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Abstract

I develop a simple dynamic model of reputation-based transactions between a buyer in one country and a supplier in another. I use the model to study the impact of a more stringent regulation on the buyer optimal purchase volume within an existing buyer-seller partnership. A more stringent standard affects the volume of trade in two intuitive ways: directly, a stricter standard affects the supply of quality goods and indirectly through reputation. I refer to the former effect as the *regulation effect*, and to the latter as the *reputation effect*. I show that, whereas most of the empirical literature has so far assumed that more stringent standards would be likely to reduce trade, the net effect is in fact non-monotone, even without taking into account endogenous technological upgrading in the supplier country. It varies with the belief the buyer holds about his seller at the time the change in regulation takes place. For both very low and very high seller's reputation, the *reputation effect* is negligible vis-à-vis the *regulation effect*. For intermediate levels of the supplier's reputation, reputation has the power to significantly mitigate the direct negative effect of a more stringent sanitary standard on trade. This result has significant implications for developing countries, for which access to developed countries markets is by and large, said to be disproportionately constrained by stricter standards.

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1 Introduction

Food safety issues are increasingly important in the arena of international food trade. As countries lower their tariffs and become more integrated into world markets, there is a presumption that food safety measures can be used as a protectionist tool (Beghin 2001). As consumers grow wealthier, they also focus more on the attributes of their food, its safety and nutrition.

So far, the empirical literature has assumed that enhanced stringency of food regulations have, by and large, a negative effect on developing-country exports. Is this necessarily true? The answer is no for at least three reasons. First, standards can facilitate trade by ensuring a controlled quality for processors. Second, standards may come with technical assistance that helps upgrade quality and enables exporters to target more markets. Third, even without endogenous quality upgrading, when consumers cannot evaluate the quality of a good before purchase–such goods are referred to as experience or credence goods in the literature[1]– more stringent standards may improve the information available to purchasers on product safety, increasing their confidence. This may in turn affect their optimal purchase volumes which go counter to the first-approach intuition. The impact of a food regulation on trade thus stems from the direct cost of compliance incurred by suppliers, and indirect supply and demand effects.

What I set up to do in this paper, is design a simple model that highlights the potential non monotonicity in the relationship between regulation stringency and purchase volumes. I develop a simple dynamic model of reputation-based transactions between a buyer in one country and a supplier in another. I study a situation where quality is imperfectly observable.

What do we know about the impact of food standards on trade?[2] Empirical evidence shows that the proliferation and enhanced stringency of food safety standards do impede exports of developing countries, either because explicit bans are placed on imports or because the costs of compliance diminish their export competitiveness. Maskus et al (2005) show that complying with regulations entails non-trivial fixed costs and raises significantly short-run production costs. Also the lack of institutional capacity limit developing countries ability to cope with stringent standards (Kim and Reinert 2009). A large set of studies has focused on the quantitative as-

[1] An experience or a credence good is distinguished by the fact that its quality cannot be determined by consumers at the time of purchase. The true quality of an experience good is revealed later when consumers experience its consumption. The true quality of a credence good is never fully revealed to consumers.(Nelson, 1970; Darby and Karni 1973)

[2] Throughout the paper, the terms regulation and standards are used interchangeably.

sessment of the impact of food regulations on trade. Using gravity-based analysis they provide evidence of the detrimental effect of domestic regulations on developing countries agricultural and food exports (Otsuki et al., 2001; Fontagné et al.,2005; Fulponi, 2006). Crucially, the impact of importing country standards varies with the level of income of the exporting country, with negative effects being more likely vis-à-vis developing country exporters (Disdier et al., 2008). An alternative and less pessimistic view, largely based on detailed case studies, emphasizes the potential catalyzing role product standards may play in the promotion of quality upgrading and technical progress in developing countries (Diaz Rios and Jaffee 2008; Henson and Jaffee 2008).

From a theory perspective, there is substantial evidence that the impact of regulation is more complicated than much of the empirical literature suggests. In the words of Swann (2010) "Existing econometric models represent, at best, 'black boxes' that disguise a complex of relationships." A large body of literature, derived from research in industrial economics, has made it possible to account for supply and demand-side sophisticated effects. Regulations may confer competitive advantage to complying firms, due to improved control and increased efficiency–i.e. direct positive externalities generated by the (quality) management systems adopted (Henson and Caswell, 1999). Regulations may also generate a competitive advantage if they have a positive effect on demand. Due to the credence aspect of the "safety" attribute, regulations can modify the information available to consumers (Shapiro, 1983; Donnenfeld et al., 1985). Regulations by improving the information on which to base decisions avoid or reduces the costs of assessing product quality–the lemon problem (Leland,1979; Marette,1998; Bureau and Marette, 2001).

Also, there is an extensive theoretical literature on the importance of producers' reputations as quality indicators. In environments with informational asymmetries and repeated purchases, Shapiro (1983) demonstrates how building up a reputation can improve the market outcome. Price premiums paid by consumers are necessary for producers to invest in quality and reputation, provided that these efforts are explicitly or implicitly communicated to consumers. Minimum quality standards also have a function in reducing reputation establishment costs (Falvey, 1989). Regulations may also modify the costs of signaling quality and possibly result in improved competitiveness of those that meet stringent standards (Jones and Hudson, 1996; and Hudson and Jones, 2001, 2003).

The issues of reputation has also received some attention in the trade literature. The volume of trade increases with the belief the buyer holds about suppliers' ability to successfully fill large orders (Rauch and Watson, 2003). Closer to this paper, is the work of Araujo and Ornelas (2007). They develop a model where agents build a reputation to overcome the difficulties created by the imperfect enforcement of international contracts. Unlike my paper, they consider an asymmetric information setting. Their results suggest that reputation serves only imperfectly to mitigate the negative effect of weak contract enforcement.

On the whole, the available evidence on the variety of channels through which regulation may impact trade, leaves little doubt that the net effect of a more stringent regulation on trade is not straightforward. However, this relationship has to my knowledge not been studied before. This paper seeks to fill this gap and provides a small, simple model that describes the interplay between regulation stringency, reputations and purchase volumes.
There is one buyer and sellers in two different countries. There are two types of sellers, good and bad quality sellers. This is standard in reputation models (Tirole 1996; Kreps and Wilson, 1982), where sellers are of different types and are exogenously different. The supply at various qualities is exogenous as in Leland (1979). The buyer wants to import goods from good quality sellers. He forms a partnership with a supplier in another country. The seller's type is unknown to the buyer and the seller and is revealed over time to both through experimentation. The common belief that the seller is good is termed the seller's reputation. I study situations where the acquisition of information is symmetric, unlike earlier papers, where sellers most often are aware of their product quality (Shapiro, 1983). When the buyer forms a partnership, he proposes a one-period contract to the seller and incurs a search cost. Because the buyer cannot contract on the quality of the good, his decision to import and how much to import depends only on the belief he holds about the seller's type. A pessimistic buyer may start small to uncover the nature of his supplier (Ray, 1996, Watson, 1999, 2002). This result arises endogenously from the model. The buyer's belief, in turn depends on the export history of the seller. Sellers behave mechanistically. Sellers choosing their quality and behaving strategically, would result in a moral hazard problem (Grossman and Shapiro, 1988; Falvey, 1989). The buyer's country sets a minimum quality standard against which every shipment of goods is tested for compliance before purchase[3]. The minimum quality standard is set exogenously. Realisation is publicly observable but true quality is not. The buyer and seller update their belief accordingly through Bayesian revision. Updated beliefs affect future decisions, because the buyer decides on its import volume according to his updated belief. Lastly, the buyer in a partnership with a seller learns about his type over time. The match persists until the buyer decides to break the partnership and switch to another seller for which he starts with an initial belief. In my setting,

[3]The buyer does not choose to detect quality or not as in Marette et al., (2001)

the buyer may form a partnership with only one seller at a time. This drastic simplifying assumption allows me to take down the problem to its simplest expression since bandit models are inherently complex (see Rothschild, 1994; Whittle, 1982; Bar-Isaac, 2001).

