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1 **Some insights into the screech tone of under-expanded supersonic** 2 **jets using dynamic mode decomposition**

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10 **Abstract**

11 Jet screech is an intense pure tone which has attracted decades of research interest due to its
12 possible detrimental effect to engineering structures. Its modes and closure mechanisms have
13 been investigated analytically, experimentally and numerically, however, there are still
14 outstanding questions on the generation and propagation of instabilities in the near-field region.
15 Recent studies have identified that the instabilities also travel inside the jet potential during the
16 screech process to form the complete feedback loop. Using dynamic mode decomposition on
17 a 3D pressure near-field from large eddy simulation results, the present study examines the
18 viability of the modal decomposition to provide further insights into the screech modes and its
19 associated characteristics and investigates the effect of temperature mixing in jet screech. The
20 results show that the modal decomposition approach identifies very well the helical structure
21 of the screech mode. Furthermore, a method is proposed to reveal the temporal evolution of
22 the dynamic screech mode. It was found that the bulk behaviour of the pressure field at screech
23 frequency propagates backward towards the nozzle exit.

24 **1. Introduction**

25
26 When operating at off-design conditions, a supersonic jet exhausting into a quiescent ambience
27 undergoes either an over- or under-expansion process, which is characterised by a train of
28 shock cell structures in the jet potential core (Tam, 1995). The formation of shock cells leads
29 to ‘shock associated’ noise components in supersonic jet noise, namely the broadband shock
30 associated noise (Harper-Bourne and Fisher 1973; Norum and Seiner 1982; Tam 1990; André
31

32 *et al.* 2013; Pérez Arroyo *et al.* 2019) and the screech tone (Powell 1953; Tam and Hu, 1989;
33 Powell *et al.* 1992; Shen and Tam 2002; Edgington-Mitchell *et al.* 2018; Gojon *et al.* 2018;
34 Mancinelli *et al.* 2019), as a direct result of the jet mixing layers interacting with the shock
35 structures. The jet screech is a very intense tone, first observed and reported by Powell (1953),
36 which is often considered detrimental to engineering components due to its ability to induce
37 sonic fatigue failure and structural damages.

38

39 The generation of the screech tone has been extensively studied in the literature. Under specific
40 self-resonance conditions, a screeching jet completes an acoustic feedback loop with, firstly
41 the growth of the instability wave within the mixing layer; secondly interaction between the
42 shock and turbulent structures to generate upstream traveling waves (Tam and Hu, 1989);
43 thirdly, propagation of the upstream traveling waves; and lastly re-excitation of the instability
44 waves upon the jet exit, thus closing the loop for screech tone emission. Using shock-refracted
45 wave model, Kandula (2008) argued that the thin shear layer close to the jet exit were receptive
46 to excitations, hence sustaining the intense screech tone. Furthermore, different screech modes,
47 as the jet Mach number increases and screech tone frequency jumps occur, have been identified
48 as the axisymmetric mode (A1 and A2, also referred to as toroidal mode by Tam), flapping
49 mode and helical mode (Powell 1953; Powell *et al.* 1992; Tam 1995). To understand the closure
50 mechanisms for the feedback loops produced from the different screech modes, Shen and Tam
51 (2002) examined the upstream propagation of disturbance waves and suggested the existence
52 of two possible mechanisms. The first mechanism recognized that the amplification of the
53 Kelvin-Helmholtz instabilities in the mixing layer, which subsequently generates upstream-
54 propagating acoustic perturbations outside of the jet shear layer, was re-excited at the nozzle
55 lip (Tam *et al.*, 1986). While the proposed model agreed well with the experimental and
56 numerical results for the A1 axisymmetric mode and the flapping modes, significant
57 differences were found when predicting the A2 axisymmetric and helical mode (Assunção *et*
58 *al.*, 2019). The second mechanism, proposed later by Tam and Hu (1989), emphasized on a
59 different instability wave, where it can be found both outside of the jet shear layer and inside
60 of the jet potential core. In more recent experiments conducted by Edgington-Mitchell *et al.*
61 (2018), they identified an upstream-traveling instability mode with a negative phase velocity
62 inside of the jet core flows, using a triple-decomposition method based on proper orthogonal
63 decomposition (POD). Mancinelli *et al.* (2019) and Pérez Arroyo *et al.* (2019) also observed
64 similar upstream-traveling modes in their studies of jet screech tone and broadband shock
65 associated noise.

66 To extract the most dominant dynamics in the near-field of the jet flows and identify the
67 upstream-moving modes, both Edgington-Mitchell *et al.* (2018) and Mancinelli *et al.* (2019)
68 have applied modal decomposition, *i.e.* POD, to the velocity fields. These modal
69 decomposition methods, such as proper orthogonal decomposition (Berkooz *et al.*, 1993) and
70 dynamic mode decomposition (DMD; Schmid, 2010), are powerful analyses tools for
71 identifying the either energetically or statistically important dynamics of the flow and have
72 been utilized successfully to provide further insights into many flow scenarios (Tu *et al.*, 2014),
73 which help enhance significantly our understanding of a given flow phenomenon. Lárusson *et*
74 *al.* (2014) subjected their URANS (unsteady Reynolds-Averaged Navier-Stokes) pressure field
75 to DMD and found that the dominant frequency from an optimal ranking of the DMD modes
76 agreed very well with the screech frequency, confirming that generation of screech tone is
77 dynamically dominant in the near-field. Gao *et al.* (2017) applied DMD on their numerical
78 simulation results to examine the interaction noise from twin closely-spaced supersonic jets
79 under perfectly expanded condition. The upstream shift of the DMD modes indicated clearly
80 the effect of jet interactions and an early onset of the shear layer instabilities, which provided
81 an elevated level of jet noise. Burak and Andersson (2018) analysed the velocity field results
82 obtained from their large-eddy simulation using DMD and concluded that DMD was able to
83 isolate the screech tone behaviour from the overall jet dynamics.

84

85 Compared to experimental characterisation of supersonic jet noise, which are often challenging
86 and costly to perform, high-fidelity numerical modelling, such as large eddy simulation (LES),
87 can provide rich flow field and noise information on the jets, especially so when the jets are
88 operating at ‘measurement-difficult’ conditions, *e.g.* highly heated jet (Brès and Lele, 2019).
89 Hence, there has been a growing number of numerical studies investigating the various aspects
90 of supersonic jet noise and improving understanding of the jet noise phenomenon in the past
91 decade, for instances, Liu *et al.* (2009), Gojon *et al.* (2016), Brès *et al.* (2017), Viswanath *et al.*
92 (2017) and Langenais *et al.* (2019), just to name a few. In a most recent review, Brès and Lele
93 (2019) remarked that the advances in the numerical schemes and methods in discretization,
94 meshing, boundary treatments and turbulence modelling have all contributed to the increased
95 accuracy in numerical modelling. Using numerical modelling, a relatively complete near-field
96 pressure and velocity results can be time-marched and collected with sufficiently small
97 sampling frequencies, and subsequently being subjected to modal decomposition methods to
98 extract and identify the salient features relevant to the different jet noise components.

99 To extend from the previous literature on the DMD studies of supersonic jet noise and shed
100 further lights on the screech modes of under-expanded jet flows, the present study performs
101 the dynamic mode decomposition on the three-dimensional pressure near-field results obtained
102 from large eddy simulations, with a particular focus on the ability of DMD to identify screech
103 modes and capture the corresponding upstream-traveling mechanisms for both an unheated and
104 a heated supersonic jet. Furthermore, by advancing the DMD modes in time, the characteristics
105 of the screech-dominated pressure near-field are further explored. To the authors' knowledge,
106 there has been very few attempts to show the non-axisymmetric structures of a screeching jet
107 using the dominant dynamic mode derived from three-dimensional pressure fields.

