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ORIGINAL PAPER

Evaluation of Rock Slope Stability Conditions Through Discriminant Analysis

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Abstract A methodology to predict the stability status of mine rock slopes is proposed. Two techniques of multivariate statistics are used: principal component analysis and discriminant analysis. Firstly, principal component analysis was applied in order to change the original qualitative variables into quantitative ones, as well as to reduce data dimensionality. Then, a boosting procedure was used to optimize the resulting function by the application of discriminant analysis in the principal components. In this research two analyses were performed. In the first analysis two conditions of slope stability were considered: stable and unstable. In the second analysis three conditions of slope stability were considered: stable, overall failure and failure in set of benches. A comprehensive geotechnical database consisting of 18 variables measured in 84 pit-walls all over the world was used to validate the methodology. The discriminant function was validated by two different procedures, internal and external validations. Internal

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validation presented an overall probability of success of 94.73% in the first analysis and 68.42% in the second analysis. In the second analysis the main source of errors was due to failure in set of benches. In external validation, the discriminant function was able to classify all slopes correctly, in analysis with two conditions of slope stability. In the external validation in the analysis with three conditions of slope stability, the discriminant function was able to classify six slopes correctly of a total of nine slopes. The proposed methodology provides a powerful tool for rock slope hazard assessment in open-pit mines.

Keywords Multivariate statistics · Rock slope stability · Principal component analysis · Boosting technique · Discriminant analysis

1 Introduction

Slope failures are recurring in many situations, leading to serious hazards and consequences. Landslides in urban environments, for instance, can be of great proportions, affecting large inhabited areas. In road cuts slope failures can cause accidents and traffic disruption. In open-pit mines steeper slopes are highly desired on account of their positive economic impact on cash flow of large projects. Steeper slope angles, however, result in an increased risk of failure. Traditionally, the slope failure evaluation is assessed by stability analyses such as limit equilibrium methods and 2D/3D numerical modelling (see e.g. Wyllie and Mah 2004; Read and Stacey 2009). Among the limit equilibrium methods there are deterministic and probabilistic approaches. Probabilistic methods are based on a quantitative analysis of the probability of failure, considering many different scenarios of strength parameters and loading conditions. Deterministic methods are based on the safety factor calculation of a slope in an established scenario of unchanged strength parameters and loading conditions. In many situations the discrimination of the slopes according to their stability condition might help to find out those slopes which can present serious hazards. When many slopes must be evaluated techniques of multivariate statistics are powerful tools to detect hazardous situations which require proper and detailed stability analyses.

Multivariate statistics have been used in the literature for different purposes as well as the boosting procedure. The boosting technique has been implemented in algorithms in order to optimize functions such as in Lin (2011) and Wu et al. (2015). Another example is the research developed by Massumi and Gholami (2016). They use principal component analysis for investigation of structural damage caused by seismic events. Multivariate statistical methods have also been applied in geology and in engineering geology for a variety of problems. A typical problem extensively studied is landslide susceptibility mapping. Multiple regression models, such as logistic regression, were used to improve susceptibility mapping accuracy. Researches with this focus were presented by Yilmaz (2009), Erener et al. (2017) and Ahmed and Dewan (2017). In mining, discriminant analysis was also used to develop a criterion for stability of underground excavations (Nickson 1992).

Another interesting application of multivariate analysis in mining industry was presented by Kulatilake et al. (2012). These authors used multivariate models to predict mean particle size in rock blast fragmentation. A cluster analysis was done to separate a database composed of 97 blasting data into two groups based on rock stiffness, which were confirmed by the application of discriminant analysis. Finally a neural network model was applied to predict mean particle size resulting from blasting fragmentation.

In problems involving rock slope failures, there has been few applications in the literature. This research proposes a methodology for the assessment of the rock slope stability status condition based on the application of multivariate statistical techniques. The multivariate statistical techniques used in this research are the principal component analysis (PCA) and the boosting analysis through Fisher's linear discriminant analysis.

The methodology was applied in the database built and organized by Zare Naghadehi et al. (2013). The geotechnical database presents eighteen parameters, which are ratings, related to geomechanical parameters surveyed at open pit mines around the world. Besides the 18 variables, the stability condition of the slopes (stable or unstable) is known. Zare Naghadehi et al. (2013) used this database to propose an instability index for open-pit mine slopes, defined in their research as Mine Slope Instability Index (MSII). The authors proposed the use of these eighteen parameters, based on the fact that they can be easily obtained at the field and they are important parameters related to rock slope stability.

The model proposed in this research can be used to know the slope stability condition of an untested slope. The model is also capable of predicting the most hazardous situations in a group of rock slopes.

2 Theoretical Basis

2.1 Principal Component Analysis

The principal component analysis technique (PCA) was proposed by Pearson (1901) and developed later by Hottelling (1933). Principal components can replace the original variables, besides reducing the number of variables of the model.

In principal component analysis technique the principal axes of the multi-dimensional configuration of the studied data are determined as well as the coordinates of each sample on these new axes. Therefore, the principal component analysis technique consists of a rotation of the original coordinate axes in the direction of the principal axes. The new variables (principal components) are linear combinations of the p original variables of the data set. If the database has p variables, the PCA can change the system into k components, where $k \leq p$.

The principal components are given by Eq. (1).

$$Y_j = e_{j1}X_1 + e_{j2}X_2 + \ldots + e_{jp}X_p \tag{1}$$

where $X_1, X_2, \ldots, X_p$ are the original variables;

e_{ij} are the eigenvectors of the covariance or correlation matrix of the original data.

The database represented by the matrix $\mathbf{X}$ is constituted by n rows of individuals and p columns of variables. Table 1 shows the matrix $\mathbf{X}$ where n are the samples and p are the variables.

Table 1 Matrix **X** of original variables

Sample	Variables			
	X_1	X_2	…	X_p
1	x_{11}	x_{12}	…	x_{1p}
⋮	⋮	⋮	⋮	⋮
N	x_{n1}	x_{n2}	…	x_{np}

In this research the correlation matrix was used. The number of principal components was defined using the criterion proposed by Kaiser (1958). The Kaiser's criterion uses the eigenvalues greater than one, i.e., it keeps the linear combinations that explain at least the amount of variance of an original standard variable, see Johnson and Wichern (1998). This criterion can be envisaged in the scree plot showed in Fig. 1. Another way to select the number of the principal components also uses the scree plot, as proposed by Cattell (1966). In this graph, it is possible to see from which principal component the eigenvalues are stabilized, meaning that from this point, the eigenvalues approach zero.

This research used the principal component analysis not only to reduce database dimensions, but also to change original qualitative variables into new quantitative ones (the scores).

2.2 Discriminant Analysis

The discriminant analysis is a technique used for classifying the elements of a sample in different populations. The classification rule is built using a function able to distinguish between two or more groups through original features that must be known for all the groups. Fisher (1936) presented a solution for the problem based on a linear combination of these original features.

Fisher's discriminant function is given by Eq. (2).

$$D(X) = L' \cdot X = [\mu_1 - \mu_2]' \sum^{-1} \cdot X \tag{2}$$

where $D(X)$ is the discriminant function; L' is the discriminant vector; X is the random vector $\mathbf{X}$ containing the variables of the populations; μ_1' is the multivariate average vector of the population 1; μ_2' is the multivariate average vector of the population 2; $\sum$ is the population common covariance matrix.

The Eq. (3) shows the value of the discriminant function for a particular observation x_o.

$$D(x_0) = [\mu_1 - \mu_2]' \sum^{-1} \cdot x_0 \tag{3}$$

The classification rules based on the Fisher's discriminant function for the case of two populations are given by Eqs. (4) and (5).

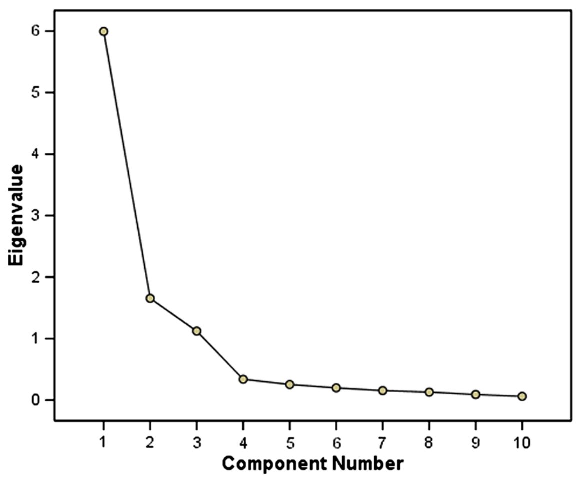


Fig. 1 Example of scree plot

$$D(x_0) = [\mu_1 - \mu_2]' \sum^{-1} \cdot x_0 \geq \frac{1}{2}[D(\mu_1) + D(\mu_2)], \quad to \quad x_0 \rightarrow \pi_1 \tag{4}$$

$$D(x_0) = [\mu_1 - \mu_2]' \sum^{-1} \cdot x_0 < \frac{1}{2}[D(\mu_1) + D(\mu_2)], \quad to \quad x_0 \rightarrow \pi_2 \tag{5}$$

A common covariance matrix $\sum$ can be estimated through the estimation of the covariance matrix of the populations π_1 and π_2. The common covariance matrix $\sum$ is calculated through Eq. (6), according to Anderson (1984), replacing $\sum$ by the known covariance matrix of the samples S. This equation can only be used when there is homocedasticity between the populations π_1 and π_2.

$$S_c = \left[\frac{n_1 - 1}{(n_1 - 1) + (n_2 - 1)}\right] \cdot S_1 + \left[\frac{n_2 - 1}{(n_1 - 1) + (n_2 - 1)}\right] \cdot S_2 \tag{6}$$

where S_c is the sample common covariance matrix; n_1 is the number of observations in π_1; n_2 is the number of observations in π_2; S_1 is the covariance matrix of π_1; S_2 is the covariance matrix of π_2.

