Classical and stochastic mine planning techniques, state of the art and trends

http://dx.doi.org/10.1590/0370-44672016710165

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Abstract

Determination of the best possible ultimate pit for an open pit mine is a fundamental subject that has undergone a highly evolutionary process, reviewed in this study, since the correct choice carries substantial economic impact for the industry. The correct choice can be very beneficial for project analysis, whereas an incorrect choice has the potential to mask huge financial and economic future losses that could render a project unfeasible. The advent of computers in the 1960s allowed sophisticated analysis for the selection of the best ultimate pit determination, under specific modifying factors such as economic, social, environmental, and political, but only in deterministic situations, i.e., when the problem and variables for the ultimate pit determinations were considered deterministically and almost always based on average values. Techniques such as the Lerchs–Grossman algorithm and mixed-integer programming are among many standard tools now used by the mineral industry. Geological uncertainty and the associated risks as well as the need to consider the appropriate time to mine a block during a mine operation have a significant impact on the net present value of the resulting ultimate pits. Stochastic aspects embed a probabilistic component that varies in time and are now under intense investigation by researchers, who are creating algorithms that can be experimented with and tested in real mine situations. One can expect that once these algorithms demonstrate their efficiency and superior results, they will readily dominate the industry.

Keywords: deterministic mine planning; stochastic mine planning; direct block schedule; uncertainty.

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1. Introduction

The development of methodologies that allow accurate orebody modeling and mining production planning is currently a recurrent research theme at important research centers worldwide. Lerchs and Grossmann (1965) proposed an algorithm based on graphic theory, which has been the most widely used mine planning algorithm.

Classical planning methodologies are developed from the obtained results and comprise the following steps: final pit and pushback determination, production scheduling, cut-off grade determination, and mixing proportions for blending. The final pit limit is determined with the assumption that all blocks must be mined within the same period, ignoring operational constraints and money value variation with time. In 2008, Osanloo *et al.* did a review of deterministic and uncertainty-based algorithms presenting the advantages and disadvantages of these algorithms, indicating the need for continuing this study.

Dimitrakopoulos (2011) showed in his work the developments of stochastic optimization for strategic mine planning, where some case studies are shown to demonstrate the gains of using this approach: stochastic optimal pit limits can be about 15% larger in terms of total tonnage when compared to the conventional optimal pit limits; higher NPV; and lower potential deviation from production targets.

Lamghari and Dimitrakopoulos (2015) cited a list of authors that demonstrated the benefits of accounting for metal uncertainty in the optimization process, in spite of the increased complexity. Figure 1 shows some benefits of using stochastic approach.

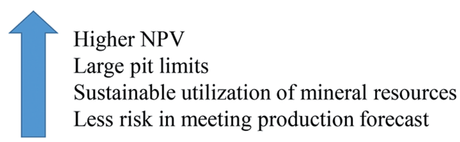

Figure 1
Benefits of using stochastic approach.

Computational development allows the use of approaches such as linear programming, integer programming, mixed-integer programming, and heuristics. The present study attempts to provide a bibliographic review of pit optimization approaches, considering conventional and stochastic methodologies.

2. Conventional and stochastic concepts

According to Dimitrakopoulos, Farrelly and Godoy (2002), the input for conventional mine planning is usually a single estimated—usually krigged—orebody model. There are some issues for using ordinary kriging to estimate a block model, like smoothing. It is a dangerous effect, in which low values are overestimated and high values underestimated. Therefore, ordinary kriging estimates do not reproduce the sample histogram, since tails of the distribution are lost in the estimation process. If the sample variance is not reproduced, the sample semivariogram will not be reproduced as well. (Yamamoto, 2008). The resulting plan obtained from the krigged model will be unreal. Dimitrakopoulos (2011) said that the main drawback of estimation techniques, be they geostatistical or not, is that they are unable to reproduce the in-situ variability of the deposit grades, as inferred from the available data. Ignoring such a consequential source of risk and uncertainty, may lead to unrealistic production expectations.

