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Formal Theodicy: Religious Determinism and the Logical Problem of Evil

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Introduction

The use of logical tools in the investigation of religious discourse can be traced back as far as the ancient times, but was particularly common in specific periods of some religious traditions. Several authors, for example Thomas Aquinas, endeavoured to use in their texts the rigour and precision that the available logical tools provided. As heirs of such a tradition, Józef Maria Bocheński, Jan Salamucha, Jan Drewnowski, and Bolesław Sobociński formed the so-called Cracow Circle (1934–1944),¹ which aimed at employing the most current methods of mathematical logic in matters of the philosophy of religion and theology. Their ideas and achievements were so important that, according to Roger Pouivet, the philosophers of religion of the Cracow Circle are "the principal precursors of what is now called the analytic philosophy of religion." According to Bocheński, the Cracow Circle postulated that:

1) the language of philosophers and theologians should exhibit the same standard of clarity and precision as the language of science;

See R. Murawski, Cracow Circle and Its Philosophy of Logic and Mathematics, "Axiomathes" 2015, Vol. 25, pp. 359–376.

R. Pouivet, On the Polish Roots of the Analytic Philosophy of Religion, "European Journal for Philosophy of Religion" 2011, Vol. 3, No. 1, p. 1.

- 2) in their scholarly practice they should replace scholastic concepts with new notions now applied by logicians, semioticians, and methodologists;
- 3) they should not shun the occasional use of symbolic language.³

In this sense, philosophical and theological doctrines can benefit greatly from employing formalization. This logical approach introduces exactness, clarity, and precision in concepts and arguments and makes it possible to eliminate several kinds of ambiguities.

Although there have been some initiatives in the second half of the past century involving this way of doing philosophy of religion and theology and an increased interest in the study of the relationship between logic and religion, as well as in the use of logical tools by analytic philosophers of religion, the goals outlined by the Cracow Circle are far from achieved. The construction of formalized systems in the philosophy of religion is an approach not as common as it could be. The belief that such an enterprise has several advantages in solving philosophical problems has propelled the authors of this paper into the investigation of a famous issue in the philosophy of religion. The approach used here can be considered a case of logic of religion, as proposed by Bocheński; particularly, it is a case of logic applied to theodicy. In what follows, we delineate this problem.

1. Systems N1, N2, and the Logical Problem of Evil

The problem of evil is one of the most prominent issues in the history of philosophy. Though many answers have been provided since ancient times, many authors still maintain that it is a challenge to belief in God. One can approach the problem in many ways; however, in the contemporary debate, there are at least two strands of this problem. Some philosophers have argued that the existence of God is very improbable, given the amount and variety of evil in the world; this proposal has been called the *evidential problem of evil.*⁵ Others claim that the propositions "God exists" and "evil exists" are mutually inconsistent, and thus theists cannot be rational. John Mackie is one such philosopher; as he says,

J.M. Bocheński, *The Cracow Circle*, in: *The Vienna Circle and the Lvov–Warsaw School*, ed. K. Szaniawski, Dordrecht 1989, pp. 9–18.

On the logic of religion, see Bocheński's seminal work, *The Logic of Religion*, New York, NY 1965.

⁵ For a comprehensive overview on this issue, cf. D. Howard-Snyder, ed., *The Evidential Argument from Evil*, Indianapolis, IN 1996.

[A] more telling criticism can be made by way of the traditional problem of evil. Here it can be shown, not that religious beliefs lack rational support, but that they are positively irrational, that the several parts of the essential theological doctrine are inconsistent with one another.⁶

The affirmation that belief in God is contradictory with the existence of evil is called the *logical problem of evil*, and it is our focus here. One of the most influential responses to Mackie is the free will defence, developed by Alvin Plantinga. He develops his defence in modal metaphysics and semantics of possible worlds, to consider whether God could have created a world with less moral evil – or even no moral evil at all. Plantinga concludes that it is possible that God could not have done this; it is possible that He had a reason to permit the existence of evil in the world, and, thus, there is no contradiction between the existence of God and the existence of evil.⁷

Although the free will defence can still be considered relevant nowadays, it relies on some assumptions that remain a matter of discussion. Just to provide an example, one of its most debatable issues is the property of "transworld depravity" (TWD), a property needed to show that it is possible that God could have had a reason to allow evil in the world. However, as Richard Otte has shown, the original version of TWD is necessarily false, and Plantinga himself recognized it. The current version of the argument relies on the version of TWD given by Otte, but one might wonder whether the argument with Otte's TWD is the definitive version of a free will defence; as there is no way to *demonstrate* its logical possibility, the success of such an account depends on the success of its underlying modal metaphysics, and to the extent that natural language allows concepts to be defined with no ambiguity.

Furthermore, even if the free will defence succeeds, other proposals may provide alternative paths to tackle the question. Meanwhile, we could also conceive

⁶ J.L. Mackie, Evil and Omnipotence, "Mind" 1955, Vol. 64 (254), p. 200.

A. Plantinga, God, Freedom, and Evil, Grand Rapids, MI 1977, pp. 7–73.

R. Otte, *Transworld Depravity and Unobtainable Worlds*, "Philosophy and Phenomenological Research" 2009, Vol. 78, No. 1, pp. 165–177; A. Plantinga, *Transworld Depravity, Transworld Sanctity, & Uncooperative Essences*, "Philosophy and Phenomenological Research" 2009, Vol. 78, No. 1, pp. 178–191.

There are plenty of proposals: both defences (possible answers that invalidate an objection from evil) and theodicies (responses to the question of why God permits evil in the world), and even arguments from evil for the existence of God. See J. McBrayer, D. Howard-Snyder, *The Blackwell Companion to the Problem of Evil*, Oxford 2013, for a comprehensive sample of such approaches.

alternative approaches that have not been explored yet; for instance, instead of just using the semantics of possible worlds and modal metaphysics, why not appeal to *full-blown* formal logic? Formal logic improves precision and clarity of arguments, removes ambiguity, brings new results that could not be attained through natural language, and even preserves truth in a way that natural languages are not able to. Why not refer to such a powerful tool? As the Cracow Circle did, concerning many relevant philosophical and religious questions of their days, we also do believe that logical questions require logical answers, and the logical problem of evil is not different. In what follows, we introduce this proposal.

Edward Nieznański developed two logical systems to deal with the task of a formal theodicy.¹⁰ His aim is, in principle, to provide an answer to the problem of evil, especially in its logical form, but his systems offered a wider framework to consider questions related to religious determinism, the attributes of God, and formal axiology. One of the merits of his systems is that they characterize the logical problem of evil, a question usually stated in terms of contradiction and consistency, as a matter of logical investigation. Much of the contemporary debate on the logical problem of evil is done within a modal metaphysics framework, with no explicit formalization,¹¹ and Nieznański's approaches have the merit of dealing with the question by developing such formal systems.

