The Geometrical Pattern in the Royal Architecture of Ancient Egypt during the Middle Kingdom

VICTOR GARDÓN-RAMOS
Universitat Autònoma de Barcelona

Abstract. The royal architecture of Ancient Egypt evolved into an obviously aesthetic way all through its history: from the primitive royal tombs at Umm el-Qa'ab through the pyramids of the Old Kingdom and the great temples of the Middle Kingdom to the Ptolemaic temples built along the Nile bank. The present paper pretends to expose the systematic use of a geometrical pattern, emerged during the Thinite Age and Old Kingdom, to design and define the main proportions of the royal architecture of the Middle Kingdom.

Keywords: royal, architecture, geometrical, pattern, ancient, Egypt

Introduction

The royal architecture evolution of ancient Egypt has been one of the main subjects of study by the Egyptology. During the Thinite Age and Old Kingdom, the first eight centuries of the third millennium BC (Table 1),[1] royal architecture (royal tombs and sacred buildings constructed by order of the pharaoh or his core family) managed to convert from simple tombs at Umm el-Qa'ab and funerary enclosures at Abydos to huge mortuary complexes largely around the ancient city of Memphis.

The great pharaohs of the Old Kingdom arranged political stability to build their great pyramids and temples. These projects involved a large part of the society and became into a community aim. The building itself responded to the greatness of the pharaoh but it is also true that this kind of projects were an

[1] The present study sets out the later Old Kingdom, First Intermediate Period and Middle Kingdom chronologies in accordance with Erik Hornung, Rolf Krauss and David. A. Warburton's chronology of 2006. See Krauss / Warburton 2006: 491-492.

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important economic and social reason for that society.[2] Furthermore, the fact they were building their first great architectonic projects was probably another powerful motive to achieve these challenges.

The pyramid construction arrived to its peak during the IV dynasty. The pharaohs of the V and VI dynasties built smaller mortuary complexes and this is interpreted by the scholars as a one more proof of the decline of the monarchy.[3] Besides, at the late Old Kingdom the country suffered an increasing independence from the provincial powers, especially in the Middle and Upper Egypt.[4]

Nevertheless, it does not seem to be enough evidence to consider an economic crisis strong enough to bring down the social structure during the V and VI dynasties.[5] The transition between these two dynasties was not traumatic. In fact, the central administration was successor and solid. It was state, provincial and local, repeating the same structures of the previous dynasties. Every time more complexes but with the same order, controlled by a small group of officials belonging to a little group of families.[6]

During the reign of Djedkare Isesi (Table 1) the provincial administrators started to be relocated from the capital, Memphis, to the provinces that they governed. The continuous changes during the V and VI Dynasties in the administrative organization, specifically in the high offices like Vizier and Overseer of Upper Egypt, helped to emerge provincial redoubts of power.[7] Keeping the central power present throughout the country, pharaohs of the VI dynasty initiated the policy of marrying their daughters with these increasingly powerful governors.[8] On the one hand, this fact gave the royal family a strong presence all over the country but, on the other hand, new suitors to the throne emerged.[9]

This situation would continue until the VIII dynasty.[10] Although Manetho separates the VII and the VIII dynasties, the transition from the end of the Old Kingdom (Table 1) to the first half of the First Intermediate Period (FIP) was neither traumatic. This transition gave rise first, to the new Heracleopolitan Kings who continued the Memphite tradition in the Lower and Middle Egypt and second, in the south, the new Theban kings. During the second half of the FIP

[2] Moreno 2004: 190-199.
[3] Barta 2015: 187.
[4] Moreno / Agut 2016: 178.
[5] Moreno 2015: 81.
[6] Seidlmayer 2000: 120-121.
[7] Warden 2015: 471.
[8] Papazian 2015: 424.
[9] Moreno 2015: 83.
[10] Papazian 2015: 93-94.

these two factions fought for control of Egypt, although there are few sources about this military campaign.[11]

It seems clear that during the FIP to the Middle Kingdom the country of the two lands underwent major economic and social changes, including the separation of the Lower and Upper Egypt, and it is also clear that their royal architecture was not an exception. Their royal buildings evolved into an obviously aesthetic way, even though it is very possible that its geometry responds to an architectural tradition initiated by the first pharaohs.

The first pharaohs created a geometrical pattern which was systematically used in the architectural design of Thinite Age and Old Kingdom royal buildings.[12] This geometrical pattern was thought in order to represent the union between the earthly and divine spheres in their sacred architecture. They did it using the proportion 11x14[13] which has a geometrical peculiarity that is entirely accidental. A rectangle of 11x14 cubits defines a circle with a radius of 14 and a perimeter of 87.964 cubits, and a square with a half side of 11 cubits and a perimeter of 88 cubits (Figure 1).[14] The intersections of the square and the circle, with the same center, determine the rectangles R.α and R.ß that seems to define the main geometry of the royal buildings built at Thinite Age and Old kingdom mixed with ordinary rectangles as R.1x2, R.1x3 or R.2x3.[15]

In order to rule out coincidences between the royal architecture proportions and the GPAD, a further study of the sacred buildings presents two arguments that seem to confirm the GPAD hypothesis. On the one hand, their main lengths are multiples of 14 cubits and, on the other hand, their main widths appear to be multiples of 8.66 (rectangle R.α) and 11 (rectangle R.ß). Moreover, the royal cubit is divided into 28 digits (the diameter of the circle in the GPAD) which makes sense to the design of the main tool of length in ancient Egypt, even more when this civilization developed a decimal method to their way of accounting and mathematics.[16]

[11] Wolfram 2006: 7-8.
[12] Gardón-Ramos 2021
[13] Corinna Rossi presents the traditional use of this proportion as an independent triangle used by the architects in ancient Egypt since the beginning of the fourth dynasty. See Rossi 2003: 215-216.
[14] Gardón-Ramos 2021.
[15] Gardón-Ramos 2021.
[16] Gardón-Ramos 2021. See also Sánchez 2000: 143; Clagett 1999: 2.

1. The Geometrical Pattern in the Architectural Design (GPAD) of the Middle Kingdom royal temples

1.1. The GPAD in the temple of Nebhepetre Montuhotep II at Deir el-Bahri

Without no doubt scholars have interpreted the beginning of the Middle Kingdom since the reign of Montuhotep II.[17] The mortuary temple of Montuhotep II at Deir el-Bahri is the great achievement of funerary architecture in the Middle Kingdom. This king, the fifth pharaoh of the Theban 11th Dynasty (Table 1), united again the country of the two lands and chose Deir el-Bahri to build his mortuary temple near Thebes, making profit of the orography and the mountain that gives shelter to his temple as if a great pyramid.[18]

Figure 2.a[19] sets out the plan of the mortuary temple of Montuhotep II at Dier el Bahri. It is noteworthy that part of the temple was rock-cut in the mountain because the rectangles R.1x2 (Figure 2.b)[20] and R.α (Figure 2.c)[21] seem to define the proportions of the temple including the parts under open sky and the rock-cut zones. The tumult[22] sited at the entrance of the temple has a side (161/4 cubits) of a quarter of the main length of the temple, being its center at 161/4 cubits from the main entrance. These concrete measurements seem to make sense to assume the R.1x2 rectangle (Figure 2.b) as a probable design principle.

