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A newly implemented upwind scheme and numerical benchmark for the resolution of the Exner equation in GAIA.

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***Abstract*— Erosion and bedload transport have a high influence on industrial facilities and water quality. These phenomena can be modelled by the Saint-Venant Exner system defined by shallow water equations and a sediment mass balance equation. This system is often handled by using a splitting method which consists in developing solvers for the hydraulic part and the morphodynamic part.**

However, this numerical resolution can lead to instability issues when complex flows are treated. Spurious oscillations can appear for different flow regimes, and in particular for supercritical flows. For example, [1] carried out hydrodynamic and morphodynamic simulations dealing with a torrential flow. They showed numerous oscillations and were forced to fix a non-erodible bed in high erodible zones with supercritical flows in their simulations.

In this work, we present the current numerical methods proposed in GAIA module to solve the Exner equation. We also propose an upwind scheme for the two dimensional Exner equation which is the 2D adaption of the upwind scheme proposed in COURLIS [6]. A coupled scheme [7] has been implemented in order to be compared with the splitting method.

In order to validate the different results and to highlight the various limitations of the current schemes, a numerical benchmark is set up, using test cases from the scientific literature. This benchmark is composed of several dunes evolutions under fluvial, transcritical and torrential flows in order to test the ability of the schemes to deal with regime changes and is also made of dam break cases which are relevant indicators for testing shock treatment.

It is shown that the centered scheme is stable most of the time, but fails on two tests including the full torrential one. The currently implemented upwind scheme does not work as soon as supercritical flow appears and the newly implemented is stable for almost all test cases.

I. INTRODUCTION

Erosion and bedload transport can be modelled by the Saint-Venant Exner system. Some numerical papers [2] and recent studies [1] have shown that using splitting methods to solve this system can lead to instabilities. Oscillations often appear for different flow regimes and pollute simulations with transcritical flows apparitions.

The aim of this work is to find when theses numerical oscillations appear and how to deal with it. For this purpose, we have listed the current numerical schemes developed in the GAIA module of the v8p1 version of TELEMAC-MASCARET. Moreover, we have implemented a two dimensional upwind scheme adapted from COURLIS, which is a one dimensional sedimentology module. A coupled scheme [7], which consists in considering the system as a whole, has also be studied in order to be compared with the other methods. Therefore, we have set up a numerical benchmark of test cases composed of dune evolutions under fluvial, transcritical and torrential regimes flows and dam break cases. These simulations have enabled us to test the stability of the different schemes and to highlight the outbreak of spurious oscillations.

This work is organized as follows: in Section 2, the mathematical model, the several numerical schemes and the numerical benchmark are introduced. In Section 3, the results on those tests cases are presented.

II. METHODS

A. Mathematical model

In this work, one considers the 2D bedload transport modelled by the Saint-Venant Exner system defined by the shallow water equations [3] and the Exner sediment mass balance equation. This system can be written as:

$$\begin{cases} \partial_t h + div(h\boldsymbol{u}) = 0 \\ \partial_t h\boldsymbol{u} + div(h\boldsymbol{u}\otimes\boldsymbol{u}) + \nabla\left(\frac{gh^2}{2}\right) = -gh\nabla b - \frac{\tau}{\rho_w}, \\ \partial_t b + \varepsilon div(\boldsymbol{q}) = 0 \end{cases} \quad (1)$$

with the water depth $h(t,\boldsymbol{x})$ in m, the velocity $\boldsymbol{u}(t,\boldsymbol{x}) = (u_1,u_2)$ in m/s and $\boldsymbol{x} = (x,y)$, g the gravitational constant in m²/s, $b(t,\boldsymbol{x})$ the bathymetry in m, the water density ρ_w in kg/m³,

$\boldsymbol{\tau} = (\tau_x, \tau_y)$ the frictional stress in $m^{1/3}$.s, the solid discharge $\boldsymbol{q} = (q_x(\boldsymbol{x}), q_y(\boldsymbol{x}))$ in m²/s and $\varepsilon = 1/(1-\Phi)$ with Φ the bed porosity. In this work, Grass formula is used [4]:

$$\boldsymbol{q} = \left(A_g u_1(u_1^2+u_2^2), A_g u_2(u_1^2+u_2^2)\right), \tag{2}$$

which is widely present in numerical papers due to its simplicity. The constant A_g takes values between 0 and 1 and models the intensity of the interaction between the fluid and the bed.

Meyer Peter & Muller formula [5], which is often used in industrial studies, has also been tested in order to ensure that the schemes also produce similar results in term of stability.