It is useful to compare the conventional mine planning concept based on deterministic models and the stochastic mine planning models based on probabilistic concepts. Yamamoto (2008) said that conditional simulation has been used as an alternative to ordinary kriging because it provides multiple equiprobable images of the phenomenon under study, reproducing in an ergodic sense both the sample histogram and the sample semivariogram. The multiple realizations also yield an assessment of global uncertainty. It means conditional simulation guarantees global accuracy. In this way, the models obtained by simulation tend to be more realistic than an estimate map affected by smoothing.

Mathematically optimal pits search for the ideal value of the constrained objective function; in mathematical terms, optimal refers to minimizing costs and maximizing income, thereby maximizing the profit (Whittle, 2011). The most widely used parameter for economic evaluation of mining projects is the net present value (NPV), owing to its realistic approach to the values involved.

Numerous approaches to mine planning optimization are treated by various software algorithms, which consider as many parameters as possible to obtain simulations that are more realistic. An optimization model must involve the costs, specially mining and processing costs, and the generated income. Both approaches use the same basic input data: a set of block values representing the net economic worth of each block. These block values are determined using the cutoff grade, costs, prices, recovery, dilution, density, operational parameters (slope angle, capacities, production, among others). Meagher, Sabour and Dimitrakopoulos (2010) pointed out the principal assumptions done in conventional mine planning: the first assumption indicates that there is perfect knowledge about the metal content of mining blocks; the second implies that market variables such as metal prices and exchange rates are fixed, i.e., do not change throughout life-of-mine (LOM) and are known with certainty. The difference in stochastic mine planning is the use of equiprobable scenarios, estimated using uncertainties that can be geological, market or operational, not a fixed value.

The example shown in Figure 2 demonstrates the gain when using the time stochastic approach to optimize mine planning, using the same parameters. The simulated model gives an improvement of 7.5% in average of NPV, comparing with the deterministic values (in green).

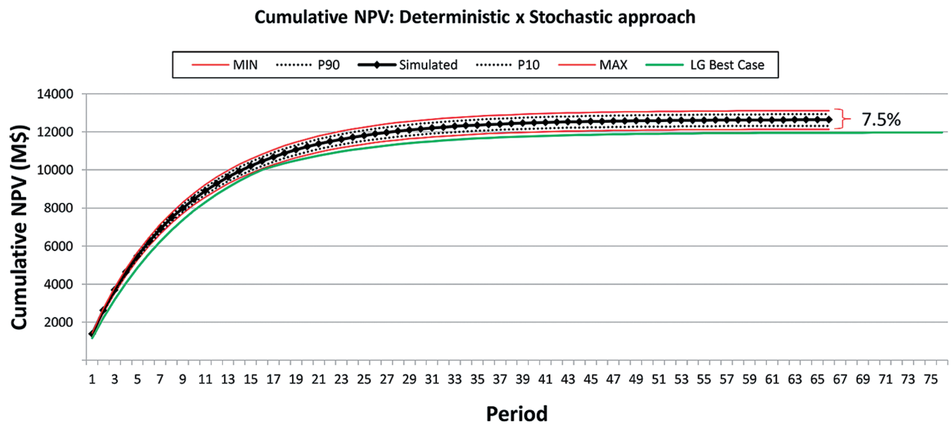

Figure 2
Cumulative NPV obtained using deterministic and stochastic approach.

2.1 The Lerchs–Grossmann algorithm

The algorithm proposed by Lerchs and Grossmann (1965) attempts to define the most economical envelopment according to the problem's constraints. However, it does not fully satisfy the industry's needs, probably due to the following factors:

• method complexity in terms of comprehension and programming

• time required to render the image due to ordering issues. This problem increases if there is a need to perform a sensitivity analysis that generates multiple designs owing to changes in variables such as costs, prices, and minimum cutoff grades. However, the arrival in recent years of powerful low-cost computers has minimized this problem

• difficulties in incorporating slope angle variations

• the optimization criterion is based on the total benefit rather than on the NPV

The original two-dimensional method proposed was later extended to the third dimension. In many cases, the solutions adopted to account for the natural limitations of the method increase the gap between the results obtained and the optimal solution.