Nieznański's systems have philosophical relevance, and, further, his general methodology is very inspiring as an application of formal tools to philosophical problems. However, some logical issues led us to revisit his systems, proposing a few changes. For instance, consider the following list of formulas, the original axioms of his first system. ¹² In these formulas, β stands for God, x is a vari-

E. Nieznański, Aksjomatyczne ujęcie problemu teodycei, "Roczniki Filozoficzne" 2007, Vol. 55, No. 1, pp. 201–217; E. Nieznański, Elements of Modal Theodicy, "Bulletin of the Section of Logic" 2008, Vol. 37, No. 3/4, pp. 253–264.

Plantinga's contribution is not only the most influential answer to the logical problem of evil, but it also determined the trend of dealing with this subject in the context of metaphysics of modality and possible worlds semantics. Among the works that follow Plantinga on dealing with the problem, but in natural language, are D. Howard-Snyder and J. O'Leary-Hawthorne, *Transworld Sanctity and Plantinga's Free Will Defense*, "International Journal for Philosophy of Religion" 1998, Vol. 44, No. 1, pp. 1–21; W.L. Rowe, *In Defense of "The Free Will Defense"*: *Response to Daniel Howard-Snyder and John O'Leary-Hawthorne*, "International Journal for Philosophy of Religion" 1998, Vol. 44, No. 2, pp. 115–120; R. Otte, *Transworld Depravity...*, op. cit.; and A.R. Pruss, *A Counterexample to Plantinga's Free Will Defense*, "Faith and Philosophy" 2012, Vol. 29, No. 4, pp. 400–415.

¹² E. Nieznański, Aksjomatyczne ujęcie problemu teodycei, op. cit., pp. 203–211.

able that stands for persons, p and q are variables that stand for situations, W is a symbol for "knows that," C is a symbol that means "wills that," D is a symbol for "permits that," $p \in d$ means "the situation p is good," and, finally, P stands for "to be the cause of":

A1.
$$\forall x \ (\exists p \ xWp \land \exists q \ xCq)$$

A2. $\forall p \ [\beta Cp \rightarrow \beta C(\beta Cp)]$

A3. $\forall p \ [\beta Dp \rightarrow \beta C(\beta Dp)]$

A4. $\forall p \ [\exists x \ \beta C(xCp) \rightarrow \forall x \ \beta D(xDp)]$

A5. $\sim \forall p \ (p \rightarrow p \ \varepsilon \ d)$

A6. $\forall p \ (\beta Pp \leftrightarrow \beta Cp)$

When we examine these axioms, from the point of view of mainstream logic, some questions naturally arise: what is the underlying logical system? What is the precise meaning of the symbol ε ? Do the symbols W, C, D and P represent functions, predicates or modal operators? And, finally, how to justify the use of variables in formulas such as those in A5? These questions are surely relevant, but there is not much discussion about the specific logical structure in Nieznański's article. A more robust account is provided in his second system, a modal account, but some of the questions listed above can also be applied to this second approach.

Nevertheless, we maintain that Nieznański's insights are philosophically penetrating, ¹⁴ and his general methodology of formalizing concepts is inventive and very inspiring. For this reason, these issues, among others, led us to work on a detailed treatment. Thus, we have proposed two revisited systems, as a revisiting (or remaking) of Nieznański's approaches; the first of these systems is **N1**,

¹³ E. Nieznański, *Elements of Modal Theodicy*, op. cit.

It is not possible to summarize all of the relevant insights of Nieznański's works here. We encourage interested readers to acquaint themselves with his works in detail to see how the philosophical concepts that he explores rely upon good intuitions concerning the nature of God, the relation between situations, and the relation between will and other features of agents, such as coherence and responsibility; see E. Nieznański, *Aksjomatyczne ujęcie problemu teodycei*, op. cit.; E. Nieznański, *Elements of Modal Theodicy*, op. cit.

published recently in an article, and the second is N2, to be published soon. Of course, the line that distinguishes a "revisiting" from a "remaking" is sometimes obscure, but our proceeding with both systems can be synthesized as follows: first, we re-established the formal language to create one that, according to our vision, is more adequate for accomplishing Nieznański's tasks; we described both N1 and N2 as first-order modal systems, with two modal operators: \mathcal{W}_{θ} ("God knows") and \mathcal{C}_{θ} ("God wills"). In our interpretation, these operators are sufficient for dealing with some of the main subjects that Nieznański is concerned with in his articles. Furthermore, we established the formal language, the rules of inference, and other features. Then, we defined a new set of axiom schemes, many of them inspired by Nieznański's work, but with a new formulation, to finally prove some theorems.

The resulting systems have much of the original basic structure, and many axioms, definitions and theorems remain unchanged – yet some new results are obtained. In particular, both N1 and N2 characterize the attributes of God, provide a formal axiology, and give an answer to a form of religious determinism; all of these results are also obtained from their distinct sets of axioms. However, when considered together, one of the outcomes is surprising: the systems are mutually *contradictory*. Let us first consider the following formulas:

- (1) $\forall p \ [\beta Dp \rightarrow \beta C(\beta Dp)]$
- (2) $\sim \forall p \ [\beta Dp \rightarrow \beta C(\beta Dp)]^{16}$

These formulas are axioms of Nieznański's systems: formula (1) belongs to the first one, and (2) belongs to the second. The first-order modal versions of them, with the correspondent standard interpretations, are the following:

(1')
$$\forall p \ (\mathcal{D}_{\scriptscriptstyle \theta} \alpha(p) \to \mathcal{C}_{\scriptscriptstyle \theta} \mathcal{D}_{\scriptscriptstyle \theta} \alpha(p))$$

(For all situations, if God permits some state of affairs, then God wills to permit such state of affairs.)

G.B. da Silva, F.M. Bertato, A First-Order Modal Theodicy: God, Evil, and Religious Determinism,
 "South American Journal of Logic" 2019, Vol. 5, No. 1, pp. 49–80; G.B. da Silva, F.M. Bertato,
 God, Evil, and Religious Determinism: Another First-Order Modal Theodicy, forthcoming.

E. Nieznański, *Elements of Modal Theodicy*, op. cit., p. 259. The formula written here in the notation of Nieznański's first system is presented in his second system as axiom A6, which in another notation is given by ~∀p(Dbp → CbDbp).

(2')
$$\neg \forall p(\mathcal{D}_{\theta} \alpha(p) \to \mathcal{C}_{\theta} \mathcal{D}_{\theta} \alpha(p))$$

(Not all situations are such that if God permits some state of affairs, then God wills to permit such state of affairs.)

As is evident, the axioms are explicitly contradictory: (1) contradicts (2), and (1') contradicts (2'). These contradictions lead us to at least two possible conclusions: either one or both systems are trivial, and thus all of the results could be trivially obtained, or, there is a set of axioms which is sufficient to reach some of the most interesting conclusions for both systems, avoiding contradictions.

