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# Interactions between gas slug ascent and exchange flow in the conduit of persistently active volcanoes

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## Key Points:

- Experiments and simulations demonstrate the value of coupling slug ascent and exchange flow in models of persistently active volcanoes.
- A single slug can pass through an exchange flow without creating a lasting disruption because it ascends much faster than the core magma.
- Sequential slug ascent disrupts the mass balance of the exchange flow, potentially contributing to transitions in eruptive regimes.

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**Abstract**

Many volcanoes around the world are persistently active with continuous degassing for years or even centuries, sometimes exceeding historic records. Such long-term stability contrasts with short-term instability, reflected in eruptive episodes that punctuate passive degassing. These two aspects of persistent activity, long-term stability as opposed to short-term instability, are often conceptualized through two distinct model frameworks: Exchange-flow in volcanic conduits is commonly invoked to explain the long-term thermal balance and sustained passive degassing, while the ascent of large gas slugs is called upon to understand explosive eruptions. While typically considered separately, we propose here that both flow processes could occur jointly in the conduits of persistently active volcanoes and in transient connections between subvolcanic melt lenses. To understand the dynamic interplay between exchange flow and slug ascent, we link analogue laboratory experiments with direct numerical simulations. We find that the two flows superimpose without creating major disruptions when only considering the ascent of a single gas slug. However, the sequential ascent of multiple gas slugs is disruptive to the ambient exchange flow, because it may entail continual buildup of buoyant magma at depth. While our study focuses on the laboratory scale, we propose that the dependence of exchange-flow stability on sequential slug ascent is relevant for understanding why explosive sequences are sometimes followed by effusive eruptions. Taken together, our work suggests that integrating exchange flow and slug ascent could provide a more complete understanding of persistently active volcanoes than either model framework offers in isolation.

**Plain Language Summary**

Many volcanoes around the world erupt frequently, some of them on a daily basis, emitting large quantities of gas, but erupting relatively little magma. Gas fluxes remain high even when there is no eruptive activity, suggesting that magma acts as a conveyor belt that transports gas bubbles to the surface and returns to depth degassed, having lost its buoyant cargo. The framework quantifying this process is called the exchange-flow model. It explains observations of high heat and gas flux, but raises a new question: If gas transport is efficient enough to allow continuous gas emission without eruptive activity, why does not all of the gas escape in this way? One possibility is that gas accumulates into large bubbles that exceed the capacity of the magmatic conveyor belt and

51 rise to the surface by themselves, expanding and accelerating on the way. The frame-  
52 work quantifying this process is called the slug model. Here, we integrate these two mod-  
53 els by studying their interactions in laboratory experiments and computer simulations.  
54 We find that one can impede the functionality of the other, which could be relevant for  
55 transitions in eruptive regimes, such as a switch from episodic explosions to outpourings  
56 of magma.

## 57 **1 Introduction**

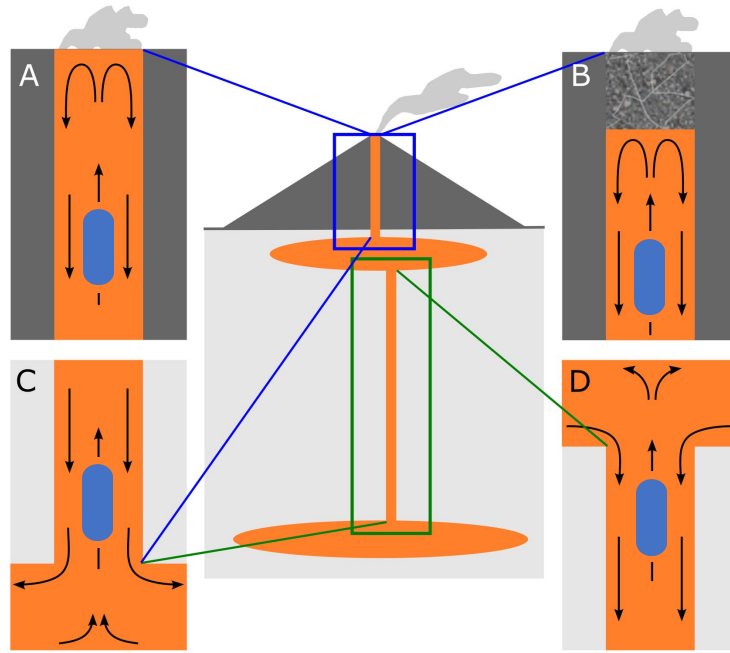
58 Many volcanoes around the world erupt frequently, sometimes as frequently as ev-  
59 ery few minutes (Ripepe et al., 2002; Harris & Ripepe, 2007; Palma et al., 2008; C. Op-  
60 penheimer et al., 2011). Periods of unrest tend to alternate with extended periods of rel-  
61 ative quiescence (Shinohara, 2008), creating a wide spectrum of behavior from contin-  
62 uous passive degassing to intermittent explosive or effusive eruptions and more rare, parox-  
63 ysmal eruptions that emerge with little or no clear precursory activity (e.g., Albert et  
64 al., 2016; Passarelli & Brodsky, 2012). The transitions between different eruptive regimes  
65 are sudden and unpredictable, creating significant uncertainty in risk assessments (e.g.,  
66 Ripepe et al., 2017). A central challenge in understanding the activity at persistently  
67 active volcanoes is hence to explain the great variety in eruptive styles and the processes  
68 that govern the transition between them.

69 Frequent or persistent activity is often closely related to excessive degassing (Shinohara,  
70 2008), which means that the volcanoes emit larger quantities of gas than can be dissolved  
71 in the volume of magma erupted during the various eruptive regimes. Example volca-  
72 noes exhibiting this kind of behavior include, but are not limited to, Stromboli (Italy),  
73 Masaya (Nicaragua), Villarrica (Chile), Izu Oshima (Japan), Erebus (Antarctica), and  
74 Yazur (Vanuatu) (Allard et al., 1994; Palma et al., 2008; Kazahaya et al., 1994; C. Op-  
75 penheimer et al., 2011; Stoiber & Williams, 1986; Firth et al., 2014). Given that these  
76 volcanoes degas much more magma than they erupt, the majority of degassed magma  
77 must be transported back down to depth after degassing. Passive degassing occurs con-  
78 tinuously, so the ascent of volatile-rich and the descent of volatile-poor magma must oc-  
79 cur simultaneously in the volcanic conduit, leading to what is commonly referred to as  
80 exchange flow.

81 In exchange flow, the upward flux of volatile-rich magma approximately equals the  
82 downward flux of degassed magma, which can create a balanced, relatively steady flow  
83 field (Stevenson & Blake, 1998). It provides both the relatively large surface gas flux sug-  
84 gested by observations and the heat flux necessary to explain how persistently active vol-  
85 canoes remain intermittently active over centuries or even millennia (Francis et al., 1993;  
86 Kazahaya et al., 1994). However, exchange flow does not offer an obvious explanation  
87 for why persistently active volcanoes erupt in the first place, rather than emitting all of  
88 the gas passively. It is hence not surprising that the eruptive behavior at persistently  
89 active volcanoes is typically explained through an entirely different conceptual frame-  
90 work, commonly referred to as the slug model.

91 The slug model is based on the insight that exsolved volatiles separate from the  
92 magmatic phase, forming a gas phase with its own dynamics (Vergnolle & Jaupart, 1986).  
93 The existence of a separate gas phase creates different dynamic regimes, from bubbly  
94 flow to slug flow and churn flow, that could be associated with different eruptive regimes,  
95 as first suggested by Jaupart and Vergnolle (1988). However, other factors such as gas  
96 overpressure (James et al., 2004, 2008; Del Bello et al., 2012; Allard, 2010) and crystal-  
97 lization in the upper portion of the conduit (Del Bello et al., 2015; Suckale et al., 2016;  
98 J. Oppenheimer et al., 2020) may contribute as well. While the presence of a separate  
99 gas phase is undoubtedly important for eruptions dynamics, it is not entirely clear how  
100 the transition between different eruptive regimes, and the variability of behavior within  
101 a given eruptive regime, is related to flow processes in the volcanic conduit.

102 Since persistently active volcanoes are characterized by both large heat and gas flux  
103 on the multi-decadal or century scale and by sometimes violent eruptive activity on the  
104 minute or hourly scale, it would be desirable to develop a unified model framework able  
105 to capture both limits. The two models integrate naturally in the limit of long tempo-  
106 ral scales: In exchange flow, the buoyancy of the ascending magma is due to the pres-  
107 ence of small gas bubbles that decouple and degas upon reaching the free surface, leav-  
108 ing behind degassed, dense magma that sinks back down to depth (Francis et al., 1993;  
109 Kazahaya et al., 1994; Stevenson & Blake, 1998). The slug model also associates pas-  
110 sive degassing with the steady ascent of small bubbles (e.g., Jaupart & Vergnolle, 1988,  
111 1989; Vergnolle & Mangan, 2000). In the limit of short temporal scales, the integration  
112 of the two frameworks is less obvious: A rapidly ascending gas slug could perturb the



**Figure 1.** Conceptual sketch of where exchange flow and slug ascent could occur within and below the volcanic edifice. Possible exchange-flow domains include both the uppermost portion of the volcanic plumbing system (blue box) and intermediate depths where vertical connections might transiently connect multiple melt lenses (green box). Ascending slugs are shown in light blue. A: At open-vent volcanoes or lava lakes such as at Kīlauea, Hawaii, backflow can sometimes be observed directly (Stovall et al., 2009). B: At some persistently active volcanoes such as Stromboli volcano, Italy, the upper conduit is partially crystallized and exchange flow likely occurs underneath, removed from direct observation. C: Base of the model domain where exchange flow connects to a storage reservoir or melt lens. D: Top of the model domain when exchange flow at intermediate depth connects two melt lenses.

precarious balance of the exchange flow interface (Hickox, 1971), potentially triggering flow switching (Suckale et al., 2018).

The goal of this paper is to understand the dynamic interplay between slug ascent in exchange flow by linking analogue laboratory experiments and direct numerical simulations. Our study is motivated by the possibility that the integration of slug ascent and exchange flow into a single model framework may provide the basis for a more complete understanding of persistently active volcanoes than either of the two models in iso-

120 lation. We hypothesize that the presence of exchange flow may destabilize an ascend-  
121 ing gas slug and, vice versa, that the ascent of a gas slug may destabilize exchange flow.  
122 To test this hypothesis, we first investigate the dynamic interactions between exchange  
123 flow and slug ascent through analogue experiments, use these experimental results to ground-  
124 truth numerical simulations for the same setup, and then use the numerical simulations  
125 to generalize our laboratory findings.