2.2 Conventional mining production scheduling

According to Boland, Dumitrescu and Froyland (2008), the open pit mine production scheduling problem studied in recent years is usually based on a single geological estimate of material to be excavated and processed over a number of decades. This problem consists of finding the sequence in which the blocks should be removed from the pit during the lifetime of the mine, so that the NPV of the operation is maximized. The solution of this problem provides a basis for the strategic future development of a new or existing mine.

Geological and economical models rely on mathematical and technical methods and must address economical and operational constraints. Solutions for the final pit problem and production scheduling must also account for the uncertainty raised because of the stochastic nature of the variables involved. Production scheduling is proposed according to goals previously set by the nested pits.

2.3 Direct block schedule (DBS)

DBS is an evolution of the classical methodology in which the process of scheduling is obtained block by block, using discounted values for the beneficial function, therefore considering the time period the block is actually planned to be mined, in an attempt to minimize the differences between the estimated and the actual value associated with the pit. DBS proposes a new approach to the optimal pit problem, starting with scheduling and obtaining pushbacks and the final pit as a natural result of the methodology as it actually occurs.

DBS is a sequential analysis methodology involving the associated value of blocks, considering all simulated scenarios. The search process treats each period as a stage with individual operational requirements and restrictions to be respected. The methodology can also determine possible combinations for making the block economically feasible, complying with the constraints of grades and contaminants, and thus rendering the concept of cut-off grades obsolete.

The following proposed formulations must be executed successively to include the different simulated scenarios. When the best solution for a period is found, the pit surface is respected in the following time period (*t+1*), and this goes on until the exhaustion of the economically feasible blocks. The final pit is then obtained as a natural result of the simulation.

2.3.1 Scheduling modeling using mixed-integer programming

The formulation of the problem is based on the operational income and costs. Models considering mixed-integer programming provide a formulation for optimized production scheduling and for the control of target production and grades.

Ramazan and Dimitrakopoulos (2013) proposed a stochastic model based on the following objective function (Equation 1):

$$\text{Max} \sum_{t=1}^{P}\Big[\sum_{i=1}^{N} E\{(NPV)_i^t\} b_i^t - \sum_{j=1}^{U} E\{(NPV)_j^t + MC_j^t\} w_j^t + \sum_{s=1}^{M}\left(\frac{SV^t}{M}\right) k_s^t - \sum_{s=1}^{M}\left(C_u^{to} d_{su}^{to} + C_l^{to} d_{sl}^{to} + C_u^{tg} d_{su}^{tg} + C_l^{tg} d_{sl}^{tg} + C_u^{tq} d_{su}^{tq} + C_l^{tq} d_{sl}^{tq}\right)\Big] \quad (1)$$

Where p is the total number of production periods; n is the number of blocks; b_i^t is the decision variable for when to mine block i (if mined in period t, b_i^t is one; otherwise bit is zero); $E\{(NPV)_j^t\}$ is the expected (average) NPV to be generated if block i is mined in period t.

U is the number of blocks considered for stockpiling; Mc_j^t is the mining cost of block j incurred in period t and discounted to time zero; and w_j^t is a variable representing the fraction of block j sent to the stockpile in period t.

SV^t is the profit to be generated by processing a ton of material from the stockpile in period t and discounted to time zero; M is the number of simulated orebody models; and k_s^t is the amount of material (ore) in tons processed from the stockpile with respect to orebody model s in period t.

The c variables are the unit costs of deviation (represented by the d variables) from production targets for grades and ore tons. The subscripts u and l correspond to the deviations and costs from excess production (upper bound) and shortage in production (lower bound), respectively, while s is the simulated orebody model number, and g and o are grade and ore production targets.

The function is divided into four main parts. The first accumulates the resultant NPV for all mined and processed ore blocks. The second reflects the opportunity cost involved when the ore is stockpiled rather than processed and sold, with the result that the function accounts only for mining costs for these blocks at the respective period. The third represents the discounted income obtained with the resumption and processing of the stockpiled material within the same period. The fourth part attempts to manage the geological risks (Ramazan and Dimitrakopoulos, 2013).

