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Abstract

We examine how moral hazard impacts risk-sharing when risk-taking can be part of the mechanism design. In a two-agent model with binary effort, we show that moral hazard always increases risk-taking (that is the amount of wealth invested in a risky project) whereas the effect on risk-sharing (the amount of wealth transferred between agents) is ambiguous. Risk-taking therefore appears as a useful incentive tool. In particular, in the case of preferences exhibiting Constant Absolute Risk Aversion (CARA), moral hazard has no impact on risk-sharing and risk-taking is the unique mechanism used to solve moral hazard. Thus, risk-taking appears to be the prevailing incentive tool.

Keywords: Risk-Taking, Informal Insurance, Moral Hazard

JEL: D82, D86, G22

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1   Introduction

When investing in a risky project, one should be concerned by two main aspects: what amount to invest and how to get insured against the risk of failure. If these two aspects are pretty easy to deal with separately, it appears that there are difficult to tackle jointly especially in presence of an other key feature of project financing: moral hazard.

Consider an agent who can invest in a project that increases her expected wealth but also increases its variance (through the use of new technology or entrepreneurship for instance). Everything equal, this agent will be willing to invest more in this project if he is insured against the risk of failure. Consider now the easiest way to get insured: mutual revenue sharing. Agents insure themselves by elaborating bilateral transfers contingent to the state of nature, without relying on a third part, as an insurance company.[1] When agents are equally wealthy and have same risk aversion, and if individual risks are idiosyncratic, then, under complete information, the optimal risk-sharing agreement consists in full equal sharing of wealth. By aggregating and dividing equally, such a contract minimizes risk in the sense of the mean-preserving spread criterion. Hence, full equal sharing would provide an incentive to take risk.

However, when agents can exert an effort to enhance the probability of success of their risky project, equal sharing of wealth can be subject to moral hazard if effort is unobservable. Hence, in the absence of efficient peer monitoring, a mechanism design should foster incentives to exert effort. Basically, such a contract must be such that agents' revenues are more sensitive to their efforts. The traditional tool is the reduction of transfers. Indeed, by increasing exposure to own risk, the reduction of risk-sharing should in principle restore incentives to exert effort. In a context of endogeneous risk-taking decision, a direct consequence of reduced risk-sharing would be the decrease of risk-taking.

Now, suppose that agents set up risk-sharing contracts and take risk *simultaneously*. In that context, risk-taking can itself be used to create incentives to exert effort. Intuition suggests that to restore incentives, agents should increase risk-taking, because this increases the impact of their own effort on their revenue. In a word, to increase exposure

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[1]Such mechanisms are often used in village economies where private insurance is not developed, but also appears among insurers for reinsurance propose.

to own risk, taking more risk is a solution. Therefore, the joint use of transfers and risk-taking as incentive tools induces a priori ambiguous prediction on risk-taking.

This article analyzes optimal risk-taking and risk-sharing decisions in presence of moral hazard. We consider a two-agent model in which agents can affect their wealth distribution through two decisions: first they decide upon a share of wealth to invest in a risky projet (that we interpret as the level of risk); second they can exert some costly effort to increase the probability of success of their project. We show that risk-taking is indeed an incentive tool: introducing moral hazard always increases the level of risk. Moreover, it appears that – contrary to cases where risk-taking is exogenous – moral hazard has an ambiguous effect on (absolute) transfer. In particular, for Constant Absolute Risk Aversion (CARA) preferences, the level of wealth transferred between agents does not depend on moral hazard, and risk-taking is the unique tool used to solve moral hazard problems. Risk-taking appears then to be the prevailing incentive tool.

We briefly discuss the relationship of this paper with the literature. First, our work fits into the mechanism design literature. Our paper can indeed by related to works that analyze the sharing of wealth between a principal and an agent when the aggregate wealth depends on the effort exerted by the agent. For example, Braveman and Stiglitz (1982) and Thiele and Wambach (1999) study the mechanism a principal (a landowner or an entrepreneur) has to design to induce the agent (a tenant or an employee) to exert the required level of effort. In particular, Braveman and Stiglitz (1982) use such a model to give a rational to interlinking contracts (that is contracts with interlinkages among the land, labor, credit, and product markets) in situation where the agent can choose a level of effort[2] Our work differs from this brand literature in the definition of the contracting parties. Whereas Braveman and Stiglitz (1982) and Thiele and Wambach (1999) model a risk-neutral profit-maximizer principal that wants to design an incentive compatible contract, we focus here on the incentive compatible agreement that can emerge between two risk averse agents who bargain over the sharing of aggregate wealth. Therefore, in opposite to the papers cited above, our work is based on

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[2]In Braveman and Stiglitz (1982) the agent can also choose a risk level, but the analysis of optimal effort and risk choices are conducted separately.

the maximization of an ex-ante utilitarist criterion.

This notion of risk-sharing has been first developed by Borch (1960) who modeled risk-sharing agreements as a two-person cooperative agreement similar to ours. His main point was then to state (Borch 1962) the mutualization principle: under complete information, the optimal agreement makes individual wealth only depend on state of nature insofar as the aggregate wealth in that state is concerned. However, full equal sharing is not often observed (Townsend 1994). Enforceability is a standard economic rationale to explain limited commitment (Coate and Ravaillon 1993, Ligon, Thomas and Worrall 2002, Dubois, Jullien and Magnac 2008). Alternatively, recent work on risk-sharing under asymmetric information (Bourlès and Henriet, 2010) however shows that the mutualization principle no longer holds when agents have private information on their individual distribution of wealth (*i.e.* their risk type) if heterogeneity is high and risk aversion is low. Regarding moral hazard, works by Laffont and Martimort (2002) or Demange (2008) have also pointed out that consistent with the intuition developed above, moral hazard reduces risk-sharing, in the sense that it decreases the amount of transfers among agents for each given state of nature. Our paper shows that the introduction of risk-taking in the mechanism design changes this pattern.