108

109 **2. Numerical methodology**

110 The simulations in this study were performed using an in-house compressible flow solver
111 developed in the OpenFOAM framework. The inviscid fluxes were calculated using an
112 efficient hybrid flux methodology (U S Vevek *et al.*, 2019) that blends a MUSCL-type HLLC
113 flux scheme and a characteristic HLL flux scheme using a modified shock sensor. The spatial
114 reconstruction was performed using a seventh-order finite volume WENO (weighted
115 essentially non-oscillatory) scheme (Jiang and Shu, 1996) that has been implemented using the
116 dimension-by-dimension approach. The present implementation is capable of handling non-
117 uniform block-structured meshes. An adaptive mapping procedure (U S Vevek *et al.*, 2019)
118 was used to enhance the spectral resolution of the WENO reconstruction. The simulations were
119 performed using the implicit LES (iLES) approach (Garnier *et al.*, 1999; Fureby and Grinstein,
120 1999) whereby the numerical dissipation of the spatial discretization scheme acts as an implicit
121 SGS (sub-grid scale) model that serves to dissipate the energy contained in the eddies that
122 cannot be resolved on the mesh. The viscous fluxes were calculated using a second-order
123 central difference scheme. The explicit third-order TVD (total-variation-diminishing) Runge-
124 Kutta scheme (Gottlieb and Shu, 1998) was used for time integration.

125

126 **2.1. Case Setup**

127 A schematic of the computational domain for LES is shown in Fig. 1. Since the domain is
128 axisymmetric, only a half-slice is shown. The domain extends $40D$ downstream and $5D$
129 upstream of the nozzle outlet. Its radial extent varies from $8D$ at the nozzle outlet plane to $10D$
130 near the end of the domain. A close-up view of the region close to the nozzle outlet is shown
131 at the bottom of the Fig. 1. The nozzle wall thickness is $t_w = 0.5mm$. A small portion of the

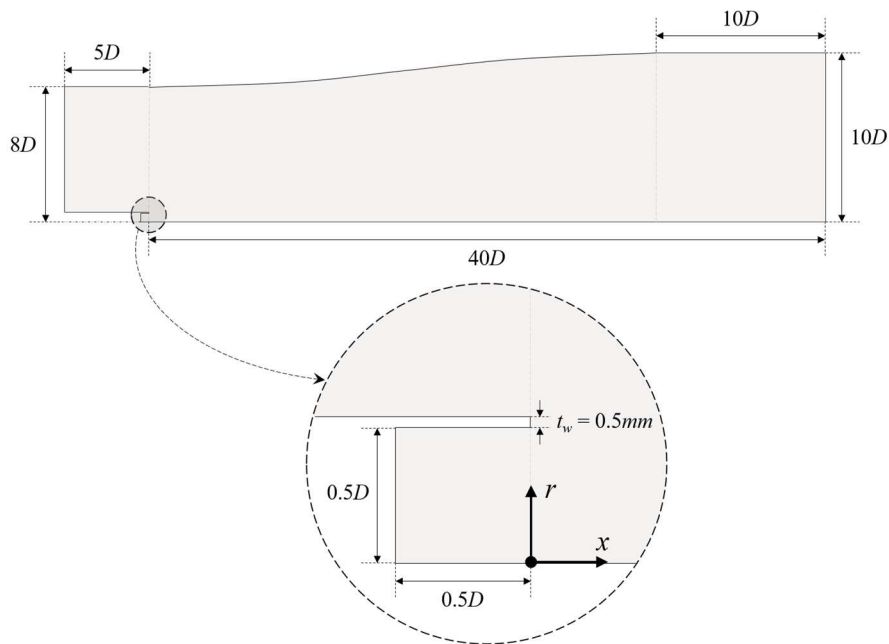
132 nozzle straightening section measuring $0.5D$ in length is retained in the domain since the
 133 nozzle lip provides the required receptivity for the acoustic waves (Bodony and Lele, 2008).

134

135 Figure 2 shows the mesh topology used for LES with mesh refinements along the shear layer.
 136 The streamwise cell spacings, Δx , at various streamwise locations are given in Fig. 2(a). The
 137 streamwise cell spacings smoothly vary across the length of the domain. The streamwise
 138 resolution near the nozzle exit and the overall mesh size are comparable to the past studies as
 139 summarized in Table 1. The near wall spacing at the nozzle outlet is kept at $\Delta y_w = 0.005mm$.
 140 Figures 2(b) and 2(c) show the yz plane cross-sections at $x = 0D$ and $x = 40D$, respectively.
 141 The core region was meshed using an OH block topology and care was taken to reduce the
 142 skewness of cells by smoothing the grid lines near the corners of the central block.

143

144 The initial and boundary conditions for the simulations are listed in Table 2. Wave-transmissive
 145 boundary conditions were applied at the far-field boundaries to minimize reflection of acoustic
 146 waves. The nozzle wall was modelled as an adiabatic, no-slip wall. The boundary conditions
 147 for the inlet were sampled from the results of RANS simulations of the CD (converging-
 148 diverging) nozzle that were performed earlier using *rhoCentralFoam*, one of the native solvers
 149 in OpenFOAM. More details on the CD nozzle geometry and the RANS simulation setup can
 150 be found in Wu and New (2017) and Zang *et al.* (2018), respectively. The RANS results
 151 showed that the y_+ values close to the nozzle outlet were smaller than 5 for both cases which
 152 indicate that the first cell lies within the viscous sub-layer.



153

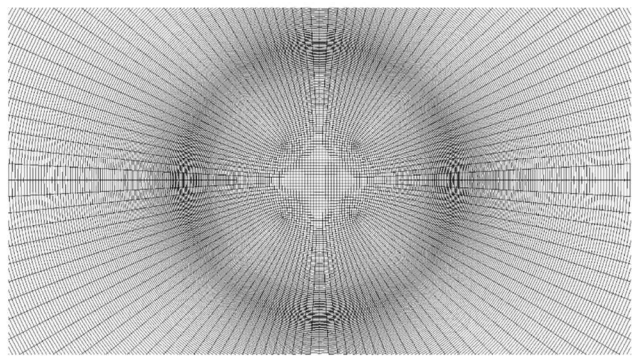
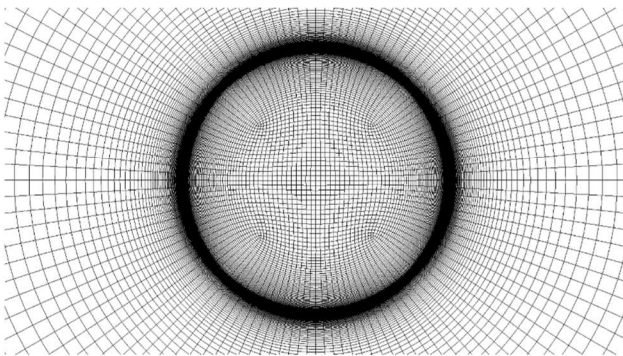
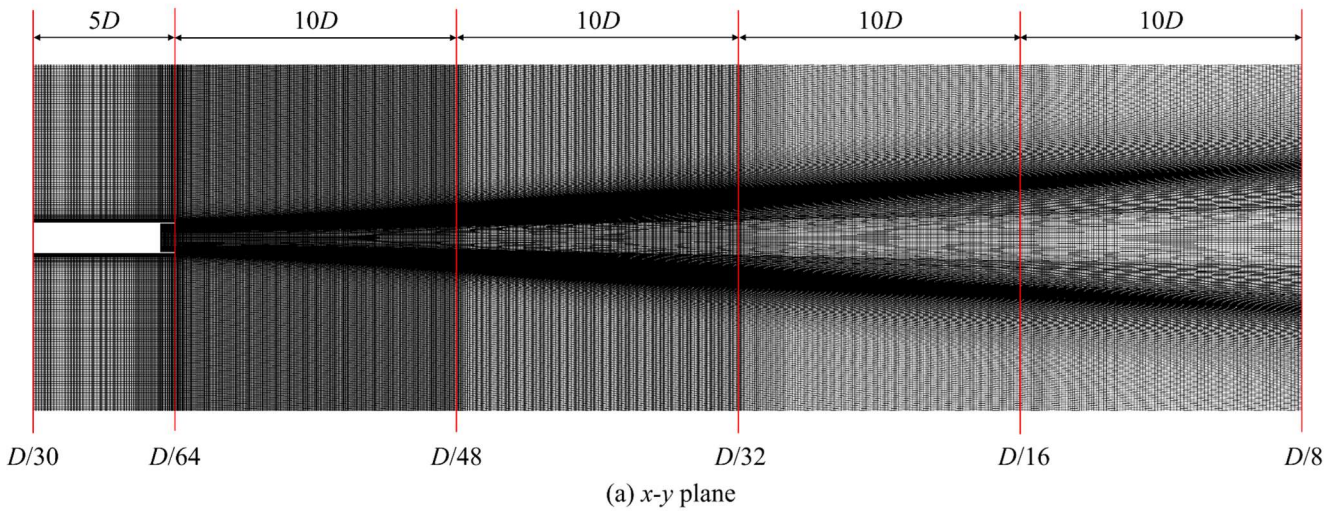
154 Figure 1. Schematics of the LES computational domain of the under-expanded jets