The chapels sited between the tumult and the open sky courtyard have their corresponding entries facing the geometrical half of the R.α (Figure 2.c). Furthermore, point c determines the cut-rock area of the temple in its north side (Figure 2.c). Therefore, the rectangle R.α (Figure 2.c) seems to be the perfect complement for the rectangle R.1x2 to define the main proportions of the temple. Meanwhile, rectangle R.ß delimits the great hypostyle chamber which houses the sanctuary (Figure 2.c).

The mixture of the rectangles R.1x2 and R.α makes sense again when their longitudinal axes are studied. On the one hand, the main axis is the central one of the rectangle R.1x2 (Figure 2.b). It starts at the entrance of the temple and arrives to the center of the sanctuary. On the other hand, the secondary axis is the central one of the rectangle R.α (Figure 2.c). It starts where the first and lower row of

[17] Diego Espinel 2009: 209.
[18] Alexander 1966: 53-54.
[19] Arnold 1979: Pl. 1.a.
[20] Arnold 1979: Pl. 39.
[21] Gabolde 2015: Fig. 12.b.
[22] Several speculations have been developed about the final aesthetic of the Mentuhoteps's Temple at Deir el-Bahri. See Arnold 1979: 34; Gabolde 2015: 155-158; Stadelmann 1997: 232.

columns appear and seems to mark the secondary entrance and the width of the entrance ramp (point d,[23] Figure 2.c).

Finally, Figure 2.d[24] shows how the concrete measures of the GPAD were a tool of design by the architects of Montuhotep II. The main heights of the temple seem to be multiples of 14. Firstly, 7 cubits seems to be the height of the columns and the sacred chapel. Secondly, 42 cubits is the height from the floor of the burial chamber to the outer side of the temple's roof. Table 2 presents all the temple measurements and proportions.

1.2. The GPAD in the royal temples of the XII Dynasty

The XI dynasty lasted about 140 years and Montuhotep II reigned during more than a third (Table 1). Nevertheless, during the first half of this dynasty, great local governors, such as Intef I, II and III, had a remarkable importance, actually some of them had taken on the full pharaonic titularity years before of the reunification of Egypt.[25] They were buried at el-Tarif, few kilometers to the north of Deir el-Bahri, in a type of grave called *saff*-tombs, a form of rock tombs with huge courtyards to open sky. This kind of tombs were built along much the Nile Valley in Upper Egypt, particularly in Theban area and some of them go back to the Old Kingdom.[26] These three tombs and the mortuary temple of Montuhotep's II could be considered the most important royal architecture of the XI Dynasty.

After Montuhotep II, the reign of Montuhotep III was a time of continuity. Actually, both Montuthoep III and Montuhotep IV (Table 1) carried on the administrative political of Montuhotep II, it is to say, eliminating unfair governors especially in the middle Egypt.[27] A representative sample of this continuity among these reigns could be the temple of Montu sited at el-Tod, 20 km south from Thebes. Its oldest remains are dated from the Old Kingdom but the most important part of the temple was built by Montuhotep II and III. Even so, the first two pharaohs of the XII Dynasty, Amenemhat I and Senusert I, rebuilt the temple.[28]

The transition between the XI and XII Dynasties was likely more troublesome. The successor of Montuhotep IV, Amenemhat I, was probably one of the leaders

[23] Gabolde 2015: Fig. 5.
[24] Arnold 1979: Pl. 40.
[25] Papazian 2015: 423.
[26] The huge courtyards of Intef's (I, II and III) tombs have a rectangular shape and their measurements are about 300x75 m (R1x4). See Arnold 2003: 206. This type of tombs found its monumental expression during the late Middle Kingdom in the Wahka's I and II tombs, a nomarch leaders from Qau el-Kebir. See Bard 1999: 1011.
[27] Diego Espinel 1999: 222-224.
[28] Bard 1999: 1025.

of Wadi Hammamat's missions for mining work, which this pharaoh ordered during his reign. That instability in the royal politics can be observed with the arrival of pharaoh Senusert I, Amenemhat's I son, who founded a new administrative capital of the kingdom called Iti-Tawy,[29] in the Lower Egypt far away from Thebes.[30]

Figure 3.a[31] sets out the plan of the temple that Amenemhat I and his son, Senusert I, not only rebuilt but refounded as well.[32] Table 3 provides the measurements of Rossi&Imhausen's[33] work and its correspondences with the GPAD. Rossi&Imhausen use the length of 0.525 m by royal Egyptian cubit[34] but this study proceeds on the basis that a royal cubit is 0.523 m.[35] With all this in mind, checking Figure 3.a and Table 3, the proportions of the Senusert's I platform of the temple at Tod seem to be based on the GPAD.

Another important building of Senusert I was the so-called White Chapel at Karnak (Figure 3.b).[36] Nowadays the White Chapel is rebuilt inside Karnak complex due to most of its blocks were found inside the third pylon of Karnak's temple.[37] Figure 3.b.1[38] represents the plan of the White Chapel and its measurements (13.75 x 35 cubits) seem to respond to the rectangle R.ß'[39] proportions ($\frac{11}{2}$ x 14) which is the longitudinal half of the rectangle R.ß.

Some temples built in the Middle Kingdom and throughout the whold history of ancient Egypt have not been identified as easily as the royal buildings exposed above. One of these temples is the one found at Qasr el-Sagha (Figure 3.c).[40] It is an unfinished building sited in the Middle Kingdom settlement on the northern

[29] Diego Espinel 2009: 225.
[30] Snape 2014: 167.
[31] Arnold 1975: 184-186. Corinna Rossi compares the original plans of Badawy and Arnold, see Rossi 2003: 46-47. She arrives at the conclusion that Badawy's process about the study of the architectural geometry of the royal buildings, using isosceles triangles, does not respond to a design system used by ancient architects. So, it is just about a geometrical description of the royal constructions.
[32] Bard 1999: 1025.
[33] Rossi / Imhausen 2009: 455.
[34] Lepsius 1865: 8.
[35] In order to be consistent with previous works, the present study proceeds on the basis that the royal cubit is 0.523 m. See Gardón-Ramos 2021.
[36] Lacau / Chevrier 1956: 14, 19.
[37] Lacau / Chevrier 1956: 11-12.
[38] Badawy 1966: 79.
[39] Rectangle R.ß' was used by the very first pharaohs to design their funerary enclosures at Abydos or, for example, the proportions of the Khafra's pyramid temple at Giza. See Gardón-Ramos 2021: Fig. 4. It is true that the rectangle R.ß' proportions (5.5 x 14) is very close to a rectangle R.2x5. But the fact that the concrete length measurements (of all the royal buildings studied in the present and previous works which respond to the R.ß' proportions) were multiples of 14 cubits seems to reject the rectangle R2x5 as a tool of design in these cases.
[40] Badawy 1966: 72.

part of the Fayum depression[41] which was possible designed based on the rectangle R.ß' as well. The dating of this temple has been reason for debate because it does not have inscriptions and its megalithic construction is reminiscent of Old Kingdom's. Nevertheless, nowadays scholars agree that it is a late XII Dynasty temple with a religious function based on the archaeological remains and architectonic decorations motives.[42]