But the conclusions obtained are not trivial. Indeed, our research finds that there is a set of axioms which satisfies the demands of a new axiomatic system, similar to N1 and N2. The new system, called N3, is based on both previous systems, but has only three axioms (much less than N1 or N2, each one with eleven axioms). We think that these axioms are sufficient to prove the most relevant results of N1 and N2, as well as the results Nieznański aimed at: N3 proposes an answer to the problem of evil through the refutation of a version of religious determinism, showing that the attributes of God in classical theism, namely, those of omniscience, omnipotence, infallibility, and omnibenevolence, when adequately formalized, are consistent with the existence of evil in the world. On the one hand, these questions are also tackled by Nieznański's systems, but, on the other hand, they are obtained in N3 with fewer assumptions.

In the following, we present the formal structure of N3.

2. N3: A Minimal System

2.1. The Language, Rules, and Axioms of N3

The first-order modal language $\mathfrak{L}_{\rm N3}$ of N3 is composed of the following symbols as primitives: 17

- (i) Unary predicate symbols: B, P, δ , ξ ;
- (ii) A constant symbol (a distinguished element): θ ;

Concerning first-order modal logic, see G.E. Hughes, M.J. Cresswell, A New Introduction to Modal Logic, London 1996; W. Carnielli, C. Pizzi, Modalities and Multimodalities, Logic, Epistemology, and the Unity of Science 12, Dordrecht 2008; M. Fitting, R.L. Mendelsohn, First-Order Modal Logic, Dordrecht 2012. These works have provided useful guidance in the development of this work.

- (iii) Variables for situations: *p*, *q*, *r*, possibly with subscripts;¹⁸
- (iv) The symbols for connectives: \neg , \rightarrow , \lor , \land , \leftrightarrow ;
- (v) The symbols for operators: \forall , \exists ;
- (vi) Two symbols for specific modal operators: \mathcal{C}_{θ} , \mathcal{W}_{θ} . 19

The definition of a well-formed formula (*wff*) is the usual, with the expected extensions. The interdefinibility of connectives and operators is also the usual.

The basic rules of deduction of **N3** are: *Modus Ponens* (MP), *Uniform Substitution* (US), *Rule of Necessitation* (Nec) and *Substitution of Equivalents* (Eq). They are stated below:

(MP)
$$\phi$$
, $\phi \rightarrow \psi \vdash_{N3} \psi$.

(US) The result of uniformly replacing any variable or variables $p_1, ..., p_n$ in a theorem by any $wff \phi_1, ..., \phi_n$, respectively, is itself a theorem.

(Nec) If
$$\vdash_{N_3} \phi$$
, then $\vdash_{N_3} \mathcal{W}_{\theta} \phi$ and $\vdash_{N_3} \mathcal{C}_{\theta} \phi$.

(Eq) If ϕ is a theorem and ψ differs from ϕ in having some $wff \mu$ as a subformula in one or more places where ϕ has a $wff \gamma$ as a subformula, then if $\mu \leftrightarrow \gamma$ is a theorem, ψ is also a theorem.²⁰

As a *convention*, in **N3**, $\alpha(p)$ stands for any *wff* that involves *only* the variable p, where p is free. Thus, we distinguish in **N3** two types of situations, namely, *basic situations* (of the world) and *situations involving situations* (which are here called *states of affairs*). The variables for situations correspond, therefore, to basic situations. In turn, formulas that contain occurrences of free variables for situations represent states of affairs. We are particularly interested here in a specific type of state of affairs, namely, one in which a certain situation p is the case, represented by P(p). Thus, formulas in the form $\alpha(p)$ with only one free variable, p, are the

Despite considering variables for situations, a variable can assume the value θ . It is not intuitive to say that God is a situation, and therefore we could say that the symbols p, q, and r represent either God or situations. **N3** deals explicitly only with God and the (possible) situations of the world. Thus, in addition to dealing with situations, we can express in **N3** sentences such as "God is good," "God is not contingent," etc.

¹⁹ N3 is a type of First-Order Epistemic-Boulomaic Modal Logic. \mathcal{W}_{θ} is an epistemic operator that represents *knowledge* and \mathcal{C}_{θ} is a boulomaic operator that represents *will*.

This version of Eq is inspired by an equivalent rule formulated in G.E. Hughes, M.J. Cresswell, A New Introduction to Modal Logic, op. cit., p. 32.

most important. However, we could deal with states of affairs involving more than one situation, say $\alpha(p, q)$, but this is dispensable for our purposes.

For ease of reading, we establish a standard interpretation for each *wff*, according to the following semantic in natural language, which can be considered as a set of abbreviations:

```
\theta := "God";

P(p) := "p is the case";

B(\theta) := "\theta is divine";

\delta(p) := "p is good";

\xi(p) := "p is evil";

\mathcal{C}_{\theta} \alpha(p) := "God wills the state of affairs \alpha(p)";

\mathcal{W}_{\alpha} \alpha(p) := "God knows the state of affairs \alpha(p)."
```

In the following, we give an account of some of the divine attributes in order to provide a formal treatment of the questions raised in section 1.

2.2. The Attributes of God

We begin by presenting the definition of divinity:

Def. 1. (Divinity).
$$B(\theta) : \leftrightarrow (WW \land NM \land WM \land DB)$$

(God is divine *iff* He is omniscient, infallible, omnipotent, and omnibenevolent.)

The following definitions aim at formally expressing some relations that characterize the divine attributes of classical theism according to our approach. They are essentially proposed by Nieznański and they have the advantage of their precision and of a certain correspondence with the traditional concepts in the historical debate.²¹

Def. 2.
$$WW : \leftrightarrow \forall p \ (\alpha(p) \to \mathcal{W}_{\theta} \ \alpha(p))$$

(God is omniscient *iff* for all situations, if a state of affairs is the case, then God knows it.)

²¹ Cf. E. Nieznański, *Aksjomatyczne ujęcie problemu teodycei*, op. cit., pp. 204–205; E. Nieznański, *Elements of Modal Theodicy*, op. cit., pp. 255–256.

Def. 3.
$$NM : \leftrightarrow \forall p \ (\mathcal{W}_{_{\theta}} \alpha(p) \to \alpha(p))$$

(God is infallible *iff*, for all situations, if God knows a state of affairs, then it is the case.)

Def. 4.
$$WM : \leftrightarrow \forall p \ (\mathcal{C}_{\alpha} \alpha(p) \to \alpha(p))$$

(God is omnipotent *iff*, for all situations, if God wills a state of affairs, then it is the case.)

Def. 5.
$$DB : \leftrightarrow \forall p \ (\mathcal{C}_{a} P(p) \rightarrow \delta(p))$$

(God is omnibenevolent *iff*, for all situations, if God wills a situation to be the case, then such situation is good.)