| Study | Re | $\min(\Delta x)$ | Total cells ($\times 10^6$) |
|-------------------------------|------------|------------------|-------------------------------|
| Daupain <i>et al.</i> (2010) | $\sim 1e6$ | $D/40$ | 22 |
| Munday <i>et al.</i> (2011) | $\sim 5e6$ | $D/50$ | – |
| Mendez <i>et al.</i> (2012) | $\sim 2e5$ | $D/80$ | 28 |
| Vuorinen <i>et al.</i> (2013) | $\sim 2e5$ | $D/70$ | 12 |
| Li <i>et al.</i> (2014) | $\sim 1e5$ | $D/67$ | 27.3 |
| Present | $\sim 1e6$ | $D/64$ | 33.5 |

155 Table 1. Comparison of the jet Reynolds number, streamwise cell size (Δx) and total number
 156 of cells between past literature and the present study on supersonic jet simulations

157

158



159 Figure 2. Cross-sectional view of the mesh topology of the LES for (a) entire streamwise
 160 domain, (b) yz -plane at the nozzle exit and (c) yz -plane at domain outlet

161

162

163

164

| | pressure, p | velocity, \mathbf{U} | temperature, T |
|-------------|--|------------------------|------------------|
| Initial | 100 kPa | (0 0 0) m/s | 300 K |
| Far-field | wave-transmissive | zero-gradient | zero-gradient |
| Nozzle wall | zero-gradient | no-slip | zero-gradient |
| Inlet | Sampled from RANS simulations of CD nozzle | | |

165 Table 2. Initial and boundary conditions applied to the LES of under-expanded jets

166

167 2.2. Jet operating conditions

168 In earlier studies by Wu and New (2017), Wei *et al.* (2019) and Lim *et al.* (2020), the CD
169 nozzle used in the present simulations was observed to achieve perfect expansion at a nozzle
170 pressure ratio ($\eta = P_t/P_\infty$, where P_t and P_∞ are the total pressure and ambient pressure,
171 respectively) of 3.4 and at $\eta = 5$, the under-expanded jet began to produce intense screech tone.
172 Thus, the nozzle pressure ratio of the under-expanded jet is set at $\eta = P_t/P_\infty = 5$, of which
173 the jet Mach number, M_j , jet velocity, U_j , and fully-expanded to design diameter ratio, D_j/D
174 can then be calculated according to Eq. 1 to 3 shown below:

$$M_j = \sqrt{\frac{2(\eta^{(\gamma-1)/\gamma} - 1)}{\gamma - 1}} \quad (1)$$

$$U_j = M_j \sqrt{\frac{\gamma R T_0}{1 + \frac{1}{2}(\gamma - 1)M_j^2}} \quad (2)$$

$$\frac{D_j}{D} = \left(\frac{1 + \frac{1}{2}(\gamma - 1)M_j^2}{1 + \frac{1}{2}(\gamma - 1)M_d^2} \right)^{\frac{(\gamma+1)}{4(\gamma-1)}} \sqrt{\frac{M_d}{M_j}} \quad (3)$$

175 where, $\gamma = 1.402$, $M_d = 1.45$, $D = 0.0127m$, $T_\infty = 300K$ and $\eta = 5$. Two temperature
176 ratios of $T_0/T_\infty = 1$ and 2 were chosen for the LES simulations, which corresponds to unheated
177 and heated jets respectively, to firstly examine the validity of the dynamic mode decomposition
178 approach applied to the 3D pressure field under different jet operating conditions, and secondly
179 to investigate the effects of temperature on the temporal evolution of the screech modes. Table
180 3 lists the various operating conditions of the unheated and heated jets.

181

182 The jet characteristic time is defined as $t_j = D/U_j$, where U_j refers to the jet exit velocity. Each
 183 LES was performed for a total of $200t_j$. The jet flows were assumed to have settled down to a
 184 quasi-steady state by $100t_j$, after which a total of 300 sets of data points were stored for each
 185 LES case at intervals of $\Delta t = 10^{-5}s$.

186

| η | T_0/T_∞ | T_0 | U_j | M_j | D_j |
|--------|----------------|-------|---------|-------|-------|
| 5 | 1 | 300 K | 471 m/s | 1.708 | 1.08 |
| 5 | 2 | 300 K | 666 m/s | 1.708 | 1.08 |

187 Table 3. Operating conditions of the unheated and heated under-expanded jets

188

189 2.3. Dynamic mode decomposition

190 The dynamic mode decomposition (DMD) is designed to extract flow structures, which
 191 represent (in case of linearized flows) or approximate (in case of non-linear flows) the
 192 dominant dynamic behaviour of the measured data. Schmid (2010) first proposed the method
 193 based on the snapshots of experimental and numerical data. Different from the companion
 194 matrix in the Koopman analysis, Schmid (2010) utilized the numerically more stable approach
 195 of single value decomposition to formulate the method, and this has become the defining DMD
 196 algorithm. Following the method outlined by Schmid (2010) and Baruk and Andersson (2018),
 197 the data sequence, sampled at a frequency of $1/\Delta t$, can be represented in matrix form as:

198

$$\mathbf{Z}_1^N = \{\mathbf{z}_0, \mathbf{z}_1, \mathbf{z}_2 \dots \mathbf{z}_{N-1}, \mathbf{z}_N\}$$

199 where each \mathbf{z}_i represents a vector of M measurement points, *i.e.* $\mathbf{z}_i \in \mathbb{R}^M$. Assuming that the
 200 data sequence is generated in a linear time-invariant system, there exists a mapping \mathbf{A} that
 201 yields:

$$\mathbf{z}_t = \mathbf{A}\mathbf{z}_{t-1}, \quad \forall t = 0, 1, \dots, N-1 \quad (4)$$

203 The linear operator \mathbf{A} describes the dynamics of the system.

204

205 Subsequently, two matrices of identical dimensions can be constructed from the snapshots:

$$206 \quad X \equiv \{\mathbf{z}_0 \dots \mathbf{z}_{N-1}\} \in \mathbb{R}^{M \times N} \text{ and } Y \equiv \{\mathbf{z}_1 \dots \mathbf{z}_N\} \in \mathbb{R}^{M \times N}$$

207 And the matrix X can be factorised with the single value decomposition as:

$$208 \quad X = U\Sigma V^* \quad (5)$$

209 Note that U and V contains the spatial modes (i.e. the proper orthogonal decomposition of the
 210 snapshots in matrix X) and the temporal information, respectively. By projecting the linear
 211 operator \mathbf{A} onto the basis spanned by POD modes of X , the optimal approximation $\tilde{\mathbf{A}}$ to the
 212 linear operator \mathbf{A} can be determined using Eq. 5:

$$213 \quad \tilde{\mathbf{A}} = U^* \mathbf{A} U = U^* Y V \Sigma^{-1} \quad (6)$$

214 Finally, the dynamic modes of the snapshots ϕ_i are calculated by solving the eigen-value
 215 problem and projecting the eigenvectors back onto the U basis as:

$$216 \quad \tilde{\mathbf{A}} \psi_i = \lambda_i \psi_i \quad \text{and} \quad \phi_i = U \omega_i \quad (7)$$