Our work is also related to the literature addressing group incentives. Analyzing moral hazard in teams or clubs, Holmstrom (1982) and Prescott and Townsend (2006) model situations where agents work on a joint project, whose outcome depend on joint efforts. Used to analyze how firms emerge and operate, these works differ from ours in a key aspect. In the literature on group incentives, the efforts of all agents determine the distribution of the aggregate outcome that has to be split among them. In our model, the effort of each agent determines the distribution of its own outcome from which she can transfer wealth to others. Therefore, in contrast with the (benevolent) principal of our model, in a model of group incentive, the principal is unable to infer from the outcome the contribution of each agents.

Moreover, our work contributes to the literature on risk-taking and more precisely on the standard portfolio problem. In this widely used model, agents choose to allocate their available wealth between a safe (i.e. with constant return) project and a risky project (whose return is stochastic). Our main contribution to this literature is the modeling of moral hazard and risk-sharing in the standard portfolio problem. We first allow agents to enhance the probability of success of the risky project (whose return

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is supposed to be binomial). Our paper is therefore related to Fishburn and Porter (1976) or Hadar and Seo (1990), who study the effect of a shift of distribution of return. They show that a stochastic dominating shift in the return of the risky asset does not necessarily increase investment in that asset.[3] To tackle this issue, Fishburn and Porter (1976) and Hadar and Seo (1990) provide some conditions on the investor's utility function for a dominating shift not to decrease the investment in the risky asset. Taking another approach, Landsberger and Meilijson (1990) conclude that stochastic dominance is not a good measure of risk in this case. They show that a shift in the distribution of returns of the risky asset in the sense of likelihood ratio leads to an increase in demand for this asset by all investors with nondecreasing utilities. Our work contributes to this discussion by stating that the inclusion of moral hazard on an effort that increases the expected return of the risky asset always leads to an increase in the demand for this asset.

To our knowledge, only few papers analyze risk-taking and risk-sharing simultaneously. One exception is Pratt and Zeckhauser (1989) that look at group decision on risk-taking. In their paper, the group has to choose among monetary risks and payoff-sharing rules. Their main result is then to show that under HARA (harmonic absolute risk aversion) preferences, the efficient group choices are independent of payoff sharing. Our paper seems to indicate that when moral hazard is modeled this is no longer the case. Still regarding this relationship between risk-sharing and risk taking, a recent work by Belhaj and Deroïan (2009) shows that when agents share risk through an exogenous risk-sharing network, an increase of risk-sharing does not necessarily lead to an increase of risk taking. By adding to the purpose moral hazard and by endogeneizing risk sharing, we support this conclusion in a two-agent model as we show that moral hazard can increase risk-taking and decrease risk sharing.

The rest of this paper is structured as follows. Section 2 introduces the two-agent model of risk-taking and discusses the benchmark case of complete information. In Section 3, we incorporate moral hazard to the model and characterize the optimal incentive-compatible sharing rule and level of risk taking. Our concluding remarks and

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[3]Fishburn and Porter focus on first order stochastic dominance and on the case of only one risky asset, whereas Hadar and Seo generalize this conclusion to second and third order stochastic dominance and to the case of more than one risky asset

suggestions for future research are in Section 4.

2   The model

Two identical risk-averse agents can make transfers to cope with volatile revenue. They can affect the distribution of their revenue through two decisions. First, agents face the standard portfolio choice problem, that is, they have to allocate their wealth between two investments, one risk-free and the other risky. Second, agents choose a level of effort that affects the probability of success of their risky investment.

More precisely, each agent is endowed with wealth ω, and can invest a share $\alpha \leq 1$ of this wealth in the risky project. We interpret α as the level of risk taken by the agents. The remaining part is invested in the risk-free project whose gross return is normalized to 1. The risky project gives a return μ with probability p (in case of success) and 0 otherwise. For the risky project to be profitable, we assume that the expected gross return on investment is higher than the cost: $p\alpha\omega\mu > \alpha\omega$, i.e. $p\mu > 1$. In the absence of risk-sharing, a single agent chooses α in order to maximize his expected utility:

$$\max_{\alpha} \left\{ (1-p)u\big(\omega(1-\alpha)\big) + pu\big(\omega(1+\alpha(\mu-1))\big) \right\} \tag{1}$$

whose solution is given by α_0 such that:

$$\frac{u'\big(\omega(1-\alpha_0)\big)}{u'\big(\omega(1+\alpha_0(\mu-1))\big)} = \frac{p}{1-p}(\mu-1) \tag{2}$$

The LHS of equation (2) is increasing in α_0 for all $\mu > 1$. The condition $p\mu > 1$ implies $\frac{1-p}{p} < \mu - 1$. Therefore, the solution of problem (1) is unique. Note that, since the LHS of (2) is decreasing in ω if $-\frac{u''(\omega(1-\alpha))}{u'(\omega(1-\alpha))} > -\frac{u''(\omega(1+\alpha(\mu-1)))}{u'(\omega(1+\alpha(\mu-1)))}$, the amount invested in the risky project is increasing with wealth ω if the utility function exhibit decreasing absolute risk aversion (DARA) (see Pratt [1961]).

Now consider two identical individuals that face this basic investment problem and can share risk through monetary transfers. We assume that agents can observe both the levels of risk and transfers. They can therefore set up contract on risk levels and

transfers[4]. Formally, if agents choose different risk levels four net transfers would be designed. Here, we only focus on the symmetric equilibrium (that is an equilibrium where both agents choose the same risk level α). This corresponds to a utilitarian criterion proposed by a benevolent principal that weights equally both agents.[5] From symmetry, no net transfers optimally take place when the two risky projects succeeds or fail. We denote by t the transfer from the successful agent to the one that failed. The maximization problem of utilitarian criterion writes:

$$\max_{\alpha,t} (1-p)^2 u\big(\omega(1-\alpha)\big) + p(1-p)\Big(u\big(\omega(1-\alpha)+t\big) + u\big(\omega(1+\alpha(\mu-1))-t\big)\Big)$$

$$+p^2 u\big(\omega(1+\alpha(\mu-1))\big) \tag{3}$$

We then find:

$$\begin{cases} t^* = \frac{\alpha^*\omega\mu}{2} \\ \frac{pu'\left(\omega+\frac{\alpha^*\omega(\mu-2)}{2}\right)+(1-p)u'(\omega-\alpha^*\omega)}{pu'(\omega+\alpha^*\omega(\mu-1))+(1-p)u'\left(\omega+\frac{\alpha^*\omega(\mu-2)}{2}\right)} = \frac{p}{1-p}(\mu-1) \end{cases} \tag{4}$$

Result 1 *The optimal transfer is such that agents share their revenue equally. Furthermore, risk-sharing enhances risk-taking with respect to the no sharing case.*

Not surprisingly, risk mutualization entails that agents share wealth equally.