The mentioned authors also define constraints to the objective function due to mining and processing capacities, to blending requirements, and to maximum desired volume of the stockpiles, along with other logical constraints. Slope angle constraints also ensure a precedence relationship between blocks.

2.3.2 Formulation of the DBS

The DBS methodology consists in grouping blocks in vertical columns arranged within a matrix. A fundamental aspect of this methodology is the minimization of the difference between the proposed and the actual pit surfaces due to the approximation of the superior faces of the blocks to the actual pit surface. This approach might allow a more realistic reproduction of the pit surface when compared to the sub-blocking technique.

Marinho (2013) proposed a new objective function to be maximized, shown in Eq. (2):

$$Max\ \frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{c=1}^{M}\sum_{z=1}^{Z} V_{c,t,s}^{z}\left(X_{c,t}^{Z} - X_{c,t-1}^{Z}\right) \quad (2)$$

Where S is the number of simulated orebody models considered; s: simulation index, s =1, …, S. T is the number of periods over which the orebody is being scheduled and also defines the number of surfaces considered; t: period index, t =1, …, T. M is the number of cells in each surface, where M = x × y represents the number of mining blocks in x and y dimensions; c: cell index corresponding to each (x, y) block/cell location, c=1, …, M. Z is the number of levels in the orebody model; z: level index, z =1, …, *Z*. $V^z_{c,t,s}$ is the cumulative discounted economic value of block (c, z) and all blocks above it in scenario s and period t. $X^Z_{c,t}$ is the binary variable that is one (1) if block (c, z) is the last block being mined in period t over c, and zero (0) otherwise.

According to Marinho (2013), the proposed objective function in Equation 2 maximizes the expected NPV from mining and processing selected blocks over all considered mine production periods. The objective function accumulates the discounted income according to the mining period for each block, relating the obtained value to the difference in the consecutive proposed surfaces.

Marinho (2013) has also proposed some constraints, such as surface, slope angles, and production targets. The system restrictions can be classified into those that are dependent and independent of the simulation. Operational restrictions, as mentioned, are independent of the considered scenario.

2.3.3 Slope angle management

The mining schedule for the three-dimensional block model must follow a logical and operational order of block precedence: a block can be removed only if a list of precedence blocks was previously mined. Figure 3 illustrates the variation of deviations on the slope angles between the simulated models, based on a block centroids approach, and the reality.

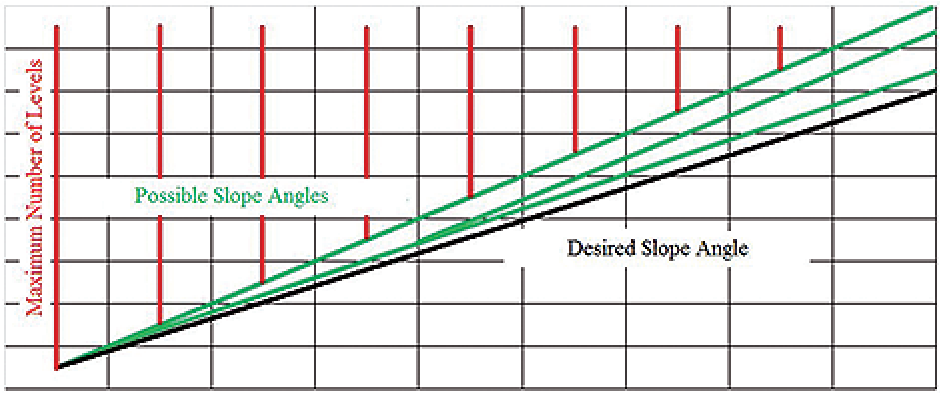

Figure 3
Inaccuracy of slope angles obtained by the precedence method (adapted from Whittle, 1998 cited in Beretta and Marinho, 2014).