Solving (1) by a splitting approach consists in building a solver for the hydrodynamical part and another for the morphodynamical part that communicate together. Solving this system by a coupled approach consists in treating all three equations at the same time. This splitting method allows to add some complex physical processes and is easier to set up in an industrial context than the coupled approach. However as shown in [2], it can lead to some instabilities issues because the eigenvalues of the fluid part with a zero fixed bottom evolution are not always convenient approximations of the eigenvalues of the full system. In particular, with supercritical flows, one of the eigenvalues of the full system is always negative which can be interpreted as information propagating upstream. The use of splitting strategy in such situation cannot take into account this information since the two eigenvalues of the shallow water equations are always positive in this case.

B. Numerical methods

1) Current numerical methods in GAIA

GAIA is a sediment transport and bed evolution module of the TELEMAC-MASCARET modelling system [16]. It manages different sediment classes and numerous physical processes for both 2D and 3D spatial dimensions. In this paper, we only focus on finite volumes schemes solving bedload transport without suspension. The finite elements centered scheme of GAIA has shown the same stability results as the finite volumes one.

To solve the two-dimensional Saint-Venant Exner system on an unstructured mesh, a control volume C_i is built around each node P_i as shown on Figure 1. It passes through the gravity center of each element adjacent to that node.

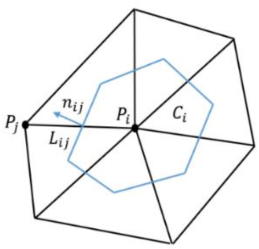

Figure 1: Control volume on an unstructured mesh [15].

Exner equation can be discretised by a finite volume method as:

E. $$b_i^{n+1} = b_i^n - \Delta t \sum_{K_{ij}} \sigma_{ij} q_{ij}^n \tag{3}$$

with $\sigma_{ij} = \frac{L_{ij}\varepsilon}{|C_i|}$ and $\sum_{K_{ij}} \sigma_{ij} q_{ij}^n \cong \int_{C_i} \varepsilon div(q)$.

b_i^n is the bottom variable discretized at the time n, C_i is the cell i and $|C_i|$ its area, K_{ij} is the interface between the cells i and j, L_{ij} its length. q_{ij}^n is the solid discharge at the interface between cells i and j.

There are two finite volumes schemes implemented in GAIA, a decentring scheme and a centered scheme.

1.1) GAIA decentring scheme

The decentring is chosen according to the sign of the projected solid discharge at the interface between two cells:

F. $$(q_{proj})_{ij}^n = n_{x,ij} q_{x,ij} + n_{y,ij} q_{y,ij} \tag{4}$$

G. with $q_{x,ij}$ and $q_{y,ij}$ the mean of the x and y solid discharge components at each side of the interface, $n_{x,ij}$ and $n_{y,ij}$ the components of the normal of the interface.

H. If (4) is positive i.e the solid discharge comes from the cell i so we take $q_{ij}^n = q_i^n$, the solid discharge at the node i. If (4) is negative i.e the solid discharge comes from the cell j so we take $q_{ij}^n = q_j^n$, the solid discharge at the node j.

This gives us the following scheme:

$$q_{ij}^n = \begin{cases} q_j^n, \text{ if } (q_{proj})_{ij}^n < 0 \\ q_i^n, \text{ if } (q_{proj})_{ij}^n \geq 0\text{'} \end{cases} \tag{5}$$

with $q_i \cong \frac{1}{|C_i|} \int_{C_i} q$.

1.2) GAIA centered scheme

The numerical flux for the centered scheme is defined by:

$$q_{ij}^n = \frac{q_i^n + q_j^n}{2}.$$

2) Newly implemented COURLIS adapted scheme

An upwind scheme based on the same idea as one proposed in the module COURLIS has been implemented [6]. COURLIS is a 1D-sedimentology module coupled with MASCARET, the 1D hydraulic code of TELEMAC-MASCARET. The proposed numerical scheme in this module is an upwind scheme based on the Froude number. The Froude number is defined as:

$$F_r = \frac{u}{\sqrt{gh}}, \tag{7}$$

and indicates the flow regime. If the regime is fluvial the Froude number is smaller than one, otherwise the regime is torrential. In the fluvial case, the information concerning bed evolution propagates in the direction of the fluid stream so an upstream decentring is made. In the torrential case, solid flow information propagates upstream so a downstream decentring is made. It is known that some instabilities can appear with transcritical flows [2,7] so the main idea behind this scheme is to capture regimes changes in order to adapt the stream to the flow regime. We need to construct velocities and water height at the interfaces in order to calculate an associated Froude number discretized as:

$$(F_r)_{i+\frac{1}{2}} = \frac{u^{Roe}_{i+\frac{1}{2}}}{\sqrt{gh^{Roe}_{i+\frac{1}{2}}}}, \tag{8}$$

