 4.2: Equilibrium of political competition

that at least one time, this function intersects the x-axis. There exists at least one equilibrium for the electoral competition game. We represent the equilibrium results in Figure (4.2). Notice that the midpoint of the equilibrium policies increases with s.

4.3 The dynamic analysis of party membership decisions and ideologies

The previous section has described the political competition model and given the equilibrium results based on two assumptions. First, we suppose that the militants in each party compute their policy proposals according to the results of the electoral game. In a sense they are supposed to know their supporters before they give their policy proposals and before the parties give their policy proposals. The determination of the ideology of the party does not depend on any intra-party process. Second, the distribution of opportunists in the parties is given. The equilibrium results are provided as a function of the parameter s i.e. the share of opportunists in party L.

In this section, we introduce an adjustment process to the membership decision of opportunist candidates and to the preference formation of the militants. The opportunists are allowed to review their membership decisions before the elections by randomly drawing another opportunist candidate and imitating his decision if this decision performs better than his own previous decision. The militants are supposed to compute their policy proposals before the intra-party preference formation by adjusting their policy proposals based on the results of the previous elections. In this manner, the model of the previous section is improved since the ad hoc determination of the preferences of the party by the preferences of the average citizen supporting the party is replaced by a dynamic adjustment process and the distribution of opportunists politicians is endogenised. These two dynamic adjustment processes vary at different rates. The militants will review their proposals at the rate v_1 and the opportunists will review their membership decisions at the rate v_2.

4.3.1 The membership decisions of the opportunists

Before the elections the opportunists review their membership decisions. They have two alternatives: being a candidate of party L or being a candidate of party R. Since before each election they review their membership decisions and change their parties if necessary, the share of the opportunist politicians in each party will vary through time. As we denote by s the share of the opportunist politicians in the party L we will define an adjustment process for the membership decisions by defining the change in s.

While they review their membership decisions the opportunist candidates ignore the dynamic consequences of their actions and adjust their behaviours by imitating other opportunist politicians. The imitation is based purely on the payoff of the alternatives. Consequently, we have to define the payoffs of both alternatives in order to define the adjustment process. The utility of being a candidate of the party L is $u_L(x_L(s), x_R(s))$ $(u_L(s))$. The utility of being a candidate of the party R is given by $u_R(x_L(s), x_R(s))$ $(u_R(s))$. We suppose that not all the opportunist politicians review their membership decisions and before each election an opportunist politician is randomly drawn from the population and matched with an opportunist politician from the other party. If an opportunist candidate of the party L is drawn from the population and is matched with an opportunist candidate of the party R, then he will weight the payoff of his strategy by comparing the payoffs of the two alternatives.

We obtain the change in s as follows:

$$\begin{aligned} \dot{s} &= v_1 s\left(u_L(s) - s u_L(s) - (1-s)u_R(s)\right) \\ &= v_1 s\left(u_L(s) - \bar{u}(s)\right) \end{aligned} \tag{4.15}$$

where $\bar{u}(s)$ is the average payoff in the total opportunist population.

4.3.2 The preference adjustment process

The militants compute the average policy preference of their supporters to adjust their policy preferences for the following elections. We denote by x_L^M, x_R^M the policy proposals of the militants. Given these proposals, we can compute the equilibrium outcome of the electoral competition. Solving for $x_L^O(s) = x_R(s)$ and $x_R^O(s) = x_L(s)$ in equations (4.3) will result in a Nash equilibrium of the game which is a pair of policies $(x_L^E(s), x_R^E(s))$ such that:

$$x_L^E(s) = \frac{l(r+n(1-s))}{rl+rns+ln-lns} x_L^M + \frac{rns}{rl+rns+ln-lns} x_R^M \tag{4.16}$$

$$x_R^E(s) = \frac{nl(1-s)}{rl+rns+ln-lns} x_L^M + \frac{r(l+ns)}{rl+rns+ln-lns} x_R^M \tag{4.17}$$

The middle point of the equilibrium policy proposals is $x^E = \frac{x_L^E + x_R^E}{2}$.

$$x^E(s) = (1 - \frac{\alpha(s)}{2}) x_L^M + \frac{\alpha(s)}{2} x_R^M \tag{4.18}$$

The militants do not act strategically. When they review their policy proposals, they try to represent more efficiently their supporters in the elections, The proposal of the left militants is determined according to the following dynamic process:

$$\dot{x}_L^M = v_2\left(\frac{\int_0^{x^E(s)} x dF(x)}{F(x^E(s))} - x_L^M\right) \tag{4.19}$$

Notice that in this process the militants adjust by considering the distance of their proposals and the policy of the average citizen supporting the party. By the same manner, the proposal of the right militants is determined according to the following dynamic process:

$$\dot{x}_R^M = v_2\left(\frac{\int_{x^E(s)}^{1} x dF(x)}{1 - F(x^E(s))} - x_R^M\right) \tag{4.20}$$

These two dynamic processes determine how population shares corresponding to different pure strategies evolve over time and the ideologies of the parties are adjusted. The dynamics are functions of the utilities of the opportunist politicians and the distribution of the voters. The stability of the rest points of these dynamic processes will be studied in the following section.

4.3.3 The stability of the rest points

First, we have to define the utilities of the opportunist candidates since the dynamics governing the changes in the proportion of the opportunists in each party, are defined based on the utility of opportunist politicians. The utility of an opportunist candidate is a function of the benefits of power. These benefits are determined according to the political system which in our case is proportional. In the proportional system, the rents from power are split among the candidates proportionally to their share of the vote. Each party gets seats in the parliament equal to its vote share. The utility of an opportunist candidate is the share of the votes per candidate of his party.

Formally, the utility of an opportunist candidate of the party L is $u_L(x_L, x_R)$

$$u_L(x_L, x_R) = \frac{1}{l + ns} F(\frac{x_L + x_R}{2}) \tag{4.21}$$

and the utility of an opportunist candidate of the party R is given by $u_R(x_L, x_R)$.

$$u_R(x_L, x_R) = \frac{1}{r + n(1 - s)} (1 - F(\frac{x_L + x_R}{2})) \tag{4.22}$$

The next step will be to determine the rest points of the dynamic processes (4.15, 4.19 and 4.20) and analyse their stability. The dynamics (4.15) has the trivial rest points at $s = 0$ and $s = 1$. The other rest points of the evolutionary dynamics are given by the following equation:

$$F(x^E(s)) = l + ns$$

which is the case where the number of votes of the party L is equal to the share of left party candidates in total candidate population. For each of these values, the rest points of the dynamic processes (4.19 and 4.20) are given by the equations (4.3 and 4.4), the equilibrium ideologies of the political competition, satisfying the following condition:

$$x^E(s) = x^N(s)$$

Proposition 24 *In the proportional system*

- *the state where all opportunists are in party L* ($s = 1$) *is stable if* $F(x^N(1)) > l + n$ *and* $K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2}\Big|_{s=1, x^E(1)=x^N(1)} < 1$.
- *the state where all opportunists are in party R* ($s = 0$) *is stable if* $F(x^N(0)) < l$ *and* $K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2}\Big|_{s=0, x^E(0)=x^N(0)} < 1$.
- *the state where the opportunists are in party L and party R* ($0 < s^* < 1$) *where* s^* *satisfies the equation* $F(x^E(s^*)) = l + ns^*$ *is stable if*

$$\left. \begin{array}{c} v_1 \left(K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2} - 1 \right) \\ -v_2 \left(\frac{\partial F(x^E(s))}{\partial x^E(s)} \frac{\partial x^E(s)}{\partial s} - n \right) \end{array} \right|_{s=s^*, x^E(s^*)=x^N(s^*)} < 0$$

and

$$\left.\begin{array}{c}\left(H(s)(\frac{2-\alpha}{2})^2-1\right)\frac{\partial F(x^E(s))}{\partial x^E(s)}\frac{\partial x^E(s)}{\partial s}\\ -n\left(K(s)\frac{\alpha}{2}+H(s)\frac{2-\alpha}{2}-1\right)\end{array}\right|_{s=s^*,x^E(s^*)=x^N(s^*)}<0$$

where $K(s)=\frac{\partial(\frac{K(x^E(s))}{F(x^E(s))})}{\partial x^E(s)}$ *with* $K(x^E(s))=\int\limits_{x^E(s)}^{1} xdF(x)$ *and* $H(s)=\frac{\partial(\frac{H(x^E(s))}{F(x^E(s))})}{\partial x^E(s)}$ *with* $H(x^E(s))=\int\limits_{0}^{x^E(s)} xdF(x)$.