The red lines indicate the number of precedence blocks for mining the next block within that column. Analyzing the chart, it is easy to see that the larger the quantity of precedence blocks is, the closer to the desired slope angle the green line will be. Thus, the method's accuracy increases with the number of precedence blocks, and the model cannot ensure a constant overall slope angle.

To ensure that no steep angles occur, a conservative methodology can be adopted to force the slope angle to 38.7°. The conservative approach might reflect a decrease in mineral recovery, also decreasing the total value associated with the project (Figure 4).

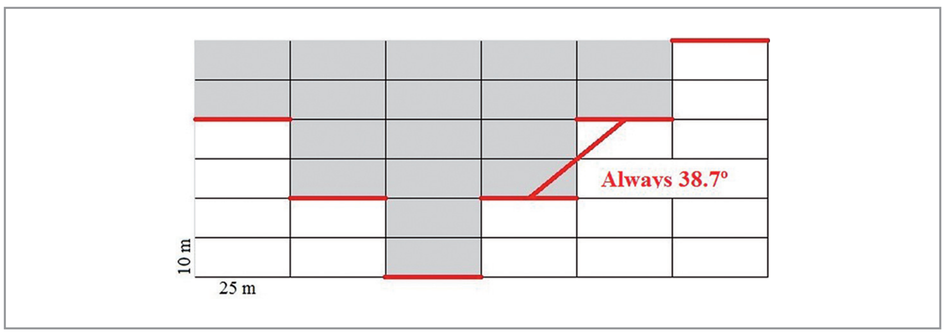

Figure 4
Constant slope angles obtained through precedence blocks (adapted from Beretta and Marinho, 2014).

An alternative for adequacy of the slope angles is surface control. This methodology is based on the triangulation achieved between points $e_{c,t}$ (elevation of the cell c in the period *t*). Therefore, the three-dimensional surface obtained always guarantees the maintenance of the slope angle with 100% accuracy, with no need for adjustments, even in deposits with multiple slope angles, as shown in Figure 5 (Beretta and Marinho, 2014).

Figure 5
Example of constrained surface slope angle at 45° (adapted from Beretta and Marinho, 2014).

2.3.4 Multi-period optimization

Multi-period optimization presents a refining methodology for an initial solution considering the precedence relationship between the adjacent blocks within the model. Summarizing, the search continues analyzing two consecutive periods for the blocks in the transition region between these periods, aiming to mine the most valuable blocks first. The possible shifts between the blocks must comply with the geotechnical and other constraints of the objective function (Lamghari, Dimitrakopoulos and Ferland, 2014).

The present methodology attempts to guarantee, sequentially, the optimal solution between the periods until the final pit limit. The process approaches the problem on both the global and local scales simultaneously, with the periods *i* and *i+1* being analyzed globally to increase project value (Marinho, 2013).

2.4 Stochastic DBS

The stochastic DBS accounts for geological uncertainty and the variability of the parameters, realistically addressing the problem once it quantifies the probability of occurrence of the operational feasibility for each block. The major limitations of the method are poor resolution when considering the mineral processing and the mining requirements for, i.e., blending stages, stockpiles, multi-element deposits, and multiple destinations.

Mining operation scheduling is traditionally based on a single deterministic model of the orebody. Traditionally, the methods used in designing these systems cannot properly incorporate the uncertainty associated with an estimated value. Ordinary Kriging, for example, results in best local estimates, but with a smooth global variance (Matheron, 1963). The problem of deriving mine planning from a single model is that the chosen model will be unable to reproduce the intrinsic variability of the reality of the deposit (Silva, 2008).

Geological uncertainty represents risks for the investment related to spatial distribution, to the variability and quality of the ore, and to operational and final slope stability. These characteristics are inferred from a limited database that might not be representative for the deposit. In stochastic simulation, the local values within the model are generated randomly in accordance with the probability distribution inferred from the sampled data. This statistical approach allows the determination of equally probable scenarios. Each scenario can be analyzed separately, and the probability distribution allows estimation of their limits (Spleit, 2014), thereby minimizing the geological risk.