I use the model to study the impact of a more stringent regulation on the buyer optimal purchase volume within an existing buyer-seller partnership. A more stringent standard affects the volume of trade in two intuitive ways. First, there is a direct supply-side effect. A stricter regulation induces a higher rate of rejection for shipments following inspection. The effect decreases with the seller's reputation. Second, there is an indirect demand-side effect. A stricter regulation facilitates the formation of reputations. The buyer imports more as he becomes more optimistic about his seller. Hence, a more stringent regulation has an indirect positive effect on future trade volumes. This in turn moderates the direct negative effect of the regulation on trade. I refer to the former effect as the *regulation effect*, and to the latter as the *reputation effect*. Overall, I show that the net effect of a stricter regulation on trade volume varies non monotonically with the belief the buyer holds about his seller at the time the change in regulation takes place. This is so even without taking into account endogenous technological upgrading in the supplier country. For both very low and very high seller's reputation, the *reputation effect* is negligible vis-à-vis the *regulation effect*. For intermediate levels of the supplier's reputation, the *reputation effect* may significantly water down the direct negative effect.

The remainder of the paper proceeds as follows. Section 2 sets up the model. Section 3 describes the dynamics of a partnership. Section 4 formalises the relationship between reputation and trade. And Section 5 studies the implications of a change in regulation on trade flows. Finally, Section 6 concludes. The appendix contains all formal proofs.

2 The basic Framework

Consider one buyer and an infinite pool of sellers in two countries. The horizon is infinite, with discounting factor $\gamma \in [0, 1]$.[4] The seller can be a good-quality seller or a bad-quality one. Let $j = G, B$ be the seller's 'type'.[5] In each discrete time period,

[4] Note that for $\gamma \in (0, 1)$ all qualitative results derived below hold, though the discount factor would affect the level of the critical reputation discussed in proposition (1). However for $\gamma \in \{0, 1\}$, results need not hold. If $\gamma = 0$, the game is a one shot game and dynamic reputational concerns are irrelevant, and if $\gamma = 1$, there are potentially infinite future rewards.

[5] The distribution of types in the pool of sellers is unknown to the both buyer and sellers.

each seller can produce a single, homogenous good under a constant-marginal cost technology with random quality $\phi \in [0,1]$ drawn from one of two possible uniform distributions $\theta^G(\underline{\phi}^G, \overline{\phi}^G)$, $\theta^B(\underline{\phi}^B, \overline{\phi}^B)$, such that:

$$\theta^j(\phi) = \begin{cases} \frac{1}{\overline{\phi}^j - \underline{\phi}^j} & \text{for } \underline{\phi}^j < \phi < \overline{\phi}^j \\ 0 & \text{otherwise} \end{cases}, \ j = G, B.$$

where $0 < \underline{\phi}^B < \underline{\phi}^G < \overline{\phi}^B < \overline{\phi}^G < 1$, and $\overline{\phi}^B - \underline{\phi}^B = \overline{\phi}^G - \underline{\phi}^G$. The seller type is unknown to buyer and seller and is revealed over time to both through experimentation.

At the beginning of the game, the buyer's government sets a minimum quality standard ϕ_m. The buyer implements sanitary control procedures and at the beginning of each period t, he decides to import from a seller. Before purchase, each shipment is tested for compliance with the buyer's standard. If a shipment fails the test, it is rejected. The event of a test failure by the incumbent supplier is marked by a binary variable

$$\xi_t = \begin{cases} 1 & \text{if the test is failed at } t \\ 0 & \text{otherwise.} \end{cases}$$

The realisation is publicly observable. However quality ϕ is unobservable to both seller and buyer. The test does not reveal the true quality of the good ϕ, instead it reveals whether ϕ is below or above the quality standard.

In each period, a type-j seller has a probability of failure:

$$\Pr(\xi_t = 1 | j) = \Theta^j(\phi_m)$$

and,

$$\Theta^j(\phi_m) = \begin{cases} 0 & \text{for } \phi_m < \underline{\phi}^j \\ \frac{\phi_m - \underline{\phi}^G}{\overline{\phi}^G - \underline{\phi}^G} & \text{for } \underline{\phi}^j < \phi_m < \overline{\phi}^j \\ 1 & \text{for } \overline{\phi}^j < \phi_m \end{cases}, \ j = G, B$$

where $0 < \Theta^G < \Theta^B < 1$. Θ^G and Θ^B are common knowledge. The probability of failure is independent and identical across time and depends only on the type of the seller and the level of the minimum quality standard. In fact, the two types of sellers differ only in that each time a good seller produces a good it will meet the buyer's quality standard with a higher probability $1 - \Theta^G$. In order to avoid zones of complete revelation $\Theta^j \in \{0, 1\}$, $j = G, B$, I restrict $\phi_m \in [\underline{\phi}^G, \overline{\phi}^B]$. On the common support $[\underline{\phi}^G, \overline{\phi}^B]$, $\theta^G(\phi) = \theta^B(\phi) > 0$. Since $\partial\Theta^j / \partial\phi_m = \theta^j(\phi)$, two important implications follow, (i) a more stringent standard raises the probability of failure, and (ii) a change in the regulation affects both types equally.

The buyer is risk neutral and seeks to maximize the discounted present value of profits. At the beginning of period t, the buyer offers a one-period contract[6] to the seller he is in partnership with. Buyer and seller share a common belief that with probability λ_t the seller is good–this belief is termed the seller's reputation.
The contract specifies a quantity q, together with a clause stating that payment will be made only upon successful inspection. The shipment is tested for compliance. If it passes inspection (event $\xi_t = 0$), the buyer pays a constant unit price p and receives $u(.)$ characterised by diminishing marginal returns on q.[7] If it fails it is rejected and the buyer receives nothing. Also, in both cases, he pays a per unit inspection cost c, and writing the contract involves a fixed cost $f > 0$ that is incurred no matter what.[8] The existence of the fixed cost paid in each period, induces the buyer to trade with one seller at a time. The buyer's per-period profit is:

$$\pi(p,q,c,f) = \begin{cases} u(p,q) - cq - f & \text{if } \xi_t = 0 \\ -cq - f & \text{otherwise.} \end{cases} \tag{1}$$

with $u_q > 0$ and $u_{qq} < 0$. The quantity q is independent of the outcome of the inspection in the same period though it varies with the history of the seller's previous successes and failures.

