Consistency improvement with a feedback recommendation in personalized linguistic group decision making

Cong-Cong Li, Haiming Liang, Yucheng Dong, Francisco Chiclana, Enrique Herrera-Viedma

***Abstract*—Consistency is an important issue in linguistic decision making with various consistency measures and consistency improving methods available in the literature. However, existing linguistic consistency studies omit the fact that words mean different things for different people, i.e. decision makers' personalized individual semantics (PISs) over their expressed linguistic preferences are ignored. Therefore, the aim of this paper is to propose a novel consistency improving approach based on PISs in linguistic group decision making. The proposed approach combines the characteristics of personalized representation and integrates the PIS-based model in measuring and improving the consistency of linguistic preference relations. A detailed numerical and comparative analysis to support the feasibility of the proposed approach is provided.**

***Index Terms*—Personalized individual semantics, linguistic preference relation, consistency, group decision making**

I. Introduction

Preference relation is the most commonly used preference representation structure in group decision making (GDM). There are various types of preference relations: additive preference relation [24, 33], multiplicative preference relation [3, 25, 30], and linguistic preference relation [9, 11].

In real decision making activities, it is common that decision makers provide their knowledge and preferences using words (linguistically) rather than numbers (numerically). Generally, consistency of information is important in GDM problems because its lack may lead to the inconsistent results [6, 7, 8, 18, 38, 42]. Existing studies in the literature measure the consistency of linguistic preference relations mainly by computing the difference between the original linguistic preferences and their estimated consistent ones [1, 21]. If the consistency of a linguistic preference relation is unacceptable, then methods to improve the consistency degree are applied. Generally, two types of consistency improving approaches are often used in decision making with linguistic preference relations [21]:

(1) The iterative approach, which improves the consistency degree by helping decision makers to construct a new linguistic preference relation according to the consistent linguistic preference relation.

(2) The optimization method, which deals with inconsistent linguistic preference relation by finding a suitable RPR with acceptable consistency to preserve the original information as much as possible.

Dong et al. [6] proposed an iterative algorithm to improve the consistency degree of linguistic preference relations by constructing a new linguistic preference relation with acceptable consistency, and also suggested a non-linear programming model to improve the consistency. Jin et al. [17] proposed two automatic iterative algorithms to help decision makers improve additive consistency level until it is acceptable. Wu et al. [40] proposed an integer optimization model for improving consistency by deriving the acceptably consistent linguistic preference relation. More research regarding the consistency improving methods can be found in the recent review [21].

It is a fact that words mean different things for different people [26, 27]. Mathematically, this has been addressed in linguistic GDM by using type-2 fuzzy sets [26] and the multi-granular linguistic model [14, 28]. Although they are useful in processing the multiple meanings of words, they are unable to represent the specific meaning of words for each decision maker. Therefore, the personalized individual semantics (PISs) model was proposed in [19] to obtain the personalized numerical scales of linguistic terms for decision makers. Furthermore, Li et al. [20, 22], Zhang et al. [43] and Tang et al. [34,35] studied the consistency-driven approaches to show the PISs in hesitant linguistic GDM, large-scale

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linguistic GDM, and distribution linguistic GDM, respectively. The application of the PIS model were studied in failure modes and effects analysis [44] and opinion dynamics [23].

The PISs among decision makers can influence the measurement of consistency for linguistic expressions. For example, let $S = \{s_0 = very\ poor, s_1 = poor, s_2 = medium, s_3 = good, s_4 = very\ good\}$ be an established linguistic term set. A decision maker who assesses the preference of alternative x_i over alternative x_j with the s_3 value, the preference of the alternative x_j over the alternative x_z with the s_2 value, and the preference of the alternative x_i over the alternative x_z with the s_2 value, is actually providing, based on the additive transitivity [32, 33] and the 2-tuple linguistic computational model [10], is additive consistent linguistic preferences on the set of alternatives $\{x_i, x_j, x_z\}$. However, if the PISs of words are considered, then these linguistic preferences may not satisfy the additive consistency requirement for some decision makers.

Although the existing consistency improving approaches have been investigated intensively, the decision makers' PISs are not considered. Therefore, this paper revisits the linguistic consistency improving methodologies from the PISs perspective. Specifically, we propose a consistency improving method with a feedback recommendation based on PISs in linguistic GDM, in which the feedback recommendation help decision makers revise their preferences to improve the consistency. The main goal of the proposed consistency improving method is to construct a new linguistic preference relation that has acceptable consistency taking into account the decision makers' PISs. This proposal includes the following stages:

(1) By constructing a consistency-driven optimization model, personalized numerical scales of linguistic terms are set for different decision makers to personalize individual semantics; this is followed by the developing of a novel consistency index of linguistic preference relations based on the PISs.

(2) A PIS-based consistency improving method is proposed. A theoretical analysis shows (i) that the method's adjusted linguistic preference relations are of acceptable consistency, and (ii) the convergence of the consistency improving process.

(3) A comparative study with the existing consistency improving methods based on experimental simulations is included. The obtained results show that the integration of the PIS model can help improve the consistency of linguistic preference relations more rapidly.

The rest of this paper is arranged as follows. Section II introduces the necessary preliminaries to develop the proposed PIS-based consistency improving method of linguistic preference relation in Section III. Section IV includes numerical examples to illustrate the PIS-based consistency improving process, while Section V is devoted to an experimental comparative study of the propose approach performance with respect to the existing approaches in the literature. Finally, Section VI concludes the paper with final remarks.

II. Preliminaries

This section introduces preliminary material necessary to build the proposed consistency improving process: the 2-tuple linguistic model and the numerical scale with PISs.

A. *The 2-tuple linguistic model*

The 2-tuple linguistic model, proposed by Herrera and Martínez [10], is widely used in computing with words frameworks.

Definition 1 [10]. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation. The 2-tuple linguistic model comprises the transformation function between symbolic aggregation numerical values and 2-tuples:

$$\Delta: [0, g] \to \bar{S} \quad (1)$$

$$\Delta(\beta) = (s_i, \alpha), \quad (2)$$

where $i = round(\beta)$ and $\alpha = \beta - i,\ \alpha \epsilon [-0.5, 0.5)$.

The 2-tuple negation operator is defined as $Neg((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha)))$, where $\Delta^{-1}(s_i, \alpha) = i + \alpha$ is the inverse function of Δ.

Linguistic preference relations, as defined below, are widely used in decision making.

Definition 2 [12, 13]. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. A linguistic preference relation on a set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$ is represented by a matrix $L = (l_{ij})_{n\times n}$, whose element $l_{ij} \in S$ is the preference degree of alternative x_i over x_j, subject to $l_{ij} = Neg(l_{ji})$ for $i, j = 1,2 \ldots, n$.

The consistency of a linguistic preference relation based on the 2-tuple linguistic model is measured as follows:

Definition 3 [1]. A linguistic preference relation on a linguistic term set S, $L = (l_{ij})_{n\times n}$, is consistent if

$$\Delta^{-1}(l_{ij}) + \Delta^{-1}(l_{jk}) - \Delta^{-1}(l_{iz}) = \frac{g}{2}\ \ \forall i, j, z = 1,2, \ldots, n.$$

The consistency index of L is defined as follows,

$$CI(L) = 1 - \frac{2}{3gn(n-1)(n-2)} \sum_{i,j,z=1}^{n} \left(\Delta^{-1}(l_{ij}) + \Delta^{-1}(l_{jz}) - \Delta^{-1}(l_{iz}) - \frac{g}{2}\right) \quad (3)$$

A larger value of $CI(L) \in [0,1]$ indicates a better consistency of L.

B. *PIS based on numerical scale*

Dong et al. [4] extended the 2-tuple linguistic model with the concept of the numerical sale.

Definition 4 [4]. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and $\mathbb{R}$ be the set of real numbers. A function $NS: S \to \mathbb{R}$ is called a numerical scale of S, and $NS(s_i)$ is referred to as the numerical index of s_i.

If $NS(s_i) < NS(s_{i+1})$ ($\forall i = 0,1, \ldots, g-1$), then the numerical scale NS on S is ordered.