Thus, the most relevant divine attributes are defined in order to approach the problem of evil, namely, omniscience, infallibility, omnipotence, and omnibenevolence.

As we have stated in the last section, **N3** is composed of only three axioms. These axioms are not difficult to assume in the context of the problem of evil; we believe that to some extent they correspond to what Mackie intends to do when he proposes the contemporary version of the problem of evil.²²

A1.
$$B(\theta)$$

(God is divine.)

A2.
$$\neg \forall p \ (P(p) \rightarrow \delta(p))$$

(Not all situations are such that, if a situation is the case, then such situation is good.)²³

A3.
$$\forall p \ (\delta(p) \rightarrow \neg \xi(p))$$

(For all situations, if a situation is good, then it is not evil.)

Mackie states some of the attributes of God, affirms that there is evil, and relates evil situations to good ones. Then, he argues for the incompatibility of those concepts. See J.L. Mackie, Evil and Omnipotence, op. cit., pp. 200–201.

²³ An equivalent formulation of A2 would be $\exists p \ (P(p) \land \neg \delta(p))$, i.e., there is at least one situation that is the case but is not good.

As an immediate consequence of Def. 1 and A1, the validity of the four attributes considered is established:

T1. $WW \wedge NM \wedge WM \wedge DB$

(God is omniscient, infallible, omnipotent, and omnibenevolent.)

Proof.

1.
$$B(\theta)$$
 [A1]

$$2. B(\theta) : \leftrightarrow WW \land NM \land WM \land DB$$
 [Def. 1]

3.
$$WW \wedge NM \wedge WM \wedge DB$$
 [PC, 1, 2]²⁴

The following theorems are corollaries of T1 and can be easily obtained:

T1.1.
$$\forall p \ (\alpha(p) \to \mathcal{W}_{_{\theta}} \ \alpha(p))$$

(For all situations, if a state of affairs is the case, then God knows such a state of affairs.)

T1.2.
$$\forall p \ (\mathcal{W}_{\alpha} \alpha(p) \rightarrow \alpha(p))$$

(For all situations, if God wills a state of affairs, then such state of affairs is the case.)

T1.3.
$$\forall p \ (\mathcal{C}_{\theta} \alpha(p) \to \alpha(p))$$

(For all situations, if God knows a state of affairs, then such state of affairs is the case.)

T1.4.
$$\forall p \ (\mathcal{C}_{\theta} P(p) \to \delta(p))$$

(For all situations, if God wills some situation to be the case, then such situation is good.)

²⁴ We use **PC** to indicate the use of some theorems or results from the Propositional Calculus.

Summing up, in all situations, and for arbitrary states of affairs, if a state of affairs is the case, then God knows it, for He is omniscient; if God knows a state of affairs, then it is the case, for God is infallible in His knowledge; if God wills a state of affairs, then it is the case, for he is omnipotent; and, finally, as God is omnibenevolent, if he wills a state of affairs, then it is good.

In this regard, David Hume asks: why is there evil in the world? As he remarks in his classical statement on the problem of evil:

Epicurus's old questions are yet unanswered. Is he willing to prevent evil, but not able? then is he impotent. Is he able, but not willing? then is he malevolent. Is he both able and willing? whence then is evil?²⁵

Hume seems to assume that if God does not will a state of affairs, then such a state of affairs is not the case. But this is equivalent to saying that if a state of affairs is the case, then God wills such a state of affairs. Such a statement is the reciprocal of the attribute of omnipotence and is not necessarily valid. In fact, we will prove that such a hypothesis – which is here called *religious determinism*, or, simply, *determinism* (**DET1**) – is not a theorem in **N3**.

Another version of determinism is more related to God's omniscience, since it supposes that if a state of affairs is the case, then He wills such a state of affairs (DET2). Surprisingly, it can be easily proved in N3 that the negation of DET2 is derived from the negation of DET1.

In this way, we formally state two deterministic hypotheses, which are refuted in **N3**.

(DET1).
$$\forall p \ (P(p) \rightarrow \mathcal{C}_{\theta} P(p))$$

(For all situations, if a situation is the case, then God wills such situation to be the case.)

(DET2).
$$\forall p \ (\mathcal{W}_{\theta} P(p) \rightarrow \mathcal{C}_{\theta} P(p))$$

(For all situations, if God knows a situation to be the case, then God wills such situation to be the case.)

Next, we proceed to refute these two versions of religious determinism.

D. Hume, Dialogues Concerning Natural Religion, in: David Hume: Dialogues Concerning Natural Religion in Focus, ed. S. Tweyman, London 2013 [1779], p. 273.

2.3. Religious Determinism Defeated

As we have already said, N3 is a minimal system, in the sense of assuming a very small number of axioms in comparison to N1 and N2, in order for us to be able to propose a solution to the famous logical problem of evil. Therefore, we will present some results about the existence of contingent situations and the fact that there are states of affairs that are not willed by God, but that are permitted. It can be considered that such results show the possibility of an investigation on free will. However, this is outside the scope of this article.

That said, we now proceed to formally refute the two versions of determinism.

T2 (
$$\neg$$
DET1). $\neg \forall p \ (P(p) \rightarrow \mathcal{C}_{\theta} P(p))$

(Not all situations are such that, if a situation is the case, then God wills such situation to be the case.)

Proof.

<i>y</i>	
1. $\neg\neg \forall p \ (P(p) \to \mathcal{C}_{\theta} P(p))$	[Hip] ²⁶
2. $\forall p \ (P(p) \to \mathcal{C}_{\theta} \ P(p))$	[PC , 1]
3. $P(p) \to \mathcal{C}_{\theta} P(p)$	[2, Spec] ²⁷
4. $C_{\theta} P(p) \rightarrow \delta(p)$	[T1.4, Spec]
5. $P(p) \rightarrow \delta(p)$	[PC, 3, 4]
6. $\forall p \ (P(p) \to \delta(p))$	[Gen, 5] ²⁸
7. $\neg \forall p \ (P(p) \rightarrow \delta(p))$	[A2]
8. $\neg\neg\neg\forall p\ (P(p)\to \mathcal{C}_{\theta}\ P(p))$	[¬Hip, 6, 7]
9. $\neg \forall p \ (P(p) \to \mathcal{C}_{\theta} P(p))$	[PC , 8]

T3 (¬DET2). ¬
$$\forall p \ (\mathcal{W}_{\theta} \ P(p) \to \mathcal{C}_{\theta} \ P(p))$$

(Not all situations are such that if God knows a situation to be the case, then God wills such a situation to be the case.)

²⁶ Hypothesis.

²⁷ Specification Rule.

²⁸ Generalization Rule.

