The time asymmetry of quantum mechanics and concepts of physical directionality of time.

Part 1. T asymmetry of quantum probability laws.

Andrew Holster, ATASA Research
Te Kuiti, New Zealand.
Email: ATASA030@gmail.com
(Draft 1.) 31/12/2017.

Introduction to Part 1.

This is Part 1 of a four part paper, intended to redress some of the most fundamental confusions in the subject of *physical time directionality,* and present the concepts accurately. There are widespread fallacies in the subject that need to be corrected in introductory courses for physics students and philosophers. Parts 1 and 2 are about quantum mechanics, Part 3 is about fundamental concepts, and Part 4 is about cosmology.

We start here by immediately analyzing the *time reversal symmetry of quantum probability laws. Time reversal symmetry* is defined as the property of *invariance under the time reversal transformation, T: t* → *-t.* This property is also variously called *time symmetry, time reversibility*, or just *reversibility*. Two main analyses are given, first in Part 1 here and second in Part 2, showing that:

- *Quantum mechanics (classical or relativistic) is strongly time asymmetric in its probability laws.*
- *Quantum mechanics (classical) is time asymmetric in its deterministic laws.*

This contradicts the orthodox analysis, found throughout the conventional literature on physical time, which claims that quantum mechanics is *time symmetric* or *reversible*. This is widely claimed as settled scientific fact, and large philosophical and scientific conclusions are drawn from it. But it is an error. The fact is that:

- While quantum mechanics is widely claimed to be *reversible* on the basis of two formal mathematical properties (that it does have), these properties *do not represent invariance under the time reversal transformation.* They are independent of time symmetry.

Most physicists remain unaware of the errors, decades after they were first demonstrated. Many orthodox specialists in the philosophy of physics are aware of the errors, but continue to refer to the 'time symmetry' or 'reversibility' of quantum mechanics anyway – and exploit the ambiguity to claim false implications about physical time reversal symmetry in nature. The excuse for perpetrating the confusion is that, since it is has now become customary to refer to the formal properties of quantum mechanics as 'reversibility' or 'time reversal symmetry', we should just keep referring to them by this name, *even though they are not time reversal symmetry*.

This causes endless confusion, especially in attempts to explain the physical irreversibility of our universe, and in philosophical discussions of the implications for the nature of time. The failure of genuine *time reversal symmetry in quantum mechanics* changes the interpretation of modern physics in a deep way. It changes the problem of explaining the real irreversibility found throughout nature.

Continuing to obfuscate this issue with semantic games is not acceptable for science. We may as well call a convicted man 'guilty', long after he has been proved innocent of his crime, on the grounds that it has become a popular convention, and it would embarrass the authorities to change their language to call him 'innocent'. We can save the academic authorities the embarrassment of admitting their mistake about time symmetry by saying: "quantum mechanics is time symmetric (in the sense of having certain formal properties), but not time symmetric (in the sense that it is not symmetric under time reversal)". But only by denying the truth. Scientists cry foul if this semantic obfuscation is seen in other subjects: and it is unacceptable in physics.

Partly as a result of such obfuscation, the broader *conceptual analysis* of 'time symmetry', 'time directionality' and related concepts is fatally confused. It needs fundamental revision too. But rather than starting with the conceptual analysis (which physicists find boring), we start directly with a real analysis of quantum mechanics, to see immediately why there is such a problem.

Part 1. Irreversibility of probabilistic laws of quantum mechanics.

1.1 The orthodox analysis.

Quantum mechanics is a *probabilistic theory*, and there is a long-standing controversy about the time symmetry of its *probabilistic laws*. The orthodox analysis claims that *the probabilistic laws of quantum mechanics are reversible.* The usual argument is quite simple. It start with this premise:

(1*) The condition for time reversal symmetry of a probabilistic theory of physics is:

$$prob(s_1 \rightarrow s_2) = prob(Ts_2 \rightarrow Ts_1)^1$$

(future directed probability) *(future directed probability)*

This states a *criterion for reversibility or time symmetry:* i.e. that *the probability of a state transition from an earlier to a later state must be equal, in all cases, to the probability of the reversed transition, from the reversed later state to the reversed earlier state.* Quantum mechanics predicts such probabilities, and a simple argument shows that it satisfies the condition (1*). It can be summarized in a couple of lines:

(2) The quantum mechanics probability for: $prob(s_1 \rightarrow s_2)$ equals the square of the inner product: $|<s_2|s_1>|^2$. Likewise: $prob(Ts_2 \rightarrow Ts_1) = |<Ts_1|Ts_2>|^2$. It is a simple mathematical fact that: $|<s_2|s_1>|^2 = |<s_1|s_2>|^2 = |<Ts_1|Ts_2>|^2$. Hence the condition required by (1*) is satisfied.[2]

Argument done and dusted. What could possibly be wrong with that? Well, there is a hole in it you could drive a truck through. It is not the inference in Step (2). It is the *assumption* in Step (1*). This is false. The correct condition is in fact:

---

[1] The term: $prob(s_1 \rightarrow s_2)$ refers to the transition probability from an initial state s_1 to a final state s_2 when measured a short time later. Ts_1 and Ts_2 are the *time reversals* of states s_1 and s_2.

[2] Note this is independent of whether we choose T or T^* as the time reversal operator – the question raised in Part 2 – for: $|<Ts_1|Ts_2>|^2 = |<T^*s_1|T^*s_2>|^2$. So we can ignore this subsequent question until Part 2.

The correct criterion for reversibility of a probabilistic theory.

(1) The condition for time reversal symmetry of a probabilistic theory is:

$$prob(s_2(t+\Delta t)|s_1(t)) = prob(Ts_2(t-\Delta t)|Ts_1(t))^3$$

(*future-directed probability*) (*past-directed probability*)[4]

How do we know this is the correct criterion, rather than (1*)? Because *it has been proven by careful analysis.*[5] We will review this shortly. But first we emphasize that *quantum mechanics fails the condition (1)*. This requires *past-directed probabilities* in symmetry with the normal *future-directed probabilities* of quantum mechanics. But quantum mechanics supports no *past-directed probabilities* like this at all – except in the special condition of thermodynamic equilibrium. This law would imply that a state Ts_1 is *preceded* by the state Ts_2 with a law-like probability. But there are no general quantum mechanical laws relating present states to *earlier states.* There are general laws relating present states to *subsequent states.*

Indeed it is quite certain that quantum mechanics fails (1), from the fact that *the universe is in thermodynamic disequilibrium.* For the symmetry (1) combined with symmetry (1*) entails a state of equilibrium. We will see this next, but first we examine (1). Where did the *past-directed probabilities* in (1) come from? The right-hand term in the criterion for reversibility must be the *time reversed image* of the left hand term: $prob(s_2(t+\Delta t)|s_1(t))$. This left hand term is a *future-directed probability law*. Watanabe (1955, 1965, 1966) seems to be the first to fully understand that the time reversal of a *future-directed law:* $prob(s_2(t+\Delta t)|s_1(t))$ must be a *past-directed law:* $prob(Ts_2(t-\Delta t)|Ts_1(t))$. He proved this. But his proofs of this were ignored, and almost every writer on the time reversal of quantum physics over the last 60 years have implicitly or explicitly assumed (1*) without question.

---

[3] We cannot use the abbreviated arrow notation of (1*) to write this – we have to expand it into its more explicit conditional probability form.

