2010

Applying cognitively guided instruction through word problem probes

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APPLYING COGNITIVELY GUIDED INSTRUCTION THROUGH WORD PROBLEM PROBES

An Abstract of a Thesis

Submitted

in Partial Fulfillment

of the Requirements for the Degree

Specialist in Education

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May 2010

ABSTRACT

Professional development programs have the opportunity to engage educators with new knowledge and pedagogical implications. Cognitively Guided Instruction (CGI) is a teacher professional development program focused on elementary mathematics instruction (Carpenter, Fennema, Franke, Levi, & Empson, 1999). The program's foundation is based upon research of how students learn to solve basic addition, subtraction, multiplication, and division word problems. Using children's common solutions strategies, CGI leads to teaching practices that support children in generating their own meaningful problem-solving skills while naturally learning mathematics. The paper reviews the literature on CGI and examines the initial findings of interrater reliability of CGI-based word problem probes, which were created for this research project. The limitations and implications of this project will also be discussed.

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This Thesis by: James Jackson Ploen

Entitled: Applying Cognitively Guided Instruction through Word Problem Probes

has been approved as meeting the thesis requirement for the

Degree of Specialist in Education of School Psychology

3·24·10	
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DEDICATION

I would like to dedicate this thesis specifically to my wife, Brooke; my parents, Jack and Judy; and my siblings, Jennifer, Jessica, and Jared. Their love and support through this entire thesis process has been monumental, and I was able to accomplish this thesis and the rest of my work because of that love and support. This thesis is also dedicated broadly to all of the other individuals who assisted me through this process including family, friends, and faculty who are too numerous to mention. I truly appreciate everyone's support.

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CHAPTER 1

INTRODUCTION

For more than 20 years, Cognitively Guided Instruction (CGI) has grown as a research-supported, teacher professional development program designed to address mathematics instruction in kindergarten through sixth grade (Carpenter, Fennema, & Franke, 1996; Carpenter, Fennema, Franke, Levi, & Empson, 1999; 2000). The foundations of CGI are based on the child's intuitive understanding about mathematics. The teachers' role is to use their knowledge of mathematics along with the students' performance on basic arithmetic word problems to facilitate mathematics instruction in the classroom. The classroom is designed as a learning environment that encourages children's thoughts, questions, and solutions when working with real mathematical scenarios.

The thesis of CGI is that all children enter school with a great deal of understanding about mathematics. The children's intuitive understanding should serve as the foundation and building point for primary school mathematics instruction and curriculum (Carpenter et al., 1996; 1999; Peterson, Fennema, Carpenter, & Loef, 1989). The understanding of mathematics formed before starting school comes from normal, everyday experiences. Mathematical word problems more closely resemble those normal, everyday experiences than simple number sentences that are posed in traditional instruction. An example of a word problem is "A boy has 2 shells, and he gets 3 more shells. How many shells does he have?" An example of a number sentence problem is "2 + 3 = ___." The principles of CGI would still encourage introducing more complex mathematical concepts, such as number sentences, place value, fractions, percents, and remainders, during instruction because those concepts build upon the great deal of knowledge that children bring to school.

CGI is not a formal curriculum or any sort of a textbook that is to be delivered in a classroom. The professional development program is designed to help teachers understand children's mathematical thinking and problem solving capabilities. The understanding about children's thinking is gained by enhancing the teacher's skills in understanding about the different types of arithmetic word problems and the solution strategies that children commonly use to solve a word problem. With the professional development workshop activities, group discussions, and school-based team support, the elementary teachers create learning environments that encourage children to express and share their ideas of how to solve the mathematical word problems. With the framework of CGI principles and school-based team support, teachers can better assess and evaluate elementary students' development in mathematical problem solving skills (Carey, Fennema, Carpenter, & Franke, 1995; Carpenter et al., 1999).

CGI research has expanded into two related areas. The research areas could be described as investigating learning and teaching (Fennema, Franke, Carpenter, & Carey, 1993). The learning component has focused on how young school children solve arithmetic word problems. The research has focused on breaking down basic addition, subtraction, multiplication, and division word problem types and on analyzing the solution strategies that children employ with those word problems. The learning research is the staple of the CGI framework that is introduced to teachers through the professional development workshops.

The research area about teaching has focused on how the teachers facilitate student learning by using the mathematical understanding that children bring to school. The research on teaching has examined what teaching beliefs more effectively support the use of CGI in the classrooms and what teaching methods experienced CGI teachers currently use in their classrooms. The latter teaching research topic is important because it lends insight to the common concern of how teachers can effectively employ instructional methods that embrace principles of CGI.

Research about the CGI professional development program has been based primarily upon analyzing basic arithmetic operations, which are addition, subtraction, multiplication, and division, and examining the common solution strategies for those arithmetic operations. Recently, the authors and proponents of CGI have extended their investigations to understanding how algebra can be presented along with arithmetic concepts even in early elementary classrooms (Carpenter, Franke, & Levi, 2003; Carpenter, & Levi, 2000; Carpenter, Levi, & Farnsworth, 2000; Falkner, Levi, & Carpenter, 1999; Farnsworth, 2003).

In the following sections, the literature on CGI will be further reviewed. First, an overview of the professional development workshops will be explained. Second, the frameworks of student learning with elementary mathematics will be discussed. This section will include an outline of the arithmetic word problem types, a brief section about the number of digits within problems, and a description of possible solution strategies.

Third, the impact of CGI on teaching will be explained which shall include a section about how CGI has commonly appeared across various CGI classrooms.

After the foundational overview of CGI, the main intention of this paper is to introduce and examine the current research. The research project is a collaboration between the author and an educational consultant of the Iowa Department of Education. The educational consultant created a collection of arithmetic word problem probes based upon CGI principles. The word problem probes were administered by a group of Iowa elementary teachers, who were in their third year of CGI professional development training during the data collection period, to their students as part of regular mathematics instruction. The teachers then assessed their students' work on the word problem probes. One of the areas that the teachers assessed was to classify the students' solution strategy that was used to solve the probe. After the teachers completed the probe administration and made their evaluations, the educational consultant, who was also in her third year of CGI training during the data collection period, assessed the word problem probes for the same standards as the teachers. Based upon the available data, the interrater reliability of the evaluators' judgments on the probes will be analyzed to better understand the research question. Specifically, the research question is to what extent do equally, extensively CGI trained raters agree on CGI- based word problem probe classifications regarding accuracy, correctness of solution strategy, and type of solution strategy? Finally, results will be discussed for implications of the probes and CGI.

CHAPTER 2

COGNITIVELY GUIDED INSTRUCTION

To fundamentally understand what CGI is, the CGI teacher professional development program needs to be described. The professional development workshops focus on improving student learning and achievement through first working to develop the teacher's knowledge and skills. The relatively intensive workshop sequence engages teachers in various ways to educate them about important differences within arithmetic word problems and students' solution strategies.

CGI Professional Development Workshops

The purposes of the professional development workshops have been (a) to enhance teachers' knowledge about the basic types of arithmetic word problems and the various sophistications of modeling within children's solution strategies and (b) to encourage teachers to apply that knowledge when teaching their students (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Carpenter, Franke, Jacobs, Fennema, & Empson, 1997; Fennema et al., 1996; Villaseñor & Kepner, 1993; Warfield, 2001). CGI is not a curriculum or textbook to be prescribed for use in a classroom for elementary mathematics education. The professional development workshops are conducted to broaden teachers' mental framework and pedagogical philosophy about their elementary students. Concepts, ideas, and learning materials are shared with the teachers at the workshops, however, no materials are provided to the teacher participants with the sole intention of being used as explicit instruction activities. Planning time is built into the workshops to encourage the participants to generate ideas and materials that can be used when they return to their classrooms, such as lesson plans. CGI workshops are designed to bring about change in the classroom by educating teachers with CGI's research-supported frameworks of student learning. Similar to any other program that is designed to improve staff skills rather than modify teaching materials such as textbooks, initial and continued change relies upon teachers who embrace and implement the new pedagogy with their students and upon administrators who support the teachers with all of the resources needed to effectively and efficiently make the change.

