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Abstract

This study introduces GARCH models with cross-sectional market volatility, which we call GARCHX model. The cross-sectional market volatility is equlvalent to common heteroskedasticity in asset specific returns, which was suggested by Connor and Linton (2001) as an important component in individual asset volatility. Using UK and US data, we find that daily return volatility can be better specified with GARCHX models, but GARCHX models do not necessarily perform better than conventional GARCH models in forecasting.

Keyword Cross-sectional Volatility, GARCHX, Forecasting Volatility

JEL Code C21, C22

1 Introduction

Volatility has long been used as a risk measure. Traditionally, volatility represented by variance (or standard deviation) is decomposed into diversifiable and non-diversifiable components. This measure of volatility, however, is unconditional and does not recognize that there are interesting patterns in asset volatility; e.g., time-varying and clustering properties. Over the last two decades financial econometricians have introduced various models to explain and predict these patterns in volatility. One of the important volatility models is the autoregressive conditional heteroskedasticity (ARCH) family of models introduced by Engle (1982). Another important model is the stochastic volatility (SV) model introduced by Taylor (1986) and Hull and White (1987) among others.

In this study, we introduce a GARCH model that includes market volatility as an additional explanatory variable. The motivation of using the market volatility is to include cross-sectional relationship between asset returns and market returns in linear factor models. With the proposed model, we are able to investigate if there is a market wide common component in volatility and find if the common component is useful for explaining and forecasting individual stock volatility.

Since Fama and French (1993) introduced some accounting factors in linear regression framework, linear factor models have been increasingly popular. When we accept linear factor models as our return process, volatility of a stock is cross-sectionally decomposed into multiple components of the factors. A recent study on this area by Campbell, Lettau, Malkiel, and Xu (2001) decomposed the total volatility of a stock into three components, market volatility, industry volatility, and firm specific (idiosyncratic) volatility, and then showed that the market volatility is an important component of the stock volatility and tends to lead the idiosyncratic volatility.

One major problem in including market volatility into the GARCH framework is the appropriate choice of market volatility. The squared values of market returns or factor values are highly noisy and may not work in conditional volatility models; see Andersen and Bollerslev (1997) and Hwang (2001b) for example. An increasingly popular method to obtain less noisy and parameter free volatility is to use high frequency data; for example, daily volatility can be obtained by measuring summations of intra-day squared returns. For studies using this method, see Andersen and Bollerslev (1997), Andersen, Bollerslev, Diebold, and Labys (1999) and Campbell, Lettau, Malkiel, and Xu (2001) among others.

In this study we use cross-sectional market volatility. The cross-sectional market volatility is a measure of disperse of individual asset returns with respect to the market return for a given time. Hwang (2001a), using linear factor models, showed that there is an analytical relationship between cross-sectional market volatility and time-series market volatility. He compared the properties of the cross-sectional market volatility with those of time-series market volatility such as squared market returns and conditional market volatility in the UK and US markets. The empirical results showed that cross-sectional market volatility is not only highly correlated with time-series market volatility but also more informative than squared market returns, suggesting that cross-sectional market volatility can be useful for the explanation and forecasting of time-series market volatility.

The cross-sectional market volatility can be considered common volatility in market. Recently, Connor and Linton (2001), using a large number of monthly UK equity returns, showed that the dynamic heteroskedasticity can be decomposed into three components; common factor-related heteroskedasticity, common heteroskedasticity in asset-specific returns, and purely asset specific heteroskedasticity. They found evidence of common heteroskedasticity in asset-specific returns. The cross-sectional market volatility in our study is equivalent to the common heteroskedasticity in

asset-specific returns which is a cross-sectional average of individual volatility. In addition, since the cross-sectional and time-series market volatility is related both analytically and empirically (Hwang, 2001a), the common factor-related (market factor related) heteroskedasticity may also be explained with the cross-sectional market volatility.

We call our model GARCHX models since the constant in GARCH models is replaced by an extra term, i.e., the lagged cross-sectional market volatility, and thus the GARCHX model does not need additional parameters. Note that the cross-sectional market volatility is lagged to make the GARCHX mode conditional. The GARCHX model is simple, but includes information on some important factors, especially the market factor, via the cross-sectional volatility. Our model is a special case of the multivariate Factor-GARCH model of Engle, Ng, and Rothchild (1990) in the sense that only one factor, i.e., the market factor is included. Note that the main problem in the multivariate GARCH models is that the number of parameters to be estimated grows very fast and we need to impose some restrictions to make the conditional covariance matrix positive definite. Several methods have been suggested to solve these problems; see chapter 12, Campbell, Lo, and MacKinlay (1997).

GARCHX models are also a generalised version of models by Braun, Nelson, and Sunier (1995) and Glosten, Jagannathan, and Runkle (1993), but different from Braun, Nelson, and Sunier (1995) in the sense that cross-sectional volatility is used. A different version of GARCHX models was introduced by Apergis (1998) to investigate how short-run deviations from the relationship between stock prices and certain macroeconomic fundamentals affect stock market volatility. In the Apergis model, the squared past error-correction term which represents the short run deviations is added to the GARCH conditional volatility. GARCHX models may be considered a simplified version of Connor and Linton (2001). Without using complicated econometric models, we examine if the inclusion of the common heteroskedasticity

in factors and asset specific returns can improve modelling volatility and also if it can be used for forecasting volatilities.

Using 10 years UK and US daily data, we show that the return volatility (i.e., squared returns) of an individual stock can be better specified with GARCHX models than with GARCH models. The maximum likelihood values, significance of the parameter on the cross-sectional market volatility, and some other statistics show that GARCHX models perform better than GARCH models in general. We also use GARCHX models for forecasting return volatility. As expected, GARCHX models show little improvement in forecasting performance for return volatility which is highly noisy in the GARCH framework. See Andersen and Bollerslev (1997) for the detailed explanation on the difficulties of forecasting return volatility with GARCH models.

This study shows that common cross-sectional heteroskedasticity in stock returns is an important component in conditional volatility models. We find evidence that more than three-quarters of individual stocks included in the FTSE350 and S&P500 show a significant coefficient on the lagged cross-sectional market volatility. If we allowed current cross-sectional market volatility, we may get stronger evidence of common volatility for the explanation of individual stock volatility. This finding is consistent with Jones (2001) and Connor and Linton (2001) who suggested common volatility as an important source of individual stock volatility. We also show how much of an individual stock volatility can be explained by common volatility. We find that on average 12% to 16% of individual volatility is explained with the market common volatility. This is what Campbell, Lettau, Malkiel, and Xu (2001) found during a similar sample period with monthly data.