The proof of the proposition is provided in the Appendix (Annexe C).

Notice that the stability of the states where all the opportunist politicians are in party L or party R does not depend on the relative velocity of the two dynamics. When the opportunists are spread in party L and party R the stability will depend on the relative speed of the two adjustment processes i.e. on how often the militants review their proposals compared with how often the opportunists review their membership decisions.

As the equations do not make very much sense like this we will provide and example.

Example 25 *We consider an economy where the following parameters apply for the politicians:* $r=0.40$, $l=0.15$ *and* $n=0.45$. *The population distribution is given by the distribution function* $F(x,m)=mx^2+(1-m)x$ *and we consider two different population represented by* $m=0.5$ *and* $m=-0.5$.

When $m=0.5$, the median is 0.62 and the midpoint of the equilibrium policies is given by the equation: $4(x^N(s))^3+14(x^N(s))^2+4(3-2\alpha)x^N(s)-7\alpha=0$. When

$s = 0$, $x^N(0) = 0.34$. When $s = 1$, $x^N(1) = 0.89$. We check for the stability of these states using the previous conditions. The first state is not stable. The second state is stable. When $m = -0.5$, the median is 0.38 and the midpoint of the equilibrium policies is given by the equation: $4(x^N(s))^3 + 14(x^N(s))^2 + 4(3 - 2\alpha)x^N(s) - 7\alpha = 0$. When $s = 0$, $x^N(0) = 0.20$. When $s = 1$, $x^N(1) = 0.86$. We check for the stability of these states using the previous conditions. The first state is not stable. The second state is stable. The population distribution changes from an orientation to left to an orientation to right. The change does not affect the stability results. The state where all the opportunists are in party L is stable. this is due to the fact that the opportunists must also share the victory with the other politicians in the party and all the opportunists invading the less populated party is a stable outcome.

4.4 Conclusion

The results of the paper may be classified in two parts. First we studied a one shot electoral competition game and we defined the endogenous party equilibrium. The equilibrium results are analysed given the party composition and the population distribution. Second we defined a dynamic setup where the agents adjust their actions based on random encounters or information from previous period. Through this dynamic setup we studied the stability of the equilibrium we defined in the first section and we have given the conditions of stability of three population states of the opportunist politicians i.e. when all the opportunist politicians prefer party L, when they prefer party R and when they are in both parties. The stability depends not only on the dynamics governing the change in the share of the population of opportunist politicians but also the adjustment dynamics we defined for the militant politicians. The relative speed of both dynamics affect the stability of the state where the opportunist are distributed in both parties.

ANNEXES

A. Annexe du Chapitre 2

For the convenience, the propositions are restated before their proofs.

Proof of **(Proposition 3)**

(Proposition 3) *The set of rest points $\Phi(s)$ is non-empty for all values of s_i for all $i = 1, 2, 3$.*

Proof. Given $s \in [0,1]^3$, define a continuous function $F(p)$ as follows:

$F_i(p) = p_i + (1 - p_i)p_{i+2}a_i - (s_i + p_i - 1)p_{i+1}a_{i+1}$

Since $p_{i+2} \geqslant 1 - s_{i+2}$ and $p_{i+1} \leqslant 1$ we have the following inequality:

$F_i(p) \geqslant p_i + (1 - p_i)(1 - s_{i+2})a_i - (s_i + p_i - 1)a_{i+1}$

$= p_i(1 - a_{i+1}) + (1 - p_i)(1 - s_{i+2})a_i + (1 - s_i)a_{i+1}$

To check whether $F_i(p) \geqslant 1 - s_i$:

$p_i(1 - a_i) + (1 - p_i)(1 - s_{i+2})a_i + (1 - s_i)a_{i+1} \overset{?}{\geqslant} 1 - s_i$

$p_i(1 - a_i) + (1 - p_i)(1 - s_{i+2})a_i \overset{?}{\geqslant} (1 - a_{i+1})(1 - s_i)$

Dividing both sides by $(1 - a_{i+1})$ we get the following inequality.

$p_i + (1-p_i)(1-s_{i+2})\frac{a_i}{1-a_{i+1}} \overset{?}{\geqslant} (1-s_i)$

As $p_i \geqslant (1-s_i)$ and $(1-p_i)(1-s_{i+2})\frac{a_i}{1-a_{i+1}} \geqslant 0$ from the fact that p_i and s_{i+2} lie both in the unit interval and $\pi_i \leqslant 1$ for all $i = 1, 2, 3$ we conclude that $F_i(p) \geqslant 1 - s_i$.

Since $p_{i+1} \geqslant 1 - s_{i+1}$ and $p_{i+2} \leqslant 1$ we have the following inequality:

$F_i(p) \leqslant p_i + (1-p_i)a_i - (s_i + p_i - 1)(1-s_{i+1})a_{i+1}$

$= p_i(1-a_i) + a_i - (s_i + p_i - 1)(1-s_{i+1})a_{i+1}$

To check whether $F_i(p) \leqslant 1$:

$p_i(1-a_i) + a_i - (s_i + p_i - 1)(1-s_{i+1})a_{i+1} \overset{?}{\leqslant} 1$

$p_i(1-a_i) - (s_i + p_i - 1)(1-s_{i+1})a_{i+1} \overset{?}{\leqslant} 1 - a_i$

Diving both sides by $(1-a_i)$ we get the following inequality:

$p_i - (s_i + p_i - 1)(1-s_{i+1})\frac{a_{i+1}}{1-a_i} \overset{?}{\leqslant} 1$

As $p_i \leqslant 1$ and $(s_i + p_i - 1)(1-s_{i+1})\frac{a_{i+1}}{1-a_i} \geqslant 0$ from the fact that $s_i + p_i - 1$ and s_{i+1} lie both in the unit interval and $\pi_i \leqslant 1$ for all $i = 1, 2, 3$ we conclude that $F_i(p) \leqslant 1$.

Hence $F : \Delta^s \to \Delta^s$. Since Δ^s is a compact set and F is continuous, we have by Brouwer's Fixed Point Theorem the existence of a p^* satisfying $F(p^*) = p^*$. At any such point we have $(1-p_i)p_{i+2}a_i - (s_i + p_i - 1)p_{i+1}a_{i+1} = 0$ by the definition of F so p^* is a fixed point of the inventory dynamics defined by equation (2.1). ■

Proof of **(Proposition 4)**

(Proposition 4) *Suppose* $s \neq (1,1,1)$*, the set of rest points* $\Phi(s)$ *contains a single element.*

The proof of three preliminary results will lead to the proof of the proposition.