Note 1. The concept of the numerical scale was first proposed in [4]. The established range of the numerical scale will not influence its essence, and in the original definition [4] the value of numerical scale is defined on the real number set in a general way, which provides a connect framework for computing with words [5]: setting $NS(s_i) = i (i = 0,1,\dots,g)$ yields the 2-tuple linguistic model [10]; setting $NS(s_i) = CCV(s_i) (i = 0,1,\dots,g)$ yields the Wang and Hao model [36]; setting $NS(s_i) = \Delta^{-1}(s_{I'(i)}^{n(t_m)})$ ($i = 0,1,\dots,g$) yields the unbalanced linguistic model [15].

Definition 5 [4]. Let S be defined as above. The 2-tuple numerical scale $NS: \overline{S} \rightarrow \mathbb{R}$ is:

$$NS(s_i, \alpha) = \begin{cases} NS(s_i) + \alpha \times \left(NS(s_{i+1}) - NS(s_i)\right), \alpha \geq 0 \\ NS(s_i) + \alpha \times \left(NS(s_i) - NS(s_{i-1})\right), \alpha < 0 \end{cases} \quad (4)$$

The inverse of a 2-tuple numerical scale NS is $NS^{-1}: \mathbb{R} \rightarrow \overline{S}$

$$NS^{-1}(r) = \begin{cases} \left(s_i, \frac{r - NS(s_i)}{NS^k(s_{i+1}) - NS^k(s_i)}\right), & NS(s_i) < r < \frac{NS(s_i)+NS(s_{i+1})}{2} \\ \left(s_i, \frac{r - NS(s_i)}{NS(s_i) - NS(s_{i-1})}\right), & \frac{NS(s_{i-1})+NS(s_i)}{2} \leq r \leq NS(s_i) \end{cases} \quad (5)$$

In [5], the authors showed that the numerical scale model provides a unified framework to connect the 2-tuple linguistic model [10], the proportional 2-tuple linguistic model [36] and the unbalanced linguistic model [15]. To address the fact that words mean different things for different people, Li et al. [19] proposed numerical scale based consistency-driven optimization models to derive the different decision makers' PISs. They also presented the linguistic GDM with PISs framework shown in Fig.1.

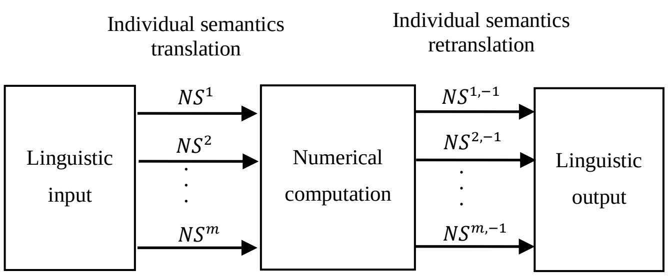


Fig.1. The framework for the linguistic model with PISs

In Fig. 1, NS^k is an ordered numerical scale on S associated with decision maker e_k $(k = 1,2,..,m)$, and the value of $NS^k(s_i)$ represents the individual semantics of decision maker e_k on the term s_i $(i = 0,1,..,g)$. The optimization models to obtain the PISs of decision makers under different decision making environments were proposed in [20] and [21]. Without loss of generality, in this paper, the decision makers' numerical scales range is set as $[0,1]$, instead of $\mathbb{R}$.

III. Consistency Improving Approach Based on PISs with Linguistic Preference Relation

This section presents a novel consistency index based on the personalized numerical scales for linguistic preference relations, and a consistency improving method with PISs in linguistic GDM.

A. *Description of the decision problem*

In the linguistic GDM, $X = \{x_1, x_2, \dots, x_n\}$ $(n \geq 2)$ denotes a set of alternatives and $E = \{e_1, e_2, \dots, e_m\}$ $(m \geq 2)$ a set of decision makers, who express their preferences using linguistic terms in set $S = \{s_0, s_1, \dots, s_g\}$ $(g \geq 2)$: $L^k = (l_{ij}^k)_{n\times n}$ denotes the linguistic preference relation over X provided by decision maker e_k. Decision makers have their own, possibly different, personalized numerical scales over S: NS^kdenotes the PIS of decision maker e_k. The problem to address is how to improve the consistency of a linguistic preference relation in GDM taking into account the decision maker's PIS.

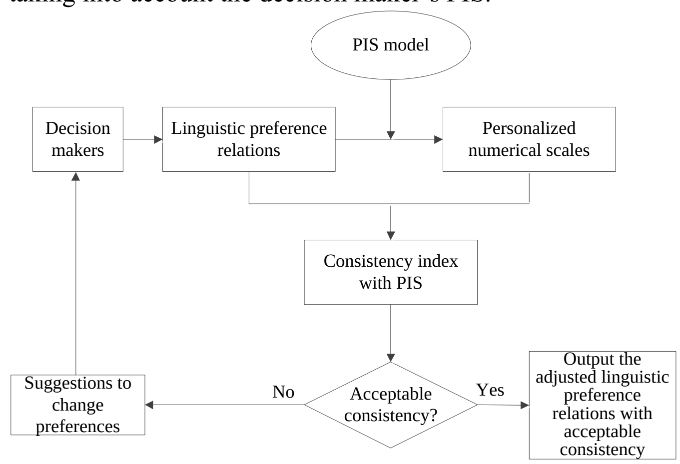


Fig.2. The framework of the consistency improving process with PISs

Fig. 2 illustrates the three phases consistency improving framework with PISs:

(1) *PIS process*. A decision maker's PIS is obtained by solving the corresponding linguistic preference relation with consistency-driven optimization model.

(2) *Consistency measurements based on the PISs*. The consistency of linguistic preference relations with PISs are measured to judge whether their consistency is acceptable within the PIS context.

(3) *Feedback recommendation for improving consistency*. Decision makers with unacceptable consistency based on PIS values receive feedback on how to improve their linguistic preference relations' consistency.

B. *Consistency-based PIS model with linguistic preference relations*

Additive transitivity is commonly used to define consistency of preferences. The concept of additive consistent linguistic preference relation based on numerical scale has been defined as follows:

Definition 6 [16, 21]. A linguistic preference relation on a linguistic term set S, $L = (l_{ij})_{n\times n}$, is a consistent based on a numerical scale, $NS: S \rightarrow [0,1]$, if $NS(l_{ij}) + NS(l_{jz}) - NS(l_{iz}) = 0.5$ $\forall\, i,j,z = 1,2,\dots,n$.

The following definitions are introduced for measuring the consistency of linguistic preference relations.

Definition 7. The distance between linguistic preference relations on a linguistic term set S based on a numerical scale NS is computed as follows:

$$d_{NS}(L^1, L^2) = \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left| NS(l_{ij}^1) - NS(l_{ij}^2) \right|.$$

Definition 8. Let M_n be the set of $n \times n$ linguistic preference relations on a linguistic term set S consistent based on a numerical scale NS. The distance between a linguistic preference relation on a linguistic term set S and set M_n is

$$d_{NS}(L, M_n) = \min_{\bar{L} \in M_n} d_{NS}(L, \bar{L}).$$

The proximity of a linguistic preference relation on a linguistic term set S to the set M_n is proposed as a measure of its consistency index (CI):

$$CI_{NS}(L) = 1 - d_{NS}(L, M_n). \quad (6)$$

The larger the value $CI_{NS}(L) \in [0,1]$, the better the consistency of L.

Proposition 1. The consistency index of a linguistic preference relation based on the numerical scale

$$NS(s_i) = \frac{1}{g} \cdot \Delta^{-1}(s_i)$$

as per expression (6) coincides with the consistency index of the 2-tuple linguistic model as per expression (3).

Proof: Omitted.

In the following, the PISs of a decision maker in linguistic GDM is obtained by developing a consistency-driven optimization model with objective function

$$\max CI_{NS^k}(L^k) = \max_{\bar{L}^k \in M_n} 1 - d_{NS^k}(L^k, \bar{L}^k). \quad (7)$$

with $\bar{L}^k = (\bar{l}_{ij}^k)_{n\times n} \in M_n$ being a consistent linguistic preference relation on a linguistic term set S based on a numerical scale NS^k, i.e.

$$NS^k(\bar{l}_{ij}^k) + NS^k(\bar{l}_{jz}^k) - NS^k(\bar{l}_{iz}^k) = 0.5 \ \forall\, i, j, z \quad (8)$$

and $\bar{l}_{ij}^k = Neg(\bar{l}_{ji}^k) \ \forall\, i, j$.

