Pollution Control Instruments in the Presence of an Informal Sector

Sudeshna Chattopadhyay, Sarmila Banerjee, Katrin Millock

Documents de Travail du
Centre d'Economie de la Sorbonne

CES Working Papers

Pollution Control Instruments in the Presence of an Informal Sector

Sudeshna CHATTOPADHYAY, Sarmila BANERJEE, Katrin MILLOCK

2010.103

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://centredeconomiesorbonne.univ-paris1.fr/bandeau-haut/documents-de-travail/
ISSN : 1955-611X

Pollution Control Instruments in the Presence of an Informal Sector

Sudeshna Chattopadhyay
Assistant Professor
Bidhannagar College
EB-2 Sector - I
Salt Lake City, Kolkata
India 700064
sudeshna.chattopadhyay@gmail.com

Sarmila Banerjee
Rajiv Gandhi Chair Professor
University of Calcutta,
56A, B.T. Road, Kolkata
India 700050
sbeco@caluniv.ac.in

Katrin Millock
Paris School of Economics, CNRS
Centre d'Economie de la Sorbonne, Université Paris 1 Panthéon-Sorbonne
106/112 Boulevard de l'Hôpital
75 467 Paris Cedex 13
France
millock@univ-paris1.fr

Pollution Control Instruments in the Presence of an Informal Sector

Abstract

The paper examines the challenges faced by the regulator in managing pollution when there is a linkage between a formal and an informal industrial sector across the stages of production. The formal sector is more productive than the informal sector and the latter saves cost by evading pollution regulation due to incomplete monitoring. This creates a natural tendency for the more polluting processes to be concentrated in the informal sector. We show the unintended effects of the standard Pigouvian tax (emission fee), which might lead to further deterioration by encouraging the shift of stages in favour of the informal sector. Instead, we propose a second-best hybrid instrument, comprised of a tax on polluting input and a subsidy on proper disposal of residual waste.

JEL codes: O17; Q53; Q58
Keywords: emissions tax, informal sector, pollution control, vertical production.

1. Introduction

The industrial sector in many developing countries is characterized by a certain degree of informalization in the production structure. Not only do the informal sector units[1] account for a significant percentage of total employment and output in the economy, even some units which legally belong to the formal sector are actively involved in informal practices as far as employment, labour conditions and standards are concerned (Damodaran, 2001). In fact the boundary line between the formal and the informal sector is often blurred. In India for example, a manufacturing unit to be labeled as strictly formal has to obtain a host of permissions from different government authorities such as a trade license from the Directorate of Commercial Taxes of State Government to carry out any remunerative activity, sanction from local (municipal) bodies to set up factory/shop and use public utilities, a manufacturing license from the Directorate of Industries, a consent to operate certificate for green clearance from the Pollution Regulatory Authority and so on. Hence, a formal unit must be registered with a number of official bodies like the Municipal Corporation, the Pollution Control Board, etc. However, in general most of the units even when 'formal' are registered with some but not all the designated authorities. Thus it is often difficult to categorize a unit as strictly formal or strictly informal. Damodaran (2001) has shown that many leather making units which belong to the formal sector in legal terms or in terms of capital size follow informal labour recruitment procedures. Even large firms, whose direct production and employment activities are sufficiently well organized and formal, are linked to the informal sector through an extensive chain of subcontracting and job work. Thus the formal and informal sectors are

[1] The informal sector consists of very small scale units in urban areas operated mostly by self employed producers. The activities carried out in these units are neither immoral nor illegal. However, the degree of compliance with formal regulation is generally very low. These units are characterized by low capital intensity, lack of modern technology and largely unskilled labour force and low productivity. In developing countries the informal sector accounts for a significant percentage of total output and employment. For example, the informal sector in India has continued to absorb more than 90% of the workforce for more than two decades (Oberai and Chadha, 2001). In India the contribution of the informal sector in the total net domestic product is also very high. In 2000-2001, the contribution was 47.7 per cent (Kolli and Hazra, 2005).

intricately connected with each other through the production chain. An interesting question is then what economic factors determine the tendency and extent of informalization of a particular production process. If capital and labour are taken as two standard inputs of production, the formal sector definitely enjoys some advantage over the informal sector in terms of better access to formal credit, which is not only cheaper but also secured. However, the informal sector - being outside the jurisdiction of formal regulation - enjoys assured supply of cheaper labour[2] since it avoids all minimum wage legislations and safety standards. In spite of different relative factor rewards, both factors are more productive in the formal sector compared to their informal counterpart mostly because of better quality of infrastructure. It is quite natural to observe a relative concentration of capital intensive production processes in the formal sector and labour intensive ones in the informal sector. Thus, the division of production stages between the formal and informal sector takes place on the basis of the relative cost advantage of the two sectors.

Apart from labour, an area where the informal sector is expected to have a cost advantage is pollution regulation. Due to the unorganized and diffuse nature of the informal sector, the cost of monitoring and enforcement of environmental laws in the informal sector is extremely high. As a result, the informal sector is able to evade environmental regulation rather easily. In fact, the high pollution intensity of the informal sector units is well recognized in the literature. Blackman (2000) shows that in developing countries informal brick kilns or urban tanning units can create serious environmental pollution. In India a large number of informal sector units operate in pollution intensive sectors like leather, textile, food processing and gems and jewellery. In most of these units the pollution control norms are not followed and the workers are forced to work in an extremely hazardous environment. In the informal

[2] The wages in the informal sector are often abysmally low. Dewan (2001) for example reports that in the informal bakery units in Mumbai the share of wages in total circulating capital is as low as 3.86.

leather tanning units of Kolkata for example the workers are forced to stand in waist deep chemical solutions in pits to soak raw hides. The pits are not cleaned and have accumulated material that is 40-50 years old. As a result the workers often suffer from skin infections, respiratory disorders, tuberculosis and other diseases (Damodaran, 2001). In the gems and jewellery making units the workers are forced to work in small dingy rooms filled with smoke from the acid that is used during refining and polishing processes. Cadmium is used for shouldering in gold ornaments in these small informal ornament making units though according to the World Gold Council cadmium fumes are highly toxic (Basu, 2007).

The informal sector in developing countries is by definition largely outside the purview of formal regulation. This regulatory weakness might induce the formal sector to try to escape pollution control costs by shifting the pollution intensive part of the production process to the informal sector through subcontracting. If this happens, then even with stricter environmental regulations developing countries might experience a deteriorating environment. Till date, the literature has considered the formal and informal sectors as two disjoint structures of production. In the present paper we aim at analysing the interdependence between the two at the organizational level encouraged by the weakness of the regulatory institutions that is typical for developing economies. We also suggest a feasible policy for pollution control in the informal sector, accounting for this interlinkage.

The rest of the paper is organized as follows: Section 2 contains a review of the existing literature on formal informal linkages in order to situate the contribution of the paper, and some stylized facts on pollution emanating from the informal sector. The model set-up is presented in Section 3. In Section 4, we analyse the decision problem of a production unit in the formal and in the informal sector facing a Pigouvian tax, or a tax per unit emissions, and we study the comparative statics of the shifting of pollution from the formal to the informal sector. Section 5 then discusses the alternative pure policy instruments and Section 6 proposes a hybrid policy instrument in order to avoid the perverse effects of a Pigouvian tax in the presence of a formal-informal sector linkage. Section 7 concludes and suggests future extensions of the model. All proofs that are not apparent from the text are put in an Appendix.

2. Existing Literature and Stylized Facts on Pollution from the Informal Sector

The characteristics of the informal sector in general have been analyzed quite extensively. Mazumdar (1976) surveyed the wage structure in the informal sector and found evidence of lower wages in the informal sector in some countries, even when adjusting for selection effects. Chaudhuri (1989) developed a general equilibrium model of an urban economy comprised of a formal and an informal sector, where the informal sector produces intermediate goods for the formal sector and is characterized by lower wages and no access to the organized credit market. Rauch (1991) in fact explained the size of the informal sector as a result of a binding minimum wage constraint in the formal sector. Azuma and Grossman (2008) explain the occurrence of a large informal sector by asymmetric information on behalf of government which makes it unable to exercise differential taxation according to resource endowments.