To support teachers in using this new knowledge of student learning, the CGI workshops can be described by two primary methods of engagement used with the teacher participants at the workshops. One of the primary engagement methods involves teachers learning through experiences with student examples, which occurs mainly by observing videotaped sessions of children. The student examples provide an opportunity for teachers to observe how children respond and solve some mathematical word problems. The student examples contain both how students effectively and ineffectively attempt to solve the problems.

The other primary method has the teachers engaged in open discussion groups where they talk with other teachers about how CGI may or has affected their classroom. While the teachers are engaged with discussion, they are able to have personal questions answered and personal experiences used in educating the other teacher participants. The workshop facilitator uses discussion points to more appropriately meet the professional development needs and concerns of the teacher participants.

Complete participation in the CGI workshops is relatively rigorous. Professional development workshops sponsored by CGI's original authors have between two to five professional development days per year. Workshop participation will also last for three years, which will add up to over ten days of CGI training in its intended workshop design (Levi, 2007). However, some of the training programs conducted for research by the authors of CGI were even more intensive; Carpenter, Fennema, Peterson, Chiang, and Loef (1989) used a four-week long summer training session for their experimental group in that particular study. The author-sponsored workshops also require a school-based team to be involved with training. The school-based team is meant to ensure more structure, support, and resources for the teachers while they transition to using CGI principles. Experienced CGI teachers have noted that change was difficult initially for them, but the early struggles were worth the eventual outcomes (Carpenter et al., 1999).

As a basic overview of CGI and its professional development workshops have been addressed, the following sections will examine the knowledge bases of how young children learn mathematics. The areas of learning that will be covered include descriptions of the basic arithmetic word problem types, a brief discussion of the number of digits used in problems, and explanations of the various solution strategies that are commonly used by children.

Learning Mathematics through Word Problems and Solution Strategies

The concepts underlying how children naturally learn to use mathematics are a key focus of CGI and its knowledge basis. A major research contribution and primary focus of the professional development workshops of CGI has been its organization of mathematical word problem types and the subsequent solution strategies. This knowledge basis of student learning is vital for teachers to enable them to more effectively use CGI in their elementary classrooms. This knowledge basis allows teachers to better understand how children typically work with arithmetic word problems, which is different from how adults tend to work with the same problems. Carpenter et al. (1999) simply stated that "children have different conceptions of addition, subtraction, multiplication, and division than adults do" (p. 1). Young children are still learning how to structure mathematical concepts in their minds, while adults have already formed many effective methods for working with simple arithmetic problems. Even though adults tend to have effective strategies of working with simple mathematics, it is important for children to find and use their own meaningful methods to solve mathematical problems. Teachers are important in fostering a learning environment that allows students use their own unique, intuitive understanding of mathematics. To better prepare teachers to foster such an environment, the research basis of CGI has outlined some of the important differences between the addition, subtraction, multiplication, and division word problem types, considered how the number of digits influences problems, and described the different forms of modeling within the solution strategies.

Basic Word Problem Types

The word problem types are focused on the arithmetic operations, which are addition, subtraction, multiplication, and division. The arithmetic operations are the major focus of the word problems because those mathematical operations are a beginning level of mathematical understanding and serve as a primary focus of the elementary grades. The basic word problem types are categorized by how the question is presented, including the known and unknown components in the problem. Essentially, the specific word problem types are classified by the part of the problem that is unknown, which affects how children solve the problem.

Addition and subtraction problems. The types of mathematical word problems for addition and subtraction have been broken down into four basic categories, which are join problems, separate problems, part-part-whole problems, and compare problems (Carpenter et al., 1996; 1999; Carpenter & Moser, 1984; Carpenter, Moser, & Bebout, 1988; Fuson, 1992; McClain, Cobb, & Bowers, 1998). The addition and subtraction problems tend to increase in difficulty in that respective order.

Join problems. Join problems involve a direct or implied action of increasing a set by a particular amount over time, which are fundamentally presented as an addition problem. Each join word problem has three components. Those components are the Start, Change, and Result. A join problem example with all parts known would be the following: "Nicole has 3 cookies (Start). Then, she gets 2 more cookies (Change). Nicole now has 5 cookies all together (Result)."

There are three classifications of join problems. Join, Result Unknown problems occur when the result is the unknown part of the word problem. The starting point of the problem and the additional changing factor of the problem are known. Join, Change Unknown problems have the starting factor and the result of the problem as known components, but the changing factor in the word problem is unknown. Join, Start Unknown problems occur when the starting number of the problem is known; however, the word problem contains the result and the changing factors of the problem. Table 1 contains examples of each join problem type.

Separate problems. Separate problems are only different from Join problems in the notion that they are presented as subtraction problems. The separate problems have a direct or implied action of decreasing a set of numbers by a particular amount over time. Every separate word problem has three components, which are a Start, Change, and Result. A separate problem example with all parts known would be said, "Troy has 6 cookies, (Start) but he eats 2 cookies (Change). Now, Troy has 4 cookies (Result)."

Separate problems also have three classifications similar to Join problems. Separate, Result Unknown problems are presented as word problems where the result of the situation is unknown but the starting factor and changing factor are known. Separate, Change Unknown problems lack the change factor but contain the starting factor and result of the word problem. Separate, Start Unknown problems are the problems where the starting factor of the situation is unknown but the word problem has the changing factor and the resulting factor. Table 2 contains examples of each separate problem type.

Part-part-whole problems. Part-part-whole problems are somewhat different from Join and Separate problems. Part-part-whole problems have no change over time, do not have a direct or implied action, and the factors are not exactly the same categories, i.e. a whole of tennis players with a part of male tennis players and a part of female tennis players. In other words, part-part-whole problems involve a fixed relationship between one larger group and two similar but slightly different subgroups. A more specific example of a part-part-whole problem with all parts known would be the following:

"Emily has 3 sugar cookies (Part) and 4 brownies (Part). She has 7 treats (Whole) in total."

Since the part factors are considered to do the same task, there are only two distinctions for problem types. Part-Part-Whole, Whole Unknown problems have the two part factors known but the resultant whole factor unknown. The Part-Part-Whole, Part Unknown word problem has one of the part factors as unknown but the other part factor and the whole factor are known. Table 3 contains examples of each part-part-whole problem type.

Compare problems. Compare problems are the final category of problem types for addition and subtraction problems. Compare problems are similar to part-part-whole problems because they both have relationships between different groups as the focus of their distinction. As the title of this category suggests, compare problems have a comparison relationship between three number groups, which are also known as the components of a compare problem. The three components are the Referent, the Compared Quantity, and the Difference. An example of a compare problem with all of the components known would be the following: "Rick has 5 cookies (Referent), and Amanda has 7 cookies (Compared Quantity). Amanda has 2 more cookies than Rick (Difference)."

Corresponding with the three components of a compare problem, there are three compare problem types. Compare, Difference Unknown problems have the difference unknown while the referent and the compared quantity are known. A Compare, Compared Quantity Unknown word problem has the referent set and difference present in the problem, but the compared quantity is unknown. The Compare, Referent Unknown problems have the compared set and the difference known in the word problem but not the referent. Table 4 contains examples of each compare problem type.

Multiplication and division problems. The essential components of multiplication and division word problems can be described by one basic problem type, while addition and subtraction word problems have four basic problem types (Carpenter et al., 1999; Greer, 1992). This basic problem type involves the assumption that the word problem can be grouped or partitioned into equal groups and does not have a remainder. That assumption is made to simplify the explanation of multiplication and division problems. In addition, children tend to view and solve most possible manifestations of multiplication and division problems in the three different forms of the basic problem types.

The one basic multiplication and division word problem type can be used as three different problem types. Those problem types are Multiplication, Measurement Division, and Partitive Division. Basic multiplication and division problems have the three components of the number of groups, number of items per group, and the total number of items. An example of a multiplication and division word problem with all parts known would be the following: "Nick has 4 bags of cookies (number of groups). There are 6 cookies in each bag (number of items per group). In total, he has 24 cookies (total number of items)."