3 Risk taking, risk-sharing and moral hazard

We incorporate moral hazard in the analysis. We model a situation in which each agent can exert a costly effort that increases the probability of success of her risky project. The traditional tool to induce effort is to reduce transfer. The economic intuition behind this mechanism is that reducing transfer increases the exposure of each agent toward her own risk. Moreover, if we allowed risk-taking to be used as an incentive tool, economic intuition would suggest that, for a given level of transfer, an increase in risk-taking would induce effort. Since risk-taking and risk-sharing are complementary, in

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[4]We assume in the following that contracts are enforceable. For a discussion on enforceability see Coate and Ravaillon 1993, Ligon, Thomas and Worrall 2002 or Dubois, Jullien and Magnac 2008

[5]The solution can be interpreted as the outcome of a bargaining Nash solution with equal outside option and equal bargaining power.

the sense that more risk-sharing enhances risk taking, the optimal mechanism remains unclear: reducing insurance shall decrease risk-taking by complementarity, reducing thus the impact of risk-taking on efforts' incentives; conversely increasing risk-taking shall induce more insurance, thus limiting its impact on efforts incentives. How do agents, investing in a risky project and building up optimal transfers, use jointly these tools to enhance incentives to exert effort?

For the sake of simplicity, we consider two effort levels $\underline{e}$ and $\overline{e}$, with $\underline{e} < \overline{e}$, that lead respectively to probabilities of success $\underline{p}$ and $\overline{p}$ such that $\overline{p} - \underline{p} > 0$. The cost for providing high effort $\overline{e}$ (rather than $\underline{e}$) is denoted ψ, and $C = \frac{\psi}{\overline{p} - \underline{p}}$. This fits for instance with informational cost.[6]

The timing of the game is as follows (see figure 1). First, agents agree on a level of investment (which is assumed to be observable) and contract upon transfers. Second agents choose an unobservable level of effort. Third nature generates realizations (given levels of investment and efforts). Last agents proceed to transfers. Let $\pi_i(e_i, e_j, \alpha, t)$ by
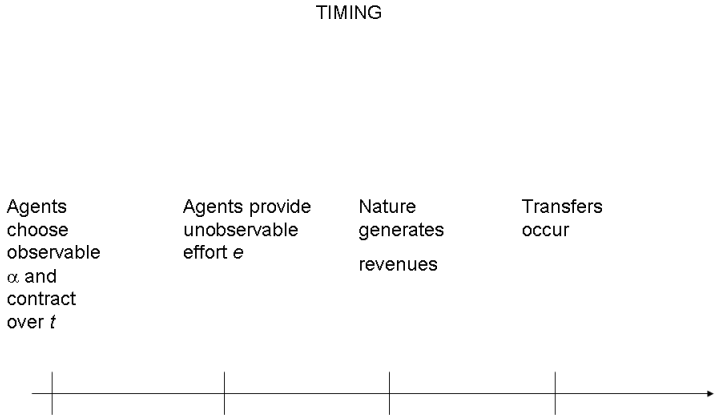
TIMING

Agents choose observable α and contract over t

Agents provide unobservable effort e

Nature generates revenues

Transfers occur

Figure 1: Timing of the game with moral hazard

the utility of agent i when agent i (resp. j) exert effort e_i (resp. e_j), under symmetric

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[6]The analysis is extended locally to affine cost of effort, i.e. to $C = C_0 + \alpha C_1$, with C_1 small enough.

risk level α and symmetric transfer t. Let also $\alpha(e)$ (resp. $t(e)$) represents a symmetric risk level (resp. transfer) under symmetric effort e. To make the problem non-trivial, we make the following two assumptions (the first imposes C to be low enough, the second to be high enough):

Assumption 1 *An isolated agent is always interested in undertaking effort. This implies that the optimal incentive compatible transfer is nonnegative. Indeed, by concavity of utility functions, negative transfers are dominated by null transfer, which is the autarky case.*

Assumption 2 *The first best equilibrium is not incentive compatible:*

$$\pi(\overline{e}, \overline{e}, x^*(\overline{e}), t^*(\overline{e})) < \pi(\underline{e}, \overline{e}, x^*(\overline{e}), t^*(\overline{e}))$$

That is, given that investments and transfers are observable, defecting upon effort from the first-best equilibrium is individually beneficial.

We focus, in the following, on the incentive compatible (second-best) optimum which represents the overall optimum if it provides the agents with higher expected utility than when they both agents exert low effort and share wealth equally. This again corresponds to an upward bound for C.

The following lemma is useful to describe the maximization program of agents.

Lemma 1 *In the case of symmetric behavior, the only incentive compatibility constraint that has to be checked is the one when the other agent exerts high effort $\overline{e}$.*

The intuition behind Lemma 1 is the following. The incentive to free ride on effort is higher when the opportunity of receiving a transfer is high. As an agent will receive a transfer only if the other agent succeeds, the probability of getting a transfer increases with the effort of the other agent. Therefore the incentive to shirk is higher when the other agent exerts effort.