P_{1}	00	f.

$$\begin{aligned} &1. \ \forall p \ (\mathcal{W}_{\theta} P(p) \rightarrow \mathcal{C}_{\theta} P(p)) & [\text{Hip}] \\ &2. \ \mathcal{W}_{\theta} P(p) \rightarrow \mathcal{C}_{\theta} P(p) & [1, \text{Spec}] \\ &3. \ P(p) \rightarrow \mathcal{W}_{\theta} P(p) & [\text{T1.1, } \alpha(p)/P(p), \text{Spec}] \\ &4. \ \mathcal{W}_{\theta} P(p) \rightarrow P(p) & [\text{T1.2, } \alpha(p)/P(p), \text{Spec}] \\ &5. \ P(p) \leftrightarrow \mathcal{W}_{\theta} P(p) & [\text{PC, 3, 4}] \\ &6. \ P(p) \rightarrow \mathcal{C}_{\theta} P(p) & [\text{Eq, 5 in 2}] \\ &7. \ \forall p \ (P(p) \rightarrow \mathcal{C}_{\theta} P(p)) & [\text{Gen, 6}] \\ &8. \ \neg \forall p \ (\mathcal{W}_{\theta} P(p) \rightarrow \mathcal{C}_{\theta} P(p)) & [\text{T2}] \\ &9. \ \neg \forall p \ (\mathcal{W}_{\theta} P(p) \rightarrow \mathcal{C}_{\theta} P(p)) & [\text{PC, 9}] \end{aligned}$$

Thus, we easily refuted the two versions of determinism which are fundamental for the discussion about the logical problem of evil. To solve the question of determinism without needing many previous results is a remarkable characteristic of **N3**.

2.4. Further Consequences: God, Values and Determinism

Given the results of the previous section, we now explore some elementary results with respect to the problem of evil in itself. We think these theorems and definitions are self-explanatory: they, in conjunction, provide a framework to reconsider whether there is compatibility between the existence of evil and the existence of God.

T4.
$$\forall p \ (\xi(p) \rightarrow \neg \delta(p))$$

(For all situations, if a situation is evil, then such situation is not good.)

Proof. Easily deduced from A3, by contraposition.

T5.
$$\forall p \ (\neg \delta(p) \rightarrow \neg \mathcal{C}_{\alpha} P(p))$$

(For all situations, if a situation is not good, then it is not the case that God wills it to be the case.)

Proof. Easily deduced from T1.4, by contraposition.

T6.
$$\forall p \ (\xi(p) \rightarrow \neg \mathcal{C}_{\alpha} P(p))$$

(For all situations, if a situation is evil, then it is not the case that God wills it to be the case.)

Proof.

1.
$$\xi(p) \rightarrow \neg \delta(p)$$
 [T4, Spec]

2.
$$\neg \delta(p) \rightarrow \neg C_{\theta} P(p)$$
 [T5, Spec]

3.
$$\xi(p) \rightarrow \neg C_a P(p)$$
 [PC, 1, 2]

4.
$$\forall p \ (\xi(p) \to \neg \mathcal{C}_{\theta} P(p))$$
 [Gen, 3]

T7.
$$\forall p \ \neg \mathcal{C}_{\theta} (\alpha(p) \land \neg \alpha(p))$$

(For all situations, it is not the case that God wills some contradiction.)

Proof.

1.
$$C_{\alpha}(\alpha(p) \land \neg \alpha(p)) \rightarrow (\alpha(p) \land \neg \alpha(p)))$$
 [T1.3, $\alpha(p)/(\alpha(p) \land \neg \alpha(p))$, Spec]

2.
$$\neg(\alpha(p) \land \neg \alpha(p)) \rightarrow \neg C_{\theta}(\alpha(p) \land \neg \alpha(p))$$
 [PC, 1]

3.
$$\neg(\alpha(p) \land \neg \alpha(p))$$
 [PC-Theorem]

4.
$$\neg C_{\alpha}(\alpha(p) \land \neg \alpha(p))$$
 [MP, 3, 2]

5.
$$\forall p \ \neg \mathcal{C}_{\theta} \ (\alpha(p) \land \neg \alpha(p))$$
 [Gen, 4]

T8.
$$\forall p \ \mathcal{C}_{\scriptscriptstyle{\theta}} \neg (\alpha(p) \land \neg \alpha(p))$$

(All situations are such that God wills non-contradictions.)

Proof.

1.
$$\neg(\alpha(p) \land \neg \alpha(p))$$
 [PC-Theorem]

2.
$$C_{\theta} \neg (\alpha(p) \land \neg \alpha(p))$$
 [Nec, 1]

3.
$$\forall p \ \mathcal{C}_{\theta} \neg (\alpha(p) \land \neg \alpha(p))$$
 [Gen, 2]

Permission is defined in **N3** as the dual operator of \mathcal{C}_{θ} :

Def. 6 (Permission).
$$\mathcal{D}_{\theta} \alpha(p) : \leftrightarrow \neg \mathcal{C}_{\theta} \neg \alpha(p)$$

(God permits a state of affairs iff He does not will the opposite.)

T9.
$$\forall p \ (\mathcal{C}_{\scriptscriptstyle{\theta}} \ \alpha(p) \leftrightarrow \neg \mathcal{D}_{\scriptscriptstyle{\theta}} \neg \alpha(p))$$

(For all situations, God wills a state of affairs *iff* He does not permit the opposite.)

Proof.

1.
$$\mathcal{D}_{\theta} \neg \alpha(p) \leftrightarrow \neg \mathcal{C}_{\theta} \neg \neg \alpha(p)$$
 [Def. 6, $\alpha(p)/\neg \alpha(p)$, Spec]

2.
$$\mathcal{D}_{\theta} \neg \alpha(p) \leftrightarrow \neg \mathcal{C}_{\theta} \alpha(p)$$
 [PC, 1]

3.
$$C_{\theta} \alpha(p) \leftrightarrow \neg \mathcal{D}_{\theta} \neg \alpha(p)$$
 [PC, 2]

4.
$$\forall p \ (\mathcal{C}_{\theta} \ \alpha(p) \leftrightarrow \neg \mathcal{D}_{\theta} \neg \alpha(p))$$
 [Gen, 3]

T10. $\forall p \ (\alpha(p) \to \mathcal{D}_{\theta} \ \alpha(p))$

(For all situations, if a state of affairs is the case, then it is permitted by God.)

Proof.

1.
$$C_{\theta} \neg \alpha(p) \rightarrow \neg \alpha(p)$$
 [T1.3, $\alpha(p)/\neg \alpha(p)$, Spec]

2.
$$\mathcal{D}_{\alpha} \alpha(p) \leftrightarrow \neg \mathcal{C}_{\alpha} \neg \alpha(p)$$
 [Def. 6, Spec]

3.
$$\alpha(p) \rightarrow \mathcal{D}_{\alpha} \alpha(p)$$
 [PC, 1, 2]

4.
$$\forall p \ (\alpha(p) \to \mathcal{D}_{\theta} \ \alpha(p))$$
 [Gen, 3]

T11. $\neg \forall p \ (\mathcal{D}_{\scriptscriptstyle \theta} \ \alpha(p) \to \alpha(p))$

(Not all situations are such that, if God permits some state of affairs, then it is the case.)