After Senusert I, Amenemhat II reigned more than 30 years. Few constructions remain from this pharaoh, the most significant one is his pyramid at Dahshur. The reasons that took Amenemhat II to leave el-Lisht[43] (ancient Ity-Tawy) are unknown. In fact, not much of his reign is known. He was probably Senusert's I son but the lack of sources makes difficult to confirm this.[44] The same problem exists with his successor, Seusert II. He was possibly Amenemhat's II son but there are not enough evidences to confirm that. However, he chose the eastern area of the entrance to the Fayum to build his pyramid, in the proximities of the current el-Lahun.[45] His courtyard is not well attested although most were buried around the pyramid complex. Even so, during his reign for the first time the *nebty* name and Horus name of the king were not identical.[46]

The first half of the XII dynasty was a political stable time, although it was a time of substantial changes in the internal organization of the country's administration as, for example, the capital relocation of the country.[47] Several changes, not only in the daily life, art and culture but in the foreign policy, took place with the arrival of Senusert III, Senusert's II son. Senusert III became a king as famous and notorious as Montuhotep II. Up to the point that his campaigns at Nubia led these people to consider him as a local god.[48]

Figure 4.a[49] sets out the mortuary complex built by Senusert III at Abydos. This mortuary complex is made up of a mortuary temple to the north, near the Nile River, and a funerary enclosure with a subterranean tomb to the south, just near the cliff base. Furthermore, it could be considered one of the main royal architectural projects in ancient Egypt notwithstanding it is not well known[50] by non-specialists and architectural historian researchers. Firstly, Senusert III built

[41] Sliwa 1992: 177-191.
[42] Arnold 2003: 191.
[43] Baines / Málek 1991: 133.
[44] Grajetzki 2006: 45-46.
[45] Baines / Málek 1991: 130.
[46] Grajetzki 2006: 49-51.
[47] Manley 1996: 44.
[48] Grajetzki 2006: 51-54.
[49] Wegner 2009: 105.
[50] One of the most complete works about the mortuary complex of Senusert III at Abydos is Wegner's. See Wegner 2009: 103-169.

his mortuary complex at Abydos, in a clear reference to the architectural tradition of the Thinite Age, which was focused on the pharaohs' tombs at Umm el-Qa'ab and their great funerary enclosures built at north Abydos. Meanwhile he ordered the construction of his pyramid at el-Lahun, approaching his pyramid project to those of Old Kingdom.

Secondly, he built his funerary enclosure and subterranean tomb near the cliff base, just like Montuhotep II did at Deir el-Bahri. Once again, the shape of the mountain's summit reminds the pyramidal architecture and enhances the mortuary complex' solar symbolism.[51] Finally, his subterranean tomb not only refers to those of the first pharaohs at Umm el-Qa'ab but it is a precursor to the royal tombs built at the Valley of the Kings as well.

The study of the geometrical design of this mortuary complex through the GPAD seems now to be essential. On the one hand, Figure 4.b shows how rectangle R.α describes the main wall of the mortuary temple and rectangle R.ß defines its surrounding wall, and on the other hand, Figure 4.c presents how the rectangle R.ß seems to indicate de main proportions of the mortuary enclosure to the south. According to the subterranean tomb, Figure 4.d can confirm that the architects used the GPAD to define the proportions of the subterranean chambers as well. It is to say, the architects of Senusert III used the GPAD as a tool of design for his mortuary temple, funerary enclosure and subterranean tomb, although a priori these three constructions have nothing to do architecturally with each other.

Table 4 summarizes the proportions and measurements of Senusert's III mortuary complex. Most of measurements in this temple and the temples above may seem rare.[52] It is fundamental in this discussion to recall that every plan in the present work has its original scale; however, the measurements are not closer to significant numbers like tens or hundreds and, when they are, they do not respond to ordinary rectangle proportions (R.1x2, R.2x3, R.2x5, etc).

Senusert's III architectural project was designed in order to take profit of the orography to enhance their solar symbolism (in the same way than Montuhotep's II was). Architecturally, both were made up by:

- A valley temple just along the Nile shore.
- A causeway from the valley temple to a mortuary temple, in Montuhotep's II case,[53] or to a mortuary enclosure, in Senusert's III case.

[51] Wegner 2009: 142.
[52] Some of these values (Table 4), multiples of 14, are rather sophisticated, although still conforming to basic arithmetic operations. See Gardón-Ramos 2021.
[53] Arnold 1979: Pl. 5-6.

- Finally, a subterranean burial chamber, in Montuhotep's II case, or subterranean tomb, in Senusert's III case.

After Senusert III, his successor Amenemhat III, continued with several changes and upgrades in the administration of the country. This pharaoh built two pyramids but only the one at Dahshur has been well excavated. Few tombs of high officials are known from Amenemhat's III kingdom and those of his successors, Amenemhat IV and Sobekneferu. Furthermore, the burial places of the last two pharaohs of the XII Dynasty remain unknown. At this point, the political and cultural unity of the country was lost at the beginning of the XIII dynasty. The Second Intermediate Period, in terms of dynastic chronology, is framed between the XIII and XVII dynasties and several reigns arose during the late Middle Kingdom (Dynasties XIII-XIV). The kingdom of Avaris and Kerma are a very proof of that, which implies that local kings grabbed power again.[54]

2. The GPAD in the pyramid complexes of the Middle Kingdom

The pyramids of the Middle Kingdom are not notorious as those of Old Kingdom. During the fourth dynasty the pyramid construction reached its highest sophistication. The following pharaohs of V and VI dynasties did not bestow the same resources for their pyramids. Moreover, these pharaohs overturned many of their efforts building great solar temples at the Abu Sir area[55] and built their pyramid complexes nearby the great mortuary complexes of their antecessors between Dahshur and Abu Rowash, on the lower Egypt.

The famous pyramids of the fourth dynasty were built with a solid limestone core. Since then, the constructive procedures even the quality of the pyramid cores, have been impoverishing. Actually, the pyramid cores of Middle Kingdom consist of accumulated masses of rough blocks or fieldstones with a stepped outer face.[56] This is why the pyramids of this epoch are in such poor conditions.

Figure 6[57] sets out the main pyramids built during the Middle Kingdom. The first pharaoh of Middle Kingdom to build his pyramid complex was Amenemhat I (Figure 6.a),[58] at the ancient Ity-Tawy in present el-Lisht area. He did not create a solid core for his pyramid but he seems to have focused his efforts on founding a new capital for his kingdom. Even though, he took profit of the place orography

[54] Ilin-Tomich 2016: 45.
[55] Edwards 1955: 133-137.
[56] Arnold 1991: 159.
[57] Rainer Stadelmann published in 1997 one of the most important archaeological works about the pyramid architecture. The present work has used his plans in order to confirm if pyramids of Middle Kingdom were designed based on the GPAD. Stadelmann 1997: 233-247.
[58] Stadelmann 1997: 233.

in the same way than Montuhotep II at Deir el-Bahri[59] and the outer face of his pyramid complex surrounding wall responds to the R.ß proportions (Figure 6.a.1). After him, Senusert I (Figure 6.b),[60] who mainly rebuilt several royal buildings,[61] chose the same place as his father did to build his pyramid, specifically two km to the south from Amenemhat's I pyramid. Senusert I emulated to the fifth and sixth pharaohs[62] designing his pyramid temple and courtyard based on the GPAD (Figure 6.b.1). Furthermore, rectangle R.α frames the first surrounding wall of the pyramid and its temple and courtyard (Figures 6.b and 6.b.1).