Lemma 1 *Suppose* $s \neq (1,1,1)$*,* $\pi_i \neq 0$ *and* $\pi_i \neq 1$ *for all* $i = 1,2,3$*. If* $p \in \Phi(s)$ *then* $p_i > 0$ *for all* $i = 1,2,3$*.*

Proof. If $p \in \Phi(s)$ then from equation (2) we have

$(1 - p_i)p_{i+2}a_i - (s_i + p_i - 1)p_{i+1}a_{i+1} = 0$

Suppose $p_1 = 0$. This requires that $s_1 = 1$. Then the above equation results in $p_3 = 0$ implying $s_3 = 1$. If $p_3 = 0$ from the above equation we have $p_2 = 0$ implying $s_2 = 1$. This contradicts the assumption $s \neq (1,1,1)$. ■

Lemma 2 *Suppose* $s \neq (1,1,1)$*,* $\pi_i \neq 0$ *and* $\pi_i \neq 1$*. If* $p \in \Phi(s)$ *and* $p' \in \Phi(s)$ *then either* $p = p'$ *or* $p_i \neq p'_i$ *for all* $i = 1,2,3$*.*

Proof. As p and $p' \in \Phi(s)$ they satisfy equation (2.1). Rearranging the terms we have the following equation that is well defined under the assumptions and $p_i > 0$ from Lemma 1.

$$s_i = (1 - p_i)\left[1 + \frac{a_i p_{i+2}}{a_{i+1}p_{i+1}}\right] \qquad \text{(A.23)}$$

Suppose that p and p' have at least one common element. Without loss of generality, suppose $p_1 = p'_1$. Then either (i) $p_2 = p'_2$ or (ii) $p_2 < p'_2$ or (iii) $p_2 > p'_2$.

In case (i) it must also be true that $p_3 = p'_3$ otherwise equation (A.23) could not be satisfied for both p and p'. In case (ii), equation (A.23) implies that $p_3 < p'_3$. But $p_2 < p'_2$ and $p_3 < p'_3$ are inconsistent if $p = p'$, so case (ii) is impossible. In case (iii), equation (A.23) implies that $p_3 > p'_3$. But $p_2 > p'_2$ and $p_3 > p'_3$ are inconsistent with equation (A.23) if $p = p'$, so case (iii) is impossible. Hence if p and p' have a common element, they are identical. ■

Lemma 3 *Suppose $s \neq (1,1,1)$, $\pi_i \neq 0$ and $\pi_i \neq 1$. If $p \in \Phi(s)$ and $p' \in \Phi(s)$ then either $p > p'$ or $p < p'$ or $p = p'$.*

Proof. Suppose $s \neq (1,1,1)$ and $p \in \Phi(s)$ and $p' \in \Phi(s)$. Suppose $p \neq p'$. Then from the previous lemma, we know that all their elements differ. Consequently, at least two elements of one vector must be strictly greater than the corresponding two elements of the other. Suppose, without loss of generality, that $p_1 > p'_1$ and $p_2 > p'_2$. This implies that $p_3 > p'_3$ and $p > p'$. Reversing the inequalities yields the remainder of the lemma. Suppose $p \in \Phi(s)$ and $p' \in \Phi(s)$, with $p \neq p'$. Following the previous lemma, we have either $p > p'$ or $p < p'$. Without loss of generality, assume that $p < p'$ with $p_1 < p'_1$. This implies $\frac{p_3}{p_2} < \frac{p'_3}{p'_2}$. By equation (4.2) we have $\frac{p_3}{p_2} > \frac{p'_3}{p'_2}$. If $p \in \Phi(s)$ and $p' \in \Phi(s)$, $p \neq p'$ is impossible. ■

Proof of **(Proposition 5)**

(Proposition 5) *Suppose $s = (1,1,1)$, $\pi_i \neq 0$ and $\pi_i \neq 1$. The set of rest points $\Phi(s)$ contains exactly two elements, exactly one of which is stable with* *respect to the dynamics of equation (4.1).*

Proof. If $s = (1,1,1)$ then $\dot{p}_i = (1-p_i)p_{i+2}a_i - p_i p_{i+1} a_{i+1}$. Setting $\dot{p}_i = 0$ we get two rest points: $p = (0,0,0)$ and $(\rho_1^4, \rho_2^4, \rho_3^4)$. In order to check if these rest points are asymptotically stable we calculate the eigenvalues of the following Jacobian:

$$\frac{\partial F}{\partial p} = \begin{bmatrix} -a_1 p_3 - a_2 p_2 & -a_2 p_1 & a_1(1-p_1) \\ a_2(1-p_2) & -a_2 p_1 - a_3 p_3 & -a_3 p_2 \\ -a_1 p_3 & a_3(1-p_3) & -a_3 p_2 - a_1 p_1 \end{bmatrix}$$

At $p = (0,0,0)$ the eigenvalues of the Jacobian are w, $w(-1/2 \pm i\sqrt{3}/2)$ where $w = (a_1 a_2 a_3)^{1/3} \geqslant 0$ as π_i lie in the unit interval for all $i = 1,2,3$. p is unstable at $p = (0,0,0)$. At $p = (\rho_1^4, \rho_2^4, \rho_3^4)$ the eigenvalues of the Jacobian are $w' - \frac{w''}{3}$, $-\frac{1}{2}w' - \frac{w''}{3} \pm i\frac{\sqrt{3}}{2}w'$ where $w'' = (a_1+a_2)\rho_1^4 + (a_2+a_3)\rho_2^4 + (a_1+a_3)\rho_3^4 \geqslant 0$ and as π_i and p_i lie in the unit interval for all $i = 1,2,3$. So p is stable at $p = (\rho_1^4, \rho_2^4, \rho_3^4)$.

■

Proof of **(Proposition 6)**

(Proposition 6) *Suppose $\pi_i \neq 0$ and $\pi_i \neq 1$. The function $\rho(s)$ is continuous at all monomorphic states and at all polymorphic states of the type $s = (x,1,0)$ where $x \in (0,1)$.*

Proof. Define the function $F(s,p) : R^3 \times R^3 \to R^3$ as follows:

$F_i(s,p) = (1-p_i)p_{i+2}a_i - (s_i + p_i - 1)p_{i+1}a_{i+1}$ for all $i = 1,2,3$.