Similar to the addition and subtraction problems, the difference between the three multiplication and division problem types is based on the unknown component in the problem. Multiplication word problems have the total number of items as the unknown factor, while the number of groups and number of items per group are known. Measurement Division problems have the number of items per group and the total number of items known, but the number of groups is unknown. Partitive Division problems have the number of items per group as the unknown. The number of groups and the total number of items are known in Partitive Division word problems. Table 5 contains examples of each multiplication and division problem type.

Multiplication and division word problems can be fashioned to have remainders. Problems with remainders are generally not much more difficult for children than multiplication and division problems that have equal groups and no remainder. When children begin working with multiplication and division problems that have remainders, the possible meaning of a remainder should be explored with the children to ensure that they understand what a remainder means in different scenarios.

Even though fundamentals of multiplication and division problems can be described by one basic problem type, there are several ways to present related concepts with multiplication and division word problems. Some related concepts include problems that involve rate, price, area, array, or multiplicative comparisons. When multiplication and division problems are presented for those different concepts, the problems tend to follow the three fundamental components of the basic multiplication and division problem types. Even when those related concepts are incorporated into the word problem, children tend to solve multiplication and division problems with similar solution strategies (Carpenter et al., 1999).

Number of digits. CGI teachers have two basic considerations when creating an arithmetic word problem for their students. First, the teacher must consider which problem type is most suited for the current instructional goals. For example, the teacher must figure if a Join, Result Unknown problem or a Compare, Difference Unknown problem is more suitable for mathematics instruction. Second, the CGI teacher considers how small or large of numbers will be used with the chosen word problem type. The type of the word problem and the number of digits in the word problem are the two basic elements that affect the complexity and difficulty of the word problem.

The most fundamental problems have one-digit numbers for components of the word problem. One-digit problems have been the majority of the previously presented example problems with all parts known. One-digit problems tend to be the foundation of known facts once a child's mathematical understanding develops into more efficient and abstract solution strategies. When the number facts about a problem are unknown, children can use other known facts and adapt those facts to fit the problem's scenario.

When problems are based on addendums or factors that have two, three, or more digits, those problems are considered multidigit problems. Since those problems contain larger numbers, children tend to use strategies that are based on their developed understanding of base tens (Carpenter et al., 1997; 1999; Fuson et al., 1997; McClain et al., 1998). When children approach these more difficult, multidigit problems, they tend to use algorithms. Those can be standard algorithms or invented algorithms. Invented algorithms allow students to practice and learn different ways of problem solving within mathematics. Invented algorithms represent a developed sense of abstract thought for mathematics; however, standard algorithms that are known to a person represent the most abstract and efficient way to solve particular multidigit problems.

Solution Strategies

There are various solution strategies for the different arithmetic word problems previously discussed. The solution strategies have also been broken down into three categories of distinguishable strategies of modeling, and the categories are Direct Modeling, Counting, and Number Facts/Algorithms (Carpenter et al., 1999; Fuson, 1992; Ginsburg, Klein, & Starkey, 1998; Kouba, 1989). It may be easiest to picture the solutions strategies as forms of modeling which originate as very concrete, physical forms of modeling, then become mentally-based forms of modeling, and eventually are solid mental concepts of number facts and their relationships in formal procedures. It is important to understand the different problems types because children tend to view and work with each kind of the word problem types in important and different ways (Ginsburg et al.). Some solution strategies are more appropriate and common for particular problem types (Carpenter et al., 1993; 1999; Fuson). The following sections include detailed description of the three categories of modeling along with narratives of the commonly used solution strategies that have been observed for the Direct Modeling and Counting categories.

Direct modeling. Direct Modeling is the most concrete category of solution strategies. Direct Modeling is a directly observable task where the child manipulates physical tools including fingers or blocks to organize and represent the problem's components. Direct Modeling can include physically producing objects such as shapes or tally marks on a piece of paper. These physical objects are used to manage all of the information and quantities for solving the word problem.

Children can solve mathematical word problems in an infinite number of ways; however, some common solution strategies have been observed when children solve the basic word problems. For the addition and subtraction word problems, six common Direct Modeling solution strategies have been observed and categorized. Those six strategies are (a) Joining All, (b) Joining To, (c) Separating From, (d) Separating To, (e) Matching, and (f) Trial and Error. Table 6 contains examples of each Direct Modeling solution strategy for addition and subtraction.

Joining All strategy. Joining All is a solution strategy that occurs when a child uses objects to represent each number in the word problem. After the child physically represents all known parts of the word problem with objects, he then *joins all* of the objects together and counts the total of the objects to find the correct answer. The Joining All solution strategy is used to solve Join, Result Unknown and Part-Part-Whole, Whole Unknown problems.

Joining To strategy. Joining To is a solution strategy where the child begins with a group of objects based upon of the smaller number presented in the problem. Then, he uses more objects to *join to* the smaller number until it totals the larger number that was presented in the problem. Finally, the child counts the group that was added to the smaller group to find the correct answer. The Joining To solution strategy is used to solve Join, Change Unknown problems.

Separating From strategy. Separating From is a solution strategy that begins with the child representing the larger number presented in the problem. Then, the smaller number is *separated from* the larger group. The remaining objects are counted to find the correct answer to the problem. The Separating From solution strategy is used to solve Separate, Results Unknown problems.

Separating To strategy. Separating To is a direct modeling solution strategy where the child starts by using objects to represent the larger number presented in the problem. Then, objects are removed from the larger group until the original set equals the smaller number presented in the problem. In other words, the child *separates to* the smaller number from the larger number. Finally, the child counts how many objects were removed to find the correct answer. The Separating To solution strategy is used for Separate, Change Unknown problems.

Matching strategy. Matching is a solution strategy used to make one-to-one correspondence between two sets of numbers. First, the child creates two sets of objects, each set representing a respective number as presented in the problem. Then, the child *matches* pairs between the two groups. Finally, the child counts the remaining unmatched objects from the larger group to find the answer of how many more objects are in the one group. The Matching solution strategy is used for Compare, Difference Unknown problems.

Trial and Error strategy. Trial and Error is a solution strategy that happens when a child attempts to systematically guess and check possible solutions for the problem until the appropriate relationship is found. To begin, the child will use objects to *try* different scenarios, while possibly making *errors*, to discover the correct answer. The Trial and Error solution strategy is used for Join, Start Unknown and Separate, Start Unknown problems.

As the six Direct Modeling solution strategies have been observed for eight of the eleven addition and subtraction problem types, the three remaining problem types have not been observed with a single commonly used strategy. However, those three problem types have been observed with solution strategies that are usually or generally used for solving each specific problem type. The Joining All strategy is usually used for Compare, Compare Quantity Unknown problems. The Joining To and Separating From strategies are generally used for Part-Part-Whole, Part Unknown and Compare, Referent Unknown problems.

For the three multiplication and division problem types, three common Direct Modeling solution strategies have been observed and categorized. Each problem type has its own common solution strategy. The three strategies are (a) Grouping, (b) Measurement, and (c) Partitive. Table 7 contains examples of each Direct Modeling solution strategy for multiplication and division.

Grouping strategy. Grouping is a solution strategy where the child uses objects to model each group presented in the problem with the respective number of items in each group. Then, the child will simply count the total of the objects to find the correct answer. The Grouping solution strategy is used for Multiplication problems.

Measurement strategy. Measurement is a solution strategy that occurs when the number of groups is unknown to the child in the problem. In other words, the child obtains the number of items per group and the total number of items from the word problem. With the known information, the child uses objects to make groups that contain the number of items per group based upon the total number of items. A slight variation of this solution strategy involves whether the child counts the objects for the total number of items at the beginning or ending of the solution process. The child will count the number of groups to find the correct answer. The Measurement solution strategy is used for Measurement Division problems.

