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Abstract

This paper considers the optimal management of exchange rate target zones by regarding the operation of a target zone as a dynamic signalling game between the monetary authorities and the financial markets. A Sequential Open Loop (Feedback) policy of sterilised intervention is proposed that depends critically on the evolution of the policy maker's credibility as opposed to the open loop precommitment strategy that has been implemented, for instance, in the Exchange Rate Mechanism of the EMS and in the Bretton Woods system. The width of the target zone and re-alignments are in turn determined optimally given the policy maker's credibility. This *flexible target*

*This paper is a substantially revised version of an earlier paper with the same title. The revisions in this version have been stimulated by the consideration of the design of crawling pegs as in place in countries such as Hungary. Work on this revision was carried out while the third author was involved in an ACE project; P96-6149-R on Policy Making in a small open economy aimed at joining the European Union. We thank the partners on this project, in particular Axel Weber and Istvan Szekely, for a number of discussions on the issues raised by the formal design of Crawling Bands. A further revision of this paper which also considers the optimal rate of depreciation as well as the band width is currently in progress.

zone proposal is shown through simulation to stabilize the exchange rate to a substantial degree while retaining considerable flexibility and robustness in response to shocks.

Key Words: Exchange Rate Target Zones, Credibility, Flexibility, Sterilised Intervention, Signalling, Dynamic Policy Games.

1 Introduction

The history of international monetary arrangements from the Gold Standard through Bretton Woods to the Exchange Rate Mechanism of the EMS has been characterised by periodic recourse to what have in effect been target zones for exchange rates, although at times obviously with such narrow band widths that they are considered fixed exchange rate systems. The open loop precommitment to maintaining an exchange rate within such bands has led to rigid and inflexible systems that over time lose credibility and eventually collapse. Yet there is no inherent theoretical reason why target zones *if properly managed* need to collapse. The dependence on a strategy of open loop precommitment would seem to be based on the *misplaced* belief that only such policies can attract credibility and that feedback strategies or flexible target zones, that respond the state of the economic system and recognise asymmetric shocks, would either not stabilise exchange rates or not be seen to be credible by the financial markets.

While the underlying reasons for the loss of credibility in previous systems may have differed there appears to be no well developed theory for the *operation* of such target zones that may prevent periodic crises and the ensuing instability in financial markets[1]. This paper seeks to provide one such theory that rests on endogenising the credibility of the policy maker within the target zone and requires that his policy actions with respect to the maintenance of the target zone are

[1] Calls to widen the 1% bandwidths of the Bretton Woods System so as to make it more flexible and less " brittle" were clear in the mid 1960's, see Bergsten et al [1970], the " Burgenstock Papers" but there was little theoretical rationalisation for the extent to which the bands should be widened. John Williamson [1983],[1986],[1987],[1989] has more recently called for the establishment of target zones for the world's major currencies and his proposals have much in common with the *flexible* target zone model we develop below.

determined by this credibility. Of course, the objective of requiring the relative fixity of exchange rates has been seen as one part of a wider context which involves the coordination of monetary and fiscal policies internationally and ultimately it may only be the success or failure of such coordination that determines the viability of any partially fixed exchange rate system. However given the ever present concern that freely floating exchange rates demonstrate excessive volatility and sustained misalignments from equilibrium it would seem there is a clear need to consider how such target zones may be *optimally* managed with *feedback* policies within the context of internationally coordinated economic policy.

The recent technical literature on stochastic target zone models, following the formalisation offered by Krugman [1991][2], has not yet addressed a number of important practical issues such as the optimal width of a target zone[3] or the manner by which realignments may be *optimally* carried out to pre-empt such attacks that led to the events in the EMS during 1992/3. Since both of these issues can be seen to depend on the credibility of the system it would appear necessary to consider an explicit mechanism that determines the policy-maker's credibility and its evolution. This is all the more important given that the process which determines the financial market's expectations is central and generates the so called " honeymoon effect" which provides one of the basic *raison d'être* for exchange rate target zones. Secondly there is a need to consider the trade off that exists between a policy-maker's basic desire for flexibility (i.e. divergence in monetary policy) and credibility in terms of the degree of stabilization offered by a given band width. Incorporating these elements into the analysis of target zones provides a clearer understanding of the stresses that operate within a rigid target zone system and indicates how such systems could be reformed and managed[4].

In this paper we consider these issues by formulating the problem as a stage-wise signalling game through sterilised intervention between the financial markets and the policy-maker. This approach is distinct but has obvious similarities in objectives

[2] See for instance the volume edited by Krugman and Miller[1992]

[3] It is, for instance, interesting to note how often a band width of $2\frac{1}{4}\%$ has appeared historically without apparently any formal justification. This band width was suggested both at the time the reform of Bretton Woods was considered, in the "Snake" and of course more recently in the ERM.

[4] We stress at the outset that we see the issue of how to determine the central parity in a target zone as depending on the wider policy issues involving policy coordination which we do not consider in this paper and which the target zone alone may not be able to achieve.

with the analysis of realignments provided, for instance, by Miller and Weller [1989], Bertola and Caballero [1991],[1992] and Driffill and Miller [1993] amongst others. However, in this work on realignments, the analysis of credibility has invariably been implicit rather than explicit and moreover it has not been developed within an optimising model of exchange rate policy. It is the *evolution* of credibility and the dynamic implications of partial credibility on policy within a target zone that largely determines the sustainability of the target zone system *given* international coordination of policy and the ensuing choice of central parity in the band.

It appears that central bank intervention is, in reality, relatively frequent, occurs intra-marginally and is largely, if not completely, sterilised [5]. Hence we depart from standard target zone models in the way that we consider the monetary authorities acting so as to influence the exchange rate through frequent *fully sterilised* intervention which leaves the fundamentals unaffected. The role of such intervention in signalling and hence adjusting the market's expectations of future monetary policy or the policy maker's preferences has been considered, more generally, by a number of authors, see for instance, Dominguez [1992] and Dominguez K.M. and J.A. Frankel [1993]. While these aspects are built into our model of the monetary authority's behaviour we also allow the authority to use direct unsterilised intervention on the fundamentals at the margins of the band which again seems to correspond well with reality.

In the next section we describe how partial credibility may be introduced into the standard forward looking target zone model. We distinguish between two forms of credibility, *absolute* and *relative*. Absolute credibility refers essentially to the absolute degree of commitment the policy-maker makes to exchange rate stabilisation. So a free float regime would correspond to zero commitment or absolute credibility and a fixed exchange rate regime would correspond to full commitment or perfect absolute credibility. We assume full absolute credibility is gained from an extraneous commitment or an institutional agreement that ensures full credibility and hence it serves as the natural base from which partially (absolute) credible target zones may be gauged. Relative credibility on the other hand refers to the degree of confidence the markets hold in the particular announced exchange rate band which

[5] See *inter alia* Weber[1994] or Mastropasqua et al[1989] for an analysis of sterilised intervention within the ERM and Rogoff (1984) for evidence of the absence of a portfolio balance effect from sterilised intervention.

lies somewhere between the free float and the fixed regimes. So given any particular band-width the corresponding target zone may obtain full relative credibility while only representing partial absolute credibility.

It is important to notice that the policy maker's commitment in our framework is intended to be much weaker than that assumed in the standard target zone model. The policy maker's initial announcement corresponds to a commitment to direct policy, both inside the band limits and at the margins, towards the objective of keeping the exchange rate within the announced band given the shocks hitting the system. While this is clearly a much weaker commitment than usual it may still be fully credible and indeed it may be easier to be more credible with a weaker commitment. The possibility of the exchange rate passing beyond the announced limits on the exchange rate for a period is not precluded in our model as we demonstrate below, and we therefore provide one formalisation of the notion of " Soft Buffers" originally put forward by John Williamson [1983].