Senusert's I successor, Amenemhat II, sited his pyramid at Dahshur and incorporated a new elongated rectangle for the pyramid complex surrounding wall (Figure 6.c).[63] These long rectangular enclosures did not go unnoticed by scholars,[64] and their proportions are probably based on rectangle R.ß' (Figure 6.c.1). Otherwise, Senusert II sited his pyramid at il-Lahun, the entrance of the Fayum. This region took importance during the Middle Kingdom due to its agricultural potential. Senusert II did not add important aesthetic updates to the pyramid architecture (Figure 6.d[65] and Figure 6.d.1). Instead, his pyramid was the first with a core built in mud brick.[66]

Senusert III recovered rectangle R.α for the proportions of the pyramid complex surrounding wall (Figure 6.d[67] and Figure 6.d.1). He turned to Dahshur to build his pyramid and continued building the pyramid core in mud brick. After him, Amenemhat III built his pyramid at Dahshur with a core built in mud brick as well. He also ordered the construction of a second pyramid at Hawara (Figure 6.d) and emulated to Amenemhat's II designing the surrounding wall based on rectangle R.ß' (Figure 6.d.1).[68] Finally, Table 5[69] summarizes all pyramid complex proportions and their slope designs.

Few pyramids were built during the later XII Dynasty but they cannot be identified with certainty.[70] The lack of sources determine the poor information about the end of the Middle Kingdom and the beginning of the Second

[59] Edwards 1955: 177.
[60] Stadelmann 1997: 235.
[61] Senusert I rebuilt and renovated several temples. One of the most famous of these works is his obelisk at Heliopolis, the oldest obelisk preserved in its original place. Grajetzki 2006: 36.
[62] Gardón-Ramos 2021.
[63] Stadelmann 1997: 236.
[64] Lehner 1997: 174.
[65] Lehner 1997: 175.
[66] Grajetzki 2006: 49-50.
[67] Stadelmann 1997: 242.
[68] Stadelmann 1997: 247.
[69] Table 5 shows the slopes of the pyramids, see Lehner 1997: 19, and the proportions of their main temples and walls, see Stadelmann 1997*:* 233-247.
[70] Edwards 1955: 177.

Intermediate Period. Nubia was lost, the delta was divided in several kingdoms (XII-XIII Dynasties) and finally new local powers at Thebes emerged during the XVII Dynasty.[71] New Kingdom started with the XVIII dynasty meanwhile the pyramid construction continued to evolve.

3. Conclusions

The collapsed structures of Middle Kingdom royal architecture do not arouse the same interest to non-scholars than the notorious pyramids of the Old Kingdom or the great Ramesside temples of New Kingdom. However, Middle Kingdom royal buildings seem to be an inflection point in the aesthetic and constructive procedures in the royal architecture of ancient Egypt. This period of the ancient Egypt architectural history shows itself as a transition between, first, the Old Kingdom great pyramid complexes and the temples of billions of years in the New Kingdom. Secondly, between the subterranean mortuary chambers of the pharaohs sited under the pyramids or the mortuary temples and the subterranean royal tombs in the Kings Valley.

Looking closer the royal architecture of Middle Kingdom, their designs can be explained through the GPAD. The systematic apply of this design tool seems to determine the main proportions of the royal buildings from the first tombs of the very first pharaohs to the royal tombs of the Middle Kingdom kings, a thousand years later. As in previous works, the plans that this study uses to review the royal building proportions of the Middle Kingdom—obviously—were not performed before the GPAD. So, once again, the fact that the GPAD proportions fit perfectly in with all of them upgrades the present research.

All lengths of the royal buildings are multiples of 14 and their weight are multiples of 8.66 (rectangle R.α) and 11 (rectangle R.ß). It is true that some of these measurements are so close to significant numbers as tens or hundreds, but the fact that the proportions fit in all temple plans and their lengths are multiples of 14 seem to discard these approximations as a coincidence. The measurements of Figures in Table 4 are a very proof of that.

Moreover, the great similarity between rectangle R.α (14/8.66 = 1.62) and R.5x8 (8/5 = 1.6) could be a topic of debate about if the priest architects used one or the other. Although, all measurements seem to be multiples of 14, some of them are very close to be multiples of 8 (Tables 3 and 4). Only the measurements of the smallest buildings and the great sampling of temples studied since Thinite

[71] Manley 1996: 52-53.

Age[72] have allowed to discard the rectangle R.5x8 as a tool of design opposite to rectangle R.α in these royal buildings.

The evolution of ancient Egypt royal architecture makes sense when all its buildings are put chronologically together. Likewise, that is when the GPAD can be proved and shown as a design tool.

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[72] The present study and previous works add 26 examples of royal buildings in which the rectangle R.α defines their proportions. See Gardón-Ramos 2021.

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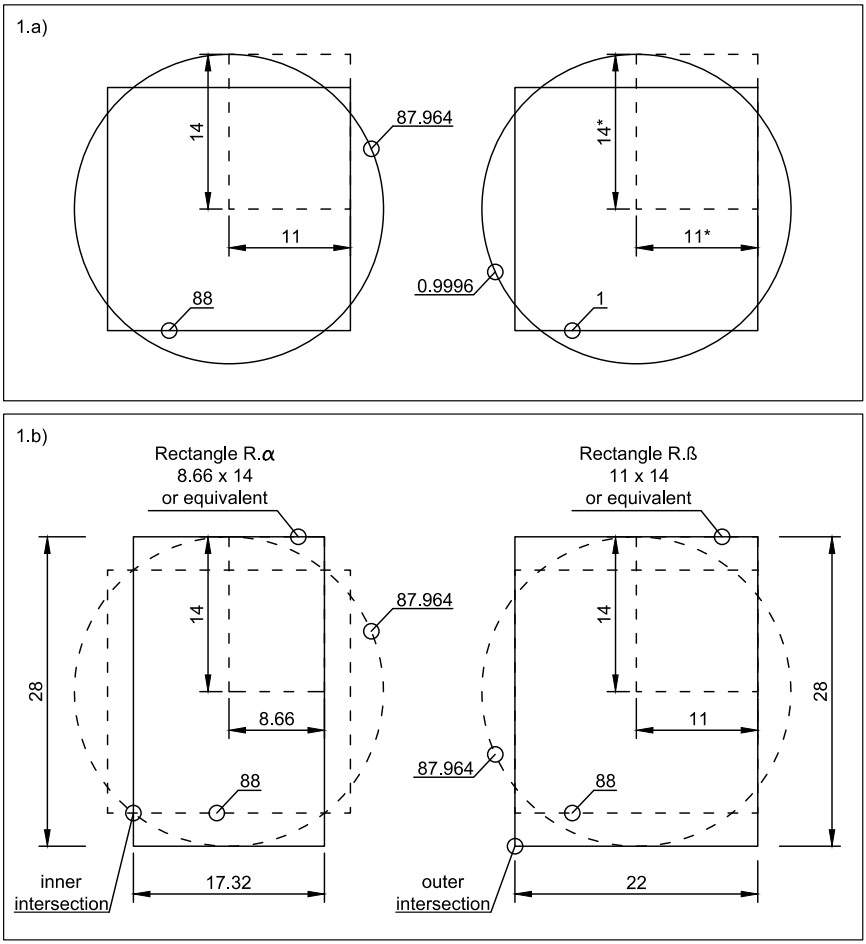


Figure 1. Geometrical pattern of the arch design (GPAD) in royal buildings that emerged during the Thinite Age