Supposing that each seller begins with some initial reputation λ_0[9], the game can be summarized as the repetition of the following steps:

1. At the beginning of period t, the buyer decides to import from a seller with reputation λ_t.

2. The buyer offers a one-period contract to the seller specifying a quantity q.

[6]Such short-term contract conforms well to actual agri-food business pratices. The class of the contract the buyer offers to suppliers does not allow the buyer to extract any information on the type of the supplier.

[7]In this setup I restrict the price of the good p, to be constant. Instead it is quantity that plays an important role and depends on the seller's reputation.

[8]I assume that the buyer pays c to inspect every shipment whether the shipment passed or failed inspection. In practice, the buyer may be reimbursed by the seller for lots that fail inspection, or the cost may be shared with the seller.

[9]The buyer has no information on the distribution of types among its potential sellers. He assigns the same initial prior λ_0 to any seller he initiates a partnership with.

3. The seller produces q units of homogeneous good of quality ϕ drawn from either θ^G, θ^B, depending on the seller's type.

4. The shipment is tested for compliance with the buyer's standard ϕ_m. A type-j supplier passes inspection with probability $1 - \Theta^j$, and fails with probability Θ^j. The seller is deemed good with probability λ_t and bad with probability $1 - \lambda_t$. Then, the buyer per-period expected profit is:

$$E(\pi|\,\lambda_t) = [\lambda_t(1 - \Theta^G) + (1 - \lambda_t)(1 - \Theta^B)]u(p, q) - cq - f \tag{2}$$

5. The realisation of ξ_t as a success or a failure is publicly observed. On the basis of the realisation the buyer revises his belief with respect to the seller's type, so that the seller's end of period reputation is λ_{t+1}.

   He then decides at the beginning of period $t + 1$ whether to keep dealing with the incumbent seller or switch to a new one. If he switches from an incumbent to a new seller, he initiates a new partnership and incurs a one time fixed switching cost, s, that captures in a simple way the buyer's cost to engage in business. If the current partnership ends the seller looses his ability to sell to the buyer and is replaced by a new active seller.

Throughout I suppose that the information structure is common knowledge. The seller has no private information. Instead, the buyer and potential seller share the same beliefs and update these beliefs identically. Quality is not observable, only inspection realisations convey information about the seller's type. The buyer strategy is simply to decide whether or not to stick to the incumbent seller given the realisation of ξ_t, and how much to purchase at $t + 1$ from either the incumbent, if he kept him, or an alternative seller, if he switched. Sellers have no strategy, they behave mechanistically. This framework ignores any forms of signalling and allows me to focus on the role of seller's reputation on (i) the decision of the buyer to trade (Section 3) and (ii) the volume of trade (Section 4 and Section 5). The buyer necessarily terminates the partnership when $\lambda = 0$. Stated differently, let $q^\star(\lambda)$ be the buyer's optimal purchase volume given belief λ. Then $q^\star(0) = 0$ and $q^\star(1) > 0$; that is, a buyer would always buy from a good type and never buy from a bad type if he knew.

3 Buyer policy

In this section I consider the buyer's optimal policy as concerns whether or not to stick to the incumbent seller given the realisation in the previous period. In deciding whether or not to switch sellers, the buyer takes into account both the profit he expects to earn in that period and the value of information generated about the seller's type in a further trading experience. At the beginning of each period t, the incumbent seller's failure history is

$$h_{t-1} = \{\xi_{\tau_0}, \xi_{\tau_0+1}, ..., \xi_{t-1}\}$$

where τ_0 is the first period in which the buyer bought from him. Given h_{t-1} and ξ_t, i.e. at the *end* of period t, the common belief that the seller is good is

$$\lambda_t(h_{t-1}, \xi_t) = \Pr(\, j = G |\, h_t).$$

Following a success or failure, the seller's reputation is updated according to Bayes' rule:

$$\ln \frac{\lambda_t}{1-\lambda_t} = \ln \frac{\lambda_{t-1}}{1-\lambda_{t-1}} + \begin{cases} \ln \frac{\Theta^G}{\Theta^B} & \text{if } \xi_t = 0 \\ \ln \frac{1-\Theta^G}{1-\Theta^B} & \text{otherwise.} \end{cases} \tag{3}$$

The posterior belief follows a two element distribution, such that the reputation after a success or a failure in period t, are given respectively by λ_t^s and ${\lambda^f}_t$.
Symmetric information implies that the buyer's value of trading only depends on the current reputation of the seller he is in partnership with. Since the buyer may decide to switch to a new seller in any period, at the beginning of each period t the optimal policy given reputation λ_{t-1} is to stick to the incumbent whenever

$$V(\lambda_{t-1}) \geq V(\lambda_0) - s$$

where

$$V(\lambda_{t-1}) = E(\pi |\, \lambda_{t-1}) + \gamma \max\{E(V(\lambda_t)); V(\lambda_0) - s\} \tag{4}$$

is the buyer's expected value of trading with the current incumbent given a reputation λ_{t-1}. And $V(\lambda_0) - s$ is the expected value of switching to a new seller and incurring a switching cost s, with $V(\lambda_0) - s \geq 0$ otherwise the buyer would not initiate a partnership. In expression (4) $E(\pi |\, \lambda_{t-1})$ is the profit in the current period. Future expected profits, whether the buyer stayed with the incumbent or switched, $\max\{E(V(\lambda_t)); V(\lambda_0) - s\}$, are discounted at the discount rate γ. Expanding expression (4) using equation (2) gives:

$$\begin{aligned} V(\lambda_{t-1}) =& ((1-\Theta^G)\lambda_{t-1} + (1-\Theta^B)(1-\lambda_{t-1}))u(p,q) - cq - f \\ &+ \gamma \max\{V(\lambda_t^s); V(\lambda_0) - s\}[(1-\Theta^G)\lambda_{t-1} + (1-\Theta^B)(1-\lambda_{t-1})] \\ &+ \gamma \max\{V(\lambda_t^f); V(\lambda_0) - s\}[\Theta^G \lambda_{t-1} + \Theta^B(1-\lambda_{t-1})] \end{aligned}$$

where $(1-\Theta^G)\lambda_{t-1}+(1-\Theta^B)(1-\lambda_{t-1})$ is the probability the seller will prove successful in period t, and $\Theta^G\lambda_{t-1}+\Theta^B(1-\lambda_{t-1})$ that he will fail. In the former case expected profits, whether the buyer kept his incumbent or switched are $\max\{V(\lambda_t^s); V(\lambda_0)-s\}$, in the latter expected profits are $\max\{V(\lambda_t^f); V(\lambda_0)-s\}$. If at the beginning of period t, there is a very low belief that the seller is good, λ_{t-1}, the buyer may buy at a loss in the current period and would not expect much valuable information to be generated by further trial with the same seller[10]. Equivalently, at a sufficiently low reputation, both the current expected gains and the option to continue selling will be so low as not to compensate the seller for the cost of trading and he will switch. This intuition is formalized in the proposition below.