Having drawn this distinction between absolute and relative credibility we then describe, in section 3, how we view the financial market forming its expectations in the aggregate given an underlying atomistic and heterogeneous structure in which the monetary authority acts a single large player. The degree of market power given to the policy maker in shifting the views of the market via his interventions and other signals is determined by an explicit market power function that depends on the relative credibility of the policy maker.

In section 4 the stage-wise asymmetric information game that exists between the policy-maker and the markets is described. The policy-maker in general faces a trade-off between a desire for greater flexibility in monetary policy and a desire to maintain credibility. Flexibility is obtained through a greater divergence in fundamentals than would be consistent with the announced band width. Whereas a loss in credibility results if the markets recognise any deviation from the announced policy and adjust their expectations accordingly. When the markets recognise that the policy-maker has lost all relative credibility they will ignore the existence of the announced target zone and adjust their expectations removing the benefit of the honeymoon effect by moving to the free float value of the exchange rate and inducing a realignment of the band. The net benefit of the political costs incurred in balancing this trade-off between credibility and flexibility is weighed in the policy-maker's objective function together with the basic desire to stabilise the exchange rate around the central parity [6].

Finally we demonstrate the potential power of the proposed flexible target zone structure to stabilise the exchange rate, through a series of simulation exercises, before drawing some conclusions.

2 Partial Credibility and the Target Zone Model

Several attempts have been made to endogenise and formally introduce partial credibility into the standard target zone model. Krugman [1991] and Pesenti [1991] suggested a similar approach to modelling partial credibility to that we use below as the weighted average of two solutions of the basic target zone model but they did not develop their analysis so as to endogenise credibility nor did they consider the policy implications for the management of a target zone as we do below. Bertola and Svensson [1993] developed a model with time varying realignment risk with an exogeneous devaluation probability and Svensson [1991] describes a simple method of assessing credibility in a target zone by checking whether expected future exchange rates fall inside the announced band. Recent contributions that consider learning and the role of credibility to varying degrees are Driffill and Miller [1993] and Cukierman *et al.* [1993],[1994]. Driffill and Miller, while considering the Bayesian updating of realignment expectations, do not consider wider issues relating to the private sector's uncertainty about the government's preferences and hence the evolution of credibility in an optimising context. Cuckierman *et al.* [1993] develop an analysis which has similar objectives to that given below, but they assume that all the relevant dynamics take place outside the announced band and hence there are no forward looking target zone dynamics and expectations are essentially regressive. Cuckierman *et al.* [1994] is concerned with the trade-off between credibility and flexibility in stabilisation programs and does not consider target zones.

The "canonical" target zone model is based on two equations which specify the evolution of the exchange rate $s(t)$ as the present discounted value of expected

[6] One contribution of this paper lies in the analysis of the policy-maker's loss function given in section 4 since we deviate substantially from the proportional costs of intervention model that has been considered elsewhere and which does not have a clear theoretical justification. For instance Miller and Zhang [1994] developed an analysis of the benefits of precommitment over discretion a target zone context with proportional costs of intervention while ignoring the evolution of credibility

fundamentals $k(t)$ based on information available today:

$$s(t) = \frac{1}{\alpha} e^{\frac{t}{\alpha}} \left\{ \mathbf{E} \int_t^\infty e^{-\frac{\tau}{\alpha}} k(\tau) d\tau | \mathcal{F}_t \right\} \tag{1}$$

where α is the semi-elasticity of money demand, and a law of motion for fundamentals,

$$dk(t) = \eta dt + \sigma dW(t) + d\xi^+ + d\xi^- \tag{2}$$

where η is a given drift, σ is the diffusion parameter and $W(t)$ is a standard Wiener process. The terms $d\xi^+$ and $d\xi^-$ represent unsterilised interventions the policy maker applies to the fundamentals when they reach either the upper or lower limits of the announced band. Given the basic model described in equation (1) there are in principle only two ways in which the policy maker can affect the exchange rate, either by direct action on the fundamentals or via the expectations operator. We consider both channels of policy action below as we assume the presence of standard marginal unsterilised inteventions but also sterilised intramarginal intervention[7]. While our analysis includes both elements we emphasise the signalling channel that affects the expectations operator noting that this latter route has largely been ignored in the existing target zone literature. The σ-algebra, $\mathcal{F}_t$, summarizes the information set available at time t, which as we emphasise below includes the degree of credibility granted to the policy-maker and hence the band width profile the market believes will be relevant in the future.

Each given pair of values for U and L, respectively the upper and lower limits of the announced target zone defines the regime in place. A useful way to explicitly indicate this dependence of the exchange rate path on the information available to the financial markets is to re-express (1) as:

$$s(t) = g(k) = \frac{1}{\alpha} \int_0^\infty \left[\; E \int_t^\infty e^{-\frac{(\tau - t)}{\alpha}} k(\tau) d\tau \;\; | \; U - L \right] p(U - L \,|\, \eta, \sigma) d(U - L) \tag{3}$$

which emphasises that the standard solution is conditional on a given band width $(U - L)$ and that the agents in the financial markets need to form their expectations of the future course of fundamentals based on their perception of the distribution of (future) band-widths given by $p(U - L|\eta, \sigma)$. In the standard target zone model

[7] General policy announcements or policy *inaction* may in fact be as powerful as policy action in sending signals to the financial markets and thereby changing their expectations of future government policy

it is assumed that the policy-maker announces given values for U and L and his intention to defend these limits which is regarded as being perfectly credible. If this open loop pre-commitment loses credibility a realignment may in due course become necessary leading to new values for U and L and the market will in reality start to anticipate this when forming its expectations. A reaction which is not permitted in the standard fully credible model. In our set up we assume that the market has to learn the policy-maker's degree of commitment to the announced band-width given imperfect observations on his actions. His actions, through sterilised intervention reflect the implicit band width the policy maker sees as being desirable at any point in time. Hence the evolution of credibility will essentially turn on the ability of the market to learn the band-width distribution which is also clearly the subject of the policy-maker's interest if he is seeking to manipulate the private sector's beliefs. As we describe below the policy maker will invariably have an incentive to cheat on his initially announced band-width to gain greater flexibility in monetary policy.

If a fixed, fully credible and binding band, (U, L) given, is announced and placed on the exchange rate, the standard derivation (see for instance Bertola [1993]) leads to the following solution for the exchange rate:

$$s = g_{pc}(k) = k + \alpha\eta + Ae^{\lambda_1(k-\underline{k})} + Be^{\lambda_2(k-\underline{k})} \tag{4}$$

where $\underline{k}$ is the lower limit on the desired band for the fundamentals. If instead the exchange rate is free to move unconstrained by a band then, given the earlier assumptions, it will evolve as a random walk with a drift. In this free float case the solution for the exchange rate is:

$$s = g_{ff}(k) = k + \alpha\eta \tag{5}$$

We identify the solution given in (4) with the subscript, pc , since it represents the solution under the assumption that the announced band is perfectly credible and given that it is obtained as the general solution to the following second order differential equation with fixed coefficients:

$$\frac{\alpha\sigma^2}{2} g\prime\prime(k) + \alpha\eta g\prime(k) - g(k) = -k \tag{6}$$

where the roots

$$\lambda_1 = \lambda_1(\alpha, \sigma, \eta) \qquad \lambda_2 = \lambda_2(\alpha, \sigma, \eta) \tag{7}$$

with $\lambda_1 > 0$ and $\lambda_2 < 0$ are found from the quadratic equation;

$$\frac{\lambda^2 \alpha \sigma^2}{2} + \lambda \alpha \eta - 1 = 0 \tag{8}$$

and the constants,

$$A = A(\alpha, \sigma, \eta, L, U) \qquad B = B(\alpha, \sigma, \eta, L, U) \tag{9}$$

are determined together with $\bar{k}$ and $\underline{k}$ from the following boundary conditions *given* values for U and L:

$$g(\overline{k}) = U \qquad g(\underline{k}) = L \qquad g\prime(\overline{k}) = 0 \qquad g\prime(\underline{k}) = 0 \tag{10}$$

The general form of $g_{pc}(k)$ then provides a family of individually perfectly credible solutions as the band-width $(U,\ L)$ and the corresponding boundary conditions vary, as indicated in Figure 1.