Since $F(s,p)$ is continuously differentiable at any (s,p), we have the following matrix.

$$\frac{\partial F}{\partial p} = \begin{bmatrix} -a_1p_3 - a_2p_2 & -a_2(s_1+p_1-1) & a_1(1-p_1) \\ a_2(1-p_2) & -a_2p_1 - a_3p_3 & -a_3(s_2+p_2-1) \\ -a_1(s_3+p_3-1) & a_3(1-p_3) & -a_3p_2 - a_1p_1 \end{bmatrix}$$

If this matrix is invertible at (s,p), then by the implicit function theorem, there are open sets $U \in R^3$ and $V \in R^3$ with $(s,p) \in U \times V$ and a unique continuously differentiable function, $\rho : U \to V$ such that $F(s', \rho(s')) = 0$ for all $s' \in U$. $\rho(s)$ is continuous at all monomorphic population states if $\frac{\partial F}{\partial p}$ is invertible at all such states. By symmetry, it is sufficient to check this for states $s = (0,0,0)$, $s = (0,1,0)$, $s = (1,1,0)$, $s = (1,1,1)$. At the values of inventories corresponding to these states (Table 2), the determinant of $\frac{\partial F}{\partial p}$ is equal to $-(a_1+a_2)(a_1+a_3)(a_2+a_3)$, $(1-a_2)^{-2}(a_2(1-a_2)+a_3^2)(a_1+a_3)(a_2^2-a_1a_3-a_2)$, $-(a_1\rho_3^3+a_2\rho_2^3)(a_1a_2+a_1a_3\rho_3^3+a_2a_3\rho_2^3+a_3^2\rho_2^3)$, $a_1a_2a_3 - a_1a_2a_3\rho_1^4 - a_1a_2a_3\rho_2^4 - a_1a_2a_3\rho_3^4 - a_1a_2^2(\rho_1^4)^2 - a_2a_3^2(\rho_2^4)^2 + a_1a_2a_3\rho_1^4\rho_2^4 + a_1a_2a_3\rho_1^4\rho_3^4 + a_1a_2a_3\rho_2^4\rho_3^4 - a_2^2a_3\rho_1^4\rho_2^4 - a_1a_3^2\rho_2^4\rho_3^4 - a_1^2a_2\rho_1^4 \rho_3^4 - a_1^2a_3(\rho_3^4)^2 - 4a_1a_2a_3\rho_1^4\rho_2^4\rho_3^4$ respectively. In each case the determinant is non zero, so $\frac{\partial F}{\partial p}$ is invertible.

We now prove continuity at all points $s = (z,1,0)$. If $s = (z,1,0)$, then

$p = (\frac{a_3 w}{a_2(1-w)}, w, 1)$ where w is the root of the following second degree equation:

$$w^2(a_2^2(z-1) - a_2 a_3) + w(-a_1(a_2+a_3) - a_2^2(z-1)) + a_1 a_2$$

At these values, the determinant of the Jacobian is:

$$-\frac{w a_2 a_3 \left(a_2(1-w)+a_1\right)\left(a_2^2\left(1-z\right) - a_1 a_3 - a_2 w \left(a_2(1-z)+a_3\right)(2-w)\right)}{(1-w)^2}$$

Since the determinant can not be zero for all values of $0 < w < 1$, the Jacobian is invertible. ■

Proof of **(Proposition 7)**

(Proposition 7) Suppose $c_1 < c_2 < c_3$.

1. If $(c_3 - c_2) > a_1 - a_2(1-\rho_2^2)$ there is an asymptotically stable rest point at $s = (0, 1, 0)$.

2. If $(c_3 - c_2) < a_1 - a_2(1-\rho_2^3)$ there is an asymptotically stable rest point at $s = (1, 1, 0)$.

To prove the proposition, we need to prove the following lemma.

Lemma 4 *Given a monomorphic population state s and a monotonic selection dynamic ξ_i, s is asymptotically stable if $(2s_i - 1)(u_{\beta i}(s) - u_{\alpha i}(s)) > 0$ for all i.*

Proof. Suppose $J = \{i : s_i^* = 1\}$ and $K = \{i : s_i^* = 0\}$. If for all i, $(2s_i - 1)(u_{i\beta}(s) - u_{i\alpha}(s)) > 0$, then $u_{i\beta}(s^*) > u_{i\alpha}(s^*)$ when $i \in J$ and $u_{i\beta}(s) < u_{i\alpha}(s)$ when $i \in K$. By continuity of payoffs in population shares, there exists a neighborhood N of s^* such that, for all $s \in N - s^*$, $u_{i\beta}(s^*) > u_{i\alpha}(s^*)$ when $i \in J$ and $u_{i\beta}(s) < u_{i\alpha}(s)$ when $i \in K$. If the dynamics $f_i(s)$ are monotonic, then $f_i(s) > 0$ when $i \in J$ and $f_i(s) < 0$ when $i \in K$. ■

The Lemma states simply that a monomorphic population state s is asymptotically stable if $u_{\beta i}(s^*) > u_{i\alpha}(s^*)$ in those sub-populations i which consist exclusively of β-types, and $u_{\beta i}(s^*) < u_{i\alpha}(s^*)$ in those sub-populations i which consist exclusively of α-types. The proposition now may be proved.

Proof. For $s = (0, 1, 0)$ to be asymptotically stable we need the following conditions: $u_{1\alpha}(s) > u_{1\beta}(s)$, $u_{2\alpha}(s) < u_{2\beta}(s)$, $u_{3\alpha}(s) > u_{3\beta}(s)$. For $s = (0, 1, 0)$, the equilibrium inventory holdings are $\rho(s) = (1, \rho_2^2, 1)$ where ρ_2^2 is equal to $\frac{a_2}{a_3+a_2}$.

$$u_{1\alpha}(s) - u_{1\beta}(s) = \frac{\left(a_2 - a_1 - c_2 + c_3 - a_2\rho_2^2\right)a_2\rho_2^2}{a_1 + a_2\rho_2^2}$$

$$u_{2\alpha}(s) - u_{2\beta}(s) = -\frac{(a_2 + c_3 - c_1)a_3}{a_3 + a_2}$$

$$u_{3\alpha}(s) - u_{3\beta}(s) = \frac{(c_2 - c_1)a_1}{a_1 + \rho_2^2 a_3}$$

For $s = (1, 1, 0)$ to be asymptotically stable we need the following conditions: $u_{1\alpha}(s) < u_{1\beta}(s)$, $u_{2\alpha}(s) < u_{2\beta}(s)$, $u_{3\alpha}(s) > u_{3\beta}(s)$. For $s = (1, 1, 0)$, the equilibrium inventory holdings are $\rho(s) = (\rho_1^3, \rho_2^3, 1)$ where ρ_1^3 is given by the equation (2.2) and ρ_2^3 is by the equation (2.3).

$$u_{1\alpha}(s) - u_{1\beta}(s) = \frac{\left(a_2 - a_1 - c_2 + c_3 - a_2\rho_2^3\right)a_2\rho_2^3}{a_1 + a_2\rho_2^3}$$

$u_{2\alpha}(s) - u_{2\beta}(s) = -\frac{(c_3-c_1)a_3}{a_3+\rho_1^3 a_2}$

$u_{3\alpha}(s) - u_{3\beta}(s) = \frac{(a_1-c_1+c_2-a_1\rho_1)a_1\rho_1}{a_1\rho_1^3+\rho_2^3 a_3}$ ■

Proof of **(Proposition 8)**

(Proposition 8) *Suppose* $c_1 < c_2 < c_3$.

If $a_1 - a_2(1-\rho_2^3) < (c_3 - c_2) < a_1 - a_2(1-\rho_2^2)$ *then there is an asymptotically stable rest point at* $s = (x, 1, 0)$ *where* $x \in (0, 1)$.

