Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 60 (2015) 936 – 943

Procedia Computer Science

19th International Conference on Knowledge Based and Intelligent Information and Engineering Systems

Fuzzy random weighted Weber problems in facility location

Takeshi Uno[a,*], Kosuke Kato[b], Hideki Katagiri[c]

[a]*Institute of Socio-Arts and Sciences, Tokushima University, 1-1, Minamijosanjima-cho, Tokushima-shi, Tokushima, 770-8502, Japan*
[b]*Faculty of Applied Information Science, Hiroshima Institute of Technology, 2-1-1, Miyake, Saeki-ku, Hiroshima, 731-5193, Japan*
[c]*Graduate School of Engineering, Hiroshima University, 1-4-1, Kagamiyama, Higashihiroshima-shi, Hiroshima, 739-8527, Japan*

Abstract

This article considers facility location in a Weber problem with weights including both uncertainty and vagueness. By representing its weights as fuzzy random variables, it can be extended to a fuzzy random weighted Weber problem, and then formulated as a fuzzy random programming problem. By introducing possibility and necessity measures and chance constraints, the extended problem is reformulated to new two types of Weber problems. Based upon characteristics of facility location, theorem for solving the reformulated problems are shown.

Peer-review under responsibility of KES International

Keywords: Location; Fuzzy Random Programming; Possibility and Necessity Measures.

1. Introduction

Weber problem is one of optimal location problems, for finding a facility location minimizing the sum of distances from/to all considering points. If the points have weights, it is extended to a weighted Weber problem, to minimize the sum of weighted distances from/to them. Weber problem can be applied to locating delivery centers, fire stations, and so on. Weber problems and their applications have been considered from a long time ago, and their recent references are written by Dendievel[1], Hosseininezhad et al.[3], Uno et al.[13], and so on.

In this paper, we extend Weber problems by considering both uncertainty and vagueness in their weights. We show a significance of extension by using the above first example, locating delivery centers. For this example, weights on points represent demands to deliver some goods to shops, store, and so on. An objective of locating delivery centers is to deliver efficiently, and then we can regard it as to minimize the sum of weighted distances to addresses for delivery. Hence their location problem can be represented as an ordinary weighted Weber problem for the case that demands of delivery can be estimated accurately. However, it is usually hard to estimate them because of including both uncertainty and vagueness.

For uncertainty, facility location with random demands is considered by Wagnera et al.[14], Shiode and Drezner[11], Uno et al.[13] and so on. On the other hand, for vagueness, facility location with fuzzy demands is considered by Moreno

* Corresponding author. Tel.: +81-88-656-7294 ; fax: +81-88-656-7294.
E-mail address: uno.takeshi@tokushima-u.ac.jp

1877-0509
Peer-review under responsibility of KES International
doi:10.1016/j.procs.2015.08.257

Pérez[8], represented them as fuzzy numbers proposed by Dubois and Prade[2]. For considering both uncertainty and vagueness, Kwakernaak[7] proposed fuzzy random variables including both randomness and fuzziness. For the details of fuzzy random variable, the reader can refer to the book of Kruse and Meyer[6]. For application of fuzzy random variables to mathematical programming, Katagiri et al.[4] considered fuzzy random bottleneck spanning tree problems, and Katagiri et al.[5] considered multiobjective fuzzy random 0-1 programming.

In this paper, we extend weighted Weber problems by representing their weights as fuzzy random variables, called fuzzy random weighted (FRW) Weber problems. As previous studies of evaluation for fuzzy random variables, Katagiri et al.[4,5] proposed solving fuzzy random programming problems by using possibility and necessity measures for them under some chance constraints. We propose to utilize possibility and necessity measures for the FRW Weber problems under some chance constraints. Then, we can reformulate two types of new deterministic Weber problems. Moreover, we show their theorem for solving them based upon characteristics of facility location.

The remaining structure of this article is organized as follows. The next section devotes to introducing the definition of fuzzy random variables. In Section 3, we introduce a location problem with demands given uncertainly and vaguely as a fuzzy random weighted Weber problem. By reformulating it with possibility and necessity measures under some chance constraints, we propose two types of new Weber problems in Section 4. In Section 5, we show a theorem for solving the new Weber problems. An example of fuzzy random weighted Weber problem and finding its optimal locations are illustrated in Section 6. Finally, conclusions and future studies are summarized in Section 7.