In this one-dimensional scheme, the left cell is centered on the node i, the right cell is centered on the node $i+1$ and $i+\frac{1}{2}$ is the cells interface. Velocities and water heights are computed with the following Roe intermediate states:

$$h^{Roe}_{i+\frac{1}{2}} = \sqrt{h_i h_{i+1}}\,, \tag{9}$$

$$u^{Roe}_{i+\frac{1}{2}} = \frac{\sqrt{h_i}u_i+\sqrt{h_{i+1}}u_{i+1}}{\sqrt{h_i}+\sqrt{h_{i+1}}}, \tag{10}$$

The numerical flux is defined as:

$$q^n_{i+\frac{1}{2}} = \begin{cases} q^n_i, \text{ if } (F_r)_{i+\frac{1}{2}} < 1 \\ q^n_{i+1}, \text{ if } (F_r)_{i+\frac{1}{2}} \geq 1 \end{cases}. \tag{11}$$

To extend this resolution in two dimensions, we now have to take into consideration the tangential velocity at the interface, indeed not to consider this quantity could result to an underestimation of the Froude number and to decenter in the wrong direction.

We have the new Froude number and Roe intermediate states:

$$(F_r)_{ij} = \frac{||\boldsymbol{u_{ij}}||}{\sqrt{gh^{Roe}_{ij}}}\ , \tag{13}$$

$$u^N_{ij} = \frac{\sqrt{h_i u^N_i}+\sqrt{h_j u^N_j}}{\sqrt{h_i}+\sqrt{h_j}}\ , \tag{14}$$

$$u^T_{ij} = \frac{\sqrt{h_i u^T_i}+\sqrt{h_j u^T_j}}{\sqrt{h_i}+\sqrt{h_j}}, \tag{15}$$

with $\boldsymbol{u_{ij}} = (u^N_{ij}, u^T_{ij})$.

We are now able to calculate (11) with these new variables.

3) Coupled implemented scheme

The coupled approached has been proved to be more stable than the splitting one and can be used as a reference in terms of stability. In order to compare the stability of the two methods, two numerical schemes have been implemented. The first is an approached Riemann solver for the shallow water equations [14] and the second is its extension to the Saint-Venant Exner system [7]. They satisfy several essential properties. These schemes guaranty water depths positivity and preserve the steady state of the lake at rest, the wet-dry and dry-wet transition. Numerical fluxes are derivatives of the Harten-Lan-van Leer flux [9] with a specific discretization of source terms. The numerical scheme for the shallow waters equations will be used in TELEMAC-2D to solve the hydrodynamical part when the splitting approach will be considered for the test cases. Its extension to the whole system (1) will be used when the coupled approach will be considered.

4) Second order extension

A simple second order extension of the newly implemented upwind scheme has been tested. For the Saint-Venant Exner system (1), the main idea is to calculate the numerical fluxes with reconstructed variables [13]. A MUSCL reconstruction (Monotonic Upstream-centered Scheme for Conservation Laws) is made in TELEMAC-2D when second order scheme are used. It consists in replacing the piecewise constant approximation of the variables by reconstructed ones. Reconstructed left and right states are obtained by linear or parabolic approximation computed with the previous time steps states. These corrected variables are now used to calculate second order flux.

Our naïve approach has consisted in using the reconstructed variables computed by TELEMAC-2D and to send it to GAIA. They are used to calculate the Froude number and the solid discharge in the Grass formula. Unfortunately, spurious oscillations appeared even with the most diffusive flux limiter.

C. Numerical Benchmark

The scientific literature on this subject has shown that instabilities can appear with flow regime changes and shock apparitions [1,2,14]. Therefore, we have selected test cases that include all these configurations. It will enable to link theoretical assumptions on the instabilities apparitions with practical results and to highlight the limitations of the each scheme.

1) Dune evolutions under various flows

Dune evolutions are classical test cases for morphodynamical simulations and deal with different flow regimes, which is interesting for our work. We have set up five dune evolution tests, two full fluvial cases with a strong and a weak interaction, two transcritical cases with and without a shock and a full torrential one.