Proof.

1.
$$\forall p \ (\mathcal{D}_{\theta} \ \alpha(p) \to \alpha(p))$$
 [Hip]

2.
$$\mathcal{D}_{\alpha} \alpha(p) \rightarrow \alpha(p)$$
 [Spec, 1]

3.
$$\mathcal{D}_{\theta} \alpha(p) \leftrightarrow \neg \mathcal{C}_{\theta} \neg \alpha(p)$$
 [Def. 6, Spec]

4.
$$\neg C_{\theta} \neg \alpha(p) \rightarrow \alpha(p)$$
 [Eq. 3 in 2]

5.
$$\neg \alpha(p) \rightarrow C_{\theta} \neg \alpha(p)$$
 [PC, 4]

6.
$$\alpha(p) \to \mathcal{C}_{\theta} \alpha(p)$$
 [5, $\alpha(p)/\neg \alpha(p)$, **PC**]

7.
$$P(p) \rightarrow \mathcal{C}_{\theta} P(p)$$
 [6, $\alpha(p)/P(p)$]

8.
$$\forall p \ (P(p) \to \mathcal{C}_{\theta} P(p))$$
 [Gen, 7]

9.
$$\neg \forall p \ (P(p) \rightarrow \mathcal{C}_{\scriptscriptstyle \theta} P(p))$$
 [T2]

10.
$$\neg \forall p \ (\mathcal{D}_{\theta} \ \alpha(p) \to \alpha(p))$$
 [¬Hip, 1]

A *contingent* state of affairs, according to the philosophical tradition, is one such that this state of affairs and its complementary are both possible, or, as we define here, permitted by God:

Def. 7.
$$K(p) : \leftrightarrow (\mathcal{D}_{\theta} P(p) \wedge \mathcal{D}_{\theta} \neg P(p))$$

(A situation is contingent *iff* God permits it to be or not to be the case.)

T12.
$$\forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\alpha} P(p) \land \neg \mathcal{C}_{\alpha} \neg P(p)))$$

(For all situations, a situation is contingent *iff* neither God wills such situation to be the case, nor wills it not to be the case.)

Proof.

$$\begin{aligned} &1. \ K(p) \leftrightarrow (\mathcal{D}_{\theta} \ P(p) \land \mathcal{D}_{\theta} \neg P(p)) & & & & & & & \\ &2. \ \mathcal{D}_{\theta} \ P(p) \leftrightarrow \neg \mathcal{C}_{\theta} \neg P(p) & & & & & & \\ &3. \ \mathcal{C}_{\theta} \ P(p) \leftrightarrow \neg \mathcal{D}_{\theta} \neg P(p) & & & & & & \\ &4. \ \mathcal{D}_{\theta} \neg P(p) \leftrightarrow \neg \mathcal{C}_{\theta} \ P(p) & & & & & & \\ &4. \ \mathcal{D}_{\theta} \neg P(p) \leftrightarrow \neg \mathcal{C}_{\theta} \ P(p) & & & & & & \\ &5. \ K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p)) & & & & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & & & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) & \\ &6. \ \forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} \ P(p) \land \neg \mathcal{C}_{\theta} \neg P(p))) &$$

The following corollaries are easily deduced from T12:

T12.1.
$$\forall p \ (K(p) \leftrightarrow \neg (\mathcal{C}_{a} P(p) \vee \mathcal{C}_{a} \neg P(p)))$$

(For all situations, a situation is contingent *iff* either it is not the case that God wills such situation to be the case or He wills such situation not to be the case.)

T12.2.
$$\exists p \ (K(p) \leftrightarrow \neg \forall p \ (\mathcal{C}_{\theta} P(p) \lor \mathcal{C}_{\theta} \neg P(p)))$$

(There is a contingent situation *iff* it is not the case that, for all situations, either God wills a situation to be the case or He is opposed to that.)

T12.3.
$$\forall p \ (K(p) \leftrightarrow (\neg \mathcal{C}_{\alpha} P(p) \land \mathcal{D}_{\alpha} P(p)))$$

(For all situations, a situation is contingent *iff* either it is not the case that God wills such situation to be the case or He permits such situation not to be the case.)

Now, we proceed to prove that at least one contingent situation exists. In order to do this, we use the following theorem:

T13.
$$\neg \forall p \ (\mathcal{C}_{\scriptscriptstyle \theta} \alpha(p) \vee \mathcal{C}_{\scriptscriptstyle \theta} \neg \alpha(p))$$

(Not all situations are such that either God wills a state of affairs or its opposite.)

Proof.

1.
$$\forall p \ (\mathcal{C}_{\alpha} \alpha(p) \vee \mathcal{C}_{\alpha} \neg \alpha(p))$$
 [Hip]

2.
$$C_{\alpha} \alpha(p) \vee C_{\alpha} \neg \alpha(p)$$
 [1, Spec]

3.
$$\neg C_{\alpha} \alpha(p) \rightarrow C_{\alpha} \neg \alpha(p)$$
 [PC, 2]

4.
$$C_{\alpha} \neg \alpha(p) \rightarrow \neg \alpha(p)$$
 [T1.3, $\alpha(p)/\neg \alpha(p)$, Spec]

5.
$$\neg \mathcal{C}_{\alpha} \alpha(p) \rightarrow \neg \alpha(p)$$
 [PC, 3, 4]

6.
$$\alpha(p) \rightarrow \mathcal{C}_{\theta} \alpha(p)$$
 [PC, 5]

7.
$$\forall p \ (\alpha(p) \to \mathcal{C}_{\theta} \ \alpha(p))$$
 [Gen, 6]

8.
$$\forall p \ (P(p) \to \mathcal{C}_{\theta} \ P(p))$$
 [7, $\alpha(p)/P(p)$]

9.
$$\neg \forall p \ (P(p) \rightarrow \mathcal{C}_{o} P(p))$$
 [T2]

10.
$$\neg \forall p \ (\mathcal{C}_{\theta} \alpha(p) \vee \mathcal{C}_{\theta} \neg \alpha(p))$$
 [¬Hip, 1]

T14. $\exists p \ K(p)$

(There is at least one situation that is contingent.)

Proof.