126 Where within volcanic systems could exchange flow and slug ascent interact? Mo-  
127 tivated by the relatively short time of 10-100 years that magma appears to reside beneath  
128 basaltic volcanoes (Pyle, 1992), Francis et al. (1993) argued for convective exchange be-  
129 tween deep crustal reservoirs and the surface. At crustal depth, flow would be driven pri-  
130 marily by carbon dioxide, which is significantly less soluble in magma than water (e.g.,  
131 Javoy & Pineau, 1991; Dixon et al., 1995) and exsolves well before the magma reaches  
132 the volcanic edifice (Métrich & Wallace, 2008). While exchange flow has been used to  
133 explain shallow degassing (Kazahaya et al., 1994), it is not necessarily limited to the shal-  
134 low depth range, as illustrated in the conceptual sketch in Figure 1. Interactions between  
135 exchange flow and slug ascent could hence occur over the upper few kilometers of the  
136 plumbing system for deeply originating slugs as is the case, for example, at Stromboli  
137 volcano, Italy (Burton et al., 2007).

138 The ascent of a gas slug, or Taylor bubble, in initially stagnant liquid has been stud-  
139 ied extensively in the laboratory as reviewed in the text book by Grace and Clift (1979).  
140 The ascent speed of the slug depends on whether inertial forces dominate over viscous  
141 and capillary forces (e.g., Dumitrescu, 1943; Davies & Taylor, 1950; White & Beardmore,  
142 1962; R. Brown, 1965; Zukoski, 1966) as opposed to viscous forces dominating over in-  
143 ertial and capillary forces (e.g., Goldsmith & Mason, 1962; R. Brown, 1965; Wallis, 1969)  
144 or capillary forces dominating over viscous and inertial forces (e.g., Bretherton, 1961; Reinelt,  
145 1987). Viana et al. (2003) compiled published data from 255 previous laboratory exper-  
146 iments, complemented by 7 new experiments, in an attempt to bridge these different regimes.  
147 Slug ascent has also been studied in the context of Poiseuille flow (Nicklin et al., 1962).  
148 Our work is the first attempt to combine slug ascent and exchange flow and assess whether  
149 the presence of a slug amplifies or reduces these potential instabilities.

150 The most common exchange-flow configuration for liquids with different viscosity  
151 in both laboratory experiments (Stevenson & Blake, 1998; Huppert & Hallworth, 2007;

152 Beckett et al., 2011) and numerical simulations (Suckale et al., 2018) is core-annular flow  
153 with the less dense liquid occupying the core and the more dense liquid descending in  
154 the outer annulus. For the laboratory portion of our study, we generate this flow field  
155 by using an experimental setup consisting of two reservoirs filled with unstably strat-  
156 ified liquid that are connected by a vertical glass pipe (see Figure 2). The setup mim-  
157 ics the experimental configuration of Beckett et al. (2011). We initiate core-annular flow  
158 by removing a rubber stopper at the top of the pipe connecting the two reservoirs and  
159 then inject gas slugs into the flow. We note that the evolving density stratification in  
160 the two reservoirs introduces a transient dynamics into the flow, but prior work shows  
161 that the flow profiles in the pipe closely approximate steady state (Beckett et al., 2011;  
162 Suckale et al., 2018).

163 There is no doubt that laboratory experiments have greatly advanced our under-  
164 standing of the fluid dynamics contributing to eruptive processes in volcanic systems (Jaupart  
165 & Vergnolle, 1988; Seyfried & Freundt, 2000; James et al., 2006; Del Bello et al., 2015;  
166 J. Oppenheimer et al., 2020), but experimental insights are notoriously difficult to scale  
167 up to volcanic systems. Non-dimensional numbers can help attain greater generality, but  
168 laboratory experiments and volcanic systems are rarely, if ever, described by the exact  
169 same non-dimensional numbers, which would be required for scale invariance. This bar-  
170 rier is particularly pertinent for interface-dominated flows such as those considered here,  
171 because interface and body forces scale differently with spatial length,  $l$  (i.e., interface  
172 forces scale as  $l^2$  and body forces as  $l^3$ ). As a result, a change in spatial scale implies a  
173 change in the relative importance of interface to body forces. More specifically, inter-  
174 face forces tend to play a much larger role at laboratory as compared to volcanic scales.

175 To generalize our laboratory findings, we combine them with direct numerical sim-  
176 ulations following the methodology of Qin and Suckale (2017). The advantage of direct  
177 numerical simulations is that they resolve the interactions between multiple different phases  
178 at the scale of individual interfaces (Suckale, Nave, & Hager, 2010; Qin et al., 2015; Qin  
179 & Suckale, 2017; Suckale et al., 2018), obviating the need for approximate parametriza-  
180 tions of drag, rise speed, or long-range hydrodynamic coupling. After reproducing the  
181 experimentally observed flow behavior to validate our numerical simulations, we can then  
182 investigate a broader range of parameters and behavior than is easily accessible in the  
183 laboratory. Despite this attempt to generalize, we emphasize that the primary focus of  
184 this study is to improve our process-based understanding of slug ascent in exchange flow.



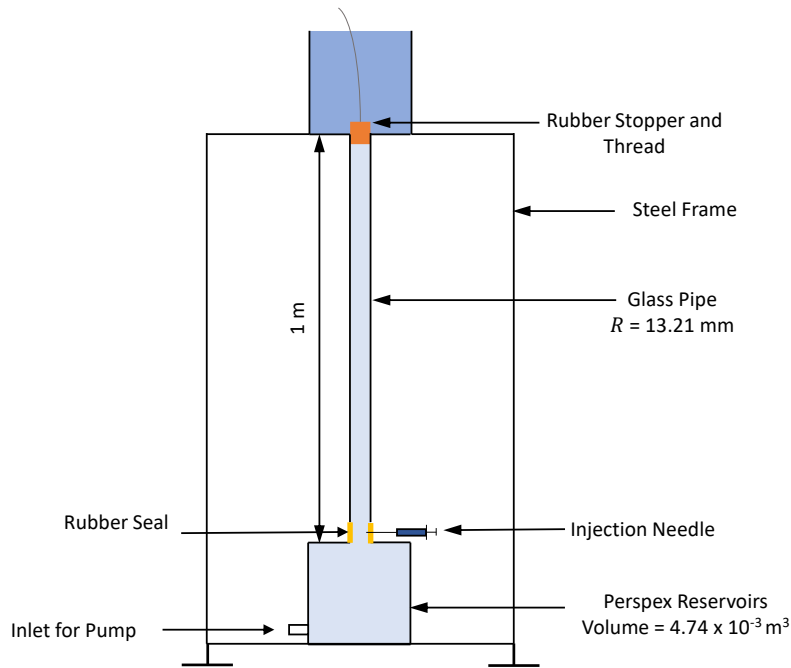
185 The laboratory scale is uniquely suited for that specific purpose, providing a valuable  
186 stepping stone towards a more complete understanding of the dynamic interplay between  
187 exchange flow and slug ascent eventually.

## 188 **2 Analogue laboratory experiments**

189 We perform analogue laboratory experiments to better understand the dynamic  
190 coupling between exchange flow and slug ascent. Our work builds on previous experi-  
191 mental studies of exchange flow in a vertical pipe between two liquid reservoirs (Huppert  
192 & Hallworth, 2007; Beckett et al., 2011). We use an experimental setup that resembles  
193 that of Beckett et al. (2011), but additionally entails a rubber seal at the base of the pipe  
194 connecting the two liquid reservoirs to inject gas slugs into the flow. Figure 2 shows a  
195 diagram of the apparatus used. It consists of two  $4.74 \times 10^{-3} \text{ m}^3$  perspex reservoirs con-  
196 nected by a glass pipe of inner radius  $R = 13.2 \text{ mm}$  and height  $H = 1.00 \text{ m}$ . The top  
197 of the upper reservoir is open. The whole setup is attached to a steel frame which is mounted  
198 on adjustable legs to ensure the pipe is vertical.

199 The two reservoirs allow us to generate different background flows in the vertical  
200 pipe connecting the reservoirs. To study slug ascent in initially stagnant liquid, we fill  
201 both the lower reservoir and the pipe with the same liquid, golden syrup-water mixture.  
202 To generate unidirectional Poiseuille flow, we use the same setup but connect a hydraulic  
203 pump to the lower reservoir to pump liquid up through the apparatus at a constant vol-  
204 ume flux. In contrast, core-annular-geometry exchange flow requires two different liq-  
205 uids that are initially unstably stratified. We fill the lower reservoir and pipe with a mix-  
206 ture of water and Golden Syrup (Tate and Lyle Ltd.), dyed red using food colouring. We  
207 then seal the top of the pipe (i.e., the base of the upper reservoir) with a rubber stop-  
208 per and add pure Golden Syrup with higher density to the upper reservoir. We initiate  
209 core-annular flow by pulling the stopper up through the pure syrup and out the top of  
210 the apparatus with the aid of a thread.

211 The liquid properties for all experiments in the initially stagnant liquid and Poiseuille  
212 flow are summarized in Table 1 and for the core-annular flow in Table 2. We measured  
213 liquid viscosities using a Haake RV20 Rotational Rheometer with a Searle-type concen-  
214 tric cylinder system and calculated the liquid densities from calibration curves for Golden



**Figure 2.** The experimental apparatus. As shown it is set up to generate an exchange flow by pulling up the stopper which initially separates the denser and more viscous liquid (darker blue) in the upper reservoirs from the less dense liquid in the pipe and lower reservoir (lighter blue). After the core-annular exchange flow became steady, a gas slug was injected through a rubber seal between the base of the pipe and the top of the lower reservoir. The inlet into the lower reservoir was used for experiments with a (unidirectional, upwards) background Poiseuille flow (with a single liquid).

215 Syrup and a wide range of Golden Syrup-water solutions relating density and viscosity  
 216 (Beckett et al., 2011).

217 In all the experiments with an initially stagnant liquid, Poiseuille flow, and core-  
 218 annular flow, gas slugs were injected through an annular rubber seal at the base of the  
 219 pipe using a 50 ml syringe and  $0.8 \times 50$  mm surgical needle. Only one slug was intro-  
 220 duced to the flow at a time. We injected the slugs manually and the rate of injection was  
 221 not controlled or measured, although subsequent modelling indicates that the rate of in-  
 222 jection likely affects some results with core-annular background flow. We videoed the ex-  
 223 periments and used Tracker image analysis software (D. Brown, Accessed 2011) to de-  
 224 termine the vertical displacement of the front of the gas slug with time (see videos "Slug-  
 225 Core-Annular-V-1", "Slug-Core-Annular-V-2" and "Slug-Core-Annular-V-3" in the on-

**Table 1.** Fluid properties and slug ascent speed in the initially stagnant liquid ( $U$ ) and the Poiseuille flow ( $U_P$ ). In this case, we define the aspect ratio, AR, as the ratio of slug length,  $l_s$ , to the pipe radius,  $R$ . The uncertainty in the liquid viscosity,  $\mu$ , is  $\sim 5\%$  in Pa s, and the liquid densities,  $\rho$ , have an uncertainty of  $\pm 1$  kg m $^{-3}$ . The speed of the Poiseuille flow  $\bar{U}_L$  is calculated from the known volume flux of the pump, which has an uncertainty of  $\sim 1\%$ . All velocities are in m s $^{-1}$ .