Fluvial flow: These classic test cases of sedimentology model the evolution of a dune under a fluvial flow. The first one simulates a strong interaction between the flow and the bed river [7]. The second models a weak interaction [10] and results are compared with an asymptotic solution given in [11]. The channel is 1000 m long and 10 m wide, the initial data are given by:

$$\begin{cases} b(0,x) = \begin{cases} 0.1 + sin\left(\dfrac{(x-300)\pi}{200}\right)^2 if\ 300 \le x \le 500\ , \\ \quad 0.1 \ \text{ elsewhere}\ , \end{cases} \\ h(0,x) = 10 - b(0,x), \\ u(0,x) = \dfrac{q_0}{h(0,x)}, \end{cases}$$

with $q_0 = 10$ m²/s the inflow discharge. Grass formula (2) is used with $A_g = 1.0$ for the strong interaction and 0.001 for the weak interaction case. The asymptotic solution is valid for a low interaction of the riverbed with water, $A_g < 0.01$ and a flow rate less than 10 m²/s. It is given by:

$$b(t,x) = \begin{cases} 0.1 + \sin\left(\dfrac{(x_0-300)\pi}{200}\right)^2 & \text{if } 300 \le x_0 \le 500, \\ 0.1 & , \quad \text{otherwise} \end{cases},$$

with x_0 solution of:

$$x = x_0 + A_g \epsilon m_g {q_0}^{m_g} t \left(10 - \sin\left(\frac{(x_0-300)\pi}{200}\right)^2\right)^{-(m_g+1)}, \quad \text{if } 300 \le x_0 \le 500$$

$$x = x_0 + A_g \epsilon m_g {q_0}^{m_g} t 10^{-(m_g+1)}, \quad \text{otherwise}$$

This solution is valid until $t < t_0$ where t_0 is the time at which the characteristics cross. It is estimated at $t_0 = 23827912.4$ s with $m_g = 3$.

Transcritical flow without shock: it corresponds to the transition from a subcritical regime (Froude number <1) to a supercritical regime (Froude number >1).This test evaluates the robustness of schemes that may be sensitive to flow regime changes. The channel is 10 meters long and 1 meter wide, the initial data are given by:

$$\begin{cases} b(0,x) = 0.1 + 0.1e^{(x-5)^2} \\ h(0,x) = 0.4 - b(0,x) \\ u(0,x) = \dfrac{q_0}{h(0,x)} \end{cases},$$

with $q_0 = 10$m²/s the inflow discharge..
Grass formula (2) is used with $A_g = 0.0005$ on a 2300 elements unstructured mesh.

Transcritical flow with a shock: it models a hydraulic jump that is characterized by two regime flow changes, subcritical to supercritical then supercritical to subcritical. The channel is 20 m high and 2 m wide. The initial bathymetry is:

$$b(0,x) = 0.25e^{-0.5(x-10)^2}.$$

We evaluate the initial water height in a steady state with the equations in [12] with $q(t,0) = q_0 = 0.45m^2/s$ and $h(t,0) = 0.5$ m. Grass formula (2) is used with $A_g = 0.0005$ on a 1270 elements unstructured mesh.

Torrential flow: Under a torrential flow, we expect the sand dune to move upstream. The initial bathymetry is given by:

$$b(0,x) = \begin{cases} 0.2 - 0.05(x-10)^2, & \text{if } 8 \le x \le 2 \\ 0\ , & \text{otherwise} \end{cases}.$$

The inflow discharge is $q(t,0) = 2$ m²/s and $h(t,0) = 0.5$ m. The channel is 10 m long and 1 m wide and we use Grass formula with $A_g = 0.001$ on a 2680 elements unstructured mesh.

2) Dam break tests

Dambreak test cases are useful to evaluate a scheme ability to deal with shock and rarefaction waves. The initial data of the wet case are:

$$\begin{cases} b(0,x) = 0\ , \\ h(0,x) = \begin{cases} 2\ if\ x \le 5\ , \\ 0.125 \text{ otherwise}\ , \end{cases} \\ u(0,x) = 0 \end{cases}$$

with Grass formula (2) and $A_g = 0.005$.
For the dry bottom case, the water height now includes a dry zone:

$$b(0,x) = \begin{cases} 2 & if \quad x \le 5 \\ 0 & \text{otherwise} \end{cases}.$$

Moreover we include a friction term based on the Strickler formula with $K_s = 50$.

III. RESULTS

In all the different test cases results, the centered scheme of GAIA will be named as CENTER, the decentring scheme of GAIA as GAIA DECENTRING, the newly implemented scheme as COURLIS_2D and the coupled scheme as ACU.

A. Dune evolutions under fluvial flow

Long simulations are useful to highlight diffusivity or lack of stability of a scheme. The first result on Figure 2 is obtained after a 1000-second simulation on the strong interaction case. We notice that GAIA DECENTRING and COURLIS_2D give same results, which seems logical because the Froude number is always smaller than one and the solid flow is always positive. However, CENTER oscillates and is less diffusive.