217 For the present LES results, the pressure near-field data were used to construct the \mathbf{Z} matrices.
 218 It is also worthwhile mentioning that the modes can be further scaled and weighted by solving
 219 an optimization problem involving the Vandermonde matrix constructed from $\tilde{\mathbf{A}}$. Nevertheless,
 220 the screech tone is an intense and discrete tone, which if present, can be clearly identified from
 221 the near-field pressure spectrum. Therefore, no ‘ranking’ of the DMD modes have been carried
 222 out in the following analyses. Instead, the time evolution of a single mode is scrutinized in this
 223 work. The pressure field \mathbf{z}_0 can be approximately reconstructed from the eigenvectors ϕ_i as:

$$224 \quad \mathbf{z}_0 \approx \sum_{i=1}^N \alpha_i \phi_i \quad (8)$$

225 where α_i are the amplitudes of the modes at $t = 0$. From Eq. 4, it follows that

$$226 \quad \mathbf{z}(n\Delta t) = \mathbf{z}_n \approx \mathbf{A}^n \mathbf{z}_0 = \sum_{i=1}^N \alpha_i \lambda_i^n \phi_i \quad (9)$$

227 whereby the result on the right is obtained from the definition $\mathbf{A} \phi_i = \lambda_i \phi_i$. The above
 228 result can be generalized for any arbitrary time t (not only integer multiples of Δt) as given
 229 below:

$$230 \quad \mathbf{z}(t) \approx \sum_{i=1}^N \alpha_i \exp\left(\ln(\lambda_i) \frac{t}{\Delta t}\right) \phi_i \quad (10)$$

231 Focusing on a single mode i , its temporal evolution of the mode, denoted as $\phi_{\tau,i}$, is
 232 captured by the following term:

$$233 \quad \phi_{\tau,i} = \exp\left(\ln(\lambda_i) \frac{t}{\Delta t}\right) \phi_i \quad (11)$$

234 Expressing $\lambda_i = e^{j\theta_i}$ (i.e. eigenvalue lies on the unit circle), the period of the mode is given
 235 by $T_i = \frac{2\pi\Delta t}{\theta_i}$. Using this result, Eq. 11 simplifies to the following:

236

$$\phi_{\tau,i} = \exp\left(j \frac{2\pi t}{T_i}\right) \phi_i \quad (12)$$

237

Eq. 12 can be used to capture the evolution of the mode at several instants over its period

238

$t \in [0, T_i]$ at arbitrarily small intervals.

239

240 3. Results and discussion

241 3.1. Shock cell and time-frequency spectrum validation

242

Before discussing the LES results and applying modal decomposition to its pressure and

243

velocity fields to further investigate the screech tone behaviour, it is essential to first examine

244

the jet flows captured by the LES and validate not only the overall dynamics, but also its

245

accuracy in time-frequency spectrum with the experiments. Note that limited by the

246

experimental results of the jet flow, only the LES results from unheated jet will be validated

247

with the experimental measurements. Figure 3 shows the comparison between the mean

248

streamwise density gradient, $(\nabla \rho)_z$ from LES ('numerical schlieren') and the schlieren images

249

from experimental work also conducted in-house at similar nozzle pressure ratio of $\eta = 5$. The

250

formation of quasi-periodic shock cells after a series of jet expansion to restore pressure

251

balance is representative of the overall jet dynamics along the potential core. The lengths of

252

the first shock cell, defined as the distance from the nozzle exit to the location where the

253

reflected shock reaches the jet boundary, are shown in Figure 3. It can be seen that there is

254

excellent agreement between LES and experiments for the under-expanded jet.

255

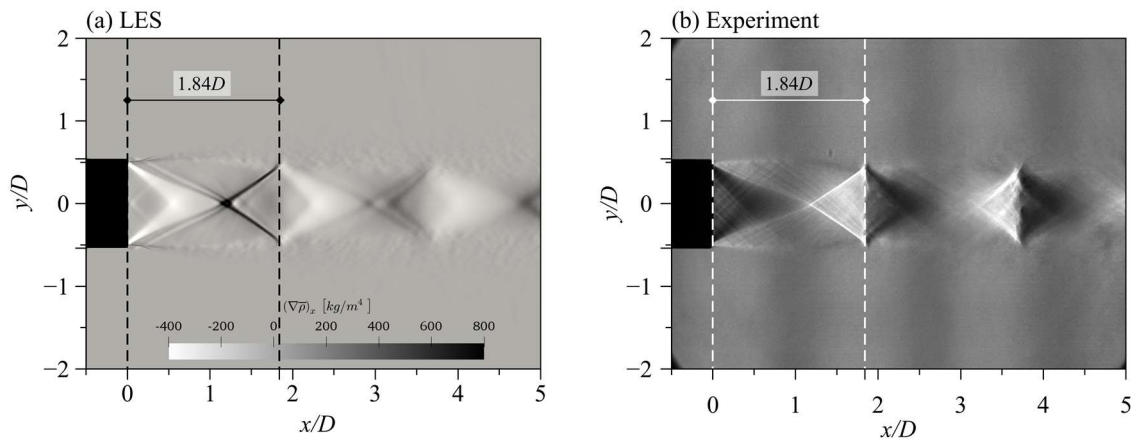


Figure 3. Comparison of shock cell structures and streamwise density gradients from LES with experimental schlieren image

256

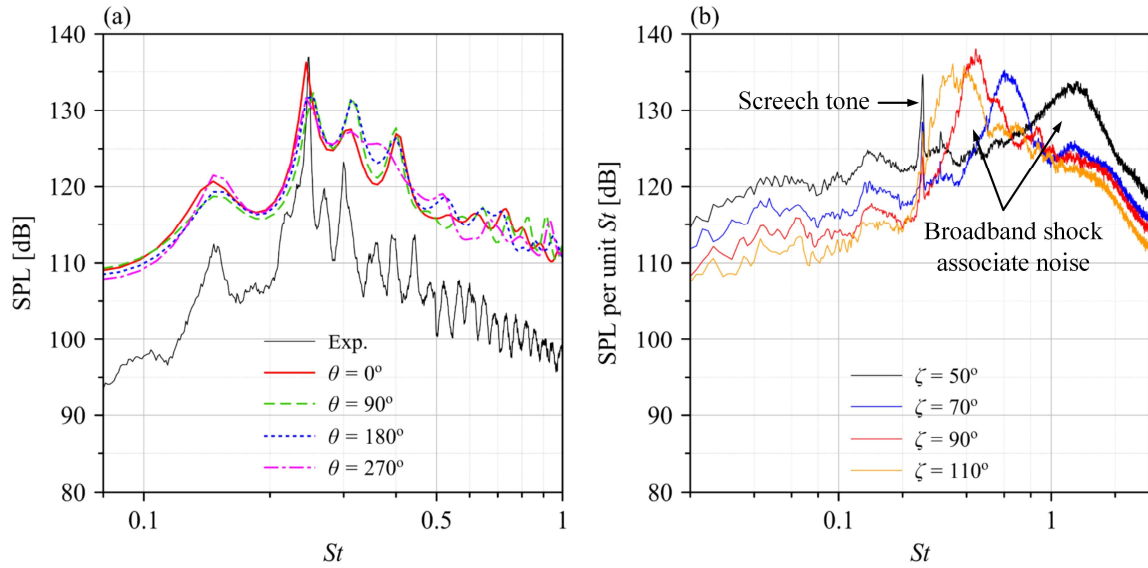


Figure 4. Comparison of the near-field pressure fluctuations at four azimuthal angles with experimental measurement (a) and far-field noise spectra obtained from experimental microphone array (b).

257

258 Time-frequency analysis is subsequently performed on the results sampled after the jet have
 259 reached quasi steady-state. The probes were placed at four azimuthal locations, $\theta =$
 260 $0^\circ, 90^\circ, 180^\circ$ and 270° , at a radial distance of $r = 0.8D$ from the jet centreline line on the
 261 nozzle exit plane of $x = 0$. A total of $N = 30001$ time samples were collected. To compensate
 262 for the short sample time, the maximum entropy spectral analysis method proposed by Burg
 263 (1967) is used to process the sampled unsteady pressure fluctuations. Burg's method is well-
 264 suited for processing acoustic data from LES (Larchevêque *et al.* 2004; Levasseur *et al.* 2008)
 265 due to its ability to resolve peaks accurately even for short sample times. Figure 4(a) shows the
 266 power spectral density of the unsteady pressure fluctuations close to the nozzle, together with
 267 experimental results from near-field microphone measurements. The microphone was placed
 268 at the same radial distance of $r = 0.8D$ from the jet centreline on the nozzle exit plane at a
 269 single azimuthal location. It can be observed from Fig. 4(a) that, despite the short sampling
 270 duration compared to experiments, LES captures well the general trend of the time-frequency
 271 spectra over the entire Strouhal number range of $0.1 \leq St \leq 1$, where shock-associated jet
 272 noise are prominent. Note that the jet Strouhal number is defined as $St = fU_j/D_j$. The three
 273 prominent peaks at $St = 0.15, 0.25$ and 0.3 agree very well with the experimental
 274 measurements. For the unheated jet at $\eta = 5$, the peak at $St = 0.25$ is identified as the screech
 275 tone from the experimental far-field noise measurement as its frequency remains unchanged

276 with directivity, as shown in Fig. 4(b). All the near- and far-field experimental measurements
 277 were conducted by Wei *et al.* (2019) and reproduced for LES validation here.