As μ_1, μ_2 are also unknown the Fisher's discriminant function is obtained replacing them by the respective sample quantities, as shown in the Eq. (7).

$$D(x) = \hat{L}' \cdot x = [\overline{x_1} - \overline{x_2}]' S_c^{-1} \cdot x \tag{7}$$

where $D(x)$ is the Fisher's discriminant function; $\hat{L}'$ is the estimated discriminant vector; x is the random vector containing the variables of the samples; $\overline{x_1}$ is the multivariate average vector of the sample 1; $\overline{x_2}$ is the multivariate average vector of the sample 2; S_c is the common covariance matrix.

Johnson and Wichern (1998) explained that there are two types of errors in the classification of two populations. The error 1 is defined when the sample element belongs to the population 1 but the classification rule allocates it in the population 2. The error 2 is defined when the sample element belongs to the population 2 but the classification rule allocates it in the population 1. The Eqs. (8) and (9) show the probability of these two types of errors.

$$Prob(Error1) = p(2|1) \tag{8}$$

$$Prob(Error2) = p(1|2) \tag{9}$$

The overall probability of success (OPS) is shown in Eq. (10). The apparent error rate (AER) is show in Eq. (11).

$$OPS = \frac{n_{11} + n_{22}}{n_1 + n_2} \tag{10}$$

$$AER = 1.0 - OPS \tag{11}$$

where n_{11} is the number of sample elements of the population 1 which were correctly allocated in population 1; n_{22} is the number of sample elements of the population 2 which were correctly allocated in population 2; n_1 is the number of samples of the population 1; n_2 is the number of samples of the population 2.

This research used the discriminant analysis to obtain a classifier that is the starting classifier of the boosting procedure. This technique is referred as the boosting algorithm via discriminant analysis. The boosting procedure improves this classifier, expanding its discrimination power and reducing the errors 1 and 2.

2.3 Boosting Procedure

As pointed up by Schapire (1990) the boosting procedure has the purpose of improving the performance of classifiers. According to Skurichina and Duin (2000), the boosting algorithm is flexible and simple to implement in various scenarios, presenting high potential of classification.

Okada et al. (2010) explained that the boosting algorithm consists of applying sequentially a classification rule called basic classifier. The application of this classification rule is made iteratively in the training sample. On each iteration, the algorithm recognizes the incorrectly classified observations and attributes to them higher weights. The final strong classifier is obtained by a linear combination of the updating classifiers on each iteration.

Among the versions of the boosting procedure the AdaBoost algorithm (Adaptive Boosting) stands out in the literature. When the problem comprises two populations, the algorithm is named Discrete AdaBoost. Two binary classifier values were yielded by this algorithm.

Adhikari et al. (2011) described the AdaBoost algorithm procedure in the following way: suppose a training set $L = (x_1, y_1), \ldots, (x_N, y_N)$, where the classes are labeled $\{-1,1\}$, i.e., $C = \{-1, 1\}$. $F(x) = \sum_1^M c_m f_m(x)$ can be defined, where f_m is a basic classifier that returns values between -1 and 1, the c_m values are constants and the corresponding prediction is the signal of $F(x)$, i.e., $sign(F(x))$. The AdaBoost algorithm adjusts basic classifiers f_m to the weighted samples of the training set, assigning greater weights to the cases that they were wrongly classified. The weights are adjusted at each iteration and the final classifier is a linear combination of the classifiers f_m.

Adhikari et al. (2011) presented the steps of the AdaBoost algorithm:

1. Entering the weights $w_i = 1/N, i = 1, 2, \ldots, N$;
2. Repeat for $m = 1, 2, \ldots, M$:
   (a) Adjust the classifier $f_m(x) \in \{-1, 1\}$ using the weights w_i of the training data;
   (b) Calculate $\varepsilon_m = E_w\left[I_{(y \neq f_{m(x)})}\right], c_m = \log((1-\varepsilon_m)/\varepsilon_m)$;
   (c) Do $w_i \leftarrow w_i exp\left(c_m I_{(y \neq f_{m(x)})}\right), i = 1, 2, \ldots, N$and update for $\sum_i w_i = 1$;
3. Obtain the final classifier $sign(F(x)) = sign \sum_{m=1}^{M} c_m f_m(x)$.

Adhikari et al. (2011) explained that in the algorithm, E_w represents the mathematical expectation of database training set with weights $w = (w_i, \ldots, w_N)$. M is the number of iterations required for stabilization of the classifier, i.e., for the classifier becomes a strong classifier.

In this research, the starting basic classifier was obtained by Fisher's linear discriminant analysis.

3 Methodology

The script developed for both statistical techniques, principal component analysis and boosting technique via discriminant analysis was implemented in the freeware R (2006). The methodology was applied to the database compiled and organized by Zare Naghadehi et al. (2013).

The first part of the methodology is the application of principal component analysis, see Fig. 2. The database was randomly partitioned into two parts, the training sample 1 and the test sample. The training sample 1 contains 90% of the original database and consequently the test sample contains 10% of the database. This partition was necessary to carry out the model external validation; therefore the test sample has not been used to create the model. This sample can be considered a new set of slopes to be tested by the model.

The results of principal component analysis allowed the selection of the most significant principal components creating the data training sample 2 and the data validation sample (Fig. 2).

The second part of methodology is the application of the boosting procedure via discriminant analysis, see Fig. 3. The procedure was applied to training sample 2, which is a partition containing 75% of training sample 1. Boosting procedure via discriminant analysis yields a function able to classify the slopes according to their stability status condition.

Figure 4 presents the procedure used for validating the results of the function obtained by the boosting via discriminant analysis. There are two validations, the internal validation and the external validation. The internal validation was done in the validation sample (Fig. 2), which corresponds to 25% of the training sample 1. The external validation was done in the test sample, which corresponds to 10% of the original database.

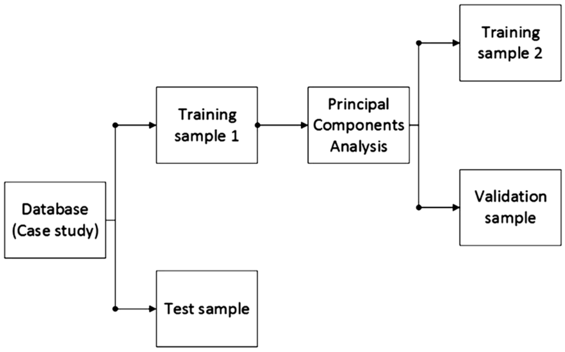


Fig. 2 First part of methodology, the application of principal component analysis

Fig. 3 Second part of methodology, the application of boosting via discriminant analysis

Confidence intervals for the apparent error rate, overall probability of success and errors were estimated using the bootstrap technique in the validation sample (Fig. 4). Figure 5 presents this part of methodology.

4 Results and Discussions

4.1 Geotechnical Database

The database used in this research was presented by Zare Naghadehi et al. (2013). The authors have used published articles and books which encompass many worldwide open pit slope stability case histories to build an extensive database. Zare Naghadehi et al. (2013) pointed out that the selection of the collected information was based on Hudson (1992), which had proposed an atlas of the parameters that directly influence the stability of rock slopes. The selection of the parameters among those proposed by Hudson (1992) was made based on recommendations from the literature and on the experience of the authors in open-pit mine slope stability. The authors also reported that the selection of the parameters took into account the facility of surveying them at the field.

The database organized by Zare Naghadehi et al. (2013) contains 84 slopes located in different mines around the world. Table 2 presents information of the number of slopes collected in each mine listing the names of every mine and their respective countries.

A total of eighteen variables comprise the database organized by Zare Naghadehi et al. (2013). These variables are: rock type (lithology), intact rock strength, rock quality designation (%), weathering, tectonic regime, groundwater condition, number of major discontinuity sets, discontinuity persistence, discontinuity spacing, discontinuity orientation, discontinuity aperture, discontinuity roughness, discontinuity filling, slope (pit-wall) angle, slope (pit-wall) height, blasting method, precipitation, previous instability. Besides the eighteen variables, the stability condition is known. Zare Naghadehi et al. (2013) use three types of stability conditions; stable slopes, unstable inter-ramp slopes and unstable global slopes.