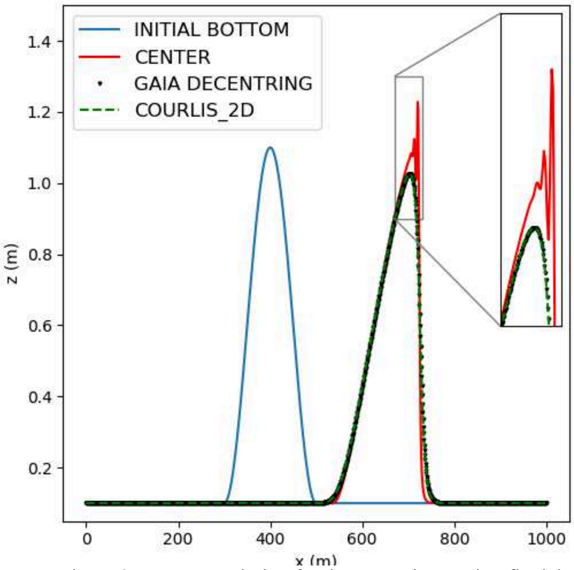

Figure 2: Bottom evolution for the strong interaction fluvial case.

The weak interaction case shown on Figure 3 is obtained with a 238 080 s simulation. Results are similar and we can see that ACU and CENTER begin to oscillate. It shows that even on weak interaction case, spurious instabilities can appear. Mesh convergence has been made on this case and a one-order accuracy has been highlighted for COURLIS_2D on Figure 4 that explains the numerical diffusion.

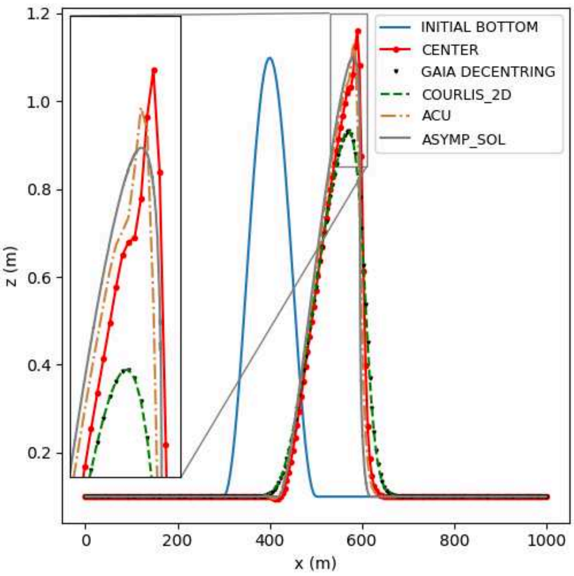

Figure 3: Bottom evolution for the weak interaction fluvial case.

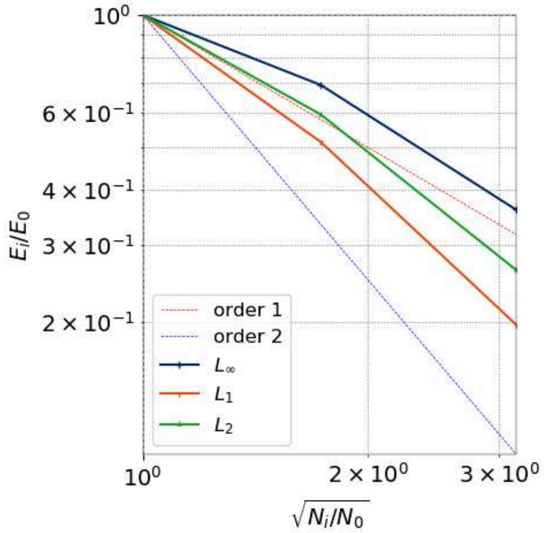

Figure 4: Strong interaction fluvial case: relative error on the bottom.

C. Dune evolution under a shock-free transcritical flow

Figure 5 illustrates the initial and final states at T = 20s, GAIA DECENTRING is not shown for clarity sake due to its high oscillations. The inflow is subcritical and the outflow is supercritical.

Bottom evolution for all scheme can be seen on Figure 6, we notice that GAIA DECENTRING is the only one to produce oscillations in particular in the torrential zone and that CENTER shows a small bump at the critical outflow that increases with time.

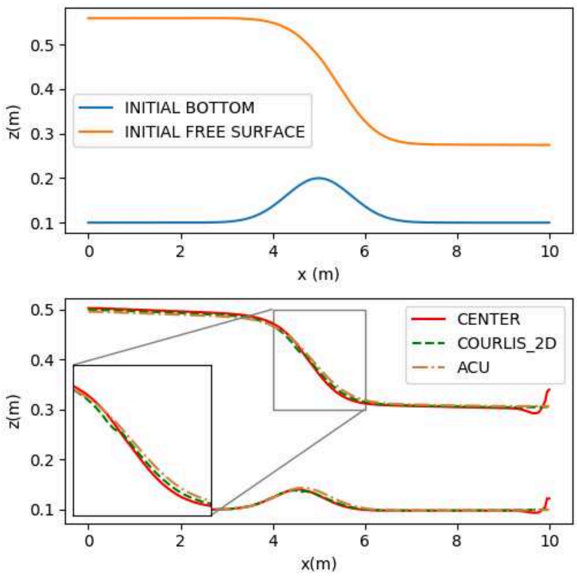

Figure 5: Free surface and bottom at initial and final time for the shock-free transcritical flow.

