Age-Dependent Employment Protection

Arnaud Chéron, Jean-Olivier Hairault, François Langot

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Abstract

This paper examines the age-related design of firing taxes by extending the theory of job creation and job destruction to account for a finite working life-time. We first argue that the potential employment gains related to employment protection are high for older workers, as they are magnified by the proximity to retirement. But higher firing taxes for these workers increase job destruction rates for the younger generations. Furthermore, from a normative standpoint, when firms cannot ex-ante age-direct their search, the impact of each generation of unemployed workers on the average return on vacancies makes the internalization of the search costs for the other generations imperfect through the ex-post Nash bargaining process. We show that the first best age-profile of firing taxes is typically hump-shaped, partially in contradiction with existing policies in some European countries. Taking into account the fact that the human capital of older workers is more specific than general tends to exacerbate these results.

JEL Classification : J63, J68

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1 Introduction

Faced with low employment rates for older workers, most OECD countries have experimented with specific older worker employment protection in the form of taxes on firing and subsidies on hiring (see OECD [2006]). Additional penalties for firms that lay off older workers have been introduced, either in the form of a tax or higher social security contributions (*e.g.* Austria, Finland, France and Spain) or in the form of paying part or all of the costs of outplacement services to help workers find new jobs (*e.g.* Belgium and Korea). Older workers are also protected to a greater extent than younger workers by tenure-related provisions: workers with longer tenure (more likely to be older workers) are often required to be given longer notice periods in the case of dismissals and higher severance payments. In Sweden, the Last In-First Out rule implies that older workers are more protected than younger workers in the event of lay-offs since they usually have longer tenure. At the same time, various hiring and wage subsidy schemes encouraging employers to hire and to retain older workers have been introduced. The hirings of older workers lead to permanent reduction or exonerations in social security contributions in Austria, Belgium, Netherlands, Norway and Spain. Direct subsidies to employers who hire older workers also exist in Denmark, Germany, Japan and Sweden. Employment subsidy schemes for older workers (in-work benefits) are present in Austria, Germany, Japan, the United Kingdom and the United States.

What are the predicted effects of the higher employment protection on the older workers? If older workers benefit from the higher protection put in place in most developed countries, what are the consequences of this policy on younger workers? Can this policy be legitimized by welfare arguments? Surprisingly enough, the theoretical foundations and implications of the age-dependent employment protection have not yet been addressed. This contrasts with the extensive use of this policy in OECD countries (OECD [2006]). From this point of view, our paper aims at filling this gap.

Following Mortensen and Pissarides [1999a], our theoretical framework embodies search frictions, idiosyncratic productivity shocks, endogenous separations and Nash-bargained wages. We only depart from this traditional framework by considering a life cycle setting characterized by an exogenous age at which workers exit the labour market. The difference in distance to retirement creates an intrinsic heterogeneity across workers. A crucial assumption in our model is that firms cannot ex-ante age-direct their search, the vacancy creations are independent of age. There are two main reasons to justify the absence of an age-segmented search in our framework. On the one hand, there is legislation prohibiting age-discrimination in the US and European countries as far as vacancies are concerned. If some firms clearly state experience requirements in their vacancy postings, these requirements are often labeled in terms of some minimal years of experience. This means that the youngest workers may be excluded from the global search sector, but not the older workers, as these requirement are never in terms of some maximal years of experience. On the other hand, search equilibrium with separate markets does not exist in our economy without any job age-requirements. We do indeed consider that job productivity is not age-specific, neither senior-specific nor junior-specific: technology diffusion and standardization, training programmes and worker experience are likely to make age requirements irrelevant to most jobs, except maybe for jobs requiring the use of the most recent wave of technology. More generally, there is no ability requirement associated with the job positions in our economy. The focus is on the implications of ageing in a given class of ability, leaving aside the role of skill heterogeneity, which has already been studied in Mortensen and Pissarides [1999b] and Albrecht and Vroman [2002] for instance.