Figure 1: Perfectly Credible Solutions with Different Band Widths

We also obtain, for each band–width on the exchange rate, an implied band on the movement of the fundamentals, ($\overline{k}$ and $\underline{k}$) which can then be interpreted as the range of policy independence granted to policymakers for a given choice of exchange rate band. As the width of the band on the exchange rate increases the implied band on the fundamentals becomes wider allowing a greater potential divergence between the two economies.

Emphasising this flexibility to diverge in fundamentals provides a slightly different interpretation for the original policy problem facing the monetary authority. A free float is characterized by what is, in effect, a potentially unbounded deviation between the fundamentals of the two countries but at the same time it suffers from the lack of any positive feedback stabilisation induced through the market's forward looking expectations given the existence of a perfectly credible band; the *honeymoon effect*. A tighter band-width in a fully credible target zone constrains domestic monetary policy more but induces a greater stabilising effect on the exchange rate than a wider band. On the one hand the policy-maker wants to have the greatest degree of stabilisation provided by the markets themselves through the honeymoon effect while on the other he would like to have the greatest freedom in domestic monetary policy[8]. One way of viewing the question of credibility in a target zone can then be seen as the temptation facing a policy-maker to exploit his established credibility for supporting the announced band-width while adopting a domestic policy position and intervention strategy that is ultimately inconsistent with the implied degree of flexibility offered by the announced band. This gain in flexibility can be seen in Figure 1 as a movement towards the free float solution. Given that it is costly to establish and maintain a reputation the policy maker may at any instant prefer to exploit some of his prior investment in credibility and allow the exchange rate and fundamentals to move away from the perfectly credible path. The policy maker is faced with this temptation at the risk that the financial markets may perceive this deviation as implying a lack of will to support the announced band.

Let us consider the compromise facing the policy-maker more closely. As suggested above the trade-off between flexibility and stabilising the exchange rate in effect represents a choice between two alternative solutions with different band-widths from the family given above in (3) including the free float solution which is also a member of this family. Moving towards a position that would be consistent

[8] It is this compromise of course that has provided the argument for the *disciplinary mechanism* of the EMS.

with a wider band-width then represents a loss in credibility with a gain in flexibility that can be seen from Figure 1 either as a relaxation in the need to intervene or greater ultimate range of monetary freedom, $\overline{\overline{k}} - \overline{k}$. To model the degree of credibility we can then consider linear combinations of any two members of this family representing relatively constrained and unconstrained solutions where the weight applied, $w(t)$, $0 \leq w(t) \leq 1$, is time varying and measures the degree of absolute credibility (A-credibility) corresponding to the degree of commitment to exchange rate stabilization[9]. Notice that any convex combination of two solutions of the original family of solutions is also a member of the original family but with a different band width both on the fundamentals and the exchange rate.

Zero absolute credibility is then given by the free float solution and full absolute credibility is given by what we refer to as a fixed exchange rate regime that attains its full credibility from an extraneous commitment device or institution. This "fully" A-credibile solution could also of course correspond to Monetary Union and so we are also interested in considering how the band-width can be systematically reduced given the evolution of credibility and the existence of policy coordination to remove the pressure from divergent fundamentals. The class of partially A-credible solutions to the target zone is then given by:

$$g_w(k) = wg_{pc}(k) + (1-w)g_{ff}(k) = k + \alpha\eta + w[Ae^{\lambda_1(k-\underline{k})} + Be^{\lambda_2(k-\underline{k})}] \qquad (11)$$

The intuition should be clear in that a high value of w ensures that more weight is given to the fixed regime and less to the free float solution and by decreasing w we are able to recover all the solutions which could be found by defining increasing widths for the exchange rate band. For each of these solutions we will have a series of $\overline{k}_w$, $\underline{k}_w$ which are further and further apart. Each of these different target zones, while having limited absolute credibility in principle correspond to a particular policy choice through a band-width announced by the policy maker, w_a, given his loss function in a process we discuss more fully in the next section.

We now define relative credibility (R-credibility) as the degree of commitment to this announced target zone as reflected in the realised exchange rate in the market. We need to distinguish between these two distinct notions of credibility in describing the strategic game since target zones with different band-widths (i.e.,

[9] To simplify the notation, from now on reference will be made to w, rather than $w(t)$, keeping implicit the dependence on time.

different A-credibility) may equally be fully credibile (same R-credibility). So we choose absolute credibility to describe the " absolute" degree of stabilisation aimed for and "relative" credibility to describe the perceived degree of commitment to this absolute objective. The simplest expression for relative credibility is then just the ratio between the band width implied by the current observed exchange rate and that originally announced. This follows immediately from (11) as;

$$rc_t = \frac{w}{w_a} = \frac{s_t - g_{ff}(k_t)}{g_a(k_t) - g_{ff}(k_t)} = 1 - \frac{s_t - g_a(k_t)}{g_{ff}(k_t) - g_a(k_t)} \tag{12}$$

Figure 2 characterises this nonlinear function in terms of the divergence between the observed exchange rate and that expected given the announced band. We can see in particular that $rc = 1$ when $s = g_a(k)$, and $rc = 0$ for $s = g_{ff}$; a symmetric function holds for the lower part of the band.

Figure 2: Relative Credibility

In the absence of learning the policy maker could expect to hold full relative credibility when he adheres to the (s, k) mapping consistent with the initially announced band-width, ie. along the path A $(s = g_a(k))$ in the left hand subpanel of the figure. Beneath this "s-mapping" he may actually increase his credibility as he would be potentially adopting a tighter intervention or monetary policy than required given his initial announcement. Such credibility building is important if we wish to consider how policy can be directed towards achieving a narrower band. Above the path A, there will be a loss of relative credibility, in the region AB, which becomes more dramatic as the exchange rate passes over the upper threshold of the announced band, U_a, through B and into the region BC. The policy-maker achieves both zero relative and absolute credibility when the exchange rate coincides with the free float value.

An important element of the flexible target zone being put forward is that it captures in some respects Williamson's " Soft Buffers" in that fundamental intervention is not necessarily required when *the exchange rate* passes through the upper or lower limits of the announced target zone but only when the *fundamentals* themselves breach either their announced upper or the lower limits. In the fully credible target zone model these two points will coincide but they do not in our partially credible case as will become apparent below. Moreover unlike the perfectly credible target zone a speculative attack will not necessarily emerge immediately when the exchange rate bands are breached as the policy maker will still retain a degree of credibility and only as this credibility evaporates will the market move rapidly towards the free float solution. However if the policy maker recognises this cost of losing credibility in his policy optimisation it should lead him to act more conservatively before the exchange rate crosses the band limits and hence intervene earlier to send the appropriate signals to the market and eventually to use direct unsterilised intervention on the fundamentals. How fast relative credibility is lost and hence how rapidly a speculative attack is built up is determined by the slope of the relative credibility function in the range betweenBC in Figure 2.

So, in this framework, the monetary authority can be seen to have the incentive to exploit its established reputation by moving away from the announced exchange rate-fundamentals path into the region AB in Figure 2 and possibly even into region BC in order to gain a degree of monetary independence. If the financial markets could observe or interpret the policy-maker's actions directly and with certainty there would be no game to be played. However we assume that they are in general unable to directly observe the interventions the monetary authorities make or able to precisely infer the band-width it implicitly uses when accomodating domestic shocks through domestic monetary policy or when intervening. An assumption that would seem to correspond well with reality given the evidence that exists as to how central banks intervene at times.