278
 279 Examining the comparison between LES and experimental results, while LES captures well
 280 the overall trends in the spectra, notable discrepancies exist in terms of spectra magnitude.
 281 Results from LES tend to overpredict the power spectra density by about $10dB/St$. The
 282 disagreements could possibly stem from the relatively short sample time (Morris *et al.*, 2002).
 283 However, the simulation has identified the screech tone relatively accurately, allowing further
 284 investigations and analysis into the modes and propagation of screech tone in the under-
 285 expanded jets.

286 287 **3.2. Characterisation of screech tone**

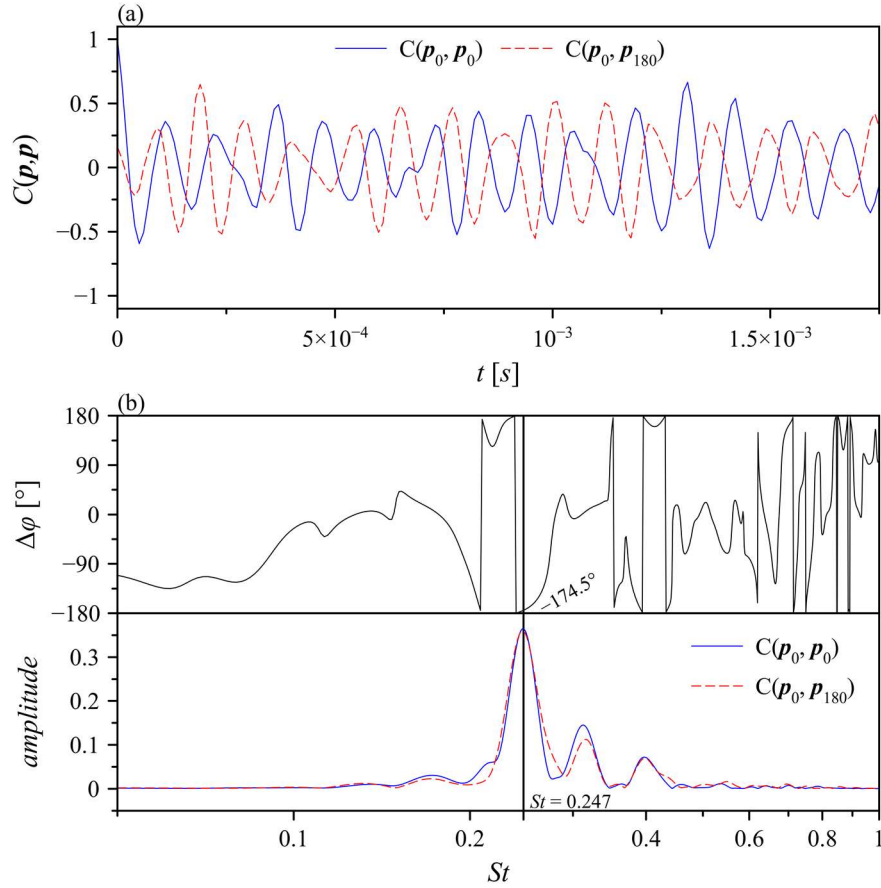
288 Since instabilities associated with the screech modes can be axisymmetric, flapping or helical,
 289 it is useful to analyse the cross spectrum of the pressure fluctuations at two different locations
 290 close to the jet shear layers, and examine the relative phase between the signals (Gutmark *et al.*
 291 *et al.*, 1989). Given two time series of pressure data $\mathbf{p} = \{p_0, p_1, \dots\}$ and $\mathbf{q} = \{q_0, q_1, \dots\}$ with a
 292 sampling rate of $1/\Delta t$, the normalised cross-correlation coefficient $C \in [-1, 1]$ can be
 293 determined as:

$$C(\mathbf{p}, \mathbf{q}) = \frac{\sum_{k=0}^{N-1} p_k q_{k+M}}{\sqrt{\sum_{k=0}^{N-1} p_k^2} \sqrt{\sum_{k=0}^{N-1} q_k^2}} \quad (13)$$

294 Here, $N = 10^4$ and the time shift M varies from $0s$ to $0.002s$ at intervals of $10^{-5}s$.

295
 296 Denoting the pressure time-series at the azimuthal location θ to be \mathbf{p}_θ , the cross-correlation
 297 coefficients $C(\mathbf{p}_0, \mathbf{p}_0)$ and $C(\mathbf{p}_0, \mathbf{p}_{180})$ are computed and plotted in Fig. 5(a). Both cross-
 298 spectrum coefficients show periodic behaviour with nearly identical dominant frequency. In
 299 addition, temporal evolution of the coefficients appears to be nearly anti-phase. Hence, the
 300 dominant frequency and phase difference, $\Delta\varphi$, between the two pairs of coefficients are
 301 determined by subjecting the coefficients to fast Fourier-transform (FFT). As can be observed
 302 in Fig. 5(b), the dominant frequency, identified by the peak amplitude, occurs at corresponding
 303 exactly to the screech frequency. The phase difference between the two coefficients is close to
 304 $\Delta\varphi = -180^\circ$. The cross-correlation coefficient and its FFT clearly reveals that the screech
 305 mode in the unheated jet LES is non-axisymmetric, i.e. flapping or helical.

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312 3.3. Dynamic modes from pressure near-field

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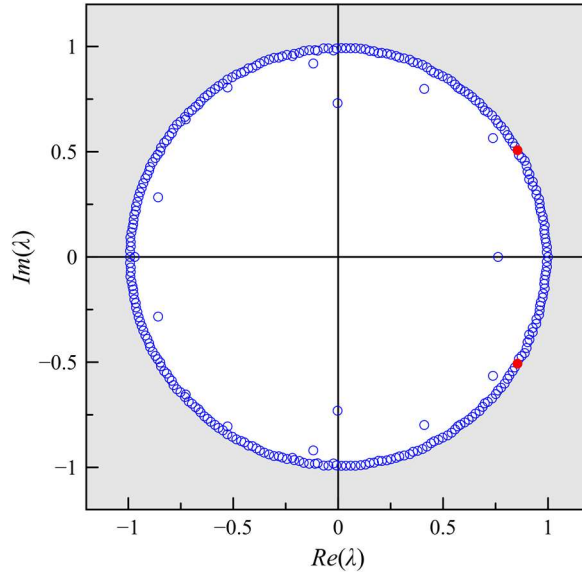
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Figure 5. Cross-correlation coefficients $C(\mathbf{p}, \mathbf{p})$ of the time-series pressure data between two probe locations of the unheated jet (a) and FFT spectra and phase differences computed directly from the coefficients C (b).