Cross-sectional volatility can also be used in stochastic volatility (SV) models; SVX models. We expect such models work as well as in the GARCHX context. In this study we consider the market factor only. However, other factors may also be

included. We leave these for future research.

In the next section, we explain why cross-sectional market volatility may be included for conditional volatility models and introduce GARCHX models. In section 3, using UK and US data, we show that squared return can be better specified with GARCHX models than with GARCH models and how GARCHX models can be used for volatility forecasting. Conclusions are in section 4.

2 GARCH Models with Cross-sectional market Volatility

The introduction of GARCHX models needs explanation of cross-sectional expectation for cross-sectional statistics such as mean and variance.

2.1 Time-series and Cross-sectional Expectation

In this section, we define a notion of expections essentially equivalent to weighted moments. For any variable x_{it}, where $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, the time-series expectation is defined as

$$E^T(x_{it}) = \frac{1}{T}\sum_{t=1}^{T} x_{it} \text{ for } i = 1, 2, ..., N.$$

On the other hand, the cross-sectional expectation for the variable x_{it} is defined to be

$$E^C(x_{it}) = \sum_{i=1}^{N} w_{it}x_{it}, \text{ for } t = 1, 2, ..., T, \tag{1}$$

where w_{it} is an appropriate cross-sectional weight on asset i at time t. This weight may be a probability measure if $w_{it} \geq 0$ for all i and t, and $\sum_{i=1}^{N} w_{it} = 1$ for all t. One simple example is $w_{it} = \frac{1}{N}$ for all i so that $E^C(x_{it}) = \frac{1}{N}\sum_{i=1}^{N} x_{it}$ which is an equally weighted cross-sectional average at time t, corresponding to a discrete version of the uniform distribution.

Cross-sectional expectations can therefore be thought of as taking expectations with respect to a measure where the weights are determined by the market. Note that we have the time-series expectation for each asset so that we have N expectations, whilst we have T cross-sectional expectations. Using the definition of expectations above, we can calculate the mean, variance, skewness, and kurtosis in both the time-series and cross-sectional world.

A simple, but widely used method to obtain time-series volatility is to apply a return process to calculate errors and then square them. For example, the AR(1) process may be used as the return process to calculate errors. That is, for returns, r_{it},

$$r_{it} = a_i + b_i r_{it-1} + \varepsilon_{it}, \tag{2}$$

where a and b are parameters, volatility is calculated by estimating ε_{it}^2. Skewness and kurtosis can also be obtained by estimating ε_{it}^3 and ε_{it}^4, respectively.

On the other hand, cross-sectional market volatility needs many asset returns in the market. Suppose that there are N assets in the market. Then, the cross-sectional market return at time t is

$$r_{mt} = \sum_{i=1}^{N} w_{it} r_{it}, \tag{3}$$

which is equivalent to the (equally or value weighted) market return at time t. Thus, one definition of cross-sectional market volatility, $\sigma_{C,mt}^2$, is

$$\sigma_{C,mt}^2 = \sum_{i=1}^{N} w_{it}(r_{it} - r_{mt})^2. \tag{4}$$

Therefore, the cross-sectional volatility represents volatility calculated across a universe of stocks. When residuals obtained with a linear factor model are used, the calculated cross-sectional market volatility is equivalent to the common asset specific volatility of Connor and Linton (2001).

Cross-sectional skewness and kurtosis can also be calculated accordingly. These statistics can be calculated for an industry level, or a specific set of stocks such as

a portfolio. In the special case that $w_{it} = \frac{1}{N}$, equation (4) gives the practitioners notion of cross-sectional volatility of a portfolio. Therefore, an individual asset's cross-sectional volatility does not exist.

2.2 GARCHX Models

The model we propose uses the lagged cross-sectional market volatility as an additional explanatory variable in the conventional GARCH model. This is a special case of the factor-ARCH model by Engle, Ng, and Rothchild (1990). However, Hwang (2001a) argues that the cross-sectional volatility provides more information on the time-varying factors than the time-series volatility. Thus, we propose a simple conditional volatility model including the cross-sectional market volatility. Our model avoids the econometric hurdle of ensuring the positivity of conditional volatility in the factor-ARCH model, while providing better results than the conventional GARCH model.

Let us first consider the following cross-sectional relationship between the returns of asset i and the returns of the market portfolio and K factors;

$$r_{it} = \beta_{0i} + \beta_{mi} r_{mt} + \beta_{1i} f_{1t} +, ..., + \beta_{Ki} f_{Kt} + \zeta_{it} \tag{5}$$

where β_{0i}, β_{mi} and β_{ki}, $k = 1, ..., K$, are parameters, and ζ_{it} is a mean zero error process, $\zeta_{it} \tilde{} N(0, \sigma^2_{i\zeta})$, and r_{mt} and f_{kt} are the market portfolio return and the realised value of factor k at time t. As in conventional linear factor models, we assume that the explanatory variables, r_{mt} and f_{kt} are orthogonal, and r_{it} and r_{mt} represent excess returns calculated with risk free returns. Factors may be macroeconomic, firm specific characteristics, or based on market micro-structure, see Chen, Roll, and Ross (1986), Schwert (1989), Fama and French (1992), Lakonishok, Shleifer and Vishny (1994), Basu (1977) for example.