The realised exchange rate is then assumed to be determined by that value of w that arises from a weighted average of the financial markets' aggregate estimate of the band-width, $\hat{w}_g$, and that employed implicitly by the policy-maker, w_g ;

$$w = \nu(rc_t)w_g + (1 - \nu(rc_t))\hat{w}_g \tag{13}$$

The weighting parameter $\nu(rc_t)$, represents the degree of market power held by the government in the determination of the exchange rate and it is natural that this should depend on the degree of relative credibility ascribed by the market to the policy maker. The parameter w, measuring the effective band width, determines the expectations operator which in turn determines the exchange rate through equation (1) or equivalently equation(11). Equation (13) could therefore be re-interpreted as the aggregation of positions taken about the exchange rate in the market leading to the realised exchange rate through the use of w (in the expectations operator) . Initially, with no deviation from w_a, market power will be a constant, determined by the degree of absolute credibility, and $w_g = \hat{w}_g$ and hence from (13) $w_a = w_g = \hat{w}_g$. However when the policy-maker chooses to deviate from w_a he will in effect be willing to give more power to the financial markets. Eventually the policy-maker could in this way lose complete control of the exchange rate as his relative credibility collapsed.

Individual agents in the financial markets are assumed not to know the exact value of ν_t since we regard the market as being heterogeneous and atomistic. As they are faced with the exchange rate being determined, at each point in time, effectively through equation (13) which involves two unknowns, ν_t and w_g, individual agents in the market are unable to identify the policy maker's actions and therefore intentions exactly [10] . It is this aspect of asymmetric information that provides the leverage which the monetary authorities may exploit in the strategic game below. Notice that we assume the policy maker does have knowledge of $\nu(rc_t)$ for two reasons; in the first place being a single large player in the market he recognises his potential to control the market, an opportunity which is not available

[10] It is well documented in the literature how difficult it is, at times, to precisely determine the monetary authorities actions in the foreign exchange markets. At other times they clearly wish to advertise the fact that they are intervening or feel the exchange rate is at an "inappropriate level", see Dominguez and Frankel (1993) and references therein.

to the individual smaller traders. Secondly we assume the monetary authorities, through their regulatory role, have the ability to monitor events in the market and the market *as a whole* more effectively than the individual traders. The function, ν_t, that determines the degree of effective market power reflects certain intuitive consistency conditions such as ensuring that when the policy-maker has full relative credibility the exchange rate will, in large part, be determined by beliefs consistent with w_g and when it has lost all relative credibility then the expectations of the market, i.e. the free float value of the exchange rate, will dominate. The atomistic nature of the market could be formalised further by letting the individual agents hold heterogeneous rational beliefs as in Kurz (1994)[11]. In particular since it is the wide heterogeneity of individual beliefs in the market that gives the policy maker its power it would be interesting to explore the implications more formally when beliefs in the market converged on a single view as the strategic nature of the game would then be radically altered. However the essential elements we need at present are given by the structure assumed above. In principle any monotonic increasing function of relative credibility such as the following exponential form, and shown in Figure 3, captures the required effects on market power and is used in the simulations below.

$$\nu(rc_t) = 1 - e^{rc_t(\ln(1-\chi))} \tag{14}$$

[11] Some recent work along similar lines has been done by Alberola[1994].

Figure 3: Market Power

The fixed value χ determines the degree of control over market power held by the monetary authority when it has full relative credibility. Notice that this can also be interpreted as the market power held by those in the market whose beliefs coincide with the monetary authority and not necessarily just the authority itself.

3 The Financial Markets and the Adjustment of Expectations

There are two aspects to the financial markets' aggregate expectation formation problem.

(i) The choice of $\hat{w}_g$ reflecting absolute and hence relative credibility;

(ii) The use of the (s, k) mapping corresponding to the selected value of $\hat{w}_g$.

Notice first that the use of the (s, k) mapping corresponding to the value of $\hat{w}_g$ estimated by the markets is formally correct since each implicit band width implied by $\hat{w}_g$ is by definition fully credible from the market's point of view. Hence the appropriate mathematical function to use to determine the market's expectation of the exchange rate is that given by the standard solution given by (11). It can also be seen from (13) that the expected value of w given $\nu(rc_t)$ is simply $\hat{w}_g$ hence the private sector minimises their mean square forecast errors on the exchange rate by determining their best estimate of w_g. In reality it almost certainly takes time to build reputation, therefore the evolution of credibility must have some stickyness. In addition the markets are faced with the difficult task of predicting the evolution of w_g which is a non-linear function of the *state* of a system which is itself continuously changing. Moreover they do not have direct observations on the quantity they are trying to estimate w_g, only on the observed exchange rate.

This is quite a different type of expectation formation and signal extraction problem than considered in the monetary policy and inflation games considered by Cuckierman and Meltzer [1990] and Basar and Salmon [1990]. In that class of problems the markets were left with the problem of estimating the evolution of a stochastic policy preference parameter which follows a fixed law of motion. The natural approach in that case was to let the market learn this policy preference parameter through the Wiener or Kalman filters. These filters are justified as optimal in the mean square sense and correspond with this quadratic loss to the use of unconditional and conditional expectations for the unknown parameters. Given the stable linear environment a learning mechanism based on those filters converges as the Kalman Gain effectively goes towards zero and credibility reaches its steady state value. Intuitively this means that the learning algorithm *turns off* when all parameters in the fixed structure have been estimated. In our case this is would be clearly suboptimal as the structure generating the exchange rate is in reality changing over time as the band width implicitly being tracked moves with the policy-maker's optimisation problem. In this non-constant environment it is important that the learning algorithm adopted by the financial markets has the capability to track this instability and does not turn itself off as time passes. Optimal learning schemes in this nonlinear, time varying situation are derived from the theory of stochastic approximation and we propose that the market uses an algorithm of this type when attempting to minimise its prediction error loss function[12]. Therefore we specify that *in the aggregate* the market learns by adjusting

[12] see White [1992],Kuan and White [1991], [1994] and Robbins and Munro [1951].

its previous estimate of $\hat{w}_g$ using the following stochastic approximation algorithm:

$$d\hat{w}_g = \gamma_t(w - \hat{w}_g)\ dt \tag{15}$$

where γ_t measures the "viscosity" of the financial market which we assume is a constant rather than decreasing to zero at rate $T^{-1/2}$ and thereby ensure continuous learning. The use of the term viscosity is intentional as it indicates the speed with which we assume an atomistic and heterogeneous market can process information efficiently. This learning mechanism is clearly an essential element in a signalling game of the type we have envisaged above and indeed the monetary authorities at various times in reality do make a clear decision to make public their interventions in the market. Hence the parameter γ_t could be seen to change depending on how discrete the authorities wished to be in their actions and hence how fast the market was able to learn. Alternatively γ_t could be state dependent and itself reflect relative credibility as in adaptive learning schemes, see Marimon and McGratten [1994].The information set the individual agents have available is the inferred value of w from the observed values of the exchange rate and the fundamentals. This in turn depends on the unobserved w_g through equation (13) and the learning mechanism can in fact then be written as;

$$d\hat{w}_g = \gamma_t \nu(rc_t)(w_g - \hat{w}_g)\ dt \tag{16}$$

This formulation allows us to see how relative credibility directly affects the financial markets ability to learn. Given the nonlinearity, when the policy-maker has low reputation he must make a much greater effort proportionately to recover his reputation than if he had a high degree of credibility. We may *a priori* have expected that as the level of relative credibility decreased with the exchange rate perhaps being close to, or beyond, the limit of the announced band the markets may more rapidly adjust their expectations. However learning about w_g slows down somewhat paradoxically when the market power is low as there is *less* information in the observed exchange rate regarding the policy maker's actions and w_g. The exchange rate is dominated by the market's beliefs as it moves to the free float value of the exchange rate and the policy maker must work harder to re-establish its reputation than if he had a high degree of relative credibility. Similarly when relative credibility is close to unity then we can expect a relatively fast adjustment of expectations and again the $\nu(rc_t)$ function which maps relative credibility into the $(0, 1)$ interval will pick these effects up.