Exp string	AR	$\mu$	$\rho$	$EO$	$N_f$	$U$	$Re$	$\bar{U}_L$	$U_P$
1p	5.8	49.5	1440	123	0.39	1.70E-3	0.001	4.21E-3	1.10E-2
2p	5.2	49.5	1440	123	0.39	1.70E-3	0.001	3.72E-3	9.47E-3
3p	5.2	49.5	1440	123	0.39	1.70E-3	0.001	4.03E-3	1.00E-2
4p	4.8	2.50	1393	119	7.49	3.05E-2	0.525	1.58E-3	3.31E-2
5p	4.6	2.30	1392	119	8.14	4.02E-2	0.615	2.87E-3	4.48E-2
6p	4.2	1.08	1381	118	17.2	7.17E-2	2.413	2.87E-3	7.63E-2
7p	3.8	0.623	1373	117	29.6	9.21E-2	5.896	2.87E-3	9.48E-1
8p	4.2	0.152	1352	116	120	1.50E-1	36.80	2.52E-3	1.53E-1
9p	3.8	0.127	1346	115	142	1.51E-1	44.78	2.36E-3	1.54E-1
10p	3.6	0.126	1349	115	144	1.53E-1	45.28	2.31E-3	1.58E-1
11p	3.8	0.0916	1342	115	197	1.54E-1	63.66	2.87E-3	1.59E-1
12p	3.4	0.0806	1339	114	223	1.57E-1	72.77	1.66E-3	1.60E-1
13p	3.2	0.0256	1326	113	696	1.60E-1	234.6	1.30E-3	1.62E-1

226 line supplement). Slug ascent speed was measured at distances  $\geq 0.5H$  from the base  
 227 of the pipe to ensure the slug has attained its terminal speed.

The experiments of gas slugs ascending through an initially stagnant liquid as well as a Poiseuille and core-annular flow have Reynolds numbers between  $10^{-3}$  and  $10^2$ . The Reynolds number captures the ratio of inertial to viscous forces

$$Re = \frac{\rho_f U L}{\mu_f} . \quad (1)$$

228 where  $U$  represents the slug ascent speed in an initially stagnant liquid, while  $\rho_f$ ,  $L$  and  
 229  $\mu_f$  are the density, width and viscosity of the liquid,  $f$ , that the slug rises through. In  
 230 the initially stagnant liquid and the Poiseuille flow, there is only one liquid that fills the  
 231 entire pipe and  $L$  is equal to the pipe diameter  $2R$ . In the case of the core-annular flow,  
 232 we use the material properties of the core liquid to determine the Reynolds number and  
 233 set  $L$  to the diameter of the core,  $2R_c$ , as determined by the analytical solution (Suckale  
 234 et al., 2018).

In volcanic systems and in our laboratory experiments, it is difficult to estimate the Reynolds number a priori, because the characteristic speed,  $U$  is unknown a priori. We thus use the inverse viscosity,  $N_f$ , to characterize the nondimensional slug dynamics. The inverse viscosity differs from the Reynolds number only in assuming that the relevant characteristic velocity is the ascent speed of a slug in an initially stagnant liquid. It is defined as

$$N_f = \frac{(L^3 \rho_f^2 g)^{1/2}}{\mu_f} , \quad (2)$$

where  $g$  is the gravity. The range of  $N_f$  in the present work is  $0.39 < N_f < 694$ . Additionally, we use Eötvös numbers to present the ratio of inertia force to interfacial force, which is defined as,

$$Eo = \frac{\rho_f g L^2}{\sigma} , \quad (3)$$

235 where  $\sigma$  is the surface tension. In this study, we define  $\sigma$  as  $0.08 \text{ N m}^{-1}$  according to the  
 236 surface tension of golden syrup (E. W. Llewellyn et al., 2002). We do not expect the sur-  
 237 face tension of the dilute syrup to differ significantly from that of pure golden syrup given  
 238 that the surface tension of water is  $0.07 \text{ N m}^{-1}$ . Similar to the Reynolds number, we de-  
 239 termine the inverse viscosity and Eötvös numbers of the core-annular flow using the prop-  
 240 erties of the core liquid. We list the values of the inverse viscosity and Eötvös numbers  
 241 of our experiments for the initially stagnant liquid, the Poiseuille flow and the core-annular  
 242 flow in Table 1 and Table 2, respectively.

**Table 2.** Fluid properties, and experimental measurements for gas slug ascent through the core-annular flow in a vertical pipe. The unit of the core liquid viscosity  $\mu_c$  and annular liquid viscosity  $\mu_a$  are Pa s. The density of the core liquid  $\rho_c$  and annular liquid  $\rho_a$  are in  $\text{kg m}^{-3}$ . The ascent slug increases the core radius, in m, from  $R_c$  to  $R'_c$ . In this case, we define the aspect ratio, AR, by dividing the slug length,  $l_s$ , to the core radius,  $R_c$ . The dimensionless numbers,  $EO$  and  $N_f$ , are defined using the parameters of core liquid. The unit of both the averaged pure core speed  $\bar{U}_L$  and slug ascent speed in the the core-annular flow  $U_x$  is  $\text{m s}^{-1}$ . The column "State" lists whether the slug was always stable (S) or experienced breakup (B) in the experiments.

Exp string	State	AR	$\mu_c$	$\mu_a$	$\rho_c$	$\rho_a$	$R_c$	$R'_c$	$EO$	$N_f$	$\bar{U}_L$	$U_x$
1e	S	13.2	0.0845	52.2	1341	1438	5.6E-3	6.1E-3	20.2	58.1	3.82E-3	7.76E-2
2e	S	9.6	0.0200	65.2	1323	1442	4.8E-3	5.3E-3	14.7	192	1.30E-3	8.10E-2
3e	S	6.1	0.126	60.6	1349	1440	5.8E-3	6.4E-3	22.3	42.0	1.60E-3	6.08E-2
4e	B	-	0.0776	64	1343	1441	5.0E-3	-	16.6	54.5	-	-
5e	B	-	0.151	65.4	1352	1442	5.6E-3	-	20.4	32.5	-	-
6e	B	-	0.175	60.3	1354	1440	-	-	-	-	-	-
7e	B	-	0.0806	54.3	1340	1441	-	-	-	-	-	-

### 3 Numerical Simulations

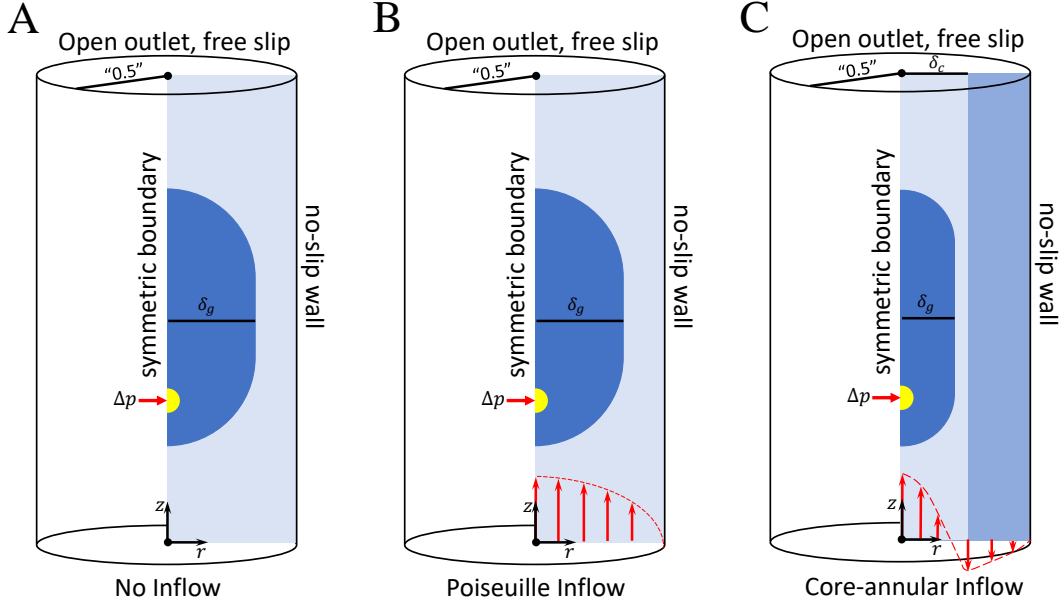
In our simulations, we only model the flow in the vertical pipe connecting the two reservoirs, but not the flow in the reservoirs. Instead, we can enforce the three different flow fields through the boundary conditions. The advantage of this approach is that it eliminates the transient dynamics associated with the gradual draining of the two reservoirs over time and ensures steady-state conditions. Assuming axisymmetry in the flow, we reduce the three-dimensional problem to a quasi-two-dimensional computation with a symmetry boundary condition in the center of the pipe, as illustrated in Figure 3.

#### 3.1 Governing Equations

We solve for conservation of momentum and mass in incompressible flow,

$$\nabla \cdot \mathbf{v} = 0, \quad (4)$$

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p + \nabla \cdot \left[ \mu (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) \right] + \rho \mathbf{g}. \quad (5)$$



**Figure 3.** Setup of the 2-D axisymmetrical numerical simulations for a gas slug (dark blue) ascending through the three different flow fields considered. The model domain is not shown to scale. A. In an initially stagnant liquid (light blue), there is no inflow at the lower boundary; B. Poiseuille flow (light blue) is driven by a parabolic velocity profile at the lower boundary; C. Core-annular flow requires two liquids, the core liquid (light blue) and the annulus liquid (intermediate blue). In all three models, the slug is injected by applying an elevated pressure,  $\Delta p$ , at a small spherical region at the inner boundary (marked in yellow).

254 where  $\mathbf{v}$  is the velocity,  $p$  the pressure,  $\mathbf{g}$  the gravitational acceleration,  $\rho$  and  $\mu$  are the  
 255 density and viscosity.

256 Density and viscosity represent individual phase properties that change discontinu-  
 257 ously and often dramatically at the interface. For example, the viscosity changes dis-  
 258 continuously at the phase boundary between fluid and gas,

$$\mu(\mathbf{x}) = \begin{cases} \mu_g & \text{in the gas slug} \\ \mu_l & \text{in the liquid} \end{cases}, \quad (6)$$

259 where  $\mu_g$  and  $\mu_l$  are the gas and liquid viscosity, respectively. In the core-annular flow  
 260 with ascending slug, the viscosity profile is,

$$\mu(\mathbf{x}) = \begin{cases} \mu_g & \text{in the gas slug} \\ \mu_c & \text{in the core flow} \\ \mu_a & \text{in the annular flow} \end{cases}, \quad (7)$$

261 where  $\mu_g$  represents the gas viscosity in core-annular flow, and  $\mu_c$  and  $\mu_a$  are the viscos-  
 262 ity of core and annular liquids, respectively. In the present work, we assume that all phases,  
 263 including liquids and the gas, abide by a Newtonian rheology.