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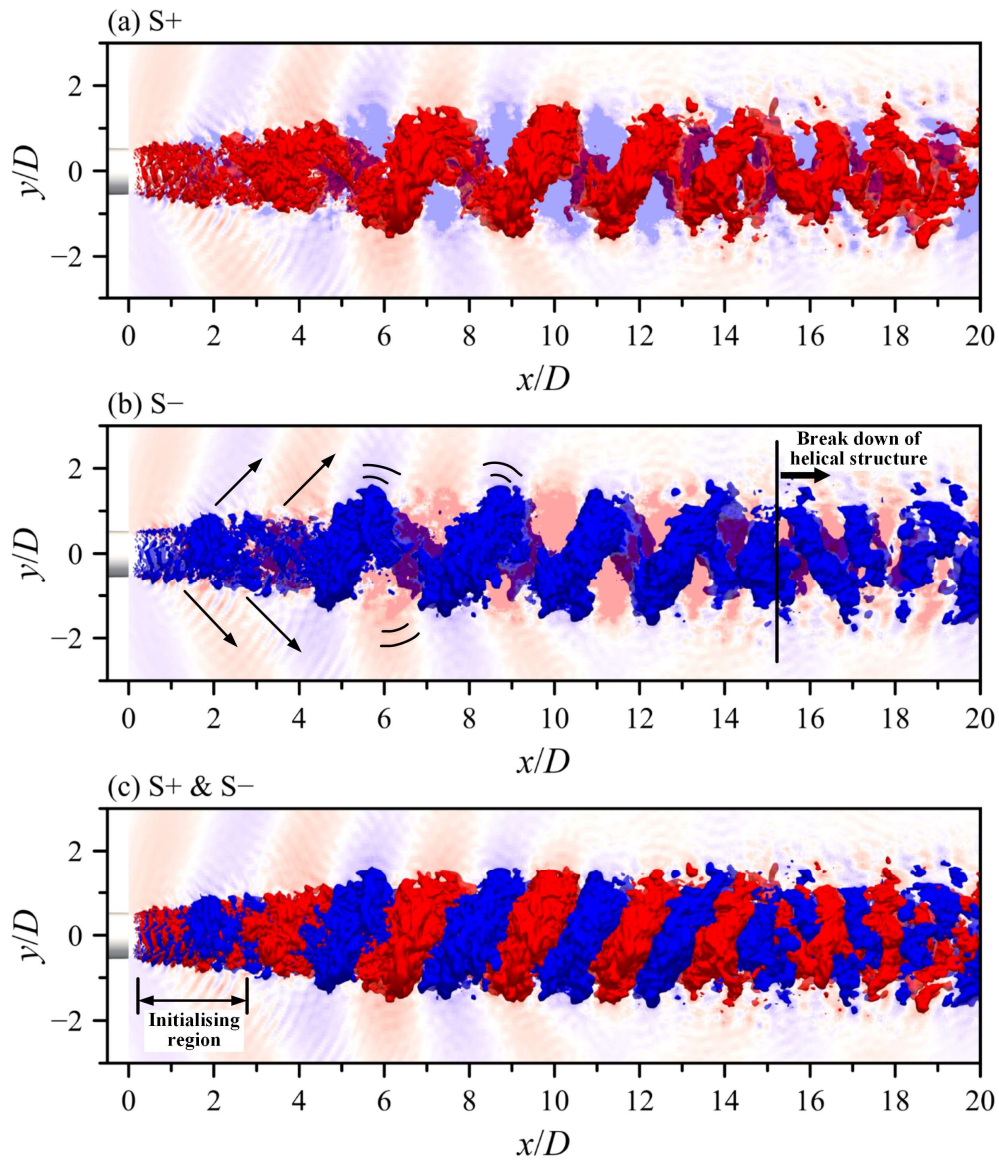
Figure 6. Distribution of Eigenvalues from the DMD modes around a unit circle

327

328 effects have already settled and the screech feedback loop has developed and sustained), the
 329 majority of the modes are expected to fall onto the unit circle, without significant grow or decay.

330

331 The screech mode, which is the dynamic mode having closest frequency to the screech
 332 frequency, is then extracted from the DMD analysis, and the isosurfaces $S+$ and $S-$ are plotted
 333 and shown in Fig. 7. The $S+$ and $S-$ are the positive and negative values of the real part of the
 334 mode, i.e. $Re(\phi_i)$. Note that the streamwise cross-section of the DMD mode is superimposed
 335 to the isosurfaces to illustrate the acoustic emission of screech tone. There are several notable
 336 observations from the dynamically decomposed mode. Firstly, the dominant structure derived
 337 from the time-series pressure data is clearly helical, with the positive and negative structures
 338 remaining out of phase for all azimuthal angles. Secondly, the helical structure remains
 339 relatively stable from $3 \leq x/D \leq 14$. With reference to the shock cell structures (see Fig. 3),
 340 the helical structure of the screech mode develops approximately at the end of the second shock
 341 cell, where the shock oscillations due to coherent structure roll-ups in the shear layer become
 342 pronounced (Suzuki and Lele, 2003). Moreover, the organized helical structure begins to break
 343 up at a downstream distance of about $x/D = 14$, which interestingly corresponds quite well
 344 with the potential core length of the unheated jet. The relatively stable helical mode over the
 345 length of the jet potential core suggests that the screech tone is likely to be produced over a
 346 relatively long distance as the shear layer interacts with the shock cells, hence, accurate
 347 modelling and prediction of its far-field noise levels would require complete flow field



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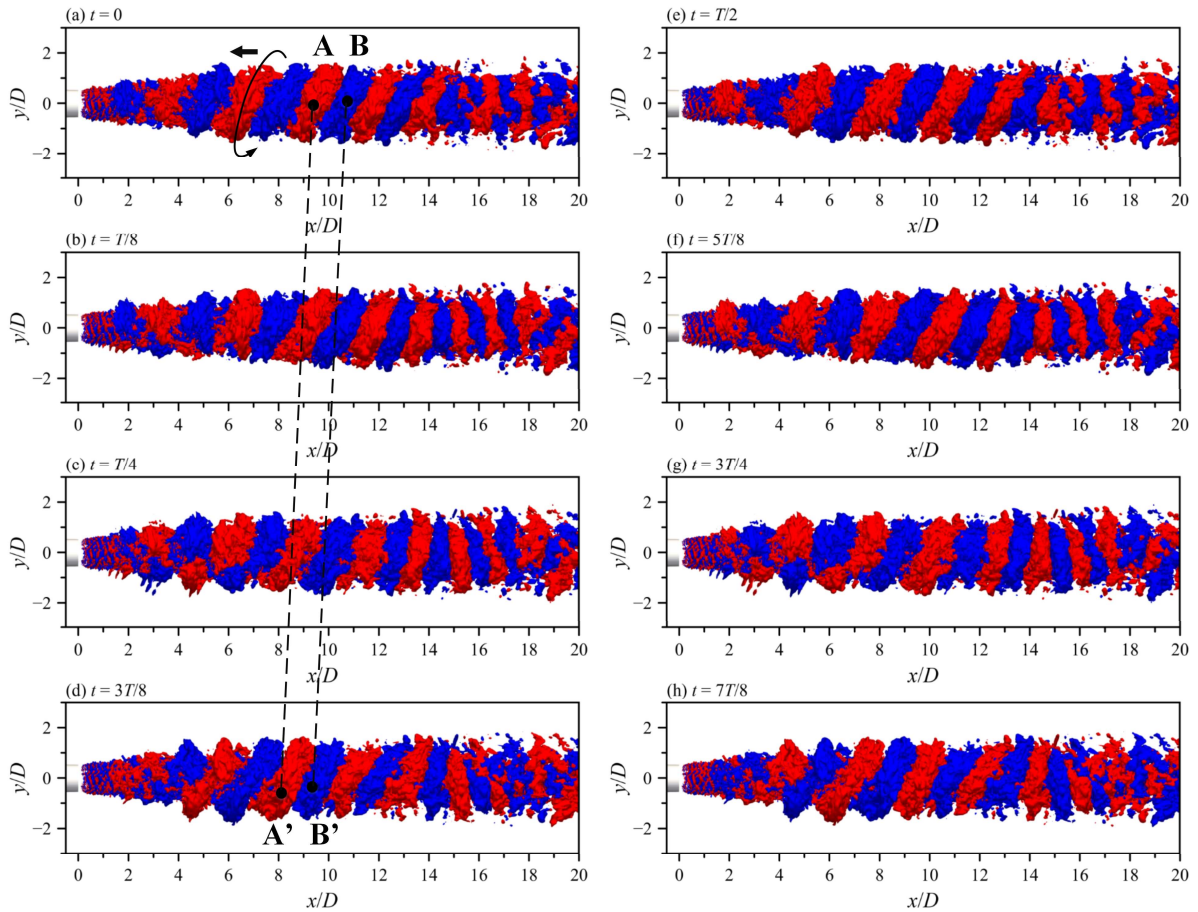
349 Figure 7. Isosurfaces of the DMD mode at corresponding screech frequency of the unheated
 350 jet

351

352 information in the near-field. More importantly, the helical structure is not confined merely to
 353 the shear layer of the jet but, in fact, initialises both within the jet potential core as well as
 354 outside of the jet shear layer, which agrees remarkably well with the recent studies on the
 355 propagation of *jet neutral modes*, believed to be responsible for the helical screech mode
 356 (Edgington-Mitchell *et al.*, 2018). Thus, the DMD mode reinforces the notion that
 357 measurements in the jet potential core is equally important in order to fully understand the
 358 screech generation mechanisms.