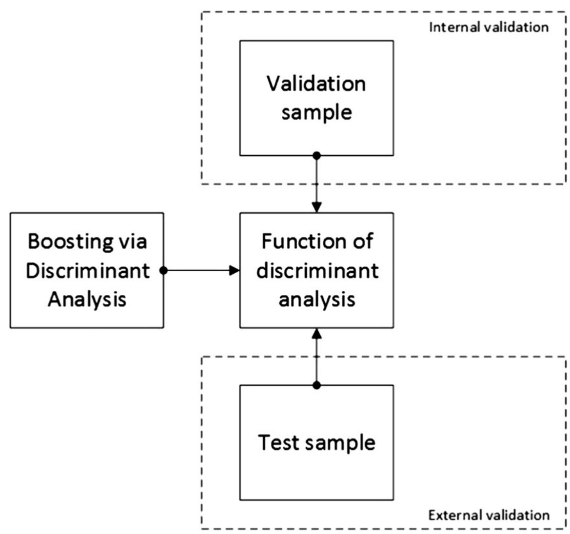


Fig. 4 Validation of function

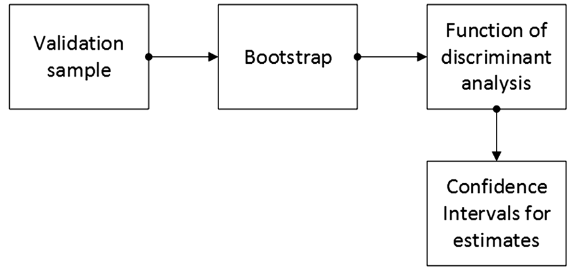


Fig. 5 Confidence intervals for estimate rates

Table 2 Information of studied slopes, according to Zare Naghadehi et al. (2013)

Number of slopes	Country	Mine	Number of slopes	Country	Mine
4	Iran	Angooran	5	Australia	Cadia-Hill
5	Iran	Chadormalou	6	Sweden	Aitik
5	Iran	Choghart	7	Chile	Escondida
4	Iran	Golegohar	5	Spain	Aznalcollar
4	Iran	Sarcheshmeh	5	USA	Betze-Post, Goldstrike
4	Iran	Sungun	2	Spain	La Yesa
7	South Africa	Venetia	1	Chile	Ujina, Collahuasi
5	Brazil	Águas Claras	1	Canada	Panda, Ekati
5	Chile	Chuquicamata	1	USA	Esperanza, Phelps-Dosge
6	South Africa	Sandsloot	2	Papua New Guine	Ok-Tedi

Table 3 Database geotechnical parameters, according to Zare Naghadehi et al. (2013)

Variable	Name	Nature	Type	Unit
V1	Rock type	Overall environment	Qualitative	Lithology
V2	Intact rock strength	Rock quality	Quantitative	MPa
V3	RQD	Rock mass properties	Quantitative	%
V4	Weathering		Qualitative	Classification
V5	Tectonic Regime	In-situ rock stress	Qualitative	Classification
V6	Groundwater	Hydraulic conditions	Qualitative	Classification
V7	Number of sets	Discontinuity properties	Qualitative	Unit
V8	Persistence		Quantitative	Meters
V9	Spacing		Quantitative	Meters
V10	Orientation		Quantitative	Degrees
V11	Aperture		Quantitative	Millimeters
V12	Roughness		Qualitative	Classification JRC
V13	Filling		Qualitative	Classification
V14	Overall Angle	Pit-wall geometry	Quantitative	Degrees
V15	Overall Height		Quantitative	Meters
V16	Blasting method	Construction	Qualitative	Type
V17	Precipitation	Overall environment	Qualitative	Millimeters/year
V18	Previous Instability	History	Qualitative	Classification

Table 3 shows the geotechnical parameters of the database and their classifications according to the nature, typology and units.

Different failure mechanisms are found in this database. Overall failures are landslides and wedges. There is only one bi-planar and one combined mechanism toppling/circular failure. Wedge, planar and toppling are failure types in set of benches. Thus most of failure mechanisms occurred along discontinuity planes, even in case of overall failures. It assures a direct relation between the variables used in the analyses and the failure mechanisms.

Zare Naghadehi et al. (2013) used a rating system to classify the collected information into classes. This rating system is presented in Table 4. The rating of each variable varies between 0 and 1 and it is directly related to the slope instability. In other words, higher weights are assigned to values that lead to a higher degree of slope instability than lower ones.

Table 4 Classification system and parameter ratings proposed by Zare Naghadehi et al. (2013)

Parameters	Categorical classification and weights					
Rock type (lithology)	Igneous: Granite Granodiorite Diorite and Gabbro Metamorphic: Gneiss Quartzite, Amphibolite	Sedimentary: Breccia, Greywacke Sandstone and Conglomerate Metamorphic: Hornfels Igneous: Dolerite Obsidian, Andesite, Norite and Agglomerate	Sedimentary:Anhydrite and Gypstone; Igneous: Tuff,Basalt Breccia, Dacite and Rhyolite	Sedimentary: Limestone shale Dolomite Limestone Chalk and Siltstone Metamorphic: Slate, Phyllites and Marble	Metamorphic: Schists and Mylonites	Sedimentary: Clay shale Mudstone, Claystone and Marble
	0.0	0.2	0.4	0.6	0.8	1.0
Intact rock strength-UCS (MPa)	> 150	150–100	100–75	75–50	50–25	< 25
	0.0	0.2	0.4	0.6	0.8	1.0
RQD (%)	100–75	75–50	50–25	25–10	< 10	
	0.0	0.3	0.6	0.8	1.0	
Weathering	W_1	W_2	W_3	W_4	W_5	
	Unweathered	Slightly weathered	Moderately weathered	Highly weathered	Completely weathered	
	0.0	0.3	0.6	0.8	1.0	
Tectonic regime	Weak (almost absence of meaningful tectonic events)	Moderate (presence of foliation, schistosity and cleavage)	Strong (presence of folds, faults and discontinuities)	Very strong (high fractured zones)	Intense (Imbrications and over thrusts)	
	0.0	0.3	0.6	0.8	1.0	
Groundwater condition	Completely dry	Damp	Wet	Dripping	Flowing	
	0.0	0.3	0.6	0.8	1.0	
Number of sets	0	1	2	3	> 3	
	0.0	0.3	0.6	0.8	1.0	
Discontinuity persistence (m)	< 5	5–10	10–25	25–40	> 40	
	0.0	0.3	0.6	0.8	1.0	
Discontinuity spacing (hb is bench height)	> 3hb	2hb–3hb	1hb–2hb	1/5hb–1hb	< 1/5hb	
	0.0	0.3	0.6	0.8	1.0	

Table 4 continued

Parameters	Categorical classification and weights				
Discontinuity orientation	Very favorable	Favorable	Fair	Unfavorable	Very unfavorable
αs-discontinuity dip direction αd-slope dip direction βd-discontinuity dip	$\beta d > \beta s$ e $\alpha d - \alpha s > 30°$	$\beta d > \beta s$ e $\alpha d - \alpha s < 30°$	$0 \leq \beta d < \beta s/4$ ou $\alpha d - \alpha s > 30°$	$\beta s/4 \leq \beta d < \beta s/2$ e $\alpha d - \alpha s < 30°$	$\beta s/2 \leq \beta d < \beta s$ e $\alpha d - \alpha s < 30°$
βs- slope dip	0.0	0.3	0.6	0.8	1.0
Discontinuity aperture	No separation	< 0.1	0.1–1	1–5	> 5
	0.0	0.3	0.6	0.8	1.0
Discontinuity roughness (JRC_{Macro})	Very rough	Rough	Slightly rough	Smooth	Slickensided
	0.0	0.3	0.6	0.8	1.0
Discontinuity filling	Not filled	Very hard filling	Hard filling	Soft filling	Very soft filling
	0.0	0.3	0.6	0.8	1.0
Slope (pit-wall) angle	< 30°	30°–40°	41°–50°	51°–60°	> 60°
	0.0	0.3	0.6	0.8	1.0
Slope (pit-wall) height (m)	< 50	50–100	100–200	200–300	> 300
	0.0	0.3	0.6	0.8	1.0
Blasting method	Presplitting	Postsplit	Smoothwall/cushion	Modified production blast	Regular blasting/ mechanical
	0.0	0.3	0.6	0.8	1.0
Precipitation (annual rainfall and snow) (mm/y)	< 150	150–300	300–450	450–600	> 600
	0.0	0.3	0.6	0.8	1.0
Previous instability	None	Inactive	Quiescent	Relatively active	Highly (obviously) Active
	0.0	0.3	0.6	0.8	1.0

4.2 Application of Multivariate Statistics

4.2.1 Principal Component Analysis

The first part of the methodology was the application of the principal component analysis in training sample 1. This technique allowed the transformation of original qualitative variables into quantitative ones; the scores of the principal components. This transformation was necessary because the boosting procedure via discriminant analysis can only be accomplished in a quantitative database. Table 5 presents the significance of the 18 estimated components.

The first component explained 22.9% of the total variance of the database; the second principal component explained 13.2% of the total variance and so forth. It is important to highlight that approximately 70% of the total variance of the database is explained by the first six principal components.

Kaiser's criterion (1958) was used to reduce the database dimension. The principal components that presented eigenvalues greater than one were selected, i.e., the linear combinations that explained at least the amount of variance of one original standardized variable. Figure 6 presents the scree-plot for the first ten principal components. According to Kaiser's criterion the first six principal components should be kept.

Figure 7 shows the relation between the first two principal components which accounted for 36.0% of the total database variability. Different symbols for stable and unstable slopes were used in Fig. 7. It is evident by observing Fig. 7 that the first principal component was directly related to the stability status condition of each slope. Slopes with positive scores in the first principal component have a stable status and slopes with negative scores have an unstable status. It was correct for 71 slopes of the 75 analyzed, representing an accuracy of 94.67%, so an error of 5.33%.

The set of variables with higher weights in the first component are the slope height, blasting method, previous instability, precipitation, weathering grade of the rock and orientation of discontinuities. This is in a good accordance with the results of systems analyzed by Zare Naghadehi et al. (2013) in which most of the selected parameters of current research have shown high interaction intensity values and hence high importance in the subject of slope stability.