Partitive strategy. Partitive is a solution strategy commonly used when the total number of items and the number of groups are presented in the problem to obtain the solution. Some children may sort the objects one-by-one into the specified number of groups until the object are used up. Other children may start with more than one object in each group. If the child starts with too few objects per group, he will sort out the remaining objects until no objects remain. If the child begins with too many objects per group, he will remove objects from the groups that were created until the correct number relationship or action is obtained. Finally, the child will count the objects in one group to find the correct answer. The Partitive solution strategy is used for Partitive Division problems.

<u>Counting</u>. Once Direct Modeling concepts become more grounded and instilled during the child's development, Counting solution strategies become more frequent. Counting strategies are more abstract because the child moves from using physical

objects to mental representations of numbers in the word problems. Otherwise, the idea of organizing the information and modeling of objects is still rather similar for Counting as it was for Direct Modeling. A child may use physical tools while using a Counting strategy, but the tool is only used for organization, such as to keep track of a number count. For a Counting strategy, the mental representations of the child's mathematical understanding must occur while an organizational tool is used.

Distinct counting strategies commonly used by children have also been observed. Five common counting strategies for addition and subtraction word problems have been categorized. They consist of (a) Counting On, (b) Counting On To, (c) Counting Down, (d) Counting Down To, and (e) Trial and Error. Table 8 contains examples of each Counting solution strategy for addition and subtraction.

Counting On strategy. Counting On is a solution strategy where the child mentally counts on from one number with the other number presented in the problem. There are two slightly different variation of Counting On, which are Counting On From First and Counting On From Larger. Counting On From First occurs when the child simply *counts on* from the first number presented in the problem with the second number. Counting On From Larger happens when the child begins counting from the larger number presented in the problem by the smaller number. For both variations of the Counting On strategy, the final correct answer is the total of both numbers that were presented in the problem. This solution strategy is used for Join, Result Unknown and Part, Part, Whole, Whole Unknown problems.

For example, a child may encounter a problem such as “Timmy has 2 apples. He gets 4 more apples. How many does he have all together?” Some children when using the counting on strategy will begin at two and then mentally count on four more units to get a total answer of six. Other children will commonly begin at the larger number, which is four in this example, and add on the smaller amount, two, to find the answer, six. If the larger number is the first number presented in the problem, the two slight variations of the Counting On strategy make no difference for this form of a problem.

Counting On To strategy. Counting On To is a solution strategy where the child starts with the smaller number in the problem. Next, the child counts up to the larger number beginning at the smaller number. The child pays attention to the amount that was *counted on to* the smaller number to reach the larger number to find the correct answer to the problem. The Counting On To solution strategy is used for Join, Change Unknown problems.

Counting Down strategy. Counting Down is a solution strategy that involves a child counting backwards to find the answer. The child starts with the larger number. From the larger number, the child *counts down* the amount of the smaller number. The number that the child stops at is the correct answer to the problem. The Counting Down solution strategy is used for Separate, Result Unknown problems.

Counting Down To strategy. Counting Down To is a solution strategy where the child again counts backward to find the answer. The child begins at the larger number. Then, the child *counts down to* the smaller number. The solution to the problem is the amount that lies between the larger number and the smaller number. The Counting Down To solution strategy is used for Separate, Change Unknown problems.

Trial and Error strategy. Trial and Error is essentially the same solution strategy to the Direct Modeling strategy of the same title, but the distinct difference between the two strategies is the level of abstractness used by the child. The Counting strategy of Trial and Error involves systematically guessing and checking to find the correct solution relationship. When Trial and Error is considered a Counting strategy, the child does not use objects to physically represent numbers. The child may use objects, such as fingers or blocks; however, the objects are only for organization and keeping track of the counting sequence. The child will use this strategy of systematic guesses and checks until the correct number relationship is found. The Trial and Error solution strategy is used for Join, Start Unknown and Separate, Start Unknown Problems.

As the five Counting solution strategies have been observed for seven of the eleven addition and subtraction problem types, there are four remaining problem types have not been observed with a single commonly used strategy. However, the four problem types have been observed with corresponding solution strategies that are usually or generally used for solving each specific problem type. The Counting On strategy is usually used for Compare, Compare Quantity Unknown problems. The Counting On To and Counting Down strategies are generally used for Part-Part-Whole, Part Unknown; Compare, Referent Unknown; and Compare, Difference Unknown problems.

Children will gradually begin to use counting strategies to replace the less efficient direct modeling strategies. Similar to the direct modeling strategies, children tend to use some common counting solution strategies for solving the multiplication and division problem types. One important note to make is that some problems are more difficult and less conforming to use counting strategies. Three common counting solution strategies for multiplication and division problem types have been observed. They include (a) Skip-Counting, (b) Addition and Subtraction strategies, and (c) Trial and Error. Table 9 contains examples of each Counting solution strategy for multiplication and division.

Skip-Counting strategy. Skip-Counting is a solution strategy where the child counts by a particular number group while skipping other numbers. The child will tend to use number groups that are better known to child, such as counting by threes or fives. For numbers that are not well known, the child may begin skip counting as far as he can but will need to finish counting by ones instead. The Skip-Counting solution strategy is used for Multiplication and Measurement Division problems.

Addition and Subtraction strategies. Addition and Subtraction strategies are solution strategies that are some children use to solve multiplication and division word problems. A child will repeatedly add or subtract the components of the problem to find the solution. Fundamentally, repeated addition is the same as Skip-Counting, and repeated subtraction is the same as Skip-Counting in reverse. However, many children tend to think of Skip-Counting and Addition and Subtraction strategies as different methods. Doubling is an addition strategy where the child repeatedly adds the components of the problem by adding doubles as much as possible and includes any remainder in the final answer. Addition and Subtraction strategies are used for Multiplication and Measurement Division problems.

Trial and Error strategy. Trial and Error is a solution strategy of systematic guesses and checks to find the correct answer. Children will attempt different relationships and actions in the pursuit of answering a word problem. For example, a child may learn from a problem that there are four equal groups and twenty-eight total items but the number of items per group is unknown. A child may attempt using equal groups of different numbers, such as five, six, eight, or nine items per group, with the eventual finding that seven items per group fits the relationship of four equal groups and twenty-eight total items. The Trial and Error solution strategy is used for Partitive Division problems.

Number facts/algorithms. Number Facts and Algorithms both demonstrate the most complex, abstract, and efficient solution strategies for solving the mathematical word problems for children. For clarification, Number Facts are more specific to problems with factors and addendums that are below ten and use one-digit concepts, while Algorithms are relevant to problems with factors and addendums that are greater than ten, use base concepts of ten, and have multidigit concepts. Even with the respective differences, Number Facts and Algorithms are fundamentally similar process when considered as a solution strategy category.

Known Number Facts and Algorithms tend to start with a few facts and algorithms that tend to become settled, known, and standard for the child. Children then will use invented strategies that are derived from the known facts and algorithms as a source in solving a more complex problem. Those invented Number Facts and Algorithm strategies develop into other known mental understandings with additional experience. Based on efficiency and overall comprehension, known facts and standard algorithms are considered higher forms of developed mathematical thinking than derived facts and invented algorithms. An example of a derived fact would be if a child knows that five groups of six equals thirty and two more groups of six to that group would make forty-two. An example of a known fact would be simply that the child knows from experience that seven groups of six equals forty-two, which is more established and efficient for that problem than generating that fact based on other known facts. It is important to note that derived facts and invented algorithms are very helpful strategies for children to use for resourcefully solving various word problems; however, known facts and standard algorithms are the most abstract and efficient methods for solving a word problem.

There are some more points that need to be discussed about solution strategies beyond their definitions in relation to CGI. Children tend to revert back to the earlier, more concrete strategies of Direct Modeling and Counting when working with more complex problems and larger numbers that are not normal mathematical encounters. For example, a child can know the mathematical fact of two plus two equals four when solving a word problem; however, that same child may need to directly model with objects for a word problem based upon the mathematical face of twelve plus twelve equals twenty-four.