[4] We refer to *future-directed* or *predictive* laws, and *past-directed* or *retrodictive laws.* These pairs are almost synonymous, and used interchangeably here. However *future-directed* and *past-directed* has the connotation of objective probabilities. *Predictive* and *retrodictive* has the connotation of epistemic probabilities, and predicting and retrodicting, a human activity. The difference is slight, but important to philosophers.

[5] Originally by Watanabe 1955; a more comprehensive treatment with rigorous proofs is given in Holster 2003. Healy 1993 also understood the principle.

Proof of the criterion.

The *criterion* we adopt to judge the reversibility of a theory has to be *proven* to correctly represent time reversal invariance. A rigorous proof of (1) and disproof of (1*), verifying Watanabe's less formal proofs, is given in Holster (2003); I will explain only the key points of it here. But to begin with, simply the realization that *a proof of the criterion has to be given* is the major step forward. A generation of authorities would claim (1*) without analysis, like Sklarr (1974):

> "If the laws of nature are time reversal invariant, then for any process that occurs in the world, the process that consists in starting with the time-reversed final state of the original evolution and ending up with the time-reversed initial state of the original process is equally compossible with the laws of nature. . .. Remember, of course, that in the quantum-theoretic context it is the transition probabilities between reversed states which must equal the probabilities of the unreversed states taken in opposite temporal order for the laws to be time reversal invariant. The physics of time reversal invariance, then, is quite simple." Sklarr (1974) p 368.

Similarly Davies (1974) just knows it is a fact:

> "*the probabilistic principle of micro-reversibility"* requires: "$\omega = \omega_{\mathrm{rev}}$, where ω is the transition probability $|(\psi, \varphi)|^2$, and $\omega_{\mathrm{rev}} = |(T\varphi, T\psi|^2$".

These are just two different statements of (1*). However simple examples show (1*) must be wrong in principle. First it is not a *necessary* condition for time symmetry.

Counter-example 1: a simple reversible theory that fails (1*) and satisfies (1).

We define a very simple time symmetric probabilistic theory. This governs a process that has just three possible states, *A, B, C,* which are their own time reversals:

$$TA = A, \qquad TB = B, \qquad TC = C$$

The state jumps at a fixed interval (every second) from one state to another. From state *A* it jumps to *B* or *C* with equal probabilities of ½. From state *B* or *C,* it always jumps back to state *A.* I.e.

$$prob(B(t+1)|A(t)) = ½$$

$$prob(C(t+1)|A(t)) = ½$$

$$prob(A(t+1)|B(t)) = 1$$

$$prob(A(t+1)|C(t)) = 1.$$

And identically for their time reversals of course: $prob(TB(t+1)|TA(t)) = ½$, etc.

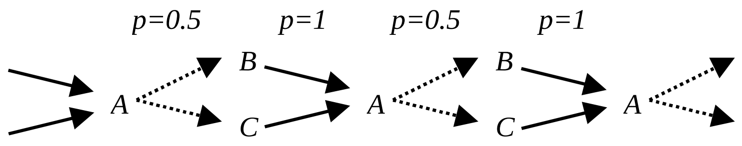


Figure 1. Future transition probabilities for a simple reversible theory.

Now the theory is clearly *time symmetric* or *reversible:* for the probability of any sequence of states is exactly the same as the probability for the reversed sequence. E.g. an actual sub-sequence might go:

$$\ldots ACABABACABACACA\ldots$$

Its reversal would be:

$$\ldots ACACABACABABACA\ldots$$

These are two different sequences of course, but they have identical probabilities.
To prove this:

- Any sequence with N occurrences of either B or C has a probability: $p = (½)^N$.
- The time reversal of any sequence with N occurrences of either B or C is another sequence with N occurrences of either B or C, and hence also has: $p = (½)^N$.

Therefore:

- The theory entails that the probability of any process is exactly same as the probability of the time reversed process.

Indeed the theory is obviously time symmetric. However it fails the orthodox criterion, (1*). E.g.

predictive: $prob(B(t+1)|A(t)) = ½$

≠ predictive: $prob(TA(t+1)|TB(t)) = 1$

And similarly for all other transitions. According to the orthodox criterion, these should be equal for a reversible theory, but they ain't. However the theory does satisfy the correct criterion, *(1)*, e.g.

predictive: $prob(B(t+1)|A(t)) = ½$

= retrodictive: $prob(TB(t-1)|TA(t)) = ½$

And similarly for all the other possible transitions. It is obvious enough that the retrodictive probability that *state A came from the state B,* is the same as the predictive probability that state *A* will go *to the state B.* But we should give a proof just to be thorough.

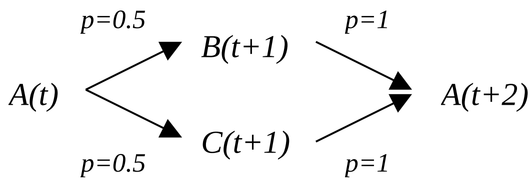


Figure 2. All the paths to reach *A.*

If the state *A* occurs at a time *t+2,* it is always preceded by *A* at *t,* i.e. two state transitions previously, because the only immediate states preceding *A* are *B* or *C,* and both must be preceded by *A.* I.e. every *A* originates either from: $A \rightarrow B \rightarrow A$, or from: $A \rightarrow C \rightarrow A$. Both have the probability of ½.

This simple *model system* shows in principle that:

- *(1*)* is *not a necessary condition for time reversal symmetry*

since *(1*)* fails for this reversible theory. By contrast, (1) is consistent with it.

A full proof of course requires a proof that *(1) is both a necessary and sufficient condition for time symmetry;* and a proof that *(1*) is neither a necessary nor sufficient condition for time symmetry.* Here we will give a simple derivation of *(1),* and then another counter-example showing (1*) is not sufficient.

The essential problem is to determine the *time reversed image* of a predictive probability law:

- *L (predictive law):* $prob(s_2(t+\Delta t)|s_1(t)) = p$

The normal procedure is to apply the transformation $T: t \rightarrow -t$ to all the terms in the expression, including taking time reversal of all the states. Doing this immediately gives: $prob(Ts_2(-t-\Delta t)|Ts_1(-t))=p$. And since the term *t* is universally quantified, this is equivalent to:

- *TL (retrodictive law):* $prob(Ts_2(t-\Delta t)|Ts_1(t)) = p$

Notice that in the time reversal *the time order of the reversed states* is the opposite of the time order of the initial states. However this is achieved by reversing the times.

The order of the *states in the conditional probability expression* itself is not reversed (as it is in the orthodox analysis – an error in apply the time reversal transformation).

Now if a theory entails the first law, *L*, to be time symmetric it must also entail the time reversal of that law. And *vice versa*. Thus the condition for reversibility is:

$$prob(s_2(t+\Delta t)|s_1(t)) = prob(Ts_2(t-\Delta t)|Ts_1(t))$$

This is the criterion (1).

If there is still a doubt about this, we can decompose the conditional probabilities in more detail as absolute probabilities, using Bayes theorem:

$$prob(s_2(t+\Delta t)|s_1(t)) =_{df} \frac{prob(s_2(t+\Delta t)\ \&\ s_1(t))}{prob(s_1(t))}$$

and repeat the exercise of substituting terms with their time reversals; with same result. Or we can look at it in terms of statistical ensembles, again with the same result. It can be quickly verified that *the time reversal of any ensemble conforming to the predictive law must conform to the retrodictive law*. Let us do this with an example to illustrate.

Counter-example 2. An irreversible theory that satisfies (1*) and fails (1).