Geostatistical algorithms are used to obtain the variability of the model, a process referred to as stochastic simulation. According to Deutsch and Journel (1998), stochastic simulation is the process of building alternative, equally probable, high-resolution models of the spatial distribution of a variable. Such a simulation can be considered a spatial extension of the concept of Monte Carlo simulation and has the objective of estimating geological variability and the possible values for each block. This method is adverse to the classical theory, which uses interpolation algorithms, smoothing the spatial variations of the estimated attributes.

The combination of stochastic simulation with the DBS methodology minimizes uncertainty, since diversified scenarios are scheduled, and hence it maximizes the adherence probability of long-term mine planning to the daily production of the mine.

2.5 Simulated annealing

Simulated annealing (SA) (Kirkpatrick *et al*, 1983) involves the application of probabilistic techniques to move the objective function closer to an optimal global solution. The method is characterized as a metaheuristic owing to the wide search space it uses to reach the solution.

The use of SA in long-term mine scheduling, developed by Godoy (2003), incorporates the uncertainties in considering a single final pit model among a set of stochastically simulated models subject to constant production rates. The primary objective is determining an optimal schedule, in order to minimize deviations between the production rates and goals through the management of the uncertainty of the mining blocks in a time period. The steps of this approach are as follows:

• final pit determination based on the estimated Kriging model

• production schedule until the final pit limit, predefined for each simulated model

• utilization of SA in conformance with the production targets and restrictions, such as number of mining sequences, parameters related to the annealing algorithm, and production targets

The proposed objective function is defined as (Equation 3):

$$O = \sum_{n=1}^{N} O_n$$
$$O_n = \frac{1}{S}\sum_{s=1}^{S} |\theta_n^*(s) - \theta_n(s)| + \frac{1}{S}\sum_{s=1}^{S} |\omega_n^*(s) - \omega_n(s)| \qquad (3)$$
$$\Delta O = \sum_{n=1}^{N} \Delta O_n$$

The proposition is to minimize the gap between the obtained production, for both ore (θ) and waste (ω), reasoning by the total number of simulated models S. The method addresses the uncertainty in the spatial grade distribution within the deposit and minimizes the deviation possibility.

2.6 Stochastic integer programming (SIP)

SIP is based on mathematical modeling and considers various equally probable scenarios, obtaining the best feasible result for a predefined set of goals within the space delimited by a set of constraints. Birge and Louveaux (2011) discuss various SIP approaches, but scientific advances related in literature are not always applicable to mine-planning problem resolution.

The NPV of a block is calculated for all simulated models to find the average value later. The values are discounted by periods using the geological risk factor developed by Dimitrakopoulos and Ramazan (2008). The discount rate due to geological risk (GRD) enables risk management among the periods. If a high value of GRD is specified, the lower risk areas will be mined first, and the mining of the higher risk areas is postponed. If a very low or null GRD is specified, the risk is distributed at a more balanced rate between the time periods, depending on the uncertainty distribution within the deposit (Dimitrakopoulos and Ramazan, 2008).

2.6.1 SIP increment

Dimitrakopoulos and Ramazan (2008) evidenced in their work the possibility of applying a methodology based on SIP to the solution of the long-term mine-planning problem. Menabde (2008) observed that the currently developed work successfully addressed questions related to geological uncertainty and schedule optimization, considering predecessor and successor blocks. Benndorf and Dimitrakopoulos (2013) significantly progressed in relation to the required operational constraints to solve the problem, obtaining more operational variables when considering operational restrictions.

Additional formulations attempt to minimize the costs considering truck operation, based on the distance between the mining front and the processing plant. The optimization dynamically decides between two destinations for each block: processing plant or waste disposal. The restrictions smooth possible variations on the annual fleet sizing in order to avoid sub-use of the fleet in the following time period (Spleit, 2014).

2.6.2 Beneficial function increment

The method renders the convergence of the system easier since, however, even when the system does not obtain a solution within the required parameters, it is possible to ease the requirements in order to find the best possible solution. An objective function based on SIP considers applied discounts as penalties due to deviations of the target (Benndorf and Dimitrakopoulos, 2013).