Table 5 Significance of the components and proportion of explained variance

Components	Standard deviation	Proportion of explained variance	
		By component	Accumulated
Comp. 1	2.029	0.229	0.229
Comp. 2	1.54	0.132	0.360
Comp. 3	1.378	0.105	0.466
Comp. 4	1.253	0.087	0.553
Comp. 5	1.107	0.068	0.621
Comp. 6	1.006	0.056	0.677
Comp. 7	0.964	0.052	0.729
Comp. 8	0.958	0.051	0.780
Comp. 9	0.854	0.040	0.820
Comp. 10	0.779	0.034	0.854
Comp. 11	0.702	0.027	0.881
Comp. 12	0.674	0.025	0.907
Comp. 13	0.624	0.022	0.928
Comp. 14	0.595	0.020	0.948
Comp. 15	0.573	0.018	0.966
Comp. 16	0.497	0.014	0.980
Comp. 17	0.457	0.012	0.992
Comp. 18	0.388	0.008	1.000

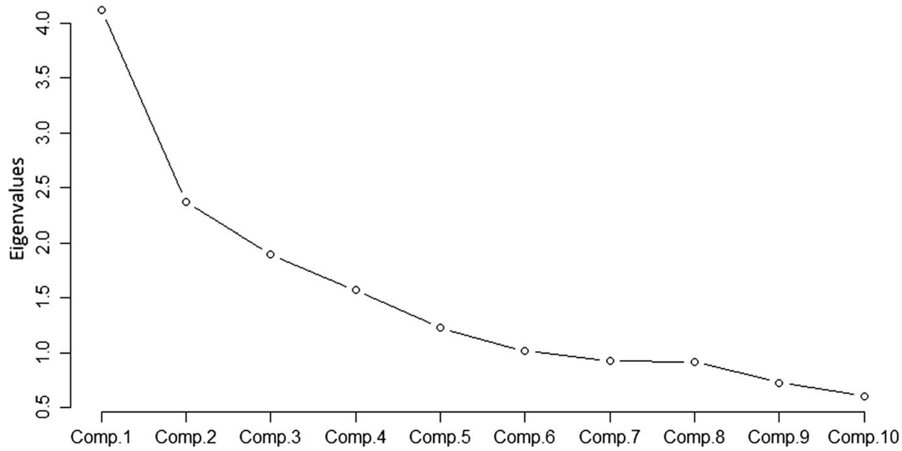
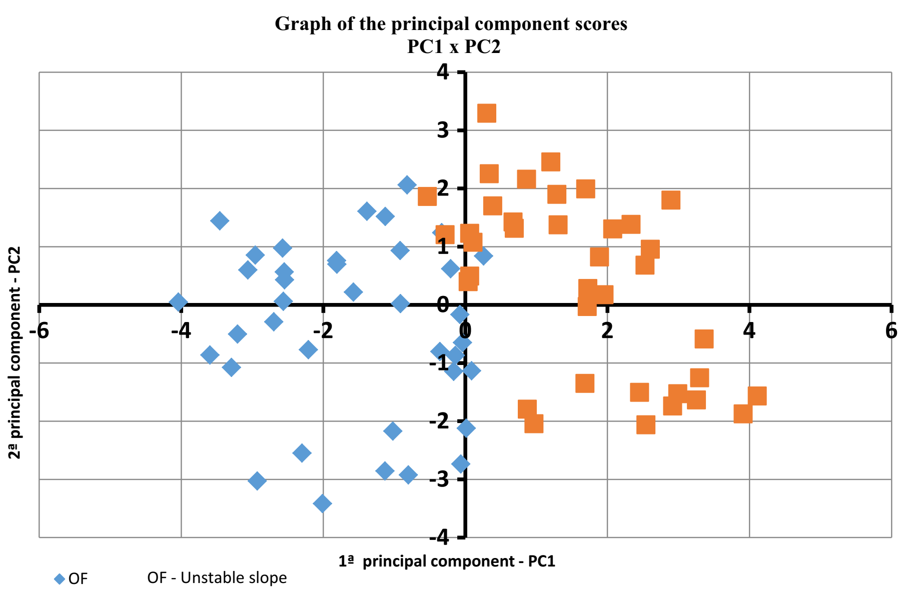


Fig. 6 Scree-plot of the first ten principal components

Fig. 7 Scores of the first two principal components



In the second component the variables with higher weights are: the rock type, slope angle and the tectonic regime.

The first two components account for 36.0% of the total variability of the database. The meaning of the other four selected principal component was not straightforwardly interpreted.

4.2.2 Boosting Procedure via Discriminant Analysis for Two Classes: Stable and Unstable

The discriminant analysis was used to determine the starting basic classifier for the boosting procedure. A number of 100 iterations for the stabilization of the classifier were defined in the boosting procedure. The procedure stabilized the classifiers in the seventh iteration. Figure 8 shows the iterations and classifier values.

The analysis of Fig. 8 allows observing the relation between the values of the classifiers in each iteration. The value of the classifier for coefficient 1 is greater than the values of classifiers for the other coefficients because of the variance explained by the first component is 0.229 (see Table 5). This variance is high comparing to the variance of other components; so the first component explains significant part of the original

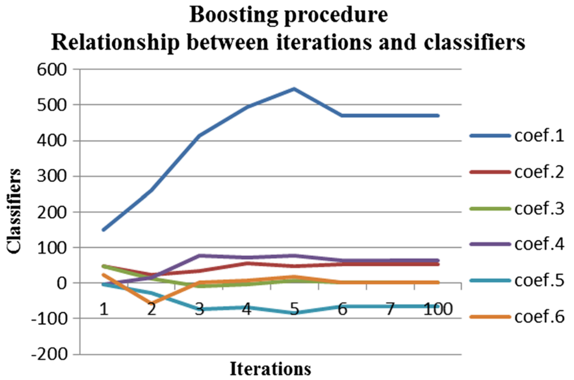


Fig. 8 Relationship between iterations and classifier values

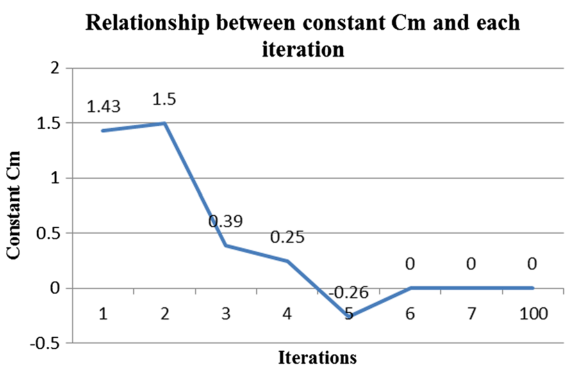


Fig. 9 Relationship between constant c_m and each iteration

data set variability, which reflects on a greater weight of its classifier. As the values of the variances explained by the other components are close, the classifiers presented a slight variation.

Figure 9 presents the values of the constant c_m for each iteration of the boosting procedure. It is observed that nearby the sixth iteration the constant value became null, demonstrating the stabilization of classifiers.

Once classifiers are stabilized, the final classifier is defined by its result after six iterations. It is given by the linear combination of classifiers f_m, Eq. (12), i.e.

$$sign(F(x)) = sign \sum_{m=1}^{M} c_m f_m(x).$$

$$\begin{aligned} sign(F(x)) = {} & 745.97(Comp1) + 118.40(Comp2) \\ & + 78.14(Comp3) + 46.31(Comp4) \\ & - 70.55(Comp5) - 54.85(Comp6) \end{aligned} \tag{12}$$

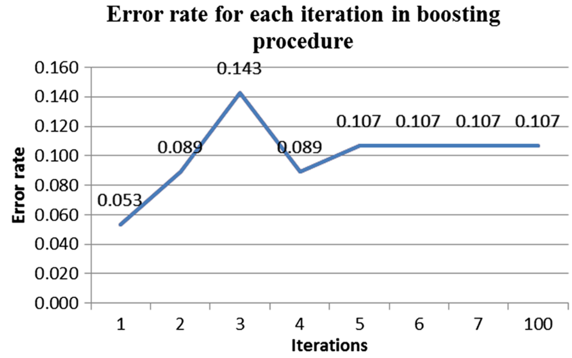


Fig. 10 Error rate of the boosting procedure

The sign of the function in Eq. (12), negative or positive, indicates the classification status of the slope. When the sign is negative, the slope is stable and when the sign is positive, the slope is unstable. Figure 10 presents the errors related to the classification in each iteration. It is possible to see that the larger error is equal to 0.14 in the third iteration. The error stabilized at the fifth iteration.

By observing Eq. (12) it is possible to see that the largest classifiers are related to the first two components. This result demonstrated that the classifiers of the components 1 and 2 define the slope stability condition status, confirming the results yielded by principal component analysis.

4.2.3 Validation Results of the Function Obtained by the Boosting via Discriminant Analysis for Two Classes: Stable and Unstable Conditions

The validation sample (Fig. 2) was used to perform the internal validation. Table 6 presents the results of this internal validation. The point estimates of the apparent error rate, the overall probability of success, error 1 (probability of an unstable slope be classified as stable) and error 2 (probability of a stable slope be classified as unstable) are shown.

The overall probability of success is 94.73% in the database used for internal validation. Consequently, the apparent error rate is 5.26%. The errors 1 and 2 are 0 and 10.0%, respectively. This result, once again, demonstrated the reliability of the method.

For this research, the two errors are important but the error 1 is more serious. The error 1 occurs when an unstable slope is classified as a stable slope. This error

Table 6 Results of the internal validation

Internal validation				
Point estimate	Apparent error rate	Overall probability of success	Error 1	Error 2
	0.0526	0.9473	0.0000	0.1000

Table 7 Results of the confidence intervals

Rates	Lower limit	Point estimate	Upper limit
Probability	2.5%		97.5%
Apparent error rate	0.05263158	0.05263158	0.058389792
Overall probability of success	0.94736842	0.94736840	0.953126634
Error 1	0.00000000	0.00000000	0.005758213
Error 2	0.03703704	0.10000000	0.105794500

could lead to safety problems in open pit slopes. The error 2 occurs when a stable slope is classified as an unstable slope. This error denotes a conservative estimation of the slope stability status.

The error 1 is close to zero, which is in favor of security. The error 2 is equal to 10.0%. Therefore, the function obtained by boosting procedure via discriminant analysis proved to be little conservative, once error 2 comprised the total error rate of the function.