The range of numerical scale NS^k for linguistic terms s_r $(r = 0,1, \dots, g)$ can be set as follows:

$$NS^k(s_r) \begin{cases} = 0, \ r = 0 \\ \in \left[\frac{r-1}{g}, \frac{r+1}{g}\right], \ r = 1,2, \dots, g-1; r \neq \frac{g}{2} \\ = 0.5, \ r = \frac{g}{2} \\ = 1, r = g \end{cases} \quad (9)$$

Note 2. The set of the range of the numerical scales does not influence the essence of the PIS model. The core of the PIS model is to discuss the distribution of the personalized numerical scale values of linguistic terms within the established range. The semantics of linguistic terms are often defined in the interval [0,1], and thus in this study we set the values of numerical scale for linguistic term in the interval [0,1].

To make NS^k ordered, the following constraint value λ between numerical scales is introduced:

$$NS^k(s_{r+1}) - NS^k(s_r) \geq \lambda, \text{ for } r = 0,1, \dots, g-1 \quad (10)$$

In this paper, we set $\lambda = 0.01$.

Based on Eqs. (7)-(10), the following consistency-driven optimization model derive the PIS of decision maker e_k:

$$\begin{cases} \max CI_{NS^k}(L^k) = 1 - \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left| NS^k(l_{ij}^k) - NS^k(\bar{l}_{ij}^k) \right| \\ s.t. \\ NS^k(\bar{l}_{ij}^k) + NS^k(\bar{l}_{jz}^k) - NS^k(\bar{l}_{iz}^k) = 0.5 \quad \text{for } i, j, z = 1,2, \dots, n \\ \bar{l}_{ij}^k \in S \quad \text{for } i, j = 1,2, \dots, n \\ \bar{l}_{ij}^k = Neg(\bar{l}_{ji}^k) \quad \text{for } i, j = 1,2, \dots, n \\ NS^k(s_0) = 0 \\ NS^k(s_r) \in \left[\frac{r-1}{g}, \frac{r+1}{g}\right], r = 1,2, \dots, g-1; r \neq \frac{g}{2} \\ NS^k\left(s_{\frac{g}{2}}\right) = 0.5 \\ NS^k(s_g) = 1 \\ NS^k(s_{r+1}) - NS^k(s_r) \geq \lambda, \quad r = 0,1, \dots, g-1 \end{cases} \quad (11)$$

In Model (11), $NS^k(s_r)$ $(r = 0,1, \dots, g)$ and $\bar{l}_{ij}^k$ $(i, j = 1,2, \dots, n)$ are decision variables. By solving Model (11), we can obtain the personalized numerical scales of linguistic terms for decision makers, i.e., $NS^k(s_r)$ $(r = 0,1, \dots, g)$. In addition, we can also obtain the associated consistent linguistic preference relations associated with L^k, i.e., $\bar{L}^k = (\bar{l}_{ij}^k)_{n\times n}$. The decision variable $NS^k(\bar{l}_{ij}^k)$ $(i, j = 1,2, \dots, n)$ with the associated consistent numerical preference relation $\bar{L}^k$ show the difference between Model (11) and the existing PIS models [19,20,22].

By solving Model (11), the personalized numerical scales for the different decision makers based on their personal understanding of words for decision makers, as represented by their provided linguistic preference relations, are obtained.

Note 3. Model (11) can be easily transformed a linear programming model, and thus the Weierstrass theorem guarantees the existence of the optimal solution(s) in Model (11) because it has a closed bounded nonempty feasible region. There exists a two-stage general procedure [2] to deal with the case that multiple optimal solutions exist in linear programming models. This procedure can directly be applied in Model (11), and for details, see [2]. In this paper, we focus on the consistency improving of linguistic preference relations, which is an iterative process with a feedback recommendation. The obtained optimal solution(s) just provide a reference for decision makers to modify their preferences, and thus the uniqueness of the solution is not the focus of our model.

Following novel consistency index of linguistic preference relations based on PISs is now introduced:

Definition 9. Let NS^k and L^k be defined as before, and $\bar{L}^k = (\bar{l}_{ij}^k)_{n\times n}$ be the consistent linguistic preference relation obtained from Model (11). The consistency index of L^k based on the PIS is computed as

$$CI_{NS^k}(L^k) = 1 - \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left| NS^k(l_{ij}^k) - NS^k(\bar{l}_{ij}^k) \right| \quad (12)$$

A larger value of $CI_{NS}(L^k)$ indicates a better consistency of L^k. When $CI_{NS^k}(L^k) = 1$, L^k is fully consistent.

C. *PIS-based consistency improving algorithm*

Next, we describe in detail the algorithm to improve the consistency of linguistic preference relations with PISs.

(1) *PIS process*. Apply the optimization Model (11) to obtain the PIS of L^k, $\{NS^k(s_0), NS^k(s_1), \dots, NS^k(s_g)\}$, and its consistency index, $CI_{NS^k}(L^k)$.

(2) Feedback recommendation for improving consistency. Let $\bar{L}^k = (\bar{l}_{ij}^k)_{n\times n}$, obtained from Model (11), be the consistent linguistic preference relation associated to L^k. *A* new linguistic preference relation $L'^k = (l'^k_{ij})_{n\times n}$ is constructed based on L^k and $\bar{L}^k$:

- When $l_{ij}^k < \bar{l}_{ij}^k$, the decision maker e_k should increase the preference value l_{ij}^k to be closer to $\bar{l}_{ij}^k$, i.e., $l'^k_{ij} \in (l_{ij}^k, \bar{l}_{ij}^k]$;
- When $l_{ij}^k > \bar{l}_{ij}^k$, the decision maker e_k should decrease the preference value l_{ij}^k to be closer to $\bar{l}_{ij}^k$, i.e., $l'^k_{ij} \in [\bar{l}_{ij}^k, l_{ij}^k)$;
- When $l_{ij}^k = \bar{l}_{ij}^k$, then e_k should not change the preference value l_{ij}^k, i.e., $l'^k_{ij} = l_{ij}^k = \bar{l}_{ij}^k$.

The PIS-based consistency improving algorithm is summarized in Algorithm 1 below:

ALGORITHM 1
PIS-BASED CONSISTENCY IMPROVING ALGORITHM

Input: The linguistic term set $S = \{s_0, s_1, \dots, s_g\}$; the set of decision makers $E = \{e_1, e_2, \dots, e_m\}$; the linguistic preference relations $\{L^k = (l_{ij}^k)_{n\times n} | k = 1, \dots, m\}$; the consistency threshold $\overline{CI}$; and the maximum number of iterations T.

Output: The adjusted linguistic preference relations $\{L'^k = (l'^k_{ij})_{n\times n} | k = 1, \dots, m\}$ and their consistency indices $\{CI_{NS^{k,t}}(L'^k) | k = 1, \dots, m\}$.

Step 1: Let $t = 0$, and $L^{k,t} = (l_{ij}^{k,t})_{n\times n} = L^{k,0} = (l_{ij}^k)_{n\times n}$.

Step 2: Solve Model (11) to obtain the PISs of $\{L^{k,t} | k = 1, \dots, m\}$, $\{NS^{k,t}(s_0), NS^{k,t}(s_1), \dots, NS^{k,t}(s_g) | k = 1, \dots, m\}$, the associated consistent linguistic preference relation $\bar{L}^{k,t} = \left(\bar{l}_{ij}^{k,t}\right)_{n\times n}$ with $\bar{l}_{ij}^{k,t} = NS^{-1,k}\left(NS^{k,t}(\bar{l}_{ij}^{k,t})\right)$, and their consistency indices $\{CI_{NS^{k,t}}(L^{k,t}) | k = 1, \dots, m\}$. If $CI_{NS^{k,t}}(L^{k,t}) \geq \overline{CI}$ $\forall k$ or $t = T$, then go to Step 4; otherwise, go to Step 3.