The model in (5) implies that there are multiple factors; the market portfolio

and K other factors. Therefore, the model is a multi-factor-GARCH model. The expected return conditional on $_{t-1}$ available up to time $t-1$ is

$$E_{t-1}^{T}[r_{it}|\ _{t-1}] = \beta_{0i} + \beta_{mi} E_{t-1}^{T}[r_{mt}|\ _{t-1}] + \beta_{1i} E_{t-1}^{T}[f_{1t}|\ _{t-1}] +, ..., + \beta_{Ki} E_{t-1}^{T}[f_{Kt}|\ _{t-1}]. \tag{6}$$

Using (5) and (6), we have the volatility of the return on asset i at time t[1]

$$\epsilon_{it}^2 = \beta_{mi}^2 \epsilon_{mt}^2 + \beta_{1i}^2 \epsilon_{1t}^2 +, ..., + \beta_{Ki}^2 \epsilon_{Kt}^2 + \zeta_{it}^2$$

where $\epsilon_{it} = r_{it} - E_{t-1}^{T}[r_{it}|\ _{t-1}]$, $\epsilon_{mt} = r_{mt} - E_{t-1}^{T}[r_{mt}|\ _{t-1}]$ and $\epsilon_{kt} = f_{kt} - E_{t-1}^{T}[f_{kt}|\ _{t-1}]$. The above equation shows the cross-sectional relationship between individual asset's time-series volatility and factor volatilities.

Note that there is a significant difference between β_{mi} and β_{ki}. The cross-sectional average of β_{mi} is one, whilst those of β_{ki} are zero; see Hwang and Salmon (2001) for further discussion. This means that β_{mi} is distributed around one, whilst β_{ki} is distributed around zero. In addition, in many cases, the coefficients on factors other than the market factor are not significantly different from zero; see Hall, Hwang, and Satchell (2001) for example. Campbell, Lettau, Malkiel, and Xu (2001) also showed that market volatility and firm asset specific volatility are the important components for the explanation of individual asset volatility.[2] Furthermore, when we take squared values of β_{mi} and β_{ki}, β_{mi}^2 may still be significant in many cases but β_{ki}^2 may not. In addition, the identification of factors other than the market factor is generally difficult and controversial. Thus as long as our universe is one rational market, we may approximate asset i's return volatility with the following

[1] Generally if factors are not orthogonal to each other, then there should be cross-product terms in the equation, resulting in multi-factor volatility models as in Engle, Ng, and Rothchild (1990).

[2] We do not use industry volatility in our study. However, the industry volatility is also an important component in the individual asset volatility as shown in Campbell, Lettau, Malkiel, and Xu (2001), and can be included in our model.

cross-sectional relationship with market volatility;

$$\epsilon_{it}^2 \approx \beta_{mi}^2 \epsilon_{mt}^2 + \zeta_{it}^2. \tag{7}$$

The popular GARCH(1,1) process for asset i is

$$h_{it} = \alpha_{i0} + \alpha_{i1}\epsilon_{it-1}^2 + \alpha_{i2}h_{it-1}, \tag{8}$$

where h_{it} is the conditional volatility of asset i at time t. GARCH models uses past volatility and conditional volatility to explain the current conditional volatility. However, in GARCH models we do not find market volatility which may be an important component in volatility. That is, the important cross-sectional relationship between the market volatility and the individual asset's volatility in (7) is not included in the conventional GARCH model.

One way to include market volatility in the GARCH (1,1) model is

$$h_{it} = \alpha_{i0} + \alpha_{i1}\epsilon_{it-1}^2 + \alpha_{i2}h_{it-1} + \alpha_{i3}\epsilon_{mt-1}^2. \tag{9}$$

This is the same as the GARCH-X model proposed by Apergis (1998). Throughout this study, we call the model in (9) GARCHX-T(1,1) model to differentiate from our GARCHX models. For the GARCHX-T(1,1) model to be stationary, we need

$$\alpha_{i1} + \alpha_{i2} < 1 \tag{10}$$

as in the ordinary GARCH (1,1) model. However, one major problem in the above model is the non-negativity condition. For the conditional volatility process in (9) to be always positive, we need

$$\alpha_{i0} > 0, \alpha_{ij} \geq 0, j = 1, 2, 3. \tag{11}$$

However, during empirical estimation, we face many cases which have $\alpha_{i0} \leq 0$ but still

$$\frac{1}{T}\sum_{t=2}^{T}(\alpha_{i0} + \alpha_{i3}\epsilon_{mt-1}^2) > 0. \tag{12}$$

The condition in (12) is not necessary and sufficient for the positiveness of the conditional volatility h_{it} in (9), but the conditions in (11) seem to be too strong in empirical calculation. We tried to estimate model (9) using conditions (11) and (12) and found many cases where the strict nonnegativity condition of (11) is too strong. In addition, one more parameter needs to be estimated than the GARCH(1,1) model.[3]

An alternative model is

$$h_{it} = \alpha_{i0}\epsilon^2_{mt-1} + \alpha_{i1}\epsilon^2_{it-1} + \alpha_{i2}h_{it-1}. \tag{13}$$

Here, for stationarity and nonnegativity

$$\alpha_{i1} + \alpha_{i2} < 1, \alpha_{i0} > 0, \alpha_{i1} \geq 0, \alpha_{i2} \geq 0, \tag{14}$$

which is exactly the same as the ordinary GARCH model. The unconditional variance of asset i is

$$E^T[h_{it}] = \frac{\alpha_{i0}E^T[\epsilon^2_{mt-1}]}{1 - \alpha_{i1} - \alpha_{i2}}.$$

One problem of the above (9) and (13) models is that ϵ^2_{mt} is highly noisy over time in the GARCH frame work; see Andersen and Bollerslev (1998) for example. Therefore allowing one lag for ϵ^2_{mt} and thus using ϵ^2_{mt-1} instead of ϵ^2_{mt} may not improve the model.

The empirical test results in Hwang (2001a) suggest that the cross-sectional market volatility, $\sigma^2_{C,mt}$, is more informative than the time-series market volatility, ϵ^2_{mt}, and highly persistent. Connor and Linton (2001) and Jones (2001) suggested that there is common heteroskedasticity in asset specific returns, which may be replicated

[3] To aviod the nonnegativity constraints, we may use logarithmic function as in exponential GARCH; see Braun, Nelson, and Sunier (1995) for example. We applied the logarithmic function to the volatility process to make the voalitlity process such as log-ARFIMA model (Hwang and Satchell, 1998), but the model does not seem to work well. In this study we confine our studies on cross-sectional volatility to GARCH models. The EGARCH-cross-sectional volatility process is not too difficult to set up and left for future studies.

by the cross-sectional market volatility. Therefore, we propose the GARCH-cross-sectional (GARCHX) model;

$$h_{it} = \alpha_{i0}\sigma^2_{C,mt-1} + \alpha_{i1}\epsilon^2_{it-1} + \alpha_{i2}h_{it-1}. \tag{15}$$

where, using $E^C[.]$ as defined in (1),

$$\begin{aligned} \sigma^2_{C,mt-1} &= E^C[(r_{it-1} - r_{mt-1})^2], \\ r_{mt-1} &= E^C[r_{it-1}], \end{aligned}$$

and the stationarity and nonnegativity conditions are in (14). Note that we allow one lag for the cross-sectional market volatility as $\sigma^2_{C,mt-1}$ in equation (15). We expect the GARCHX model performs better than the GARCHX-T model because $\sigma^2_{C,mt}$ is more informative and highly persistent.