1.
$$\exists p \ (K(p) \leftrightarrow \neg \forall p \ (\mathcal{C}_{_{\theta}} P(p) \lor \mathcal{C}_{_{\theta}} \neg P(p)))$$
 [T12.2]

2.
$$\neg \forall p \ (\mathcal{C}_{\scriptscriptstyle \theta} P(p) \lor \mathcal{C}_{\scriptscriptstyle \theta} \neg P(p))$$
 [T13, $\alpha(p)/P(p)$]

3.
$$\exists p \ K(p)$$
 [PC, 1, 2]

T15. $\forall p \ ((\xi(p) \land \mathcal{D}_{\theta} P(p)) \rightarrow K(p))$

1. $\neg \forall p ((\xi(p) \land \mathcal{D}_{\theta} P(p)) \rightarrow K(p))$

11. $\neg \mathcal{C}_{\theta} P(p) \wedge \mathcal{D}_{\theta} P(p)$

(For all situations, if a situation is evil, and God permits it, then such situation is contingent.)

[Hip]

[PC, 10, 8]

Proof.

2. $\exists p \neg ((\xi(p) \land \mathcal{D}_{\theta} P(p)) \rightarrow K(p))$	$[FOL, 1]^{29}$
3. $\neg ((\xi(p) \land \mathcal{D}_{\theta} P(p)) \rightarrow K(p))$	[FOL, 2]
4. $(\xi(p) \wedge \mathcal{D}_{\theta} P(p)) \wedge \neg K(p)$	[PC , 3]
5. $\neg K(p)$	[PC , 4]
6. $\xi(p) \wedge \mathcal{D}_{\theta} P(p)$	[PC , 4]
7. $\xi(p)$	[PC , 6]
8. $\mathcal{D}_{\theta} P(p)$	[PC , 6]
9. $\xi(p) \to \neg \mathcal{C}_{\theta} P(p)$	[T6, Spec]
10. $\neg \mathcal{C}_{\theta} P(p)$	[MP, 7, 9]

12.
$$K(p) \leftrightarrow (\neg \mathcal{C}_{\theta} P(p) \wedge \mathcal{D}_{\theta} P(p))$$
 [T12.3, Spec]

14.
$$\neg \neg \forall p ((\xi(p) \land \mathcal{D}_{\theta} P(p)) \rightarrow K(p))$$
 $[\neg \text{Hip, 5, 13}]$

²⁹ We use **FOL** to indicate the use of some theorems or results from the First-Order Logic.

15.
$$\forall p \ ((\xi(p) \land \mathcal{D}_{\theta} P(p)) \rightarrow K(p))$$
 [Gen, 15]

Theorem T15 is equivalent to $\forall p \ ((\xi(p) \land \neg \mathcal{C}_{\theta} \neg P(p)) \to K(p))$. It means, therefore, that if there is evil in the world and God seems to not will the opposite, then this very situation is contingent and thus does not depend on his good will. In conjunction with the refutation of **DET1** and **DET2**, these theorems explain how the existence of evil can be consistent with the attributes of God. The answer is quite simple: as determinism fails (T4 and T5), if there is evil in the world, God does not will such an evil to be the case (T6), but if He permits evil, then it must be contingent (T15).

Finally, we present a semantics for N3.

2.5. Semantics of N3

As usual, the model for the system N3 is a structure $\langle W, R, D, V \rangle$, where W is a set of possible worlds; $R \subseteq W^2$ is a relation of accessibility; D is the domain of objects; and $V: WFF \times W \to \{0,1\}$ is a function of valuation, where $\phi, \psi \in WFF$ and $w, w' \in W$, determined by an assignment μ , such that, for each variable p of \mathcal{L}_{N3} , $\mu(p) \in D$. In particular, the valuations for the modal operators \mathcal{W}_{θ} and \mathcal{C}_{θ} are given by the following conditions:

(i)
$$V(\mathcal{W}_{\theta} \phi, w) = 1 \text{ iff } V(\phi, w) = 1$$

(ii) If
$$V(\mathcal{C}_{\theta} \phi, w) = 1$$
 then $V(\phi, w') = 1$ for every $w' \in W$ such that wRw' .

Let $W = \{w_0, w_1, ..., w_n, ...\}$, where $n \in \mathbb{N}$, be a set of possible worlds, $R = W^2$, and $D = \mathbb{N} \cup \{-1\}$, and for every assignment μ we fix that $\mu(\theta) = 0$. The function of valuation V is such that:

$$\begin{split} V(B) &= \{(0, w_n) \colon n \in \mathbb{N}\}; \\ V(\xi) &= \{(-1, w_n) \colon n \in \mathbb{N}\}; \\ V(P) &= V(B) \cup V(\xi) \cup \{(2n, w_n) \colon n \in \mathbb{N}\}; \\ V(\delta) &= V(P) - V(\xi). \end{split}$$

We proceed to show that $\mathfrak{M} = \langle W, R, D, V \rangle$, thus defining a model for the axioms of **N3**. It is worth noting that this model does not explicitly distinguish objects from situations. Furthermore, we can say that an object p satisfies a predicate P_i in a possible world w iff $(\mu(p), w) \in V(P_i)$.

Let us begin with axioms A2 and A3, because we use A2 to show A1.

[A2:]
$$\mathfrak{M} \models \forall p \ (\delta(p) \rightarrow \neg \xi(p))$$

Proof. Since $V(\delta) = V(P) - V(\xi)$, then, $V(\delta) \cap V(\xi) = \emptyset$. Thus, for every p, it is not the case that $(\mu(p), w) \in V(\delta)$ and $(\mu(p), w) \in V(\xi)$. Thus, for every p, it is not the case that $V(\delta(p), w) = 1$ and $V(\xi(p), w) = 1$. Then, by De Morgan's law, we have that, for every p, $V(\delta(p), w) = 0$ or $V(\xi(p), w) = 0$. Therefore, for every p, $V(\delta(p), w) = 0$ or $V(\neg \xi(p), w) = 1$. So, for every p, $V(\delta(p) \rightarrow \neg \xi(p), w) = 1$. It follows that $V(\forall p \ (\delta(p) \rightarrow \neg \xi(p)), w) = 1$. \square

[A3:]
$$\mathfrak{M} \models \neg \forall p \ (P(p) \rightarrow \delta(p))$$

Proof. We have that $V(\neg \forall p \ (P(p) \rightarrow \delta(p)), \ w) = 1$ *iff* $V(\forall p \ (P(p) \rightarrow \delta(p)), \ w) = 0$. The latter is the case, *iff* there is an object p, such that $V(P(p), \ w) = 1$ and $V(\delta(p), \ w) = 0$. We have that $(-1, \ w) \in V(\xi) \cap V(P)$, because $(-1, \ w_n) \in V(\xi)$, for every $n \in \mathbb{N}$, and $V(\xi) \subset V(P)$. Thus, since V and μ are well-defined functions, there is an object p in the domain, such that $\mu(p) = -1$. Therefore, we have an object p, such that $(\mu(p), \ w) \in V(P)$ and $(\mu(p), \ w) \in V(\xi)$. Since $V(\delta) \cap V(\xi) = \emptyset$, so we have that $(\mu(p), \ w) \in V(P)$ and $(\mu(p), \ w) \notin V(\delta)$. But this implies that $V(P(p), \ w) = 1$ and $V(\delta(p), \ w) = 0$. Therefore, $V(\neg \forall p \ (P(p) \rightarrow \delta(p), \ w) = 1$. □