Consider the following fragment of a theory *which is defined to satisfy (1*)*, i.e. in this theory: $p(s_2(t)|s_1(t-\Delta t))= p(Ts_1(t)|Ts_2(t-\Delta t))$ for all states. All transition probabilities involving a state *B* or *TB* are shown. All transition probabilities are just ½, and the theory looks nicely symmetric. (Since *(1*)* is satisfied, this could match the quantum transitions for some simple quantum system.)

Transition Probabilities:	Inverse Transition Probabilities (1*):
$p(D(t+\Delta t)|B(t)) = ½$	$p(TD(t+\Delta t)|TB(t)) = ½$
$p(E(t+\Delta t)|B(t)) = ½$	$p(TE(t+\Delta t)|TB(t)) = ½$
$p(B(t+\Delta t)|A(t)) = ½$	$p(TA(t+\Delta t)|TB(t)) = ½$
$p(B(t+\Delta t)|Z(t)) = ½$	$p(TZ(t+\Delta t)|TB(t)) = ½$

(The probabilities in blue are tested in the hypothetical sample below.) These are all states relevant to *B*. There are of course further transitions between other states, e.g.

$p(C(t+\Delta t)|A(t)) = ½$ $\qquad$ $p(TA(t+\Delta t)|TC(t)) = ½$

But we need only be concerned with transitions involving *B* or *TB.*

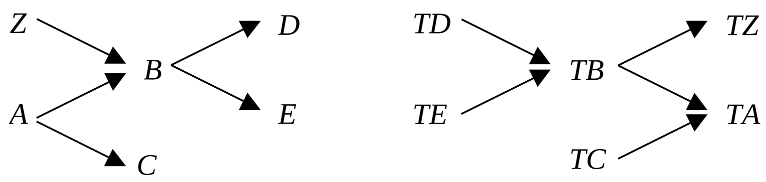


Figure 3. All the possible transitions from *B* or *TB*, and to *B* or *TB* are illustrated (and one extra: $A \rightarrow C$). All arrows represent probabilities of ½.

To test these laws, we now take a system that we can observe in our environment, observe it over a period of time, and *select all occurrences of processes that go into either state B or TB.* We record the previous and subsequent states. We then analyze the frequencies of state transitions to verify the probability laws above.

Let us suppose that we empirically observe the following sample: does this confirm or disconfirm the probability laws?

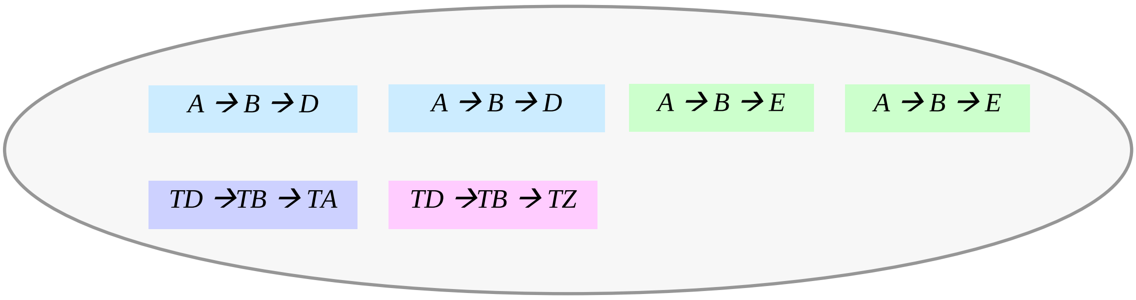


Figure 4. A sample S of transitions consistent with the probability laws. Different transition sequences are in different colours.

We can count the frequencies in our sample of states *B* and *TB* and their transitions.

$freq(B) = 4$

$freq(B \rightarrow D) = 2$ $\qquad freq(B \rightarrow D)/ freq(B) = ½$

$freq(B \rightarrow E) = 2$ $\qquad freq(B \rightarrow E)/ freq(B) = ½$

$freq(TB) = 2$

$freq(TB \rightarrow TA) = 1$ $\qquad freq(TB \rightarrow TA)/ freq(TB) = ½$

$freq(TB \rightarrow TZ) = 1$ $\qquad freq(TB \rightarrow TZ)/ freq(TB) = ½$

This sample is obviously consistent with the probability laws. So we may imagine it is typical of samples in a universe satisfying those laws. Of course this sample is small (for illustration), but imagine we take a much larger sample, let say with 1,000 times the numbers here, and the frequencies conform quite closely to those here (within expected statistical fluctuations). Then we would say we have good confirmation of the probability laws. Suppose our sample instead produced frequencies like: $freq(B) = 4{,}000$, $freq(B \rightarrow D) = 3{,}000$, $freq(B \rightarrow E) = 1{,}000$. This would surely disconfirm the claim that: $p(D(t+\Delta t)|B(t)) = ½$. (It would appear to confirm: $p(D(t+\Delta t)|B(t)) \approx ¾$ instead).

To confirm the probabilities are *general laws of nature,* we would also need to confirm it many times, with different systems. If we found the probabilities broke down sometimes, we would reject the probabilities as *general laws.*

Now it is important of course that we select our sample *by choosing all cases of B and TB.* E.g. if we '*post-selected' it* by only allowing cases with transitions like: $B \rightarrow D$, for example, then of course it is not valid to test the laws: $p(D(t)|B(t-\Delta t)) = ½$ and $p(D(t)|B(t-\Delta t)) = ½$, because *we have excluded most cases of:* $A \rightarrow C$ through our selection criterion.

But we have not *post-selected or pre-selected the sample by deliberately choosing either special states that the B states transition into, or special states the B states have transitioned from.* We have included *all cases of B or TB that occur, independently of the pre-states or post-states.* We have allowed *nature* to select the transitions to and from *B* and *TB.* The selection criterion is unbiased and time symmetric w.r.t. *B* and *TB.* If appropriate frequencies failed to occur for states following *B*, we would dismiss the probabilities as laws. Indeed, how can we even conceive testing probabilistic laws (like those of quantum mechanics or thermodynamics), except by such experiments?

We now consider the *time reversal* of the sample. This is as follows:

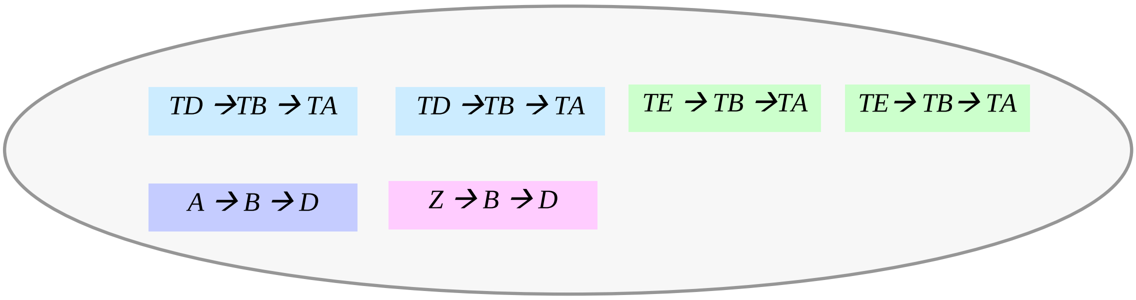


Figure 5. Sample TS is the time reversal of sample S.
Time reversed images of original sequences are shown in the same colors.