Finally with this estimated value of the band-width parameter the financial markets will in aggregate, use the standard forward looking target zone mapping between the fundamentals and the exchange rate to generate its forecast of the exchange rate which is then combined effectively with the policy-maker's forecast through the market power function (13) to generate the observed exchange rate given the observed fundamentals.

4 The Strategic Game with the Markets

As in a standard asymmetric information Stackelberg game we assume we start from an initially announced and credible band on the exchange rate determined by U,L or w_a. The market then forms its expectations of the exchange rate so as to minimise its mean squared forecast error taking into account the observed exchange rate, the policy-maker's credibility and the initial announcement. The policy-maker then optimises an objective function at each point in time *given* the private sector's expectations to determine the band-width he will implicitly use to set his policy actions (sterilised intervention, policy announcements or inaction) in the next period. In doing so he will trade-off the potential costs of reneging on his initial announcement and the potentially increased volatility in the exchange rate against the benefits of gaining extra monetary freedom[13]. The policy maker also has a direct incentive to reinforce his credibility if he wishes to maintain the stabilising influence of the honeymoon effect consistent with the announced band-width. In adjusting their expectations the agents in the financial markets may reduce the policy-maker's credibility and hence also the degree of stabilisation provided by the honeymoon effect.

Given that the reputational costs of reneging on the announced policy are explicitly taken into account in the monetary authority's policy optimisation problem described above the solution that follows represents a sequential open loop strategy which characterises a feedback policy from which at any point in time there would be no incentive to renege. The use of such innovations contingent policies to

[13]The structure of this strategic game between the policy-maker and the financial markets reflects exactly the analysis of credibility and reputation in the optimal monetary policy and inflation problem studied by Barro and Gordon [1986], Cuckierman and Meltzer [1990] and Basar and Salmon[1990] amongst others.

mimic feedback policy in forward looking models has been considered for instance by Buiter [1981] and adaptive policies of this kind have a long history in control theory, see for instance, Tse [1974].

As explained above the essential element of asymmetric information in this process lies in that individual agents in the financial markets are not able to exactly recover the actions of the policy-maker. If an asymmetric shock hits the exchange rate moving it away from the expected relationship with the fundamentals as shown in either small panel in Figure 2, the market will, being uncertain as to whether the policy maker has acted consistently with his declared policy or not, start to revise its view that the policy-maker is perfectly credible in his intention to ultimately defend the announced band. Hence some weight will be added to the opinion that the policy-maker would like to have greater flexibility in managing the evolution of its own fundamentals and the market reduces its estimate of w_g and gives a larger weight to the free float solution compared with the originally announced band. The observed exchange rate is then determined through the band-width that results from the market power relation (13). If the policymaker does not in due course undertake adjustments which bring the exchange rate back to the credible path, he has to be ready either to bear larger costs when the exchange rate hits the limit of the band or pay a political cost in order to obtain an realignment of the initially announced band and hence reduce his level of absolute credibility. The policy-maker therefore recognises at each point a cost which is the present discounted value of the credibility gap measured on the boundary for the announced w_a. When the exchange rate is close to the central parity, a small deviation from the perfectly credible path implies a large deviation at the boundary and this implies a big reduction in the flexibility enjoyed by the policymaker. The existence of an "anticipative" political cost of this type connected with deviation from the credible path should encourage early adjustment and intra-marginal intervention.

4.1 The Structure of the Objective Function

The general strategic aspects in the game between the policy-maker and the markets have been outlined above but the critical element in this game is clearly the nature of the policy-maker's objective function. We assume that the policy-maker has five considerations in mind at each point in time;

(i) a basic desire to stabilise the volatility of exchange rate movements,

(ii) a desire to prevent the exchange rate deviating from some absolute level that may be a fixed central parity or a variable target offered by an estimate of the FEER,

(iii) a temptation to gain more flexibility than is offered by the existing band and degree of credibility,

(iv) a desire to reduce the political cost of the loss of credibility that may arise when the markets learn that he has deviated from his announced policy, this can also be seen to reflect the signalling costs, RC_{cost},

(v) and finally a desire not to be forced to realign and thereby lose absolute credibility, AC_{cost}.

The objective function the policymaker seeks to maximize through his choice of w_g is then given by

$$J_1(w_g) = E \int_0^T e^{-\beta t} \left(\alpha_1 (s - s^e) + \alpha_2 (\frac{ds}{dt})^2 + \alpha_3 s^2 + \alpha_4 RC_{cost} + \alpha_5 AC_{cost} \right) \; dt \tag{17}$$

where $\alpha_1 > 0$ and $\alpha_2, \alpha_3, \alpha_4, \alpha_5 < 0$. At the same time the financial markets through their expectation formation seek to minimize their prediction errors as suggested in the following loss function,

$$J_2(\hat{w}_g) = \int_0^T (s - s^e)^2 \; dt \tag{18}$$

which acts as a surrogate for more fundamental profit seeking motives presumably driving behaviour in the financial markets.

The first term in the policy-maker's objective function represents the incentive to surprise the financial markets since it is through an exchange rate surprise that the policy-maker ultimately gains flexibility. How does this come about? Two immediate measures of flexibility could be put forward. The first is a direct comparison of the increase in the potential divergence in the fundamentals from adopting a band-width w_g that is greater than that originally announced; in Figure 4 this is indicated by, A, $(\overline{\overline{k}} - \overline{k})$. The two points indicated, $\overline{\overline{k}}$ and $\overline{k}$, correspond to the smooth pasting tangency points where the slope of the (s, k) mapping is, in each case, zero. This comparison reflects a *potential* increase in flexibility in the implied range in the fundamentals since it is only meaningful when the announced band has been broken. Essentially flexibility can then be seen to be reflected in the curvature of the relevant (s, k) mappings at any particular reference point and an equivalent measure within the announced or expected band can be given by the difference in the level of fundamentals at which same observed gradient on the announced or expected (s, k) mapping occurs on a different mapping; in Figure 4 this is indicated by the distance, B, between $k_{\hat{w}_g}$ and k. In other words the increased flexibility comes from adopting a band width for which the observed sensitivity in the exchange rate (to changes in the fundamentals) occurs at a higher level of fundamentals divergence than expected. However both these measures compare aspects of *ex ante* or potential flexibility since they compare aspects of the observed fundamentals divergence with another somewhat abstract level of fundamentals. Another comparison could be based on the difference between the observed exchange rate and that consistent with the $k_{\hat{w}_g}$ value, D, in Figure 4. but again the degree of *realised* flexibility should perhaps depend more naturally on the market's belief as to the relevant band-width, *given the observed fundamentals* position. Notice that two (s, k) mappings are shown in Figure 4, one corresponding to the realised exchange rate/fundamentals position and the other corresponding to the expected mapping given the market's expectation of $\hat{w}_g$. The easiest way to capture realised flexibility and the one chosen here would then simply seem to be to calculate the difference between the expected and observed level of the exchange rate at the observed level of the fundamentals, i.e. the surprise in the exchange rate. This surprise value, C, will in general understate the potential flexibility measure given by, D.