The bootstrap technique was used to estimate the confidence intervals for the apparent error rate, the overall probability of success, the error 1 and the error 2, Efron and Tibshirani (1993). In this technique 10,000 samples were randomly picked from the validation sample (Fig. 4).

The confidence interval results provided the estimation of the lower and upper limits of the apparent error rate, the overall probability of success and the errors 1 and 2. Table 7 shows the results of the confidence intervals.

The confidence intervals were very strict around the point estimates (Table 7). This observation confirmed that the boosting procedure improved the classifier to the maximum. Figure 11 presents graphically the results for the confidence intervals.

The external validation was performed in the test sample (Fig. 2) which comprised 10% of the original database.

The function generated by boosting procedure via discriminant analysis classified correctly all the slopes of the test sample. Then, the overall probability of success obtained was nearby 100%, so the apparent error rate was nearby 0%. Table 8 shows the results of the model external validation.

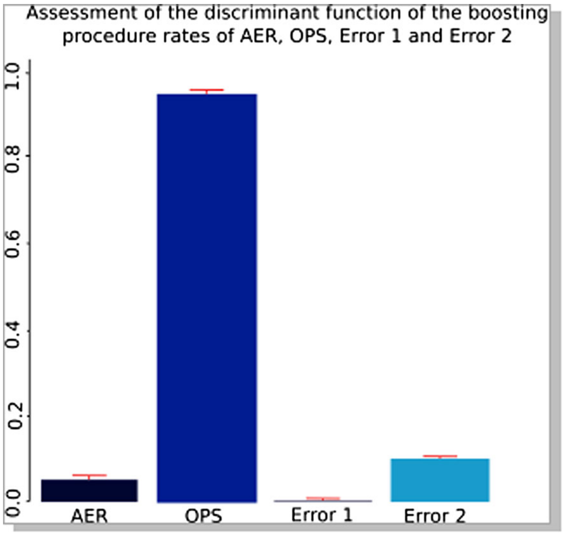


Fig. 11 Rates of AER, OPS, error 1 and error 2

4.2.4 *Boosting Procedure via Discriminant Analysis for Three Classes: Stable, Overall Failure and Failure in Set of Benches*

The multiclass boosting procedure was used to classify the slopes in three classes: stable, overall failure and failure in set of benches. It was necessary to use the multiclass boosting algorithm for two discriminant functions, considering that the problem involves the classification in three different classes.

Table 8 Results of the model external validation

External validation				
Point estimate	Apparent error rate	Overall probability of success	Error 1	Error 2
	0.0000	1.0000	0.0000	0.0000

Table 9 Results of the internal validation

Internal Validation for the analysis with three classes								
Point estimate	Apparent error rate	Overall probability of success	Error1	Error 2	Error 3	Error 4	Error 5	Error 6
	0.3157	0.6842	0.8000	0.2000	0.0000	0.0000	0.1000	0.0000

The multiclass boosting procedure was performed with a number of 1000 iterations, but in the second iteration the strongest classifiers were achieved. Thus, after the second iteration classifiers do not change anymore. The Eqs. 13 and 14 were obtained in the second iteration:

$$F_{LD1}(x) = -63.56(Comp1) + 14.65(Comp2) + 14.71(Comp3) - 4.83(Comp4) - 5.44(Comp5) - 1.56(Comp6) \quad (13)$$

$$F_{LD2}(x) = -3.74(Comp1) - 14.66(Comp2) - 13.49(Comp3) - 17.20(Comp4) - 18.98(Comp5) - 32.66(Comp6) \quad (14)$$

4.2.5 Validation Results of the Function Obtained by the Boosting via Discriminant Analysis for Three Classes: Stable, Overall Failure and Failure in Set of Benches

The validation sample (Fig. 2) was used to perform the internal validation. In this validation, there are six types of errors:

1. Error 1: probability of a failure in set of benches be classified as an overall failure
2. Error 2: probability of a failure in set of benches be classified as a stable slope
3. Error 3: probability of an overall failure be classified as a failure in set of benches
4. Error 4: probability of an overall failure be classified as a stable slope
5. Error 5: probability of a stable slope be classified as a failure in set of benches
6. Error 6: probability of a stable slope be classified as an overall failure slope

Table 9 presents the results of the internal validation. The point estimates of the apparent error rate, the overall probability of success and the six types of errors are shown.

The overall probability of success is 68.45%. This value is less than the value for the boosting with two classes, probably due to the inclusion of one more class. Figure 12 shows the comparison between apparent error rate and overall probability of success for the two analyses, with two classes and three classes.

The apparent error rate is 31.57%. This value is large comparing to the one obtained by boosting procedure with two classes.

It is important to note that errors 1, 2 and 5 are relative, and these probabilities are calculated according to the number of slopes that are randomly selected for the internal validation sample. In this validation the algorithm selected 5 slopes of the class Failure in set of benches and 4 of these slopes were classified by the algorithm as Overall failure class and 1 slope as Stable class. Therefore, Error 1 has a value of 80% and Error 2 has a value of 20%, which reflects a lack of accuracy of the boosting multiclass to distinguish the failure in set of benches.

The confidence interval results provided the estimation of the lower and upper limits of the apparent error rate, the overall probability of success and the errors 1, 2, 3, 4, 5 and 6. The range of values of the confidence intervals is low; hence the point estimate is significant. Table 10 shows the results of the confidence intervals, and Fig. 13 presents the results for the confidence intervals.

In the external validation, nine slopes of the sample test (Fig. 2) were used, and three slopes were not

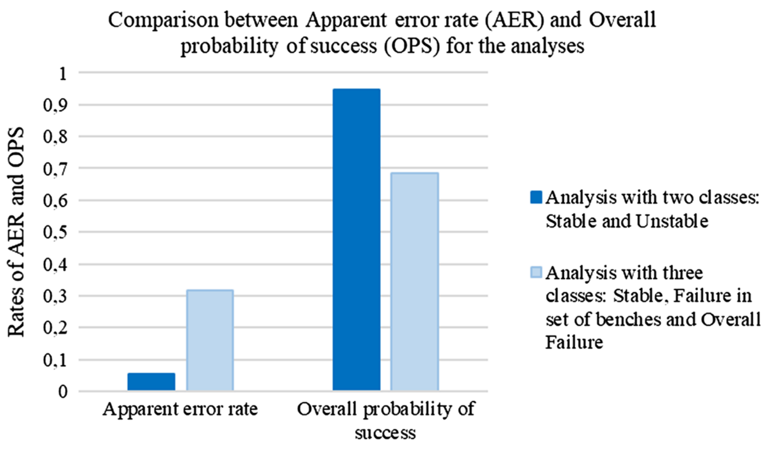


Fig. 12 Comparation of rates of AER an OPS for the analyses

Table 10 Results of the confidence intervals

Rates	Lower limit	Point estimate	Upper limit
Probability	2.5%		97.5%
Apparent error rate	0.3170063	0.3157895	0.3218735
Overall probability of success	0.6896861	0.6842105	0.6945534
Error 1	0.8000000	0.8000000	0.8214734
Error 2	0.2000000	0.2000000	0.2143156
Error 3	0.0000000	0.0000000	0.0000000
Error 4	0.0000000	0.0000000	0.0000000
Error 5	0.1000000	0.1000000	0.1035455
Error 6	0.0000000	0.0000000	0.0000000

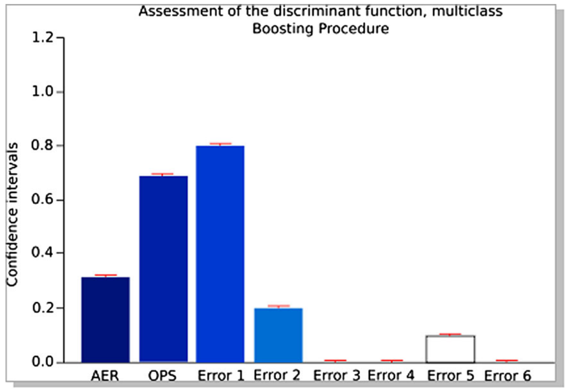


Fig. 13 Rates of AER, OPS, error 1, error 2, error 3, error 4, error 5 and error 6

classified correctly, all three due to error 1. Thus, the discriminant functions presented values of apparent error rate (AER) of 0.333, overall probability success (OPS) of 0.666 and error 1 of 0.187. The other errors presented value of 0.00. The Table 11 shows the result for external validation.

The results of external validation confirmed the difficulty of the model to distinguish the failure in set of benches, the same evidence shown by internal validation. However the error 1 is much smaller in the external validation, which is a good indication of the model ability to explain the phenomenon.

The methodology proposed by Zare Naghadehi et al. (2013) in the Mine Slope Instability Index (MSII) was validated by comparing predictions of the MSII with the known status for each slope. This procedure classified 11 slopes correctly of 12 slopes used in the sample test. In this research, the first analysis, with two classes, the sample test used 10 slopes and as result of validation all slopes were classified correctly. In the second analysis, with three classes, the validation test used nine slopes and as result of validation 6 slopes were classified correctly. In this research, the boosting for two classes presented accurate results as those presented by MSII. The boosting for three classes presented errors that are strongly related to the prediction of failure in set of benches.

Table 11 Results of the external validation of the model

External validation								
Point estimate	Apparent error rate	Overall probability of success	Error 1	Error 2	Error 3	Error 4	Error 5	Error 6
	0.3333	0.6666	0.1875	0.0000	0.0000	0.0000	0.0000	0.0000

The scripts developed in R for the two analyses are presented in "Appendix 1".