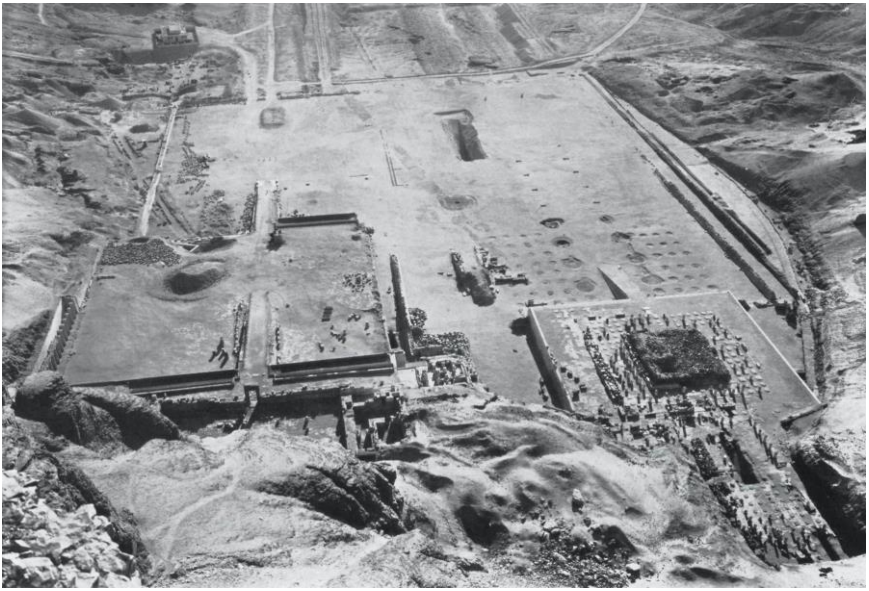
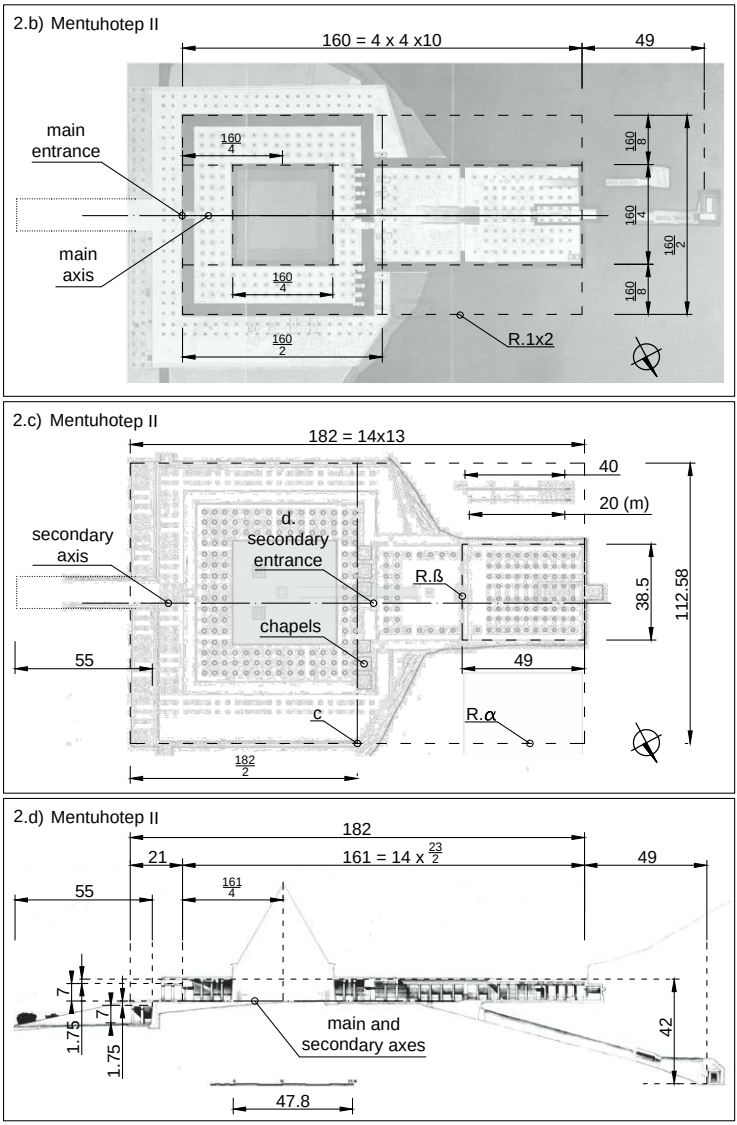
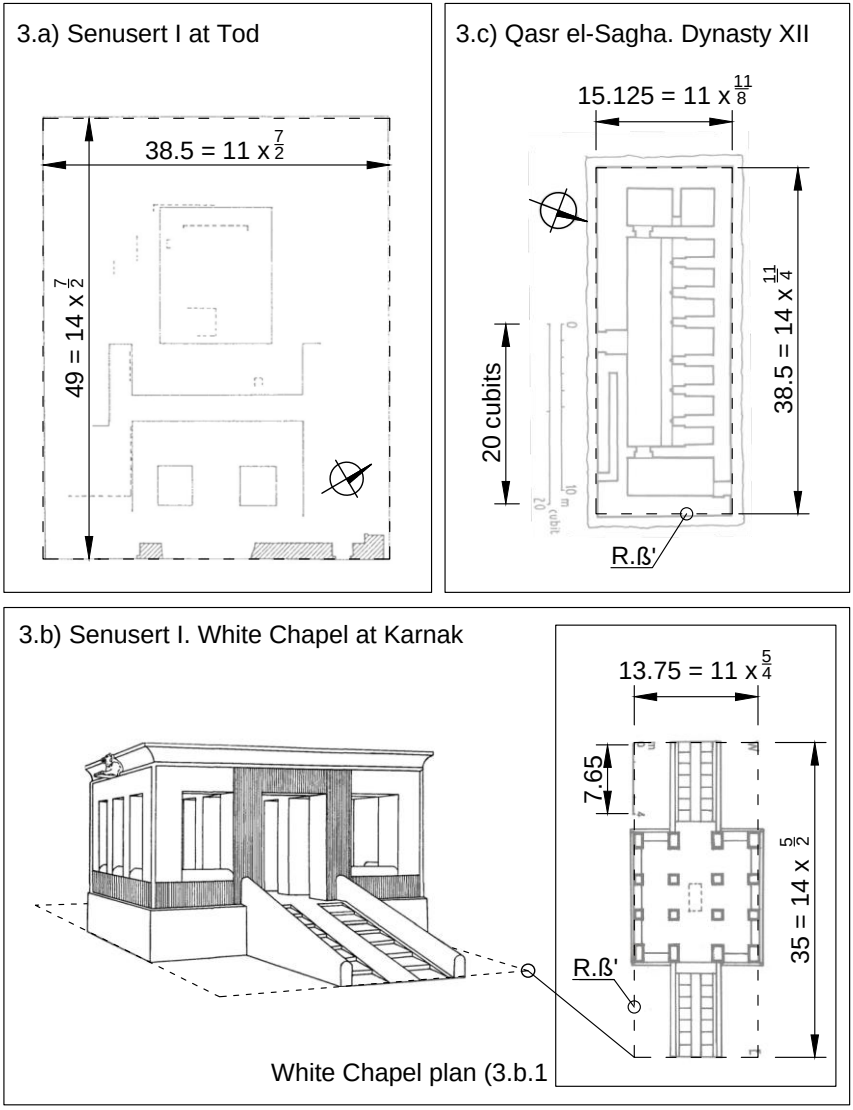