$freq(B) = 2$

$freq(B \rightarrow D) = 2$	$freq(B \rightarrow D)/ freq(B) = 1$
$freq(B \rightarrow E) = 0$	$freq(B \rightarrow E)/ freq(B) = 0$

$freq(TB) = 4$

$freq(TB \rightarrow TA) = 4$	$freq(TB \rightarrow TA)/ freq(TB) = 1$
$freq(TB \rightarrow TZ) = 0$	$freq(TB \rightarrow TZ)/ freq(TB) = 0$

Yikes! This *time reversed* sample does not reflect the original probability laws at all! E.g. it would indicate that: $p(TA(t+\Delta t)|TB(t)) = 1$. The laws are *not time symmetric.* If they were time symmetric then *the time reversal of any process satisfying those laws must also satisfy those laws.*

Since those laws *do satisfy the condition (1*),* we see that:

- *The condition (1*) does not ensure time reversibility. It is not a sufficient condition.*

By such means, it can be confirmed that *the orthodox criterion (1*) is wrong.* The probability: $prob(Ts_1(t+\Delta t)|Ts_2(t))$ *is logically independent of the time reversal of* $prob(s_2(t+\Delta t)|s_1(t))$. (1*) is not an 'alternative choice of criterion for time reversal' or an 'alternative concept of time reversal'. (1*) is simply a false criterion: *it has nothing to do with time reversal symmetry.* In fact it is not even a *symmetry* in the proper sense*:* for it is not based on any fundamental transformation.

We now show why quantum mechanics fails the criterion for time reversal (1).

Proof that QM fails time reversibility.

Watanabe also demonstrated that past-directed probabilities required by (1) for time reversibility of quantum mechanics or thermodynamics are not law-like in nature. The simplest proof is as follows: it shows that (1) combined with (1*) entails thermodynamic equilibrium. Since our universe is far from equilibrium, this is *empirically contradicted in nature,* and (1) must be false.

Proof. The condition (1*) actually holds of quantum mechanics. If (1) also held we could immediately infer (equating the RH Sides of (1) and (1*)):

$$prob(Ts_1(t+\Delta t)|Ts_2(t)) = prob(Ts_2(t-\Delta t)|Ts_1(t))$$

Since s_1 *and* s_2 range over all states, including reversed states, this is equivalent to:

$$prob(s_1(t+\Delta t)|s_2(t)) = prob(s_2(t-\Delta t)|s_1(t))$$

Because this holds for all *t*, this is equivalent to:

$$prob(s_1(t+\Delta t)|s_2(t)) = prob(s_2(t)|s_1(t+\Delta t))$$

Now expanding these as conditional probabilities:

$$\frac{prob(s_1(t+\Delta t)\ \&\ s_2(t))}{prob(s_2(t))} = \frac{prob(s_2(t)\ \&\ s_1(t+\Delta t))}{prob(s_1(t+\Delta t))}$$

Then assuming we are dealing with non-zero probabilities, this entails:

$$prob(s_2(t)) = prob(s_1(t+\Delta t))$$

Since this holds for all states, we have:

$$prob(s_1(t)) = prob(s_1(t+\Delta t))$$

and hence:

$$prob(s_1(t)) = prob(s_2(t))$$

This means that *the probability of all states is equal at all times.* But this is the condition for thermodynamic equilibrium.

Hence (1*) and (1) together imply that *all systems are expected to be in thermodynamic equilibrium.* But it is an empirical fact that most systems are not in equilibrium: far from it. This is very strong empirical evidence against the conjunction: (1*) & (1). Since the condition (1*) is true, the condition (1) is false.

Now this derivation may seem a little abstract, but we can understand it more intuitively if we consider what a closed non-equilibrium system means for *past-directed probabilities.* Such a system evolves towards equilibrium in the future – by the laws of thermodynamics. This is ensured in quantum mechanics by the law-like probabilities of state transitions towards the future 'randomizing' mixtures of states.[6] But a non-equilibrium system relaxing to equilibrium towards the future will be seen to evolve *away from equilibrium* towards the past. This must contradict the possibility

[6] While classical thermodynamics relies on an assumption of a randomized mixtures of states, and it is logically possible for entropy to decrease from special initial conditions, this is only because it has a fully deterministic micro-theory. In quantum mechanics, with an intrinsically probabilistic micro-theory, entropy increase is ensured by law.

of past-directed law-like probabilities identical to the future directed laws. Such probabilities simply do not exist in real physics.

As Watanabe emphasized, this equally reflects the fact that *causation in the real world is temporally asymmetric.* Causation only works in a law-like manner *in one direction*: from the present towards the future, not from the present towards the past. The failure of quantum mechanics to satisfy (1) corresponds to the temporal directedness of causation, and this allows our ability to deliberately control initial conditions, which allows us to set up experiments that explicitly contradict any past directed probability laws. By contrast, we cannot set up experiments to contradict future directed probability laws of quantum mechanics (as least, not as far as most physicist know[7]) – if we could, quantum mechanics would be proved wrong. It is not ‘merely human manipulation’ that allows us to experimentally contradict past-directed probability laws. It is *the absence of past-directed probability laws in nature* that allows ‘human manipulation’ to be possible in the first place.

The popular fallacy of reversibility.

The error of calling quantum mechanics ‘reversible’ or ‘time symmetric’ because it has the symmetry (1*) has led to a vast confusion in the scientific-philosophical literature on time. The popular fallacy, repeated endlessly in popular lectures as well academic articles, is that “fundamental physics is reversible, and hence the time reversal of our whole universe and all its processes is equally consistent with the laws of nature as the actual universe. If we could reverse the entire present state of the universe (perfectly), it would all run backwards. All the normal processes we see running forwards (eggs breaking, rivers running downhill, etc) would undo themselves: eggs would mend, rivers would run uphill, etc.”

Now this was first envisaged in classical physics, with its *purely deterministic reversible laws.* In this context it is justifiable. But in quantum mechanics with its *intrinsically probabilistic laws* it is a fallacy.

[7] Experiments claiming quantum probabilities can be influenced by mental intentions would contradict this – but these are rejected by most physicists.

- If we took the exact time reversal of the present state of the world, and evolved it according to quantum mechanics (and other laws, e.g. GTR), it would *not evolve backwards through its original process.*

The basic reason it can work in classical physics is because the theory is *fully deterministic,* and if we took the *exact state reversal of the classical universe,* by definition it is precisely tuned to run backwards. But the reversed state has to be essentially *perfect.* If we made the tiniest perturbations in the reversed state, e.g. giving a single atom in each star a slightly different momentum, these would rapidly magnify, and thermodynamic chaos would ensue. The time reversed state *would return to thermodynamic chaos,* and after a brief period of entropy reduction (or processes running backwards), it would go back into a process of entropy increase.

However in quantum mechanics, *such perturbations are inevitable and pervasive.* Quantum thermodynamics is intrinsically irreversible (like quantum mechanics). *Processes cannot be made to go backwards in quantum mechanics.* The expected behavior is illustrated in Figures 6-8.

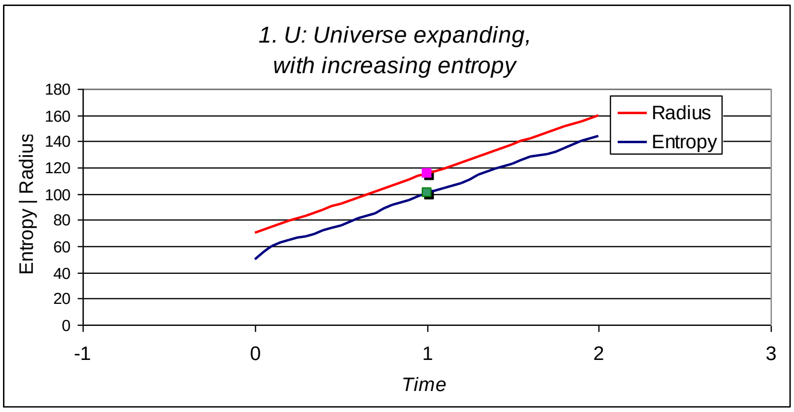


Figure 6. Universe between two times, with expanding radius and increasing entropy – like our own. This may be classical quantum mechanical.