5 Conclusions

The discriminant function obtained in this research was able to classify the slopes according to their stability conditions, stable and unstable. The differentiation between overall failure and failure in set of benches is perceptible by multiclass boosting only in external validation. Internal validation of this model did not yield reliable results. Boosting multiclass yielded worse results than boosting with two classes. A more detailed analysis should be addressed to explore the causes for this result.

The technique of principal component analysis was applied to the database allowing the selection of the first six components, according to Kaiser's criterion, keeping approximately 70% of the total variability of the database. The first component could be interpreted as an index of the stability condition of slopes, represented mainly by the variables: slope height, blasting method, previous instability, precipitation, weathering and orientation of discontinuities. The second component is not easily interpretable.

The discriminant function obtained by boosting procedure via discriminant analysis with two classes, stable and unstable, presented interesting results because the classifiers assigned higher weights to the first two principal components, which were the most significant ones.

The functions obtained by boosting procedure were validated in two different ways: internal validation and external validation. In the analysis with two stability conditions the function obtained in internal validation presented an overall probability of success of 94.73% and an apparent error rate of 5.26%. Error 1 (unstable slopes classified as stable slopes) did not present significant values. The error 2 (stable slopes classified as unstable slopes) presented an error of 10.0%. In the external validation, the function was able to classify correctly all the slopes and it presented an overall probability of success nearby 100% and error rates close to zero.

The results of this research allowed the comparison between the two applied boosting algorithms. It must be highlighted that the PCA analysis allowed the clear identification of two classes, stable and unstable. This result indicates a preferential clustering of the slopes in only two classes, i.e. the scale of failure is not distinguishable by the analysis. Boosting analyses confirm this clustering because the inclusion of three classes in boosting increased the errors.

This research has demonstrated that multivariate statistics can be a powerful tool, helping engineers to cope with geotechnical problems in open pit slopes, especially because information from the database used in this study is absolutely wide, providing it was representative of many different situations all over the world.

Acknowledgements The authors wish to thank CNPq (National Counsel of Technological and Scientific Development) and Fapemig (Foundation for Research Support of Minas Gerais) for supporting this work.

Appendix 1

SCRIPT FOR TWO CLASSES, STABLE AND UNSTABLE

```
## Algorithm Boosting via Discriminant Analysis
## install.packages('ks',dependencies=TRUE)
library('ks')

## Discriminant function
flda <- function(xtrain, group)
{
  train <- cbind(xtrain, group)
  x1 <- as.matrix(train[group==-1, 1:(dim(train)[2]-1)])
  x2 <- as.matrix(train[group==1, 1:(dim(train)[2]-1)])
  mean_x1 <- apply(x1, 2, mean)
  mean_x2 <- apply(x2, 2, mean)
  S1 <- cov(x1)
  S2 <- cov(x2)
  Sc <- (((dim(x1)[1]-1)*S1)+((dim(x2)[1]-1)*S2))/(dim(x1)[1]+dim(x2)[1]-2)
  l_hat <- t(mean_x1 - mean_x2) %*% solve(Sc)
  m_hat <- .5 * t(mean_x1 - mean_x2) %*% solve(Sc) %*% (mean_x1 + mean_x2)
  means <- rbind(mean_x1, mean_x2)
  rownames(means) <- list("group 1 ", "group 2 ")
  result <- list(means, S1, S2, Sc, l_hat, m_hat)
  names(result) <- list("means", "s1", "s2", "sc", "coef", "threshold")
  result
}

## Discriminant function prediction
predict.lda <- function(fit, newdata)
{
  x <- newdata
  n <- dim(x)[1]
  coef <- fit$coef
  threshold <- fit$threshold
  y_hat <- numeric(n)
  class <- numeric(n)
  for(t in 1:n)
  {
    y_hat[t] <- coef%*%x[t,]
    if(y_hat[t]>=threshold)
      class[t] <- -1
    else
      class[t] <- 1
  }
  class
}



## Boosting via discriminant linear Fisher function
BoostLDA <- function(xtrn, ytrn, M)
{
  N <- dim(xtrn)[1]
  p <- dim(xtrn)[2]
  w <- rep((1/N),N)
  f.trn <- matrix(0,N,M)
  F.trn <- numeric(N)
  I <- matrix(0,N,M)
  error <- numeric(M)
  c <- numeric(M)
  coefs <- matrix(0,M,p)
  thresholds <- numeric(M)
  for (m in 1:M) {
    xweighted <- w*xtrn
    classifier <- flda(xweighted,ytrn)
    coefs[m,] <- classifier$coef
    thresholds[m] <- classifier$threshold
    f.trn[,m] <- predict.lda(classifier,xtrn)
    I[,m] <- (f.trn[,m] != ytrn)
    error[m] <- w%*%I[,m]
    c[m] <- .5*log((1-error[m])/error[m])
    w <- w*exp(-c[m]*ytrn*f.trn[,m])
    w <- pmax(w/sum(w),1e-24)
```

```
    F.trn <- F.trn+(c[m]*f.trn[,m])
  }
  class.trn <- sign(F.trn)
  result <- list(coefs, thresholds, error, w, c, class.trn, error.trn = apply(I, 2, mean))
  names(result) <- list("coefs", "thresholds", "error","weights", "const", "class.trn", "error.trn")
  result
}

## boosting to predict discriminant function
predict.boostlda <- function(fit, newdata)
{
  x <- newdata
  n <- dim(x)[1]
  p <- dim(x)[2]
  coefs <- fit$coefs
  thresholds <- fit$thresholds
  boost.weights <- fit$const
  M <- length(thresholds)
  y_hat <- matrix(0,n,M)
  class <- matrix(0,n,M)
  for (i in 1:M) {
    for (j in 1:n) {
      y_hat[j,i] <- coefs[i, ]%*%x[j, ]
      if (y_hat[j,i] >= thresholds[i])
        class[j,i] <- -1
      else
        class[j,i] <- 1
    }
  }
  class.final <- sign(t(t(boost.weights)%*%t(class)))
  result <- list(class, class.final)
  names(result) <- list("class", "class.final")
  result
}


########## Boosting estimation start

dados = read.table("dados_allan.txt",header=TRUE,row.names=1)
head(dados)

n = dim(dados)[1]

## PCA analysis with 90% of data; 10% to test FDA
prop = 0.9

## Sample to estimate PCA
set.seed(1234)
trind_acp = sample(1:n, floor(prop*n),replace = FALSE)
Xtreino_acp = dados[trind_acp,-19]
Ytreino_acp = dados[trind_acp,19]

## Sample for external validation
teind_acp = setdiff(1:n,trind_acp)
Xteste_acp = dados[teind_acp,-19]
Yteste_acp = dados[teind_acp,19]
Yteste_acp_cod = ifelse(Yteste_acp=='ST',-1,1)

## Estimation of PCA components
acp = princomp(Xtreino_acp, cor = TRUE, scores=TRUE)
y_acp = acp$scores[,1:6]
y_acp

## Sample to external validation
y_teste = predict(acp,Xteste_acp)[,1:6]
y_teste

## Boosting function estiomation via discriminant analysis

## Training sample
m = dim(y_acp)[1]
prop_lda = 0.75

set.seed(432100)
trind_lda = sample(1:m, floor(prop_lda*m),replace = FALSE)
Xtreino_lda = y_acp[trind_lda,]
```

```
Ytreino_lda = Ytreino_acp[trind_lda]
Ytreino_lda_cod = ifelse(Ytreino_lda=='ST',-1,1)

## Sample to test FDA, internal cross validation
teind_lda = setdiff(1:m,trind_lda)
Xteste_lda = y_acp[teind_lda,]
Yteste_lda = Ytreino_acp[teind_lda]
Yteste_lda_cod = ifelse(Yteste_lda=='ST',-1,1)


## Boosting model estimation via discrimant function
M <- 100
fit1 <- BoostLDA(Xtreino_lda, Ytreino_lda_cod,M)

## AER estimation: Sample for punctual validation
pred1.tes <- predict.boostlda(fit1, Xteste_lda)$class.final
tab1.tes <- compare(pred1.tes, Yteste_lda_cod)

TEA_tes_i <- tab1.tes$error
PGAcerto_tes_i <- 1-TEA_tes_i
Pstof_tes_i <- tab1.tes$cross[2,1]/sum(Ytreino_lda_cod==1)
Pofst_tes_i <- tab1.tes$cross[1,2]/sum(Ytreino_lda_cod==-1)

resultado.validacao.interna <- matrix(c(TEA_tes_i, PGAcerto_tes_i,
                                        Pstof_tes_i, Pofst_tes_i),1, 4)

colnames(resultado.validacao.interna) <- c("TEA", "PGAcerto","P(ST|OF)","P(OF|ST)")
rownames(resultado.validacao.interna) <- "Estimacao Pontual"

resultado.validacao.interna

### Confidence interval estimation - bootstrap

B = 10000

V_TEA_tes_i = matrix(NA,B,1)
V_PGAcerto_tes_i = matrix(NA,B,1)
V_Pstof_tes_i = matrix(NA,B,1)
V_Pofst_tes_i = matrix(NA,B,1)


for(i in 1:B){


 ## CI for boosting, internal validation
 teind_lda_boot = sample(teind_lda, length(teind_lda),replace = TRUE)
 Xteste_lda_boot = y_acp[teind_lda_boot,]
 Yteste_lda_boot = Ytreino_acp[teind_lda_boot]
 Yteste_lda_boot_cod = ifelse(Yteste_lda_boot=='ST',-1,1)

 pred1.tes <- predict.boostlda(fit1, Xteste_lda_boot)$class.final
 tab1.tes <- compare(pred1.tes, Yteste_lda_boot_cod)
 V_TEA_tes_i[i] <- tab1.tes$error
 V_PGAcerto_tes_i[i] <- 1-V_TEA_tes_i[i]
 V_Pstof_tes_i[i] <- tab1.tes$cross[2,1]/sum(Yteste_lda_boot_cod==1)
 V_Pofst_tes_i[i] <- tab1.tes$cross[1,2]/sum(Yteste_lda_boot_cod==-1)

}

## confidence interval via resampling

talpha1  <- quantile(V_TEA_tes_i,0.025)
talpha2 <- quantile(V_TEA_tes_i,0.975)
s <- sd(V_TEA_tes_i)
c(TEA_tes_i+talpha1*mean(s)/sqrt(n),TEA_tes_i+talpha2*mean(s)/sqrt(n))


talpha1  <- quantile(V_PGAcerto_tes_i,0.025)
talpha2 <- quantile(V_PGAcerto_tes_i,0.975)
s <- sd(V_PGAcerto_tes_i)
c(PGAcerto_tes_i+talpha1*mean(s)/sqrt(n),PGAcerto_tes_i+talpha2*mean(s)/sqrt(n))


talpha1  <- quantile(V_Pstof_tes_i,0.025)
talpha2 <- quantile(V_Pstof_tes_i,0.975)
s <- sd(V_Pstof_tes_i)
```

```
c(Pstof_tes_i+talpha1*mean(s)/sqrt(n),Pstof_tes_i+talpha2*mean(s)/sqrt(n))


talpha1  <- quantile(V_Pofst_tes_i,0.025)
talpha2 <- quantile(V_Pofst_tes_i,0.975)
s <- sd(V_Pofst_tes_i)
c(Pofst_tes_i+talpha1*mean(s)/sqrt(n),Pofst_tes_i+talpha2*mean(s)/sqrt(n))



## External validation: test sampling
pred2.tes <- predict.boostlda(fit1, y_teste)$class.final
tab2.tes <- compare(pred2.tes, Yteste_acp_cod)

TEA_tes_e <- tab2.tes$error

PGAcerto_tes_e <- 1-TEA_tes_e


Pstof_tes_e <- tab2.tes$cross[2,1]/sum(Ytreino_lda_cod==1)
Pofst_tes_e <- tab2.tes$cross[1,2]/sum(Ytreino_lda_cod==-1)

resultado.validacao.externa <- matrix(c(TEA_tes_e, PGAcerto_tes_e,
                                  Pstof_tes_e, Pofst_tes_e),1, 4)

colnames(resultado.validacao.externa) <- c("TEA", "PGAcerto","P(ST|OF)","P(OF|ST)")
rownames(resultado.validacao.externa) <- "Estimacao Pontual"

resultado.validacao.externa


#### Drawing graphs with CI

## Graph of AER (Apparent error rate), PGAcerto, P(ST|OF) e P(OF|ST)

ep = c(TEA_tes_i, PGAcerto_tes_i, Pstof_tes_i, Pofst_tes_i)
ep

names(ep) <- c('TEA', 'PGAcerto', 'P(ST|OF)','P(OF|ST)')

ylim <- c(0, max(ep)*1.5)

color <- colorRampPalette(c("darkblue","lightblue"))
color(4)

bp = barplot(ep, ylim = ylim,col=color(4),main = 'Medidas de AvaliaÃ§Ã£o da FDLF via Boosting')

lim_inf = c(TEA_tes_i+talpha1*mean(s)/sqrt(n),PGAcerto_tes_i+talpha1*mean(s)/sqrt(n),
        Pstof_tes_i+talpha1*mean(s)/sqrt(n),Pofst_tes_i+talpha1*mean(s)/sqrt(n))

lim_sup = c(TEA_tes_i+talpha2*mean(s)/sqrt(n),PGAcerto_tes_i+talpha2*mean(s)/sqrt(n),
        Pstof_tes_i+talpha2*mean(s)/sqrt(n),Pofst_tes_i+talpha2*mean(s)/sqrt(n))

arrows(bp, lim_sup, bp, lim_inf, code=3, angle=90,col='red')
```