Proposition 1 *The importer maintains an existing partnership as long as his belief on his supplier is greater than or equal to a critical reputation level $\underline{\lambda}$, where $\underline{\lambda} \in [0,1]$ and $\underline{\lambda} < \lambda_0$. Otherwise he terminates the partnership.*

Proof. See Appendix. ■

Although Proposition (1) is similar to a well-known result (see for example Rothschild, 1974 and Whittle, 1982) the details of the proof appear in the Appendix.[11] The key elements of the proof involve the use of standard recursive techniques to show that $V(.)$ is a unique, well-defined, continuous and increasing function. Abstracting from time indices, it can be readily verified that for any $\lambda > \lambda_0$, $V(\lambda) > V(\lambda_0) > V(\lambda_0) - s$, and by the continuity and monotonicity of $V(.)$, there is a unique $\underline{\lambda}$, such that $V(\underline{\lambda}) = V(\lambda_0) - s$. Since $V(\lambda)$ is monotonically increasing in λ, it follows that for all $\lambda \geq \underline{\lambda}$, $V(\lambda) \geq V(\lambda_0) - s$, so that the buyer will remain in partnership with the current seller. Whereas for $\lambda < \underline{\lambda}$, $V(\lambda) < V(\lambda_0) - s$ and the buyer would prefer to initiate a partnership with a new seller. Note that even when in partnership with a good seller, the buyer might switch to a new seller, this is because even a good type could suffer a run of such bad luck that it would drive his reputation down below the critical level. Further note that $\underline{\lambda}$ can be thought of as a "barrier for reputation"–if ever reputation falls below this level, the buyer ceases trading with his current seller. From now on, I consider the situation where the buyer is and remain in partnership with the same seller.

[10] If the buyer prior belief is close to zero then the posterior belief derived as above according to Bayes' rule would not be much different.

[11] The framework here is similar to a one-and-a-half-armed bandit. Both arms are risky, paying positive profits and sometimes negative ones, with a probability that is different and that can be learned through experience. But the arm with initial prior always pays the same value, $V(\lambda_0) - s$, as evaluated by the buyer before being tried.

4 Reputation and trade

In this section I investigate the role of seller's reputation on the quantity decision of the buyer within an existing buyer-seller partnership. At the beginning of each period t, the buyer offers a one-period contract to the seller he is in partnership with, specifying a quantity q. The buyer holds a belief λ_{t-1} that his seller is good. The shipment passes inspection with probability $(1-\Theta^G)\lambda_{t-1} + (1-\Theta^B)(1-\lambda_{t-1})$, in which case the buyer receives $u(p,q)$. Regardless of the inspection realisation, the buyer incurs a per unit inspection cost and the fixed cost f for writing the contract. The buyer's chooses q to maximise his expected per-period profit:

$$\underset{q}{max} E(\pi(p,q,c,f)|\,\lambda_{t-1}) = [(1-\Theta^G)\lambda_{t-1} + (1-\Theta^B)(1-\lambda_{t-1})]u(p,q) - cq - f \quad (5)$$

Let q_t^* be the quantity imported by the buyer at time t, that maximizes $E(\pi|\,\lambda_{t-1})$ given λ_{t-1}. The optimal purchase volume is independent of the outcome of the inspection in period t. The buyer optimal quantity decision depends only on the belief he holds about the seller's type in t and updated beliefs affect his decision in the following period. Intuitively, following a success, an improved reputation would lead the buyer to import higher volumes from his seller, while a failure resulting in a deterioration of the reputation would lead to decreased quantity imported. This intuition is formalised in the proposition below.

Proposition 2 *Whenever a partnership is formed, the optimal quantity q^* the buyer imports increases as he becomes more optimistic about his supplier. i.e. $\partial q^*/\partial\lambda > 0$.*

Proof. After a partnership is initiated, the buyer chooses q to maximise $E(\,\pi(p,q,c,f)|\,\lambda_{t-1})$. Denoting the buyer's optimal decision at time t by q_t^*, the first-order necessary condition yields:

$$[(1-\Theta^G)\lambda_{t-1} + (1-\Theta^B)(1-\lambda_{t-1})]u_q(p,q_t^*) - c = 0 \quad (6)$$

Condition (6) requires $u_q(p,q^\star) > 0$. The second-order necessary condition requires $u_{qq}(p,q^\star) < 0$. Diminishing marginal returns in quantity is both necessary and sufficient for the existence of an interior solution.

The optimal quantity imported $q_t^* = q(\lambda_{t-1},\Theta^G,\Theta^B,p,c)$, depends upon the buyer's prior λ_{t-1}, the probability that the shipment imported is rejected for both types, and the marginal cost of inspection. In particular it follows from condition (6) that:

$$\frac{\partial q_t^*}{\partial\lambda_{t-1}} = -\frac{(\Theta^B-\Theta^G)u_q(p,q_t^*)}{[(1-\Theta^G)\lambda_{t-1} + (1-\Theta^B)(1-\lambda_{t-1})]u_{qq}(p,q_t^*)} > 0 \quad (7)$$

Thus, the optimal quantity imported increases as the belief that the supplier is of the good type increases. ■

5 Food regulation, reputation and trade

I now move on to the main results of the paper. In this section I examine the impact of an increase in the domestic regulation ϕ_m on the optimal volume purchased within a partnership. In order to allow for a role of reputation, I consider an existing partnership at time t, that remained in existence over the following k periods, and explore the changes in the optimal volume of import at time $t+k$, given a change in the regulation at time t. I uncover two channels through which regulation stringency affects international trade.

As shown in proposition (2), at time $t+k$ the optimal volume of import, $q^*_{t+k} = q(\lambda_{t+k-1}, \Theta^G, \Theta^B, p, c)$, depends both on the reputation of the supplier λ_{t+k-1}, and his type–because good and bad quality suppliers have different probability of supplying defective products.
A change in the stringency of the regulation ϕ_m at time t affects the optimal quantity imported at time $t+k$, both directly and indirectly.

$$\frac{\partial q^*_{t+k}}{\partial \phi_m} = \underbrace{\frac{\partial \Theta^G}{\partial \phi_m} \left.\frac{\partial q^*_{t+k}}{\partial \Theta^G}\right|_{\lambda_{t+k-1},\Theta^B} + \frac{\partial \Theta^B}{\partial \phi_m} \left.\frac{\partial q^*_{t+k}}{\partial \Theta^B}\right|_{\lambda_{t+k-1},\Theta^G}}_{\text{Direct effect}} + \underbrace{\frac{\partial \lambda_{t+k-1}}{\partial \phi_m} \left.\frac{\partial q^*_{t+k}}{\partial \lambda_{t+k-1}}\right|_{\Theta^G,\Theta^B}}_{\text{Indirect effect}} \quad (8)$$

The direct effect is a supply-side effect and is termed the *regulation effect.* The indirect effect is a demand-side effect and is termed the *reputation effect.* I now examine each effects in order to then derive the net effect of a change in the stringency of the regulation on optimal trade volumes.