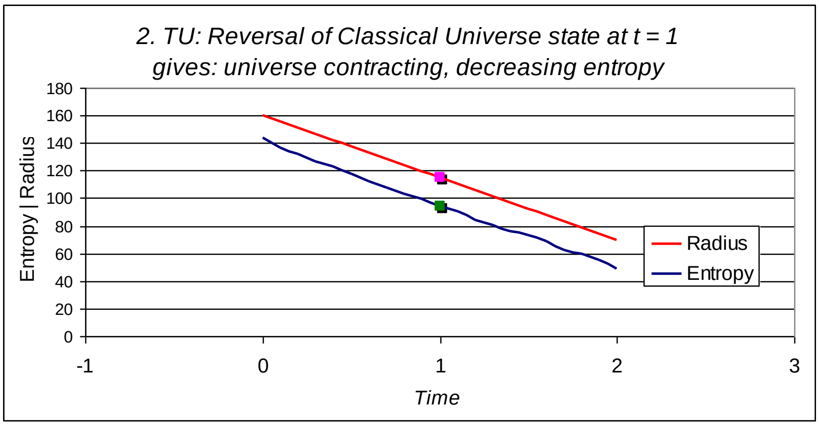


Figure 7. Classical universe, generated by full micro-state reversal of U at t =1. Past and future are projected from the reversed state at t = 1, using laws of classical physics. This is the exact time reversal of original universe U.

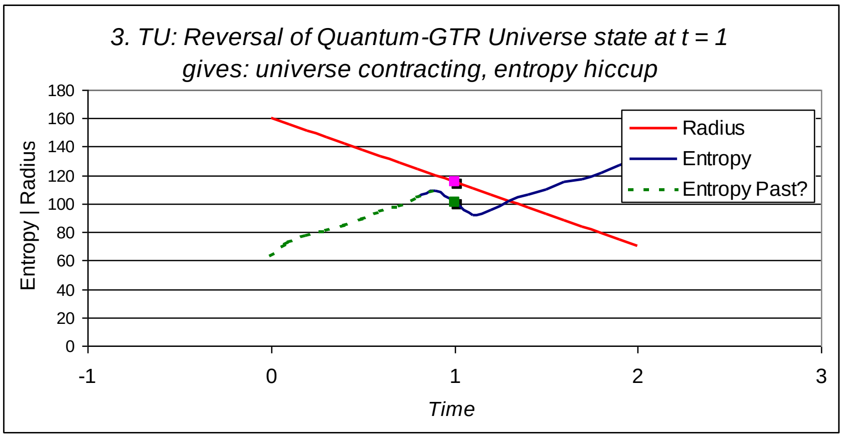


Figure 8. Quantum universe, generated by full micro-state reversal of U at t =1. Past and future are projected from the reversed state at t = 1, using laws of quantum mechanics. This is very different to the time reversal of original universe U.

Figures 6-8 illustrate the *reversibility of classical dynamics* and the *irreversibility of quantum dynamics.* We assume first a universe *U* with smoothly expanding radius and (slightly wobbly) increasing entropy, as considered typical of our own cosmological period. We then take *the time reversal of the exact micro-state at* $t = 1$, and use it to define another universe, *TU*. We project the past and future of *TU* using the laws of physics acting on the reversed micro-state at $t = 1$. However we get two radically different results, depending on whether we assume *reversible deterministic (classical) laws,* or *probabilistic (quantum) laws.*

- Figure 7. In the classical context, the reversed universe we produce is the exact time reversal of *U. It has contracting radius and decreasing entropy.*
- Figure 8. But in the quantum context, the reversed universe we produce is very different to the time reversal of *U. It has contracting radius* (assuming laws governing cosmic expansion are reversible). *But the entropy only decreases in the future for a short period.* Chaos soon reasserts, and entropy increases again, just as in real life physics.
- The projection from the reversed quantum state at $t = 1$ *to the past* is highly problematic – quantum mechanics does not tell us how to make any such projection! If we make the usual 'past hypothesis', that the highly ordered system at $t = 1$ is the causal product of the past, then we could (possibly) project decreasing entropy in the past. However there is no clear answer to what should be retrodicted about the past. The fact is the universe with the 'entropy hiccup' is so peculiar that it appears to break all the laws of physics! So indeed, the time reversal of the *state* of our universe is hardly a physically possible state.

Reversing the entire state in the quantum context (or indeed *any context where some processes are intrinsically probabilistic)* only produces a kind of 'hiccup' in the entropy climb. The reversed system 'runs backwards' for a short time – reducing entropy momentarily – but then processes reassert their normal directions again. E.g. a reversed river will flow *uphill* for a short period – but chaos ensues, the *perfectly arranged conditions needed for reverse flow to continue* quickly breaks down, and very soon the river will start flowing back downhill. This would apply to small processes as well – if we could actually produce the time reversal of a small, closed quantum process that exhibits thermodynamic behavior, we should find this only gives a brief 'entropy hiccup', and entropy does not continue to fall for long. (We will revisit this in more detail in Part 4.)

The illustrations we have seen also illustrate different but equivalent ways of analyzing reversibility.

- One is to take the time reversal of a *theory*, say *TL*, and compare it to the original theory, *L*. If they are equivalent, i.e. $L = TL$, then *L* is reversible. (Invariant under time reversal). If $L \neq TL$, then *L* is irreversible. We saw this is the earlier derivation of (1).
- Another is to take a solution to *L* (a process, call it *U*), obtain its time reversal, *TU*, and then see if *TU* obeys *L*. If so for every *U*, then *L* is reversible. If not, *L* is irreversible. We saw this in the second counter-example above. We need only one such counter example to show *L* is irreversible.
- Another, illustrated above, is to take a specific instantaneous state of an *L* process, *U*, take the state reversal, and use *L* to project the past and future states. If this produces a new process, *TU* that statistically conforms to the original *U*, then *L* *may be reversible.* But if it produces a process that is radically incompatible with *TU*, then *L* is irreversible. This is what Figure 8 illustrates.

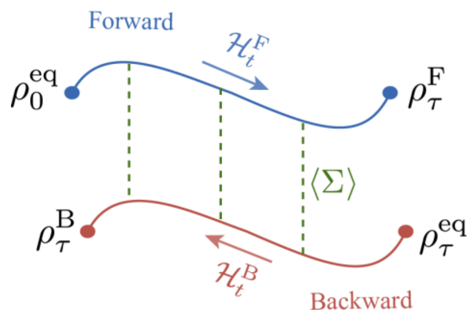
Experimental confirmation of irreversibility of quantum thermodynamics.

In the last ten years, there has been a renewed interest in the physics of irreversibility by research physicists. But the subject of time reversal in *quantum thermodynamics* has still been theoretical, not experimental – until the last couple of years. This is because it is so difficult to experimentally produce the time reversal of any real quantum system. However in 2015 a team from Brazil, Austria and Ireland confirmed the *irreversibility of quantum thermodynamics* by direct experiment (Batalhão *et aila*, 2015). This is the first time such an experiment has been done. They have actually produced the time reversal of a thermodynamic process, and measured the entropy changes of both the forwards and reversed processes. To understand their results, they are trying to experimentally confirm a key equation:

“$<\Sigma> = S(\rho^F{}_t||\rho^B{}_{t-\tau})$ *(1)*

“The above equation quantifies irreversibility at the microscopic quantum level and for the most general dynamical process responsible for the evolution of a driven closed system. A process is thus reversible … if forward and backward microscopic dynamics are indistinguishable. (Batalhão *et aila*, 2015, p.1).