When it comes to the pollution emanating from the informal sector, Blackman (2000) discusses feasible policies for pollution control. Based on case studies of emissions control options for brick kilns in 4 cities in Northern Mexico he underlines the strong lobbying power of the informal sector and the potential use of private sector-led initiatives to implement pollution control such as new technology adoption. Another salient factor that he emphasizes is the high cost of monitoring of pollution from the informal sector, which suggests a potential for peer monitoring to render command and control regulation effective. The few cost benefit analyses that have been performed on pollution control options for the informal sector suggest that the net benefits are positive. Chakrabarti and Mitra (2005) undertook a cost benefit analysis of the installation of control devices to clean flue gas and dust from secondary lead smelting units in the informal sector in Kolkata, India, and found net positive social, and private, benefits. Blackman et al. (2006) did a cost benefit analysis of informal sector control options to reduce particulate emissions versus formal industrial source control and found that the net benefits from controlling informal sector brick kilns exceeded the net benefits of controlling formal industrial sources.[3]

Apart from case studies from different countries, there are very few theoretical models of the linkage between the formal and the informal industrial sectors and its consequences for pollution regulation. Gupta (2006), using a 4-sector general equilibrium model, shows that a more stringent pollution standard may lead to an increase in both output and emissions of the urban informal sector. A reduction in the permissible level of pollution in the model has a positive impact on the nutritional efficiency of the workers and leads to an increase in the effective labour endowment. The increase in labour endowment leads to an expansion of output and emissions from the informal sector through the Rybczynski effect. Chaudhuri and Mukhopadhyay (2006) obtained a similar result by using a 3-sector general equilibrium model where an increase in the tax rate on excessive emissions may result in a contraction of the formal sector and a consequent shrinkage of the informal sector in the short run. However, in the long run, indirect effects may dominate the direct ones and lead to an expansion of the polluting informal sector and an increase in aggregate pollution. The crucial assumption to generate such a result in both Gupta (2006) and Chaudhuri and Mukhopadhyay (2006) is the

[3] As the authors point out, the analysis did not include monitoring and enforcement costs, though.

proposed inverse relation between the (nutritional) efficiency of workers with the aggregate pollution in the economy. The models however, do not suggest any specific use of the tax revenues. This issue is taken up in Chaudhuri (2006), which shows the welfare impact of the change in the pollution standard in the same general equilibrium framework, and suggests that the national welfare may improve even if aggregate pollution increases when the efficiency effect dominates.[4] A recent paper by Baksi and Bose (2010) add to the theoretical modelling and share one feature with the current paper (which was developed independently): the linkage between the formal and informal sector through the production of an intermediate good for the formal sector. The Baksi and Bose paper's main contribution is to endogenize the compliance of the informal sector, which enables the authors to determine not only the size of the informal sector but also its share of noncompliant firms. The authors find that the presence of the formal-informal linkage can explain pollution leakage to the informal sector. They model the industrial organisation, and analyse the negative effect of price differentiation by the formal sector that purchases the intermediate good, in the case when there are noncompliant firms in the informal sector. In contrast, our model adds an alternative feasible policy, after having determined the unintended effects of a Pigouvian tax.

The previous models all visualize the informal sector as a supplier of intermediate input to the formal sector. This characterization, which separates the two sectors in two distinct compartments, ignores the fact that in the case of many commodities like leather products, gold and silver jewellery, textile, etc., production is organized in stages with different formal and informal sector units specializing in different stages of the process. This linkage between the formal and the informal sector has been captured in Sarkar and Roy (1989). They developed a model where the formal and the informal sector are linked through a vertical

[4] The national welfare function does not include any direct disutility from pollution, only an indirect effect from a reduction in the efficiency of labour due to adverse health effects caused by pollution.

production structure. As in Gupta (2006), Chaudhuri and Mukhopadhyay (2006) or Chaudhuri (2006), in this model also the advantages of the two sectors have been defined in terms of their access to cheaper inputs (i.e., labour is cheaper in the informal sector while capital is cheaper in the formal sector). The allocation of stages of production between the two sectors, however, is determined endogenously in the Sarkar-Roy model. If production takes place in stages, then the distribution of stages between the sectors, i.e., the extent of informal organization of the production process is often influenced by government policies. In fact, there is a two way relationship between the extent of informalisation of a production process and government policies. On the one hand, the extent of informal operation is affected by government policies, and, on the other hand, the presence of an informal sector may also affect the success of certain government policies. For example, a stricter environmental standard will increase the pollution control cost of the formal sector but might not significantly affect the pollution control cost of the informal sector as the informal sector is largely outside government control with low probability of auditing[5]. So, the formal sector might find it profitable to transfer the pollution intensive production processes to the informal sector through subcontracting to avoid environmental compliance costs. Knutsen (2000) cites the example of the dyestuff and tanning industry to argue that large multinational companies often subcontract polluting stages of production to small scale manufacturers in developing countries. A stricter environmental policy may then lead to greater informalisation and consequently the overall pollution in the economy may rise making it eventually non-viable. Thus, it is necessary to recognize that for the industry, the extent of informal organization of a particular production activity might be a decision variable. The contribution of the model

[5] An environmental audit is compulsory for the formal sector firms to obtain the no objection clearance necessary to continue operation. However, the unregistered informal sector firms do not face any such requirement. The regulator may nevertheless intervene provided some complaints regarding environmental degradation are lodged on 'public interest ground'. Since such events are rather infrequent in a developing economy, the probability of an environmental audit is very low in the informal sector in such economies.

presented here is thus to study pollution regulation in a vertical production structure where the optimal distribution of stages between the formal and the informal sector is endogenous.

In the production of a number of commodities, production takes place in stages, where each stage of production relies on the preceding stage, and each stage differs in pollution intensity. In the leather industry, for example, the raw skins and hides undergo processing in the tanning units before they can be used for producing leather products. The tanning units use toxic chemicals, the most important being chromium. Different types of small tanneries are involved in tanning operations. These tanneries are connected to each other and to the larger tanneries through a long subcontracting chain. Small wet blue tanneries produce semi finished leather from the raw skin and hides. Finished leather is produced by finishing tanneries which acquire semi finished leather from the wet blue tanneries. Sometimes both the stages, i.e., wet blue tanning and leather finishing are carried out in single integrated tanneries. The finished leather is then procured from the tanneries by leather product manufacturing units to produce leather goods (see Sahasranaman, 1998). The manufacturing of gold and silver jewellery also consists of a large number of stages starting with refining of gold and silver before proceeding to shouldering, crafting and polishing. In India, numerous small informal sector units perform work for larger formal sector jewellery firms. Sometimes traders act as agents or intermediaries between the informal units and the larger firms. The small informal units are connected with each other and the larger firms through a long subcontracting chain with each unit specializing in a particular stage of production of jewellery. The units which specialize in crafting of jewellery for example obtain the refined metal from refining units. Sometimes these units also obtain refined gold from the agents or larger formal sector firms. After crafting of the jewellery is complete, the jewellery is polished in units specializing in polishing operations. Then, the jewellery is delivered to the larger firm or the agent. The jewellery making units use a number of hazardous substances and chemicals during the production process. The units engaged in refining and polishing operations use nitric and sulphuric acid (KMDA, 2003; Chattopadhyay, 2009), whereas the units involved in crafting of jewellery use cadmium. In the textile industry also, production is carried out in stages with different units specializing in different stages. In textile production sizing, bleaching and dyeing precedes the finishing process. The textile industry uses a large number of toxic chemicals. Chlorine bleach, which is often used to bleach fabric, is extremely toxic. The dyes used for dyeing fabrics contain organically bound chlorine, a known carcinogen. Mordants, such as chromium, are used in the textile dyeing units to fix the colour on the fabric and are also very toxic (Karthikeyan and Alexander, 2008).

3. The Model

Our aim is to model the vertical production structure that links the informal sector to the formal sector, and analyze the endogenous stage where production shifts from one sector to the other. The stylized facts that we wish to capture in the model are the following:

- Both sectors are subject to environmental regulation, but detection of pollution is more difficult in the informal sector. Hence, the expected pollution tax payment, or regulatory burden of environmental regulation, is smaller in the informal sector for a given penalty or fee;
- The formal sector is more efficient in the use of inputs and thus has a productivity advantage over the informal sector (because of access to superior technology).