Let π^f (resp. π^s) represent the agent's expected payoff in case of failure (resp. success):

$$
\begin{aligned}
\pi^f(\alpha, t) &= (1 - \overline{p})u(\omega(1 - \alpha)) + \overline{p}u(\omega(1 - \alpha) + t) \\
\pi^s(\alpha, t) &= (1 - \overline{p})u(\omega(1 + \alpha(\mu - 1)) - t) + \overline{p}u(\omega(1 + \alpha(\mu - 1)))
\end{aligned}
$$

By lemma 1, the maximization program becomes:

$$\max_{\alpha,t} \quad (1-\bar{p})\pi^f(\alpha,t) + \bar{p}\pi^s(\alpha,t)$$

$$s.t. \quad \pi^s(\alpha,t) - \pi^f(\alpha,t) \geq C$$

Hence, agents want to maximize their overall expected payoffs, under the condition of maintaining a minimal difference C between expected payoffs in case of success and failure. As mentioned before, the optimal contract involving both transfer and risk level is not trivial. Indeed, let us examine how individual profit varies along the incentive constraint (see figure 2).
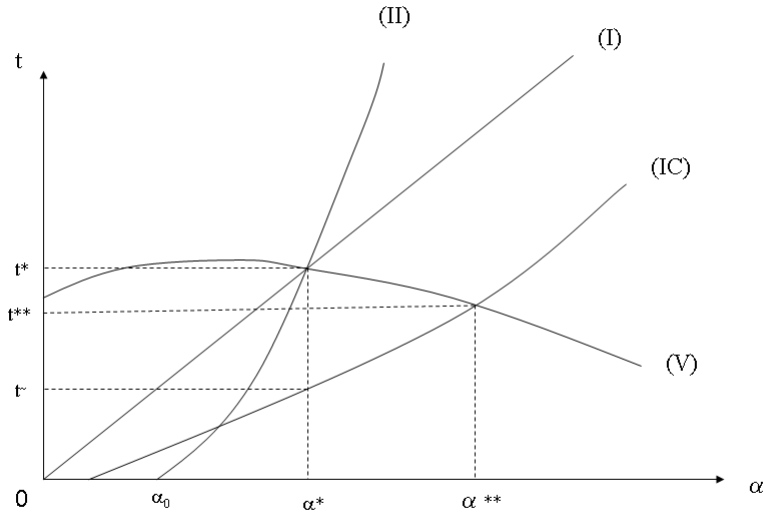

Figure 2: The FOCs and IC in the plan (α, t)

The curves (I) and (II) depicted in the plan (α,t) on figure 2 represent the FOCs (4) of the system without moral hazard. Their intersection corresponds to the first-best optimum (α^*, t^*). The curve (IC) is given by the equation $IC(\alpha,t) = 0$ that represents the binding incentive constraint. First, it can be easily shown that the implicit relationship between risk and transfers on the curve $IC(\alpha,t)$ is positive. Second, by assumption 2, (IC) is below (I) at α^*. Third, the intersection between curve (II) and α-axis, α_0, is below the curve (IC) since producing high effort in autarky is optimal.

The Lagrangian, that takes into account the incentive constraint, is maximized when the curve (V), given by the equation $V(\alpha, t) = 0$, intersects the curve (IC). [7]

From the first-best equilibrium (α^*, t^*), let us modify transfers, without modifying risk, in order to restore incentive compatibility. This corresponds to the point $(\alpha^*, \tilde{t})$ on curve (IC). Basically, $\tilde{t} < t^*$, since restoring efforts' incentives requires to lower insurance. Now, from this point, how does profit vary on curve (IC) locally? Noticing that curve (IC) is increasing, should the agents both increase transfer and risk, or both decrease transfer and risk? Increasing (resp. decreasing) both risk and transfer, both π^f and π^s are affected ambiguously. Indeed, increasing risk level α is detrimental to π^f and beneficial to π^s. Conversely, increasing transfer t is detrimental to π^s and beneficial to π^f. Simple local analysis indicates that profit locally increases on curve IC from the point $(\alpha^*, \tilde{t})$ if risk and transfer are increased. Then, locally, risk-taking should be increased. Next theorem, that delivers the main message of the paper, expresses that this result is not only local, but also global:

Theorem 1 *The optimal incentive compatible contract involves an increase in risk-taking with respect to the first-best equilibrium, i.e. $\alpha^{**} > \alpha^*$.*

The message of the theorem is simple. To increase incentive to exert effort, the contract must increase the dependence of the revenue to effort. Under exogenous risk taking, this appropriate mechanism is to reduce transfers. However, when risk is endogenous, the appropriate mechanism consists in both increasing risk and reducing relative transfers (with respect to full equal sharing).

If the impact of moral hazard on risk-taking entails a clear-cut result, its impact on absolute transfer is ambiguous. Indeed, as can be seen in figure 2, if the curve (V) is decreasing with the level of risk (for $\alpha > \alpha^*$), then $t^{**} < t^*$. This point is related to the convexity of the index of absolute risk aversion; however there does not seem to have a simple condition (parameter-free).[8] Simuations indicate that $t^{**} < t^*$ in most

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[7]Function V is defined in the proof of the theorem - equation (16) -, it is a combination of first derivatives of the Lagrangien with respect to risk and transfer.

[8]This feature seems to be linked with the work of Eeckhoudt et al. (1996) that look at the effect of a zero-mean background risk (that is a risk uncorrelated with the return of the risky asset) in the basic standard portfolio problem (with is without risk-sharing). risk-sharing can be understood as an

of the cases, but the following example illustrates that $t^{**} > t^*$ is a possible outcome.

Example 1 *Consider $u(\omega) = -\frac{1}{a}exp(-a.\omega) + k.\omega^4$, with $k = 0.00005$, $a = 0.1$. Then u is concave for $\omega < 8$. We find $t^{**} > t^*$ for $C = 1.9$, $\omega = 5$, $\mu = 2.2$, $p = 0.455$ (the participation constraint is effective in this example).*

The next illustration focuses on CARA utility functions. It is shown that the presence of moral hazard does not affect transfers; i.e. only risk-taking is used as an incentive tool.