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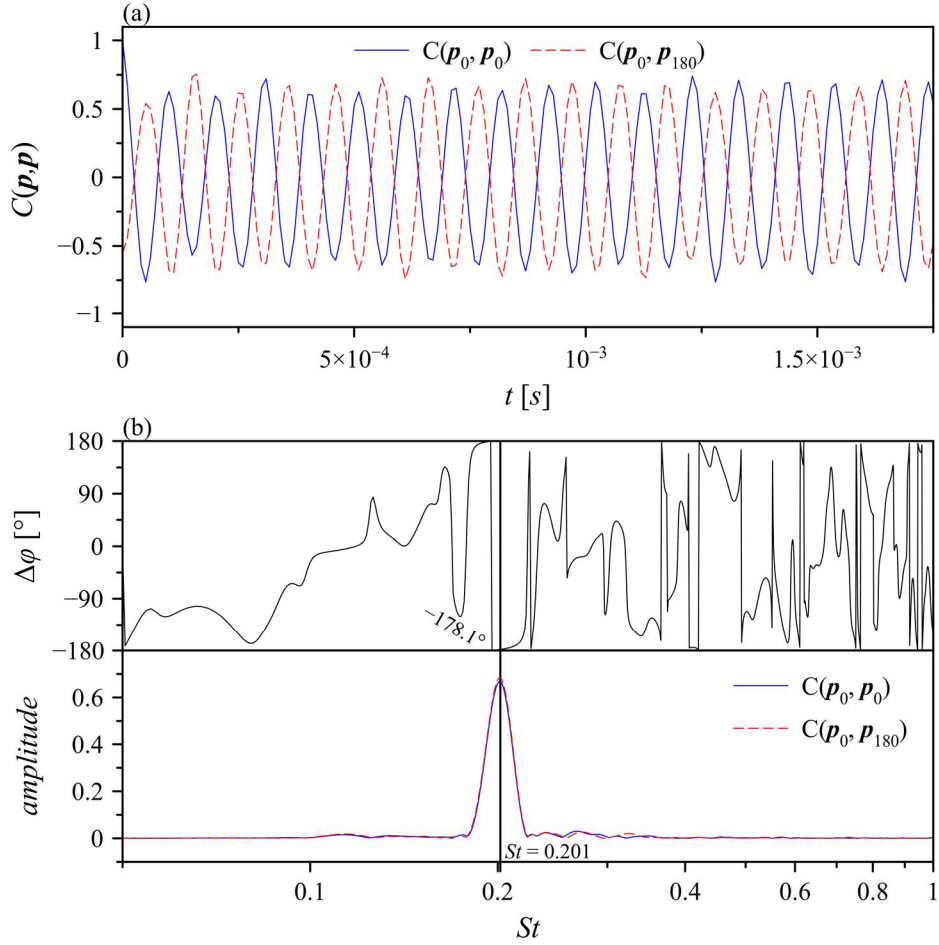
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Figure 8. Temporal evolution of the DMD mode corresponding the screech frequency of the unheated jet over a period of T at $T/8$ interval

365 As proposed in Section 2.3, on top of the spatial DMD mode obtained from the standard
366 analysis, it is possible to obtain the temporal evolution of a given mode by introducing an
367 arbitrary and yet sufficiently small time instant. Following Eq. 12, the temporal evolution of
368 the DMD mode corresponding to the screech frequency over a period T , can be obtained.
369 Figure 8 shows the resulting eight ‘instantaneous’ modes over the period (i.e. $T/8$ interval).
370 Observing closely the helical structure, it can be observed that both the S+ and S- structures
371 are propagating backward towards the nozzle exit. Note that point A-A’ and B-B’ indicated in
372 Figs. 8(a) to (d) are pairs of the same feature separated by $T/8$, which have been identified to
373 help track the movement of the helical structure. The dynamics of the temporal evolution of
374 the DMD mode can be more clearly discerned in the movie ‘*DMD_temporal_unheated*’ of the
375 supplementary materials. The backward propagation is both a validation to the DMD analysis
376 applied to the pressure near-field to capture the screech mode, since screech is characterised
377 by the complete feedback loop and a possible means to estimate the bulk propagation speed of
378 the instabilities. For the present analysis, it is estimated that the helical structure propagates at



379

380 Figure 9. Cross-correlation coefficients $C(\mathbf{p}, \mathbf{p})$ of the time-series pressure data between two
 381 probe locations of the heated jet (a) and FFT spectra and phase differences computed directly
 382 from the coefficients C (b).

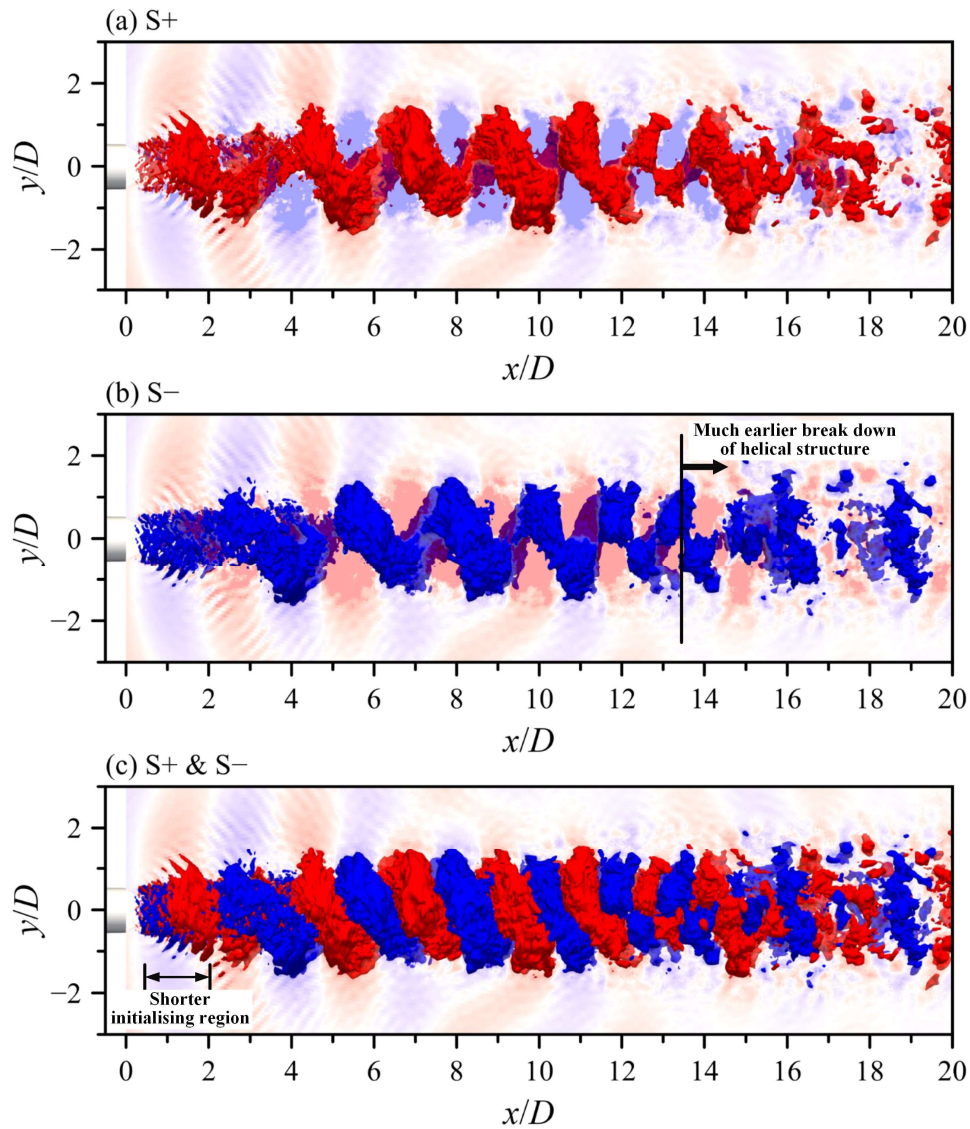
383

384 approximately $0.7U_j$, comparable to that of the jet convective velocity (Papamoschou, 1989).
 385 However, it should be cautiously noted that additional processing (for instance, image cross-
 386 correlation) is necessary to estimate the propagation speed more accurately as the visual-based
 387 approach tends to produce noticeable uncertainties.