In order to capture these characteristics we draw on the multi-stage production model (Dixit and Grossman, 1982; Sanyal, 1983) that relies on differences in factor proportions between different stages of production. The production process consists of a succession of stages where some value is added at each stage to an intermediate product. The production stages are denoted *i* and range from 0 to 1, with *i*=0 indicating the upstream stage and *i*=1 indicating the downstream stage. We assume a continuum of production stages so that *i* is a continuous variable. So, following Dixit and Grossman (1982), stage $i + di$ can be referred to as stage *i*. Goods indexed with *i<1* are intermediate goods. For the final product it is necessary that it should go through all the stages. In order to focus the analysis on the stylized facts we make some simplifying assumptions on the industry, which is assumed perfectly competitive. Both formal and informal sector units take the prices of the output and inputs as given.

The production process uses labour (*l*) and a polluting input (*s*), for example, chlorine-based dyes in the textile industry, or chromium in the leather tanning industry, or yet cadmium or acid in the gold jewellery industry. Production stage $i + di$ uses one unit of intermediate output from stage *i* and a mix of polluting input and labour to produce a unit of intermediate output. The stage production function denoting the value added at any intermediate stage *i* is $q(i) = f\left(s(i), l(i)\right)$ with the standard assumptions on positive but decreasing marginal productivity of input. All production units are price takers and the unit cost of polluting input is p^S. The per unit labour cost is *w*.

A central characteristic of the formal-informal sector linkage is the difference in productivity. Since we are interested in highlighting the tendency of pollution load shifting from the formal to the informal sector we will deliberately abstract from input related advantages in terms of input price differentials. Instead we assume higher total factor productivity (TFP) for the formal sector and a common input price for both sectors. The stage production function will thus be written as $q_F(i) = \theta f\left(s(i), l(i)\right)$ in the formal sector, with $\theta > 1$.[6] Throughout, we indicate the formal and the informal sector by subscript *j*, $j = F, IF$, respectively.

In general, pollution is some function of applied input[7], but for simplicity, we will work with the case where one unit of applied input creates one unit of pollution. Input use, denoted *s*, is a function of the production stage *i*, where we assume that the use of polluting input decreases with downstream production: $\frac{\partial s}{\partial i} < 0$. We order the production stages so that earlier, i.e., upstream stages are more pollution intensive; for any given input price ratio the optimal polluting input-labour ratio thus falls with an increase in *i*: $\frac{\partial (s/l)}{\partial i} < 0$.[8]

Pollution can be abated, and the amount of abatement per unit of applied input is bounded by the extreme points of doing nothing, and thus emit the entire polluting input, or abating a maximum of the input: $a \in [0,1]$. The abatement cost function per unit of output depends on the amount of abatement, *a*, and the production stage $i: c(a,i)$, with $\frac{\partial c}{\partial a} > 0, \frac{\partial^2 c}{\partial a^2} > 0, \frac{\partial c}{\partial i} < 0, \frac{\partial^2 c}{\partial a \partial i} > 0$. The marginal cost of abatement is thus higher in the cleaner downstream stages.

---

[6] The analysis will show that there also exists an upper bound on θ to ensure existence of the informal sector.
[7] Khanna and Zilberman (1997) argue plausibly for this kind of specification, which is well adapted to the industries studied here, like leather tanning, the textiles or jewellery industry.
[8] The rearrangement of the order of the physical production stages is done for modeling purposes. With close to zero transportation costs (which is the case between the informal and formal sectors), it is irrelevant if the economic ordering differs from the real physical ordering of production stages.

Pollution, when detected, is imposed a tax T per unit of emissions.[9] We assume that the emissions tax is imposed with probability α_j. In order to capture the essence of the difference in detection probabilities between the two sectors, we put $\alpha_F = 1$ in the formal sector and $\alpha_{IF} = \alpha, \alpha \in (0,1)$.

The marginal damage cost of pollution, assumed constant over the relevant range studied here, is denoted δ. Pollution damage from one unit of polluting input use is thus equal to $\delta(1-a_F)s_F$ in the formal sector and $\delta(1-a_{IF})s_{IF}$ in the informal sector.

4. The Shift of Production between Formal and Informal Production Units

We start by analysing the choice of allocation of production between the two sectors of production. Since our aim is to analyse the allocation of production between the two sectors in a vertical production setting, we can focus on per unit output costs here. At each stage, total costs are minimized for one unit of output. A production unit in sector j thus solves the following problem:

$$\underset{s_j, a_j, l_j}{Min} \quad C_j = wl_j(i) + p^S s_j(i) + c(a_j, i)a_j s_j(i) + \alpha_j T(1-a_j)s_j(i) \qquad \forall i,\ j = F, IF$$

s.t. $\quad \theta f\left(s_j(i), l_j(i)\right) \geq 1, \quad a_j(i) \geq 0, \quad s_j(i) \geq 0, \quad l_j(i) \geq 0,$

where C_j denotes total costs of production of one unit of intermediate output at stage i.

[9] In reality, firms self-report emissions and are subject to random inspections that are more frequent in the formal sector. Since the main focus of our analysis is the pollution load shifting, we abstract from a detailed modeling of the exact tax levying procedure and simplify the model to keep an effective tax payment that is smaller in the informal sector.

For any production stage, the optimal values of input use and abatement in the formal sector unit can be determined by solving the above problem.[10] A production unit in sector *j* chooses an abatement level according to the following conditions:

$$\frac{\partial L}{\partial a_j(i)} = \left[c(a_j, i) + \frac{\partial c(a_j, i)}{\partial a_j} a_j - \alpha_j T \right] s_j(i) \geq 0 \,,\; a_j(i) \geq 0 \,,\; a_j(i) \frac{\partial L}{\partial a_j(i)} = 0 \,,\; j = F, IF \,. \tag{1}$$

If the marginal abatement cost exceeds the tax rate, then abatement is zero. For an interior solution, abatement is undertaken up to the point where the marginal abatement cost equals the Pigouvian tax.

The second-order condition for a cost-minimizing abatement level holds from the properties of the abatement cost function:

$$2 \frac{\partial c(a_j, i)}{\partial a_j} + a_j \frac{\partial^2 c(a_j, i)}{\partial {a_j}^2} \geq 0 \,. \tag{2}$$

For an interior solution, the difference in behaviour in the two sectors comes down to the level of the effective tax rate: $\alpha T < T$. A comparison of the first-order conditions for optimal abatement choice thus yields that $a_{IF}<a_F$, given the effective tax rates and the convexity of the abatement cost function. The endogenous switch stage where production switches from the formal to the informal sector implies equalized unit production costs at the optimal switch stage:

$$C_F(l_F^*, s_F^*, a_F^*, i^*) = C_{IF}(l_{IF}^*, s_{IF}^*, a_{IF}^*, i^*) \,, \tag{3}$$

[10] The complete first order conditions are put in an appendix.

where the asterisk indicates the optimally chosen level of the variable resulting from the sector unit level optimization.