Step 3: Based on $\bar{L}^{k,t} = \left(\bar{l}_{ij}^{k,t}\right)_{n*n}$, to obtain $L^{k,t+1} = \left(l_{ij}^{k,t+1}\right)_{n\times n}$, it is required that,

$$l_{ij}^{k,t+1} \begin{cases} \in \left(l_{ij}^{k,t}, \bar{l}_{ij}^{k,t}\right], & If\ l_{ij}^{k,t} < \bar{l}_{ij}^{k,t} \\ \in \left[\bar{l}_{ij}^{k,t}, l_{ij}^{k,t}\right), & If\ l_{ij}^{k,t} > \bar{l}_{ij}^{k,t} \\ = l_{ij}^{k,t}, & If\ l_{ij}^{k,t} = \bar{l}_{ij}^{k,t} \end{cases} \tag{13}$$

Let $t = t + 1$, return to Step 2.

Step 4: Let $L'^k = L^{k,t}$. Output the adjusted linguistic preference relation with acceptable consistency $\{L'^k = (l'^k_{ij})_{n\times n} | k = 1, \dots, m\}$ and their consistency indices $\{CI_{NS^{k,t}}(L'^k) | k = 1, \dots, m\}$.

The below results prove that Algorithm 1 increases the consistency index values.

Theorem 1. Let $\overline{CI}$ be the consistency threshold in Algorithm 1. Let $L^{k,t} = (l_{ij}^{k,t})_{n\times n}$ be the linguistic preference relations generated by Algorithm 1 and $CI_{NS^{k,t}}(L^{k,t})$ its consistency index. Then, $CI_{NS^{k,t}}(L^{k,t}) \geq \overline{CI}$ $\forall k$; otherwise, if $\exists k$: $CI_{NS^{k,t}}(L^{k,t}) < \overline{CI}$, then $CI_{NS^{k,t}}\left(L_{ij}^{k,t}\right)$ is monotone increasing, with respect to t, towards $\overline{CI}$.

Proof: In Algorithm 1, by solving Model (11), we obtain the consistency index of $L^{k,t}$: $CI_{NS^{k,t}}(L^{k,t})$. If $\exists k$: $CI_{NS^{k,t}}(L^{k,t}) < \overline{CI}$, then a consistent linguistic preference relation $\bar{L}^{k,t}$ associated to $L^{k,t}$, is constructed: $CI_{NS^{k,t}}(\bar{L}^{k,t}) = 1$. Based on Eq. (13),

$l_{ij}^{k,t+1} \in \left[min\left(l_{ij}^{k,t}, \bar{l}_{ij}^{k,t}\right), max\left(l_{ij}^{k,t}, \bar{l}_{ij}^{k,t}\right)\right] \Rightarrow NS(l_{ij}^{k,t+1}) \in [NS(min\left(l_{ij}^{k,t}, \bar{l}_{ij}^{k,t}\right)), NS(max\left(l_{ij}^{k,t}, \bar{l}_{ij}^{k,t}\right))]$.

$d\left(l_{ij}^{k,t}, \bar{l}_{ij}^{k,t}\right) \geq d\left(l_{ij}^{k,t+1}, \bar{l}_{ij}^{k,t}\right) \geq d\left(l_{ij}^{k,t+1}, \bar{l}_{ij}^{k,t+1}\right)$ $\forall i, j$.

From Definition 3, it is

$$d\left(L_{ij}^{k,t}, \bar{L}_{ij}^{k,t}\right) \geq d\left(L_{ij}^{k,t+1}, \bar{L}_{ij}^{k,t}\right) \geq d\left(L_{ij}^{k,t+1}, \bar{L}_{ij}^{k,t+1}\right) \Rightarrow CI_{NS^{k,t}}\left(L_{ij}^{k,t}\right) \leq CI_{NS^{k,t+1}}\left(L_{ij}^{k,t+1}\right).$$

The sequence $\{CI_{NS^{k,t}}\left(L_{ij}^{k,t}\right) | t = 0,1,2, \dots, T\}$ is monotone increasing towards $\overline{CI}$.

Theorem 1 guarantees that the adjusted linguistic preference relations, obtained by the PIS-based consistency improving algorithm (Algorithm 1) will have the acceptable consistency or a higher consistency degree close to the threshold value $\overline{CI}$.

Note 4. The value of $\overline{CI}$ is to determine whether the consistency of a linguistic preference relation is reached. The value of $\overline{CI}$ is different to different decision making problems, and it should be set according to the specific decision making contexts. While Algorithm 1 provides a general approach to improve the consistency of linguistic preference relations based on PISs, and it works when setting different threshold values $\overline{CI}$.

IV. NUMERICAL ANALYSIS

In this section, numerical examples are included to illustrate the use of the consistency improving algorithm with PISs using the linguistic term set $S = \{s_0 = extremely\ poor, s_1 = very\ poor, s_2 = poor, s_3 = fair, s_4 = good,\ s_5 = very\ good, s_6 = extremely\ good\}$, a set of four decision makers, $E = \{e_1, e_2, e_3, e_4\}$, and a set of five alternatives, $X = \{x_1, x_2, x_3, x_4, x_5\}$. The decision makers provide the below linguistic preference relations based on S, $L^k = (l_{ij}^k)_{5\times 5}$ $(k = 1,2,3,4)$, to express their preferences over X.

$$L^1 = \begin{pmatrix} null & s_4 & s_1 & s_6 & s_5 \\ null & null & s_2 & s_3 & s_3 \\ null & null & null & s_0 & s_5 \\ null & null & null & null & s_2 \\ null & null & null & null & null \end{pmatrix}$$

$$L^2 = \begin{pmatrix} null & s_6 & s_4 & s_0 & s_0 \\ null & null & s_6 & s_3 & s_2 \\ null & null & null & s_5 & s_1 \\ null & null & null & null & s_6 \\ null & null & null & null & null \end{pmatrix}$$

$$L^3 = \begin{pmatrix} null & s_0 & s_6 & s_0 & s_4 \\ null & null & s_1 & s_5 & s_0 \\ null & null & null & s_6 & s_2 \\ null & null & null & null & s_5 \\ null & null & null & null & null \end{pmatrix}$$

$$L^4 = \begin{pmatrix} null & s_1 & s_6 & s_6 & s_0 \\ null & null & s_0 & s_5 & s_1 \\ null & null & null & s_4 & s_2 \\ null & null & null & null & s_6 \\ null & null & null & null & null \end{pmatrix}$$

(1) *The first iteration with PISs.* Let $L^1 = L^{1,0}$, $L^2 = L^{2,0}$, $L^3 = L^{3,0}$ and $L^4 = L^{4,0}$. Solving model (11) with linguistic preference relations $L^{k,0}\ (k = 1,2,3,4)$, the PISs for the linguistic terms for the four decision makers, $NS^{k,0}(s_i)(k = 1,2,3,4; i = 0,1,\ldots,6)$, are obtained, and listed in Table I.

TABLE I
VALUES OF $NS^{k,0}(s_i)$ $(k = 1,2,3,4; i = 0,1,\ldots,6)$

	$k = 1$	$k = 2$	$k = 3$	$k = 4$
$NS^{k,1}(s_0)$	0	0	0	0
$NS^{k,1}(s_1)$	0.333	0.01	0.333	0.333
$NS^{k,1}(s_2)$	0.49	0.49	0.49	0.343
$NS^{k,1}(s_3)$	0.5	0.5	0.5	0.5
$NS^{k,1}(s_4)$	0.657	0.51	0.51	0.657
$NS^{k,1}(s_5)$	0.667	0.667	0.667	0.667
$NS^{k,1}(s_6)$	1	1	1	1

The consistency indices based on the PISs are: $CI_{NS^{1,0}}(L^{1,0}) = 0.866$, $CI_{,NS^{2,0}}(L^{2,0}) = 0.78$, $CI_{NS^{3,0}}(L^{3,0}) = 0.698$ and $CI_{NS^{4,0}}(L^{4,0}) = 0.731$.