I.e. $<\Sigma> = 0$ implies *reversibility*. They confirm instead $<\Sigma> \neq 0$. In fact this is the result expected theoretically: that quantum thermodynamics is *irreversible!* I reproduce the following diagram from their paper:



“Figure 1. A quantum system … is initially prepared in a thermal state … at inverse temperature. It is driven by a fast quench into the non-equilibrium state along a forward protocol described by the Hamiltonian $H^F{}_\tau$ ... In the backward process, the system starts in the equilibrium state corresponding to the final Hamiltonian … and is driven by the time-reversed Hamiltonian. The entropy production *(Σ)* at time *t* is given by the Kullback-Leibler divergence between forward and backward states $(\rho^F{}_t||\rho^B{}_{t-\tau})$ (Eq. 1).” (Batalhão *et aila*, 2015, p.1.)

They apparently measured entropy production for a quantum thermodynamic process, and for the *time reversed process*, forcing the *time reversed final state and time reversed Hamiltonian* on the system. They confirmed an *irreversible quantum thermodynamic process* (with: $\rho^F{}_t||\rho^B{}_{t-\tau}) \neq 0)$. They introduce the study as follows:

“The microscopic laws of classical and quantum mechanics are time symmetric, and hence reversible. However, paradoxically, macroscopic phenomena are not time-reversal invariant [1,2]. This fundamental asymmetry defines a preferred direction of time that is characterized by a mean entropy production. Regardless of the details and nature of the evolution at hand, such entropy production is bound to be positive by the second law of thermodynamics [3]. Since its introduction by Eddington in 1927 [4], the thermodynamic arrow of time has not been tested experimentally at the level of a quantum system.” (Batalhão *et aila*, 2015, p.1.)

Note their primary references to *the concepts of time reversal and time symmetry* are the orthodox accounts of [1] (Lebowitz 1993), and [2] (Zeh 1989). They are testing entropy production in a quantum system for the first time. They conclude:

> "*Conclusions*. – We have assessed the emergence of the arrow of time in a thermodynamically irreversible process by using the tools provided by the framework of non equilibrium quantum thermodynamics. We have implemented a fast quenched dynamics on an effective qubit in an NMR setting, assessing both the mean entropy produced across the process and the distance between the state of the system and its reverse version, at all times of the evolution. Let us discuss the physical origin of such time-asymmetry in a closed quantum system. Using an argument put forward by Loschmidt in the classical context, its time evolution should in principle be fully reversible [1]. How can then a unitary equation, like the Schrödinger equation, lead to Eq. (1) that contains a strictly nonnegative relative entropy? The answer to this puzzling question lies in the observation that the description of physical processes requires both equations of motion and initial conditions [1, 13]. The choice of an initial thermal equilibrium state singles out a particular value of the entropy, breaks time-reversal invariance and thus leads to the arrow of time. The dynamics can only be fully reversible for a genuine equilibrium process for which the entropy production vanishes at all times." (Batalhão *et aila,* 2015, p.4.)

Now they have done a very sophisticated experiment. And I think what they have demonstrated in the laboratory for the first time is essentially the phenomenon we have seen as the 'hiccup' in entropy, and reassertion of chaos, as in Figure 8. This is what we naturally expect when we understand the *quantum mechanics is irreversible.*

But what remains unclear in their paper are their comments on the interpretation of how *irreversibility* arises. They refer to the 'paradox' that *"quantum mechanics are time symmetric, and hence reversible. However, paradoxically, macroscopic phenomena are not time-reversal invariant."* And later: *"How can then a unitary equation, like the Schrödinger equation, lead to Eq. (1) that contains a strictly nonnegative relative entropy?"* They are bothered by this, and they defer *"the answer to this puzzling question"* to the orthodox account, saying it *"lies in the observation that the description of physical processes requires both equations of motion and initial conditions [1, 13]."* And then they make the statement that: *"The choice of an initial thermal equilibrium state singles out a particular value of the entropy, breaks time-reversal invariance and thus leads to the arrow of time. The dynamics can only* *be fully reversible for a genuine equilibrium process for which the entropy production vanishes at all times."*

Now this is not very clear, and from our point of view, it is wrong. Their opening assumption that: *"The microscopic laws of classical and quantum mechanics are time symmetric, and hence reversible"* is false. Quantum mechanics is not "time symmetric or reversible". The 'paradox' they refer to is a real one: *if quantum mechanics was reversible*, the irreversible processes they have experimentally demonstrated would not be possible! Their explanation that it is *"The choice of … an initial state"* that *"breaks time-reversal invariance and thus leads to the arrow of time"* is simply a repetition of the fuzzy orthodox view, and it is wrong. If quantum mechanics really has a given symmetry property (reversibility) then nothing humans choose to do could contradict this property!

They are correct that "*The dynamics can only be fully reversible for a genuine equilibrium process for which the entropy production vanishes at all times."* This is exactly what we found above. But since they demonstrate an *irreversible process,* we must conclude that *quantum mechanics is irreversible.* A *general theory like quantum mechanics* cannot be 'reversible' sometimes (when applied to an equilibrium system) and then 'irreversible' at other times. It is simply *irreversible.*

In particular, it is the vague appeal to a distinction between 'reversibility of laws' and 'irreversibility of systems defined by special initial conditions' that 'breaks reversibility', repeated by all the orthodox theorists, that is at fault. This is wrong. This distinction only gives an illusion of resolving the 'paradox'. They rely on Lebowitz (1993) and Zeh (1989) for their understanding of this, but both those authors give typical statements of the orthodox account, and in particular, neither is aware of the crucial difference between (1*) and (1) that we have discussed in detail here. The real resolution is that the 'paradox' reflects that quantum mechanics is irreversible. Ignorance of this simple fact is why the paradox of 'irreversibility out of reversibility' never goes away, and is revisited by generation after generation of philosophers of physics, with no progress in understanding.

This is not a significant flaw in the (Batalhão *et aila,* 2015) study. Their achievement is a major step forward in experimentation. They have confirmed the expected experimental result: *quantum thermodynamics is irreversible.* Rather, it is a deep flaw that persists in the theory of time reversal.

Common objections to the proof of (1).

Orthodox theorists in the subject of time directionality will go to extreme lengths to defend their theory. The conventional view is central to their scientific world view: a paradigm that *time is intrinsically non-directional*, like space. We now look at some of the common objections offered. These have all been given by peer reviewers as reasons to repeatedly reject publication of papers I have submitted on the subject. Most are unscientific, and obvious fallacies. There are only two real issues, at the bottom of the list, that we should reply to in detail. Because they are almost all fallacies, I have organized them in groups showing typical examples of these fallacies.

A list of typical objections.