Proof. The resulting equilibrium inventories for $s = (x, 1, 0)$ is $p = (\rho_1^5, \rho_2^5, 1)$ where

$$\rho_1^5 = \frac{\rho_2^5 a_3}{(1-\rho_2^5)a_2}$$

and ρ_2^5 is given by the following equation:

$$a_2(a_3 + a_2(1-x))(\rho_2^5)^2 + (a_1(a_2 + a_3) - a_2^2(1-x))\rho_2^5 - a_1 a_2 = 0$$

From this equation we get:

$$\rho_2^5 = \frac{-a_1\frac{a_2+a_3}{a_2} + a_2(1-x) + \sqrt{(a_1\frac{a_2+a_3}{a_2} - a_2(1-x))^2 + 4a_1(a_3 + a_2(1-x))}}{2(a_3 + a_2(1-x))}$$

Rest points of the evolutionary dynamics require all surviving strategies to have equal payoffs. Then for an interior point such as $s = (x, 1, 0)$ where $x \in (0, 1)$, we must have $u_{1\alpha}(s) = u_{1\beta}(s)$. At $s = (x, 1, 0)$ and $p = (\rho_1^5, \rho_2^5, 1)$:

$u_{1\alpha}(s) = (1-\rho_2^5)a_2 - c_2$

$u_{1\beta}(s) = \frac{-a_1c_2+a_1a_2-a_2c_3\rho_2^5}{a_1+a_2\rho_2^5}$

Solving $u_{1\alpha}(s) - u_{1\beta}(s) = 0$ we get:

$$x = \frac{(c_3 - c_2 + a_2)\left((1-a_1)(c_3 - c_2 + a_2 - a_1) - a_2^2\right)}{a_2\left(c_3 - c_2 - a_1\right)\left(c_3 - c_2 + a_2 - a_1\right)}$$

We must ensure that $x > 0$ and $x < 1$. As $x \in (0,1)$, we have $\rho_1^5 > \rho_1^3$. ■

As only the solutions of linear systems may be found explicitly in order to study the stability of this nonlinear system, we approximate it by a linear one (around the equilibrium point). For an equilibrium solution, the eigenvalues of the Jacobian matrix determine stability, as long as they all have non-zero real part. If all eigenvalues have negative real part the equilibrium point is asymptotically stable. If even one eigenvalue has positive real part, the solution is not stable. If there are eigenvalues with both signs of real parts present we have a saddle point.

Define the function $\Gamma^{ij}(r)$ as follows: $\Gamma^{i1}(r) = \dot{e}_i$, $\Gamma^{i2}(r) = \dot{f}_i$, $\Gamma^{i3}(r) = \dot{g}_i$ for all $i = 1,2,3$. The fundamental state and the speculative state satisfy $\Gamma^{ij}(r^*) = 0$. Recall that for the agents of type 1 depending on the values of the difference of the costs of the good 3 and the good 2 we have two different dynamics. The Jacobian matrix will be the same for both systems except for three first rows.

Proof of **(Proposition 11)**

(Proposition 11) Suppose $c_1 < c_2 < c_3$. $v > \frac{-8k_1^2+18k_1-4}{3(8k_1-9)}$ the fundamental state is asymptotically stable.

Proof. We evaluate the Jacobian matrix at the fundamental state and we remark that: $\left.\frac{\partial \Gamma^{kj}(r)}{\partial \sigma_{k\neq i}}\right|_{r=r^*} = 0$ for $k = 1, 2, 3$. Thus the eigenvalues may be obtained by the computation of the eigenvalues of the three submatrices that we will refer to as J_1, J_2, J_3.

Case 1 $k_1 < \frac{1}{4}$

$$J_1 = \begin{bmatrix} \frac{1}{6} - \frac{2}{3}k_1 & 0 & \frac{2}{3}k_1 - v - \frac{1}{6} \\ v + \frac{1}{6} - \frac{2}{3}k_1 & -v & -v - \frac{1}{6} + \frac{2}{3}k_1 \\ -0.5 - \frac{1}{3}k_1 & 0 & -1.5v + 0.5 + k_1/3 \end{bmatrix}$$

The eigenvalues are given by the following equation: $(-v - x)(-27v + 24k_1 v - 24x + 12k_1 x + 54xv + 36x^2) = 0$. One of the eigenvalues is $-v$. We check the sum and the product of the roots for the other roots. For the eigenvalues to be negative the sum should be negative and the product should be positive. The product of the eigenvalues is $-27v + 24k_1 v$. The sum of the eigenvalues is $30 - 36k_1 - 54v$. We have $-27v + 24k_1 v < 0$. One of the eigenvalues is positive.

$$J_2 = \begin{bmatrix} -2k_3/3 - 1/2 & 0 & 2k_3/3 - v + 1/2 \\ v + k_3/6 & -v & -v - k_3/6 \\ 0 & 0 & -2v \end{bmatrix}$$

The eigenvalues of the Jacobian are $-v$, $-\frac{2}{3}k_3 - \frac{1}{2}$, $-2v$. The eigenvalues are negative.

$$J_3 = \begin{bmatrix} 0 & 5k_2/6 & -0.5v \\ 0.5v & -0.5v + 5k_2/6 & -0.5v \\ -\frac{2}{3}k_2 & 5k_2/6 & -1.5v + \frac{2}{3}k_2 \end{bmatrix}$$

The eigenvalues are given by the following equation: $(-0.5v - x)(54xv -$

$54xk_2+36x^2-42vk_2+20k_2^2)=0$. One of the eigenvalues is $-0.5v$. The product of the other eigenvalues is $-42vk_2+20k_2^2$. Their sum is $-54(v-k_2)$. The eigenvalues are negative.

We conclude that when $k_1 < \frac{1}{4}$ the fundamental state is not stable.

Case 2 $k_1 > \frac{1}{4}$

$$J_1' = \begin{bmatrix} 0 & -\frac{2}{3}k_1 + 1/6 & -v \\ v & 1/6 - \frac{2}{3}k_1 - v & -v \\ -\frac{2}{3} + k_1/3 & 1/6 - \frac{2}{3}k_1 & -1.5v + \frac{2}{3} - k_1/3 \end{bmatrix}$$

The eigenvalues are given by the following equation: $(-v-x)(-27v+4-18k_1-30x+24k_1v+8k_1^2+36k_1x+54xv+36x^2)$. One of the eigenvalues is $-v$. The product of the other eigenvalues is $-27v+4-18k_1+24k_1v+8k_1^2$. Their sum is $30-36k_1-54v$. The sum is negative and the product is positive if $-27v+4-18k_1+24k_1v+8k_1^2>0$.

Since the eigenvalues of J_2 and J_3 are the same for both cases, the eigenvalues are negative.

We conclude that for $k_1 > \frac{1}{4}$ the fundamental state is stable when the parameters in consideration satisfy the previous condition. ∎

Proof of **(Proposition 12)**

(Proposition 12) *Suppose $c_1 < c_2 < c_3$. If $v < \frac{2\left(-5+4\sqrt{2}\right)\left(k_1-1-\sqrt{2}\right)\left(7k_1+4-\sqrt{2}\right)}{21(5\sqrt{2}-4-4k_1(2-\sqrt{2}))}$ the speculative state is asymptotically stable.*

Proof. We evaluate the Jacobian matrix at the speculative state and we remark that: $\left.\frac{\partial \Gamma^{kj}(r)}{\partial \sigma_{k\neq i}}\right|_{r=r^*} = 0$ for $k = 3$. Thus the eigenvalues may be obtained by the computation of the eigenvalues of the three submatrices that we will refer to as J_1, J_2, J_3.