And from Model (11), it also obtains the associated consistent linguistic preference relations $\bar{L}^{k,0}(k = 1,2,3,4)$ as follows,

$$\bar{L}^{1,0} = \begin{pmatrix} null & (s_{5,} - 0.4) & (s_3, -0.261) & s_5 & (s_5, 0.057) \\ null & null & (s_1, 0.287) & (s_3, 0.025) & (s_3, 0.146) \\ null & null & null & (s_4, -0.204) & (s_4, -0.083) \\ null & null & null & null & (s_3, 0.121) \\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{2,0} = \begin{pmatrix} null & (s_{1,}0.053) & s_4 & (s_1, 0.053) & (s_1, 0.026) \\ null & null & (s_6, -0.06) & s_3 & s_2 \\ null & null & null & (s_1, 0.026) & s_1 \\ null & null & null & null & s_2 \\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{3,0} = \begin{pmatrix} null & (s_{2,} - 0.401) & (s_2, -0.083) & (s_1, 0.452) & (s_1, 0.038) \\ null & null & (s_4, 0.025) & (s_2, -0.083) & (s_4, 0.299) \\ null & null & null & (s_2, -0.401) & (s_4, -0.3) \\ null & null & null & null & (s_4, 0.446) \\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{4,0} = \begin{pmatrix} null & (s_6, -0.441) & (s_6, -0.471) & s_6 & (s_5, 0.057) \\ null & null & (s_3, -0.064) & (s_4, -0.064) & s_1 \\ null & null & null & s_4 & s_2 \\ null & null & null & null & (s_1, -0.442) \\ null & null & null & null & null \end{pmatrix}$$

Then, the adjusted linguistic preference relations, $L^{k,1}(k = 1,2,3,4)$, that satisfy $l_{ij}^{k,1} \in \left[\min(l_{ij}^{k,0}, \bar{l}_{ij}^{k,0}), \max(l_{ij}^{k,0}, \bar{l}_{ij}^{k,0})\right]$ and $l_{ij}^{k,1} \neq l_{ij}^{k,0}$, are

$$L^{1,1} = \begin{pmatrix} null & s_4 & s_2 & s_5 & s_5 \\ null & null & s_2 & s_3 & s_3 \\ null & null & null & s_2 & s_4 \\ null & null & null & null & s_3 \\ null & null & null & null & null \end{pmatrix}$$

$$L^{2,1} = \begin{pmatrix} null & s_4 & s_4 & s_0 & s_1 \\ null & null & s_6 & s_3 & s_2 \\ null & null & null & s_2 & s_1 \\ null & null & null & null & s_5 \\ null & null & null & null & null \end{pmatrix}$$

$$L^{3,1} = \begin{pmatrix} null & s_1 & s_5 & s_1 & s_3 \\ null & null & s_2 & s_4 & s_1 \\ null & null & null & s_5 & s_3 \\ null & null & null & null & s_4 \\ null & null & null & null & null \end{pmatrix}$$

$$L^{4,1} = \begin{pmatrix} null & s_5 & s_6 & s_6 & s_3 \\ null & null & s_2 & s_4 & s_1 \\ null & null & null & s_4 & s_2 \\ null & null & null & null & s_3 \\ null & null & null & null & null \end{pmatrix}$$

By solving Model (11) with linguistic preference relations $L^{1,1}, L^{2,1}, L^{3,1}$ and $L^{4,1}$, the PISs for linguistic terms for the four decision makers, $NS^{k,1}(s_i)(k = 1,2,3,4; i = 0,1,\ldots,6)$, are obtained and listed in Table II.

TABLE II
VALUES OF $NS^{k,1}(s_i)(k = 1,2,3,4; i = 0,1,\ldots,6)$

	$k = 1$	$k = 2$	$k = 3$	$k = 4$
$NS^{k,1}(s_0)$	0	0	0	0
$NS^{k,1}(s_1)$	0.1	0.01	0.333	0.333
$NS^{k,1}(s_2)$	0.49	0.196	0.49	0.49
$NS^{k,1}(s_3)$	0.5	0.5	0.5	0.5
$NS^{k,1}(s_4)$	0.604	0.51	0.51	0.51
$NS^{k,1}(s_5)$	0.667	0.667	0.667	0.99
$NS^{k,1}(s_6)$	1	1	1	1

The consistency indices based on the PISs are: $CI_{NS^{1,1}}(L^{1,1}) = 0.965$, $CI_{NS^{2,1}}(L^{2,1}) = 0.881$, $CI_{NS^{3,1}}(L^{3,1}) = 0.913$ and $CI_{NS^{4,1}}(L^{4,1}) = 0.9$.

(2) *The second iteration with PISs.*

By solving Model (11), we also obtain the associated consistent linguistic preference relations, $\bar{L}^{k,1}(k = 1,2,3,4)$, as follows,

$$\bar{L}^{1,1} = \begin{pmatrix} null & (s_4, 0.465) & (s_4, -0.307) & (s_4, 0.365) & (s_5, -0.19) \\ null & null & (s_2, -0.11) & (s_3, 0.125) & (s_3, 0.298) \\ null & null & null & (s_4, -0.375) & (s_4, -0.202) \\ null & null & null & null & (s_3, 0.173) \\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{2,1} = \begin{pmatrix} null & (s_{2,}0.076) & s_4 & s_1 & (s_1, 0.027) \\ null & null & (s_5, 0.372) & (s_2, 0.3125) & (s_2, 0.329) \\ null & null & null & s_0 & s_0 \\ null & null & null & null & (s_4, -0.5) \\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{3,1} = \begin{pmatrix} null & (s_{2,} - 0.452) & (s_2, -0.401) & (s_1, -0.42) & (s_2, -0.395) \\ null & null & (s_4, -0.1) & (s_4, -0.4) & (s_4, -0.1) \\ null & null & null & (s_4, -0.3) & s_3 \\ null & null & null & null & s_3 \\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{4,1} = \begin{pmatrix} null & (s_6, -0.3) & (s_5, -0.04) & (s_6, -0.1) & (s_5, -0.144) \\ null & null & (s_2, 0.1) & (s_3, 0.2) & (s_2, -0.42) \\ null & null & null & (s_4, 0.02) & (s_2, -0.363) \\ null & null & null & null & (s_2, -0.433) \\ null & null & null & null & null \end{pmatrix}$$

The adjusted linguistic preference relation, $L^{k,2}(k = 1,2,3,4)$, that satisfy $l_{ij}^{k,2} \in (l_{ij}^{k,1}, \bar{l}_{ij}^{k,1}]$ are

$$L^{1,2} = \begin{pmatrix} null & s_4 & s_3 & s_5 & s_5 \\ null & null & s_2 & s_3 & s_3 \\ null & null & null & s_3 & s_4 \\ null & null & null & null & s_3 \\ null & null & null & null & null \end{pmatrix}$$

$$L^{2,2}=\begin{pmatrix} null & s_3 & s_4 & s_1 & s_1 \\ null & null & s_6 & s_3 & s_2 \\ null & null & null & s_0 & s_0 \\ null & null & null & null & s_4 \\ null & null & null & null & null \end{pmatrix}$$
$$L^{3,2}=\begin{pmatrix} null & s_1 & s_2 & s_1 & s_2 \\ null & null & s_3 & s_4 & s_3 \\ null & null & null & s_4 & s_3 \\ null & null & null & null & s_3 \\ null & null & null & null & null \end{pmatrix}$$
$$L^{4,2}=\begin{pmatrix} null & s_5 & s_5 & s_6 & s_4 \\ null & null & s_2 & s_4 & s_1 \\ null & null & null & s_4 & s_2 \\ null & null & null & null & s_2 \\ null & null & null & null & null \end{pmatrix}$$

By solving Model (11), the PISs of $L^{k,2}(k=1,2,3,4)$ are obtained and listed un Table III.

TABLE III
VALUES OF $NS^{k,2}(s_i)(k=1,2,3,4; i=0,1,\dots,6)$

	$k=1$	$k=2$	$k=3$	$k=4$
$NS^{k,2}(s_0)$	0	0	0	0
$NS^{k,2}(s_1)$	0.1	0.333	0.157	0.333
$NS^{k,2}(s_2)$	0.343	0.49	0.167	0.343
$NS^{k,2}(s_3)$	0.5	0.5	0.5	0.5
$NS^{k,2}(s_4)$	0.657	0.51	0.51	0.671
$NS^{k,2}(s_5)$	0.667	0.828	0.667	0.828
$NS^{k,2}(s_6)$	1	1	1	1

The consistency indices based on the PISs are:

$$CI_{NS^{1,2}}(L^{1,2})=0.983,\ CI_{NS^{2,2}}(L^{2,2})=0.949,$$
$$CI_{NS^{3,2}}(L^{3,2})=0.996 \text{ and } CI_{NS^{4,2}}(L^{4,2})=0.966.$$

In accordance to Theorem 1, the numerical analysis clearly corroborates that the consistency indices of the linguistic preference relations increase in value from one round application of Algorithm 1 to the next.