Equation (15) is the GARCHX(1,1) model and we generalise this to the GARCHX(p, q) model;

$$h_{it} = \alpha_{i0}\sigma^2_{C,mt-1} + \sum_{q=1}^{Q}\alpha_{iq}\epsilon^2_{it-q} + \sum_{p=1}^{P}\gamma_{ip}h_{it-p}$$

and for stationarity and nonnegativity, we need

$$\begin{aligned} \sum_{q=1}^{Q}\alpha_{iq} + \sum_{p=1}^{P}\gamma_{ip} &< 1 \\ \alpha_{i0} &> 0, \\ \alpha_{iq} &\geq 0, q = 1, ..., Q, \\ \gamma_{ip} &\geq 0, p = 1, ..., P. \end{aligned} \tag{16}$$

In terms of comparing our analysis with Campbell, Lettau, Malkiel, and Xu (2001) who cross-sectionally decomposed volatility into market/sector/idiosyncratic level, we do not include industry factors and so a direct comparison is difficult. Our model uses a cross-sectional market volatility as a component of a time series.

3 Empirical Tests

We estimate GARCH(1,1) in (8), GARCH(1,1) with the time-series market volatility (13) denoted by GARCHX-T, and GARCHX(1,1) with the cross-sectional volatility (15) denoted by GARCHX for all individual stocks included in the FTSE350 and S&P500.

3.1 Data

We use the same data as in Hwang (2001a), which consist of individual asset returns included in the FTSE350 and the S&P500. Daily log-returns are calculated from 11 December 1989 to 9 December 1999. The data consist of 350 assets for the UK market, each of which consists of maximum 2533 log-returns during the sample period and 500 assets for the USA market, each of which has maximum 2527 log-returns during the sample period. We obtained the data from Datastream.

Note that since the components of the indices have changed during the sample period, we used equities included in the indices as of the 10th of December, 1999. Thus, the number of equities available at the beginning of our sample period is less than the number of equities in the indices at the end of our sample period. Our results may be affected by this exclusion of equities in the early period of our sample. However, the effects are not expected to be significant because of the large number of equities used here (350 in the UK market and 500 in the USA market). In addition, changes in the components of the indices are usually for small companies. The numbers of equities at the beginning of our sample period are 242 and 430 for the FTSE350 and S&P500 indices. For these equities, we have the full 10 years' daily log-returns, while for other equities, we have shorter observations.

We do not use log-returns which are larger than 50% or smaller than -50% a day. In this study we define these returns outliers. The total number of equities

removed were 17 in the FTSE350 constituents and 9 in the S&P500 constituents. For the calculation of equally weighted cross-sectional market returns and volatility, w_{it} is set to $\frac{1}{N}$ as in (3) and (4), whilst for the value weighted cross-sectional market returns and volatility, w_{it} is set to the market weights on individual assets. For the calculation of time-series volatility, we use (2) for both equally weighted market returns and value weighted market returns.

3.2 Specification Tests of GARCHX Model

Table 1 summaries some statistical properties of the cross-sectional and time-series market volatilities.[4] We find that cross-sectional returns are more dispersed than time-series returns; the average standard deviation of cross-sectional returns is larger than that of time-series returns. The time-series volatility of the value weighted return is larger than that of the equally weighted return, while the cross-sectional volatility of the value weighted return is smaller than that of the equally weighted return. This is the result of heavy weights on some large equities. Another interesting property in volatility is that the cross-sectional volatility is highly persistent, whilst the time-series volatility shows far less persistence. The high persistence of the cross-sectional volatility empirically suggests that GARCHX models perform better than GARCHX-T models. Table 1 also shows the relationship between cross-sectional and time-series volatility. The correlations between them are around 0.4 and there is little difference between the four cases. This gives a reason why we put cross-sectional volatility in the conditional volatility models such as GARCH.

We use cross-sectional market volatility in GARCH models as suggested in the previous section. For the GARCHX and the GARCHX-T models, both equally-weighted and value-weighted volatilities are used. Thus, altogether we have five

[4] The source of table 1 is table 1 of Hwang (2001). More detailed empirical results on the cross-sectional statistics can be found in Hwang (2001).

models; GARCH(1,1), GARCHX(1,1) with equally weighted cross-sectional market volatility (GARCHX(EW)), GARCHX(1,1) with value weighted cross-sectional market volatility (GARCHX(VW)), GARCHX-T(1,1) with equally weighted time-series market volatility (GARCHX-T(EW)), and GARCHX-T(1,1) with value weighted time-series market volatility (GARCHX-T(VW)).

Table 2 summarises the estimation results. The first results reveal how easily we can estimate the models. The number of converged cases provides some information about this question. For the conventional GARCH(1,1) model, we have 323 convergences out of 333 for the FTSE350 equities and 483 convergences out of 491 S&P500 equities, respectively. The numbers of converged cases for the GARCHX is nearly the same as those for the GARCH(1,1) model. In addition, the estimation time does not seem to be increased dramatically with cross-sectional volatility. However, the GARCHX-T seems to suffer convergence problems. Most of the convergence errors occur because of the non-negativity condition on the coefficient of the past market volatility, i.e., $\alpha_{i0} > 0$ in (13).

A more direct comparison between the five models is possible using the maximum likelihood (ML) values. In table 2 we reported the number of cases that the ML values of GARCHX and GARCHX-T models are larger than those of the GARCH model. Note that the comparisons are possible only when we have converged estimates. Furthermore there is no need to adjust for "the number of parameters" as both models have the same number of parameters.