2. Fuzzy random variable

In this section, we introduce the definition of fuzzy random variables. Let A be a fuzzy number[2] and $\mu_A : \mathbf{R} \to [0, 1]$ be A's membership function, where $\mathbf{R}$ is the set of real numbers. For $\alpha \in (0, 1]$, the α-level set of A is represented as the following equation:

$$A_\alpha \equiv \{z \in \mathbf{R} \mid \mu_A(z) \geq \alpha\}. \tag{1}$$

In this paper, we use the following definition of fuzzy random variables, suggested by Kruse and Meyer[6]:

Definition 2.1. *Let (Ω, B, P) be a probability space, where Ω, B, and P are a sample space, σ-algebra, and a probability measure function, respectively. Let $\mathcal{F}(\mathbf{R})$ be the set of fuzzy numbers with compact supports, and Ξ a measurable mapping $\Omega \to \mathcal{F}(\mathbf{R})$. Then Ξ is a fuzzy random variable if and only if given $\omega \in \Omega$, its α-level set $\Xi_\alpha(\omega)$ is a random interval for any $\alpha \in (0, 1]$.*

Figure 1 illustrates an example of fuzzy random variables, in which randomness is derived from changes of weather and fuzziness stems from the difficulty of exactly estimating the amount of sales.

3. Formulation of fuzzy random weighted Weber problem

In this section, we introduce a Weber problem with demands given uncertainly and vaguely. We consider that a decision maker (DM) locates m facilities on a convex set S including n points with demands for facilities, called demand points (DP), in plane $\mathbf{R}^2$. Figure 2 illustrates an example of Weber problem.

For i-th DP, $i = 1, 2, \ldots, n$, its site and demand are denoted by $v_i \in S$ and w_i^{FR}, respectively. We give all w_i^{FR}, $i = 1, 2, \ldots, n$ as fuzzy random variables with k scenarios whose probabilities are $p_1, p_2, \ldots, p_k$.

Let $x_1, x_2, \ldots, x_m \in S$ be the sites of the facilities, and $\boldsymbol{x} = (x_1, x_2, \ldots, x_m)$. The distance from j-th facility, $j = 1, 2, \ldots, m$ to i-th DP are denoted by $d_i(x_j)$. We assume that the distance from facilities to each DP are given as the minimum of the distances, e.g.,

$$d_i(\boldsymbol{x}) \equiv \min_{j=1,2,\ldots,m} \left\{d_i(x_j)\right\}. \tag{2}$$

Then, the sum of weighted distances from facilities to DPs are represented as follows:

$$f^{FR}(\boldsymbol{x}) := \sum_{i=1}^{n} w_i^{FR} d_i(\boldsymbol{x}) \tag{3}$$

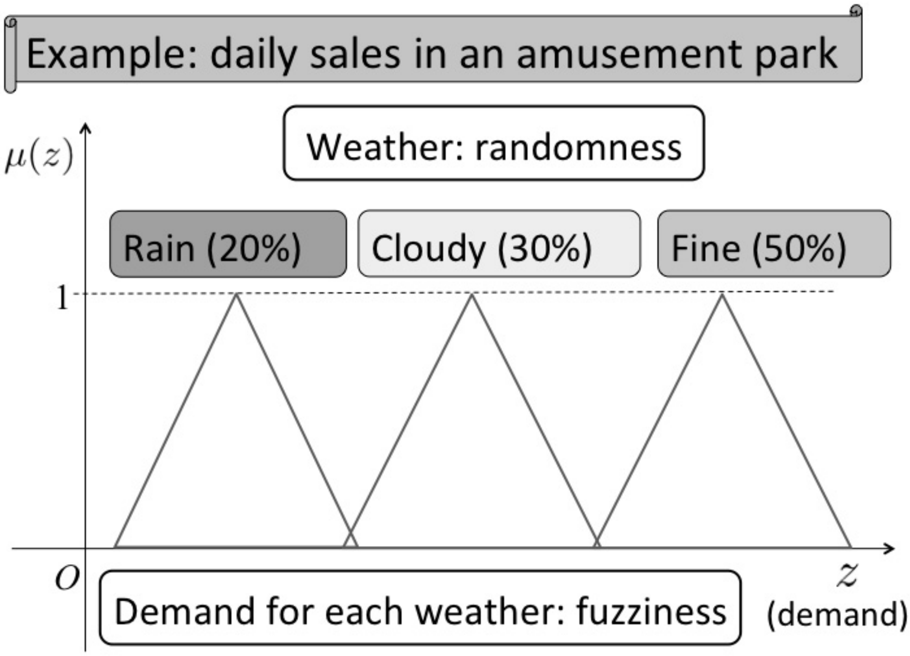

Fig. 1. An example of fuzzy random variables

Therefore, we can formulate the following fuzzy random weighted Weber problem:

$$\left.\begin{array}{ll}\text{minimize} & y = f^{FR}(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in S^m\end{array}\right\} \tag{4}$$

4. Reformulation with possibility and necessity measures

Note that (4) is an ill-defined problem because the meaning of minimizing its objective function is not well defined. An efficient method for estimating fuzziness, Katagiri et al.[4,5] proposed to define possibility and necessity measures with their goals for objective functions and constraints. By using their chance constraints for transformed stochastic programming problems, their fuzzy random programming problems can be transformed to deterministic well-defined problems. Because the DM can easily give their goal for object function, we can also utilize her/his definition to the decision making for (4) with fuzzy random variables. In this section, we propose a new Weber problem with possibility and necessity measures for her/him under some chance constraints by reformulating (4).

Taking account of vagueness of human judgment, the DM gives her/his fuzzy goal G: "I hope to make my objective function value y roughly smaller than l_1 but certainly smaller than l_0", as the following membership function:

$$\mu_G(y) = \begin{cases} 1, & \text{if } y \le l_1, \\ g(y), & \text{if } l_1 < y < l_0, \\ 0, & \text{if } y \ge l_0, \end{cases} \tag{5}$$

where $g(\cdot)$ is a decreasing function in interval (l_1, l_0) and its example is as follows:

$$g(y) = \frac{l_0 - y}{l_0 - l_1}. \tag{6}$$

Figure 3 illustrates an example of membership functions for fuzzy goal G.

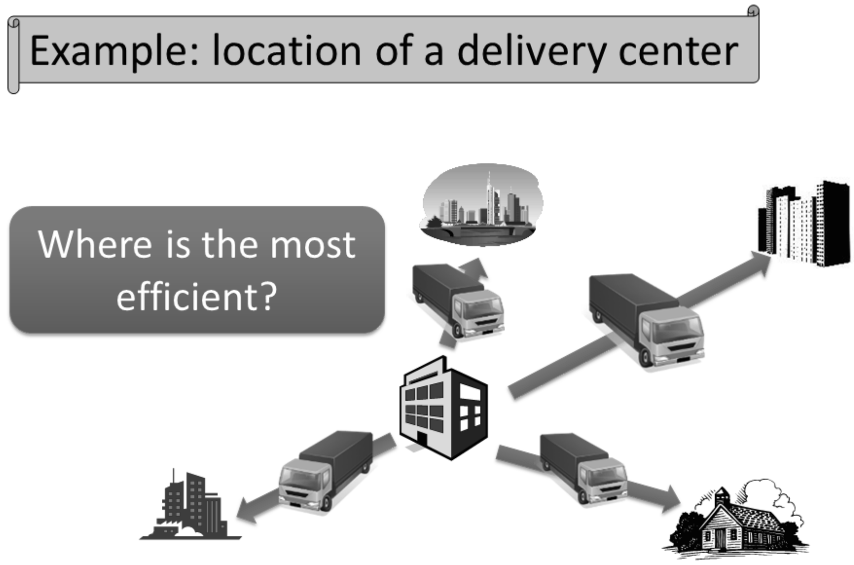

Fig. 2. An example of Weber problem

Given the DM's membership function of fuzzy random variable A and that of a fuzzy goal G, degrees of possibility and necessity that the objective function value y satisfies her/his fuzzy goal can be defined as follows, respectively:

$$\Pi_A(G) = \sup_y \min\{\mu_A(y), \mu_G(y)\}, \tag{7}$$

$$N_A(G) = \inf_y \max\{1 - \mu_A(y), \mu_G(y)\}. \tag{8}$$

Since the above degrees are represented as random variables, we use the following chance constraints suggested by Katagiri et al.[4,5]:

$$Pr(\Pi_A(G) \geq h) \geq \theta, \tag{9}$$

$$Pr(N_A(G) \geq h) \geq \theta, \tag{10}$$

where $Pr(\cdot)$ is a probability function, h is an auxiliary variable satisfying $0 \leq h \leq 1$, and $\theta \in (0, 1)$ is a permissible probability level given by the DM. Then, (4) can be reformulated as the following two types of deterministic programming problems:

$$\left.\begin{array}{ll} \text{maximize} & h \\ \text{subject to} & Pr(\Pi_A(G) \geq h) \geq \theta, \\ & \boldsymbol{x} \in S^m, \end{array}\right\} \tag{11}$$

$$\left.\begin{array}{ll} \text{maximize} & h \\ \text{subject to} & Pr(N_A(G) \geq h) \geq \theta, \\ & \boldsymbol{x} \in S^m. \end{array}\right\} \tag{12}$$

We call (11) and (12) as possibility and necessity Weber problems, respectively.

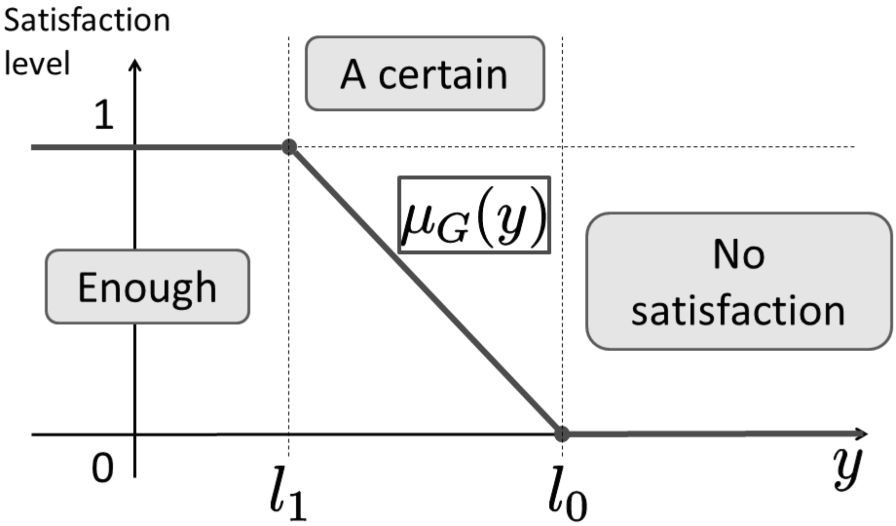

Fig. 3. An example of membership function of fuzzy goal

5. Properties of deterministic Weber problem

In this section, we show an important theorem for solving the reformulated two problems in the previous section.

Theorem 5.1. *For any given fuzzy goal G and* $\theta \in (0, 1]$*, both possibility and necessity Weber problems are convex.*

Proof: Since $d_i(\boldsymbol{x})$ is a convex function, $f^{FR}(\boldsymbol{x})$, the sum of weighted distances, is also convex if weights are deterministic values. From (9) and (10), feasible sets of (11) and (12) are convex for any given h and θ. This means that both (11) and (12) are convex.

From Theorem 5.1, possibility and necessity Weber problems can be solved by applying ordinal convex programming, e.g., Successive quadratic programming (SQP) method; for the details of the SQP method, the reader can refer to the book of Nocedal and Wright[9].

6. Numerical example

In this section, we illustrate an example of the proposed fuzzy random Weber problem and find the two optimal locations for possibility and necessity Weber problems. In the example, we set 20 DPs on a rectangle location area $S = [0, 300] \times [0, 200]$. Sites of these DPs are shown in Figure 4. For each of their demands, we give a fuzzy random variable with 100 scenarios each of whose probability is 0.01 and with triangle fuzzy numbers for each scenario, like Figure 1. Note that we add highly uncertain and vague demands to the four left lower DPs in Figure 4, that is, there are great differences between scenarios.

Next we consider an optimal location for fuzzy random weighted Weber problem on S. For fuzzy goal G, we set $(l_1, l_0) = (1.0 \times 10^6, 1.0 \times 10^7)$ and use (6). For chance constraint, we set $\theta = 0.9$. We create a program for finding optimal locations of possibility and necessity Weber problems on R version 3.1.2[10]. For the possibility and necessity Weber problems , Figures 5 and 6 shows their optimal locations, respectively.