V. COMPARATIVE STUDY

This section reports on a comparative study between the PIS based consistency improving method (Algorithm 1) and the corresponding one without implementing, which is based on the 2-tuple linguistic model (Algorithm 2).

A. The consistency improving method without PISs

When PISs have no role, decision makers are assumed to have the same words' semantics, and the 2-tuple linguistic model is used as the linguistic computational model. Algorithm 2 derives from Algorithm 1 by replacing all the NSs with the function Δ^{-1} in the representation of the semantics of linguistic expressions, i.e., we set $NS^k(s_i)=\Delta^{-1}(s_i)$ for linguistic terms $s_i\ (i=0,1,\dots,g)$ for decision makers $e_k\ (k=1,2,\dots,m)$.

ALGORITHM 2
CONSISTENCY IMPROVING ALGORITHM BASED ON THE 2-TUPLE LINGUISTIC MODEL

Input: The linguistic term set $S=\{s_0,s_1,\dots,s_g\}$; the set of decision makers $E=\{e_1,e_2,\dots,e_m\}$; the linguistic preference relations $\{L^k=(l_{ij}^k)_{n\times n}|k=1,\dots,m\}$; the consistency threshold $\overline{CI}$; and the maximum number of iterations T.

Output: The adjusted linguistic preference relations $\{L'^k=(l'^k_{ij})_{n\times n}|k=1,\dots,m\}$ and their consistency indices $\{CI(L'^k)|k=1,\dots,m\}$.

Step 1: Let $t=0$, let $L^{k,t}=(l_{ij}^{k,t})_{n\times n}=L^{k,0}=(l_{ij}^k)_{n\times n}$.

Step 2: Construct the associated numerical preference relation of $L^{k,t}$, $F^{k,t}=(f_{ij}^{k,t})_{n\times n}$, where $f_{ij}^{k,t}=\Delta^{-1}(l_{ij}^{k,t})$. If $CI(F^{k,t})\geq\overline{CI}$ or $t=T$, then go to Step 5; otherwise, go to Step 3.

Step 3 : If $CI(F^{k,t})$ is unacceptable, then construct the consistent numerical preference relation $\bar{F}^k=(\bar{f}_{ij}^k)_{n\times n}$ associated to F^k by solving the following model:

$$\begin{cases} \min\ d(F^k,\bar{F}^k) \\ s.t. \\ \bar{f}_{ij}^k+\bar{f}_{jz}^k-\bar{f}_{iz}^k=0.5 \quad \text{for } i,j,z=1,2,\dots,n \\ \bar{f}_{ij}^k\in[0,1] \quad \text{for } i,j=1,2,\dots,n \\ \bar{f}_{ij}^k+\bar{f}_{ji}^k=1 \quad \text{for } i,j=1,2,\dots,n \end{cases} \tag{14}$$

where $d(F^k,\bar{F}^k)=\frac{2}{n(n-1)}\sum_{i=1}^n\sum_{j=i+1}^n\left|f_{ij}^k-\bar{f}_{ij}^k\right|$. Solving Model (14) obtains the consistent numerical preference relation $\bar{F}^{k,t}=(\bar{f}_{ij}^{k,t})_{n\times n}$ associated to F^k.

Step 4: Construct the associated linguistic preference relation $\bar{L}^{k,t}=\left(\bar{l}_{ij}^{k,t}\right)_{n\times n}$ of $\bar{F}^{k,t}$, where $\bar{l}_{ij}^{k,t}=\Delta\left(\bar{f}_{ij}^{k,t}\right)$. For $L^{k,t+1}=\left(l_{ij}^{k,t+1}\right)_{n\times n}$, it is required that

$$l_{ij}^{k,t+1}\begin{cases} \in\left(l_{ij}^{k,t},\bar{l}_{ij}^{k,t}\right], & If\ l_{ij}^{k,t}<\bar{l}_{ij}^{k,t} \\ \in\left[\bar{l}_{ij}^{k,t},l_{ij}^{k,t}\right), & If\ l_{ij}^{k,t}>\bar{l}_{ij}^{k,t} \\ =l_{ij}^{k,t}, & If\ l_{ij}^{k,t}=\bar{l}_{ij}^{k,t} \end{cases} \tag{15}$$

Let $t=t+1$, return to Step 2.

Step 5: Let $L'^k=L^{k,t}$. Output the adjusted linguistic preference relation with acceptable consistency $\{L'^k=(l'^k_{ij})_{n\times n}|k=1,\dots,m\}$ and their consistency indices $\{CI(L'^k)|k=1,\dots,m\}$.

We apply Algorithm 2 to the same linguistic preference relations $L^k(k=1,2,3,4)$ provided in Section IV. The semantics of linguistic terms $\{s_0,s_1,\dots,s_6\}$ based on the 2-tuple linguistic model for all decision makers is: $\Delta^{-1}(s_0)=0$; $\Delta^{-1}(s_1)=0.167$; $\Delta^{-1}(s_2)=0.333$; $\Delta^{-1}(s_3)=0.5$; $\Delta^{-1}(s_4)=0.667$; $\Delta^{-1}(s_5)=0.833$ and $\Delta^{-1}(s_6)=1$.

(1) *The first iteration without considering PISs.* The linguistic preference relations are transformed into their associated numerical ones:

$$F^{1,0}=\begin{pmatrix} 0.5 & 0.667 & 0.167 & 1 & 0.833 \\ null & 0.5 & 0.333 & 0.5 & 0.5 \\ null & null & 0.5 & 0 & 0.833 \\ null & null & null & 0.5 & 0.333 \\ null & null & null & null & 0.5 \end{pmatrix}$$
$$F^{2,0}=\begin{pmatrix} 0.5 & 1 & 0.667 & 0 & 0 \\ null & 0.5 & 1 & 0.5 & 0.333 \\ null & null & 0.5 & 0.833 & 0.167 \\ null & null & null & 0.5 & 1 \\ null & null & null & null & 0.5 \end{pmatrix}$$

$$F^{3,0}=\begin{pmatrix} 0.5 & 0 & 1 & 0 & 0.667\\ \text{null} & 0.5 & 0.167 & 0.833 & 0\\ \text{null} & \text{null} & 0.5 & 1 & 0.333\\ \text{null} & \text{null} & \text{null} & 0.5 & 0.833\\ \text{null} & \text{null} & \text{null} & \text{null} & 0.5 \end{pmatrix}$$

$$F^{4,0}=\begin{pmatrix} 0.5 & 0.167 & 1 & 1 & 0\\ \text{null} & 0.5 & 0 & 0.833 & 0.167\\ \text{null} & \text{null} & 0.5 & 0.667 & 0.333\\ \text{null} & \text{null} & \text{null} & 0.5 & 1\\ \text{null} & \text{null} & \text{null} & \text{null} & 0.5 \end{pmatrix}$$

By solving Model (14), the consistent numerical preference relations are:

$$\bar{F}^{1,0}=\begin{pmatrix} 0.5 & 0.729 & 0.56 & 0.9 & 0.733\\ 0.271 & 0.5 & 0.331 & 0.67 & 0.503\\ 0.44 & 0.669 & 0.5 & 0.84 & 0.672\\ 0.1 & 0.33 & 0.16 & 0.5 & 0.333\\ 0.267 & 0.497 & 0.328 & 0.667 & 0.5 \end{pmatrix}$$

$$\bar{F}^{2,0}=\begin{pmatrix} 0.5 & 0.125 & 0.562 & 0.062 & 0.062\\ 0.875 & 0.5 & 0.937 & 0.437 & 0.437\\ 0.438 & 0.063 & 0.5 & 0 & 0\\ 0.938 & 0.563 & 1 & 0.5 & 0.5\\ 0.938 & 0.563 & 1 & 0.5 & 0.5 \end{pmatrix}$$

$$\bar{F}^{3,0}=\begin{pmatrix} 0.5 & 0.062 & 0.281 & 0 & 0.226\\ 0.938 & 0.5 & 0.718 & 0.437 & 0.663\\ 0.719 & 0.282 & 0.5 & 0.219 & 0.445\\ 1 & 0.563 & 0.781 & 0.5 & 0.726\\ 0.774 & 0.337 & 0.555 & 0.274 & 0.5 \end{pmatrix}$$

$$\bar{F}^{4,0}=\begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.75 & 0.25\\ 0.5 & 0.5 & 0.5 & 0.75 & 0.25\\ 0.5 & 0.5 & 0.5 & 0.75 & 0.25\\ 0.25 & 0.25 & 0.25 & 0.5 & 0\\ 0.75 & 0.75 & 0.75 & 1 & 0.5 \end{pmatrix}$$

Based on Eq. (3), the following consistency indices are obtained : $CI(L^{1,0})=0.817, CI(L^{2,0})=0.717, CI(L^{3,0})=0.617$ and $CI(L^{4,0})=0.683$. These values are lower than the values obtained with PISs.