5.1 Regulation effect

From equation (8) the *regulation effect* writes:

$$\left.\frac{\partial q^*_{t+k}}{\partial \phi_m}\right|_{\lambda_{t+k-1}} = \frac{\partial \Theta^G}{\partial \phi_m} \left.\frac{\partial q^*_{t+k}}{\partial \Theta^G}\right|_{\lambda_{t+k-1},\Theta^B} + \frac{\partial \Theta^B}{\partial \phi_m} \left.\frac{\partial q^*_{t+k}}{\partial \Theta^B}\right|_{\lambda_{t+k-1},\Theta^G} \quad (9)$$

A rise in the stringency of the regulation φ_m at time t, affects directly the optimal volume of trade at time $t+k$, through the rate of shipment rejection $\Theta^j, j = \{G, B\}$.

Proposition 3 *Suppose at the beginning of period t, there is an increase in the stringency of the standard ϕ_m set in the buyer's country. The direct effect of this increase is a decrease in the optimal volume of trade at time $t+k$. That is,*

$$\frac{\partial q^*_{t+k}}{\partial \phi_m} = \frac{\theta(\phi_m) u_q(p, q^*_{t+k})}{[(1-\Theta^G)\lambda_{t+k-1} + (1-\Theta^B)(1-\lambda_{t+k-1})] u_{qq}(p, q^*_{t+k})} < 0.$$

Proof. See Appendix. ■

A more stringent standard has a negative effect on the optimal quantity imported at time $t+k$. Indeed, a stricter standard affects the supply of quality goods at time $t+k$, simply because it is more difficult to pass inspection under a more stringent standard. Note that the increase in the rejection rate due to a stricter standard, does not depend either on the seller's type, $\frac{\partial \Theta^G}{\partial \phi_m} = \frac{\partial \Theta^B}{\partial \phi_m} > 0$,[12] or whether the change in regulation took place at t or $t+k$.
Importantly, the magnitude of the *regulation effect* decreases in λ_{t+k-1}, that is the belief the buyer holds when the change in regulation occurs at time t. It ranges in $[\frac{C}{(1-\Theta^B)}; \frac{C}{(1-\Theta^G)}]$, with $C = \frac{\theta(\phi_m) u_q(p,q^\star)}{u_{qq}(p,q^\star)} < 0$. It is easily verified that when both types of sellers have the same probability of rejection, i.e. $\Theta^G = \Theta^B$, there is no reputation building and the negative effect is constant equal to $2C$. Additionally, the size of the interval increases the further apart the rates of rejection for each type.
Equivalently, a seller on which the buyer is optimistic, whether accurate or not[13], is less strongly hit by a stricter regulation. Since a higher belief implies a larger purchase volume, larger sellers are less affected than smaller ones.

5.2 Reputation effect

The second channel through which regulation may influence the buyer optimal quantity decision is through the belief he holds on the seller he is trading with. From equation (8) the *reputation effect* writes:

$$\left.\frac{\partial q^*_{t+k}}{\partial \phi_m}\right|_{\Theta^G,\Theta^B} = \frac{\partial \lambda_{t+k-1}}{\partial \phi_m} \left.\frac{\partial q^*_{t+k}}{\partial \lambda_{t+k-1}}\right|_{\Theta^G,\Theta^B} \tag{10}$$

A change in the regulation at time t affects the quantity decision of the buyer through the seller's reputation at time $t+k$. The seller's reputation at t, is invariant to a

[12] Under this setting the quality distributions for both types are such that $\frac{\partial \Theta^G}{\partial \phi_m} = \frac{\partial \Theta^B}{\partial \phi_m}$, a change in the regulation affects the probability of rejection equally for both types. However, allowing for a change in the regulation to affect differently good and bad type seller would lead to a similar result.

[13] Imagine a bad type that enjoyed a run of such good luck that it would drive his reputation up.

change in the regulation at t, since reputation is a function of past events and not current ones. That is, λ_{t-1} depends only on the seller's failure history up to $t-1$. However, the rise in ϕ_m affects the seller's future reputation. An increase in the stringency of the regulation facilitates the formation of reputations, the $\frac{\partial \lambda_{t+k-1}}{\partial \phi_m}$ term. This intuition is formalised in the proposition below.

Proposition 4 *Conditional on the partnership being maintained over the following k periods, at time t an increase in the stringency of the standard ϕ_m leads to an increase in the seller's reputation at time $t+k$. That is $\partial \lambda_{t+k-1}/\partial \phi_m > 0$.*

Proof. See Appendix. ■

A rise in the stringency of the regulation ϕ_m, increases the seller's future reputation relative to what it would have been had the regulation not changed. Provided the seller remained in partnership, the effect is positive whether he enjoyed a run of successes or suffered a run of failures over the k periods.

Why is that? If a seller enjoyed a run of good luck his reputation at time $t+k$ is updated upwards. But it is updated further up than it would have been under a constant regulation regime. Simply because under a stricter regulation, it is more difficult to successfully pass inspections. Therefore a success is more informative and better rewarded. Now, if the seller suffered a run of bad luck, still his reputation in $t+k$, will be higher. While a success is more informative, a failure is less informative and less severely sanctioned. Under stricter regulation there is more merit in succeeding but less demerit in failing.

From Propositions (2) and (4), the indirect effect of a change in the regulation through reputation is positive.

$$\left.\frac{\partial q^*_{t+k}}{\partial \phi_m}\right|_{\Theta^G,\Theta^B} = \underbrace{\frac{\partial \lambda_{t+k-1}}{\partial \phi_m}}_{>0} \underbrace{\left.\frac{\partial q^*_{t+k}}{\partial \lambda_{t+k-1}}\right|_{\Theta^G,\Theta^B}}_{>0} \tag{11}$$

An increase in the stringency of the regulation facilitates the formation of reputations, $\frac{\partial \lambda_{t+k-1}}{\partial \phi_m} > 0$. Additionally, the optimal quantity decision of the buyer varies positively with the belief he holds at the beginning of period $t+k$, $\frac{\partial q^*_{t+k}}{\partial \lambda_{t+k-1}} > 0$. Thus, a change in the regulation at time t, has a positive impact on purchase volume at time $t+k$, thus moderating its direct negative effect.

Corollary 4.1 *The reputation effect is non-monotone. It varies with the supplier's reputation at the time the regulation change takes place and its failure history over the past k periods. It is increasing up to a unique value* $\overline{\lambda} \in]0, 1[$, *and decreasing thereafter. Moreover, the effect is zero when* $\lambda \in \{0, 1\}$. *Let*

$$\overline{\lambda} = \frac{1}{1 + \left(\frac{1-\Theta^G}{1-\Theta^B}\right)^{\alpha}\left(\frac{\Theta^G}{\Theta^B}\right)^{\beta}} \tag{12}$$

where α *(*β*) is the number of successes (failures) following inspection over the past* k *periods, and* $\alpha + \beta = k > 0$.