First, the GARCHX model performs very well; for example, the cases that the GARCH model performs better than the other four models are 46 (16.5%) and 33 (7.8%) for the FTSE and S&P500 equities, respectively. Other statistics such as portmanteau tests also suggest that GARCHX models perform well. The numbers of significant serial correlations in the squared standardised error in the GARCH(1,1) model for 5, 10, 20 lags are decreased in GARCHX models. For example, the

percent of significant correlations at 95% level with the first five lags (Q(5)) in the GARCH(1,1) model are 9.60% and 17.39% for the FTSE350 and S&P500, while those statistics are 6.11% and 8.92% in GARCHX(EW).[5] On the other hand, the statistics of skewness and kurtosis of the standardised residuals are not improved.

The GARCHX-T models perform poorly compared with GARCH or GARCHX models. The number of the ML values larger than those of GARCH are around 25-42% of the total number of equities. The poor performance of GARCHX-T model is not surprising when we consider the relatively large amount of noise included in the time-series volatility; see signal-to-noise ratios in table 2, Hwang (2001a).

We next compare GARCH and GARCHX models in detail in table 3. The first comparison is the number of significant α_{i0} of GARCH(1,1) in (8) and GARCHX in (15). The significance increases with cross-sectional volatility; in particular, for the S&P500, the percentage increases from 56.31% to more than 70%. This means that the lagged cross-sectional volatility is an important explanatory variable in the presence of the past volatility and past conditional volatility. This result is consistent with Connor and Linton (2001) and Jones (2001). In order to investigate further if the relationship in (7) is still important after one period lag and the replacement of ϵ_{mt}^2 (market volatility) with $\sigma_{C,mt-1}^2$ (cross-sectional volatility), we also calculate the correlation coefficient between squared betas and coefficients on the past cross-sectional volatility (α_{i0} in the GARCHX(1,1) model). The betas are estimated using a one factor (market) model. The second rows in panels A and B in Table 3 show that there is still significant positive correlation (0.16) between β_{mi}^2 and α_{i0} except for the GARCHX(VW) model of the FTSE350 which is negative but not significant. Other correlation coefficients are all positive and significant. This means that the market volatility is an important factor for forecasting individual asset volatility. The results are consistent with Campbell, Lettau, Malkiel, and Xu

[5] Note that these statistics should be around 5% since we use 95% significance level.

(2001) who found evidence that market volatility Granger-causes individual stock volatility.

We also calculate the proportion of $\alpha_{i0}\sigma^2_{C,mt-1}$ in conditional volatility, which can be interpreted as the proportion of the cross-sectional market volatility in individual stock volatility. The results in Table 3 show that these proportions range between 12% and 16%, which is consistent with Campbell, Lettau, Malkiel, and Xu (2001), who found that the market volatility accounts for about 13% for the unconditional mean of individual stock volatility for the period from January 1988 to December 1997. Our study shows that, despite differences in the definition of the market volatilities and in other detail procedures, the overall conclusion is the same; market volatility does matter for the individual stock volatility.

The next rows report results on the ML values. GARCHX(EW) has larger ML values than the GARCH model for 237 and 417 cases, for the S&P500 and the FTSE350, respectively, which are equivalent to 76.7% and 86.9% of the total cases. The average values of the changes in the ML values are all positive. Since there is no change in the number of parameters between the GARCH in (8) and the GARCHX in (15), the positive increase in ML values suggest that GARCHX model performs better than GARCH model. In addition, the average values of portmanteau statistics are decreased slightly in GARCHX models. However, the estimates of skewness show little difference, whilst those of kurtosis tend to increase for GARCHX model.

3.3 Forecasting Tests of GARCHX Model

The empirical results in the previous subsection show that return volatility (squared returns) is better specified with GARCHX models. In this subsection, we investigate the forecasting performance of the GARCHX model.

For our data, we use four daily return volatility series of three stocks, i.e., Abbey National, Unilever, British Airways, and one index, i.e., the FTSE100 index. The

return volatility (RV) is obtained by squaring de-meaned daily log-returns. The total number of 2496 daily returns from 21 January 1992 to 27 November 2001 is used to test the forecasting performance of the GARCHX model. The return series of the underlying asset is provided by Datastream.

In Table 4 GARCH(1,1) and GARCHX(1,1) estimates of the four return series are reported. The maximum likelihood values of the GARCH model are all larger than those of the GARCHX model for the four cases. Except for British Airways, the estimated coefficients on the past cross-sectional market volatility are significant at the 10% level. These are some examples of GARCHX estimates as well as other evidence that the common heteroskedasticity in asset specific returns is an important component for individual asset's volatility process.

Another important result in Table 4 is that the inclusion of the cross-sectional market volatility tends to decrease α_{i2} so that the level of persistence in the GARCHX(1,1) model, i.e., $\alpha_{i1} + \alpha_{i2}$, is less than that in the GARCH(1,1) model. This indirectly shows that the high persistence frequently found in volatility process may be due to missing time-varying components.

We use a rolling sample of the past volatilities. On day t, the conditional volatility of one period ahead, $t + 1$, is constructed by using the estimates which are obtained from only the past observations (i.e., 2197 observations in this study). By recursive substitution of the conditional volatility, forecasts for up to 60 horizons are constructed. On the next day $(t + 1)$, using 2197 recent observations (i.e., 2197 observations from the second observation to the 2198th observation), we estimate the parameters again and get another forecast for up to 60 horizons. The estimation and forecasting procedures are performed 240 times using rolling windows of 2197 observations. And then, each forecast is compared with the realised return volatility.