The objective function includes four distinct terms: (a) represents the NPV of the mined blocks; (b) accounts for the costs related to truck operation, which must be minimized; (c) represents penalties related to production targeting; and (d) accounts for loss due to the mining of non-adjacent blocks, penalizing non-functional mining sequences.

2.6.3 Fleet restriction and production targets

The objective function has one term responsible for reducing fleet operational costs through minimizing the average transport distance. This term is part of a set of restrictions that must be satisfied simultaneously. The restrictions cause the results to converge to a trend of fleet size reduction (Spleit, 2014). The formulation aims to bring about fleet replacement gradually, in order to ensure that no equipment is sub-utilized during the operation.

For each time period, the targets for minimum average grade, maximum contaminant grade, and metallurgical requirements must be respected within the system owing to the constraints.

3. Conclusions

The current study addressed some classical and stochastic mine planning techniques. Work related to the classical methodology does not account for geological uncertainty and is therefore deterministic, tending to result in final pit algorithms with impossible-to-obtain economic values resulting from a lack of discount rates with time. Osanloo et al. (2008) presented, in a review article, deterministic and uncertainty-based algorithms for long-term production planning known at that time, which demonstrates that after their work, there is great development in the stochastic methodologies, principally due to computational development and new studies.

Stochastic techniques are presented in literature as random approaches of geological scenarios. The stochastic optimization methodology allows the management of the associated risks and therefore resembles reality, ensuring greater reliability of the results obtained. The mathematical and computational support for the work requires integer and mixed-integer programming tools and heuristics to render the solution feasible.

Lamghari and Dimitrakopoulos (2015) cited in their work authors like Dimitrakopoulos, Farrelly and Godoy (2002), Godoy and Dimitrakopoulos (2004), Menabde, Froyland, Stone and Yeates (2007), Boland, Dumitrescu, and Froyland (2008), Osanloo *et al.* (2008), Albor and Dimitrakopoulos (2010), Ramazan and Dimitrakopoulos (2013), Marcotte and Caron (2013), and Koushavand, Askari-Nasab and Deutsch (2014). These authors showed that the stochastic approach could provide major improvements in NPV, reduce risk in meeting production forecasts and find large pit limits, contributing to the sustainable utilization of mineral resources.

The DBS methodology forecasts the blocks as a function of time, thus enhancing the economic analysis of the projects and, when coupled with geological simulated models, creates a new context for the mineral sector since it addresses, in a technically feasible way, the quantification of uncertainties involved in the mineral activity. The various scenarios generated by the DBS do not eliminate the need to use sampling methodologies and geological modeling consistent with the geological techniques, since primary errors can generate inconsistent geological scenarios, and thus unrealistic production planning. Furthermore, for all described methodologies, it is easy to understand the great need for frequent updates on the geological database according to mining activity progress. In this way, the mine planning could be adjusted to the best possible approximation potential, particularly owing to the implementation of structured algorithms within integrated mine planning systems, which might allow, in the near future, the routine application of these methodologies in the mineral industry. The current review presented some of the principal developing techniques across the world, which still have much evolving to do; it does not attempt to compare the algorithms developed during history, but aims to compare the differences and improvements of planning under uncertainties versus planning using fixed concepts.

Although the stochastic approach proves to be advantageous in many respects, traditional mining companies still do not use stochastic algorithms to make their stock market declarations. It happens because mining companies need to have their data audited, and audit companies use market-based software for this verification. Market-based software is commercial software that has its algorithm closed, that is, it does not allow user interference in calculations, so the results can be easily reproduced. From this point of view, the use of the classical methodology can be considered as positive, but when evaluated on the technical bias, stochastic planning is more efficient in the assertiveness of the practical results.

Acknowledgments

Special thanks for the support of the Federal University of Minas Gerais (UFMG), in association with the Instituto Tecnológico Vale (ITV).

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Received: 24 November 2016 - Accepted: 9 January 2018.