Proof. See Appendix. ■

Considering realisations over the past k periods as given, if the buyer holds a very low or very high belief about his seller's type, the magnitude of the *reputation effect* is close to zero. In fact, if there is a very low or very high belief that the seller is good, then further trial with the same seller would not generate much valuable information. According to Bayes' rule if the buyer prior belief is close to zero or one then the posterior belief derived would not be much different. The magnitude of the effect is maximum when the seller's reputation at time t equals $\overline{\lambda}$.

In equation (12), $\overline{\lambda}$ varies with the seller's failure history α and β. When $\alpha = k$, the realisations over the k periods are only successes, the value for $\overline{\lambda}$ shifts towards zero. Inspection are more informative for low levels of reputation, since it is more difficult for suppliers with a low initial reputation to pass all inspections. While if $\beta = k$, realisations are only failures, $\overline{\lambda}$ shifts towards one. In this case, inspections are more informative for high levels of reputation, since everything else being equal, a supplier with good reputation is less likely to fail future inspections.

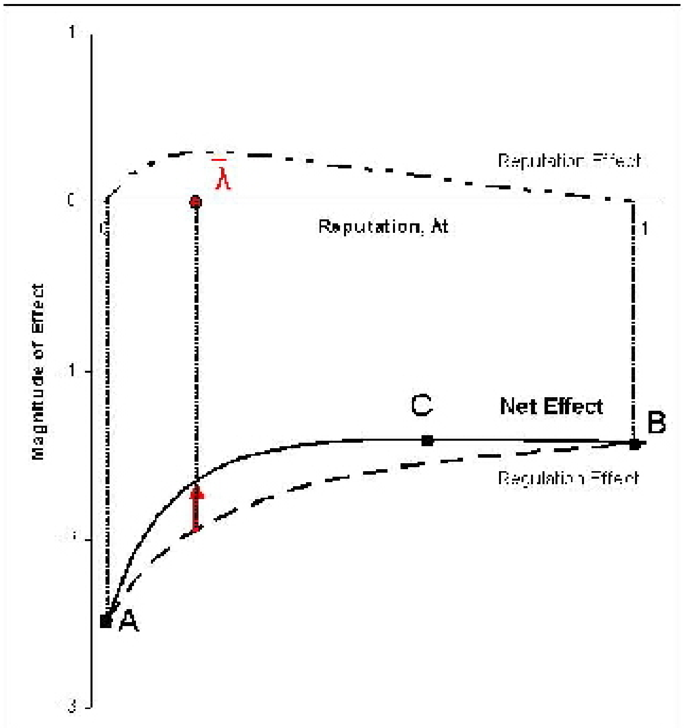
5.3 The net effect

Overall, the net effect of a more stringent standard at time t on optimal purchase volumes at time $t + k$ is the resultant of two diverging forces; a direct negative supply-side effect, and an indirect positive demand-side effect.

$$\frac{\partial q^*_{t+k}}{\partial \phi_m} = \underbrace{\left.\frac{\partial q^*_{t+k}}{\partial \phi_m}\right|_{\lambda_{t+k-1}}}_{\substack{<0 \\ \text{Regulation effect}}} + \underbrace{\left.\frac{\partial \lambda_{t+k-1}}{\partial \phi_m}\frac{\partial q^*_{t+k}}{\partial \lambda_{t+k-1}}\right|_{\Theta^G,\Theta^B}}_{\substack{>0 \\ \text{Reputation effect}}} \tag{13}$$

An increase in the regulation at time t, decreases trade volumes within a partnership at date $t+k$, below its original trend if $\frac{\partial \lambda_{t+k-1}}{\partial \varphi_m}$ is different from zero.[14] That is, it shifts the original trend upwards (Figure 1).

Figure 1: Net Effect of a More Stringent Standard on Trade Volume.



The intuition goes as follows. Suppose a supplier started off in time t, with a low reputation, but successfully passed inspections in the following k periods. His reputation at $t+k$ is updated upwards. Still, the direct effect of a change in the regulation at time t is negative and significant. However, the *reputation effect* is large, given he behaved well over the past k periods. It partly mitigates the negative *regulation effect*. The overall net impact on trade is as if the supplier's reputation was higher than it actually is.

[14]The magnitude of the reputation effect, $\frac{\partial \lambda_{t+k}}{\partial \phi_m}$, is maximum when the supplier's reputation at time t, equals $\overline{\lambda}$, and null if it equals zero or one (corollary 4.1).

The extent to which the *reputation effect* attenuates the adverse direct effect of a more stringent regulation depends on the seller's reputation in t, and on how well the seller behaved over the past k periods. That is, whether he proved to be a promising supplier. Now, since the *reputation effect* is non monotone in λ_t (corollary 4.1), the overall net effect is itself non monotone in λ_t.

- At the assymptotes, the net effect equals the regulation effect (point A and B in Figure 1).

$$\lim_{\lambda_t \to 0} \frac{\partial q^*_{t+k}}{\partial \phi_m} = \lim_{\lambda_t \to 1} \frac{\partial q^*_{t+k}}{\partial \phi_m} = \left. \frac{\partial q^*_{t+k}}{\partial \phi_m} \right|_{\lambda_{t+k-1}}$$

  That is, at time t when the regulation changes, if the buyer's belief is zero or one, the net effect of a stricter standard on the optimal volume of import at time $t + k$ narrows down to the direct effect. Thus, for very low or very high levels of the supplier's reputation, the direct negative effect dominates.

- For intermediate levels of the supplier's reputation, the *reputation effect* significantly attenuates the direct negative effect. The attenuation is maximum for $\lambda_t = \overline{\lambda}$. The net effect reaches a maximum for $\lambda_t \in]0, 1[$ (point C in Figure 1).

Intuitively, $\overline{\lambda}$ can be thought of as a breaking point, when inspection is the most informative and crucial in determining whether the supplier falls in the good-type rather than the bad-type category.

6 Concluding remarks

In this paper I uncover two channels through which regulation stringency matters for international trade: directly, a stricter standard affects the supply of quality goods and indirectly through reputation. I refer to the former effect as the *regulation effect*, and to the latter as the *reputation effect*. I show that in presence of heterogeneous quality, reputation matters for trade. But more importantly, reputation has the power to mitigate the direct negative effect of a more stringent sanitary standard on trade. Results arise from a framework reduced to its simplest form. I develop a model of reputation-based transactions between a buyer in one country and a supplier in another, where information on the quality of the good traded is incomplete but symmetric.