We believe that the shorter distance to retirement, named hereafter the horizon effect, is the key feature to understanding the economics of older worker employment, as recently argued by Ljungqvist and Sargent [2008], Saint-Paul [2009] and Hairault, Langot and Sopraseuth [2010]. The fundamental feature of this shorter horizon is that the remaining time on the labour market is short. This obvious characteristic has two deep implications for older workers' employment. Firstly, the sum of any current productivity flows on the expected remaining career time is particularly low, implying that, ceteris paribus, the intertemporal values attached to the older workers are lower within the firm (the job value) or outside the firm (the search value): as for any investment, the longer the payoff, the higher the return. Secondly, older workers have less time to have new opportunities in the future. This implies that the separation decisions for older workers depend more crucially on the instantaneous profit. Conversely, future opportunities matter for younger workers inside the firm, implying that a longer horizon can lead to more labour hoarding, but they also matter outside the firm in the search sector, making a longer horizon synonymous with a higher reservation wage. Overall, the horizon effect implies that the selection of the profitable productivity levels through the determination of the productivity reservation is age-dependent. Moreover, in our life-cycle framework, the productivity reservation at each age depends on their expected values at older ages, as the latter determine the range of the profitable productivities in the future, generating a dynamic selection effect.

Firstly, we propose a positive analysis of older worker employment protection. We argue that the impact of a firing tax is greater for older workers than for younger ones. Indeed introducing a firing tax at the end of the working life increases the present firing cost without any future consequences on the job value as the worker will be retired in the following periods. In some sense, retirement allows firms to avoid the firing tax, leading to more labour hoarding of older workers. Secondly, we emphasize the perverse effects on younger workers of higher employment protection for older workers. Even though higher employment protection for older workers decreases the job destruction rate for this age group, it increases the firings of younger ones by raising the firing taxes they expect to bear in the future.

The second step is related to normative considerations. The horizon effect is at the origin of an intrinsic heterogeneity on the labour market, across workers. Worker heterogeneity can have important implications for the efficiency of the matching allocation when there is only one matching function, as has been known since Davis [2001]. When firms cannot ex-ante age-direct their search, beyond the standard search externality, there is an additional externality related to the heterogeneity among workers in the matching process. This is why the Hosios condition[1] is not enough to restore the social optimality of the labour market equilibrium. More precisely, the impact of each generation of unemployed workers on the average return on vacancies, named hereafter an intergenerational externality, makes the internalization of the vacancy costs for the other generations imperfect through the ex-post Nash bargaining process. We show that the unemployed older workers decrease the average value of job vacancies, as their individual value of hiring is weaker than that of younger workers due to their shorter horizon. This is the key implication of the horizon effect. Through the ex-post Nash bargaining, this leads separation decisions for older workers to under-estimate the costs created by older workers in the search process. There are too many separations for older workers at the market equilibrium with regard to their level at the social optimum. Older worker employment protection is then welfare-improving. It implies a hump-shaped age-profile for the firing taxes and hiring subsidies. It gives firms incentives to fire fewer older workers by expecting lower firing taxes in the future. These policy recommendations are rather at odds with the existing policy, which never implements firing taxes that decrease progressively as far as retirement comes closer. It is, however, the natural implication of a finite working life-time.

It must be emphasized that our normative results relate to the horizon effect only, without relying on the highly debated assumption that the relative productivity of older workers is lower. Whereas Borghans and Ter Weel [2002] and Friedberg [2003] find no significant influence of technical changes on old workers' employment, Aubert, Caroli and Roger [2006] observe such an influence and emphasize both organisational and technological changes. In Section 5, we take into account human capital accumulation, either general, related to experience, or specific, related to job tenure. For simplicity, they are both considered as arising from exogenous learning-by-doing accumulation. On the one hand, in the case of specific human capital, as it is not transferable to a new job, it is quite intuitive that the horizon effect is still the only dimension that determines the relative hiring value across ages. On the other hand, the expected hiring value of older workers can be higher than that of younger workers if their higher level of general human capital dominates their shorter horizon. In that case, there would be too many destructions of younger worker jobs at the equilibrium and these jobs would have to be protected. But this case is quite unrealistic, as the age-profile of general human capital accumulation is typically considered as hump-shaped with a peak at about age forty (Kotlikoff and Gokhale [1992]).

It is fairly intuitive that the persistence level of the productivity shocks deeply interacts with the horizon effect. When the persistence is relatively low, i.e. the arrival rate of new productivity

[1] This condition states that the elasticity of the matching friction with respect to vacancies should be equal to the worker's bargaining power (Hosios [1990]).

draws inside the firm is higher than the arrival rate of job offers, workers are more likely to have new opportunities within the firm than outside. Labour hoarding is then valuable and the shorter the horizon the more stringent the productivity selection, both in the separation and in the hiring processes. On the other hand, when the persistence is high enough to make new opportunities higher outside the firm, the productivity selection can be less stringent for older workers, due to their shorter horizon, leading to less older worker separations. But it is all the more true that their expected hiring value remains lower, despite the selection process, and because only younger workers can benefit from the persistence of profitable jobs. Whatever the level of persistence, the shorter horizon of older worker can lead to a lower value in the search sector. This is why we first consider the case of purely transitory productivity shocks as a baseline model, in order to derive in a very tractable framework the implications and the foundations of the older workers' employment protection. In that baseline model, age is the only state variable, since the job duration reduces to only one period. We then show in Section 6 why the persistence of the productivity shocks does not ultimately matter for our policy arguments.