Example 2 (CARA utilities) *In the case of CARA utilities, the absolute transfer is not impacted by the presence of moral hazard (proof in the appendix). For $u(\omega) = \frac{-1}{\rho}exp(-\rho\omega)$, where ρ is the index of absolute risk aversion, $t^* = t^{**} = \frac{1}{\rho}ln\left[(\mu-1)\frac{\bar{p}}{1-\bar{p}}\right]$. The optimal transfer is thus increasing in the level of risk aversion ρ, the probability of success $\bar{p}$ and the return on the risky technology μ.*

It appears for this last example that, when risk-taking decision is endogenous and part of the mechanism design, (absolute) transfer may not be used to provide an incentive to exert effort. As we have shown in theorem 1 that risk-taking is always an effective tool to provide this incentive, it therefore seems that risk-taking is the prevailing incentive mechanism in presence of moral hazard and risk sharing.

Moreover, example 1 illustrate the fact that allowing for risk-taking to be endogeneous change a lot the story. Indeed, when risk-taking is not part of the mechanism design, to provide the agents an incentive to exert the effort, the principal necessarily decreases transfer (with respect to first). This would in turn reduce risk-taking (as it reduces the insurance provided to the agents). In example 1 both variables go in the opposite direction. When allowing for risk-taking to be part of the mechanism design, in this example, the contract provides an incentive to exert effort by increasing both risk-taking and absolute transfer (with respect to the first-best situation).

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additional risk, that corresponds to the risk of having to transfer (or receiving) some wealth. However, this additional risk is clearly negatively correlated with the return of the risky asset as optimally, the probability of having to transfer wealth is higher when the realized return on the risk asset/project is high (and the probability of receiving wealth is higher when realized return on the risk asset/project is high). Eeckhoudt et al. (1996) show that a zero-mean background risk reduces the demand for the risky asset if absolute risk aversion is decreasing and convex (for a discussion on this last condition see Gollier and Scarmure 1994 or Hatchondo 2008).

4   Conclusion

We have considered a two-agent model in which agents invest in a risky and specific project, and set up optimal transfers simultaneously in the presence of moral hazard. We have shown that for all increasing concave utilities, risk-taking is always enhanced while (absolute) transfers are not always decreased. Further, for CARA utilities, moral hazard has no impact on transfers, it only increases risk. Hence, risk-taking is a prevailing incentive tool.

The model may be extended in several direction, like increasing the number of alternatives of the lotterie, or considering cases where effort affects realizations. Also, our results apply when the cost of effort is independent of the level of risk, like informational costs; the study does not cover situations where the cost of effort varies with the risk level. Further, it could be interesting to extend this analysis to applications in which our three ingredients (risk taking, risk-sharing and moral hazard) apply, like microcredit or reinsurance markets.

5   Proofs

Proof of result 1. The first order conditions are:

$$
\begin{cases}
-(1-p)^2 u^{'}(\omega - \alpha\omega) + p(1-p)\left(-u^{'}(\omega - \alpha\omega + t) + (\mu - 1)u^{'}(\omega + \alpha\omega(\mu - 1) - t)\right) \\
\qquad +p^2(\mu - 1)u^{'}(\omega + \alpha\omega(\mu - 1)) = 0 \\
u'(\omega - \alpha\omega + t) = u'(\omega + \alpha\omega(\mu - 1) - t)
\end{cases}
\tag{5}
$$

From the second equation it is therefore optimal to state $t^* = \frac{\alpha\omega\mu}{2}$ (that is equal sharing of wealth) and the first FOC becomes:

$$
-(1-p)^2 u^{'}(\omega - \alpha^*\omega) + p(1-p)(\mu - 2)u^{'}(\omega + \tfrac{\alpha^*\omega(\mu - 2)}{2}) + p^2(\mu - 1)u^{'}(\omega + \alpha^*\omega(\mu - 1)) = 0 \tag{6}
$$

That is:
$$
\frac{(1-p)u^{'}(\omega - \alpha^*\omega)}{pu^{'}(\omega + \alpha^*\omega(\mu - 1))} = (\mu - 2)u^{'}(\omega + \tfrac{\alpha^*\omega(\mu - 2)}{2}) + \tfrac{p}{1-p}(\mu - 1) \tag{7}
$$

Let us note α_0 and α^* the respective solution of equations (2) and (7). Note first that (6) defines a function $f(\alpha)$ decreasing in α such that $f(\alpha^*) = 0$. Moreover, (6) can be

written as

$$p(\mu - 1)\left[pu'(\omega + \alpha^*\omega(\mu - 1)) + (1 - p)u'(\omega + \tfrac{\alpha^*\omega(\mu-2)}{2})\right]$$

$$-(1 - p)\left[(1 - p)u'(\omega - \alpha^*\omega) + pu'(\omega + \tfrac{\alpha^*\omega(\mu-2)}{2})\right] = 0 \tag{8}$$

That is, defining function g such that

$$g(\alpha) = \frac{pu'(\omega + \tfrac{\alpha\omega(\mu-2)}{2}) + (1 - p)u'(\omega - \alpha\omega)}{pu'(\omega + \alpha\omega(\mu - 1)) + (1 - p)u'(\omega + \tfrac{\alpha\omega(\mu-2)}{2})} \tag{9}$$

we have $g(\alpha^*) = \frac{p}{1-p}(\mu - 1)$. Conversely, (2) gives

$$\frac{u'(\omega - \alpha_0\omega)}{u'(\omega + \alpha_0\omega(\mu - 1))} = \frac{p}{1 - p}(\mu - 1) \tag{10}$$

Now as u is concave, we have:

$$\begin{cases} u'(\omega - \alpha^*\omega) \geq (1 - p)u'(\omega - \alpha^*\omega) + pu'(\omega + \tfrac{\alpha^*\omega(\mu-2)}{2}) \\ u'(\omega + \alpha^*\omega(\mu - 1)) \leq pu'(\omega + \alpha\omega(\mu - 1)) + (1 - p)u'(\omega + \tfrac{\alpha\omega(\mu-2)}{2}) \end{cases}$$