Even as all of the distinctions between basic word problem types and solutions strategies have been discussed, it is important to mention the disclaimer presented by

Carpenter et al. (1996). The acknowledgement must be noted that "not all problems or all children's strategies fall neatly into distinct categories" (p. 13). However, Carpenter et al. (1996) explain that the distinctions between basic arithmetic problem types and solution strategies that were presented are still very appropriate for understanding most situations, especially for elementary mathematics instruction. Carpenter et al. (1996) also admitted that CGI itself cannot explain all levels of mathematical thinking for children. Yet, CGI contains a concentrated look at basic addition, subtraction, multiplication, and division word problems and the various basic forms of modeling in solution strategies, which should be a beginning point for a pedagogical framework of working with elementary children in mathematics.

The CGI frameworks of student learning, being the basic arithmetic word problem types and children's solution strategies, have been described at length. The focus on how students learn mathematics naturally and solve basic mathematical word problems has been large portion of the research that has affected the CGI teacher professional development program. After the teachers have participated with the CGI workshops, the usual concern arises with how the teacher will take their new knowledge and apply it effectively to their classrooms. Teachers who have effectively implemented instructional practices that support CGI principles have been interviewed and observed to better understand how they teach. The following sections will examine the effective teaching methods of CGI teachers.

Teaching Mathematics with Word Problems and Discussions

Because CGI works mainly at enhancing the teacher's pedagogical philosophy and is not a formal curriculum, instructional practices can vary from teacher to teacher and from classroom to classroom. Carpenter et al. (1999) explained that there is no typical CGI classroom because each CGI classroom has unique features when the research has been implemented into instructional practice. However, there have been some notable observations have occurred across different CGI teachers.

Similar Teaching Methods across CGI Classrooms

Even though specific instructional activities vary across CGI teachers, some general instructional practices have been observed in classrooms with teachers who have embraced CGI principles. Those general instructional practices can be broken down into four fundamental steps. First, the teacher poses a word problem to the students. Second, the teacher allows the students to choose their own method for solving the problem, with adequate time and resources. Third, the students report their solution strategies to the group to let the other students learn they each uniquely solved the problem. Finally, the teacher asks questions to clarify the children's strategies and discusses issues that signify similarities, differences, and relationships presented in the student reports (Carpenter et al., 1997; 1999; Fennema et al., 1996; Fuson et al., 1997).

A word problem is presented. To start the mathematics instruction process, an arithmetic problem will be posed to the students. Problems can be created by the teacher or by the students. If the teacher creates the problem, the teacher may guide the questions for a few particular reasons. Possible reasons could be to focus on an area that the students have been having difficulty with solving, to provide a variety of problem types for a broader experience, to enhance reporting skills with easier problems, and to encourage transitioning to a more sophisticated and efficient solution strategy, which is the ultimate goal for the teacher. The teacher is a facilitator toward meaningful knowledge and not a provider of meaningless strategies for students. Problems can also be related to other subjects and topics, such as science, social studies, and daily activities including attendance and hot lunch count.

It is important to note Carpenter et al. (1999) explained that it is common for only a few problems to be presented and solved during mathematics instruction. Only a few problems are covered because it is incredibly important in the CGI classroom to address the problem solving process thoroughly. Complete, elaborate student input about the exact solution strategy used is the cornerstone in truly understanding the mathematical thinking of each child.

One aspect of instructional decision making that affects CGI teachers is how they use the mathematics textbooks for teaching. Occasionally, CGI teachers may continue to use their textbooks for examples of suitable word problems for instruction. However, it is much more likely that mathematics textbooks are not used as the books were originally designed by teachers who use CGI principles in their classrooms. Some CGI teachers may continue to follow the mathematics textbook, but those teachers will typically supplement instruction with word problems that they created. Other CGI teachers have eliminated their mathematics textbooks completely from classroom instruction, even though this step has not been advocated by the CGI authors (Carey et al., 1995; Carpenter et al., 1999). The teachers quit using their textbooks because the textbooks did not lead mathematics instruction in the direction that the teachers desired. Instead, the CGI teachers created their own relevant, specific mathematical word problems to enhance and guide mathematics instruction for their students.

Problems are often presented to children either as a whole group or as small groups. This grouping method becomes useful once the children share their solution strategies to the group. Even though the students are in a group, the problems are typically solved individually. Students could work as a group in generating solution strategies for a particular problem; however, working as a group may allow some assertive students to use only their strategies and may not allow some passive students to have their strategies to be heard. If the students solve the problems by themselves, it should ensure that each child chooses his own solution strategy for the problem.

Student chooses own solution strategy. In the second step, each student will choose how he/she wants to answer the question autonomously. This method of allowing the children to choose their solution strategies relates closely to the thesis of CGI, which is that children have a great deal of informal knowledge about working with mathematics which should serve as the basis of their development. Word problems are a natural way that people encounter math, which includes children's experience before they start school.

There are various ways a child can solve a problem, and those various methods must be planned for before mathematics instruction. The solution strategy that generally requires the most resources is the concrete, physical manipulations of tools used with direct modeling. When direct modeling is the solution strategy of choice, students need to have tools such as paper, pencil, and various forms of simple counters. Those counters can include individual conjoining blocks, base-ten counting objects, and other apparatuses that are easy to organize clearly. If those tools are unavailable, the students' progress toward more abstract and efficient mathematical thinking can be greatly hindered. It would be similar to asking a carpenter to build a cabinet without any of his tools.

It is important to mention that teachers in CGI classrooms do not tend to give explicit instruction, traditional algorithms, or their own effective solution strategies to their students. CGI is designed to allow children to find their own meaningful and effective solution strategies; however, the teacher is crucial as a support for the students. Teachers are encouraged to guide children through their questions, but the child is to find and use the solution strategies that are understood and successful for him/her.

Students report solutions to group. For the third instructional step, the students report their solutions to the group. There are a few purposes for reporting answers. The primary purpose for the teacher is to better understand the students' cognitions as they report their solution techniques. Teachers and students will learn by hearing and seeing how other students solve a problem. The main purpose of reporting answers for the students is for the opportunity to learn from and with their peers' strategies. Then, they can have more mathematical problem solving tools to use. Students can also learn from themselves by making self-revelations when reporting their method of solving the word problem or comparing it to others' strategies. Reporting solution strategies requires students to listen respectfully to other people while a person is reporting his answer. Students must also be courteous when incorrect answers are reported because any student can make a solution error. Most teachers would appreciate a classroom rule of listening respectfully, which is a good classroom management strategy and skill for children.

Solutions can be reported to the entire class, small groups, or even individually to the teacher. Individual reporting would be uncommon in a CGI classroom, although it could be suitable for some situations. It could also be very useful for establishing trends to have each child keep a mathematics journal for the problems and solutions they cover during class. For a student to report only to a teacher, it would achieve the purpose of informing the teacher that particular student's mathematical thinking with that particular question. However, it does not achieve the purpose of allowing other students to learn from that child's particular solution. In addition, children need input from the teacher when the solution strategies are unclear, confusing, or inaccurate. Not all children need to report for every question presented, but student responses and methods should be kept in a mathematics journal to be reviewed by the teacher at a later time.

What is a teacher to do with a diverse classroom where some students will solve the problem much quicker than other students? There are a few simple ways to address that concern. One simple way would be to have those high performing children solve the problem in more than one way. That can allow the child to self-discover new solution strategies and mathematical relationships, but that should not be encouraged as a way to meaninglessly fill that student's time. There is also the simple method of changing the number size of a problem to provide a more appropriate learning experience for some students who need a more difficult challenge. A teacher can also have the student put the word problem in the form of a number sentence, which can assist transitional information from broad word problems to number-specific terms.

It should be remembered that a student who is very successful in one academic area, such as mathematics, can provide very meaningful insight and input to his classroom peers. As the teacher's understanding about the students' cognitions about mathematics is important for the teacher's success in guiding the classroom, the side benefit of the teacher learning something new from a student, of any range of skill and ability, should always be welcome. Teachers should always be learning along with their students.

Teacher asks questions for clarity and discussion. As the final step in the learning process with a posed word problem, it is very appropriate for the teacher to address any confusing techniques and to bring up any discussion points from the students' strategies. This step can be very lengthy, or it can be the shortest step depending on what the students report. It may also be very intertwined with the reporting solutions step. If the teacher is unclear about a child's solution method, it is the teacher's responsibility to probe with questions to better clarify the child's intended communication. The case can easily occur when a student reports a solution strategy that is different from what they actually did. That is an important scenario for skillful questioning from the teacher.