Overall, our paper then makes the case for a tax policy determined by age. This statement echoes recent studies which also recommend making taxation dependent on age (Kremer [1999], Conesa, Kitao and Krueger [2009], Farhi and Werning [2010], Weinzierl [2010]). We also echo recent studies which try to legitimise individual-differentiated taxation (see for instance Alesina, Ichino and Karabarbounis [2010] for a gender-based approach). Obviously, our analysis is mainly related to the literature incorporating skill heterogeneity in search models, although our paper is the first to focus on age heterogeneity. In the first group of contributions, the Hosios condition still succeeds in decentralizing the efficient allocation when there is directed search. In Mortensen and Pissarides [1999b], it is assumed that workers are able to direct their search, leading to a perfectly segmented matching process by skill. In Marimon and Zilibotti [1999], the random search assumption ensures the efficiency of the equilibrium because heterogeneity does not generate any composition effect in the matching process. In Shi [2001] and Shimer [2005], efficiency is achieved *via* a credible directed-search equilibrium because firms can commit to posted wage offers. A second group of papers shows the implications of heterogeneity across worker skills when the search is non-directed. In Davis [2001], Sattinger [1995] and Shimer and Smith [2001], the least productive agents overinvest in the matching process and the Hosios condition is not sufficient to restore the first best allocation. Blazquez and Jansen [2008] following Albrecht and Vroman [2002] are the first to show this property in the canonical Diamond-Mortensen-Pissarides model. We share with Blazquez and Jansen [2008] the failure to internalize search costs through ex-post wage bargaining. However, in their model, as worker heterogeneity relies on ability differentials, bargained wages also determine the resource allocation across productive sectors. In our case, they are only allocative across production and search sectors. Contrary to Blazquez and Jansen [2008], a directed search in our model is not incentive-compatible from the viewpoint of older workers as they would suffer from a lower probability of contact for the same expected productivity draws if they specialized their search to possible older worker-specific jobs. More generally, equilibrium with separate markets must be incentive-compatible from the viewpoint of “bad" workers (Blazquez and Jansen [2008]). Moreover, an original feature of our framework is that the separation decision is endogenous. Forcing firms and workers to split any match surplus according to the average wage (over all ages), although correct for job creations, is not enough to restore the social optimum allocation in our economy. Only the optimal firing tax policy allows the market equilibrium to internalize both the homogenous search costs and the differentiated individual hiring values[2].

The rest of the paper is organized as follows. Section 2 presents the benchmark model and the age-profile properties of the equilibrium. Section 3 addresses the impact of age-dependent employment protection on employment. Section 4 deals with the social efficiency of the equilibrium and presents the optimal age-dependent employment protection. Section 5 introduces human capital accumulation, whereas Section 6 focuses on the role of shock persistence. The final section concludes.

2 A Finite-Horizon Economy with Endogenous Job Creations and Job Destructions

The primary objective of this section is to extend the job creation - job destruction approach to take into account a finite life-time horizon of workers. We consider an economy with labour market frictions *à la* Mortensen and Pissarides [1994] with endogenous job creation and job destruction decisions, extended to take into account a finite life time horizon for workers. That is, instead of assuming infinite-lived agents, our setting is characterized by a deterministic age T at which workers exit the labour market[3]. Workers only differ respectively in their age i, and so in their distance to retirement. The model is in discrete time and at each period the older worker generation retiring from the labour market is replaced by a younger worker generation of the same size (normalized to unity) so that there is no labour force growth in the economy. The economy is at steady-state, and we do not allow for any aggregate uncertainty. We assume that each worker of the new generation enters the labour market as unemployed.

[2]Albrecht, Navarro and Vroman [2010] have recently shown that taking into account the participation decision is also crucial in the context of worker heterogeneity.

[3]Our results are robust to the introduction of some uncertainty on the retirement age, provided that firms have some exogenous information on the retirement behavior. Assuming that there is a probability of retiring at each age after some eligibility age implies that the expected and *discounted* value of any productivity flows is lower when retirement becomes possible. On the other hand, we acknowledge that it would be more cumbersome to endogenously derive these probabilities, especially in a strategic environment. Older workers could have some incentive to send a noisy signal to their employers, as they are discriminated against in the case of a short distance to retirement. This point is left for future research.