For the GARCH(1,1) model in (8), the one step ahead forecast, $h_{t+1|t}$, and the

f step ahead forecasts, $h_{t+f|t}$, are

$$\begin{aligned} h_{i,t+1|t} &= \alpha_{i0} + \alpha_{i1}\epsilon_{it}^2 + \alpha_{i2}h_{it}^2, \qquad (17) \\ h_{i,t+f|t} &= \alpha_{i0}\sum_{j=0}^{f-1}(\alpha_{i1}+\alpha_{i2})^j + (\alpha_{i1}+\alpha_{i2})^{f-1}(\alpha_{i1}\epsilon_{it}^2 + \alpha_{i2}h_{it}^2), \text{ when } f>1, \end{aligned}$$

where $h_{i,t+f|t}$ represents f step ahead volatility conditional on the information available at time t. For large f, $h_{i,t+f|t}$ approaches $\frac{\alpha_{i0}}{1-\alpha_{i1}-\alpha_{i2}}$, the unconditional variance for the case $0 < \alpha_{i1} + \alpha_{i2} < 1$. On the other hand, the one step ahead forecast, $h^X_{i,t+1|t}$, and the f step ahead forecasts, $h^X_{i,t+f|t}$, of the GARCHX(1,1) model are

$$\begin{aligned} h^X_{i,t+1|t} &= \alpha_{i0}\sigma^2_{C,mt} + \alpha_{i1}\epsilon_{it}^2 + \alpha_{i2}h_{it}, \qquad (18) \\ h^X_{i,t+f|t} &= \alpha_{i0}\sigma^2_{C,mt}\sum_{j=0}^{f-1}(\alpha_{i1}+\alpha_{i2})^j + (\alpha_{i1}+\alpha_{i2})^{f-1}(\alpha_{i1}\epsilon_{it}^2 + \alpha_{i2}h_{i,t}^2), \text{ when } f>1, \end{aligned}$$

and for large f, $h^X_{i,t+f|t}$ approaches $\frac{\alpha_{i0}\sigma^2_{C,mt}}{1-\alpha_{i1}-\alpha_{i2}}$, the unconditional variance for the case $0 < \alpha_{i1} + \alpha_{i2} < 1$. Equations (17) and (18) clearly show that the forecasting performance depends on the property of $\sigma^2_{C,mt}$. However, since the two forecasts, $h_{i,t+f|t}$ and $h^X_{i,t+f|t}$, approach the same unconditional volatility as the forecasting horizon increases, the main difference between the two forecasts lies in short forecasting horizon.

Table 5 shows that, as reported in many other studies, the GARCH model performs poorly for the forecasts of volatility. The mean absolute forecast error is close to the average value of the realised return volatility (squared returns) during the forecasted period. The high volatile return volatility seems to be hard to forecast with the conditional volatility model, and the results are consistent with previous studies such as Andersen and Bollerslev (1997). Our GARCHX model shows smaller MAFE and MSFE than the GARCH model for all four cases in short forecast horizons, but the difference is marginal.

Over the longer horizons, we may not find if the GARCHX model performs better than the GARCH model. As explained above, as the forecasting horizon increases,

the two forecasts obtained with the GARCH and GARCHX models are similar, the main difference between the two models can be found in the shorter forecasting horizons.

4 Conclusions

This study introduces a simple new conditional volatility model called GARCHX using the cross-sectional market volatility. The model is simple, but can be used to explain the proportion of market volatility included in individual stock volatility. The model can also be used to explain common heteroskedasticity in asset returns which was suggested by Jones (2001) and Connor and Linton (2001) as a significant component.

Using the UK and US markets, we find that more than three-quarter cases, the maximum likelihood values of the GARCHX(1,1) model are larger than those of the GARCHX(1,1) model and the coefficients on the cross-sectional market volatility are significant. Therefore, individual stock volatility seems to be better specified with the inclusion of an additional cross-sectional market volatility. However, GARCHX models still are not good enough for forecasting RV.

A few important findings of our study can be summarised as follows. We find that the proportion of the market volatility in an individual stock's conditional volatility ranges from 12% to 16% which is consistent with the results of Campbell, Lettau, Malkiel, and Xu (2001). In addition, the large number of the significant coefficients on the cross-sectional market volatility support what Connor and Linton (2001) found. Our study, using a simple model with daily instead of monthly returns for both the UK and US markets, confirms these recent studies.

In this study we include one cross-sectional volatility calculated for market level, but cross-sectional volatility can also be calculated for industry level or for a specific

portfolio. These cross-sectional volatilities can be incorporated with conditional volatility models such as GARCH models or stochastic volatility models.

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Table 1 Properties of Cross-sectional Volatility

A. Properties of Cross-sectional and Time-series Equally Weighted Market Volatility

		FTSE100		S&P500	
		Cross-sectional Volatility	Time Series Volatility	Cross-sectional Volatility	Time Series Volatility
Average Value		1.5852	0.6477	1.9293	0.8353
Autocorrelations of Volatility	Lag 1	0.6105	0.2651	0.7168	0.2421
	Lag 2	0.5300	0.1543	0.6474	0.1562
	Lag 3	0.4879	0.1155	0.5968	0.0983
	Lag 4	0.4771	0.0796	0.5778	0.1156
	Lag 5	0.4562	0.0516	0.5722	0.1784
	Lag 10	0.4235	0.0705	0.4971	0.0651
	Lag 20	0.3785	0.0271	0.4169	0.0706
	Lag 30	0.3769	0.0384	0.3894	0.0479
	Lag 40	0.3316	0.0397	0.3203	0.0339
	Lag 50	0.3147	0.0234	0.3197	0.0610
	Lag 100	0.2492	0.0144	0.2064	0.0148
Correlation Between Cross-sectional and Time-series Volatilities		0.4494		0.4069	

B. Properties of Cross-sectional and Time-series Value Weighted Market Volatility

		FTSE100		S&P500	
		Cross-sectional Volatility	Time Series Volatility	Cross-sectional Volatility	Time Series Volatility
Average Value		1.3995	0.8328	1.6012	0.8876
Autocorrelations of Volatility	Lag 1	0.6350	0.1682	0.6627	0.2358
	Lag 2	0.5679	0.1886	0.6336	0.1449
	Lag 3	0.5269	0.1723	0.5890	0.0799
	Lag 4	0.5306	0.1441	0.5620	0.1114
	Lag 5	0.5265	0.1357	0.5642	0.1951
	Lag 10	0.5005	0.1469	0.4891	0.0821
	Lag 15	0.4637	0.1160	0.4301	0.0747
	Lag 20	0.4770	0.0762	0.4164	0.0699
	Lag 30	0.4087	0.1203	0.4121	0.0426
	Lag 40	0.3918	0.0923	0.3492	0.0481
	Lag 50	0.3671	0.0804	0.3617	0.0715
	Lag 100	0.3349	0.0584	0.3081	0.0313
Correlation Between Cross-sectional and Time-series Volatilities		0.3990		0.3856	

Notes: The number of daily log-returns for the FTSE350 is 2532 from 11 December 1989 to 9 December 1999, and that for the S&P500 is 2527 for the same sample period. Standard deviation rather than variance is used to obtain the properties in the above table.