SCRIPT FOR BOOSTING MULTICLASS

```
##final boosting multiclass##
########################


##packages##
library(ks)
library(MASS)
```

```
library(caret)
##functions
BoostMultiLDA <- function (xtrn, ytrn, M = 100)
{
 library(MASS)
 n <- dim(xtrn)[1]
 nclases <- nlevels(ytrn)
 pesos <- rep(1/n, n)
 guardarpesos <- array(0, c(n, M))
 w <- rep(1/n, n)
 data <- cbind(pesos, xtrn)
 fds <- list()
 pond <- rep(0, M)
 pred <- data.frame(rep(0, n))

 for (m in 1:M) {
    w <<- pesos
    xweighted <- w*xtrn
    fit <- MASS::lda(xweighted,ytrn)
    flearn <- predict(fit, data = data[, -1])$class
    ind <- as.numeric(ytrn != flearn)
    err <- sum(pesos * ind)

   c <- (1/2) * log((1 - err)/err)

   guardarpesos[, m] <- pesos
   pesos <- pesos * exp(c * ind)
   pesos <- pesos/sum(pesos)
   maxerror <- 0.5
   eac <- 0.001

   if (err >= maxerror) {
    pesos <- rep(1/n, n)
    maxerror <- maxerror - eac
    c <- (1/2) * log((1 - maxerror)/maxerror)
   }
   if (err == 0) {
    pesos <- rep(1/n, n)
    c <- (1/2) * log((1 - eac)/eac)
   }
   fds[[m]] <- fit
   pond[m] <- c
   if (m == 1) {
    pred <- flearn
```

```
  }else {
    pred <- data.frame(pred, flearn)
  }
 }
 classfinal <- array(0, c(n, nlevels(ytrn)))
 for (i in 1:nlevels(ytrn)) {
  classfinal[, i] <- matrix(as.numeric(pred == levels(ytrn)[i]),
                            nrow = n) %*% as.vector(pond)
 }
 predclass <- rep("O", n)
 predclass <- factor(apply(classfinal,1,which.max), labels=c(levels(ytrn)[1],levels(ytrn)[2],levels(ytrn)[3]),
                     levels=c(1,2,3))
 probabilidades <- classfinal/apply(classfinal, 1, sum)
 ans <- list(FD = fds, Importancia_cl = pond,
             votos = classfinal, prob = probabilidades, class = predclass)
 attr(ans, "Ytreino.summary") <- summary(ytrn, maxsum = 700)
 ans$call <- match.call()
 class(ans) <- "boosting"
 ans
}

predict.BoostMultiLDA <- function (fit, newdata)
{
 x <- newdata
 n <- dim(x)[1]
 p <- dim(x)[2]
 ytrn.summary <- attributes(fit1)$Ytreino.summary
 nclasses <- length(ytrn.summary)
 pesos <- rep(1/n, n)
 pond <- fit$Importancia_cl
 M = length(fit$FD)

 for (i in 1:M) {
  prd <- predict(fit$FD[[i]],x)$class
  if (i == 1) {
    pred <- prd
  }else {
    pred <- data.frame(pred, prd)
  }
 }
 classfinal <- array(0, c(n, nclasses))

 for (i in 1:nclasses) {
  classfinal[, i] <- matrix(as.numeric(pred == names(ytrn.summary)[i]),
                            nrow = n) %*% pond
 }
 predclass <- rep("O", n)
 predclass <- factor(apply(classfinal,1,which.max), labels=c(names(ytrn.summary)[1],names(ytrn.summary)[2]
                                              ,names(ytrn.summary)[3]),
                     levels=c(1,2,3))
```

```
  prob <- classfinal/apply(classfinal, 1, sum)
  output <- list( votos = classfinal, prob = prob, class = predclass)
}
```