For a unique switch stage, the rate of decrease in total unit costs for production in a formal sector unit has to exceed the decrease in total unit costs for production in an informal sector unit:

$$\frac{d}{di} C_F(l_F^*, s_F^*, a_F^*, i^*) < \frac{d}{di} C_{IF}(l_{IF}^*, s_{IF}^*, a_{IF}^*, i^*) . \tag{4}$$

4.1 Government tax revenue

Whereas the emission tax rate may be chosen by government, the detection probability is assumed fixed for some exogenous reason, such as a fixed budget of a separate enforcement agency. Given an emission tax T and a detection probability α in the informal sector, government's total tax revenues per unit of output are

$$TR = \int_0^{i^*} \alpha T (1 - a_{IF}^*) s_{IF}^*(i) di + \int_{i^*}^{1} T (1 - a_F^*) s_F^*(i) di .$$

What is the effect of a marginal increase in the stringency of environmental regulation, in this case a marginal increase in the emissions tax rate? It is given by

$$\frac{\partial TR}{\partial T} = \underbrace{\int_0^{i^*} \alpha (1 - a_{IF}^*) s_{IF}^*(i) di + \int_{i^*}^{1} (1 - a_F^*) s_F^*(i) di}_{the\,tax\,base} + \underbrace{\frac{\partial i^*}{\partial T} \left(\alpha T (1 - a_{IF}^*) s_{IF}^*(i^*) - T (1 - a_F^*) s_F^*(i^*) \right)}_{tax\,base\,reduction}$$

The marginal effect of raising the emissions tax can thus be decomposed into an effect on the intensive margin – the marginal effect for a given tax base - and an effect on the extensive margin following a marginal switch of production from the formal to the informal sector. Increasing the emissions tax rate always increases the tax revenue for a given tax base, so the effect on the intensive margin is positive. In order to have a trade-off between the tax increase and the pollution load shifting effect, we will assume that the detection probability in the informal sector (α) lies in the interval where the extensive margin is negative:

Lemma A: If $1 < \theta < \frac{\mu_2}{\mu_1}$, then there exists an $\bar{\alpha}$, such that when $\alpha \in \left[0, \bar{\alpha}\right]$ then

$\alpha(1-a_{IF}^*)s_{IF}^*(i) - (1-a_F^*)s_F^*(i) < 0$

Proof: In Appendix.

If the detection probability in the informal sector is lower than a cutoff level $\bar{\alpha}$, then the effect on the extensive margin reduces government tax revenues. We nevertheless assume that the overall effect on total tax revenue from raising the emission tax is positive.

4.2 Comparative statics

Given that α is in the range $\left[0, \bar{\alpha}\right]$, we obtain the following comparative statics results on the allocation of production between sectors defined in Equation (3):

$$\frac{di^*}{d\alpha} = \frac{T(1-a_{IF}^*)s_{IF}^*(i)}{\Delta} < 0 , \qquad (5)$$

since, from Equation (4) we have

$$\Delta = \left(\frac{\partial C_F}{\partial q} \frac{\partial q}{\partial s_F} \frac{\partial s_F^*}{\partial i} + w \frac{\partial l_F^*}{\partial i} + \frac{\partial c(a_F^*, i)}{\partial i} * a_F^* * s_F^*(i) \right) - \left(\frac{\partial C_{IF}}{\partial q} \frac{\partial q}{\partial s_{IF}} \frac{\partial s_{IF}^*}{\partial i} + w \frac{\partial l_{IF}^*}{\partial i} + \frac{\partial c(a_{IF}^*, i)}{\partial i} * a_{IF}^* * s_{IF}^* \right) < 0 \cdot$$

An increase in the detection probability in the informal sector makes production shift from the informal to the formal sector at an earlier downstream stage,[11] i.e., it expands the share of the formal sector.

$$\frac{di^*}{dp^S} = \frac{s^*_{IF}(i) - s^*_F(i)}{\Delta} < 0 \text{ iff (A) holds.} \qquad (6)$$

An increase in the input price makes production shift from the informal to the formal sector at an earlier downstream stage, i.e., it expands the share of the formal sector[12].

$$\frac{di^*}{dT} = \frac{\alpha(1-a^*_{IF})s^*_{IF}(i) - (1-a^*_F)s^*_F}{\Delta} > 0 \text{ iff (A) holds.} \qquad (7)$$

In the relevant interval of detection probability in the informal sector, an increase in the pollution tax rate makes production shift from the informal to the formal sector at a later downstream stage[13], i.e., it shrinks the share of the formal sector.

We summarize the comparative statics results:

Proposition 1: *For a given output, the relative share of the informal sector increases when there is a) a decrease in the detection probability in the informal sector; b) a decrease in the unit price of the polluting input; or c) an increase in the emissions tax rate.*

---

[11] A lower i^* indicates a larger share of the formal sector, since the production stages between 0 and i^* take place in the informal sector and stages i^* to 1 take place in the formal sector.

[12] The proof of Lemma A shows that the condition $\theta \prec \frac{\mu_2}{\mu_1}$ is sufficient to guarantee $s_{IF} > s_F$. The condition puts an upper bound on the productivity advantage of the formal sector. In the absence of such a boundary condition, with very high θ, the informal sector may cease to exist.

[13] Alternatively, one could model a common emission standard, with a lower probability of detection of non-compliance in the informal sector. The qualitative result on the effect of a sharpening of the emission standard would be the same.

The size of the informal sector (in terms of its share of production stages) thus varies positively with the emission tax rate and negatively with the detection probability in the informal sector and the unit price of the polluting input (since the informal sector uses it more intensively).

5. Alternative Policy Instruments

Based on the stylized facts on pollution from the informal sector, we have argued that pollution emanates from use of a polluting input, more precisely, from the effluent discharge. The pollution problem may thus be solved if there were incentives for economizing on the use of this input. As the emissions tax has been shown to be ineffective, this section is devoted to finding a workable alternative economic instrument. Given the linkage between the two sectors, an input tax or a subsidy for proper disposal of polluting input cannot be designed to target any specific sector. An input tax imposed at an equal rate on both sectors will lead to a fall in pollution per unit of output but will not create any incentive towards abatement. The total cost at any stage i under an input tax scheme is given by:

$$\left(wl_j + (p^s + t^s)s_j + c(a_j, i)a_j s_j\right) \quad j = F, IF \tag{8}$$

An input tax (t^s) per unit of the polluting input will have the same effect as an increase in the price of the polluting input so the effect of the input tax on the switch over stage is given by :

$$\frac{di^*}{dt^s} = \frac{di^*}{dp^s} = \frac{s^*_{IF}(i^*) - s^*_F(i^*)}{\Delta} < 0 \tag{9}$$

Thus the input tax would lead to a shifting of stages in favour of the formal sector as the input is relatively more productive in the formal sector. The shift of stages in favour of the cleaner formal sector will lead to a fall in pollution per unit of output. We summarize the effect of an input tax:

Proposition 2: *A tax on polluting input will lead to a shift of production stages in favour of the formal sector and a consequent decrease in pollution per unit of output and an increase in overall compliance for a given output level.*

However, an input tax would fail to create any additional incentive for pollution abatement. This result is obvious from Equations (1) and (2) as the input price and hence the input tax does not affect the optimal choice of abatement intensity at any given stage.

A subsidy on proper disposal of polluting residue would create appropriate incentives for pollution reduction, as to claim the subsidy one has to give proof of abatement and thus satisfy the pre-condition of perfect compliance ($\alpha = 1$). If a subsidy ($t^a < 0$) is imposed per unit of properly disposed of input then the total cost at stage i is given by:

$$\left(wl_j + p^s s_j + (c(a_j, i) + t^a)a_j s_j\right) \qquad j = F, IF \tag{10}$$

Given the assumption of perfect observability the intensity of abatement a_j (for an interior solution) is given by the following equation:

$$\{c(a_j, i) + a_j \frac{\partial c}{\partial a_j}\}s_j + t^a s_j = 0 \qquad j = F, IF \tag{11}$$

The effect of the subsidy on the switch over stage is given by:

$$\frac{di^*}{dt^a} = \frac{s_{IF}^*(i)a_{IF}^*(i^*) - s_F^*(i^*)a_F^*}{\Delta} < 0 \tag{12}$$

A subsidy ($t^a < 0$) on proper disposal of the input residue encourages the collection of the residual waste in both sectors. Since the remaining residue has to be turned in under an abatement subsidy, perfect observability applies ($\alpha = 1$) and the intensity of abatement will be same for both the sectors. Therefore given that $s_{IF} > s_F$ it is clear that an abatement subsidy will also shift production towards the formal sector. Comparing Equation (11) with Equation (1) it follows that the subsidy on properly disposed of input residue can replicate the result of the emission tax under perfect monitoring. However, the subsidy policy may put excessive pressure on the regulator's budget. The effect of the subsidy can be summarized as follows:

Proposition 3: *A subsidy on proper disposal of residual waste will (a) create incentives for pollution abatement and (b) improve overall compliance at the cost of putting pressure on the regulatory budget.*

In the next Section, we will propose an alternative hybrid policy in the form of a combination of an input tax on the polluting input – a kind of upfront pollution fee – and a subsidy for properly disposed of input residue. Polluting residue in water polluting industries like leather tanning, textiles or jewellery manufacturing could in fact be collected in especially manufactured disposal bags, instead of being let out into the public drains. The bags taken to the especially designated dumping ground[14] will be rewarded a subsidy on a per unit basis. As Blackman (2000) emphasizes, input taxes are difficult to implement when parallel markets exist and firms can buy input using other channels. The idea here is to combine the upfront input tax payment with a subsidy that will reduce the cost to the firms and also provide the correct incentives for reducing polluting input use. To make shirking of the input tax

[14] The option of door-step collection could also be envisaged.

unattractive to the informal sector the hybrid system may be so designed that all firms are required to produce a proof that they actually procured the input from the open market to be entitled to the subsidy. Since the firms will have to turn in the polluting residue to get the subsidy, this type of hybrid policy is suited for water pollution and toxic waste problems and can be effectively used to provide incentives for proper disposal of polluting residue in the industries concerned.