Now, let us consider **A1**. Since $B(\theta)$ is obtained through a conjunction, that is, $WW \wedge NM \wedge WM \wedge DB$, it is enough to show that its constituent formulas are true according to the interpretation considered:

[A1:]
$$\mathfrak{M} \models WW \land NM$$

Proof. $WW \wedge NM$ is equivalent to $\forall p \ (\alpha(p) \leftrightarrow \mathcal{W}_{\theta} \ \alpha(p))$. By the condition (i) above, we have that, for every p, $V(\mathcal{W}_{\theta} \ \alpha(p), \ w) = 1$ iff $V(\alpha(p), \ w) = 1$. Therefore, we have that $V(\forall p \ (\alpha(p) \leftrightarrow \mathcal{W}_{\theta} \ \alpha(p)), \ w) = 1$. Therefore, $WW \wedge NM$ is true in \mathfrak{M} . \square

 $\mathfrak{M} \models WM$

Proof. By definition, WM is equivalent to $\forall p \ (\mathcal{C}_{\theta} \ \alpha(p) \to \alpha(p))$. By the condition (ii) above, we have that if $V(\mathcal{C}_{\theta} \ \alpha(p), \ w) = 1$ then $V(\alpha(p), \ w) = 1$, since R is reflexive, and, consequently, wRw. Thus, $V(\forall p \ (\mathcal{C}_{\theta} \ \alpha(p) \to \alpha(p)), \ w) = 1$. Therefore, WM is true in \mathfrak{M} . \square

 $\mathfrak{M} \models DB$

Proof. The only restriction to assignments μ in \mathfrak{M} is that $\mu(\theta) = 0$, since θ is the only distinguished object in N3. Therefore, we can consider two assignments, μ_i and μ_j , and a pair of objects, p and q, with $p \neq q \neq \theta$, such that

$$(\mu_i(p), w_{ij}) \notin V(P)$$
 and $(\mu_i(q), w_{ij}) \in V(\xi)$; and

$$(\mu_i(q), w_{in}) \notin V(P)$$
 and $(\mu_i(p), w_{in}) \notin V(\xi)$;

for some $m, n \in \mathbb{N}$, and $m \neq n$.

But that means that

(*)
$$V(P(p), w_{m}) = 0$$
 and $V(\xi(q), w_{m}) = 1$; and

(**)
$$V(P(q), w_{...}) = 0$$
 and $V(\xi(p), w_{...}) = 1$;

for some $m, n \in \mathbb{N}$, and $m \neq n$.

Now, let us suppose that an r exists such that $V(\mathcal{C}_{\theta} P(r), w) = 1$ but $V(\delta(r), w) = 0$. Then, from **A3**, we have that $V(\mathcal{C}_{\theta} P(r), w) = 1$ and $V(\xi(r), w) = 1$. So, from condition (ii), we have that $V(\mathcal{C}_{\theta} P(r), w) = 1$ for every $V(\mathcal{C}_{\theta} P(r), w) = 1$. But, if we took $V(\mathcal{C}_{\theta} P(r), w) = 1$ for some $V(\mathcal{C}_{\theta} P(r), w) = 1$. Thus, $V(\mathcal{C}_{\theta} P(r), w) = 1$. Therefore, $V(\mathcal{C}_{\theta} P(r), w) = 1$. Therefore, $V(\mathcal{C}_{\theta} P(r), w) = 1$.

Thereby, we concluded that $\mathfrak{M} \models B(\theta)$, and, therefore, \mathfrak{M} is a model for N3.

The meta-theory is assumed to be consistent. Thus, when we give a model (in this case, a set-theoretical model) for theory N3, we conclude that it is also consistent. Otherwise, if it were possible to derive a contradiction in N3, it would imply that there would be a contradiction in the model. That is, the consistency

of N3 is conditioned by the consistency of the meta-theory and/or, in this case, also by the consistency of the set theory. Thus, from the fact that $\neg DET1$ and $\neg DET2$ are theorems of N3 it is reasonable to conclude that DET1 and DET2 are not theorems of N3.

Final Remarks

In this article, we described a first-order modal system called **N3**, a system which aims at dealing with religious determinism and the logical problem of evil. On the one hand, if the results are correct, then we have an answer to these questions, now stated in formal terms with clarity and precision. On the other hand, **N3** establishes its results with many fewer assumptions than **N1**, **N2**, or those original systems developed by Nieznański,³⁰ and in a widely recognized formal language. Both outcomes, together, provide a narrower response to the logical problem of evil through the refutation of determinism, possibly offering a new pathway to the solution of this debate.

For all that has been shown, the results provide a characterization of many issues involving the logical problem of evil. Furthermore, in conjunction, they provide a framework for reconsidering whether there is compatibility between the existence of evil and the existence of God. We hope that it deals satisfactorily with the allegation of religious determinism; we think that such results provide a detailed approach that must not be ignored, as they provide a relevant response to such difficult questions. At least, for those involved in the mainstream discussion, even if the solutions available do not fulfil their pretension, there is one more solution at hand, which tackles the question in an innovative way, and with tools that may be more precise than those of just natural language.

Finally, as we remarked in the Introduction, we believe that the usage of formal systems may provide several advantages in solving philosophical and theological problems. "Standing on the shoulders" of Bocheński, of the Cracow Circle, and of Nieznański as well, we think that our approach can be considered

We recognize that, despite affirming that the goal of his systems was to deal with the problem of evil, Nieznański had much more in mind than just dealing with this question. In his systems, he deals with God's will and a number of dispositions (permission, opposition, causality, responsibility), exhibiting a full-blown treatment of these "divine properties."

an example of logic of religion, and, as such, we hope that it contributes to the advancement of the field.

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Summary

Edward Nieznański developed two logical systems to deal with the problem of evil and to refute religious determinism. However, when formalized in first-order modal logic, two axioms of each system contradict one another, revealing that there is an underlying minimal set of axioms enough to settle the questions. In this article, we develop this minimal system, called N3, which is based on Nieznański's contribution. The purpose of N3 is to solve the logical problem of evil through the defeat of a version of religious determinism. On the one hand, these questions are also addressed by Nieznański's systems, but, on the other hand, they are obtained in N3 with fewer assumptions. Our approach can be considered a case of logic of religion, that is, of logic applied to religious discourse, as proposed by Józef Maria Bocheński; in this particular case, it is a discourse in theodicy, which is situated in the context of the philosophy of religion.

Key words: logical problem of evil, theodicy, first-order modal logic, logic of religion, Edward Nieznański