Figure 4: Flexibility Measures

The second and third terms in the policy-maker's welfare function simply indicate that he wishes to minimise the volatility in the exchange rate and the deviation of the exchange rate itself from some normalising position which we have chosen to set equal to zero. The choice of the base or central parity clearly opens up a range of fundamental questions which we do not attempt to address in this paper since our objective is simply to consider the way in which partial credibility effects can be modelled. These two terms represent the non-strategic elements in his welfare function whereas the other three terms arise from the strategic game.

The fourth term represents the political costs (and benefits) from deviating from the announced band. Within a given band we introduce two separate effects here one for the upside risk and the other for the downside risk depending on whether the actual exchange rate is above or below the announced level for the observed level of fundamentals;

(i) When the policy maker contemplates a wider band width by choosing a smaller value for w than that corresponding to the announced band-width we measure political cost, as can be seen from Figure 5., by the implied amount of intervention that would be needed to return to full credibility. This can be taken as the distance along the band between $(\overline{k}-k_w)$, E, where k_w represents the intersection of the mapping on which the observed exchange rate lies and the upper band limit. Since this measure does not depend on the observed value of the state we multiply this quantity (of intervention) by the difference between the observed exchange rate $s(k)$ and the corresponding value under the announced band $(g_a(k))$, F. In this way political cost grows as you move further towards the edge of the band or the more the policy maker deviates from the announcement.

(ii) When the policy maker attempts to increase his credibility by adopting a w that implies a narrower band-width than the announced he will receive a *bonus* rather than a cost. Reversing the roles of the two (s, k) mappings in Figure 5 this is picked up by measuring the difference in the fundamentals necessary to maintain the same exchange rate in the announced and the chosen target zones, G.

Figure 5: Political Cost

The asymmetry in the upside and downside costs, apart from their sign, reflects the nonlinearity observed in the relative credibility function. We emphasise this by adopting different exponential functions for the upside and downside political costs as discussed in the context of risk sensitive decision making for instance by (Whittle [1990]) and more recently by Hansen and Sargent (1992). So we have for the loss of relative credibility term;

$$RC_{cost} = polcost_1 + polcost_2 \tag{19}$$

where

$$\begin{aligned} polcost_1 &= \exp[-\left(\frac{\phi_1}{2}(\bar{k} - k_w)(g_w - g_a)\right)\mathcal{I}_1] & (20) \\ polcost_2 &= \exp[-\left(\frac{\phi_2}{2}(k - k_a)\right)\mathcal{I}_2] & (21) \end{aligned}$$

where $\mathcal{I}_1$ and $\mathcal{I}_2$ are indicator functions which denote which case applies.

The exponential specification of these terms in the cost function with different parameters will enable us to systematically incorporate asymmetric risk issues and in particular differential upside and downside risks when close to the upper limit of the announced band[14].

In comparison with the standard LQG specification, where the linearity implies only a mean-variance trade-off, we are able to take into account, as should be important in this non linear environment, the higher moments of the loss/benefit associated with the political cost. These are shown explicitly by the expansion:

$$E[exp-(\frac{\phi_1}{2}C(k))] = [1-\frac{\phi_1}{2}E(C(k))+\frac{\phi_1^2}{4\cdot 2}E(C^2(k))-\frac{\phi_1^3}{8\cdot 4\cdot 2}E(C^3(k))+.....] \tag{22}$$

The final term in the policy-maker's objective function indicates a cumulative political cost that reflects an assumed loss from a realignment of, in general, both the central parity and new announced band-widths. The realignment process is a crucial part of the proposed flexible target zone. In a sense this term picks up absolute credibility loss when the policy maker periodically decides it is optimal to announce a new target zone. Since wider issues are involved with setting the central parity, in the present context a realignment simply takes the form of a

[14]It is again interesting to note that Chittenden [1970], when considering the reform of the Bretton Woods system, also recognised the asymmetry of costs and called for asymmetric bands.

change in the band–width and this action is seen to be contemplated only after the possibilities offered by policy coordination in reducing fundamentals divergence are exhausted. We assume that initially the announced target zone is fully credibile and that it has an initial quantity or stock of endowed credibility given by the size of the speculative attack that would instantly move the level of fundamentals from the smooth pasting point of tangency, $\bar{k}$, with the upper band to the free float level of fundamentals, k_{ff} at the same exchange rate. The policy maker may also decide through his intervention strategy to reduce the implicit band-widths as time proceeds and shocks to the fundamentals allow by effectively conducting a tighter intervention or monetary policy than required and thereby add to his stock of initial credibility. Hence he is involved with a continuous process of investment and consumption of his credibility stock. However once this stock of relative credibility is used up, perhaps sometime after the upper limit of exchange rate fluctuations has been breached, he will see it as *optimal* to realign and propose a new band width corresponding to a new level of absolute credibility and the strategic game then restarts within the new band. The band width for which the new announcement is made is such as to leave the expectations of the private sector unchanged, so that $w = \hat{w}_g$, both for widening or narrowing the band. This ensures that there will be no jumps in the expected value of the exchange rate s^e before and after the annoncement and that the relative credibility function will jump to 1 [15] and the $\nu(rc_t)$ function is reset to χ in figure 3.

These processes of the accumulation of political costs and bonuses allow flexibility to be built into the target zone since an immediate realignment is not required each time the policy maker, deviating from the announced path, lets the exchange rate go above or below the exchange rate band. It is only after "enough" political cost (bonus) is accumulated that (depending on the loss function) a realignment in the band-width is induced. This has two advantages with respect to the rigid target zone model.

In the first place we allow for the existence of soft buffers around the announced band, as suggested by Williamson. This model provides, in effect a target zone with a *flexible structure* which can absorb temporary shocks as well as accommodate temporary deviation from the announced policy. This is pretty much in line with

[15] Whenever the announcement is equal to the level perceived by the market, relative credibility is 1 by definition.

the French experience after the widening of the bands of the ERM in the Summer of 1993 and the ensuing return of the French Franc toward the old central parity, within the $\pm 2.25\%$ limits[16].

Secondly, the flexibility introduced leaves the markets without rigid bounds or fixed targets against which safe bets can be placed. After a shock hits the system and some accommodation is granted, the policy maker can revert to a more virtuous policy avoiding an immediate realignment as long as political costs have not been too heavy.

Given these arguments we model this last element in the objective function as;

$$AC_{cost} = \frac{\sum_{\tau=t_0}^{t} \beta^{t-\tau}(polcost_1(\tau) - polcost_2(\tau))}{\bar{k}(s_t) - k_{ff}(s_t)} + \psi \mathcal{I}_3 \qquad (23)$$

The numerator of (23) records the current value of the cumulative discounted net political cost. The denominator indicates the size of the speculative attack needed to bring down the announced band in one shot, and can be seen as the natural threshold to evaluate the life–span of a band (in present discounted terms). The decision rule becomes then to widen the band if the numerator of (23) (the cumulated net political cost in present value) exceeds the denominator. The announcement of a wider band also attracts a lump sum cost given by ψ as he realigns, shown by the indicator function $\mathcal{I}_3$ in expression (23). This cost reflects the loss in absolute credibility when the band is widened and serves as an integral mechanism on the otherwise proportional and derivative nature of the loss function. As mentioned above this integral mechanism acts, in many ways, as an approximation for the deeper optimisation problem involving the determination of the central parity and the design of cooperative policies to achieve the announced level of absolute credibility.

Given current fundamentals, therefore, the cost of the band–width realignment may be less than the cost of defending the older announcement, therefore inducing the policy–maker to pre-empt a speculative attack. By the same token, an announcement of a narrower band can be made when the cumulated bonuses (net negative political cost) exceed the negative of the threshold, showing that the initial amount at stake was accumulated by virtuous behavior.