The corresponding consistent linguistic preference relations are:

$$\bar{L}^{1,0}=\begin{pmatrix} null & (s_4,0.477) & (s_3,0.359) & (s_5,0.401) & (s_4,0.395)\\ null & null & (s_2,-0.012) & (s_4,0.018) & (s_3,0.0005)\\ null & null & null & (s_5,0.042) & (s_4,0.03)\\ null & null & null & null & s_2\\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{2,0}=\begin{pmatrix} null & (s_{1,}-0.251) & (s_3,0.371) & (s_0,0.371) & (s_0,0.371)\\ null & null & (s_6,-0.377) & (s_3,-0.377) & (s_3,-0.377)\\ null & null & null & s_0 & s_0\\ null & null & null & null & s_3\\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{3,0}=\begin{pmatrix} null & (s_{0,}0.371) & (s_2,-0.311) & s_0 & (s_1,0.353)\\ null & null & (s_4,0.305) & (s_3,-0.377) & (s_4,-0.024)\\ null & null & null & (s_1,0.311) & (s_3,-0.329)\\ null & null & null & null & (s_4,0.353)\\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{4,0}=\begin{pmatrix} null & s_3 & s_3 & (s_4,0.497) & (s_1,0.497)\\ null & null & s_3 & (s_4,0.497) & (s_1,0.497)\\ null & null & null & (s_4,0.497) & (s_1,0.497)\\ null & null & null & null & s_0\\ null & null & null & null & null \end{pmatrix}$$

The adjusted linguistic preference relations $L^{k,1}(k=1,2,3,4)$, which satisfy $l_{ij}^{k,1}\in(l_{ij}^{k,0},\bar{l}_{ij}^{k,0}]$, are

$$L^{1,1}=\begin{pmatrix} null & s_4 & s_3 & s_6 & s_5\\ null & null & s_2 & s_4 & s_3\\ null & null & null & s_3 & s_5\\ null & null & null & null & s_2\\ null & null & null & null & null \end{pmatrix}$$

$$L^{2,1}=\begin{pmatrix} null & s_4 & s_4 & s_0 & s_0\\ null & null & s_6 & s_3 & s_2\\ null & null & null & s_2 & s_0\\ null & null & null & null & s_4\\ null & null & null & null & null \end{pmatrix}$$

$$L^{3,1}=\begin{pmatrix} null & s_0 & s_3 & s_0 & s_3\\ null & null & s_4 & s_4 & s_2\\ null & null & null & s_2 & s_2\\ null & null & null & null & s_5\\ null & null & null & null & null \end{pmatrix}$$

$$L^{4,1}=\begin{pmatrix} null & s_2 & s_4 & s_5 & s_2\\ null & null & s_3 & s_5 & s_1\\ null & null & null & s_4 & s_2\\ null & null & null & null & s_3\\ null & null & null & null & null \end{pmatrix}$$

The consistency indices are: $CI(L^{1,1})=0.917$, $CI(L^{2,1})=0.85$, $CI(L^{3,1})=0.85$ and $CI(L^{4,1})=0.883$. These values are lower than the values obtained based on PISs.

(2) *The second iteration without considering PISs.* The linguistic preference relations $L^{k,1}(k=1,2,3,4)$ are transformed into their associated numerical preference relations, which are fed into Model (14), from which the following consistent numerical preference relations are obtained:

$$\bar{F}^{1,1}=\begin{pmatrix} 0.5 & 0.77 & 0.528 & 0.942 & 0.775\\ 0.23 & 0.5 & 0.258 & 0.672 & 0.505\\ 0.472 & 0.742 & 0.5 & 0.914 & 0.747\\ 0.058 & 0.328 & 0.086 & 0.5 & 0.333\\ 0.225 & 0.495 & 0.253 & 0.667 & 0.5 \end{pmatrix}$$

$$\bar{F}^{2,1}=\begin{pmatrix} 0.5 & 0.131 & 0.631 & 0.131 & 0.131\\ 0.869 & 0.5 & 1 & 0.5 & 0.5\\ 0.369 & 0 & 0.5 & 0 & 0\\ 0.869 & 0.5 & 1 & 0.5 & 0.5\\ 0.869 & 0.5 & 1 & 0.5 & 0.5 \end{pmatrix}$$

$$\bar{F}^{3,1}=\begin{pmatrix} 0.5 & 0.195 & 0.495 & 0.327 & 0.381\\ 0.805 & 0.5 & 0.8 & 0.631 & 0.686\\ 0.505 & 0.2 & 0.5 & 0.331 & 0.386\\ 0.673 & 0.369 & 0.669 & 0.5 & 0.555\\ 0.619 & 0.314 & 0.614 & 0.445 & 0.5 \end{pmatrix}$$

$$\bar{F}^{4,1}=\begin{pmatrix} 0.5 & 0.503 & 0.652 & 0.827 & 0.448\\ 0.497 & 0.5 & 0.649 & 0.824 & 0.444\\ 0.348 & 0.351 & 0.5 & 0.675 & 0.296\\ 0.173 & 0.176 & 0.324 & 0.5 & 0.121\\ 0.552 & 0.556 & 0.704 & 0.879 & 0.5 \end{pmatrix}$$

The corresponding consistent linguistic preference relations are:

$$\bar{L}^{1,1}=\begin{pmatrix} null & (s_5,-0.377) & (s_3,0.168) & (s_6,-0.347) & (s_5,-0.347)\\ null & null & (s_2,-0.449) & (s_4,0.03) & (s_3,0.03)\\ null & null & null & (s_5,0.485) & (s_4,0.476)\\ null & null & null & null & s_2\\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{2,1}=\begin{pmatrix} null & (s_{1,}-0.215) & (s_4,-0.215) & (s_1,-0.215) & (s_1,-0.215)\\ null & null & s_6 & s_3 & s_3\\ null & null & null & s_0 & s_0\\ null & null & null & null & s_3\\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{3,1}=\begin{pmatrix} null & (s_{1,}0.168) & (s_3,-0.03) & (s_2,-0.036) & (s_2,0.287)\\ null & null & (s_5,-0.198) & (s_4,-0.215) & (s_4,0.114)\\ null & null & null & (s_2,-0.012) & (s_2,0.317)\\ null & null & null & null & (s_3,0.329)\\ null & null & null & null & null \end{pmatrix}$$

$$\bar{L}^{4,1}=\begin{pmatrix} null & (s_3,0.018) & (s_4,-0.09) & (s_5,-0.036) & (s_3,-0.311)\\ null & null & (s_4,-0.108) & (s_5,-0.054) & (s_3,-0.335)\\ null & null & null & (s_4,0.048) & (s_2,-0.221)\\ null & null & null & null & (s_1,-0.275)\\ null & null & null & null & null \end{pmatrix}$$

The adjusted linguistic preference relations $L^{k,2}(k=1,2,3,4)$, which satisfy $l_{ij}^{k,2} \in (l_{ij}^{k,1}, \bar{l}_{ij}^{k,1}]$, are

$$L^{1,2} = \begin{pmatrix} null & s_4 & s_3 & s_6 & s_5 \\ null & null & s_2 & s_4 & s_3 \\ null & null & null & s_5 & s_5 \\ null & null & null & null & s_2 \\ null & null & null & null & null \end{pmatrix}$$

$$L^{2,2} = \begin{pmatrix} null & s_3 & s_4 & s_0 & s_0 \\ null & null & s_6 & s_3 & s_3 \\ null & null & null & s_1 & s_0 \\ null & null & null & null & s_4 \\ null & null & null & null & null \end{pmatrix}$$

$$L^{3,2} = \begin{pmatrix} null & s_0 & s_3 & s_1 & s_3 \\ null & null & s_4 & s_4 & s_4 \\ null & null & null & s_2 & s_2 \\ null & null & null & null & s_4 \\ null & null & null & null & null \end{pmatrix}$$

$$L^{4,2} = \begin{pmatrix} null & s_2 & s_4 & s_5 & s_2 \\ null & null & s_3 & s_5 & s_2 \\ null & null & null & s_4 & s_2 \\ null & null & null & null & s_1 \\ null & null & null & null & null \end{pmatrix}$$

The consistency indices obtained are: $CI(L^{1,2}) = 0.95$, $CI(L^{2,2}) = 0.9$, $CI(L^{3,2}) = 0.917$ and $CI(L^{4,2}) = 0.917$. These values are again lower than the values obtained with PISs.