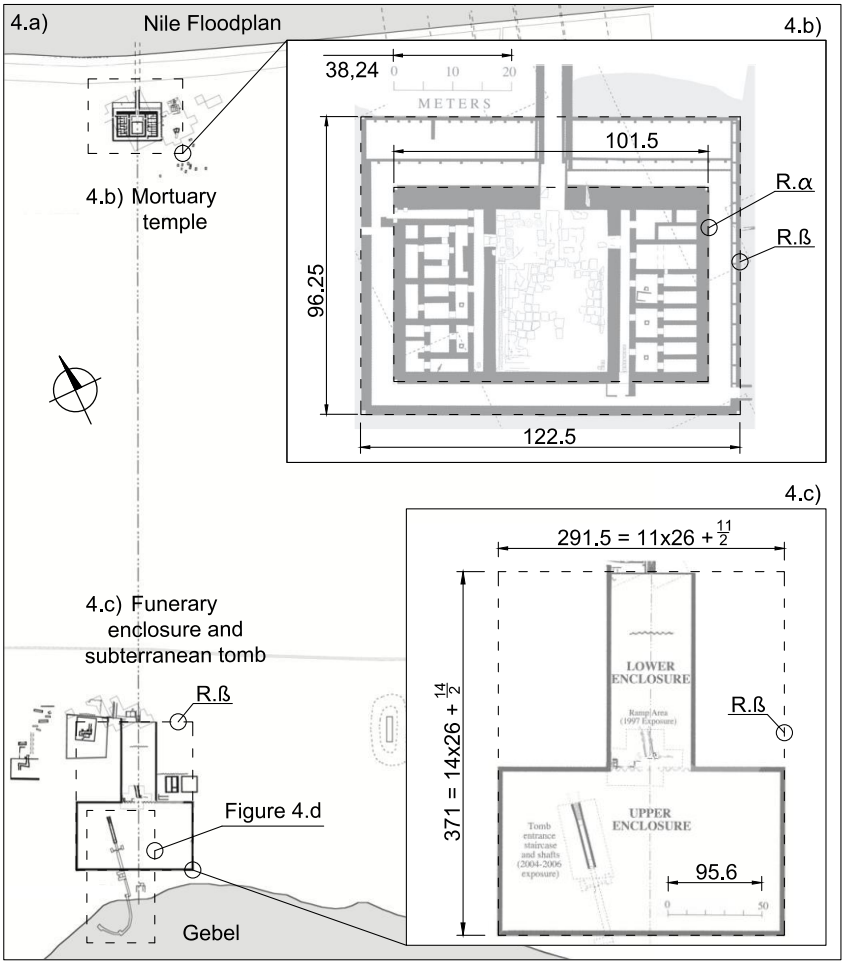
Figure 2.a. Deir el-Bahri. Temple of Montuhotep II (right)
and temple of Hatshepsut (left). After Arnold 1979



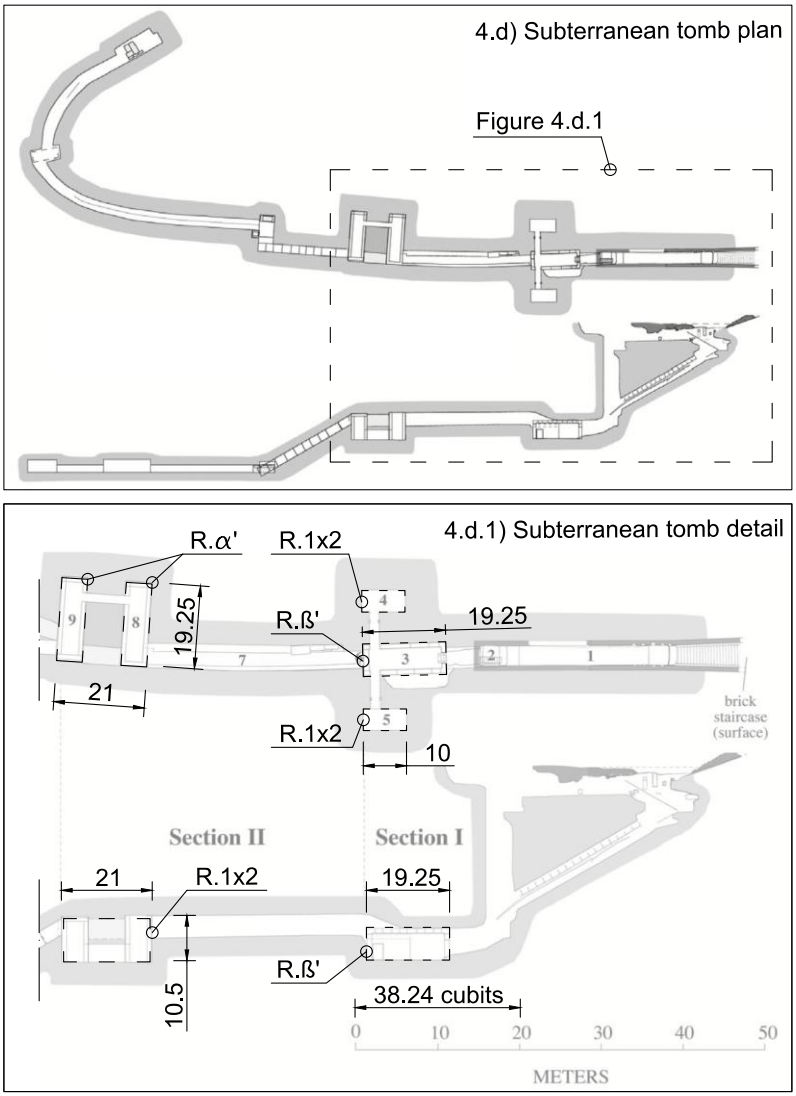
Figures 2.b, 2.c and 2.d. Temple of Montuhotep II at Deir el-Bahri according to the GPAD. Measurements in cubits



Figures 3.a, 3.b and 3.c. Royal temples of XII Dynasty according the to GPAD. Measurements in cubits



Figures 4.a, 4.b and 4.c. Mortuary complex of Senusert III at Abydos according to the GPAD. Measurements in cubits



Figures 4.d and 4.d.1. Subterranean tomb of Senusert III at Abydos according to the GPAD. Measurements in cubits