6. The Second-Best Hybrid Policy

Before analysing the proposed hybrid policy, we define the optimal stage for shifting production between the formal and the informal sector. A social planner would minimize[15] total costs per unit of production, including environmental damage costs from pollution. The optimal switch stage is defined by:

$$\begin{aligned} & wl_F^* + p^S s_F^* + a_F^* c(a_F^*, i) s_F^* + \delta(1 - a_F^*) s_F^* = \\ & = wl_{IF}^* + p^S s_{IF}^* + a_{IF}^* c(a_{IF}^*, i) s_{IF}^* + \delta(1 - a_{IF}^*) s_{IF}^* \end{aligned} \quad (13)$$

A comparison with the decentralized solution defined in Equation (3) immediately shows that a Pigouvian tax $T=\delta$ cannot obtain the optimal allocation of production unless $\alpha = 1$ in both sectors, i.e., monitoring is perfect. The input tax is not directly related to the pollution externality and hence cannot be the desired instrument if the aim is to encourage abatement. An abatement subsidy might be efficient, but, given the limited budget of the regulatory authority financing a subsidy scheme is extremely difficult in a developing economy.

[15] Since our aim is to study the allocation of production between the two sectors in a vertical production structure we assume that the social planner's objective is to design policy that would minimize the total per unit social cost of production. Nevertheless, given our assumptions of perfect competition and constant returns to scale the policy conclusions reached by our model should not be different from the case where the social planner maximizes the sum of consumer and producer surplus less the social damage cost of pollution.

A second-best hybrid instrument may be proposed to reduce pollution without placing any pressure on the regulatory budget by satisfying the enforcement constraint (Fullerton and Wolverton, 2005; Sterner and Höglund Isaksson, 2006)[16]. The regulator would choose an input tax t^s per unit of the polluting input and a subsidy t^a per unit returned residue of the polluting input to minimize the total social cost of production. The regulator's problem under the hybrid policy can be expressed as:

$$\underset{a_{IF},a_F,s_{IF},s_F,i^*}{Min} \int_0^{i^*} \left(wl_{IF} + p^s s_{IF} + c(a_{IF},i)a_{IF}s_{IF}\right)di + \int_{i^*}^{1} \left(wl_F + p^s s_F + c(a_F,i)a_F s_F\right)di +$$

$$\delta\int_0^{i^*} \left((1-a_{IF})s_{IF}\right)di + \delta\int_{i^*}^{1} \left((1-a_F)s_F\right)di$$

s.t. $\int_0^{i^*} f(s_{IF},l_{IF})di + \int_{i^*}^{1} \theta f(s_F,l_F)di \geq 1$

$$\int_0^{i^*} (t^s s_{IF} + t^a a_{IF} s_{IF})di + \int_{i^*}^{1} (t^s s_F + t^a a_F s_F)di = 0\,.$$

Given the limited budgets of most environmental protection agencies in developing countries, we impose a revenue neutrality constraint on the scheme, so that it has zero net effect on the government budget (the second constraint above). The solution to the problem yields the second-best values of input use and the switch stage (i^*) between the informal and the formal sectors.

The regulator's condition for the second-best switch stage is given by the following equation

[16] Eskeland and Devarajan (1996) also suggested combinations of instruments, e.g., an environmental tax and a technology standard.

$$\left\{wl_{IF} + p^S s_{IF} + c(a_{IF,}i)a_{IF} s_{IF} + \delta(1-a_{IF})s_{IF} - \lambda(t^s s_{IF} + t^a a_{IF} s_{IF}) - \mu f(s_{IF}, l_{IF})\right\}$$
$$-\left\{wl_F + p^S s_F + c(a_{F,}i)a_F s_F + \delta(1-a_F)s_F - \lambda(t^s s_F + t^a a_F s_F) - \mu\theta f(s_F, l_F)\right\} = 0$$

(14)

The problem for a production unit in sector *j* can be expressed as:

$$\underset{s_j, a_j, l_j}{Min} \quad \left(wl_j + (p^s + t^s)s_j + (c(a_j, i) + t^a)a_j s_j\right) \forall i, \quad j = F, IF$$

s.t. $\theta f(s_F, l_F) \geq 1$ if *j=F*

$f(s_{IF}, l_{IF}) \geq 1$ if *j=IF*

The intensity of abatement chosen by the production unit at any stage i is given by the following Kuhn-Tucker conditions:

$$\frac{\partial\phi}{\partial a_j} = \{c(a_j, i) + t^a + a_j \frac{\partial c}{\partial a_j}\}s_j \leq 0, \quad a_j(i) \geq 0, \quad a_j(i)\frac{\partial\phi}{\partial aj} = 0 \qquad (15)$$

The new decentralized condition for the switch stage is

$$wl_F^* + \left(p^s + t^s\right)s_F^* + \left(c(a_F^*, i) + t^a\right)a_F^* s_F^* = wl_{IF}^* + \left(p^s + t^s\right)s_{IF}^* + \left(c(a_{IF}^*, i) + t^a\right)a_{IF}^* s_{IF}^* \qquad (16)$$

A comparison of the first order conditions with respect to the polluting input and abatement in the regulator's problem with the respective first order conditions of the firm-level problem (in Appendix), yields the optimal tax and subsidy rates:

$$t^s = \frac{\delta}{(1+\lambda)} \text{ and } t^a = -\frac{\delta}{(1+\lambda)} \qquad (17)$$

These tax and subsidy rates also satisfy the switch condition defined in (14). Thus, the regulator can combine an input tax on the polluting input ($t^s = \frac{\delta}{(1+\lambda)}$) with a subsidy for properly disposed of input residue ($t^a = -\frac{\delta}{(1+\lambda)}$) to minimize social cost. These taxes correspond to the social marginal damage from polluting input residue, weighted by the cost of public funds, as in Fullerton and Wolverton (2005).

The effect on the switch over stage of the input tax and the refund subsidy for properly disposed of residue is given by Equations (9) and (12) respectively. On the one hand, the input tax would thus shift production towards the formal sector which is more productive in input use and thus uses less polluting input per unit of output. On the other hand, the subsidy would induce the same level of abatement in both the formal and the informal sector and lead to a shift of stages in favour of the formal sector. In fact the level of abatement induced by the hybrid policy in the formal and informal sector will be equal to the abatement level obtained when an emission tax is imposed (at a rate equal to the subsidy i.e. $t^a = -T$) under conditions of perfect monitoring. This can be easily verified if we put $t^a = -T$ in Equation 15. So, the hybrid policy will unambiguously shift production towards the cleaner formal sector. From the regulator's point of view, compared to the emission tax, the hybrid policy will be easier to implement. Unlike an emission tax, the input tax can be so designed as to discourage shirking of the input tax by the firms and the subsidy can be used to provide incentives to both the formal and informal sectors for proper disposal of the polluting residues. The effect of the hybrid policy is summarized in Proposition 4.