264 Three different phases are present in the core-annular flow, implying the need to  
 265 track two interfaces, the gas-liquid interface delineating the gas slug and the liquid-liquid  
 266 interfaces separating the ascending and descending liquids. In the numerical simulation,  
 267 we advect the interfaces separately with the flow field according to

$$\frac{\partial \Gamma_i}{\partial t} + (\mathbf{v} \cdot \nabla) \Gamma_i = 0, \quad (8)$$

268 where the subscript,  $i$ , indicates the phase interface,  $i = "gl"$  for the gas-liquid inter-  
 269 face while  $i = "ll"$  for the liquid-liquid interface. During slug ascent through an ini-  
 270 tially stagnant liquid and in Poiseuille flow, we only have the gas-liquid interface, so that  
 271 there is only  $i = "gl"$ .

272 Independent of the flow field, the jump condition of the normal stress across an in-  
 273 terface,  $\Gamma_i$ , combines the effect of the surface tension and stress,

$$[p]_{\Gamma_i} = \sigma_i \kappa_i + 2[\mu]_{\Gamma_i} \mathbf{n}_i^T \cdot \nabla \mathbf{v} \cdot \mathbf{n}_i, \quad (9)$$

274 where  $[\cdot]_{\Gamma_i}$  indicates the sharp jump across the interface,  $\kappa_i$  is the interface curvature,  
 275  $\mathbf{n}_i$  is the normal vector of the interface, and  $\sigma_i$  is the surface tension. In this work, we  
 276 set the surface tension on gas-liquid interface as  $\sigma_{gl} = 0.08 \text{ N m}^{-1}$  (E. W. Llewellyn et  
 277 al., 2002), and assume there is no surface tension on the liquid-liquid interface.

278 To non-dimensionalize our governing equations (Eqs. 4-5), we define the following  
 279 non-dimensional quantities

$$\mathbf{x}^* = \frac{\mathbf{x}}{L}, \mathbf{v}^* = \frac{\mathbf{v}}{U}, t^* = \frac{t}{L/U}, p^* = \frac{p}{\rho_f U^2}, \rho^* = \frac{\rho(\mathbf{x})}{\rho_f}, \mu^* = \frac{\mu(\mathbf{x})}{\mu_f} \quad (10)$$

280 where the parameter  $L$ ,  $U$ ,  $\rho_f$  and  $\mu_f$  are identical to the definitions in Section 2. We  
 281 substitute Eq. 10 into Eqs. 4-5, rearrange, and drop the stars, obtaining the governing  
 282 equations in non-dimensional form,

$$\nabla \cdot \mathbf{v} = 0, \quad (11)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla p}{\rho} + \frac{1}{Re \rho} \nabla \cdot [\mu(\nabla \mathbf{v} + (\nabla \mathbf{v})^T)] + \frac{1}{Fr} \mathbf{z}, \quad (12)$$

284 where  $Re$  represents Reynolds number (Eq. 1) and  $Fr$  represents Froude number,  $Fr =$   
 285  $\frac{U}{\sqrt{gL}}$ .

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### 3.2 Boundary and Initial conditions

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In our simulations, we enforce the three different flow fields representing the initially stagnant liquid, Poiseuille flow and core-annular flow through the boundary conditions. In the case of the initially stagnant liquid (Figure 3A), we apply zero inflow condition to the bottom of the computational domain. At the top boundary, we impose open outflow by enforcing constant pressure,  $p_t = \text{const}$ , and free stress,  $\partial \mathbf{v} / \partial z|_t = 0$ . We treat the outer pipe as a no-slip wall, where  $\mathbf{v}_w = 0$ , and  $\partial p / \partial \mathbf{n}|_w = 0$ , with  $\mathbf{n}_w$  the unit vector normal to the wall. At the symmetric boundary at the center of the pipe, we apply a free-slip boundary, as  $\partial \mathbf{v} / \partial \mathbf{n}|_s = 0$ , and  $\partial p / \partial \mathbf{n}|_s = 0$ , with  $\mathbf{n}_s$  the unit vector normal to the symmetric boundary.

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Our simulations for the case of a Poiseuille flow and core-annular flow employ the same boundary conditions, except for the boundary condition at the bottom of the computational domain. To generate Poiseuille flow (Figure 3B), we set the inflow at the bottom of the computational domain to be a parabolic velocity profile as,

$$u_b = 2\hat{U}_L(1 - (2r)^2) \quad (13)$$

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where  $r$  ( $\in [0, 0.5]$ ) represents the dimensionless radial coordinate and  $\hat{U}_L$  is the dimensionless, average speed. We define  $\hat{U}_L$  as  $\hat{U}_L = \bar{U}_L / U$ , where  $\bar{U}_L$  is the known volume flux of the pump used in the experiments.

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Modeling exchange flow requires two separate liquids illustrated in light and intermediate blue (Figure 3C). The presence of an additional interface complicates the boundary conditions slightly, because it requires enforcing not only the flow field itself but also the position of the liquid-liquid interface. We define the inflow profile based on the experimentally measured half core thickness,  $\delta$ , non-dimensionalized by the pipe diameter and use the analytical model proposed by Suckale et al. (2018) for the dimensionless speed profiles in the core ( $u_c$ ) and annulus ( $u_a$ ),

$$u_c(r) = \frac{M(P-1)}{\alpha}(r^2 - \delta^2) + u_i, \quad r \in [0, \delta] \quad (14)$$

$$u_a(r) = \frac{1}{\alpha} \left[ \frac{P}{4}(4r^2 - 1) - 2\delta^2 \log(2r) \right], \quad r \in [\delta, 0.5] \quad (15)$$

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where  $\alpha$  is the ratio of the velocity scale used in the present work,  $U$ , to the velocity scale used by Suckale et al. (2018),  $(\rho_a - \rho_c)gR^2 / \mu_a$ ,  $M = \mu_a / \mu_c$  is the viscosity ratio,  $P$  is



312 the non-dimensional pressure drop driving the descending phase, defined as,

$$P = 4\delta^2 \frac{2(4\delta^2 - 1) - 4M\delta^2}{(16\delta^4 - 1) - 16M\delta^4}, \quad (16)$$

313 and  $u_i = u_a(\delta) = u_c(\delta)$  is the vertical flow speed at the interface given by

$$u_i = \frac{1}{\alpha} \left[ \frac{P}{4}(4\delta^2 - 1) - 2\delta^2 \log(2\delta) \right]. \quad (17)$$

314 In all of our simulations, we assume that only the thick-core solution is realized since it  
315 is most commonly observed in experiments (Stevenson & Blake, 1998; Picchi et al., 2020).

316 At the beginning of all simulations, we mimic the experimental injection of a gas  
317 slug by applying an additional injection pressure term to the momentum equation (Equa-  
318 tion 5) inside a small, semi-spherical region (highlighted in yellow in Figure 3 A-C). The  
319 injection region includes a portion of the boundary in the center of the pipe. We apply  
320 an elevated pressure until the the slug volume is comparable to the one used in the ex-  
321 periments. To prevent the slug from touching the lower boundary, we inject the slug at  
322 a finite height above the bottom boundary. We set this height to 1/8 of the whole do-  
323 main length.

### 324 3.3 Numerical method

325 We build our numerical model based on the computational techniques that are de-  
326 rived, verified and validated by Qin and Suckale (2017) and Qin et al. (2020). Our ap-  
327 proach is optimized for modeling multiphase flow in volcanic systems, where large con-  
328 trasts in density and viscosity are common. Our numerical approach includes three main  
329 components. The first component is a 2D axisymmetric multiphase Navier-Stokes solver  
330 proposed by Qin et al. (2020), which discretizes the governing equations 4-5 through the  
331 structured Marker And Cell method (Harlow & Welch, 1965). The velocity vector field,  
332  $\mathbf{v}$ , is defined at the grid-cell faces, and the scalar fields (e.g.,  $p$ ,  $\rho$ ,  $\mu$ ) are defined at the  
333 grid-cell centers. To maintain the accuracy and efficiency of the numerical solution, we  
334 employ an implicit implementation of the viscous term, a time-step splitting approach,  
335 and an approximate factorization of the sparse coefficient matrices (Kim & Moin, 1985).

336 The second key component of our computational technique is a level-set-based in-  
337 terface solver capturing the advection of the gas-liquid and liquid-liquid interfaces through  
338 a topology preserving strategy (Qin et al., 2015). Here, we include up to three fluids, i.e.,  
339 the gas slug, the core liquid, and the annular liquid. We hence need up to two level set

340 functions,  $\phi_i$ , to represent the gas-liquid interface,  $\Gamma_{gl}$ , and the liquid-liquid interface,  
 341  $\Gamma_{ll}$ ). These two interfaces divide the domain into three fluids. In the simulation, we ad-  
 342 vect  $\phi_{gl}$  and  $\phi_{ll}$  separately by solving the following equation,

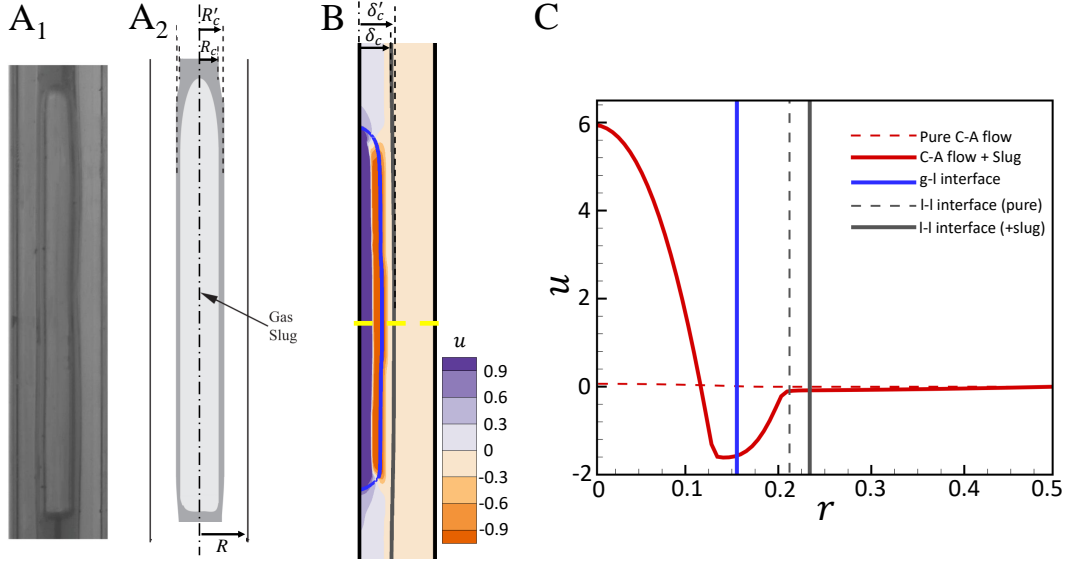
$$\frac{\partial \phi_i}{\partial t} + (\mathbf{v} \cdot \nabla) \phi_i = 0 . \quad (18)$$

343 We redistance the two level sets separately through the geometry projection scheme in-  
 344 troduced by Qin et al. (2015).