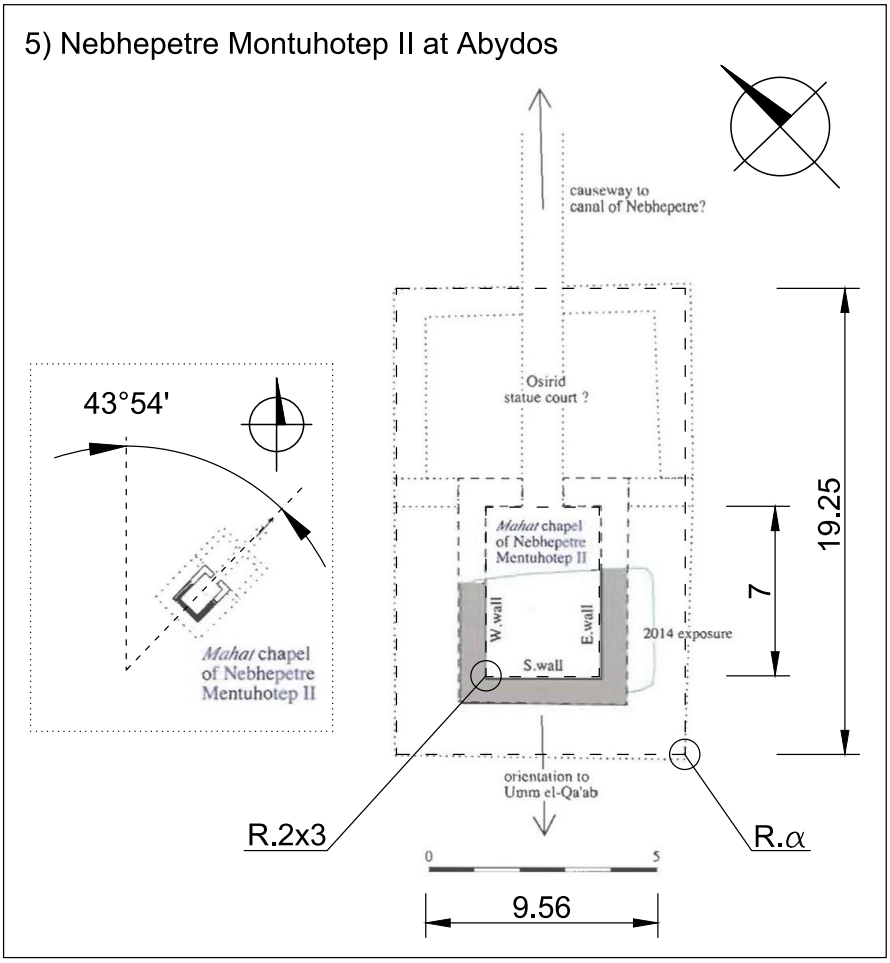
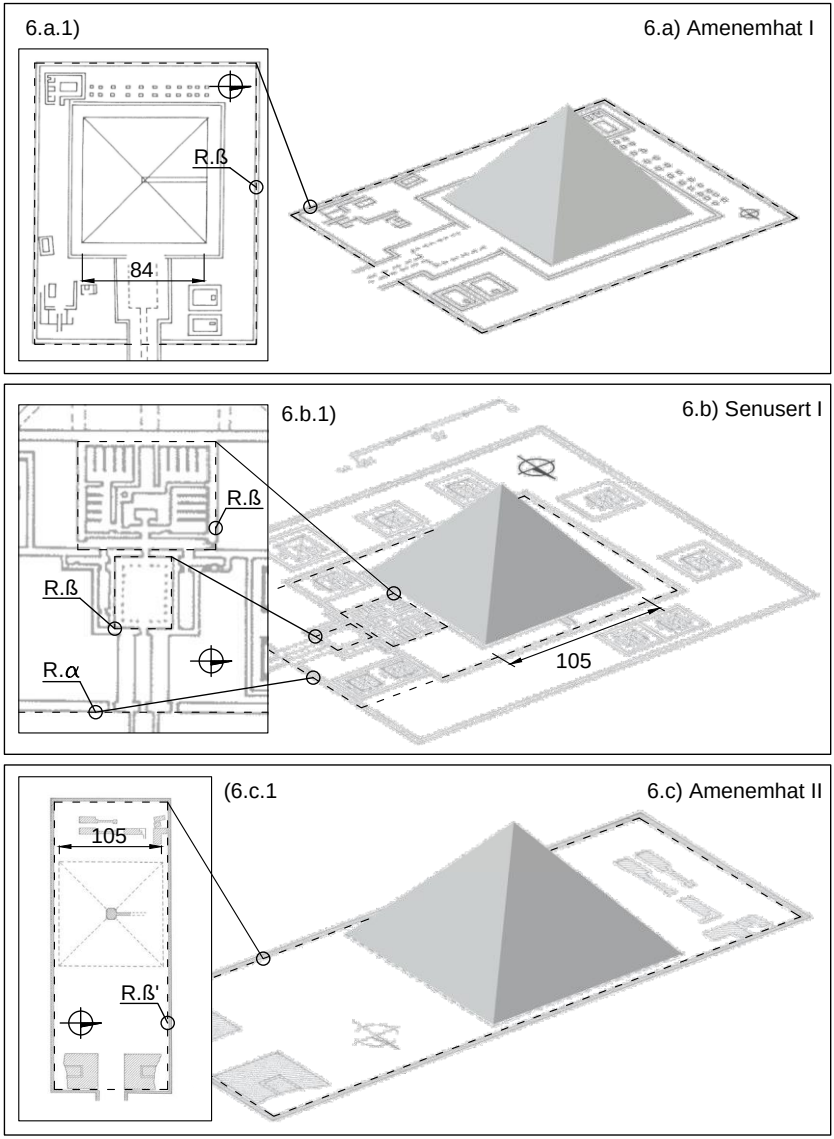
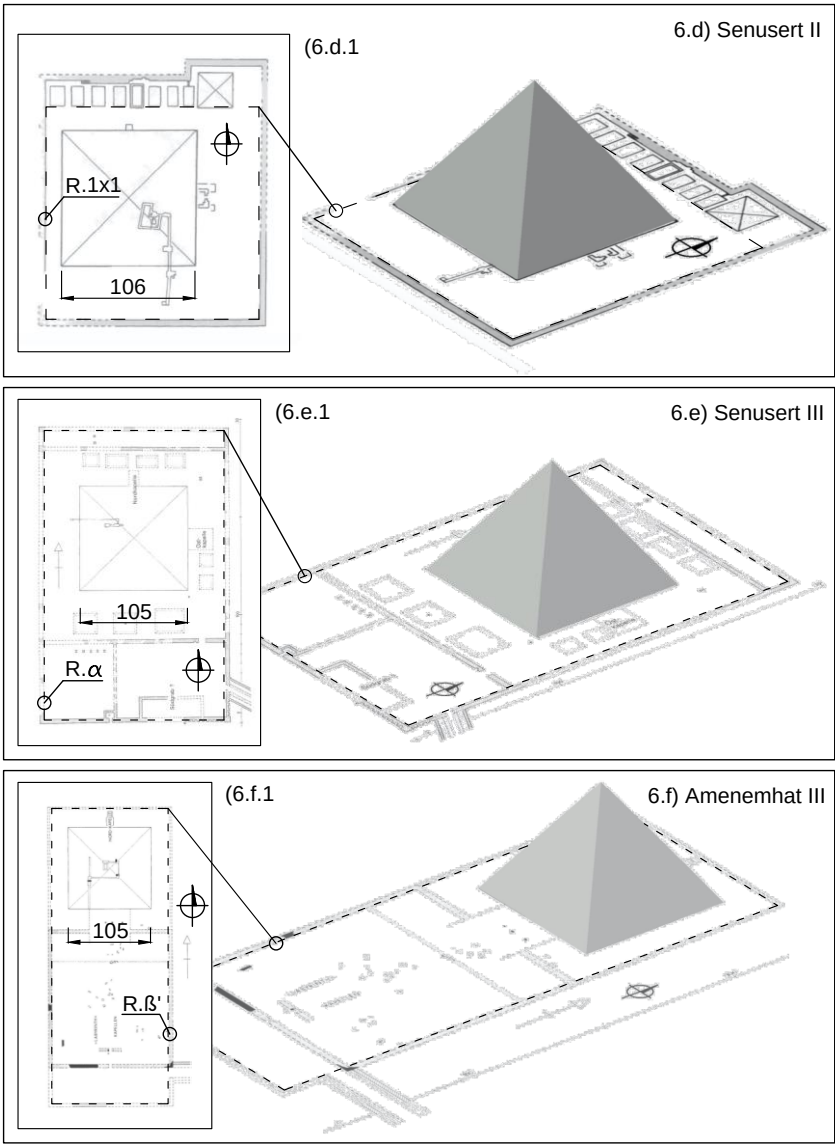