Proposition 4: *A hybrid policy with a tax on polluting input coupled with a subsidy on proper disposal of residual waste will reduce total pollution more effectively than an emission tax, and it will create conditions for better compliance.*

7. Conclusions

We have studied a model of vertical production where each stage differs in pollution intensity and the shift of production from the formal to the informal sector is determined endogenously. The model incorporated the stylized facts of informal sector pollution in developing countries: a lower rate of detection in the informal sector compared to the formal sector, and a productivity advantage of the formal sector over the informal sector. We showed that the standard Pigouvian tax in this case leads to unintended effects, as it shifts production from the formal sector to the informal one, and, therefore, increases pollution per unit of output. Alternative pollution regulation may consist of input taxation, which limits the use of the polluting input, combined with a subsidy on properly disposed of residual waste that will work as an incentive to attain more abatement in the informal sector.

The analysis centered on the endogenous shift between production in the formal versus the informal sector and thus used a model of cost minimization for producing each unit of output. We can draw some tentative conclusions regarding the effects on total pollution, though. The effect of an emissions tax is ambiguous. On the one hand, an emissions tax increases per unit costs of production and given our assumption of perfect competition this leads to a fall in the level of final output (output effect) which tends to decrease total pollution. On the other hand, the increase in pollution intensity of output (intensity effect) due to the shift in production stages towards the informal sector tends to increase total pollution. The end result on total pollution depends on the relative strength of the output effect vis-à-vis the intensity effect, making the net effect ambiguous. By comparison, in case of the hybrid policy, both output and intensity effects will tend to reduce total pollution. Moreover, the increase in the abatement intensity in the informal sector induced by the hybrid scheme will be higher than that in case of an emissions tax with imperfect monitoring. Thus, under the hybrid policy the effects would reinforce each other and total pollution would fall unambiguously.

The advantage of the hybrid scheme is that it solves the problem of non observability of emissions in case of the units participating in the scheme. However, there is no built-in mechanism in the hybrid scheme to ensure that all the informal units would choose to participate in the scheme. The probability of detection (α) is expected to vary even across the informal units, with units closer to the formal sector facing a higher probability of paying an emissions tax. If the abatement subsidy covers only part of the abatement cost then the informal units that face a lower rate of detection might find it profitable to refrain from participating in the hybrid scheme. Thus there would be a cut off level of α such that the firms facing a probability of detection above this cut off level would participate in the hybrid scheme while those facing a probability of detection that is lower than this cut off level would only pay the input tax. So, allowing α to vary across the informal sector firms could provide an interesting extension of the present model.

A drawback of the proposed hybrid scheme is that it will lead to a shift of stages in favour of the formal sector and hence will lead to closure of firms and loss of jobs in the informal sector.[17] So, in a developing economy, where the informal sector accounts for a huge share of

[17] Note that under the hybrid policy, both sectors receive the same per unit subsidy and choose equal abatement at any given stage. So, under the hybrid scheme the informal sector loses the comparative advantage it had under the emission tax where it paid a lower effective price for the polluting input compared to the formal sector. In our framework, lower environmental compliance cost was the only source of comparative advantage for the informal sector. So, in the model, under the hybrid policy the informal sector will lose all its advantages and

total employment (almost 92% in India including employment in agriculture) the regulator might find it difficult to obtain political or social consensus in favour of the hybrid scheme. Ideally, in a developing country, environmental policy should be designed to encourage cleaner growth of the informal sector by providing proper abatement incentives. Formal regulation lacks sufficient data to locate and identify infomal producers. Hence, the situation is similar to that of unknown polluters with moral hazard. So, incentive-compatible instruments constitute the only possible formal regulatory policy to encourage self revelation of polluters. This could be done more effectively if the adoption of better abatement practice enhances the efficiency of input use for the production units in both formal and informal sectors. In the present model abatement does not affect the productive efficiency of the firms. But, empirical studies have confirmed that increased abatement might sometimes be correlated with increased recycling and hence improved efficiency in input use (see Chakrabarti and Mitra, 2005). So, a possible extension would be to introduce a positive effect of abatement on the productivity of the firms due to increased efficiency in input use and analyse the welfare effects of alternative environmental policies. Another extension would be to examine the welfare effects of alternative forms of regulation that encourage adoption of cleaner production practices in the informal sector without compromising employment. If the wage advantage of the informal sector is explicitly incorporated in our model then the incentive for formalization induced by the disposal subsidy would coexist with the comparative advantage related to lower wages in the informal operation, and these effects together may lead to better pollution abatement without adversely affecting employment in the informal units.

---

cease to exist. In reality however, the informal sector has a lower wage than the formal sector and so the hybrid policy will lead to a shrinkage but not the elimination of the informal sector.

Acknowledgements

The first author is grateful to the students and the teachers of the 2008 PhD course in Environmental Policy Instruments held at Gothenburg University, Sweden, for providing useful suggestions when the idea of this model was presented by the first author as a participant of the course. The insightful comments received from Thomas Sterner, Sandra Rousseau, Malabika Roy and the participants of the 17th Annual Conference of the European Association of Environmental and Resource Economists helped in improving the contents as well as presentation. Of course, the usual disclaimer applies.

References

Azuma, Y., Grossman H., 2008. A Theory of the Informal Sector. Economics and Politics 20(1), 62-79.

Baksi, S., P. Bose, 2010. Environmental Regulation in the Presence of an Informal Sector. Department of Economics Working Paper 2010-03, The University of Winnipeg.

Basu R., 2007. Jewellers at Toxic Risk. The Telegraph 7th Nov 2007, p. 25.

Blackman, A., 2000. Informal Sector Pollution Control: What Policy Options Do We Have? World Development 28(12), 2067-2082.

Blackman, A., Shih, J.-S., Evans, D., Batz, M., Newbold, S., Cook, J., 2006. The Benefits and Costs of Informal Sector Pollution Control: Mexican Brick Kilns. Environment and Development Economics 11, 603-627.

Chakrabarti, S., Mitra, N., 2005. Economic and Environmental Impacts of Pollution Control Regulation on Small Industries: A Case Study. Ecological Economics 54, 53-66.

Chattopadhyay, S., 2009. Environmental Regulation and Process Sub-contracting: A Study of Gems and Jewellery Manufacturing Units in Kolkata. UGC Sponsored Minor Research Project, Final Report No: FPHW 007/06-07(ERO).

Chaudhuri, S., 2006. Pollution and Welfare in the Presence of Informal Sector: Is There Any Trade-Off? Keio Economic Studies 43(1), 21-42.

Chaudhuri, S., Mukhopadhyay, U., 2006. Pollution and Informal Sector: A Theoretical Analysis. Journal of Econometric Integration 21(2), 363-378.

Chaudhuri, T.M., 1989. A Theoretical Analysis of the Informal Sector. World Development 17(3), 351-355.

Damodaran, S., 2001. Informal Sector in Urban India: Case of Leather and Leather Products Industry, in: Oberai, A.S., Chadha, G.K. (Eds.), Job Creation in Urban Informal Sector in India. South Asia Multidisciplinary Advisory Team (SAAT), International Labour Organization, New Delhi, India.

Dewan, R., 2001. Unorganized Food Processing Industry in Mumbai: A Study of Bakery, Savouries and Fish Processing, in: Oberai A.S., Chadha, G.K. (Eds.), Job Creation in Urban Informal Sector in India. South Asia Multidisciplinary Advisory Team (SAAT), International Labour Organization, New Delhi, India.

Dixit, A.K., Grossman, G.M., 1982. Trade and Protection with Multistage Production. The Review of Economic Studies 49(4), 583-594.

Eskeland, G.S. and S. Devarajan, 1996. Taxing Bads by Taxing Goods: Pollution Control with Presumptive Charges. Directions in Development. The World Bank, Washington, D.C.

Fullerton, D., Wolverton, A., 2005. The Two Part Instrument in a Second Best World. Journal of Public Economics 89, 1961-1975.

Gupta, K., 2006. The Urban Informal Sector and Environmental Pollution: A Theoretical Analysis. CEDE Working Paper**:** 02-006, York University.

KMDA, 2003. Final Report on Identifying & Mapping of Industries of KMA Volume -1, February 2003; (Project sponsored by West Bengal Pollution Control Board under Industrial Pollution Control Project (1D/P/105) of Japan Bank for International Co-operation and executed by Kolkata Metropolitan Development Authority).