The net effect of a stricter regulation on trade volume varies non monotonically with the belief the buyer holds about his seller at the time the change in regulation takes place. This is so even without taking into account endogenous technological upgrading in the supplier's country. For both very low and very high seller's reputation, the *reputation effect* is negligible vis-à-vis the *regulation effect*. For intermediate levels of the supplier's reputation, the *reputation effect* may significantly water down the direct negative effect. Practically, good and bad type sellers can be thought of as complying and noncomplying firms. The model implicitly recognizes the coexistence of both types of firms, which is a situation often found in low-income countries where a small modern export-oriented segment invests in meeting foreign quality standards. An intermediate level of confidence thus matches the situation of low income countries. I can now answer the question raised in introduction. It is indeed not necessarily true that enhanced stringency of food regulations has a negative effect on developing-country exports.

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7 Appendix

Proof of Proposition (1). Let $B[0,1]$ represent the set of bounded, continuous real-valued functions with domain $[0,1]$. I begin by defining the operator $T: B[0,1] \rightarrow B[0,1]$ as follows:

$$\begin{aligned} T(f(\lambda_{t-1})) &= ((1-\Theta^G)\lambda_{t-1} + (1-\Theta^B)(1-\lambda_{t-1}))\pi \\ &+\gamma[((1-\Theta^G)\lambda_{t-1} + (1-\Theta^B)(1-\lambda_{t-1}))\max\{f(\lambda_t^s); f(\lambda_0) - s\} \\ &+(\Theta^G\lambda_{t-1} + \Theta^B(1-\lambda_{t-1}))\max\{f(\lambda_t^f); f(\lambda_0) - s\}] \end{aligned}$$

where λ_t^s and λ_t^f are as defined in equation (3) above. I then proceed as follows:

First, I prove that it is true that T takes bounded, continuous real-valued functions with domain $[0,1]$ to bounded continuous real-valued functions with domain $[0,1]$. That is, $T: B[0,1] \rightarrow B[0,1]$.

Next I prove Blackwell's two sufficiency conditions for a contraction, which ensure that the contraction mapping theorem applies and that there exists a unique solution to the recursive equation (4), this requires that:
(i) T satisfies monotonicity, that is for any $f, h \in B[0,1]$ with $f(x) \geq h(x)$ for all $x \in [0,1]$ then $(Tf)(x) \geq (Th)(x)$ for all $x \in [0,1]$
(iI) and T satisfies discounting, that is there exists some constant $\alpha \in (0,1)$ such that, $(Tf)(x) + \alpha a \geq (T(f+a))(x)$, for all constants a.

Finally I prove that S takes increasing functions to increasing functions. That T preserves continuity and preserves the monotonically increasing property is sufficient for ensuring that $V(.)$ is continuous and increasing.

- T preserves continuity since the posterior belief distribution is a discrete probability distribution and the Bayesian updating rule is continuous on the prior.

- We now apply Blackwell's sufficient conditions to show that T is a contraction.

Suppose that $f(x) \geq h(x)$ for all x, then it follows that for all x,

$$\max\{f(\tfrac{(1-\Theta^G)x}{(1-\Theta^G)x+(1-\Theta^B)(1-x)}); f(x_0) - s\} \geq \max\{h(\tfrac{(1-\Theta^G)x}{(1-\Theta^G)x+(1-\Theta^B)(1-x)}); h(x_0) - s\}$$

and

$$\max\{f(\tfrac{\Theta^G x}{\Theta^G x+\Theta^B(1-x)}); f(x_0) - s\} \geq \max\{h(\tfrac{\Theta^G x}{\Theta^G x+\Theta^B(1-x)}); h(x_0) - s\}.$$

It follows that $(Tf)(x) \geq (Th)(x)$. Thus, T is monotone.

- We turn to the discounting condition:

$$(T(f+a))(x) = ((1-\Theta^G)x + (1-\Theta^B)(1-x))u(p,q) - cq - f$$
$$+\gamma((1-\Theta^G)x + (1-\Theta^B)(1-x)) \max\{f(\tfrac{(1-\Theta^G)x}{(1-\Theta^G)x+(1-\Theta^B)(1-x)}) + a; f(x_0) + a - s\}$$
$$+\gamma(\Theta^G x + \Theta^B(1-x)) \max\{f(\tfrac{\Theta^G x}{\Theta^G x+\Theta^B(1-x)}) + a; f(x_0) + a - s\}$$

So,

$$(T(f+a))(x) \leq \gamma a + ((1-\Theta^G)x + (1-\Theta^B)(1-x))\pi$$
$$+\gamma((1-\Theta^G)x + (1-\Theta^B)(1-x)) \max\{f(\tfrac{(1-\Theta^G)x}{(1-\Theta^G)x+(1-\Theta^B)(1-x)}); f(x_0) - s\}$$
$$+\gamma(\Theta^G x + \Theta^B(1-x)) \max\{f(\tfrac{\Theta^G x}{\Theta^G x+\Theta^B(1-x)}); f(x_0) - s\}$$

or equivalently

$$(T(f+a))(x) \leq (T(f))(x) + \gamma a$$

which is precisely the discounting condition since $0 < \gamma < 1$. Therefore T satisfies Blackwell's sufficient conditions, and T is a contraction. By the contraction mapping theorem, if T is a contraction, there is a unique fixed point $f^\star \in B$ such that $f^\star = T(f^\star)$ where $f^\star$ is exactly the value function $V(\lambda)$.

- Finally, I show that T maps strictly increasing functions into strictly increasing functions. Suppose that f is strictly increasing and that $\lambda > \mu$. Consider $(Tf)(\lambda)-(Tf)(\mu)$. Then

$$(Tf)(\lambda) - (Tf)(\mu) = (\lambda-\mu)(\Theta^B - \Theta^G)\pi$$
$$+\gamma[(1-\Theta^G)\mu + (1-\Theta^B)(1-\mu)][\max\{f(\lambda^s); f(\lambda_0) - s\} - \max\{f(\mu^s); f(\mu_0) - s\}]$$
$$+\gamma[\Theta^G\lambda + \Theta^B(1-\lambda)][\max\{f(\lambda^f); f(\lambda_0) - s\} - \max\{f(\mu^f); f(\mu_0) - s\}]$$
$$+\gamma(\lambda-\mu)(\Theta^B - \Theta^G)[\max\{f(\lambda^s); f(\lambda_0) - s\} - \max\{f(\mu^f); f(\mu_0) - s\}].$$

Since $\lambda_0 = \mu_0$, $\lambda > \mu$, $\lambda^s > \mu^s$ and $\lambda^f > \mu^f$, and since f is increasing

$$\max\{f(\lambda^s); f(\lambda_0) - s\} \geq \max\{f(\mu^s); f(\mu_0) - s\}$$

and

$$\max\{f(\lambda^f); f(\lambda_0) - s\} \geq \max\{f(\mu^f); f(\mu_0) - s\}.$$

Furthermore $\lambda^s > \lambda^f > \mu^f$, so

$$\max\{f(\lambda^s); f(\lambda_0) - s\} \geq \max\{f(\mu^f); f(\mu_0) - s\}.$$

Hence,

$$(Tf)(\lambda) - (Tf)(\mu) > (\lambda - \mu)(\Theta^B - \Theta^G)\pi > 0.$$

Thus Tf is a strictly increasing function and $V(.)$ is continuous and increasing. Thus, for any $\lambda_t > \lambda_0$ and given $s > 0$, $V(\lambda_t) > V(\lambda_0) > V(\lambda_0) - s$. Therefore, there is a unique value of $\underline{\lambda}$ such that $V(\underline{\lambda}) = V(\lambda_0) - s$, and $\underline{\lambda} < \lambda_0$. ■