345 The third component is the ghost-fluid method used to capture the jump condi-  
 346 tion across the interface (Equation 9) proposed by Kang et al. (2000). We assume that  
 347 there is no surface tension at the liquid-liquid interface and smooth the viscosity pro-  
 348 file around the interface over two grid cells, following previous studies (e.g., Sussman et  
 349 al., 1999; Olsson & Kreiss, 2005; Qin & Suckale, 2017). This approach is equivalent to  
 350 neglecting any jumps that could occur at the liquid-liquid interface. We only apply the  
 351 ghost-fluid method to capture the jump condition across the gas-liquid interface, which  
 352 is caused primarily by surface tension.

### 353 3.4 Verification and Validation

354 We verify and validate our numerical model, as well as the experimental method-  
 355 ology, by comparing our results to existing theoretical solutions and empirical correla-  
 356 tions as described in detail in the supplementary material. To evaluate our ability to cap-  
 357 ture the slug ascent speed in the initially stagnant liquid, we compare our experimen-  
 358 tal measurements,  $U_s$  (Table 1), and the corresponding numerical reproduction to the  
 359 analytical solution given by Picchi et al. (2020) and the empirical speeds proposed by  
 360 Viana et al. (2003). We also use a theoretical relationship between the slug ascent speed  
 361 and the ambient flow, given by Nicklin et al. (1962), to validate our ability to predict  
 362 the effect of the ambient flow on the ascent of the slug. The results from our verifica-  
 363 tion and validation analysis in the supplementary material indicate that our simulations  
 364 are in good agreement with previous analytical and empirical solutions.



**Figure 4.** The effect of a single, stably ascending slug on core-annular flow. A<sub>1</sub>: Photo of experiment 3e showing gas slug ascent through core annular flow. A<sub>2</sub>: A sketch of A<sub>1</sub> presenting the ascent slug increases the core radius from  $R_c$  to  $R'_c$ . B: Snapshot of the numerical reproduction of experiment 3e, showing the core radius increased from  $\delta_c = 0.22$  to  $\delta'_c = 0.24$ . The color scale represents the magnitude of the vertical speed. C: Horizontal profile of the vertical speed at the cross section marked by yellow dashed line in plot B. The blue line shows the gas-liquid interface, while the gray dashed and gray solid lines represent the liquid-liquid interface without and with a slug, respectively. For comparison purposes, we plot the speed profile of the core-annular (abbreviated as C-A) flow prior to the introduction of gas slug as a red dashed line.

## 4 Results

### 4.1 The effect of core-annular flow on slug ascent

A slug ascending through core-annular flow alters the ambient flow pattern, because it has to displace fluid to move. At the same time, the slug ascent speed is altered by the ambient, non-zero flow speed. To discuss our results regarding the interaction between a slug and its surrounding core-annular flow, we focus primarily on experiment 3e and its numerical reproduction in this section. The other experiments with stable slugs, experiments 1e and 2e, are consistent with the behavior in experiment 3e and are included in the supplementary material (Section S4).

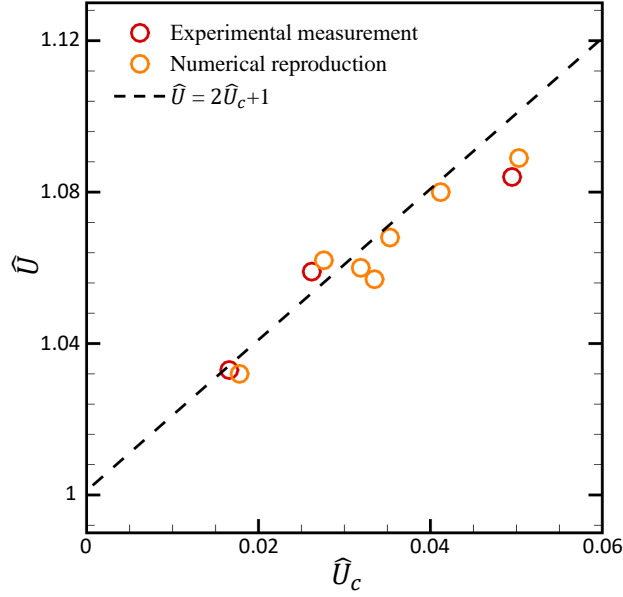
374 We observe that the gas slug always ascends in the core flow, regardless of whether  
 375 it was originally injected into the core or the annulus. In addition to the speed being upwards-  
 376 oriented, the viscosity in the core is lower, reducing the work required for slug ascent.  
 377 As shown in Figures 4, the ascending slug in experiment 3e slightly widens the core flow  
 378 (Figures 4 A<sub>1</sub> and A<sub>2</sub>), increasing the core radius,  $R_c$ , by about 10% to  $R'_c$  (see Table  
 379 2). Videos "Slug-Core-Annular-V-1" and "Slug-Core-Annular-V-3" available as supple-  
 380 mentary material also show examples of slug ascent with different slug length. Our nu-  
 381 merical simulation, based on gas injected at  $\Delta p = 10$  in  $r_{\Delta p} = 0.2$ , agrees well with  
 382 the experimental observation (Figure 4 B). However, in both the experiment and the nu-  
 383 merical simulation, the widening is only local and the core thickness above and below  
 384 the slug remain approximately equal to the thickness of the core in the absence of a slug.

385 Figure 4 B also illustrates the effect of an ascending slug on the dynamics of the  
 386 core-annular flow. Since the ascent speed of the slug is much faster than that of the core  
 387 flow in the absence of the slug, the two flows are largely decoupled. The color scale in  
 388 Figure 4 B illustrates the notable difference in the two speeds. Figure 4 C compares the  
 389 velocity profile at the cross section located in the middle of the slug to the flow profile  
 390 prior to the introduction of the slug. The background flow is almost stagnant in com-  
 391 parison to the flow speed created by the ascending slug.

392 In Figure 5, we plot the dimensionless slug ascent speeds,  $\hat{U}$ , measured in our ex-  
 393 periments and simulations, versus the dimensionless average speed of pure core flow,  $\hat{U}_c$ .  
 394 All speeds are non-dimensionalized by the slug ascent speed in an initially stagnant liq-  
 395 uid, which is calculated from the  $Fr$  predicted by the correlation given by Viana et al.  
 396 (2003) (see equation (S2) in the supplementary material), with the characteristic length,  
 397  $L$ , defined as pure core diameter before the slug pass through,  $2R_c$ . Figure 5 demonstrates  
 398 that the relationship between slug ascent speed and core speed is well described by  $\hat{U}=2\hat{U}_c+1$ .  
 399 This finding implies that the theoretical correlation originally derived by Nicklin et al.  
 400 (1962) for slugs ascending in Poiseuille flow can be generalized to core annular flow by  
 401 using the core radius in place of the tube radius.

#### 402 4.2 Stability of the injected slug

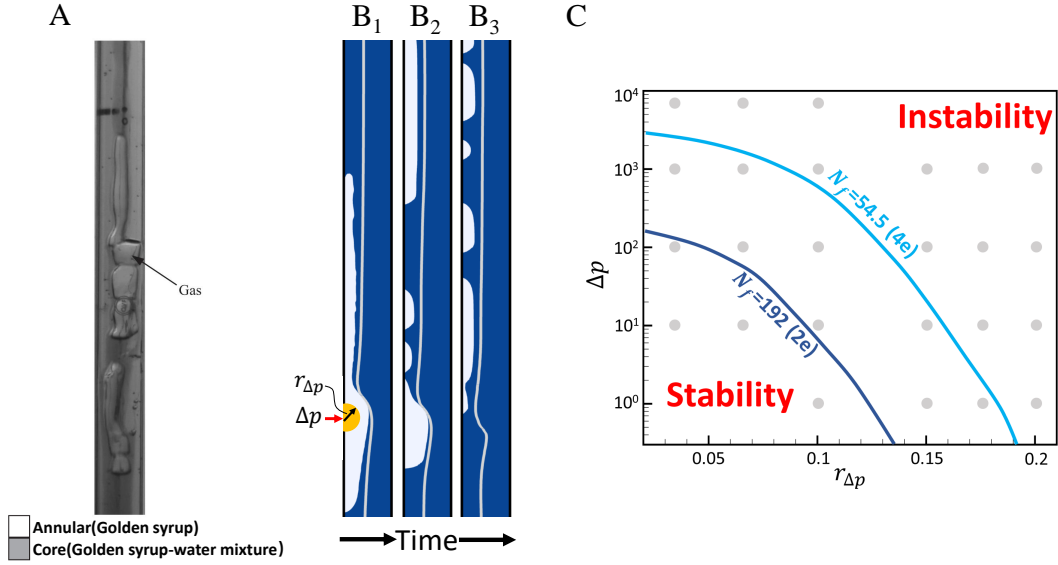
403 In experiments 1e, 2e, and 3e, the injected gas volume formed a stable slug inter-  
 404 face, reminiscent of slugs ascending in initially stagnant liquid. In some cases (i.e., ex-



**Figure 5.** Normalised ascent speed of a single gas slug through the core-annular flow. The dashed line represents the theoretical correlation,  $\hat{U} = 2\hat{U}_c + 1$ . Red circles represent the experimental data, while yellow circles represent the numerical data.

405 periments 4e-5e), however, the injected gas volume forms a slug that becomes unstable  
 406 shortly after injection and breaks up, as illustrated in Figure 6 A. For the experiments  
 407 with unstable slugs, it is hence not possible to measure a single slug ascent speed. In-  
 408 terestingly, slug instability only arises in experiments with core-annular flow. We have  
 409 not observed similar instabilities in the experiments injecting gas into an initially stag-  
 410 nant liquid or Poiseuille flow.