Karthikeyan, N., Alexander, J., 2008. Waste Minimisation in Textile Industry. Indian Textile Journal, September.

Khanna, M., Zilberman, D, 1997. Incentives, Precision Technology and Environmental Protection. Ecological Economics 23, 25-43.

Knutsen, H.M., 2000. Environmental Practice in The Commodity Chain: The Dyestuff and Tanning Industry Compared. Review of International Political Economy 7(2), 254-288.

Kolli, R., Hazra, S., 2005. Estimation of Informal Sector Contribution in the Net Domestic Product - Indian Experience. National Accounts Division, Central Statistical Organisation, India, March 2005.

Mazumdar, D., 1976. The Urban Informal Sector. World Development 4(8), 655-679.

Oberai, A.S., Chadha, G.K., 2001. An Overview of Urban Informal Sector in India, in: Oberai, A.S., Chadha, G.K. (Eds.), Job Creation in Urban Informal Sector in India. South Asia Multidisciplinary Advisory Team (SAAT), International Labour Organization, New Delhi, India.

Rauch, J., 1991. Modeling the Informal Sector Formally. Journal of Development Economics 35, January, 33-47.

Sahasranaman, A., 1998. A Survey of Indian Leather Sector (mimeo).

Sanyal, K., 1983. Vertical Specialization in a Ricardian Model with a Continuum of Stages of Production. Economica 50(197), 71-78.

Sarkar, A., Roy, M., 1992. Industrial Dualism, Vertical Specialization and Credit Policy. Journal of Quantitative Economics 8(2), 217-229.

Sterner T., Höglund Isaksson, L., 2006. Refunded Emission Payments Theory, Distribution of Costs and Swedish Experience of NO_x Abatement. Ecological Economics 57(1), 93-106.

APPENDIX

Appendix A.1. Pigouvian Tax Case

A.1.1 The problem of a formal production unit under a Pigouvian tax (page 14)

$$\underset{s_F, a_F, l_F}{Min}\ C_F = wl_F(i) + p^S s_F(i) + c(a_F, i)a_F s(i) + T(1-a_F)s_F(i) \quad \forall i$$

s.t. $\theta f(s_F(i), l_F(i)) \geq 1, \quad a_F(i) \geq 0, \quad s_F(i) \geq 0, \quad l_F(i) \geq 0,$

where C_F denotes total costs of production of one unit of intermediate output at stage *i*.

The Lagrangian can be expressed as

$$L = wl_F(i) + p^S s_F(i) + c(a_F, i)a_F s(i) + T(1-a_F)s_F(i) + \mu_1\left[1 - \theta f\left(s_F(i), l_F(i)\right)\right]$$

with μ_1 ($\mu_1 = \frac{\partial C_F}{\partial q}$) the shadow value of the production function constraint.

The Kuhn-Tucker conditions for polluting input use in the formal sector are:

$$\frac{\partial L}{\partial s_F(i)} = p^S + c(a_F, i)a_F + T(1-a_F) - \mu_1\theta\frac{\partial f}{\partial s_F(i)} \geq 0,$$

$$s_F(i) \geq 0, \quad s_F(i)\frac{\partial L}{\partial s_F(i)} = 0, \qquad \text{(A.1)}$$

$$\frac{\partial L}{\partial \mu_1} \leq 0, \quad \mu_1\frac{\partial L}{\partial \mu_1} = 0.$$

For $s_F(i) > 0$, the marginal imputed cost of using the polluting input ($\mu_1\theta\frac{\partial f}{\partial s_F(i)}$) equals the unit input price and the marginal payment for the pollution created through its use.

Labour use is determined by

$$\frac{\partial L}{\partial l_F(i)} = w - \mu_1 \theta \frac{\partial f}{\partial l_F(i)} \geq 0, \quad l_F(i) \geq 0, \quad l_F(i)\frac{\partial L}{\partial l_F(i)} = 0 \quad . \tag{A.2}$$

Equation (A2) states that in equilibrium the marginal imputed cost of labour equals the unit wage for $l_F(i) > 0$.

A production unit in the formal sector will choose an abatement level according to the following condition:

$$\frac{\partial L}{\partial a_F(i)} = \left[c(a_F, i) + \frac{\partial c(a_F, i)}{\partial a_F} a_F - T \right] s_F(i) \geq 0, \quad a_F(i) \geq 0, \quad a_F(i)\frac{\partial L}{\partial a_F(i)} = 0\,. \tag{A.3}$$

A.1.2 The problem of an informal production unit under a Pigouvian tax (page 14)

The equivalent problem for an informal sector unit is stated as

$$\underset{s_{IF}, a_{IF}, l_{IF}}{Min} \; C_{IF} = wl_{IF}(i) + p^s s_{IF}(i) + c(a_{IF}, i) a_{IF} s_{IF}(i) + \alpha T(1 - a_{IF}) s_{IF}(i) \quad \forall i$$

s.t. $\quad f\left(s_{IF}(i), l_{IF}(i)\right) \geq 1, \quad a_{IF}(i) \geq 0, \quad s_{IF}(i) \geq 0, \quad l_{IF}(i) \geq 0$

An informal production unit chooses its input use and abatement level according to the first-order conditions (with $\mu_2 = \frac{\partial C_{IF}}{\partial q}$ the shadow value of the production function constraint):

$$\frac{\partial L}{\partial s_{IF}(i)} = p^s + c(a_{IF}, i) a_{IF} + \alpha T(1 - a_{IF}) - \mu_2 \frac{\partial f}{\partial s_{IF}(i)} \geq 0$$

$$s_{IF}(i) \geq 0, \quad s_{IF}(i)\frac{\partial L}{\partial s_{IF}(i)} = 0 \tag{A.4}$$

$$\frac{\partial L}{\partial \mu_2} \leq 0, \quad \mu_2 \frac{\partial L}{\partial \mu_2} = 0$$

$$\frac{\partial L}{\partial l_{IF}(i)} = w - \mu_2 \frac{\partial f}{\partial l_{IF}(i)} \geq 0, \quad l_{IF}(i) \geq 0, \quad l_{IF}(i)\frac{\partial L}{\partial l_{IF}(i)} = 0. \tag{A.5}$$

$$\frac{\partial L}{\partial a_{IF}(i)} = \left[c(a_{IF}, i) + \frac{\partial c(a_{IF}, i)}{\partial a_{IF}} a_{IF} - \alpha T \right] s_{IF}(i) \geq 0,$$

$$a_{IF}(i) \geq 0, \quad a_{IF}(i) \frac{\partial L}{\partial a_{IF}(i)} = 0 \qquad \text{(A.6)}$$

For any production stage, the optimal values of input use and abatement are determined by Equations (A1)–(A3) for a unit in the formal sector and by Equations (A4)-(A6) for a unit in the informal sector.

Appendix A.2. Proof of Lemma A

$$\alpha(1 - a_{IF}^{*}) s_{IF}^{*}(i) - (1 - a_{F}^{*}) s_{F}^{*}(i) < 0 \qquad \text{(A.7)}$$

Recall that $0 \leq \alpha \leq 1$. For $\alpha = 0$, the inequality holds trivially. When $\alpha \to 1$, $a_{IF} \to a_F$, and a comparison of the first-order conditions for optimal input use (Equations (A1) and (A4)) yields that $s_{IF} > s_F$ if $\theta < \frac{\mu_2}{\mu_1}$. Thus, when $\alpha \to 1$, the inequality in (A7) is positive. Then, by the intermediate value theorem, there exists an $\alpha = \overline{\alpha}$, such that for values of $\alpha \in \left[0, \overline{\alpha}\right]$ inequality (A7) holds. Q.E.D.