Figure 5. Chapel of Nebhepetre Montuhotep II at Abydos according to the GPAD



Figures 6.a, 6.b and 6.c. Pyramid complexes of the XII Dynasty according to the GPAD



Figures 6.c, 6.d and 6.e. Pyramid complexes of the XII Dynasty according to the GPAD

	Regnal years BC (HKW: 2006)
Later Old Kingdom	
V Dynasty	
Userkaf	2435-2429^{+25}
Sahure	2428-2416^{+25}
Neferirkare Kakai	2415-2405^{+25}
Neferefre	2404-?$^{+25}$
Shepseskare	2403-?$^{+25}$
Niuserre	2402-2374^{+25}
Menkauhor	2373-2366^{+25}
Djedkare	2365-2322^{+25}
Unas	2321-2306^{+25}
VI Dynasty	
Teti	2305-2279^{+25}
Userkare	--
Pepi I	2276-2228^{+25}
Merenre	2227-2217^{+25}
Pepi II	2216-2153^{+25}
Nemtiemsaf II	2152-?$^{+25}$
VIII Dynasty	
Neferkare	2126-2113^{+25}
Neferkawhor	2122-2120^{+25}
Neferirkare	2119-2118^{+25}

	Regnal years BC (HKW: 2006)
First Intermediate Period	**2118-1980^{+25}**
IX and X Dyns. (Herakleopolitan)	**2118-1980^{+25}**
Middle Kingdom	
XI Dyn. (Theban)	
Montuhotep I	2080-?$^{+16}$
Intef I	?-2067^{+16}
Intef II	2066-2017^{+16}
Intef III	2016-2009^{+16}
Montuhotep II	2009-1959^{+16}
Montuhotep III	1958-1947^{+16}
Montuhotep IV	1947-1940^{+16}
XII Dynasty	
Amenemhat I	1939-1910^{+16}
Senusert I	1920-1875^{+6}
Amenemhat II	1878-1843^{+3}
Senusert II	1845-1837
Senusert III	1837-1819
Amenemhat III	1818-1773
Amenemhat IV	1772-1764
Sobekneferu	1763-1760
Sec. Int. Per.	**1759-1539**

Table 1. Chronology of the later Old Kingdom, FIP and Middle Kingdom

Part of the Temple	Figure	GPAD	Measurements and proportions weight x length (in cubits)
Main proportions of the temple	Fig. 2.b	R.1x2	80 x 160
Tumult	Fig. 2.b	R.1x1	160/4 x 160/4
Central shape. From the main entry to the sanctuary	Fig. 2.b	R.1x4	160/4 x 160
Complete proportions of the temple	Fig. 2.c	R.α	112.58 x 182 (**8.66x**13) x (**14**x13)
Hypostyle Hall	Fig. 2.c	R.ß	38.5 x 49 (**11**x3+**11**/2) x (**14**x3+**14**/2)
Horizontal distance from the sanctuary to the burial chamber	Fig. 2.d	--	49 (**14**x3+**14**/2)
Entrance columns height	Fig. 2.d	--	7 (**14**/2)
Sacred chapel height	Fig. 2.d	--	7 (**14**/2)
Roof weight	Fig. 2.d	--	1.75 (**14**/8)
Descending corridor length to the burial chamber	Fig. 2.d	--	91 (**14**x6+**14**/2)
Vertical distance between top of the roof and burial chamber floor	Fig. 2.d	--	42 (**14** x 3)

Table 2. Proportions and measurements of the Montuhotep's temple at Deir el-Bahri according to the GPAD

	Measures (in m) Rossi&Imhausen	Measures (in cubits, 1 cubit = 0.525 m) Rossi&Imhausen	Measures (in cubits, 1 cubit = 0.523 m) Gardón-Ramos	GPAD R.ß
Length	25.7	48.95 (≈ 49)	49.14 (≈ 49)	38.5 (**11**x3+**11**/2)
Weight	20	38.1 (≈ 38)	38.24 (≈ 38.5)	49 (**14**x3+**14**/2)

Table 3. Proportions and measurements of the Senusert's I foundation platform of the temple at Tod

Figure	Temple	Measures (in cubits) weight x length	GPAD Proportions weight x length
3.a	Senusert I at Tod	38.5 x 49	R.ß (**11**x3+**11**/2) x (**14**x3+**14**/2)
3.b	White Chapel of Senusert I	13.75 x 35	R.ß' (**11**x2+**11**/2)/2 x (**14**x2+**14**/2)
3.c	Qasr el-Sagha	15.125 x 38.5	R.ß' (**11**x2+**11**/2+**11**/4)/2 x (**14**x2+**14**/2+**14**/4)
4.b	Mortuary temple of Senusert III. Main wall	62.78 x 101.5	R.α (**8.66**x7+**8.66**/4) x (**14**x7+**14**/4)
	Mortuary temple of Senusert III. Surrounding wall	96.25 x 122.5	R.ß (**11**x8+**11**/2+**11**/4) x (**14**x8+**14**/2+**14**/4)
4.c	Funerary enclosure	291.5 x 371	R.ß (**11**x26+**11**/2) x (**14**x26+**14**/2)
4.d	Chamber 3. Plan and section	15.125/2 x 19.25	R.ß' (**11**+**11**/2+**11**/4)/2 x (**14**+**14**/2+**14**/4)
	Chambers 4 and 5. Plan	5 x 10	R.1x2
	Chambers 8 and 9. Plan	11.9/2 x 19.25	R.α' * (**8.66**+**8.66**/2+**8.66**/4)/2 x (**14**+**14**/2+**14**/4)

Table 4. Proportions and measurements of the royal temples of the XII Dynasty (measurements in cubits)
Note*. Rectangle R.α' is the longitudinal half of R.α.

Pyramid complex	Architecture	Proportions	Lenght (in meters)	Lenght (in cubits)	Slope
Amenemhat I Ity-Tawy	Surrounding wall	R.ß	194	371 (= 53/2 x 14)	5x7
Senusert I Ity-Tawy	First Surrounding wall	R.α	216	413 (= 59/2 x 14)	6x7
	Pyramid temple	R.ß	47.5	91 (= 13/2 x 14)	
	Pyramid complex courtyard	R.ß	25.5	49 (= 7/2 x 14)	
Amenemhat II Dahshur	Surrounding wall	R.ß'	296.5	567 (= 81/2 x 14)	--
Senusert II Lahun	Surrounding wall (inner area)	R.1x1	168.5	322 (= 23 x 14)	10x9
Senusert III Dahshur	Surrounding wall	R.α	285.5	546 (= 39 x 14)	2x3
Amenemhat III Hawara	Surrounding wall	R.ß'	377	721 (= 103/2 x 14)	10x11

Table 5. Proportions and slopes of the pyramids of the XII Dynasty