Script

```
########################################
########## Starting boosting estimation########
#########################################

dados = read.table("dados_allan_tres.txt",header=TRUE,row.names=1)
head(dados)

str(dados)

n = dim(dados)[1]

## Sample with 90%##
prop = 0.9

## Sample to estimate PCA
set.seed(1234)
trind_acp = sample(1:n, floor(prop*n),replace = FALSE)
Xtreino_acp = dados[trind_acp,-19]
Ytreino_acp = dados[trind_acp,19]

## Sample to test FDA: external validation
teind_acp = setdiff(1:n,trind_acp)
Xteste_acp = dados[teind_acp,-19]
Yteste_acp = dados[teind_acp,19]
#Yteste_acp_cod = ifelse(Yteste_acp=='ST',-1,1)

## Estimating PCA
acp = princomp(Xtreino_acp, cor = TRUE, scores=TRUE)
y_acp = acp$scores[,1:6]
y_acp

## Sample to external validation
y_teste = predict(acp,Xteste_acp)[,1:6]
y_teste

## Estimate boosting function with function discriminant

## Sample trainning
m = dim(y_acp)[1]
prop_lda = 0.75

set.seed(432100)
trind_lda = sample(1:m, floor(prop_lda*m),replace = FALSE)
Xtreino_lda = y_acp[trind_lda,]
```

```
Ytreino_lda = Ytreino_acp[trind_lda]
#Ytreino_lda_cod = ifelse(Ytreino_lda=='ST',-1,1)

## Sample to internal validation
teind_lda = setdiff(1:m,trind_lda)
Xteste_lda = y_acp[teind_lda,]
Yteste_lda = Ytreino_acp[teind_lda]
#Yteste_lda_cod = ifelse(Yteste_lda=='ST',-1,1)

## Estimating boosting with discriminantfunction
M <- 1000
fit1 <- BoostMultiLDA(Xtreino_lda, Ytreino_lda,M)

## Estimating AER
pred1.tes <- predict.BoostMultiLDA(fit1, Xteste_lda)$class
tab1.tes <- compare(Yteste_lda, pred1.tes)

TEA_tes_i = tab1.tes$error
PGAcerto_tes_i <- 1-TEA_tes_i

Pfsbof_tes_i <- tab1.tes$cross[1,2]/sum(Yteste_lda=='FSB')
Pfsbst_tes_i <- tab1.tes$cross[1,3]/sum(Yteste_lda=='FSB')
Poffsb_tes_i <- tab1.tes$cross[2,1]/sum(Yteste_lda=='OF')
Pofst_tes_i <- tab1.tes$cross[2,3]/sum(Yteste_lda=='OF')
Pstfsb_tes_i <- tab1.tes$cross[3,1]/sum(Yteste_lda=='ST')
Pstof_tes_i <- tab1.tes$cross[3,2]/sum(Yteste_lda=='ST')

library(caret)

confusionMatrix(table(Yteste_lda,pred1.tes))

fit1$Importancia_cl


### Confidence interval

B = 10000


V_TEA_tes_i = matrix(NA,B,1)
V_PGAcerto_tes_i = matrix(NA,B,1)
V_Pfsbof_tes_i = matrix(NA,B,1)
V_Pfsbst_tes_i = matrix(NA,B,1)
V_Poffsb_tes_i = matrix(NA,B,1)
V_Pofst_tes_i = matrix(NA,B,1)
V_Pstfsb_tes_i = matrix(NA,B,1)
V_Pstof_tes_i = matrix(NA,B,1)

for(i in 1:B){
 ## IC bootstrap for boosting using internal validation
 teind_lda_boot = sample(teind_lda, length(teind_lda),replace = TRUE)
```

```
 Xteste_lda_boot = y_acp[teind_lda_boot,]
 Yteste_lda_boot = Ytreino_acp[teind_lda_boot]
# Yteste_lda_boot_cod = ifelse(Yteste_lda_boot=='ST',-1,1)

 pred1.tes <- predict.BoostMultiLDA(fit1, Xteste_lda_boot)$class
 tab1.tes <- compare(Yteste_lda_boot,pred1.tes)
 V_TEA_tes_i[i] <- tab1.tes$error
 V_PGAcerto_tes_i[i] <- 1-V_TEA_tes_i[i]
 V_Pfsbof_tes_i[i] <- tab1.tes$cross[1,2]/sum(Yteste_lda_boot=='FSB')
 V_Pfsbst_tes_i[i] <- tab1.tes$cross[1,3]/sum(Yteste_lda_boot=='FSB')
 V_Poffsb_tes_i[i] <- tab1.tes$cross[2,1]/sum(Yteste_lda_boot=='OF')
 V_Pofst_tes_i[i] <- tab1.tes$cross[2,3]/sum(Yteste_lda_boot=='OF')
 V_Pstfsb_tes_i[i] <- tab1.tes$cross[3,1]/sum(Yteste_lda_boot=='ST')
 V_Pstof_tes_i[i] <- tab1.tes$cross[3,2]/sum(Yteste_lda_boot=='ST')

 print(i)
}

## confidence interval via resampling

talpha1  <- quantile(V_TEA_tes_i,0.025)
talpha2 <- quantile(V_TEA_tes_i,0.975)
s <- sd(V_TEA_tes_i)
c(TEA_tes_i+talpha1*s/sqrt(n),TEA_tes_i+talpha2*s/sqrt(n))


talpha1  <- quantile(V_PGAcerto_tes_i,0.025)
talpha2 <- quantile(V_PGAcerto_tes_i,0.975)
s <- sd(V_PGAcerto_tes_i)
c(PGAcerto_tes_i+talpha1*mean(s)/sqrt(n),PGAcerto_tes_i+talpha2*mean(s)/sqrt(n))


talpha1  <- quantile(V_Pstof_tes_i,0.025)
talpha2 <- quantile(V_Pstof_tes_i,0.975)
s <- sd(V_Pstof_tes_i)
c(Pstof_tes_i+talpha1*mean(s)/sqrt(n),Pstof_tes_i+talpha2*mean(s)/sqrt(n))


talpha1  <- quantile(V_Pofst_tes_i,0.025, na.rm = TRUE)
talpha2 <- quantile(V_Pofst_tes_i,0.975, na.rm = TRUE)
s <- sd(V_Pofst_tes_i, na.rm = TRUE)
c(Pofst_tes_i+talpha1*mean(s)/sqrt(n),Pofst_tes_i+talpha2*mean(s)/sqrt(n))



talpha1  <- quantile(V_Pfsbof_tes_i,0.025, na.rm = TRUE)
talpha2 <- quantile(V_Pfsbof_tes_i,0.975, na.rm = TRUE)
s <- sd(V_Pfsbof_tes_i, na.rm = TRUE)
c(Pfsbof_tes_i+talpha1*mean(s)/sqrt(n),Pfsbof_tes_i+talpha2*mean(s)/sqrt(n))
```

```
talpha1  <- quantile(V_Pfsbst_tes_i,0.025, na.rm = TRUE)
talpha2 <- quantile(V_Pfsbst_tes_i,0.975, na.rm = TRUE)
s <- sd(V_Pfsbst_tes_i, na.rm = TRUE)
c(Pfsbst_tes_i+talpha1*mean(s)/sqrt(n),Pfsbst_tes_i+talpha2*mean(s)/sqrt(n))


talpha1  <- quantile(V_Poffsb_tes_i,0.025, na.rm = TRUE)
talpha2 <- quantile(V_Poffsb_tes_i,0.975, na.rm = TRUE)
s <- sd(V_Poffsb_tes_i, na.rm = TRUE)
c(Poffsb_tes_i+talpha1*mean(s)/sqrt(n),Poffsb_tes_i+talpha2*mean(s)/sqrt(n))

talpha1  <- quantile(V_Pstfsb_tes_i,0.025)
talpha2 <- quantile(V_Pstfsb_tes_i,0.975)
s <- sd(V_Pstfsb_tes_i)
c(Pstfsb_tes_i+talpha1*mean(s)/sqrt(n),Pstfsb_tes_i+talpha2*mean(s)/sqrt(n))

## External Validation
pred2.tes <- predict.BoostMultiLDA(fit1, y_teste)$class
tab2.tes <- compare(Yteste_acp,pred2.tes)

TEA_tes_e <- tab2.tes$error

PGAcerto_tes_e <- 1-TEA_tes_e


Pfsbof_tes_e <- tab2.tes$cross[1,2]/sum(Ytreino_lda=='FSB')
Pfsbst_tes_e <- tab2.tes$cross[1,3]/sum(Ytreino_lda=='FSB')
Poffsb_tes_e <- tab2.tes$cross[2,1]/sum(Ytreino_lda=='OF')
Pofst_tes_e <- tab2.tes$cross[2,3]/sum(Ytreino_lda=='OF')
Pstfsb_tes_e <- tab2.tes$cross[3,1]/sum(Ytreino_lda=='ST')
Pstof_tes_e <- tab2.tes$cross[3,2]/sum(Ytreino_lda=='ST')




resultado.validacao.externa <- matrix(c(TEA_tes_e, PGAcerto_tes_e,
                                  Pfsbof_tes_e, Pfsbst_tes_e, Poffsb_tes_e, Pofst_tes_e,
                                  Pstfsb_tes_e, Pstof_tes_e),1, 8)

colnames(resultado.validacao.externa) <- c("TEA", "PGAcerto","P(FSB|OF)","P(FSB|ST)", "P(OF|FSB)",
                                   "P(OF|ST)", "P(ST|FSB)", "P(ST|OF)")
rownames(resultado.validacao.externa) <- "Estimacao Pontual"

resultado.validacao.externa

tab2.tes

#### Plots

## Graph of AER (Apparent error rate), PGAcerto, P(ST|OF) e P(OF|ST)
```

```
ep = c(TEA_tes_i, PGAcerto_tes_i,
    Pfsbof_tes_i, Pfsbst_tes_i, Poffsb_tes_i, Pofst_tes_i,
    Pstfsb_tes_i, Pstof_tes_i)
ep

names(ep) <- c("TEA", "PGAcerto","P(FSB|OF)","P(FSB|ST)", "P(OF|FSB)",
         "P(OF|ST)", "P(ST|FSB)", "P(ST|OF)")

ylim <- c(0, max(ep)*1.5)

color <- colorRampPalette(c("darkblue","lightblue"))
color(8)

bp = barplot(ep, ylim = ylim,col=color(8),main = 'Validation measures of FDL evaluation via boosting')

lim_inf = c(TEA_tes_i+talpha1*mean(s)/sqrt(n),PGAcerto_tes_i+talpha1*mean(s)/sqrt(n),
       Pfsbof_tes_i+talpha1*mean(s)/sqrt(n), Pfsbst_tes_i+talpha1*mean(s)/sqrt(n),
       Poffsb_tes_i+talpha1*mean(s)/sqrt(n), Pofst_tes_i+talpha1*mean(s)/sqrt(n),
       Pstfsb_tes_i+talpha1*mean(s)/sqrt(n), Pstof_tes_i+talpha1*mean(s)/sqrt(n))

lim_sup = c(TEA_tes_i+talpha2*mean(s)/sqrt(n),PGAcerto_tes_i+talpha2*mean(s)/sqrt(n),
       Pfsbof_tes_i+talpha2*mean(s)/sqrt(n), Pfsbst_tes_i+talpha2*mean(s)/sqrt(n),
       Poffsb_tes_i+talpha2*mean(s)/sqrt(n), Pofst_tes_i+talpha2*mean(s)/sqrt(n),
       Pstfsb_tes_i+talpha2*mean(s)/sqrt(n), Pstof_tes_i+talpha2*mean(s)/sqrt(n))


arrows(bp, lim_sup, bp, lim_inf, code=3, angle=90,col='red')
```

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