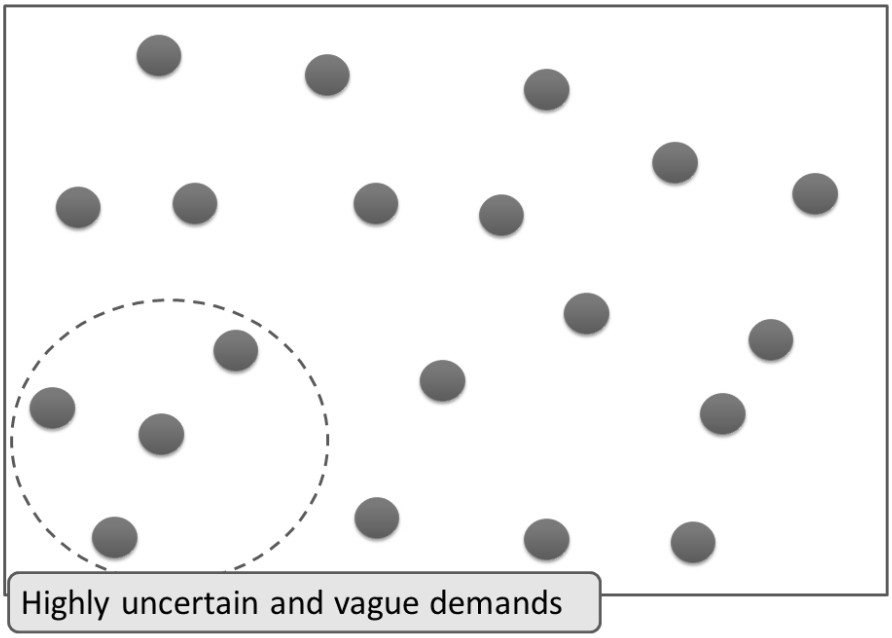

Fig. 4. Location area

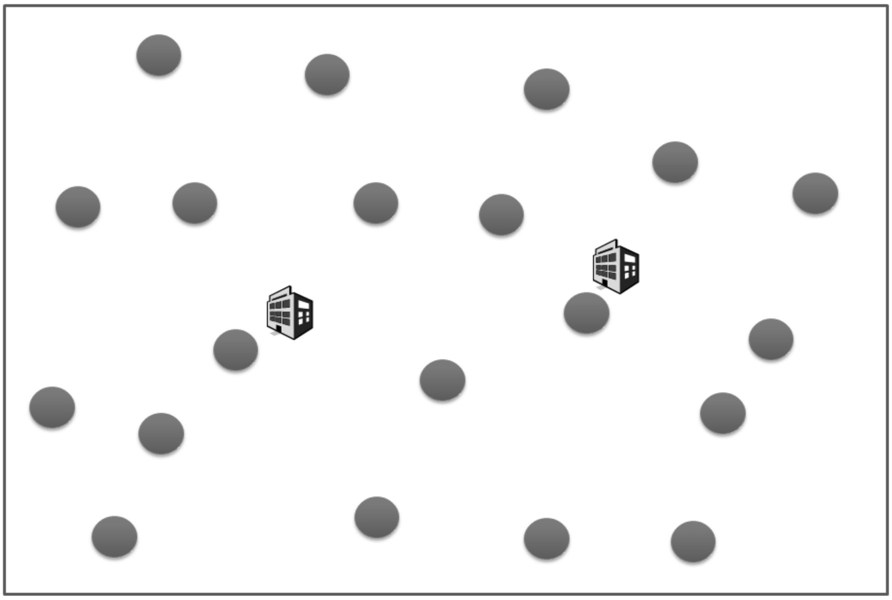

Fig. 5. Optimal location for possibility Weber problem

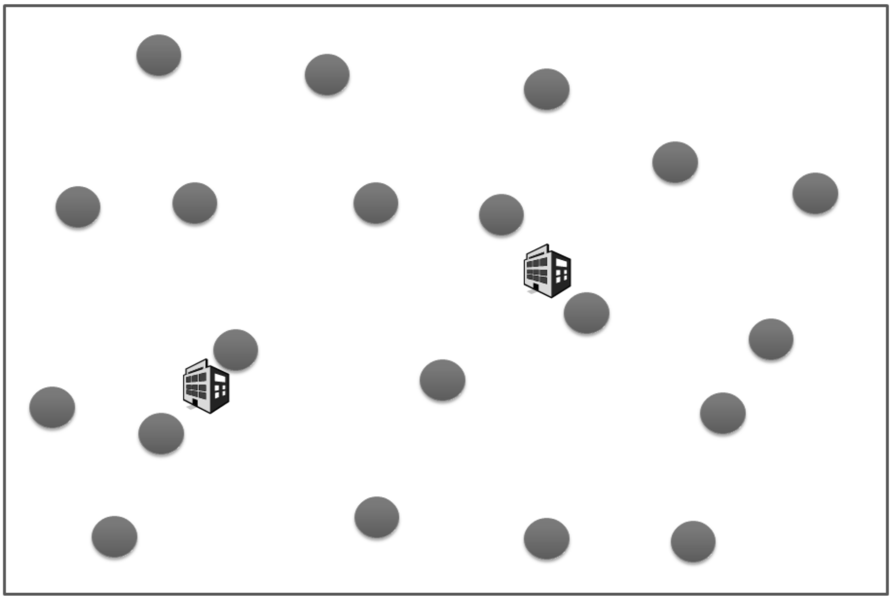

Fig. 6. Optimal location for necessity Weber problem

From Figures 5 and 6, the optimal location of necessity Weber problem makes one of sites of facilities move to a center of 4 DPs with highly uncertain and vague demands. This means that the necessity Weber problem weighs uncertainty and vagueness heavily than the possibility Weber problem.

7. Conclusions and future studies

In this paper, we have considered facility location in a Weber problem with weights including both uncertainty and vagueness. By representing its weights as fuzzy random variables, we have proposed new two types of Weber problems with possibility and necessity measures under some chance constraints. An important theorem for solving the new Weber problems have been shown based upon the characteristics of the facility location.

This paper consider Weber problem whose distance from facilities and DPs is defined by Euclid distance. However, if this model is applied to real world facility location, we need to consider the location model with other distance, such that block norm[12] or more complex distances than Euclid distance. Extending the proposed Weber problem to that of general distance is an important future study.

References

1. Dendievel R. Weber ' s optimal stopping problem and generalizations. *Statistics & Probability Lett* 2015;**97**:176-84.
2. Dubois D, Prade H. Operations on fuzzy numbers. *Int J Systems Sci* 1978;**9**:613-26.