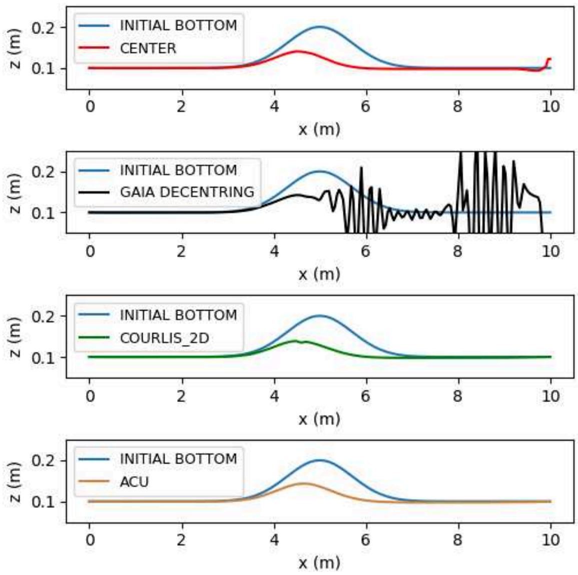

Figure 6: Bottom evolution for the shock-free transcritical flow.

B. Dune evolution under a transcritical flow with shock

This test represents a hydraulic jump characterized by two regime changes after a 20-second simulation on Figure 7. GAIA DECENTRING is not shown for clarity sake due to its high oscillations. COURLIS_2D and ACU seem to give smoother results than CENTER.

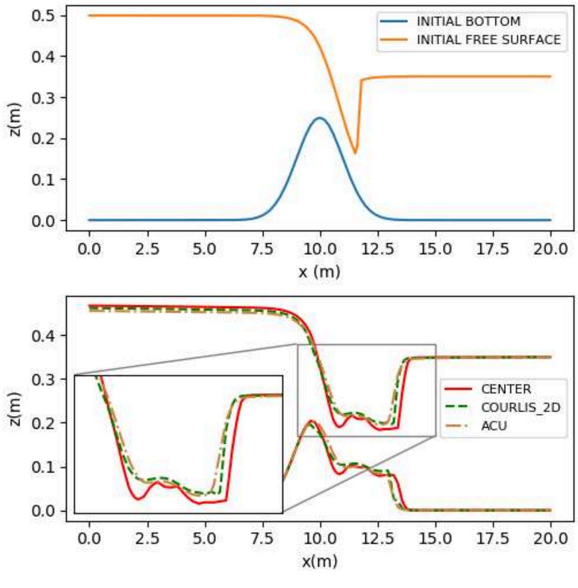

Figure 7: Free surface and bottom at initial and final time the transcritical flow with a shock case.

Once again, the upwind scheme of GAIA produces oscillations in flow regimes changes area seen on Figure 8.

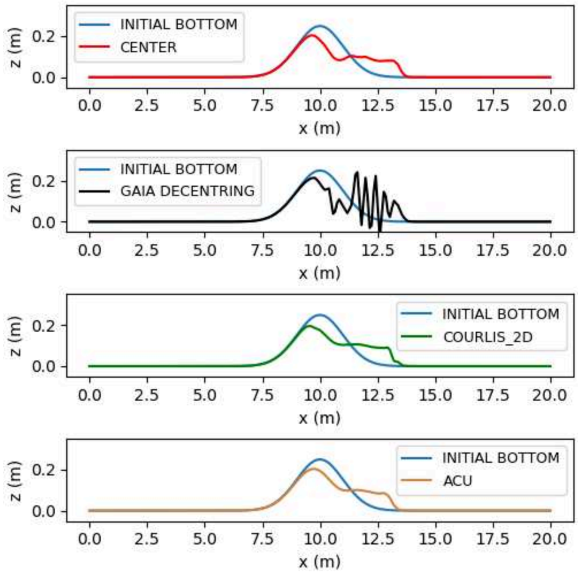

Figure 8: Bottom evolution for the transcritical flow with a shock case.

C. Dune evolution under a full torrential flow

COURLIS_2D and ACU are the only ones to finish the 20-second simulation without producing oscillations. GAIA DECENTRING was not able to finish it and CENTER produces a sediment abnormality at the right boundary shown on Figure 9. This abnormality increases with time that has forced us to stop the simulation after 20 seconds.