Conceptual fallacies:

a. Criticism of (1*) has only been demonstrated with a 'toy theory' – in the real theory of quantum mechanics, (1*) is the right criterion.

b. Any 'criterion for time reversibility' must be adapted specifically to the theory in question. It must be interpreted for the real theory being analyzed. In quantum mechanics, (1*) is the appropriate criterion, and (1) is not.

c. We need to select a criterion that works for the theory in question. (1*) works for quantum mechanics. (2) shows it is a theorem in quantum mechanics. (1) does not work: it is false for quantum mechanics.

d. There is no universal interpretation of what 'time reversal' means or what its 'criterion' is., only interpretations relative to specific theories. E.g. it is well known that the special nature of quantum mechanics means 'time reversal' must be represented by the complex state operator: T^*, not by T as in most other theories. If there was a universal definition, we would have to adopt T: but it is well known that T is impossible in quantum mechanics.

Interpretational fallacies:

e. A clear operationalist interpretation of the concept of 'reversibility' must first be given to base the argument on – until this is given the argument is unfounded.

f. The concept of 'past-directed probabilities' has no meaning or application in quantum mechanics – hence (1) is inapplicable in the quantum context.

g. quantum mechanics may be given a deterministic interpretation – on this interpretation the probabilities will disappear at the fundamental level, and the dynamics will be fully reversible, just as in classical dynamics.

h. The argument is just semantics: playing with definitions. In physics, only experimental evidence for claims like time symmetry is meaningful. The author needs to specify experiments that demonstrate 'irreversibility'.

i. The problem is just semantic: it is about the conventional choice of terms to refer to the real symmetries of quantum mechanics. Physicists have chosen to call it *reversibility* or *time symmetry,* and there is no reason to change this convention.

Scientistic fallacies:

j. This question has been dealt with by many other eminent authors who all agree that quantum mechanics is reversible. The arguments given are bound to be wrong, although I am not going to waste my time analyzing the errors.

j* (They cannot be published in a reputable journal: some readers may take them seriously even though they are wrong; questioning well established scientific facts like the reversibility of quantum mechanics undermines the authority of science.)

Irrelevance fallacies:

k. Quantum mechanics only fails the condition (1) because of the asymmetry between initial and final conditions – which is caused by the merely *de facto* thermodynamic asymmetry of the larger universe – not by irreversibility of the laws of quantum mechanics.

l. The failure of condition (1) is caused by 'measurement' or 'wave function collapse' - which depends on interference with the system by an outside 'classical' measuring system. If you reversed the measurement process completely, this measurement process must also be reversible.

Relevant objections:

m. quantum mechanics does not give conditional probabilities at all: it gives transition probabilities between states, conditional on measurements being made. (1) is inapplicable.

n. The empirical evidence for the failure of depends on irreversible systems, but these are already *pre-selected* with low entropy, and this is why the condition (1) fails. For an unbiased test we must define samples that are both *pre-selected and post-selected* in a time symmetric way. Such samples or ensembles are governed by the theory of *time symmetric quantum mechanics,* defined by (Aharonov, *et alia,* 1964*;* Cocke, 1967*).* This is called the *ABL theory.* It is fully time symmetric. This refuted the arguments raised by Watanabe, which were essentially the same as raised here.

Reply to objections.

Given such a host of objections, it may seem there are a lot of problems with the proofs given above! But almost all these are fallacies, and some are no more than facetious. However anyone who has challenged an established *scientific-philosophical paradigm* will understand that you need to face up to repeated attacks, and reply to many mistaken and facetious objections, if you wish to be heard. So here are replies. Many hardly deserve serious reply from a scientific point of view; but gate keepers in the subject readily accept them as reasons to reject publication of unwelcome views.[8]

a. It is not a 'toy theory', it is a mathematical model. We use models to give proofs. It has been proved that (1*) cannot be universally adequate as the criterion for reversibility. But (1*) has been assumed as the universal criterion for reversibility, without any proof. What is wrong with the proofs?

b. (1*) is proved to be generally inadequate for any theory defined by a class of conditional probability laws. (1) is proved to be definitely wrong. Probability theory is common across multiple theories: its mathematical laws do not change from theory to theory. The onus is on the defense to show in detail how (1*) can be considered adequate. What is wrong with the proofs?

c. This is a basic misunderstanding of concepts. If we can only select a criterion that is satisfied by the theory in question, then reversibility becomes analytic. But the problem is not to select a criterion that 'works' for the theory. The problem is to define the correct criterion to represent the symmetry, and then check if the theory has that property. What is wrong with the proofs?

[8] Interestingly though, once Holster (2003) was accepted in a serious physics journal, no objection to it has been published as far as I am aware. The orthodox analysts have made no reference to it, despite continuing to try to find ways around it – in new programs dedicated to finding 'alternative definitions of time symmetry or reversibility' that will allow them to maintain their views – without acknowledging why they need to find 'new definitions of reversibility' in the first place. The impossibility of doing this will be shown in Part 3, when we return to fundamental concepts.

d. There is a universal definition of what ‘reversibility’ or ‘time symmetry’ means: invariance under the transformation: T: t → -t. If there is no such definition then we do not know what we are talking about. The use of T* rather than T is in fact another error, analyzed in Part 2 – the error of adapting the definition of a concept to get the result we want. The definition of time symmetry is discussed in Part 3. What is wrong with the proofs?

e. The proofs given are independent of any ‘operationalist definitions’. They need to be addressed in their own terms. What is their specific error? The demand for an ‘operationalist definition’ makes the arrogant assumption that everyone must agree with a positivist theory of meaning - there is no need to give any such theory of semantics. What is wrong with the proofs?

f. The concept of ‘past-directed conditional probabilities’ has a clear meaning. It implies the statistics will match the conditional probabilities. The fact that such probabilities are not entailed by quantum mechanics, and do not match the real world, means that quantum mechanics fails (1). What is wrong with the proofs?

g. A deterministic interpretation is not quantum mechanics as we know it. It must postulate a more fine-grained set of micro-properties than represented by the standard quantum mechanics. (Most quantum physicists doubt it is possible to find a deterministic interpretation anyway; but this is not the point.) The standard theory of quantum mechanics is defined as a probabilistic theory, with irreducible probabilities. This is what the error is about: whether the standard theory of quantum mechanics is reversible. It is not whether some other hypothetical theory is reversible. What is wrong with the proofs?

h. The objection is just playing with semantics. Time symmetry is an analytic property of theories. The orthodox analysis does not specify experiments to demonstrate quantum mechanics is reversible. It specifies theorems. Physicists do experiments to demonstrate quantum mechanics is true, not to demonstrate its mathematical structure. If the argument is ‘playing with definitions’, what definitions are mistaken? For a long time there were no experiments demonstrating quantum irreversibility, but now experiments have been done, this changes nothing about the property of the theory. What is wrong with the proofs?

i. The objection is just playing with semantics. The question is about the fact of whether quantum mechanics is reversible or not. The orthodox analysis has got the facts wrong. It is unacceptable to change the meanings of words like ‘time symmetry’ to conceal mistakes in analysis. What is wrong with the proofs?

j. The ‘eminent authors’ do not consider the problem raised here: they assume (1*) is true without proof. Appeal to authority does not settle scientific questions. What is wrong with the proofs?

k. The objection is proposing an explanation for irreversibility of processes, but that is a different question. Does the objection accept (1) is the criterion for time symmetry? If not, no objection to (1) has been given. If so, the question is: does quantum mechanics satisfy the criterion (1) for time symmetry? If not, then it is not time symmetric. Speculation about the reasons it fails time symmetry is not relevant to the fact that it fails time symmetry. We may add that the assumption that the thermodynamic directionality of the larger universe is ‘merely de facto’ is made only because it has been assumed quantum mechanics is reversible. What is wrong with the proofs?