Therefore

$$\frac{u'(\omega - \alpha^*\omega)}{u'(\omega + \alpha^*\omega(\mu - 1))} \geq \frac{pu'(\omega + \tfrac{\alpha^*\omega(\mu-2)}{2}) + (1 - p)u'(\omega - \alpha^*\omega)}{pu'(\omega + \alpha^*\omega(\mu - 1)) + (1 - p)u'(\omega + \tfrac{\alpha^*\omega(\mu-2)}{2})} \tag{11}$$

This gives $g(\alpha_0) < \frac{p}{1-p}(\mu - 1)$ that is $f(\alpha_0) > 0 = f(\alpha^*)$. Therefore, as f is decreasing, $x^* > x_0$. $\square$

Proof of lemma 1. To be incentive compatible, the agreement has to give the agent an incentive to exert the high level of effort $\bar{e}$ whatever the level of effort of her opponent. Formally this give:

$$\begin{cases} (1 - \bar{p})^2 u(\omega - \alpha\omega) + \bar{p}(1 - \bar{p})\left(u(\omega - \alpha\omega + t) + u(\omega + \alpha\omega(\mu - 1) - t)\right) + \bar{p}^2 u(\omega + \alpha\omega(\mu - 1)) - \psi \\ \quad \geq (1 - \underline{p})(1 - \bar{p})u(\omega - \alpha\omega) + \underline{p}(1 - \bar{p})u(\omega + \alpha\omega(\mu - 1) - t) \\ \qquad + (1 - \underline{p})\bar{p}u(\omega - \alpha\omega + t) + \underline{p}\bar{p}u(\omega + \alpha\omega(\mu - 1)) \\ \\ (1 - \bar{p})(1 - \underline{p})u(\omega - \alpha\omega) + (1 - \bar{p})\underline{p}u(\omega - \alpha\omega + t) \\ + \bar{p}(1 - \underline{p})u(\omega + \alpha\omega(\mu - 1) - t) + \bar{p}\underline{p}u(\omega + \alpha\omega(\mu - 1)) - \psi \\ \quad \geq (1 - \underline{p})^2 u(\omega - \alpha\omega) + \underline{p}(1 - \underline{p})\left(u(\omega - \alpha\omega + t) + u(\omega + \alpha\omega(\mu - 1) - t)\right) + \underline{p}^2 u(\omega + \alpha\omega(\mu - 1)) \end{cases}$$

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That is:

$$
\begin{cases}
-(\bar{p}-\underline{p})(1-\bar{p})u(\omega-\alpha\omega) - (\bar{p}-\underline{p})\bar{p}u(\omega-\alpha\omega+t) \\
+(\bar{p}-\underline{p})(1-\bar{p})u(\omega+\alpha\omega(\mu-1)-t) + ((\bar{p}-\underline{p}))\bar{p}u(\omega+\alpha\omega(\mu-1)) \geq \psi \\
\\
-(\bar{p}-\underline{p})p(1-\underline{p})u(\omega-\alpha\omega) - (\bar{p}-\underline{p})\underline{p}u(\omega-\alpha\omega+t) \\
+(\bar{p}-\underline{p})(1-\underline{p})u(\omega+\alpha\omega(\mu-1)-t) + (\bar{p}-\underline{p})\underline{p}u(\omega+\alpha\omega(\mu-1)) \geq \psi
\end{cases}
$$

Recalling that $C = \frac{\psi}{\bar{p}-\underline{p}}$, this amounts to

$$
\begin{cases}
i(\bar{p}) \geq C \\
i(\underline{p}) \geq C
\end{cases}
$$

with $i(p) \equiv -(1-p)u(\omega-\alpha\omega)-pu(\omega-\alpha\omega+t)+(1-p)u(\omega+\alpha\omega(\mu-1)-t)+pu(\omega+\alpha\omega(\mu-1))$. As $i'(p) = [u(\omega-\alpha\omega)-u(\omega-\alpha\omega+t)] - [u(\omega+\alpha\omega(\mu-1)-t)-u(\omega+\alpha\omega(\mu-1))] \leq 0$ because of the concavity of u, if $t \geq 0$, $i(\underline{p}) \geq i(\bar{p})$ and the first condition leads to the second. $\diamond$

Proof of theorem 1. The Langrangian is written

$$
(1-p)\pi^f + p\pi^s + \lambda(\pi^s - \pi^f - C) \tag{12}
$$

with λ the Lagrange multiplier of the incentive constraint. The maximization program entails respectively $\frac{\partial L}{\partial t} = 0, \frac{\partial L}{\partial \alpha} = 0, \frac{\partial L}{\partial \lambda} = 0$, that is,

$$
\begin{cases}
\dfrac{\bar{p}u'(\omega-\alpha\omega+t)}{(1-\bar{p})u'(\omega+\alpha\omega(\mu-1)-t)} = \dfrac{\bar{p}+\lambda}{(1-\bar{p})-\lambda} \\
\dfrac{1}{\mu-1}\dfrac{(1-\bar{p})u'(\omega-\alpha\omega)+\bar{p}u'(\omega-\alpha\omega+t)}{(1-\bar{p})u'(\omega+\alpha\omega(\mu-1)-t)+\bar{p}u'(\omega+\alpha\omega(\mu-1))} = \dfrac{\bar{p}+\lambda}{(1-\bar{p})-\lambda} \\
(1-\bar{p})u(\omega+\alpha\omega(\mu-1)-t)+\bar{p}u(\omega+\alpha\omega(\mu-1)) - (1-\bar{p})u(\omega-\alpha\omega)-\bar{p}u(\omega-\alpha\omega+t) = C
\end{cases} \tag{13}
$$

This gives:

$$
\begin{cases}
\dfrac{1-\bar{p}}{\bar{p}}\dfrac{u'(\omega-\alpha\omega)}{u'(\omega-\alpha\omega+t)} + 1 = (\mu-1)\left[\dfrac{\bar{p}}{(1-\bar{p})}\dfrac{u'(\omega+\alpha\omega(\mu-1))}{u'(\omega+\alpha\omega(\mu-1)-t)} + 1\right] \\
-(1-\bar{p})u(\omega-\alpha\omega)-\bar{p}u(\omega-\alpha\omega+t)+(1-\bar{p})u(\omega+\alpha\omega(\mu-1)-t)+\bar{p}u(\omega+\alpha\omega(\mu-1)) = C
\end{cases} \tag{14}
$$