Table 2 Summary Table for Various Conditional Volatility Models

A. FTSE350

		GARCH	GARCHX(EW)	GARCH-T(EW)	GARCHX(VW)	GARCH-T(VW)
Total Number of Equities		333				
Number of Converged Cases		323	311	286	316	312
Maximum Likelihood Values	Number of Converged Cases for all Five Models	278				
	Number of Cases which have the Largest Maximum Likelihood Values among the Five Models	46	96	10	109	17
Portmanteau Tests (Percent of Significant Serial Correlation at 5% Level)	Q(5)	9.60%	6.11%	48.25%	6.33%	42.31%
	Q(10)	6.81%	5.79%	48.95%	6.33%	48.40%
	Q(20)	7.74%	8.36%	56.64%	8.54%	54.81%
Percent of Significant Skewness at 95% Level		80.80%	82.32%	86.71%	80.38%	83.33%
Percent of Significant Excess Kurtosis at 95% Level		99.07%	99.36%	99.30%	99.05%	99.36%

A. S&P500

		GARCH	GARCHX(EW)	GARCH-T(EW)	GARCHX(VW)	GARCH-T(VW)
Total Number of Equities		491				
Number of Converged Cases		483	482	436	478	430
Maximum Likelihood Values	Number of Converged Cases for all Five Models	421				
	Number of Cases which have the Largest Maximum Likelihood Values among the Five Models	33	237	12	122	17
Portmanteau Tests (Percent of Significant Serial Correlation at 5% Level)	Q(5)	17.39%	8.92%	58.72%	11.92%	59.30%
	Q(10)	13.87%	8.30%	50.46%	10.46%	51.63%
	Q(20)	11.18%	8.09%	48.62%	9.21%	49.53%
Percent of Significant Skewness at 95% Level		66.25%	65.98%	67.20%	66.95%	64.88%
Percent of Significant Excess Kurtosis at 95% Level		100.00%	99.79%	100.00%	100.00%	100.00%

Notes: The results are calculated with individual asset returns included in the FTSE350 and the S&P500 from 11 December 1989 to 9 December 1999.

Table 3 Detail Comparison of GARCH and GARCHX Models

A. FTSE350

		GARCH	GARCHX(EW)	GARCHX(VW)
Percent of Significant Coefficients on Drift (GARCH) and the Past CCS Volatility (GARCHX(EW) and GARCHX(VW))*		81.42%	87.14%	87.66%
Correlation Coefficient between Squared Betas and Coefficients on the Past Cross-sectional Volatility			0.16	-0.07
Percent of Cross-sectional Market Volatility in the GARCHX Model			13.44%	12.85%
Number of Comparisons with GARCH Model		-	309	312
Aaveraged Increase in Maximum Likelihood Values Compared with those of GARCH Model	Number of Cases Where GARCHX Has Larger Maximum Likelihood Values Than GARCH	-	237	226
	Averaged Increase in Maximum Likelihood Values in Percent	-	0.55%	0.49%
	Averaged Increase in Maximum Likelihood Values	-	19.79	17.75
Averaged Portmanteau Test Statistics for Squared Errors	Q(5)	4.70	4.15	4.36
	Q(10)	8.08	7.63	7.82
	Q(20)	15.59	15.56	15.67
Averaged Skewness for Standardised Errors		0.30	0.25	0.27
Averaged Excess Kurtosis for Standardised Errors		12.04	11.04	11.12

B. S&P500

		GARCH	GARCHX(EW)	GARCHX(VW)
Percent of Significant Coefficients on Drift (GARCH) and the Past CCS Volatility (GARCHX(EW) and GARCHX(VW))*		56.31%	73.24%	71.34%
Correlation Coefficient between Squared Betas and Coefficient on the Past Cross-sectional Volatility			0.40	0.32
Percent of Cross-sectional Market Volatility in the GARCHX Model			15.87%	12.22%
Number of Comparisons with GARCH Model		-	480	478
Aaveraged Increase in Maximum Likelihood Values Compared with those of GARCH Model	Number of Cases Where GARCHX Has Larger Maximum Likelihood Values Than GARCH	-	417	393
	Averaged Increase in Maximum Likelihood Values in Percent	-	0.30%	0.23%
	Averaged Increase in Maximum Likelihood Values	-	14.48	11.06
Averaged Portmanteau Test Statistics for Squared Errors	Q(5)	6.67	4.67	5.52
	Q(10)	10.05	8.35	9.12
	Q(20)	17.77	17.23	17.75
Averaged Skewness for Standardised Errors		-0.08	-0.08	-0.08
Averaged Excess Kurtosis for Standardised Errors		4.77	4.64	4.54

Notes: The significance test marked with * is carried at 5% level.

Table 4 Maximum Likelihood Estimates of GARCH(1,1) and GARCHX(1,1)

A. Abbey National

Models		α_0	α_1	α_2	Maximum Likelihood Values
GARCH(1,1)	Estimates	0.000003	0.0476	0.9471	6450.98
	Standard Deviation	(0.000003)	(0.0270)	(0.0339)	
GARCHX(1,1)	Estimates	0.002316	0.0769	0.8139	6469.83
	Standard Deviation	(0.001464)	(0.0236)	(0.0860)	

B. Unilever

Models		α_0	α_1	α_2	Maximum Likelihood Values
GARCH(1,1)	Estimates	0.000001	0.0471	0.9525	7196.72
	Standard Deviation	(0.000001)	(0.0156)	(0.0175)	
GARCHX(1,1)	Estimates	0.000110	0.0495	0.9438	7200.76
	Standard Deviation	(0.000081)	(0.0149)	(0.0193)	

C. British Airways

Models		α_0	α_1	α_2	Maximum Likelihood Values
GARCH(1,1)	Estimates	0.000005	0.0861	0.9079	6353.35
	Standard Deviation	(0.000008)	(0.0805)	(0.0885)	
GARCHX(1,1)	Estimates	0.001264	0.1113	0.8426	6364.04
	Standard Deviation	(0.001187)	(0.0753)	(0.1114)	

D. FTSE100 Index

Models		α_0	α_1	α_2	Maximum Likelihood Values
GARCH(1,1)	Estimates	0.000001	0.0724	0.9177	8184.52
	Standard Deviation	(0.000001)	(0.0243)	(0.0314)	
GARCHX(1,1)	Estimates	0.000182	0.0716	0.8963	8192.56
	Standard Deviation	(0.000065)	(0.0157)	(0.0225)	

Notes: Returns from 21 January 1992 to 27 November 2001 for a total of 2496 observations are used.