For $k_1 < \frac{\sqrt{2}-1}{2}$

According to the nonzero values of the Jacobian we obtain again three submatrices.

$$J_1 = \begin{bmatrix} -\frac{\left(-2+\sqrt{2}\right)\left(-1+4k_1-3\sqrt{2}\right)}{6} & 0 \\ v & -\frac{\left(3\sqrt{2}-2\right)\left(6v+7k_1-\sqrt{2}+4+9\sqrt{2}v\right)}{42} \end{bmatrix}$$

The eigenvalues, $\frac{\left(2-\sqrt{2}\right)\left(-1+4k_1-3\sqrt{2}\right)}{6}$, $\frac{\left(3\sqrt{2}-2\right)\left(-7k_1+\sqrt{2}-4-(9\sqrt{2}+6)v\right)}{42}$ are negative.

$$J_2 = \begin{bmatrix} -\sqrt{2}v & 0 & 0 & -v\left(-2+\sqrt{2}\right) \\ -\frac{1}{2}\sqrt{2}v & \frac{1}{3}k_3\left(-5+2\sqrt{2}\right) & 0 & -\frac{1}{6}\sqrt{2}\left(-5k_3\sqrt{2}+4k_3+3v\right) \\ 0 & \frac{1}{6}\sqrt{2}\left(3v+5k_3-3k_3\sqrt{2}\right) & -\frac{1}{2}\sqrt{2}v & -\frac{1}{6}\sqrt{2}\left(3v+5k_3-3k_3\sqrt{2}\right) \\ -\frac{1}{2}\sqrt{2}v & 0 & 0 & -\frac{1}{2}v\left(2+\sqrt{2}\right) \end{bmatrix}$$

The eigenvalues, $\left(-\frac{3}{4}\sqrt{2}-\frac{1}{2}+\frac{1}{4}\sqrt{22-20\sqrt{2}}\right)v$, $\left(-\frac{3}{4}\sqrt{2}-\frac{1}{2}-\frac{1}{4}\sqrt{22-20\sqrt{2}}\right)v$, $-\frac{1}{2}\sqrt{2}v$, $-\frac{5}{3}k_3+\frac{2}{3}k_3\sqrt{2}$ are negative.

$$J_3 = \begin{bmatrix} 0 & \frac{\left(-4+\sqrt{2}\right)\left(3-14k_2-\sqrt{2}\right)}{42} & -v\left(\sqrt{2}-1\right) \\ v\left(\sqrt{2}-1\right) & -\frac{\left(\sqrt{2}-1\right)\left(6v+\sqrt{2}-4k_2-6k_2\sqrt{2}\right)}{6} & -v\left(\sqrt{2}-1\right) \\ -\frac{1}{3}k_2\sqrt{2} & \frac{\left(-4+\sqrt{2}\right)\left(3-14k_2-\sqrt{2}\right)}{42} & -\frac{\left(3\sqrt{2}-2\right)\left(21v-2k_2\sqrt{2}-6k_2\right)}{42} \end{bmatrix}$$

The eigenvalues are given by the following equation: $(-\frac{v}{\sqrt{2}+1}-x)(-6v+4k_2-4k_2\sqrt{2}+6v\sqrt{2}-12vk_2-8k_2^2+16k_2^2\sqrt{2}-12\sqrt{2}vk_2+54\sqrt{2}xv+12x-36xv+36x^2-48xk_2-6\sqrt{2}x)=0$. One of the eigenvalues is $-\frac{v}{\sqrt{2}+1}$. The product of the other eigenvalues is $-6v+4k_2(1-\sqrt{2})+6v\sqrt{2}-12vk_2-8k_2^2+16k_2^2\sqrt{2}-12\sqrt{2}vk_2$.

Their sum is $-54\sqrt{2}v - 12 + 36v + 48k_2 + 6\sqrt{2}$. The eigenvalues are negative.

We conclude that when $k_1 < \frac{\sqrt{2}-1}{4}$ the speculative state is stable.

For $k_1 > \frac{\sqrt{2}-1}{2}$

$$J_1' = \begin{bmatrix} -\frac{(-2+\sqrt{2})(-\sqrt{2}+k_1-1)}{3} & -\frac{(-2+\sqrt{2})(1+2k_1-\sqrt{2})}{6} \\ v & -\frac{(-2+3\sqrt{2})(6v-\sqrt{2}+4+7k_1+9\sqrt{2}v)}{42} \end{bmatrix}$$

The eigenvalues are given by the following equation: $-4\sqrt{2} + 28k_1 - 24v - 16k_1\sqrt{2} + 30\sqrt{2}v - 12x + 30\sqrt{2}k_1x + 24\sqrt{2}k_1v - 48k_1v + 20k_1^2 - 36k_1x + 36xv + 36x^2 + 24\sqrt{2}x - 16\sqrt{2}k_1^2 + 8 = 0$. The product of the eigenvalues is $-4\sqrt{2} + 28k_1 - 24v - 16k_1\sqrt{2} + 30\sqrt{2}v + 24\sqrt{2}k_1v - 48k_1v + 20k_1^2 - 16\sqrt{2}k_1^2 + 8$. Their sum is $-54\sqrt{2}v - 12 + 36v - 48k_{12} + 6\sqrt{2}$. The eigenvalues are negative. The sum is negative and the product is positive if $-4\sqrt{2} + 28k_1 - 24v - 16k_1\sqrt{2} + 30\sqrt{2}v + 24\sqrt{2}k_1v - 48k_1v + 20k_1^2 - 16\sqrt{2}k_1^2 + 8 > 0$.

Since the eigenvalues of J_2 and J_3 are the same for both cases, the eigenvalues are negative.

We conclude that for $k_1 > \frac{\sqrt{2}-1}{4}$ the speculative state is stable according to the previous condition. ■

B. Annexe du Chapitre 3

The two-stage game for the electoral competition

In this section we analyse the model as a two stage game.We consider now that at the first stage the opportunists review their membership decisions and at the second stage the party platforms are determined. We can analyse the case where all the opportunists are in the party L or in the party R. Note that the following utilities apply at the second stage:

$$u_L(t_L(s), t_R(s)) = \frac{1}{l + ns} F(t^N(s))$$

$$u_R(t_L(s), t_R(s)) = \frac{1}{r + n(1-s)} (1 - F(t^N(s)))$$

where $t^N(s) = \frac{2nsr + lr}{2(lr + n(l(1-s) + sr))}$.

The opportunists will either choose the party L or the party R. As we would like to see whether the opportunists prefer the party L to party R, we have to analyse the difference of the utility of being in party L when they all choaose party L and the utility of being in party R when they all choose the party R. Let $\Delta = u_L(t_L(1), t_R(1)) - u_R(t_L(0), t_R(0))$. Then we have the following results.

$$\Delta = \frac{1}{l + n} F(t^N(1)) - \frac{1}{r + n} (1 - F(t^N(0)))$$

$$\Delta = \frac{1}{(1 - r)} F(\frac{2n + l}{2\,(1 - r)}) + \frac{1}{(r + n)} F(\frac{r}{2(r + n)}) - \frac{1}{(r + n)}$$

When we draw the previous function for different definitions of the distribution of voters we obtain the following graph which describe the region where the utility of being in party L when they all choose party L is greater than the utility of being in party R when all choose the party R. The region is defined according to the values of the number of opportunist politicians and the militants in party R. In the region below the solid line in the graph (4.3), the opportunists will prefer party R when the citizens are uniformly distributed along the unit line. In the region between the dashed lines in the graph (4.3), the opportunists will prefer party L when the citizens are nonuniformly distributed along the unit line.

When the distribution of voters is uniform and there are equal number of militants in each party then $\Delta = u_L(t_L(1), t_R(1)) - u_R(t_L(0), t_R(0)) = 0$. In that case the opportunist politicians will be indifferent between two parties but as we have analysed in the chapter 3 the case where they are distributed equally is not a stable outcome.

Proof of (**Proposition 16)**

(**Proposition 16)** In the proportional system the state where all opportunists are in party L ($s = 1$) is stable if $F(t^N(1)) > l + n$. The state where all opportunists are in party R ($s = 0$) is stable if $F(t^N(0)) < l$.