The incentive constraint defines an implicit relationship between t and α with:

$$
\left(\frac{\partial t}{\partial \alpha}\right)_{IC} = \omega \cdot \frac{A+B}{E+D}
$$

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with

$$\begin{cases} A = (1-p)u'(\omega - \alpha\omega) + pu'(\omega - \alpha\omega + t) \\ B = (\mu - 1)\big[(1-p)u'(\omega + (\mu - 1)\alpha\omega - t) + pu'(\omega + (\mu - 1)\alpha\omega)\big] \\ E = pu'(\omega - \alpha\omega + t) \\ D = (1-p)u'(\omega + (\mu - 1)\alpha\omega - t) \end{cases} \quad (15)$$

Note that this ratio is positive. The first equation in system (14), that takes into account the first derivatives of the Lagrangian with respect to t and α, writes as $\frac{A}{E} = \frac{B}{D}$. Therefore, in the plan (α, t), the optima are achieved when the curves $IC(\alpha, t) = 0$ (denoted curve (IC)) and $V(\alpha, t) = 0$ (denoted curve (V)) cross, with

$$\begin{cases} V(\alpha, t) = \frac{A}{E} - \frac{B}{D} \\ IC(\alpha, t) = (1-p)u(\omega + (\mu - 1)\alpha\omega - t) + pu(\omega + (\mu - 1)\alpha\omega \\ \qquad\qquad - (1-p)u(\omega - \alpha\omega) - pu(\omega - \alpha\omega + t) - C \end{cases} \quad (16)$$

. Curve (V) is not in-between curves (I) and (II). The first-best solution is the solution of $\frac{E}{D} - \frac{p}{1-p} = 0$ (curve (I), crossing $(0,0)$) and $\frac{A}{B} - \frac{p}{1-p} = 0$ (curve (II), crossing $(\alpha_0, 0)$). Then it intersects the curve (V) in the plan (α, t). Notice that both curves (I) and (II) are increasing in the plan (α, t). The areas of the plan in-between the two curves (I) and (II) are such that $\frac{A}{B} - \frac{p}{1-p} < 0$ and $\frac{E}{D} - \frac{p}{1-p} > 0$. It follows directly that $\frac{A}{B} \neq \frac{E}{D}$ in-between curves (I) and (II); equivalently, $\frac{A}{E} \neq \frac{B}{D}$ in-between curves (I) and (II), and then function $V(\alpha, t) = 0$ cannot pass in-between curves (I) and (II) in the plan (α, t).

. Curve (V) is above curves (I) and (II) when $\alpha \in [0, \alpha^*]$. We observe that function $V(\alpha, t)$ is increasing in t the plan (α, t). Indeed, function $V(\alpha, t)$ is written:

$$V(\alpha, t) = \frac{1 - \overline{p}}{\overline{p}} \frac{u'(\omega - \alpha\omega)}{u'(\omega - \alpha\omega + t)} + 1 - (\mu - 1)\left[\frac{\overline{p}}{(1 - \overline{p})} \frac{u'(\omega + \alpha\omega(\mu - 1))}{u'(\omega + \alpha\omega(\mu - 1) - t)} + 1\right] \quad (17)$$

We have:

$$\frac{\partial V(\alpha, t)}{\partial t} = \frac{1}{\overline{p}(1 - \overline{p})\left[u'(\omega - \alpha\omega + t)\right]^2 \left[u'(\omega + \alpha\omega(\mu - 1) - t)\right]^2}$$
$$\left(-(1 - \overline{p})^2 u'(\omega - \alpha\omega)u''(\omega - \alpha\omega + t)\left[u'(\omega + \alpha\omega(\mu - 1) - t)\right]^2\right.$$
$$\left. -(\mu - 1)\overline{p}^2 u'(\omega + \alpha\omega(\mu - 1))u''(\omega + \alpha\omega(\mu - 1) - t)\left[u'(\omega - \alpha\omega + t)\right]^2\right) > 0$$

Basically, $\frac{\partial V(\alpha, t)}{\partial t} > 0$ for all utility satisfying $u' > 0, u'' < 0$.

We deduce two points. First, the transfer t_0 such that $V(0, t_0) = 0$ is positive. To see this, note that $V(0,0) = \frac{1-\bar{p}\mu}{\bar{p}(1-\bar{p})}$; since $1 < \bar{p}\mu$, $V(0,0) < 0$. Since $\frac{\partial V(\alpha, t)}{\partial t} > 0$, this induces that $t_0 > 0$. From this, we know that curve (V) joins $(0, t_0)$ to (α^*, t^*), being above curves (I) and (II). Second, as (V) crosses the first-best, it may be the case that the curve goes down on the left, passing below curves (I) and (II) for some $\alpha < \alpha^*$ (see figure 3). But since $\frac{\partial V(\alpha, t)}{\partial t} > 0$, we have $V(\alpha, t) < 0$ for all (α, t) below curve (II) when $\alpha \in [0, \alpha^*]$. Thus, such a configuration cannot happen.
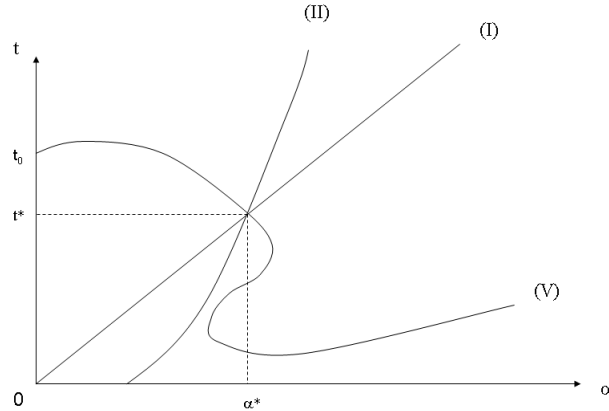

Figure 3: (V) cannot be such; it cannot pass below (I) and (II) before α^*

We conclude that curve (V) is necessarily above (I) and (II) when $\alpha \in [0, \alpha^*]$.