Table 5 Forecasting Performance of GARCH(1,1)-RV and GARCHX(1,1)-RV for RV

A. Abbey National

Forecasting Horizons	GARCH(1,1)		GARCHX(1,1)	
	MAFE	MSFE	MAFE	MSFE
1	0.1230	0.0642	0.1225	0.0636
2	0.1243	0.0647	0.1239	0.0641
3	0.1264	0.0659	0.1260	0.0654
4	0.1267	0.0659	0.1262	0.0653
5	0.1411	0.1143	0.1406	0.1141
6	0.1458	0.1203	0.1459	0.1206
7	0.1461	0.1203	0.1462	0.1205
8	0.1465	0.1202	0.1462	0.1199
9	0.1460	0.1198	0.1471	0.1201
10	0.1451	0.1194	0.1461	0.1199
15	0.1451	0.1168	0.1456	0.1178
20	0.1455	0.1168	0.1486	0.1189
25	0.1488	0.1182	0.1523	0.1209
30	0.1467	0.1163	0.1495	0.1184
40	0.1427	0.1151	0.1466	0.1156
50	0.1440	0.1142	0.1457	0.1149
60	0.1422	0.1139	0.1430	0.1145

B. Unilever

Forecasting Horizons	GARCH(1,1)		GARCHX(1,1)	
	MAFE	MSFE	MAFE	MSFE
1	0.0656	0.0112	0.0655	0.0112
2	0.0658	0.0112	0.0656	0.0112
3	0.0658	0.0111	0.0657	0.0111
4	0.0659	0.0111	0.0658	0.0111
5	0.0674	0.0117	0.0673	0.0117
6	0.0667	0.0116	0.0665	0.0115
7	0.0682	0.0120	0.0680	0.0120
8	0.0694	0.0124	0.0693	0.0123
9	0.0706	0.0128	0.0704	0.0128
10	0.0712	0.0129	0.0710	0.0128
15	0.0763	0.0168	0.0762	0.0167
20	0.0807	0.0190	0.0806	0.0189
25	0.0809	0.0192	0.0807	0.0190
30	0.0829	0.0196	0.0825	0.0194
40	0.0806	0.0187	0.0803	0.0185
50	0.0815	0.0184	0.0806	0.0181
60	0.0795	0.0161	0.0788	0.0157

C. British Airways

Forecasting Horizons	GARCH(1,1)		GARCHX(1,1)	
	MAFE	MSFE	MAFE	MSFE
1	0.1707	0.0857	0.1659	0.0851
2	0.1763	0.0932	0.1710	0.0926
3	0.1767	0.0936	0.1710	0.0930
4	0.1776	0.0944	0.1714	0.0938
5	0.2356	0.9266	0.2296	0.9263
6	0.2353	0.9264	0.2289	0.9266
7	0.2394	0.9297	0.2327	0.9293
8	0.2700	1.1549	0.2635	1.1545
9	0.2735	1.1584	0.2666	1.1586
10	0.2730	1.1590	0.2660	1.1590
15	0.3033	1.2708	0.2950	1.2714
20	0.3136	1.2776	0.3055	1.2785
25	0.3221	1.2819	0.3152	1.2848
30	0.3098	1.2580	0.3012	1.2590
40	0.3311	1.3388	0.3219	1.3416
50	0.3538	1.3616	0.3440	1.3664
60	0.3906	1.5355	0.3833	1.5479

D. FTSE100 Index

Forecasting Horizons	GARCH(1,1)		GARCHX(1,1)	
	MAFE	MSFE	MAFE	MSFE
1	0.0343	0.0028	0.0339	0.0028
2	0.0350	0.0029	0.0345	0.0029
3	0.0357	0.0030	0.0351	0.0030
4	0.0358	0.0030	0.0352	0.0030
5	0.0396	0.0061	0.0390	0.0061
6	0.0405	0.0063	0.0399	0.0063
7	0.0405	0.0063	0.0398	0.0063
8	0.0421	0.0069	0.0413	0.0068
9	0.0430	0.0071	0.0422	0.0070
10	0.0430	0.0071	0.0421	0.0070
15	0.0461	0.0082	0.0453	0.0081
20	0.0472	0.0083	0.0466	0.0083
25	0.0472	0.0084	0.0467	0.0084
30	0.0479	0.0085	0.0474	0.0085
40	0.0489	0.0087	0.0483	0.0086
50	0.0476	0.0085	0.0471	0.0085
60	0.0463	0.0084	0.0464	0.0085

Notes: Return and implied volatilities from 21 January 1992 to 27 November 2001 for a total of 2496 observations are used. Recent 2197 observations are used for estimating models and forecasting volatility over 60 horizons. MAFE and MSFE represent mean absolute forecast error and mean squared forecast error, respectively. The results are based on 240 out-of-sample forecasts.

List of other working papers:

2001

1. Soosung Hwang and Steve Satchell , GARCH Model with Cross-sectional Volatility; GARCHX Models, WP01-16
2. Soosung Hwang and Steve Satchell, Tracking Error: Ex-Ante versus Ex-Post Measures, WP01-15
3. Soosung Hwang and Steve Satchell, The Asset Allocation Decision in a Loss Aversion World, WP01-14
4. Soosung Hwang and Mark Salmon, An Analysis of Performance Measures Using Copulae, WP01-13
5. Soosung Hwang and Mark Salmon, A New Measure of Herding and Empirical Evidence, WP01-12
6. Richard Lewin and Steve Satchell, The Derivation of New Model of Equity Duration, WP01-11
7. Massimiliano Marcellino and Mark Salmon, Robust Decision Theory and the Lucas Critique, WP01-10
8. Jerry Coakley, Ana-Maria Fuertes and Maria-Teresa Perez, Numerical Issues in Threshold Autoregressive Modelling of Time Series, WP01-09
9. Jerry Coakley, Ana-Maria Fuertes and Ron Smith, Small Sample Properties of Panel Time-series Estimators with I(1) Errors, WP01-08
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