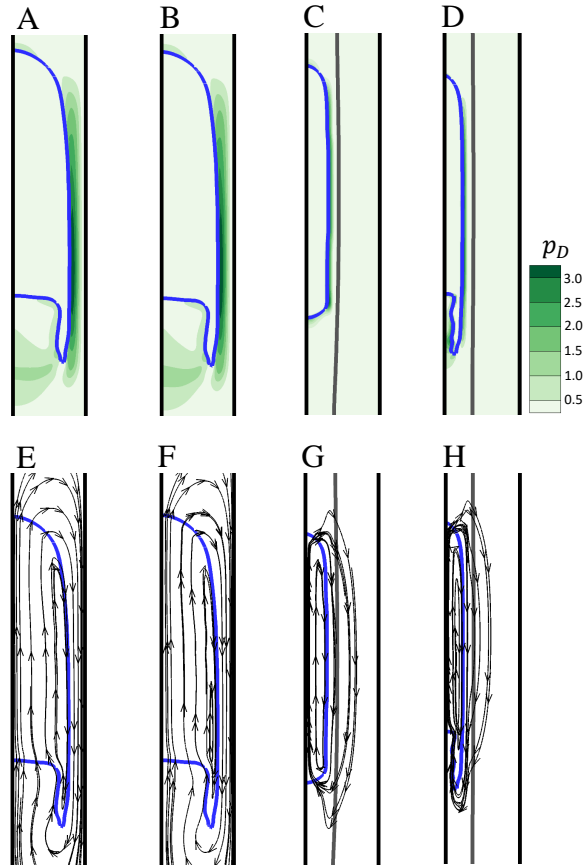
411 We hypothesize that the slug instability observed in some core-annular flow exper-  
 412 iments is a result of the gas injection method. To test this hypothesis, we numerically  
 413 reproduce the slug instability in Figure 6 B1-B3 using the parameters of experiment 4e.  
 414 In the simulation, we inject the gas slug into the core flow by applying a high injection  
 415 pressure,  $\Delta p = 10^4$ , into the injection region. The radius of the computational injec-  
 416 tion region is identical to the needle used in the experiments,  $r_{\Delta p} = 0.035$ . The high  
 417 pressure  $\Delta p$  leads to a large volume of gas suddenly displacing the core liquid. The gas-  
 418 liquid interface extends rapidly and disturbs the liquid-liquid interface. The interaction  
 419 between the two interfaces deforms the liquid-liquid interface, as shown in Figure 6 B<sub>1</sub>.  
 420 After injection, the deformed liquid-liquid interface gradually retracts to a smoother shape,



**Figure 6.** A: Photo of experiment 4e in which a single gas slug breaks up during ascent. B<sub>1</sub>-B<sub>3</sub>: Numerical reproduction of the break-up observed in experiment 4e, highlighting how the excess gas volume bends the liquid-liquid interface (B<sub>1</sub>) and affects slug break-up (B<sub>2</sub>-B<sub>3</sub>). C: Regime diagram for the instability caused by the mechanism of gas injection, where two major factors, pressure difference and the radius of the injection region, are plotted against each other. The gray circles mark the numerical simulations we performed. The blue lines indicate how the regime of stability increases as the inverse viscosity  $N_f$  decreases, based on experiments 2e and 4e.

421 but adjusts more slowly than the gas-liquid interface, creating a sustained perturbation  
 422 to the slug. In the process, the ascending slug breaks up into multiple small blobs (Fig-  
 423 ure 6 B<sub>2</sub>-B<sub>3</sub>). Similar to the experiments, the gas phase in our simulations never crosses  
 424 the liquid-liquid interface. Instead, the gas remains entrained in the core flow.

425 In addition to the injection pressure  $\Delta p$ , our simulations show that the radius of  
 426 the injection region,  $r_{\Delta p}$ , plays an important role in controlling the stability of the in-  
 427 jected slug. The closer the boundary of the injection region to the liquid-liquid interface,  
 428 the more the two interfaces interact with each other and the higher the tendency for in-  
 429 stability. We summarize the effect of  $\Delta p$  and  $r_{\Delta p}$  on the stability of the injected slug in  
 430 Figure 6 C. At a given inverse viscosity,  $N_f$ , there is a critical injection pressure above  
 431 which the instability shown in Figure 6 B<sub>1</sub>-B<sub>3</sub> occurs, and this critical injection pressure  
 432 decreases with the radius of the injection region. The comparison between the curves sep-



**Figure 7.** Comparison of the slug shape, dynamic pressure and streamlines during ascent through an initially stagnant liquid (A/E), Poiseuille flow (B/F) and core-annular flow (C/G) and (D/H). In (C/G), the properties of the core liquid is same as the liquid properties in (A/E) and (B/F). In (D/H), the inverse viscosity,  $N_f$ , is same as the one of (A/E) and (B/F). The color scale in plots A-D represents the dimensionless dynamic pressure, while the arrow lines in E-H represent the streamlines.

433 arating the stability and instability regimes for different  $N_f$  suggests that the sensitiv-  
 434 ity of the instability to the injection mechanism varies with the liquid properties.

435 Given a suitable injection process, however, the stability of gas slugs in core-annular  
 436 flow is comparable to gas slugs rising in an initially stagnant liquid or in Poiseuille flow  
 437 with matching non-dimensional conditions as demonstrated in Figure 7. We compare slug  
 438 ascent in an initially stagnant liquid (Figures 7A/E) and Poiseuille flow (Figures 7B/F)  
 439 using the properties of experiment 12p (Table 1) to core-annular flow (Figure 7C/G) us-  
 440 ing the properties of experiment 7e (Table 2) for which the slug injection process was

441 stable. The properties of the liquid in Figures 7A-B are the same as the properties of  
442 the core liquid in the core-annular flow in in Figure 7C. We set the initial slug length  
443 to be equal in all simulations, as shown in Figure 7.

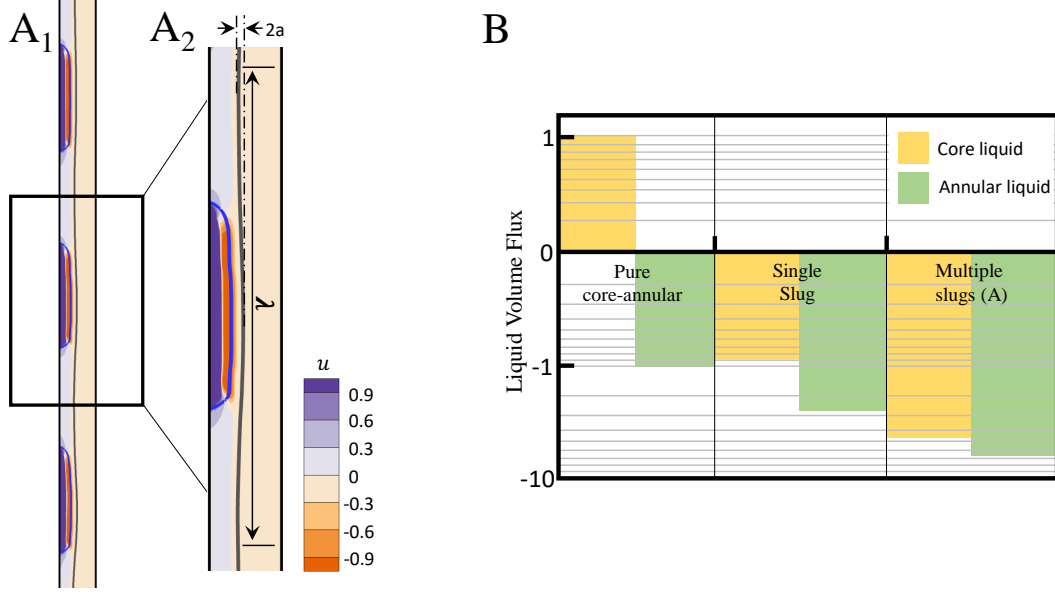
444 We find that the slugs in the initially stagnant liquid and the Poiseuille flow de-  
445 velop a similar shape (Figures 7A-B), characterized by stretched out tails at the rear of  
446 the slug. In contrast, the slug in the core-annular flow develops a blunt shape at both  
447 head and tail (Figures 7C/G). This blunt shape is a result of the slug rising through a  
448 slightly narrower domain, the core, which reduces the inverse viscosity  $N_f$ . The color scale  
449 represents the dimensionless dynamic pressure, defined as  $p_D = \frac{1}{2}\rho|\mathbf{v}|^2$ . An asymmet-  
450 ric deformation of the slug, indicative of the onset of inertia-related nonlinearity in the  
451 flow, appears in core-annular flow only (see Figures 7D/H) after reducing the core liq-  
452 uid viscosity slightly such that  $N_f$  is the same as in the initially stagnant liquid and the  
453 Poiseuille flow (Figures 7A/E and B/F). We conclude that slug instability is governed  
454 primarily by  $N_f$  rather than by the nature of the background flow field.

#### 455 4.3 The effect of slug ascent on core-annular flow

456 A characteristic of stable core-annular flow is that the flux in the core is balanced  
457 by the flux in the annulus, creating zero net flux. Slug ascent perturbs this flux balance,  
458 because it forces the core liquid in the immediate vicinity of the slug to flow downwards  
459 instead of upwards. At first sight, one might expect the conduit-wide effect of this lo-  
460 calized switch in flow direction to be negligible, because only a small portion of the core  
461 liquid is affected by it, but the ascent speed of the slug is much larger than the average  
462 ascent speed of the core liquid. As a consequence, the forced, rapid downwards motion  
463 of the core liquid around the slug could significantly alter the overall flux balance. This  
464 effect would be amplified if multiple slugs rise in succession rather than in isolation as  
465 could be the case in some volcanic systems (Pering et al., 2017).

466 To better understand potential new instabilities arising from the joint presence of  
467 both slug ascent and core-annular flow in volcanic systems, we study the effect of mul-  
468 tiple slugs on core-annular flow. To introduce multiple slugs, we generalize the injection  
469 mechanism used in our experiments. Using the same injection zone and pressure (see sec-  
470 tion 3.2), we introduce multiple slugs at a constant injection frequency,  $f$ , where we de-  
471 fine injection frequency as the ratio of the slug ascent speed to the domain length divided





**Figure 8.** A1: Snapshot of a numerical simulation for the core-annular flow involving multiply injected slugs. A2: Zoom-in image of A1 presenting the wave-formed liquid-liquid interface, caused by ascent of slugs, whose peak-to-peak amplitude is  $2a$  and wave length is  $\lambda$ . B: Comparison of the volume flux of the liquids in pure core-annular flow and core-annular flow with a single or multiple slugs of the same size injected into it. All the volume fluxes are averaged over the domain length.

472 by the number of slugs. We maintain the same boundary conditions as in our simula-  
 473 tions of single slugs ascending. We use the material properties from experiment 3e as an  
 474 example and inject each slug at  $\Delta p = 10$  in  $r_{\Delta p} = 0.2$ .