Discussion questions could include comparing and contrasting the solution strategies that were presented to the class. For example, it may be important to talk about how two students used very similar solution strategies. It may also be very important to converse about how two students used two very different methods such as when derived facts and invented algorithms are used. When several students solve a problem with the incorrect answer, an important learning experience can come from why that problem may have been so difficult for so many. A teacher can also direct attention toward mathematical concepts and terms such as odd and even numbers or other inquisitive number relationships. This opportunity for questions about clarity and discussion is a very important step for students and their mathematical learning.

Differences between Traditional Instruction and CGI

Traditional instruction is based on presenting only some of the standard algorithms to the students and having the children *plug-and-chug* with different numbers. Problems are also presented in number sentences to young children, but number sentences are not normal, real world experiences for those children and their intuitive mathematical understanding. Specific standard algorithms and number sentences are useful and needed for mathematical understanding, but they do not always explain the underlying concepts of mathematics and can be improper or complicating to use in some circumstances. It must be made clear that information such as standard algorithms and number sentences have frequent usage in materials that may appear to children, such as on state and national achievement exams. It would be important to have the children knowledgeable of mathematical practices and experiences that will be used at higher levels of mathematics, including algebra, trigonometry, and calculus.

Traditionally, teachers would lead classroom instruction by presenting students with a lecture about a mathematical concept. After the explicit instruction lecture was completed, activities such as worksheets and textbook questions are assigned to children where grades are based on the correctness of the answer. The worksheets could include the student solving several number sentences that may not be meaningful to the student's innate understanding of mathematical problem solving. The teacher is prescribed a curriculum in a rigid format and asked to solve questions that were only discussed by the teacher in front of the entire class.

A CGI classroom would be different from traditional classroom lesson format. The teacher presents a word problem that is meaningful to the students and is based on the teacher's knowledge of the students' current mathematical strengths and weaknesses. After the students are allowed time to construct their own strategy for solving the problem, the students engage in an open discussion about answers and the strategies that they used to find those answers. The teacher facilitates the open discussion and does not simply implement solution methods that may not relate to the children's current level of mathematical problem solving skills. The teacher clarifies strategy similarities and differences and mathematical concepts and relationships that the students discussed.

When compared to traditional mathematics instruction, CGI emphasizes the students' mathematical understanding through problem solving rather than rote memory that is not comprehended by the student (Carpenter et al., 1996). Children learn basic number facts through their repeated experiences with the arithmetic word problems and group discussions; however, the ability to problem solve rather than simply recall facts is the general focus of CGI. When the frameworks of CGI are used in the classroom, children are supported with a learning environment that builds upon prior knowledge and allows them to discover and use solution strategies that are useful and comprehendible. The solution strategies may be based upon real-life and everyday question or based upon other subjects so the students are able to construct meaning and learn about more than one instructional subject during a classroom lesson (Vacc & Bright, 1999). While the students learn about mathematics, teachers are also engaged in continual learning about their own understanding of mathematics while observing and understanding how students creatively solved a particular problem (Peterson et al., 1989). Also when compared to a control group of traditionally-instructed students, kindergarten students in classrooms where the teacher had attended CGI workshops had higher problem-solving achievement than their peers of the control group (Carpenter et al., 1989). Even though the CGI teacher professional development workshops cannot answer all aspects of student learning of mathematics, CGI has led to many positive outcomes for student learning and teaching of mathematics in the elementary grades (Carpenter et al., 1996).

Following this extensive literature review of CGI, the remaining sections will address the research project that was conducted in conjunction with this paper. Arithmetic word problem probes based upon CGI principles were created in order to become a useful educational tool for individuals in the state of Iowa whom have completed at least two years of CGI training that was supported by Iowa's Department of Education. To initially examine the word problem probes, the interrater reliability of the word problem probes were analyzed. In Chapter 3, the design of the study is explained. In Chapter 4, the results of the interrater reliability analyses are described. In Chapter 5, discussions concerning the results, implications, and limitations of this research project are elaborated.

CHAPTER 3

METHOD

Participants

Seven elementary teachers in the state of Iowa volunteered to participate in the research project. During the academic year of 2006-2007, the teacher participants were currently teaching in a range of elementary grades in the state of Iowa. An educational consultant with the Iowa Department of Education also volunteered to participate. During the academic year of 2006-2007, all participants were currently participating in their third year of CGI professional development training sponsored by the authors of CGI.

Materials

The educational consultant generated a collection of arithmetic word problems probes based upon CGI principles and had the input of one of the main researchers, authors, and trainers of CGI. The word problem probes varied upon grade level appropriateness and word problem type, according to CGI problem types. Each word problem probe consisted of a single, standard sheet of paper with one word problem and blank spaces for student identification, date, and time of completion, which all fit in the top one-fourth of the paper. The bottom three-fourths of the probe was open space for the student's solution strategy and answer.

Procedure

The administration of the word problem probes occurred similarly to typical mathematics instruction in a CGI classroom. Once or twice a week during the spring of 2007 the teacher used one word problem probe as part of the daily mathematics instruction. The word problem probe is posed to each student as a whole class activity where the students work individually. The students began on the word problem probe at the same time after the teacher reads the problem to all of the students. The students worked individually and chose their own method of modeling for solving the problem. Similar to typical CGI instruction, the children were provided with and were allowed to use mathematical tools, such as counting blocks, to solve the probe. Once a student finished with his/her own solution strategy and answer on the word problem probe, the student quietly notified the teacher. After all of the completed word problem probes were collected, the teacher transitioned to the final steps of CGI instruction, which include the students reporting their solution strategies to the group and the teacher asking clarification and discussion questions about the students' solution strategies.

At the teacher's next available time, the teacher assessed the word problem probes in accordance with three specific questions on a record sheet. The first question was whether or not the student had the correct answer. The second question was whether or not the student used a correct solution strategy while solving the word problem type presented in the probe. The third question was what type of solution strategy did the student use in the word problem probe. The categories of solution strategy types consisted of direct modeling, counting, derived facts, known facts, and incorrect strategy. Once the teacher finished assessing the word problem probes and completed the record sheet, the teacher mailed all of the forms to the educational consultant's secretary. The secretary separated the teacher's record sheet from the students' word problem probes.

Then, the educational consultant assessed the word problem probes for the same criteria as the teachers without being influenced by the teachers' assessments.

Analysis

Interrater reliability can be described in terms of percentage of agreement of raters or the percentage of time that the raters agreed. For example, raters could agree on a classification 85% of the time. However, this comparison does not account for change agreement. Cohen's kappa (κ) coefficient (Cohen, 1960; Sim & Wright, 2005) has been recommended by statisticians to account for chance agreement. Cohen's kappa also provides a basis for statistical significance. Cohen's unweighted kappa coefficient is suitable when the assessment categories are nominal and the comparison involved two raters or judge groups. The formula for Cohen's kappa coefficient is $\kappa = (p_o - p_c) / (1 - p_c)$ where p_o stands for the proportion of agreement observed among the raters or judge groups and p_c stands for the proportion of agreement expected by chance among the raters or judge groups. Complete agreement is observed among the raters when $\kappa = 1$. No agreement is observed among the raters when $\kappa = 0$ (Cohen; Sim & Wright). All of the elementary teachers were considered as one rater or judge group, and the educational consultant was considered as the other rater or judge group.

CHAPTER 4

RESULTS

In this section the results of the interrater reliability analyses are described. The consistency between the raters' assessments of the solution strategies on the word problem probes is examined for (a) containing the correct answer, (b) containing a correct solution strategy, and (c) the type of solution strategy used.