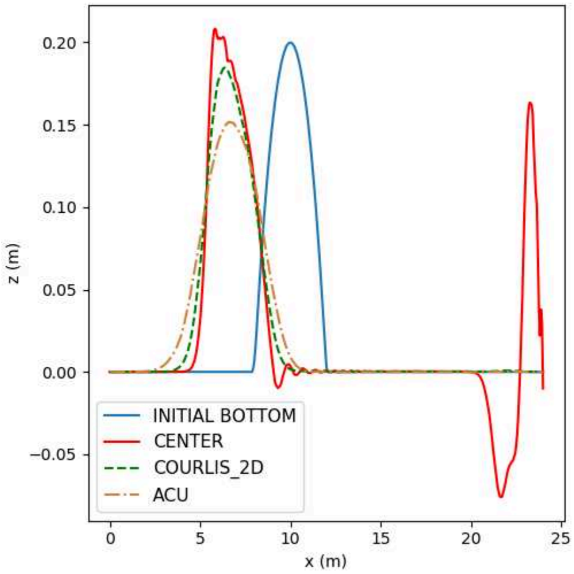

Figure 9: Bottom evolution for the anti-dune test case.

D. Dambreak over a wet bottom

Figure 10 illustrates the initial and final situation one second after the break.

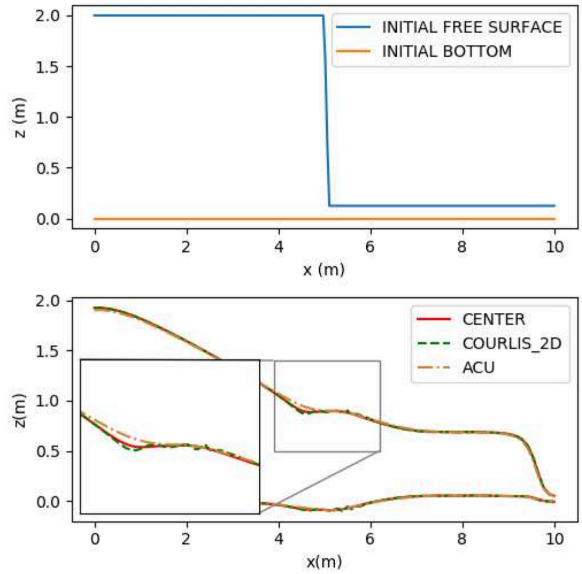

Figure 10: Free surface and bottom at initial and final time for the dambreak wet bottom case.

As shown on Figure 11, CENTER and ACU do not produce any oscillations. We notice a small peak at the shock for COURLIS_2D and bigger ones for GAIA DECENTRING. However, COURLIS_2D oscillations seem to be bounded and do not grow with longer simulations whereas those of GAIA DECENTRING do.

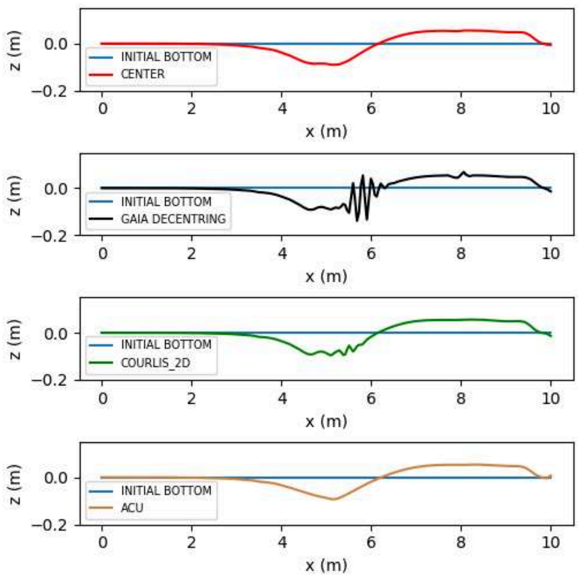

Figure 11: Bottom evolution for the dambreak wet bottom case.

E. Dambreak over a dry bottom

I. Figure 12 illustrates the initial and final situation one second after the break.

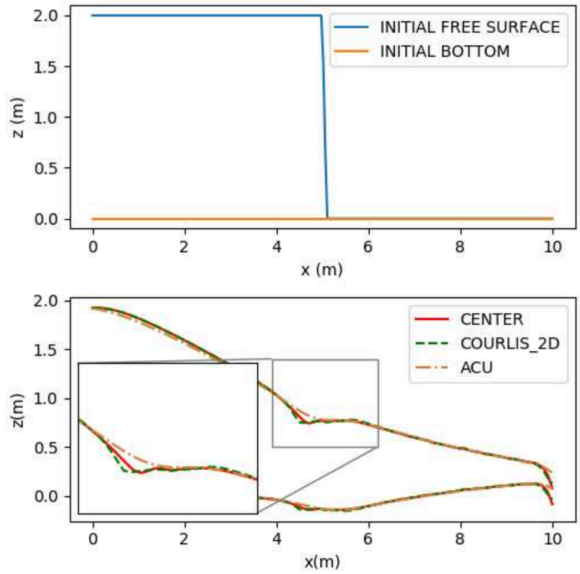

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Figure 12: Free surface and bottom at initial and final time the dambreak dry bottom case.