To prove the proposition, we need to prove the following lemma.

Lemma *Given a population state s and a monotonic selection dynamic ξ, s is asymptotically stable if* $(2s - 1)(u_L(t_L(s), t_R(s)) - u_R(t_L(s), t_R(s))) > 0$.

Proof. Let $s^* = 0$. If $(2s^* - 1)(u_L(t_L(s^*), t_R(s^*)) - u_R(t_L(s^*), t_R(s^*))) > 0$, then $u_L(t_L(s^*), t_R(s^*)) > u_R(t_L(s^*), t_R(s^*))$. By continuity of payoffs in population share, there exists a neighborhood N of s^* such that, for all $s \in N - s^*$, $u_L(t_L(s), t_R(s)) < u_R(t_L(s), t_R(s))$. If the dynamics $\xi(s)$ are monotonic, then $\dot{s} < 0$. Let $L(s) = 1 - s$. $L(s)$ attains its maximum value of 1 when $s = s^*$, and is positive and increasing in $N - s^*$. This is a strict Liapunov function for s and s is asymptotically stable by Liapunov's Stability Theorem.

Let $s^* = 1$. If $(2s^* - 1)(u_L(t_L(s^*), t_R(s^*)) - u_R(t_L(s^*), t_R(s^*))) > 0$, then $u_L(t_L(s^*), t_R(s^*)) > u_R(t_L(s^*), t_R(s^*))$. By continuity of payoffs in population share, there exists a neighborhood N of s^* such that, for all $s \in N - s^*$, $u_L(t_L(s), t_R(s)) > u_R(t_L(s), t_R(s))$. If the dynamics $\xi(s)$ are monotonic, then $\dot{s} > 0$. Let $L(s) = s$. $L(s)$ attains its maximum value of 1 when $s = s^*$, and is positive and increasing in $N - s^*$. This is a strict Liapunov function for s and s is asymptotically stable by Liapunov's Stability Theorem. ■

The lemma simply says that $s = 1$ is stable if $u_L(t_L(s), t_R(s)) > u_R(t_L(s), t_R(s))$ and $s = 0$ is stable if $u_L(t_L(s), t_R(s)) < u_R(t_L(s), t_R(s))$. The proposition now may be proved.

Proof. For $s = 1$ to be asymptotically stable we need the following condition:

$$u_L(t_L(1), t_R(1)) > u_R(t_L(1), t_R(1))$$

$u_L(t_L(1), t_R(1)) - u_R(t_L(1), t_R(1)) = \frac{1}{l+n} F(t^N(1)) - \frac{1}{r}(1 - F(t^N(1)))$

$u_L(t_L(1), t_R(1)) - u_R(t_L(1), t_R(1)) = \frac{F(t^N(1)) - l - n}{(l+n)r}$

$\frac{F(t^N(1)) - l - n}{(l+n)r} > 0 \Rightarrow F(t^N(1)) > l + n$

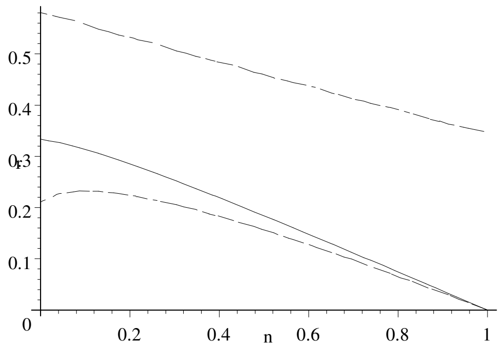

Figure 4.3: The values of Δ

For $s = 0$ to be asymptotically stable we need the following condition:

$$u_L(t_L(0), t_R(0)) < u_R(t_L(0), t_R(0))$$

$u_L(t_L(0), t_R(0)) - u_R(t_L(0), t_R(0)) = \frac{1}{l}F(t^N(0)) - \frac{1}{r+n}(1 - F(t^N(0)))$

$u_L(t_L(0), t_R(0)) - u_R(t_L(0), t_R(0)) = \frac{F(t^N(1))-l}{(r+n)l}$

$\frac{F(t^N(0))-l}{(r+n)l} > 0 \Rightarrow F(t^N(0)) > l$ ■

C. Annexe du Chapitre 4

Proof of **(Proposition 24)**

(Proposition 24)

In the proportional system

- the state where all opportunists are in party L ($s = 1$) is stable if $F(x^N(1)) > l + n$ and $K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2}\Big|_{s=1, x^E(1)=x^N(1)} < 1$.

- the state where all opportunists are in party R ($s = 0$) is stable if $F(x^N(0)) < l$ and $K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2}\Big|_{s=0, x^E(0)=x^N(0)} < 1$.

- the state where the opportunists are in party L and party R ($0 < s^* < 1$) where s^* satisfies the equation $F(x^E(s^*)) = l + ns^*$ is stable if

$$\left.\begin{array}{c} v_1\left(K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2} - 1\right) \\ -v_2\left(\frac{\partial F(x^E(s))}{\partial x^E(s)}\frac{\partial x^E(s)}{\partial s} - n\right) \end{array}\right|_{s=s^*, x^E(s^*)=x^N(s^*)} < 0$$

and

$$\left.\begin{array}{c} \left(H(s)(\frac{2-\alpha}{2})^2 - 1\right)\frac{\partial F(x^E(s))}{\partial x^E(s)}\frac{\partial x^E(s)}{\partial s} \\ -n\left(K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2} - 1\right) \end{array}\right|_{s=s^*, x^E(s^*)=x^N(s^*)} < 0$$

where $K(s) = \frac{\partial(\frac{K(x^E(s))}{F(x^E(s))})}{\partial x^E(s)}$ with $K(x^E(s)) = \int\limits_{x^E(s)}^{1} x dF(x)$ and $H(s) = \frac{\partial(\frac{H(x^E(s))}{F(x^E(s))})}{\partial x^E(s)}$ with $H(x^E(s)) = \int\limits_{0}^{x^E(s)} x dF(x)$.

Proof. Setting (4.15) yields two trivial rest points $s = 0$ and $s = 1$. These processes (4.15, 4.19 and 4.20) are non-linear. We study their stability by locally linearising the equations. The Jacobian of the system is:

$$J = \begin{bmatrix} v_2(H(s)\frac{2-\alpha}{2} - 1) & v_2 H(s)\frac{\alpha}{2} & v_2 H(s)\frac{\partial x^E(s)}{\partial s} \\ v_2 K(s)\frac{2-\alpha}{2} & v_2(K(s)\frac{\alpha}{2} - 1) & v_2 K(s)\frac{\partial x^E(s)}{\partial s} \\ v_1\frac{s(1-s)\frac{2-\alpha}{2}}{(l+ns)(r+n(1-s))}\frac{\partial F(x^E(s))}{\partial x^E(s)} & v_1\frac{s(1-s)\frac{\alpha}{2}}{(l+ns)(r+n(1-s))}\frac{\partial F(x^E(s))}{\partial x^E(s)} & v_1\frac{\partial(\frac{s(1-s)(F(x^E(s))-l-ns)}{(l+ns)(r+n(1-s))})}{\partial s} \end{bmatrix}$$

We evaluate the eigenvalues of this Jacobian at the rest points. A rest point is asymptotically stable if all eigenvalues of the Jacobian matrix have negative real parts and unstable if any eigenvalue has positive real part. After simplification, the Jacobian looks like:

$$J = \begin{bmatrix} -v_2 & v_2 H(s)\frac{\alpha}{2} & v_2 H(s)\frac{\partial x^E(s)}{\partial s} \\ 0 & v_2(K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2} - 1) & v_2(K(s) + H(s)\frac{2-\alpha}{2})\frac{\partial x^E(s)}{\partial s} \\ 0 & v_1 \frac{s(1-s)\frac{\alpha}{2}}{(l+ns)(r+n(1-s))}\frac{\partial F(x^E(s))}{\partial x^E(s)} & v_1 \frac{\partial(\frac{s(1-s)(F(x^E(s))-l-ns)}{(l+ns)(r+n(1-s))})}{\partial s} \end{bmatrix}$$

One of the eigenvalues is always $-v_2$. Notice that for $s = 0$ and $s = 1$ this matrix is a triangular matrix and the eigenvalues are given by the diagonal elements of the matrix.