Proof of Proposition (3). Differentiating equation (6) w.r.t ϕ_m yields:

$$-[\frac{\partial \Theta^G}{\partial \phi_m}\lambda_{t-1} + \frac{\partial \Theta^B}{\partial \phi_m}(1 - \lambda_{t-1})]u_q(p, q_t^*) < 0 \tag{14}$$

From the quality distributions for each type, detailed in section (2), observe that $\frac{\partial \Theta^G}{\partial \phi_m} = \frac{\partial \Theta^B}{\partial \phi_m}$. That is, a change in the regulation ϕ_m, affects both types equally. Let $\theta(\phi_m) = \frac{\partial \Theta^G}{\partial \phi_m} = \frac{\partial \Theta^B}{\partial \phi_m}$. It follows from condition (14) that:

$$\frac{\partial q_t^*}{\partial \phi_m} = \frac{\theta(\phi_m)u_q(p, q_t^*)}{[(1 - \Theta^G)\lambda_{t-1} + (1 - \Theta^B)(1 - \lambda_{t-1})]u_{qq}(p, q_t^*)} < 0$$

In an established partnership, the immediate effect of a rise in the stringency of the regulation is a decrease in the optimal volume of import. ■

Proof of Proposition (4). Consider the seller's reputation at the beginning of period $t + k$. Expressed in terms of λ_t and h_{k-1}, gives:

$$\lambda_{t+k-1}(h_{k-1}, \lambda_t) = \frac{(1 - \Theta^G)^\alpha (\Theta^G)^\beta \lambda_t}{(1 - \Theta^G)^\alpha \Theta^{G\beta} \lambda_t + (1 - \Theta^B)^\alpha \Theta^{B\beta}(1 - \lambda_t)} \tag{15}$$

where α (β) is the number of successes (failures) following inspection over the past k periods, and $\alpha + \beta = k - 1 > 0$.
Differentiating equation (15) w.r.t ϕ_m yields:

$$\frac{\partial \lambda_{t+k-1}}{\partial \phi_m} = \lambda_t(1 - \lambda_t)\theta(\phi_m)\frac{(\Theta^B - \Theta^G)[\beta(1-\Theta^G)(1-\Theta^B) + \alpha\Theta^G\Theta^B][(1-\Theta^G)(1-\Theta^B)]^{\alpha-1}(\Theta^G\Theta^B)^{\beta-1}}{[(1-\Theta^G)^\alpha(\Theta^G)^\beta\lambda_t + (1-\Theta^B)^\alpha(\Theta^B)^\beta(1-\lambda_t)]^2} \tag{16}$$

where $\theta(\phi_m) = \frac{\partial \Theta^G}{\partial \phi_m} = \frac{\partial \Theta^B}{\partial \phi_m}$. Note that the reputation λ_t is not affected by a change in regulation at date t. Given $\Theta^G, \Theta^B \in [0,1]$, and $\theta(\phi_m) > 0$, all terms in expression (16) are positive. Thus,

$$\frac{\partial \lambda_{t+k-1}}{\partial \varphi_m} \geq 0 \tag{17}$$

An increase in the stringency of regulation at time t, leads to an increase in the seller's reputation at time $t+k$, conditional on the partnership being maintained over the past k periods. ■

Proof of Corollary (4.1). Let $f : [0,1] \to [0,1]$ be equal to

$$f(\lambda_t, \lambda_{t+k-1}) = \frac{\partial \lambda_{t+k-1}}{\partial \phi_m} * \frac{\partial q^*_{t+k}}{\partial \lambda_{t+k-1}}$$

Replacing using equation (7) and (16) yields:

$$\begin{aligned} f(\lambda_t, \lambda_{t+k-1}) &= \lambda_t(1-\lambda_t) \\ &* \frac{\theta(\phi_m)(\Theta^B - \Theta^G)[\beta(1-\Theta^G)(1-\Theta^B) + \alpha\Theta^G\Theta^B][(1-\Theta^G)(1-\Theta^B)]^{\alpha-1}(\Theta^G\Theta^B)^{\beta-1}}{[(1-\Theta^G)^\alpha(\Theta^G)^\beta\lambda_t + (1-\Theta^B)^\alpha(\Theta^B)^\beta(1-\lambda_t)]^2} && (18) \\ &* - \frac{(\Theta^B - \Theta^G)u_q(p, q^*_{t+k})}{[(1-\Theta^G)\lambda_{t+k-1} + (1-\Theta^B)(1-\lambda_{t+k-1})]u_{qq}(p, q^*_{t+k})} && (19) \end{aligned}$$

Equation (19) rewrites:

$$f(\lambda_t) = C \frac{\lambda_t(1-\lambda_t)}{[A\lambda_t + B(1-\lambda_t)]^2}$$

where
$$\begin{cases} C = -(\Theta^B - \Theta^G)u_q(p, q^*_{t+k}) \\ * \frac{[\theta(\phi_m)(\Theta^B - \Theta^G)(\beta(1-\Theta^G)(1-\Theta^B) + \alpha\Theta^G\Theta^B)((1-\Theta^G)(1-\Theta^B))^{\alpha-1}(\Theta^G\Theta^B)^{\beta-1}]}{[(1-\Theta^G)\lambda_{t+k-1} + (1-\Theta^B)(1-\lambda_{t+k-1})]u_{qq}(p, q^*_{t+k})} \\ A = (1-\Theta^G)^\alpha(\Theta^G)^\beta \\ B = (1-\Theta^B)^\alpha(\Theta^B)^\beta \end{cases}$$

At the end of any period t, if the buyer knows with certainty the type of his seller, and $\lambda_t \in \{0,1\}$, the impact of an increase in ϕ_m at time t on future reputation is null, there is no more update and $f(0) = f(1) = 0$. As the buyer becomes more optimistic and λ_t approaches one (or more pessimistic and λ_t approaches zero), inspections become less informative. In which cases, there is no room for a reputation building.

Now for any $\lambda_t \in]0,1[$, $f(\lambda_t) > 0$. By the Intermediate Value Theorem, there exists a $\overline{\lambda} \in]0,1[$ such that $f'(\overline{\lambda}) = 0$. That is $f(.)$ reaches a maximum in $\overline{\lambda}$. It is a matter of algebra, to obtain:

$$\overline{\lambda} = \frac{1}{1 + \left(\frac{1-\theta_G}{1-\theta_B}\right)^{\alpha}\left(\frac{\theta_G}{\theta_B}\right)^{\beta}}$$

and $0 < \overline{\lambda} < 1$, for any $\Theta^G, \Theta^B \in [0,1]$ and $\alpha, \beta \in \mathbb{N}^+$. ■