Both Algorithms 1 and 2 improve the consistency of linguistic preference relations, being the improvement higher with PISs (Algorithm 1) than without PISs (Algorithm 2). In the next section, the difference between the two algorithms will be further analyzed with a simulation analysis.

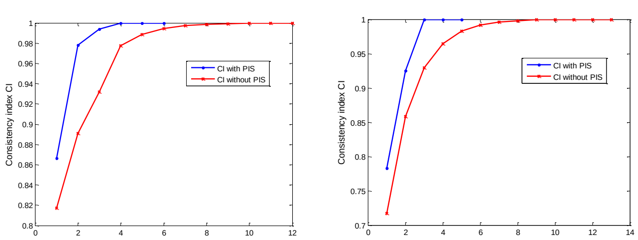
B. Simulation analysis

A simulation analysis to explore the speed of convergence to consistency of the linguistic preference relations by both Algorithms is given below. To automatically change the preferences of decision makers, Eqs. (13) and (15) in Algorithms 1 and 2 are replaced with Eqs. (16) and (17), respectively,

$$l_{ij}^{k,t+1} = NS^{-1}\left(\gamma \times NS(l_{ij}^{k,t}) + (1-\gamma) \times NS(\bar{l}_{ij}^{k,t})\right),\ \gamma \in [0,1) \quad (16)$$

$$l_{ij}^{k,t+1} = \Delta\left(\gamma \times \Delta^{-1}(l_{ij}^{k,t}) + (1-\gamma) \times \Delta^{-1}(\bar{l}_{ij}^{k,t})\right),\ \gamma \in [0,1) \quad (17)$$

The same linguistic preference relations $L^k(k=1,2,3,4)$ provided in Section IV are used with values $\gamma = 0.5; \gamma = \frac{1}{3}$; and $\gamma = \frac{2}{3}$. The consistency variation of $L^k(k=1,2,3,4)$ using Algorithm 1 and Algorithm 2 are depicted in Figs. 3, 4 and 5, respectively.



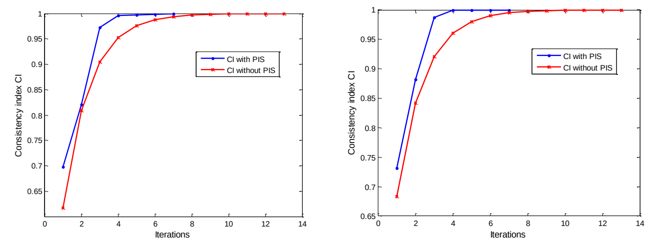


Fig.3 Consistency improvement process for L^k $(k=1,2,3,4)$ for Algorithms 1 and 2 $(\gamma = 0.5)$

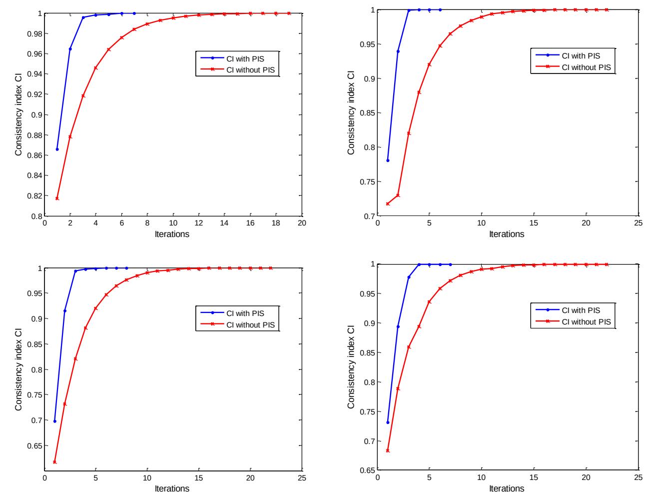


Fig. 4 Process to improve consistency of L^k $(k=1,2,3,4)$ based on Algorithms 1 and 2 $(\gamma = \frac{1}{3})$

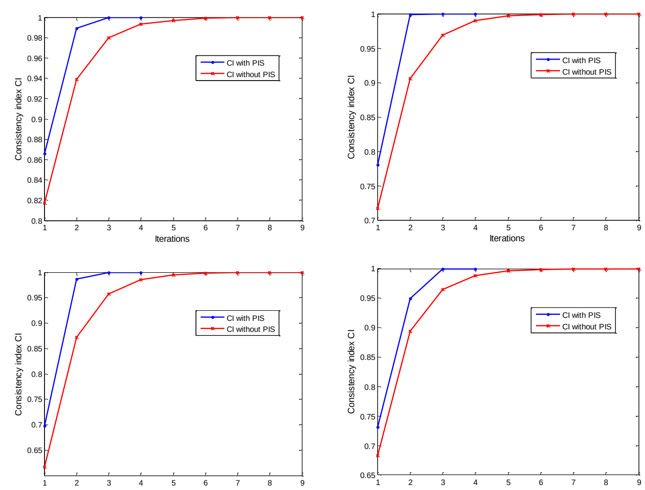


Fig. 5 Process to improve consistency of L^k $(k=1,2,3,4)$ based on Algorithms 1 and 2 $(\gamma = \frac{2}{3})$

C. Lessons learnt

The following observations are drawn:

(1) The consistency levels of linguistic preference relations improve with both Algorithms. The improvement process is increasing, and because of their boundedness property, it is convergent.

(2) Algorithm 1 improves consistency more rapidly than Algorithm 2. For $\gamma = 0.5$, the consistency index reach 1 in less than 6 iterations of Algorithm 1, while it takes 12 iterations of

Algorithm 2. For $\gamma=\frac{1}{3}$ and $\gamma=\frac{2}{3}$, the consistency index reach 1 in about 5 and 3 iterations of Algorithm 1, respectively, while it requires about 20 and 9 iterations of Algorithm 2, respectively.

(3) The number of iterations required for the consistency index to reach 1 decreases when the value of γ increases. For Algorithm: $\gamma=\frac{1}{3}$ requires about 9 iterations; $\gamma=\frac{1}{2}$ requires about 5 iterations; and $\gamma=\frac{2}{3}$ requires 3 iterations, respectively.

The above observations show that the implementation of PISs can improve consistency in GDM effectively. Particularly, from the comparisons with Algorithm 2, the PIS-based approach shows that personalized numerical meanings of words can help decision makers achieving personalized adjusted linguistic preference relations with acceptable consistency more rapidly.

VI. Conclusion

The use of PISs in linguistic GDM provides a new avenue for studying consistency issues. In this paper, a novel PIS based consistency index for linguistic preference relations is being introduced. By integrating a consistency-driven optimization model, an iterative algorithm with PISs has been developed to improve the consistency of linguistic preference relations. Finally, we provide numerical analysis to illustrate the application of the proposed model, and report on a detailed simulated analysis the differences between consistency improving process of the proposed PIS based approach and the corresponding 2-tuple linguistic model approach that does not implement PISs. The implementation of PISs leads to higher increasers of consistency and a more rapid convergence to the established consistency level than that when PISs are not considered. Therefore, the PIS-based method provides a useful tool to measure the consistency with PISs and to improve the consistency degree of linguistic preference relations.

Although the PIS-based method is performing well to manage the consistency measurement and improvement with linguistic preference relations, in GDM more complex linguistic environments than the research in this paper exist. These are based on the use of hesitant linguistic term sets [29, 39], linguistic distribution [41], multi-granular linguistic term set [31] and flexible linguistic expressions [37]. In the future, we will further study PIS-based approaches to consistency issues in such complex linguistic environments.

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