3. Hosseininezhad SJ, Salhi S, Jabalameli MS. A cross entropy-based heuristic for the capacitated multi-source Weber problem with facility fixed cost. *Computers & Industrial Engin* 2015;**83**:151-8.
4. Katagiri H, Sakawa M, Ishii H. Fuzzy random bottleneck spanning tree problems using possibility and necessity measures, *Eur J Operational Res* 2004;**152**:88-95.
5. Katagiri H, Sakawa M, Kato K, Nishizaki I. A fuzzy random multiobjective 0-1 programming based on the expectation optimization model using possibility and necessity measures, *Mathematical and Computer Modeling* 2004;**40**:411-21.
6. Kruse R, Meyer KD. *Statistics with vague data*. Dordrecht: D Riedel Publishing Company; 1987.

7. Kwakernaak H. Fuzzy random variables-I: definitions and theorems. *Information Sci* 1978;**15**:1-29.
8. Moreno Pérez JA, Marcos Moreno Vega L, Verdegay JL. Fuzzy location problems on networks. *Fuzzy Sets and Systems* 2004;**142**:393-405.
9. The R Project for Statistical Computing, http://www.r-project.org/.
10. Nocedal J, Wright S. *Numerical Optimization*. New York: Springer-Verlag; 1999.
11. Shiode S, Drezner Z. A competitive facility location problem on a tree network with stochastic weights. *Eur J Operational Res* 2003;**149**:47-52.
12. Taoa J, Gowdab MS, Sznajder R. On the block norm-P property. *Linear Algebra and its Applications* 2014;**459**:121-35.
13. Uno T, Katagiri H, Kato K. An interactive approach with four criteria for stochastic weighted Weber problems. *Communications in Computer and Information Sci* 2013;**373**:664-8.
14. Wagnera MR, Bhaduryb J, Penga S. Risk management in uncapacitated facility location models with random demands. *Computers & Operations Res* 2009;**36**:1002-11.