For $s = 1$, the eigenvalues are $K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2} - 1\Big|_{s=1,x^E(1)=x^N(1)}$ and
$\frac{\partial(\frac{s(1-s)(F(x^E(s))-l-ns)}{(l+ns)(r+n(1-s))})}{\partial s}\Bigg|_{s=1,x^E(1)=x^N(1)}$

$$\frac{\partial(\frac{s(1-s)(F(x^E(s))-l-ns)}{(l+ns)(r+n(1-s))})}{\partial s}\Bigg|_{s=1,x^E(1)=x^N(1)} < 0$$

$$\Rightarrow F(x^N(1)) > l + n$$

For $s = 0$, the eigenvalues are $K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2} - 1\Big|_{s=0,x^E(0)=x^N(0)}$ and $\frac{\partial(\frac{s(1-s)(F(x^E(s))-l-ns)}{(l+ns)(r+n(1-s))})}{\partial s}\Bigg|_{s=0,x^E(0)=x^N(0)}$

$$\frac{\partial(\frac{s(1-s)(F(x^E(s))-l-ns)}{(l+ns)(r+n(1-s))})}{\partial s}\Bigg|_{s=0,x^E(0)=x^N(0)} \Rightarrow F(x^N(0)) < l$$

For $0 < s < 1$, we have to consider the eigenvalues of the following matrix:

$$G' = \begin{bmatrix} v_2(K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2} - 1) & v_2(K(s) + H(s)\frac{2-\alpha}{2})\frac{\partial x^E(s)}{\partial s} \\ v_1 \frac{s(1-s)\frac{\alpha}{2}}{(l+ns)(r+n(1-s))}\frac{\partial F(x^E(s))}{\partial x^E(s)} & v_1 \frac{\partial(\frac{s(1-s)(F(x^E(s))-l-ns)}{(l+ns)(r+n(1-s))})}{\partial s} \end{bmatrix}$$

The eigenvalues will be the roots of a second degree equation. Rather than finding explicitly the roots of the characteristic polynomial, the product of these roots will be checked. If both of the eigenvalues are negative the product of these should be positive and their sum is negative. The first condition is obtained through the sum of the eigenvalues and we get the second condition by computing the product of the eigenvalues:

$$v_1 \left(\frac{\partial(\frac{K(x^E(s))}{1-F(x^E(s))})}{\partial x^E(s)} \frac{\alpha}{2} + H(s)\frac{2-\alpha}{2} - 1 \right) < v_2 \left(\frac{\partial F(x^E(s))}{\partial x^E(s)} \frac{\partial x^E(s)}{\partial s} - n \right)$$

$$\left(H(s)(\frac{2-\alpha}{2})^2 - 1 \right) \frac{\partial F(x^E(s))}{\partial x^E(s)} \frac{\partial x^E(s)}{\partial s} < n \left(K(s)\frac{\alpha}{2} + H(s)\frac{2-\alpha}{2} - 1 \right)$$

■

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Résumé. Dans cette thèse trois modèles en économie évolutionniste sont étudiés. Dans le chapitre 2 on présente une version évolutionniste du modèle de l'émergence de la monnaie de Kiyotaki et Wright (1989). Ce chapitre a deux objectives: Le premier objectif est de faire l'analyse des implications de l'introduction des asymétries au processus d'appariement et d'étudier la relation de l'hypothèse d'appariement aléatoire avec l'émergence d'un moyen d'échange. Cette introduction nous permet de retrouver les conditions de stabilité pour les équilibres définis dans Kiyotaki et Wright (1989). Le deuxième objectif est d'étudier les dynamiques du déséquilibre où la distribution des stocks n'est plus supposée avoir toujours sa valeur d'équilibre temporaire et déterminer l'effet de la vitesse relative des dynamiques de l'échange et des dynamiques évolutionnistes sur la stabilité des équilibres du modèle de Kiyotaki et Wright (1989). Le chapitre 3 et 4 traitent la compétition électorale entre deux partis politiques composés d'une faction opportuniste et d'une faction militante suivant Roemer (1999). Les candidats "opportunistes" ont pour seul objectif de remporter les élections ou de maximiser le nombre de voix pour bénéficier du prestige et de la puissance, alors que les candidats "militants" ont des préférences idéologiques. Dans le chapitre 3 on suppose que les militants sont inflexible et représentent une idéologie donnée et les opportunistes déterminent leurs positions en maximisant leurs probabilités d'être élu. Suivant Roemer (1999) l'équilibre de Nash d'unanimité du parti est défini. On étudie la question du choix des partis des candidats opportunistes en les permettant de changer de parti politique et en endogénéisant cette décision. Le chapitre 4 traite l'aspect dynamique de l'idéologie du parti. On suppose que les militants adoptent les préférences du citoyen moyen qui supporte le parti et les opportunistes déterminent leurs positions en maximisant leur probabilité d'être élu. Suivant Roemer (2004) l'équilibre de Nash d'unanimité du parti endogène est défini. Ensuite on introduit une analyse dynamique pour étudier la stabilité. Le premier processus dynamique est la dynamique de sélection pour endogénéiser les décisions des opportunistes. Le second processus est le processus d'ajustement des militants. Le choix des partis des opportunistes est étudié.

Summary. This thesis studies three models in evolutionary economics. Chapter 2 uses an evolutionary version of the commodity money model (Kiyotaki and Wright (1989)). The main objective of this chapter is to study the implications of introducing asymetries to the matching process. Under this set up we find new stability conditions for each kind of equilibrium (fundamental or speculative) defined in Kiyotaki et Wright (1989). The second objective is to analyse the disequilibrium dynamics, when the inventory distribution is not assumed to be continuously at its temporary equilibrium value. The inventory dynamics and the behavioural dynamics vary at different rates and we study the effect of the relative rate of these dynamics on the stability of the equilibria. Chapters 3 and 4 study electoral competition between two political parties composed of an opportunist faction and a militant faction following Roemer (1999). The office

oriented politicians, referred to as "opportunist" politicians, care only about the spoils of the office. The policy oriented politicians, referred to as "militant" politicians have ideological preferences on the policy space. In chapter 3, we study a unidimensional model of spatial competition between two parties with endogeneous party membership decisions of opportunist candidates. In this framework, we compare a winner-take-all system, where all the spoils go to the winner, to a proportional system, where the spoils of office are split among the two parties proportionally to their share of the vote. We study the existence of short term political equilibria and then within an evolutionary setup the stability of policies and party membership decisions of opportunist candidates. In Chapter 4, we analyse the spatial electoral competition between two parties when the ideology of each party is endogenously determined by the preferences of the average supporters of the parties. We study the existence of short term political equilibria and then we introduce a dynamic setup to analyse the stability of these equilibria.