Appendix A.3. Hybrid Policy

A.3.1 The problem of the regulator under the hybrid system (page 22)

The Lagrangian to the regulator's problem can be expressed as:

$$\phi =$$

$$\int_0^{i^*} \left(wl_{IF} + p^s s_{IF} + c(a_{IF}, i) a_{IF} s_{IF} \right) di + \int_{i^*}^{1} \left(wl_F + p^s s_F + c(a_F, i) a_F s_F \right) di + \delta \int_0^{i^*} \left((1 - a_{IF}) s_{IF} \right) di +$$

$$\delta \int_{i^*}^{1} ((1 - a_F) s_F) di + \mu[1 - \{ \int_0^{i^*} f(s_{IF}, l_{IF}) di + \int_{i^*}^{1} \theta f(s_F, l_F) di \}] - \lambda \{ \int_0^{i^*} (t^S s_{IF} + t^a a_{IF} s_{IF}) di + \int_{i^*}^{1} (t^s s_F + t^a a_F s_F) di \}$$

where λ is the social marginal utility of income in the public sector. The Kuhn-Tucker conditions w.r.t. l_{IF}, l_F, s_{IF}, s_F, a_{IF}, a_F , μ, λ and i^* are:

$$\frac{\partial \phi}{\partial l_{IF}} = w - \mu \frac{\partial f}{\partial l_{IF}} \geq 0$$

$$l_{IF}(i) \geq 0, \quad l_{IF}(i)\frac{\partial \phi}{\partial l_{IF}} = 0 \tag{A.8}$$

$$\frac{\partial \phi}{\partial s_{IF}} = p^s + c(a_{IF}, i)a_{IF} + \delta(1 - a_{IF}) - \lambda(t^S + t^a a_F) - \mu \frac{\partial f}{\partial s_{IF}} \geq 0$$

$$s_{IF}(i) \geq 0, \quad s_{IF}(i)\frac{\partial \phi}{\partial s_{IF}} = 0 \tag{A.9}$$

$$\frac{\partial \phi}{\partial a_{IF}} = \{c(a_{IF}, i) + a_{IF}\frac{\partial c}{\partial a_{IF}}\}s_{IF} - \delta s_{IF} - \lambda t^a s_{IF} \geq 0$$

$$a_{IF}(i) \geq 0, \quad a_{IF}(i)\frac{\partial \phi}{\partial a_{IF}} = 0 \tag{A.10}$$

$$\frac{\partial \phi}{\partial l_F} = w - \mu\theta\frac{\partial f}{\partial l_F} \geq 0 \tag{A.11}$$

$$l_F(i) \geq 0, \quad l_F(i)\frac{\partial \phi}{\partial l_F} = 0$$

$$\frac{\partial \phi}{\partial s_F} = p^s + c(a_F, i)a_F + \delta(1 - a_F) - \lambda(t^S + t^a a_F) - \mu\theta\frac{\partial f}{\partial s_F} \geq 0$$

$$s_F(i) \geq 0, \quad s_F(i)\frac{\partial \phi}{\partial s_F} = 0 \tag{A.12}$$

$$\frac{\partial \phi}{\partial a_F} = \{c(a_F, i) + a_F\frac{\partial c}{\partial a_F}\}s_F - \delta s_F - \lambda t^a s_F \geq 0$$

$$a_F(i) \geq 0, \quad a_F(i)\frac{\partial \phi}{\partial a_F} = 0 \tag{A.13}$$

$$\frac{\partial \phi}{\partial \mu} = [1 - \{\int_0^{i^*} f(s_{IF}, l_{IF})di + \int_{i^*}^1 \theta f(s_F, l_F)di\}] \leq 0,$$

$$\mu\frac{\partial \phi}{\partial \mu} = 0 \tag{A.14}$$

$$\frac{\partial\phi}{\partial\lambda} = \int_0^{i^*} (t^s s_{IF} + t^a a_{IF} s_{IF})di + \int_{i^*}^{1} (t^s s_F + t^a a_F s_F)di \leq 0,$$
$$\lambda \frac{\partial\phi}{\partial\lambda} = 0 \qquad \text{(A.15)}$$

$$\frac{\partial\phi}{\partial i^*} = \{wl_{IF} + p^S s_{IF} + c(a_{IF,}i)a_{IF} s_{IF} + \delta(1-a_{IF})s_{IF} \} - \lambda(t^s s_{IF} + t^a a_{IF} s_{IF}) - \mu f(s_{IF}, l_{IF})$$
$$-\{wl_F + p^S s_F + c(a_{F,}i)a_F s_F + \delta(1-a_F)s_F - \lambda(t^s s_F + t^a a_F s_F) - \mu\theta f(s_F, l_F) \} = 0$$
$$\text{(A.16)}$$

A.3.2. The problem of a formal production unit under the hybrid policy (page 23)

The problem of a production unit in the formal sector under the hybrid system can be expressed as

$$\underset{s_F, a_F, l_F}{Min} \quad \left(wl_F + (p^s + t^s)s_F + (c(a_F, i) + t^a)a_F s_F\right) \forall i$$

s.t. $\theta f(s_F, l_F) \geq 1$

The first order conditions for a unit operating in the formal sector are:

$$\frac{\partial\phi}{\partial l_F} = w - \mu \frac{\partial f}{\partial l_F} \leq 0,$$
$$l_F(i) \geq 0, \quad l_F(i)\frac{\partial\phi}{\partial l_F} = 0 \qquad \text{(A.17)}$$

$$\frac{\partial\phi}{\partial s_F} = p^s + c(a_F, i)a_F + (t^S + t^a a_F) - \mu\theta\frac{\partial f}{\partial s_F} \leq 0,$$
$$s_F(i) \geq 0, \quad s_F(i)\frac{\partial\phi}{\partial s_F} = 0 \qquad \text{(A.18)}$$

$$\frac{\partial\phi}{\partial a_F} = \{c(a_F, i) + t^a + a_F \frac{\partial c}{\partial a_F}\}s_F \leq 0, \ a_F(i) \geq 0, \quad a_F(i)\frac{\partial\phi}{\partial a_F} = 0 \qquad \text{(A.19)}$$

$$\frac{\partial \phi}{\partial \mu} = [1-\{\ \theta f(s_F, l_F)\}] \leq 0\,,\quad \mu \frac{\partial \phi}{\partial \mu} = 0 \qquad \text{(A.20)}$$

A.3.3 The problem of an informal sector unit under the hybrid policy (page 23)

In the informal sector, the production unit's problem under the hybrid policy can be expressed as:

$$\underset{s_{IF}, a_{IF}, l_{IF}}{Min} \quad \left(wl_{IF} + (p^s + t^s)s_{IF} + (c(a_{IF}, i) + t^a)a_{IF}s_{IF}\right) \forall i$$

s.t. $\quad f(s_{IF}, l_{IF}) \geq 1$

The Lagrangian for the production unit's problem is:

$$\phi = wl_{IF} + (p^s + t^s)s_{IF} + (c(a_{IF}, i) + t^a)a_{IF}s_{IF} + \mu[1 - f(s_{IF}, l_{IF})]$$

The first order conditions to the informal unit's problem are:

$$\frac{\partial \phi}{\partial l_{IF}} = w - \mu \frac{\partial f}{\partial l_{IF}} \leq 0\,, \qquad l_{IF}(i) \geq 0, \quad l_{IF}(i)\frac{\partial \phi}{\partial l_{IF}} = 0 \qquad \text{(A.21)}$$

$$\frac{\partial \phi}{\partial s_{IF}} = p^s + c(a_{IF}, i)a_{IF} + (t^S + t^a a_{IF}) - \mu \frac{\partial f}{\partial s_{IF}} \leq 0\,,\quad s_{IF}(i) \geq 0, s_{IF}(i)\frac{\partial \phi}{\partial s_{IF}} = 0 \qquad \text{(A.22)}$$

$$\frac{\partial \phi}{\partial a_{IF}} = \{c(a_{IF}, i) + t^a + a_{IF}\frac{\partial c}{\partial a_{IF}}\}s_{IF} \leq 0\,, \qquad a_{IF}(i) \geq 0, a_{IF}(i)\frac{\partial \phi}{\partial a_{IF}} = 0 \qquad \text{(A.23)}$$

$$\frac{\partial \phi}{\partial \mu} = [1 - f(s_{IF}, l_{IF})], \qquad \mu \frac{\partial \phi}{\partial \mu} = 0 \qquad \text{(A.24)}$$