[16] A companion paper Avesani, Gallo and Salmon[1995] considers the application of this model specifically to the crisis in the ERM in 1993.

Notice that in expression (23) the evaluation of the cumulated political cost entails a discount to an initial time t_0. This is assumed to be reset to be the time at which each announcement occurs, so that there is equivalence (in present discounted value terms) between the threshold and the cumulated political costs incurred by in deviating from the announcement.

5 Optimal Sterilised Intervention Policy and Band–Width Management: Simulation Results

The trade–off between credibility and flexibility in the stage–wise game played between the policy–maker and the markets is now studied by simulation. An analytical solution for the nonlinear stochastic optimisation problem facing the policy maker would seem to be completely infeasible given that it can be expressed as:

maximize with respect to w_g

$$J_1(w_g) = E\int_0^T e^{-\beta t}\left(\alpha_1(s-s^e)+\alpha_2(\frac{ds}{dt})^2+\alpha_3 s^2+\alpha_4 RC_{cost}+\alpha_5 AC_{cost}\right)\ dt \tag{24}$$

where $\alpha_1 > 0$ and $\alpha_2, \alpha_3, \alpha_4, \alpha_5 < 0$ and RC_{cost} and AC_{cost} are given above in equations (19)–(23)

subject to:

$$dk(t) = \eta dt + \sigma dW(t) + d\xi^+ + d\xi^- \tag{25}$$

and

$$s = k + \alpha\eta + \left[\left[1 - e^{\phi\left[\frac{s-g_{ff}}{g_a-f_{ff}}\right]}\right] w_g + \left[e^{\phi\left[\frac{s-g_{ff}}{g_a-f_{ff}}\right]}\right]\hat{w}_g\right]\left[Ae^{\lambda_1(k-\underline{k})} + Be^{\lambda_2(k-\underline{k})}\right] \tag{26}$$

where $\phi = \log(1-\chi)$. At the same time the financial markets through their expectation formation seek to minimize their prediction errors given by the following loss function,

$$J_2(\hat{w}_g) = \int_0^T (s-s^e)^2\ dt \tag{27}$$

through the use of the updating stochastic approximation rule,

$$d\hat{w}_g = \gamma_t\left[1 - e^{\phi\left[\frac{s-g_{ff}}{g_a-f_{ff}}\right]}\right](w_g - \hat{w}_g)\, dt \tag{28}$$

where g_a is the (s, k) mapping corresponding with the announced target zone.

Thus we resort to simulation of the entire game starting by simulating the fundamentals process in equation (2) as a discretization of the Brownian motion without drift (η is set to 0) and with a volatility level of 10%. The time–discretization step is taken to represent one day. At each step, the fundamentals allow for the computation of the free–float and the "Bretton–Woods" solutions which, in turn, are needed to evaluate the political cost and the flexibility terms. We treat the Bretton Woods solution as effectively corresponding to our perfectly A-credible solution given above and is given by a 1% band width. The results refer to 300 simulations performed over a 2-year period.

Given the evolution of fundamentals and an initial announced band with $w = 0.5$ (corresponding to a band–width on the exchange rate of 6.6%) the game between the markets and the policymaker is played out as described above. The policy maker maximises his welfare function at each stage in a sequential open loop manner taking into account the realisation of the fundamentals and his existing stock of credibility and the financial markets form their expectations as to the exchange rate in each period to minimise their loss through the selection of $\hat{w}_g$ and then using the appropriate (s, k) mapping.

In Figure 6a. we show a sample path (corresponding to two years) of the exchange rate resulting from this mechanism (in solid) with the corresponding announced bands (in dots–and–dashes). The free-float and the Bretton Woods solutions are also shown, to appreciate the relative effort needed to adhere to the announcement and the desire for flexibility.

There are five realignments in the two year period, the initial one leading to a tight ERM–type situation with exchange rate bands of about 2%. This regime lasts in place for almost one year with the choice of the policy maker inside the band leaning towards exploiting flexibility (cf. Fig. 6c.). This corresponds to an accumulation of political cost (within the band) which leads eventually to a widening of the band to about 8%. Notice how in the following period the policymaker is virtuous in that his choices of w are above the announced level and keep the exchange rate more towards the Bretton–Woods solution. This allows a first narrowing of the band (which lasts almost three months) and a further narrowing (fourth announcement) right after that. The apparent volatility in the series of w's corresponds to the exchange rate being well within the band and close alternatively to the free–float and the Bretton Woods solutions. In this region the surprise term dominates the political cost, in what could be called a strategy of 'hide–and–seek' pursued by the policy–maker within the band.

The evolution of the perceived band width is portrayed in Figure 6.d which shows a much lower degree of volatility or equivalently a higher degree of smoothness relative to the realized w due to the viscosity in the learning mechanism. Note that at the time of the announcement of a new band–width, there are no jumps in the perceived credibility. The volatility in w within the band arises from the fact that small changes in the level of sterilised intervention close to the central parity imply large relative changes in the effective choice of w. There is no corresponding volatility in the exchange rate as can be seen in Figure 6a. Once the free float level of the exchange rate (or equivalently the fundamentals) moves outside the band limits w stabilises at close to unity as the monetary authorities devote their efforts to increasing their credibility and defending the announced band.

From the specific paths shown, it is clear (see 6a) how the bands provide "soft-buffers" in Williamson's sense, in that the exchange rate is allowed to hover above the announced band for a period in the last quarter of the simulation. From Figure 7.b, we recognize that this last period of the simulation is characterized by some unsterilised interventions on the fundamentals. Notice that in other cases when the exchange rate lies beyond or at the band limits need not imply direct intervention on the fundamentals is necessary because the latter might not yet have reached their boundaries (a case in point is the period about six months into the simulation where the exchange rate is at the boundary while the fundamentals are well inside their band.) Of course, political cost is accumulated (as the exchange rate lies above the announced level) and relative credibility depleted, but a comparison with the free–float solution shows the benefits in terms of reduced volatility and smaller deviations from the central parity.

In Figure 6b., we show the pattern of the fundamentals/exchange rate pairs (each dot corresponds to a different period). If the announcement were followed throughout, we would neatly observe Krugman's S-shaped curve; on the other hand, when a free-float solution is followed , we observe a 45^0 line. The scatter reflects the variety of band–widths realized as the result of the interaction between w_g and $\hat{w}_g$, the outcome of the optimisation problem of the policy-maker and of the relative credibility granted by the private sector.

Figure 7a. shows the evolution of the cumulated political cost (or bonus) relative to the threshold associated with the announcement. Each time the initial endowment of relative credibility is depleted, a new regime is announced and the figure shows how proces by which this comes about. An increasing segment of the curves corresponds to a realized w below the announcement, and the reverse for decreasing segments. An interesting feature can be detected right before the last announcement where the cumulated political cost almost reaches the threshold but given the extant level of perceived credibility at that time a realignment is avoided, or rather deferred to a few weeks later.

The evolution of the $\nu(rc_t)$ function (Figure 7.d) points out that when the realised exchange rate lies within the target zone we again have frequent oscillations in market power. The last part of the simulation period, when the exchange rate is kept at around the upper limit of the band, is characterized by a smooth transition, since it corresponds to a more stable evolution of w and $\hat{w}$. The evolution of relative credibility is shown in Figure 8b and shows clearly those periods when an investment in credibility is made or lost as the band widths are adjusted. Figure 8a shows the realised relative credibility function corresponding to Figure 2 in the text.