For the analysis of whether or not the solution strategy contained the correct answer, 1,309 cases containing the assessments of both judge probes regarding the results of the word problem probes were analyzed. The percent agreement between the raters on the same classification of correct answer was 98% of the time. The strength of agreement between the two judge groups on the correct answer was very strong, $\kappa = .926$, $p < .01$. According the benchmarks established by Landis and Koch (1977), a kappa of .926 would have fallen under the strength of agreement benchmark of *almost perfect*, which was the highest benchmark and ranged from .80 to 1.00.

When analyzing the judges' consistency concerning the assessment of whether or not the solution strategy was correct, 1,308 cases were compared for investigating the word problem probes' interrater reliability. The percent agreement between the raters on the same classification of correct solution strategy was 95% of the time. The strength of agreement between the two groups for using a correct solution strategy was strong, κ $=.716$, $p < .01$. Landis and Koch (1977) would have considered a kappa of .716 to be under the *substantial* benchmark, which was the second highest benchmark and ranged from .60 to .80.

In the analysis of the interrater reliability concerning the assessments about the types of solution strategy used, 1,301 cases were analyzed between the two judge groups to investigate the consistency of the judge's assessments. The percent agreement between the raters on the same classification of solution strategy type was 76% of the time. Regarding the solution strategy type used for the word problem probe, the strength of agreement between the two judge groups was rather strong, $\kappa = .673$, $p < .01$. Following the benchmarks set by Landis and Koch (1977), a kappa of .673 would have been within the *substantial* strength of agreement benchmark. The interrater reliability results are consistent with the hypothesis of this research investigation.

CHAPTER 5

DISCUSSION

The results of this investigation provide a fundamental knowledge base for examining the reliability of the word problem probes that were created for this project. The interrater reliability results were consistent with the hypothesis of this research project. The hypothesis was that the evaluators, who were all extensively trained in CGI, will usually agree between their assessments of the solution strategies concerning the word problem probes.

The differences between the percent of agreement of the raters and Cohen's kappa coefficients for the three areas of interrater reliability occurred as expected (Sim & Wright, 2005). Due to the fact that the determination between a correct or incorrect answer is a straightforward assessment, it would be expected to be the most consistently agreed classification between the raters. Even though the assessment of a correct or incorrect solution strategy may not be as straightforward as a correct or incorrect answer, the dichotomous nature of selection a correct or incorrect solution strategy would tend to have a higher percent of agreement and kappa coefficient than an assessment with multiple categories. The percent of agreement and kappa coefficient for the type of solution strategy were fairly strong and the kappa coefficient was statistically significant even though the judges had to choose between multiple possible categories while making their assessment.

Another possible support for the strength of consistency between the judges could come from the notion that the judges were all extensively trained in and experienced with

CGI. The elementary teachers were even using CGI for mathematics instruction in their classrooms on a daily basis, and the educational consultant had a strong research interest in CGI. However, the minimum of training and experience with CGI needed by an evaluator to make reliable assessments about the solution strategies used on the word problem probes is currently unknown.

The results of this research project shall be used for understanding the effectiveness of the word problem probes as an educational tool for CGI teachers. This study investigated the reliability of the word problem probes, which is an essential component to obtain before the validity, or effectiveness, should be explained. The results of this project support the continued examination of these word problem probes as a supplementary educational tool for CGI teachers.

The implications of this study and the subsequent word problem probes affect many people in education. Most obviously, general education elementary teachers who may use the word problem probes are likely to benefit from having a time-saving tool that is based upon CGI principles. These word problem probes could also be helpful in providing data for monitoring how a student progresses in more abstract modeling of their solution strategies over a similar word problem type or how a classroom of students tended to individually solve the same word problem.

These word problem probes could also be very useful for educational staff who does not have consistent or regular contact with particular students. The information about how a student solves an arithmetic word problem could be useful for educational support staff who are trained in CGI. Even more simply, understanding CGI and its principles could be useful to educational support staff, such as school psychologists.

There are important aspects about CGI that can affect school psychologists. Traditionally, school psychologists are trained in administration and evaluation of academic and behavioral assessments. The range of those assessments can vary between behavioral observations, systematic interviews, and tests of academic achievement, cognitive abilities, and psychological, social, and emotional levels (Merrell, Ervin, & Gimpel, 2006). Those detailed assessments are intended to be part of the data collected to support proper judgments and decisions that affect students and their educational programs and settings (Merrell et al; Ysseldyke et al., 2006).

However, the roles of being an interventionist and a consultant of appropriate psychological and educational practices have developed along with being the traditional test administrator and evaluator (Merrell et al., 2006). As a consultant, school psychologists must keep themselves aware of and informed on current educational research and practices which benefit students, teachers, and schools. Yesseldyke et al. (2006) asserted that "school psychologists should be *instructional consultants* who can assist parents and teachers to understand how students learn and what effective instruction looks like" (p. 13). CGI has supportive research about student learning and documented information about what it looks like in effective classrooms of experienced CGI teachers. CGI, accompanied with its knowledge base, has demonstrated benefits for elementary teachers in organizing a framework of how to work with student knowledge to guide mathematics instruction and students in higher academic achievement.

It may appear that CGI is a method largely used in general education settings, and it mainly has been used in general education classrooms. Meanwhile, school psychologists tend to work very closely with special education needs. School psychologists can be an important link of communication and collaboration between special education teachers, general education teachers, parents, and other relevant stakeholders. The school psychologist must be aware of the different methods, concepts, and terms used by all of the people involved and capable of facilitating collaboration among all of those people.

CGI is based on the thesis that *all* children come to school with intuitive and informal methods of working with mathematics which should serve as the starting point for mathematics instruction at elementary school. That thesis would include children who are high-achieving, average, and low-achieving in relation to mathematics performance. The positive effects of CGI have been reported through teacher interviews for children with learning disabilities (Hankes, 1996). The foundation of CGI's thesis would also coincide with the view that school psychologists should work to enhance the achievement and understanding of all students (Yssseldyke et al., 2006).

While the practice of school psychology continues to develop into many broad domains of education, a program such as CGI with its information about student learning and effective teaching methods is beneficial to the knowledge base of school psychologists. Even though a program such as CGI may not have the clearest of connection with the practice of some school psychologists, it contains many applicable concepts that could aid all school psychologists and their work with children, teachers, parents, administrators, and other relevant stakeholders in the education of children.

It is evident that there are some limitations in the design and analysis of this particular research project. A clarification should be made concerning the strength of agreement benchmarks established by Landis and Koch (1977) and used in the analysis of the results in this study. Landis and Koch acknowledged that the distinctions made by the strength of assessment benchmarks were "clearly arbitrary" (p. 165). Despite the benchmarks' arbitrary basis, the benchmarks were helpful in the discussion of Landis and Koch and the analysis of the results of this study.

The generizability of this project is relatively limited. Future investigations could include more individuals trained in CGI and work with elementary students. Also, more concise data about teacher demographics and the adequate amount of CGI training needed for appropriate usage of the probes would be useful for understanding the reliability and validity of the word problem probes.

As the results of this study support that the word problem probes have fairly strong interrater reliability, the validity of the word problem probes should be investigated. If the word problem probes demonstrated strong validity, it would support to notion that the word problem probes are measuring what they are intended to measure. An intention of the word problem probes is collect data on elementary students' complex mathematical problem solving skills used while solving the word problems.

In conclusion, many educational professionals can benefit from understanding CGI and its principles. CGI supports that all elementary students are capable of creating simple and complex solution strategies for mathematical word problems. Most importantly, elementary students use solution strategies that are understood and meaningful. To enhance CGI teachers' effectiveness in using CGI in the classroom, educational tools, such as the word problem probes developed for this research project, could prove to be very useful in monitoring classroom and individual progress and for saving time for ready-to-use instructional word problems. This research project supported that the word problem probes maintain suitable interrater reliability between judges who had equal, intensive CGI training, and the consistency of assessments should be used in progressively investigating the effectiveness of the word problem probes. Future research may lend additional support for educational tools, similar to the word problem probes, to be used by individuals trained in CGI.