We can see on Figure 13 that GAIA DECENTRING still oscillates and ACU seems to produce the smoothest result.

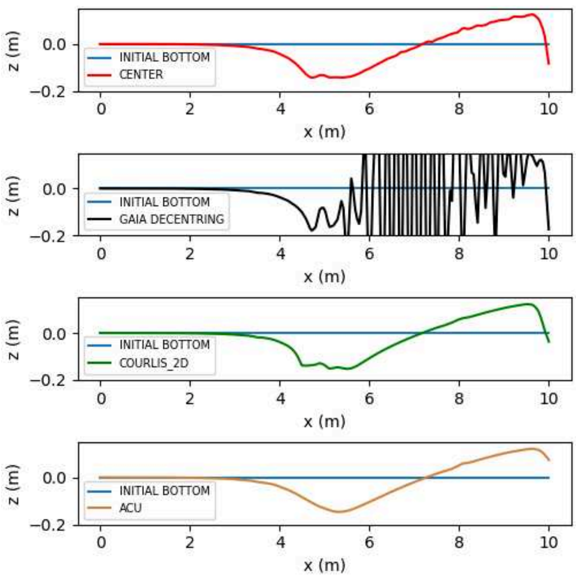

Figure 13: Bottom evolution for the dambreak dry bottom case.

F. Discussion

All the previous results are summarized in Table 1. We can see that the decentring scheme of GAIA is not stable and has only succeeded the full fluvial test. The centered scheme fails on the two dune evolutions with no regime change but handle the two dambreak cases whit no oscillations. Our newly implemented upwind scheme have passed all dune evolutions test including the full torrential one, however some small oscillations have appeared on the dambreak wet bottom test case that explains the yellow case. The coupled scheme has only failed on the fluvial test case but has handled all the other tests giving the smoothest results. For this benchmark, COURLIS_2D seems to be the strongest approach for the splitting method but is diffusive as an order one accuracy. ACU is the most stable scheme for the treatment of shocks and rarefaction waves as shown on the dambreak tests results.

TABLE 1: SUMMARY OF RESULTS (✓: SIMULATION SUCCEEDED WITHOUT OSCILLATIONS / ≈: APPARITIONS OF SMALL BOUNDED OSCILLATIONS / ×: PRESENCE OF SPURIOUS OSCILLATIONS)

	CENTER	GAIA DECENTRING	COURLIS 2D	ACU
Fluvial	×	✓	✓	×
Transcritical	✓	×	✓	✓
Transcritical with shock	✓	×	✓	✓
Torrential	×	×	✓	✓
Dambreak wet	✓	×	≈	✓
Dambreak dry	✓	×	✓	✓

IV. CONCLUSION

The aim of this work was to find when instabilities appear solving bedload transport by a splitting approach and to determine how current implemented scheme or its adaptations can handle this issue.

To do this, we have presented the current finite volumes method implemented in the v8p1 version of GAIA and adapted a one-dimensional COURLIS upwind scheme based on the evaluation of the Froude number at the interfaces. An approximate Riemann solver has also been implemented [14] to solve the shallow waters equations and its extension to the Saint-Venant Exner system based on the coupled approach. [7].

A numerical benchmark has been set up, composed of cases found in the scientific literature in order to test the ability of each scheme to deal with flow regime changes and shocks.

Results have shown that the decentring scheme of GAIA was not stable and was only able to deal with full fluvial flows. As soon that regime changes or a torrential flow arise spurious oscillations appear. The centered scheme of GAIA showed oscillations on the subcritical test case and the full torrential one but was able to handle regime changes and shocks treatment. The newly implemented 2D adaptation of the COURLIS scheme seems to be stable and only produces a small oscillation on one dam break test case. The coupled scheme has only produced instabilities on the fluvial flow test case and has presented the smoothest results. However, this approach does not allow us to use complex physical processes like managing different sediment classes, sediment slide or slope effect.

This benchmark has enabled us to highlight the apparition of oscillations in flow regime changes and torrential areas. Moreover, it has allowed us to identify the limitations of the present methods and cases where no one works. The newly implemented upwind scheme has filled in this gap by handling torrential flows and can be easily integrated into GAIA.

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