475 We quantify the effect of sequential slug ascent on core-annular flow by compar-  
 476 ing the liquid volume fluxes in the core and annulus. We compute the liquid flux of the  
 477 core flow,  $F_c$ , at each cross section by integrating in the radial direction

$$F_c = \int_{\delta_g}^{\delta_c} 2\pi r u(r) dr \approx \sum_{N_c} 2\pi r_i u_{i,J} \Delta r, \quad i = \{(1, \dots, nx) : (\phi_u)_{i,J} < 0, (\phi_{gl})_{i,J} > 0\}, \quad (19)$$

478 where  $\delta_g$  and  $\delta_c$  are the dimensionless thickness of gas region and the core flow, respec-  
 479 tively. The variable  $N_c$  is the number of discrete grids in the core liquid at an arbitrary  
 480 vertical coordinate,  $J$ . In the computation of  $N_c$ , we consider only the grid cells where  
 481 the level set function,  $\phi_{gl}$ , is positive. The variables  $r_i$  and  $u_{i,J}$  represent the width and  
 482 vertical speed on the discrete grids at  $J$ ,  $\Delta r$  is the grid size in  $r$  component of the ax-  
 483 isymmetric coordinate. Similarly, we compute the liquid flux of the annular flow,  $F_a$ , at

484 each cross section by evaluating,

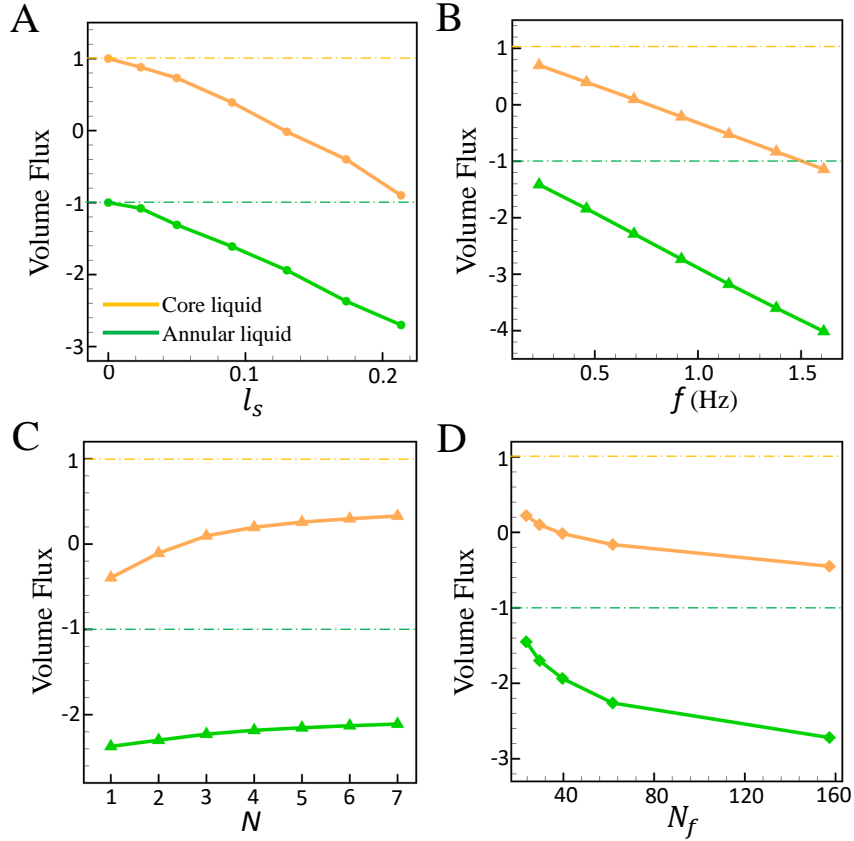
$$F_a = \int_{\delta_c}^{0.5} 2\pi r u(r) dr \approx \sum^{N_a} 2\pi r_i u_{i,J} \Delta r, \quad i = \{(1, \dots, nx) : (\phi_l)_{i,J} > 0, (\phi_{gl})_{i,J} > 0\}, \quad (20)$$

485 where  $N_a$  is the number of discrete grids inside the annular liquid at an arbitrary ver-  
486 tical coordinate,  $J$ .

487 Figures 8 A<sub>1</sub>-A<sub>2</sub> present snapshots of a simulation with multiple slugs injected at  
488 a frequency of 0.69 Hz. The presence of the slugs creates a wave-like perturbation on the  
489 exchange-flow interface. The wavelength depends on the injection frequency and the length  
490 of each slug. The peak-to-peak amplitude of the wave,  $2a$ , equals approximately the in-  
491 crease in core-thickness generated by a single slug,  $\delta'_c - \delta_c$ , shown in Figure 4, suggest-  
492 ing that each slug ascends independently. Figure 8 B shows the results of the flux com-  
493 putation on a logarithmic scale. We average the liquid fluxes computed from Eqs. 19-  
494 20 over the computational domain shown in A1 and over time. We non-dimensionalize  
495 all liquid fluxes by the core liquid flux of the pure core-annular flow computed using the  
496 speed profile (Eq. 14) from Suckale et al. (2018). It is striking that only pure core-annular  
497 flow is associated with a stable flux balance (Figure 8 B). In pure core-annular flow, the  
498 flux in the core is equal to the flux in the annulus, but has a different sign since it is ori-  
499 ented in the opposite direction. Using the properties of experiment 3e, the core and an-  
500 nular liquid fluxes of the pure core-annular flow are  $1.35 \times 10^{-3}$  and  $-1.35 \times 10^{-3}$ .

501 Our simulations suggest that even the passage of a single slug can temporarily shift  
502 the flux balance of pure core-annular flow to downwards in both liquids. The perturba-  
503 tion introduced by a single slug is transient and short-lived, partly because the passage  
504 of the slug is relatively fast. The core-annular flow recovers, avoiding the buildup of liq-  
505 uid at depth and instability. Multiple slugs create a lasting disruption with the down-  
506 wards flux in the core liquid being about three-fold higher than for a single slug, because  
507 in this specific simulation three slugs ascend simultaneously in the computational do-  
508 main. Interestingly, slug ascent in the core liquid also entails speed-up of the downwards-  
509 oriented flux in the annulus (see green bars in Figure 8 B), amplifying the flux imbal-  
510 ance.

511 Figure 9 summarizes in more detail how the liquid fluxes in the core and annulus  
512 depend on slug length (plot A), injection frequency (plot B), the number of slugs at a  
513 fixed gas volume ratio (plot C) and inverse viscosity (plot D). These liquid fluxes are com-  
514 puted in domains where slugs ascend throughout, like the one shown in Figure 8 A<sub>1</sub>. We



**Figure 9.** A: Variation of core and annular liquid volume flux with the length of a single slug, non-dimensionalized by the domain length. B: Dependence of the volume flux in the core and annulus on injection frequency for a constant non-dimensional slug length of 0.05. C: Dependence of core and annular liquid volume flux on number of slugs for a fixed gas volume to core liquid volume ratio of 0.1. The properties used in A, B and C are the same as those in experiment 3e. D: Dependence of the volume flux in the core and annulus on inverse viscosity,  $N_f$ , for an injection frequency of 0.23 Hz and a slug length of 0.12. Dashed lines mark the volume flux for a balanced exchange flow without slugs.

515 find that at a constant injection frequency (e.g.,  $f = 0.23$  Hz), the core flux switches from  
 516 positive to negative already for relatively small slugs (e.g., with the slug length non-dimensionalized  
 517 by the domain length,  $l_s$ , of 0.12). With increasing slug length, the imbalance of fluxes  
 518 increases. The injection frequency has a similar effect on the liquid fluxes. For constant  
 519 slug length, the imbalance of fluxes increases almost linearly with the injection frequency  
 520 (see Figure 9 B), suggesting that each slug ascends independently, as shown in Figure  
 521 8 A<sub>1</sub>. The approximately linear relationship between the injection frequency and the liq-

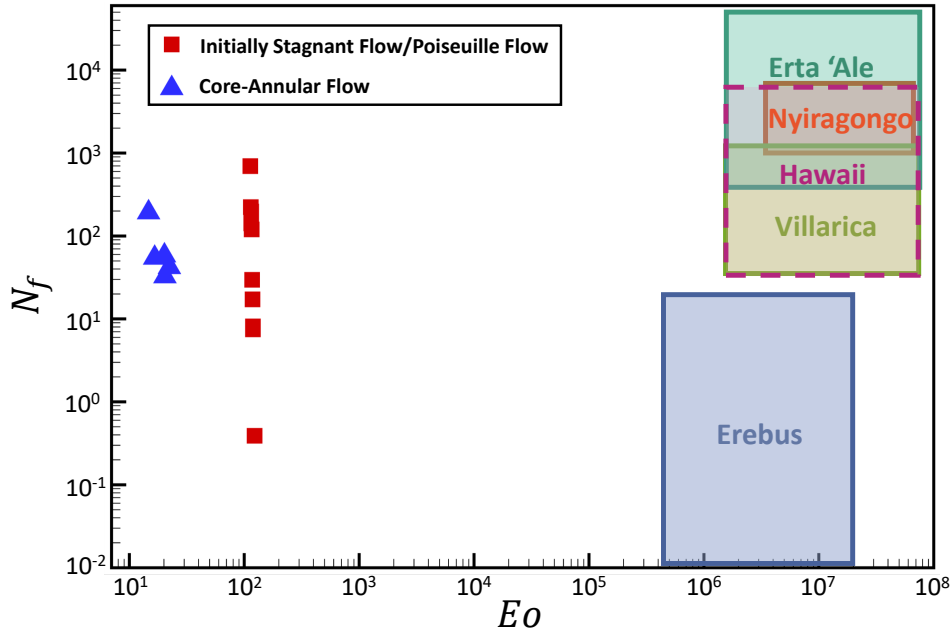
522 uid flux is consistent with the results in Figure 8 B, where three slugs create a three-fold  
523 higher disruption in the core flux than a single slug.

524 At the same total gas volume, a large number of small slugs tends to be less dis-  
525 ruptive than a small number of large slugs, but the effect is small. Figure 9 C shows that  
526 for a fixed volume ratio (10%) of gas to core liquid, increasing the number of slugs de-  
527 creases the imbalance of fluxes. This result is consistent with the velocity field shown  
528 in Figure 8 A<sub>2</sub>, where the slug promotes upward flow around the head and tail, but forces  
529 downward flow along its side. At fixed volume ratio of gas to core liquid, multiple short  
530 slugs provide more heads and tails than a single long slug, creating relatively more upward-  
531 oriented flow areas and a smaller flux imbalance. We also find that the viscosity of the  
532 core liquid has a significant effect on the flux imbalance (see Figure 9 D). Lower viscos-  
533 ity implies smaller viscous forces. As a result, the liquid fluxes increase in both core and  
534 annulus with the inverse viscosity when the slug length and injection frequency are con-  
535 stant.