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Table 1

Basic Join Problem Type Examples

Problem Type	Example
Join, Result Unknown	Jared has 3 marbles (Start). Tony gives him 5 more marbles (Change). How many marbles does Jared have altogether (Result Unknown)?
Join, Change Unknown	Jared has 3 marbles (Start). Tony gives him some more marbles. Then Jared has 8 marbles altogether (Result). How many marbles did Tony give to Jared (Change Unknown)?
Join, Start Unknown	Jared has some marbles. Tony gives him 5 more marbles (Change). Then Jared has 8 marbles in total (Result). How many marbles did Jared have to start with (Start Unknown)?

Table 2

Basic Separate Problem Type Examples

Problem Type	Example
Separate, Result Unknown	Jennifer has 7marbles (Start). She gives 4 marbles to Lacey (Change). How many marbles does Jennifer have left (Result Unknown)?
Separate, Change Unknown	Jennifer has 7 marbles (Start). She gives some marbles to Lacey. Then Jennifer has 3 marbles left (Result). How many marbles did Jennifer give to Lacey (Change Unknown)?
Separate, Start Unknown	Jennifer has some marbles. She gives Lacey 4 marbles (Change). Then Jennifer has 3 marbles left (Result). How many marbles did Jennifer begin with (Start Unknown)?

Table 3

Basic Part-Part-Whole Problem Type Examples

Problem Type	Example
Part-Part-Whole, Whole Unknown	Judy has 2 blue marbles (Part) and 3 red marbles (Part). How many marbles does she have in total (Whole Unknown)?
Part-Part-Whole, Part Unknown	Judy has 5 marbles (Whole). There are 2 blue marbles (Part), and the rest of the marbles are red. How many of the marbles are red marbles (Part Unknown)?

Table 4

Basic Compare Problem Type Examples

Problem Type	Example
Compare, Difference Unknown	Jessica has 6 marbles (Referent). Andrew has 10 marbles (Compare Quantity). How many more marbles does Andrew have than Jessica (Difference Unknown)?
Compare, Compare Quantity Unknown	Jessica has 6 marbles (Referent). Andrew has 4 more marbles than Jessica (Difference). How many marbles does Andrew have (Compare Quantity Unknown)?
Compare, Referent Unknown	Andrew has 10 marbles (Compare Quantity). He has 4 more marbles than Jessica. (Difference) How many marbles does Jessica have (Referent Unknown)?

Table 5

Basic Multiplication and Division Problem Type Examples

Problem Type	Example
Multiplication	Jack has 4 bags of marbles (Number of Groups). There are 5 marbles in each bag (Number of Items per Group). How many marbles does Jack have in all (Total Number of Items Unknown)?
Measurement Division	Jack has 20 marbles (Total Number of Items). He also has some bags. He puts 5 marbles into each bag (Number of Items per Group). How many bags does Jack fill (Number of Groups Unknown)?
Partitive Division	Jack has 20 marbles (Total Number of Items). He puts the marbles into 4 bags with the same amount of marbles in each bag (Number of Groups). How many marbles are in each bag (Number of Items per Group Unknown)?

Table 6

Direct Modeling Strategy Examples for Addition and Subtraction

Solution Strategy	Example
Joining All	Anna figures out from the problem that four and six should be combined. First, she makes a group of four blocks and a group of six blocks. Then, Anna *joins all* of the items from both groups and counts a total of ten blocks for her answer.
Joining To	Maria understands that she needs to figure out how much it takes to go from three to eight. She begins by drawing a group of three circles on a sheet of paper. She then *joins to* that group by drawing more circles until there are eight total circles. Maria counts up the additional circles to discover five circles for the answer.
Separating From	Anna believes that she needs to take four away from seven to find her answer. To start, she counts seven of her fingers to represent the problem. Next, she *separates* four fingers *from* the original group of seven. She counts up the remaining fingers to find out that three fingers is the correct answer.
Separating To	Maria needs to find the change to the problem that starts at eight and ends at six. First, she creates a group of eight with crayons. Then, she *separates* down *to* six crayons. Finally, Maria counts up the objects that she removed and has two crayons as her answer.

(Table Continues)

Solution Strategy	Example
Matching	Anna has to figure out how much more nine is than five. She starts by making a group of nine shapes on her paper. She then makes a group of five shapes on her paper. Next, she *matches* pairs of shapes between the groups, one-by-one, until there are no more available pairs. Finally, she counts up the unmatched shapes and has four shapes for the solution.
Trial and Error	Maria is given a problem where there is a loss of seven and a final point of six. First, she *tries* to begin with a group of eleven blocks. Then, she takes seven pencils away from the original group and learns that leaves her with four pencils, which is an *error* and falls short of the correct answer. Next, Maria attempts the process again with a larger starting group, a group of 13 pencils. She takes seven objects away and has six pencils remaining, which is the answer she is supposed to have. Maria discovers that 13 pencils is the correct answer to the problem.

Table 7

Direct Modeling Strategy Examples for Multiplication and Division

Solution Strategy	Example
Grouping	Bryan has a problem where there are four groups with five objects in each group. He starts by making one group with five blocks. Bryan repeats that step until he has four groups with five blocks in each group. Finally, he counts up all of the blocks to discover that twenty blocks is the answer.
Measurement	Amber needs to figure out how many groups come from having eighteen total pencils and three pencils in each group. First, she counts out eighteen pencils. Then, she makes one group with three pencils. Amber continues the same process until all of the pencils are in a group. Finally, she counts up all of the groups that she made and has six groups as her answer.
Partitive	Bryan encounters a problem that explains there are twelve total objects and three groups with the same number of items in each group. With that information, he works to find how many items are in each group. Bryan begins by constructing a group of twelve cubes. Next, he puts the cubes one-by-one into three different groups. Once he divides all of the cubes into equally sized groups, Bryan counts four cubes in each group for the answer.

Table 8

Counting Strategy Examples for Addition and Subtraction

Solution Strategy	Example
Counting On	Erik receives a problem where he must add nine and four. He starts at the number nine and orally *counts on* four more units. When he finishes at the last number, Erik realizes that the answer is thirteen.
Counting On To	Jordan must figure out how much it takes to go from seven to fifteen. He begins at seven and mentally *counts on to* seven until he gets up to fifteen. He keeps track of the amount between the two numbers and determines that eight is his answer.
Counting Down	Erik has to subtract six from eleven. First, he begins at eleven and silently *counts down* six units. Erik discovers that the amount he is left with is five, which is the correct answer.
Counting Down To	Jordan encounters a problem where he must find the amount it takes to go from seventeen to eight. He starts at seventeen and *counts down to* eight aloud. While he is counting backward, Jordan pays attention to the amount between seventeen and eight. He finds out that the solution is nine.
Trial and Error	Erik must find the starting point to a problem containing a gain of five and a final amount of fourteen. First, he *tries* beginning at eight. Next, he counts on five more units to eight to get thirteen,

(Table Continues)

Solution Strategy	Example
	which he realizes is an *error*. Erik attempts the process over again instead he begins with nine. After adding five to nine and getting fourteen, he learns that nine is the correct answer.

Table 9

Counting Strategy Examples for Multiplication and Division

Solution Strategy	Example
Skip-Counting	Anne has a problem where there are six groups and five items in each group. Since she knows how to count by fives, she begins counting aloud five, ten, fifteen, twenty, twenty-five, and finally thirty. She learns that six groups of five items equals thirty, which is the correct answer.
Addition and Subtraction Strategies	Jackson works to figure out a question that has eight groups with three items in each group. First, he writes down the number three eight times on a dry erase board. Second, he starts to double up pairs of numbers into groups. From the eight groups of three, Jackson gets four groups of six. Next, from the four groups of six, he makes two groups of twelve. Finally, he puts the two groups of twelve together to have twenty-four as his answer.
Trial and Error	Anne attempts to solve a problem where the number of items is unknown. She knows that there are three groups with the same amount in each group and that

(Table Continues)

Solution Strategy	Example
	there are twenty-seven total items. First, Anne starts by counting three groups with seven items per group, and she realizes that there are too few total items from this situation. Next, she tries counting three groups with eleven items per group and determines that there are too many total items this time. Then, Anne attempts to count three groups with nine items in each group. With nine items in each group, she finds the solution for the problem.