Various regimes can be compared with the flexible target zone proposed here. A fixed exchange rate regime is simulated by allowing a narrow $\pm 1\%$ band and forcing a fundamentals intervention at the boundary when the exchange rate hits either the upper or lower limit of its band. Although there are wider issues surrounding the simulation of an ERM equivalent system relating to their assumed full credibility we consider narrow and broad band versions which are simulated as in the fixed exchange rate case by widening the band on the exchange rate to $\pm 2.25\%$, and, $\pm 15\%$ respectively. Finally, the free-float would be characterized by the absence of interventions and by a yearly level of volatility in the exchange rate equal to that of the fundamentals. A comparison between these regimes is shown in Table 1 by comparing the non–strategic part of the objective function, namely

$$-\int_0^T e^{\beta t}\left(\alpha_2 s^2 + \alpha_3 ds^2\right) dt \tag{29}$$

relative to the same level achieved in the free float case. Next to it we present the levels of fundamentals intervention $d\xi^+$ and $d\xi^-$ (in present discounted value) needed to achieve it. Notice that at times the flexible target zone has narrower band-widths than the narrow ERM option.

Table 1: Stabilization and Intervention Comparisons

	Percentage of free float Non. Strategic Obj.Fun	Interventions Above	Interventions Below
Flexible TZ	3.69%	0.0395	0.0382
Fixed	0.50%	0.0569	0.0575
ERM (narrow)	2.47%	0.0370	0.0363
ERM (broad)	51.1%	0.0042	0.0030

Table 2: Private Sector Objective Function: Comparisons

s_t^e	*Mean*	*St.Dev.*
$s_t(\hat{w})$	0.0071	0.001
s_{t-1}	0.0101	0.003
$s_t(w^a)$	0.0368	0.012
s^{bw}	0.0896	0.015
s^{ff}	0.9042	0.442

The results show that the amount of overall stabilization achieved by the flexible target zone proposed here is much higher than the level in the Broad Band case , which in turn is not unnaturally lower than the free float result. Remember that the value of relative credibility in the ERM cases is set to unity exogenously, and no band–width realignments are allowed. The stabilising properties of the flexible target zone are impressive when compared with the narrow ERM case given that the flexible target zone spent extended periods with a wider band width than the narrow ERM.

On the financial markets' side, it is interesting to consider whether the stochastic approximation used in forming their expectations for w_g is optimal with respect to several obvious alternatives. In Table (2) we report the calculated loss over the simulation period in each case.

This simple comparison shows that the chosen rule to update the perceived credibility does indeed minimize the mean sample forecast error. The second row contains the outcome obtained by setting the expectations equal to the previous period's level of the exchange rate and we can then see that there is a clear gain from using the assumed $\hat{w}$ rule over the random walk hypothesis.

Table 3: Exchange Rate Statistical Properties

	Mean	*St.Dev.*
Skewness	-0.0012	1.070
Kurtosis	0.1861	1.851
$ARCH(4)$	548.74	43.00

The third row computes the criterion for the case in which the private sector constantly sets its expectations at the announced level of the exchange rate. Just trusting the announcement (row 3) then would entail a larger mean forecast error, given the frequent deviations of w from it. The last two rows set the benchmarks of using Bretton Woods and the free–float as the expectation updating rule, and should be read just to give an upper bound to the value of the objective function.

Finally, in Table (3) and Figure 8d we examine the properties of the exchange rates as they come out of the simulation with flexible band-widths. In particular, it is interesting to note that for the entire period the behavior of the estimated density of the exchange rate does not exhibit the U-shaped pattern as implied by the perfectly credible or " Krugman" solution, as a result of the non–constant w used. Note also that the series itself contains a high degree of conditional heteroskedasticity and clearly has a leptokurtic distribution.

Although the discussion above relates to a simulation exercise, it is possible to use the model to derive an estimated relative credibility measure for an historically observed exchange rate path. This process is discussed in more detail in Avesani, Gallo and Salmon[1995] where the French Franc/ DM experience is analysed over the period of the enlargement of the band in August 1993. The results in brief are encouraging since the estimated measure of credibility from this optimising model closely matches that which can be inferred from contemporary press accounts.

6 Conclusions

One basic rationale for a target zone is to grant policymakers a certain degree of independence in setting their monetary policy while reaping the benefits of an increased exchange rate stability, as argued for on somewhat different grounds by Williamson [1983] and Svensson [1992]. The suggestion put forward in this paper extends this view by showing how the proposed strategic game allows the desire for greater flexibility to be fulfilled at the expense of a reduction in the policymaker's credibility. In one sense this work could therefore be seen as an attempt to provide a more mathematical basis for the notion of flexible target zones as put forward by John Williamson. We have also tried to conduct a straight-forward economic modelling excercise by attempting to capture aspects of reality not present in existing target zone models. In particular we have concentrated on developing a model that implies frequent sterilised intramarginal interventions and have emphasised the signalling channel of such interventions in altering the financial market's expectations rather than unsterilised fundamental interventions.

This formalisation of partial credibility has helped us uncover a basic trade-off which lies at the heart of any target zone between credibility and flexibility in a *strategic* environment. In the proposed model the evolution of credibility and monetary flexibility emerges from the strategic interaction between policy-makers and markets instead of being determined by some exogenous process. The sequential open loop nature of the optimisation process given forward looking exchange rate determination perhaps mimics reality rather more accurately than a standard infinite horizon intertemporal dynamic game.

The learning process of the financial markets with respect to the policy-maker's objectives is crucial. In formalizing it we stressed two aspects. In the first place the announcement of the band by the policy-maker establishes the level of absolute credibility at which the system is evolving. Secondly, observations on the realised exchange rate - fundamentals relationship allow the market to evaluate the degree of commitment the policy-maker has to the announced band, in other words the policy maker's relative credibility, and to revise its own expectations through an appropriate updating mechanism.

At each stage the monetary authority determines its optimal policy in the foreign exchange market taking into account the markets view of its credibility and its basic desires for flexibility and exchange rate stabilisation. Deviations of the policy-maker from its announced policy are penalized or rewarded in a state dependent and asymmetric way. The accumulation of costs and benefits triggers a realignment and the announcement of a new band-width by the policy-maker when they reach a threshold dictated by the parameters characterizing the policy-maker's preferences and the underlying model. Therefore the process of realignment is part of an optimising strategic policy on the part of the monetary authorities rather than being forced by the markets. The contrast between the case with a fixed target zone is clear.

In terms of testable implications our approach shows that the distribution of the exchange rate within the band has a mass clustered around the central parity in accordance with observed data. In other words we do not have to resort to realignments in the central parity, as in Bertola-Caballero [1991], in order to remove the probability masses concentrated at the two boundaries which characterizes the fully credible target zone model. Secondly, the dynamic management of the band-width endogenously creates leptokurtosis and an ARCH effect in the observed exchange rate series. It is important to recognise that while the evolution of fundamentals is characterized by constant volatility it is the exchange rate policy chosen by the policy maker which defines period by period the evolving volatility of the exchange rate. So we find a time-varying exchange rate volatility even though the fundamentals volatility is constant.

The practical policy implications of our analysis emphasise the central importance of measuring credibility in the optimal management of a flexible target zone. It has been suggested that the adoption of a flexible approach with a commitment to maintain an exchange rate within an announced band to an extent determined by the policy maker's perceived credibility would stabilise exchange rates and reduce the possibility of speculative attacks that arise with inflexible target zones. The feedback nature of the intervention policy of the central bank, both on the fundamentals and the exchange rate offers a completely different rationalisation for policy than that suggested by the open loop precommitment that has historically been found in existing target zones.

This form of flexible policy response would seem, by default, to have been that practiced by the Banque de France in 1993. What we have suggested above is a framework within which such realignments may be systematically analysed and the optimal response determined. For instance we doubt whether the " new" bandwidths of 15% in the ERM are optimal in any sense. In fact in Avesani, Gallo and Salmon [1995] we calculate that the optimal band-width adjustment at that time would have been to 7.9%.

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