l. Again, the question is: does quantum mechanics satisfy the criterion for time symmetry? If not, then it is not time symmetric. If this is a criticism of the criterion we have adopted, then what is wrong with it? Views about ‘quantum measurement’ are many and various: the definition of quantum probabilities is fixed by the Born postulate. What is wrong with the proofs?

m. This is the first serious objection: it actually gives a relevant argument, not against (1), but against the application of (1) to quantum mechanics. However it is weak. First, what is the proof that quantum mechanics fails to give conditional probabilities? Do not ‘transition probabilities’ entail conditional probabilities? In any case, it is immediately open to a similar objection: time symmetry then require past-directed ‘transition probabilities’, not future-directed transition probabilities as claimed by (1*). So (1*) is still wrong. And quantum mechanics in fact gives no past-directed transition probabilities in symmetry with the future directed transition probabilities, for exactly the same reason it fails (1). The fact remains that (1*) is false and (1) is correct.

n. This is the critical objection, and a source of real controversy. Note first that this does not object to (1) or attempt to defend (1*). Rather, the ‘ABL theory’ is symmetric in the sense of (1). The problem is that this is not the real theory of quantum mechanics. It only applies to ensembles that represent equilibrium statistics, or time symmetric statistics, *by definition.* Does it apply to the real world? No. the real world is not time symmetric. So the immediate reply to this objection is that *it does not represent an objection to the arguments that real quantum statistics contradicts time symmetry. The time symmetry of ABL theory is irrelevant.*

However the most interesting issue is the argument that ‘real world’ quantum experiments are temporally biased because they depends on *pre-selection,* but not *post-selection* in an asymmetric fashion. This is ultimately an issue about what constitutes a fair or unbiased sample to test a probabilistic theory. This is why *Counter-Example 2* above was given in detail. Does the statistical sampling principle represented in this model represent fair sampling to test a set of probability laws? If so, then I think the case that quantum mechanics is asymmetric is sound. The present objection must hold that it is *not a fair sampling procedure.* If such a fundamental principle is in doubt, this goes to the heart of the scientific method itself. Given that there is disagreement about this at all shows that this needs further discussion.

It should be finally mentioned that many writers have claimed that quantum mechanics is time symmetric or reversible, *but then noted an exception: that the quantum measurement process is irreversible.* The reason usually given however is not by identifying the irreversibility of the probability theory, but by observing that ‘quantum measurement’ is a concept of an ‘irreversible’ process in its own right; a reduction of the wave function by ‘external observation’; and so on. However this is then subsequently ignored, as something to do with the peculiar role of ‘observation’ in quantum mechanics, and not reflecting ‘irreversible dynamics’. Thus it has always been left in a peculiarly uncertain subjective status (like all questions about ‘measurement’ or ‘wave function collapse’ in the orthodox Copenhagen interpretation.) However we have identified it explicitly as giving rise to irreversible probabilistic dynamics. It does not matter what the theory of ‘measurement’ or ‘wave function collapse’ is from this point of view: the probabilistic dynamics is irreversible.

Conclusion.

There is no real doubt that (1) is correct and (1*) is wrong as the criterion for time symmetry of probabilistic theories, defined by classes of conditional probability laws. Neither is there much doubt that *ordinary quantum mechanics as we know it* fails (1) and is time asymmetric. Nor is there much doubt that this gives rise to irreversible processes. This severely undermines the orthodox view that time is ‘intrinsically symmetric’ in physics. However this is just the start of the problems. We will examine further issues in subsequent parts. It should be stressed that it is not just the *irreversibility* of ordinary quantum mechanics that is key to our view of time in nature; it is the *status and interpretation* of quantum mechanics, in the wider context of physics, including cosmology. Is quantum mechanics ultimately fundamental? Is physics fundamentally probabilistic, or is there a deterministic level underlying it? These are the real questions that must be answered before we can draw inferences from the *theory of quantum mechanics* to the nature of the real world.

References. (Contains references for Part 1 and 2.)

Aharonov, Y., Bergmann, P., and Lebowitz J. (1964) “Time asymmetry in quantum process of measurement.” *Phys.Rev.* B **134** 1410–8.

Albert, David Z. *Time and Chance.* (2000) Cambridge, Mass. Harvard University Press.

Batalhão, T. B., A. M. Souza, R. S. Sarthour, I. S. Oliveira, M. Paternostro, E. Lutz, and R. M. Serra. 2015. “Irreversibility and the arrow of time in a quenched quantum system.” arXiv:1502.06704v2

de Beauregard, Olivia Costa. (1980) "CPT Invariance and Interpretation of Quantum Mechanics". *Found.Phys.* **10** 7/8, pp. 513-531.

van Benthem, Johan and ter Meulen, Alice. (1997) *A Handbook of Logic and Language.* Cambridge, Massachusetts: The MIT Press.

Callender, C. (2000) “Is Time Handed in a Quantum World?”. *Proc.Arist.Soc*, **121** pp 247-269.

Cocke, W.J. (1967). “Statistical time symmetry and two-time boundary conditions in physics and cosmology.” *Phys.Rev.* **160** 1165–70.

Davies, P.C.W. (1974.) *The Physics of Time Asymmetry.* Surrey University Press.

Gasiororowicz, S. (1966) *Elementary Particle Physics.* New York: John Wiley and

Sons.
Healey, R. (1981) “Statistical Theories, Quantum Mechanics and the Directedness of Time”, pp.99-127 In *Reduction, Time and Reality,* ed. R. Healey. Cambridge: Cambridge University Press. 1981.

Holster, A.T. (2003) “The criterion for time symmetry of probabilistic theories and the reversibility of quantum mechanics”. *New Journal of Physics* (www.njp.org), Oct. 2003. http://stacks.iop.org/1367-2630/5/130.

Holster, A.T. (2003/2017) “The quantum mechanical time reversal operator”. http://vixra.org/abs/1712.0596.

Jauch, J.M., and Rohrlich, F. (1955) *The Theory of Positrons and Electrons.* New York: Addison-Wesley. pp 88-96. [10] Penrose, R. (1989) *The Emperor's New Mind.* Oxford: Oxford University Press.

Lebowitz, J. L. (1993). “Boltzmann’s Entropy and Time’s Arrow”. Physics Today 46 (9), 32-38 (1993).

Messiah, A. *Quantum Mechanics.* (1966) New York: John Wiley and Sons.

Racah, (1937) *Nuovo Cim.* **14**.

Reichenbach, H. (1956) *The Direction of Time.* Berkeley: University of California Press.

Sachs, Robert G. (1987) *The Physics of Time Reversal.* University of Chicago Press.

Schrodinger, E. (1950) "Irreversibility". *Proc.Royal Irish Academy* **53** pp 189-195.

Sklarr, L. *Space, Time, and Spacetime.* (1974) University of California Press.

Watanabe, Satosi. (1955) "Symmetry of Physical Laws. Part 3. Prediction and Retrodiction." *Rev.Mod.Phys.* **27** (1) pp 179-186.

Watanabe, Satosi. (1965) "Conditional Probability in Physics". *Suppl.Prog.Theor.Phys. (Kyoto) Extra Number* pp 135-167.

Watanabe, Satosi. (1966) "Time and the probabilistic view of the world”. In J.T. Frazer (ed) *The Voices of Time,* 1966. 527-564.

Xiaochun, Mei. (2010). “The Current C, T Transformation Rules of Quantum Field Theory Must [be] Redefin[ed]” https://arxiv.org/ftp/arxiv/papers/1003/1003.2666.pdf

Zeh, H.D. (1989) *The Physical Basis of the Direction of Time.* Berlin: Springer-Verlag.