. We have $\alpha^{**} > \alpha^*$. Notice that, in the plan (α, t), the curve (IC) is below the first-best solution.

If curve (IC) is below curve (I) when $\alpha \in [0, \alpha^*]$, then curve V does not intersect curve (IC) when $\alpha \in [0, \alpha^*]$ and we are done.

If curve (IC) crosses over curve (I), either it does not intersect (V) and we are done, or it does. Figure 4 illustrates the case. Consider two intersections (V)-(IC), denoted t_1 (resp. t_2). Basically, utility increases when, fixing α, we pass to t_3 on curve (I) (resp. t_4) - full equal sharing is better than even more sharing -. Further, profit increases on curve (I) before α^*, that is until the second intersection (I)-(IC), denoted t_5. Last, it increases again on (IC): indeed, the next intersection is necessarily for some $\alpha > \alpha^*$ and profit is locally increasing on (IC) at the point $\tilde{t}$. ∎

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Proof of example 2. Define

$$R(\alpha,t) \equiv \frac{(1-\bar{p})u'(\omega-\alpha\omega) + \bar{p}u'(\omega-\alpha\omega+t)}{(1-\bar{p})u'(\omega+\alpha\omega(\mu-1)-t) + \bar{p}u'(\omega+\alpha\omega(\mu-1))}$$

We can write the solution of the program without moral hazard (9) as $R(\alpha^*, \frac{\alpha^*\omega\mu}{2}) = \frac{\bar{p}}{1-\bar{p}}(\mu-1)$ whereas $\left(\frac{\partial L}{\partial \alpha} = 0\right)$ becomes $R(\alpha^{**}, t^{**}) = \frac{\bar{p}+\lambda}{(1-\bar{p})-\lambda}(\mu-1)$

Suppose $t^{**} \geq \frac{\alpha^{**}\omega\mu}{2}$. In this case, $u'(\omega-\alpha\omega+t) \leq u'(\omega+\alpha\omega(\mu-1)-t)$. Therefore the RHS of $\left(\frac{\partial L}{\partial t} = 0\right)$ is lower than $\frac{\bar{p}}{1-\bar{p}}$ and $\lambda \leq 0$. This give $R(\alpha^{**}, t^{**}) < R(\alpha^*, \frac{\alpha^*\omega\mu}{2})$.

$V(\alpha,t)$ **is flat for CARA utilities.** Basically,

$$\frac{\partial V(\alpha,t)}{\partial \alpha} = \frac{\omega}{\bar{p}(1-\bar{p})\left[u'(\omega-\alpha\omega+t)\right]^2 \left[u'(\omega+\alpha\omega(\mu-1)-t)\right]^2}$$

$$\left(-(1-\bar{p})^2 u''(\omega-\alpha\omega)u'(\omega-\alpha\omega+t)\left[u'(\omega+\alpha\omega(\mu-1)-t)\right]^2 \right.$$

$$+(1-\bar{p})^2 u'(\omega-\alpha\omega)u''(\omega-\alpha\omega+t)\left[u'(\omega+\alpha\omega(\mu-1)-t)\right]^2$$

$$-(\mu-1)^2\bar{p}^2 u''(\omega+\alpha\omega(\mu-1))u'(\omega+\alpha\omega(\mu-1)-t)\left[u'(\omega-\alpha\omega+t)\right]^2$$

$$\left. +(\mu-1)^2\bar{p}^2 u'(\omega+\alpha\omega(\mu-1))u''(\omega+\alpha\omega(\mu-1)-t)\left[u'(\omega-\alpha\omega+t)\right]^2 \right)$$
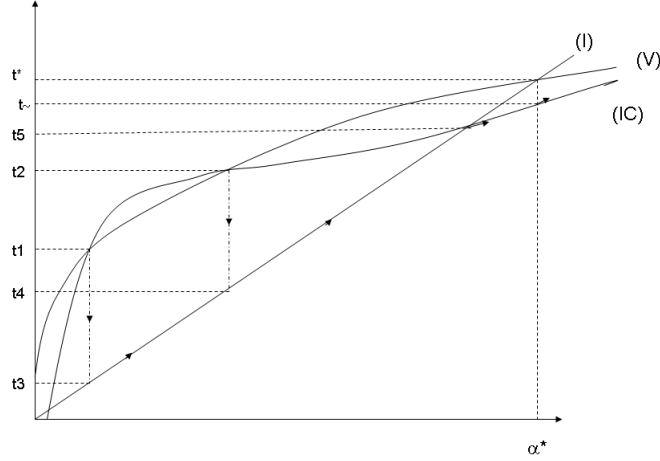

Figure 4: case where curve (IC) above curve (I) before α^*

Hence, $\frac{\partial V(\alpha,t)}{\partial \alpha} = 0 \Leftrightarrow$

$$(1 - \bar{p})^2 \left[u'(\omega + \alpha\omega(\mu - 1) - t)\right]^2 \left[u''(\omega - \alpha\omega)u'(\omega - \alpha\omega + t) - u'(\omega - \alpha\omega)u''(\omega - \alpha\omega + t)\right]$$

$$+(\mu - 1)^2 \bar{p}^2 \left[u'(\omega - \alpha\omega + t)\right]^2 \times$$
$$\left[u''(\omega + \alpha\omega(\mu - 1))u'(\omega + \alpha\omega(\mu - 1) - t) - u'(\omega + \alpha\omega(\mu - 1))u''(\omega + \alpha\omega(\mu - 1) - t)\right] = 0$$

Since $\frac{u''(.)}{u'(.)}$ is constant across wealth for CARA utilities, $\frac{\partial V(\alpha,t)}{\partial \alpha} = 0 \ \forall \alpha, t$ and $t^* = t^{**}$ can be inferred by $V(0,t) = 0$. The result follows directly. $\square$

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