## 536 **5 Implications for understanding persistently active volcanoes**

537 Persistently active volcanoes exhibit a stunning diversity in eruptive behavior. Ex-  
538 isting conceptualizations associate eruptive regimes with established flow regimes in two-  
539 phase pipe flow: Bubbly flow is associated with passive degassing, slug flow with Strombolian-  
540 type eruptions and churn flow with Hawaiian activity (Wilson, 1980; Vergnolle & Jau-  
541 part, 1986; Jaupart & Vergnolle, 1988; E. Parfitt & Wilson, 1995; E. A. Parfitt, 2004;  
542 James et al., 2009; Fowler & Robinson, 2018). Within this framework, eruptive regimes  
543 represent separate non-dimensional regimes with transitions arising if there is a system-  
544 scale change, such as an elevated influx of gas or magma from depth. Our results sug-  
545 gest that regime transitions could also occur as a consequence of the dynamic interplay  
546 between slug ascent and exchange flow in the conduits of persistently active volcanoes.

547 Our experiments indicate a wide spectrum of behavior, from stable slug ascent rem-  
548 iniscent of that seen in an initially stagnant liquid (see Section 4.1) to slug breakup as  
549 a consequence of the dynamic interactions between the slug- and the exchange-flow in-  
550 terface (see Section 4.2), and, finally, exchange-flow destabilization due to the ascent of  
551 multiple slugs (see Section 4.3). These observations highlight the wide range of possi-  
552 ble interactions that exist, even in a highly idealized laboratory setting. In volcanic sys-



**Figure 10.** Non-dimensional conditions captured in our laboratory experiments in comparison to several persistently active volcanoes.

tems, many additional factors, not considered here, could amplify these interactions, including decompression (James et al., 2004, 2008; Del Bello et al., 2012; Allard, 2010), crystallization (Del Bello et al., 2015; Suckale et al., 2016; J. Oppenheimer et al., 2020), depth variability in gas content or magma properties (Massol et al., 2001; Melnik & Sparks, 2005; Collier & Neuberg, 2006; Capponi et al., 2016), complex conduit geometries (James et al., 2006; Costa, Melnik, Sparks, & Voight, 2007; Vitturi et al., 2008), and deformation of the volcanic edifice (Maeda, 2000; Barmin et al., 2002; Costa, Melnik, & Sparks, 2007).

To provide a first step towards a more complete understanding of the flow conditions in the conduits of persistently active volcanoes, we focus on the laboratory scale, which is uniquely well suited to advance our process-based understanding of conduit flow. The non-dimensional regime covered in our experiments is motivated by volcanic systems and covers both the viscous-dominated regime and flows where inertial effects become important (see Figure 10). We emphasize that we only obtain reasonable overlap in terms of the ratio between inertial and viscous forces captured through the inverse viscosity. In contrast, the Eötvös number differs by several orders of magnitude, suggesting that surface tension and other interfaces forces are significantly more pronounced in

570 our experiments than they would be in actual volcanic systems. We note that this is-  
571 sue is common in analogue experiments of slug ascent aimed at understanding volcanic  
572 systems (James et al., 2004, 2006, 2008; E. Llewellyn et al., 2012), because of the differ-  
573 ent scaling properties of interfaces and body forces. Also, to demonstrate the capabil-  
574 ity to generalize to volcano-specific conditions, we have performed a simulation specif-  
575 ically for Erebus and presented the results in Figure S6 of the supporting information.

576 Given that interface forces such as surface tension are approximately negligible at  
577 the enormous spatial scale of volcanic systems, slug breakup may be more common in  
578 volcanoes than in the laboratory (Suckale, Nave, & Hager, 2010; Suckale, Hager, et al.,  
579 2010). However, decompression-related gas expansion and high gas fluxes may stabilize  
580 slugs at some volcanoes such as Mount Erebus, Antarctica, where inertial forces are less  
581 likely to contribute to slug breakup (Qin et al., 2018) than at some of the other well-studied  
582 open-vent volcanoes (Figure 10). Motivated by the possibility that gas slugs may form,  
583 we test how core-annular flow modulates slug ascent. We show that slug ascent in core-  
584 annular flow shares many similarities with slug ascent in an initially quiescent liquid. The  
585 dynamic interactions between the two flow fields are limited in this case, because the as-  
586 cent speed of the gas slugs is much faster than the average ascent speed of the core liq-  
587 uid (see Figure 4 C). The background flow increases the ascent speed of the slug by a  
588 few percent (see Figure 5), similar to the effect of Poiseuille flow on slug ascent (Nicklin  
589 et al., 1962), but does not fundamentally alter the ascent dynamics.

590 Despite its minimal impact on the speed and stability of gas slugs, we argue that  
591 the potential presence of exchange flow in volcanic conduits is consequential for under-  
592 standing the observed variability of eruptive patterns at persistently active volcanoes.  
593 Our analogue experiments and numerical simulations demonstrate that the presence of  
594 exchange flow entails an inherently more dynamic view of the volcanic conduit than uni-  
595 directional flow or a stagnant magma column, because exchange flow introduces an ad-  
596 ditional interface separating the gas-rich, ascending magma in the core from the degassed,  
597 descending magma in the annulus. When gas slugs rise through core-annular flow, the  
598 slug interface interacts with the exchange-flow interface with potentially significant con-  
599 sequence for the stability of flow throughout the entire conduit. Our laboratory exper-  
600 iments of rapidly injected gas slugs identify one possible consequence: Rapid extension  
601 of the slug volume in a short time creates a wave-like perturbation of the exchange-flow

602 interface that deforms the slug, leading to slug break-up soon after injection (Figure 6  
603 B<sub>1</sub>-B<sub>3</sub>).

604 We emphasize that slug breakup in our laboratory experiments is the consequence  
605 of inevitable disruption during injection, as demonstrated when reproducing the labo-  
606 ratory experiments numerically for a variety of injection conditions. In our numerical  
607 simulations, we can control injection more completely than possible in laboratory exper-  
608 iments and we find that sufficiently slow and careful injection reduces the dynamic in-  
609 teractions between the two interfaces and leads to stable slug ascent. For comparable  
610 inverse viscosity  $N_f$  and sufficiently slow injection of gas into the exchange flow, slug sta-  
611 bility is comparable for the three flow fields studied here: an initially stagnant liquid,  
612 Poiseuille flow and exchange flow (see Figure 7). The onset of breakup in Figure 6 is con-  
613 sistent with the gradual breakup identified previously (Suckale, Hager, et al., 2010). While  
614 the slug-injection mechanism used in our laboratory experiments does not translate to  
615 volcanic systems (e.g. Blackburn et al., 1976; Jaupart & Vergnolle, 1988), our finding  
616 that the dynamic interaction between the two interfaces can fundamentally disrupt the  
617 entire conduit-flow regime may. In fact, video "Slug-Core-Annular-V-2" in the online sup-  
618 plement shows the interactions of a standing wave on the exchange-flow interface lead-  
619 ing to the breakup of slug and even the core flow.

620 The delicacy of the dynamic interactions between the slug and the exchange-flow  
621 interface in a highly idealized laboratory context suggests the possibility of many addi-  
622 tional complexities in actual, inevitably more complex, volcanic systems. For example,  
623 sudden changes in the slug volume could arise in response to eruptive processes in the  
624 shallow plumbing system such as plug failure (Suckale et al., 2018) or effusive magma  
625 removal. Apart from the slug interface, disruptions could also originate at the exchange-  
626 flow interface where most of the shear is concentrated, facilitating crystal and bubble nu-  
627 cleation (Tripoli et al., 2019) and coalescence.

628 The interactions between slug ascent and exchange flow are not limited to exchange  
629 flow interrupting slug ascent (see Section 4.3). The ascending gas slug displaces the core  
630 liquid around it and, thereby, imposes a localized reversal in the core flow and an increase  
631 in the downwards-oriented annulus flow (see Figure 8). While exchange flow can recover  
632 from the transient disruption introduced by a single slug, it becomes increasingly prone  
633 to collapse or rearrangement the longer sequential slug ascent persists. Our videos of ex-

periment 4e, available in the online supplementary material, record the occurrence of this instability. After a "train" of short slugs (showed in "Slug-Core-Annular-V-1") ascends in exchange flow, a standing wave forms on the exchange flow interface, which interacts with the ascending slug, and leads to the breakup of slug and even the core flow later on (showed in "Slug-Core-Annular-V-2"). One way of re-establishing mass balance would be an effusive eruption that expunges the accumulated surplus mass. It is interesting to note in this context that Stromboli volcano appears to exhibit effusive lava flows after a particularly intense sequence of normal Strombolian eruptions (Ripepe et al., 2017), suggesting a connection between the intensity of one eruptive regime and the occurrence of another. If these normal Strombolian eruptions were to represent large gas slugs, then an intense sequence of Strombolian eruptions could be thought of as multiple, closely-spaced gas slugs and the effusive eruption could represent the expulsion of built-up magma at depth.

We emphasize that this particular interdependence between intense normal Strombolian eruptions and effusive activity is far from ubiquitous and many effusive eruptions are not related to an intense sequence of Strombolian eruptions at all. We mention this example here to highlight the possibility that different eruptive regimes are interdependent and coupled through the internal flow dynamics in the volcanic conduit. This interdependence is probably not limited to effusive and Strombolian eruptions, but would encompass passive degassing, as supported by a recent analysis of long-term eruptive trends at Stromboli, Batu Tara, and Tinakula volcanoes (Laiolo et al., 2018). More important than the specific eruptive regime is the insight that the different flow regimes that may exist in the conduits of persistently active volcanoes, such as slug ascent and core-annular flow, are interdependent with ample opportunity for mutual disruption.

## 6 Conclusion

We combine analogue experiments and numerical simulations to advance our process-based understanding of the dynamic interplay between slug ascent and exchange flow in the conduits of persistently active volcanoes. We argue that an integrated model framework considering slug ascent and exchange flow jointly, may provide a more comprehensive understanding of persistently active volcanoes than either model in isolation. More specifically, we argue that the interactions between slug ascent and exchange flow iden-



665 tified here could advance our understanding of the variability of eruptive behavior and  
666 the interdependence between different eruptive regimes.

### 667 **Author Contributions**

668 ZQ performed the numerical simulations and compared the numerical and exper-  
669 imental results. FMB performed the laboratory experiments. ACR conceptualized the  
670 laboratory experiments, supported their analysis, and provided volcanological context.  
671 JS contributed to the problem formulation, conceptualized the simulations with ZQ and  
672 contributed to the analysis of the simulations. All authors contributed to the text.

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677 The manuscript also benefited from comments from two anonymous reviewers. This manuscript  
678 do not entail any original or processed data. All results come from our simulation and  
679 experiments. The codes are developed by us for the specific purposes of this paper, which  
680 are preserved at <http://doi.org/10.5281/zenodo.5030904> ((Qin et al., 2021)).

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