388

389 3.4. Effect of heated jet

390 Similar to the unheated jet case, LES results of the heated jet with a temperature ratio of
 391 $T_0/T_\infty = 2$ were subsequently subjected to the pressure cross-spectrum and DMD analyses, to
 392 reaffirm that the present DMD approach is physically meaningful and to examine the effect of
 393 temperature mixing to the generation and propagation of jet screech tone. Figure 9 first shows
 394 the cross-correlation coefficients between two pairs of probes separated by 180° azimuthal
 395 angle at $x/D = 0$ plane and the corresponding FFT and phase differences determined from



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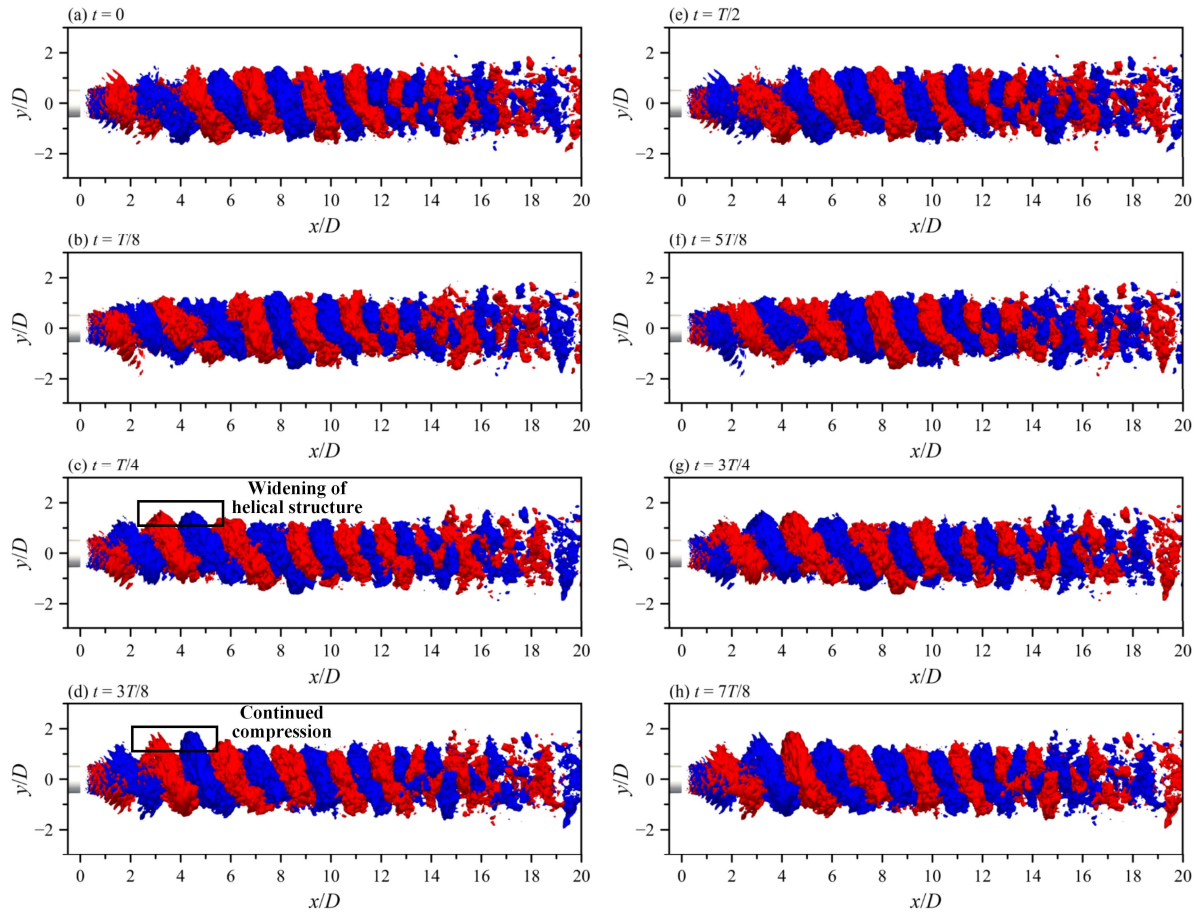
397 Figure 10. Isosurfaces of the DMD mode at corresponding screech frequency of the unheated
 398 jet – this is to be directly compared with Fig. 7
 399

400 these coefficients. The intense screech tone can be clearly identified to occur at $St = 0.201$,
 401 slightly lower than the unheated jet scenario, as expected (Massey *et al.* 1994; Chen *et al.* 2018).

402 The cross-correlation coefficient remains anti-phase, suggesting either a flapping or helical
 403 screech mode. Yet, compared to the unheated jet in Fig. 5, the single periodicity associated
 404 with jet screech is even more pronounced for the heated jet close to the nozzle exit plane.

405

406 Figures 10 and 11 shows the DMD mode associated with the screech mode and its temporal
 407 evolution over a period T at $T/8$ intervals, respectively, which can be compared directly with
 408 the unheated jet. Although the overall observations from the unheated jet still holds for the
 409 heated jet case, it is worthwhile to highlight the differences between the two. Firstly, the helical



410

411 Figure 11. Temporal evolution of the DMD mode corresponding the screech frequency of the
 412 heated jet over a period of T at $T/8$ interval - this is to be compared directly with Fig. 8
 413

414 structure develops over a shorter initialising region, suggesting a more intensified growth of
 415 the instability waves, which explains that the cross-correlation coefficients show
 416 predominantly a single frequency close to the nozzle exit plane. Secondly, the organized helical
 417 structure begins to break up at a closer downstream distance than the unheated jet. Hence, it is
 418 likely that the temperature mixing has destabilizing effect to the screech mode and the feedback
 419 loop. Thirdly, a closer examination of the temporal evolution of the dynamic mode in Fig. 11
 420 reveal an intriguing behaviour of the helical structure, where the closer to the nozzle exit, the
 421 structures widen and subsequently compress in the spanwise direction with a period of
 422 approximately $T/2$. While similar behaviour is absent from the unheated jet, it requires further
 423 study to investigate its influence on the generation of screech tone in heated jet.

424

425 Conclusion

426 In the present study of unheated and heated supersonic jets, dynamic mode decomposition
 427 (DMD) has been applied to the 3D pressure near-field obtained from LES to establish the

428 validity and effectiveness of DMD analysis in capturing the intense jet acoustics process from
429 pressure field information, and furthermore shed more light on the screech mode and its
430 instability propagation, especially when the temperature mixing is present. The results show
431 that the DMD mode is capable of capturing well the screech modes. In the present jet operating
432 conditions, the helical structure has been clearly identified over the length of the jet potential
433 core. The dynamic mode reveals additional information of the near-field regions where the
434 helical mode is dominant. It is observed that, the instability propagates both within the jet
435 potential core and along the jet shear layer, which agrees very well with recent studies in jet
436 screech phenomenon. Furthermore, a method to obtain the temporal evolution of the DMD
437 mode has been proposed, and it reveals that the bulk behaviour of the jet is dominated by back
438 propagation of the pressure field at screech frequency. Finally, compared to the unheated jet,
439 the growth of the instability wave in heated jet is more intense, as seen from the shorter
440 initialising region of the helical structure. More curiously, there appears to be periodic spanwise
441 widening of the helical structure during the screech process, which requires further
442 investigations in order to better understand the temperature mixing effect.

443

444 **Data Availability**

445 Some or all data, models, or code that support the findings of this study are available from
446 the corresponding author upon reasonable request.

447

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453

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