We consider (un)employment policies: *(i)* a firing cost F_i which refers to the costs in employment protection legislation , *(ii)* a hiring subsidy H_i, that is a lump sum paid to the employer when a worker of age i is hired.

2.1 Shocks and worker flows

Firms are small and each has one job. The destruction flows derive from idiosyncratic productivity shocks that hit the jobs at random. Once a shock arrives, the firm has no choice but either to continue production or to destroy the job. At the beginning of their age i, the realization of the productivity level on each job is revealed. Workers contacted when they were $i-1$ years old (at the end of the period) are now productive. Workers whose productivity is below the reservation productivity R_i (R_i^0) are laid off (not hired, for those previously unemployed). R_i^0 may differ from R_i since firms are not liable for the firing cost at this stage.

The shocks hit the workers uniformly, whatever their age. In that sense, we think that these shocks drive worker flows, and not job flows as in Mortensen and Pissarides [1994]. For tractability matter, shocks are then assumed to display no persistence at all: at the beginning of each age, the new productivity level ϵ of an existing match is drawn in the general distribution $G(\epsilon)$ with $\epsilon \in [0,1]$, the same as the initial productivity of a new match. In that case, the productivity of existing matches changes more frequently than job offers arrive, as there are search frictions. We will show in Section 6 that the results on age-dependent employment protection are robust to the existence of persistent shocks.

Job creation takes place when a firm and a worker meet. The flow of newly created jobs result from a matching function, $M(v,u)$, the inputs of which are vacancies v and unemployed workers u. M is increasing and concave in both its arguments, and with constant returns-to-scale. In our economy, due to the age-discrimination prohibition and the absence of job age-requirements, firms do not ex-ante age-direct their search and the matching function embodies all unemployed workers.

Let $\theta = v/u$ denote the tightness of the labour market. It is then straightforward to define the probability of unemployed workers of age i being employed at age $i+1$, as $jc_i \equiv p(\theta)[1-G(R_{i+1}^0)]$ with $p(\theta) = \frac{M(u,v)}{u}$. Similarly, we define the job destruction rate for an employed worker of age i as $jd_i = G(R_i)$. For any age i, the flow from employment to unemployment is then equal to $G(R_i)(1-u_{i-1})$. The other workers who remain employed $(1-G(R_i))(1-u_{i-1})$ can renegotiate their wage. The age-profile of unemployment is then given by:

$$u_{i+1} = u_i\left[1-p(\theta)(1-G(R_{i+1}^0))\right] + G(R_{i+1})(1-u_i) \quad \forall i \in (1, T-1) \tag{1}$$

for a given initial condition $u_1 = 1$. The overall level of unemployment is $u = \sum_{i=1}^{T-1} u_i$, so that the average unemployment rate is $u/[T-1]$.

2.2 Hiring and firing decisions

Any firm is free to open a job vacancy and engage in hiring. c denotes the flow cost of recruiting a worker[4] and $\beta \in [0,1]$ the discount factor. Let V be the expected value of a vacant position and $J_i^0(\epsilon)$ the value of a filled job with productivity ϵ:

$$V = -c + \beta q(\theta) \sum_{i=1}^{T-2} \frac{u_i}{u} \left(\int_{R_{i+1}^0}^{1} \left[J_{i+1}^0(x) + H_{i+1} \right] dG(x) + G(R_{i+1}^0) V \right) + \beta(1 - q(\theta)) V$$

where the hiring subsidy H_i is received by the firm when the job becomes productive. At this time, the age of the hired worker is perfectly observed.

The zero-profit condition $V = 0$ allows us to determine the labour market tightness from the following condition:

$$\frac{c}{q(\theta)} = \beta \sum_{i=1}^{T-2} \frac{u_i}{u} \left(\int_{R_{i+1}^0}^{1} \left[J_{i+1}^0(x) + H_{i+1} \right] dG(x) \right) \tag{2}$$

The expected value of hiring a worker depends on the composition of the unemployed population.

We follow MP by considering that the wage structure that arises as a Nash bargaining solution has two tiers. The first tier wage reflects the fact that the hiring subsidy is directly relevant to the decision to accept a match and that the possibility of incurring firing costs in the future affects the value the employer places on the match. In turn, the second tier wage applies when firing costs are directly relevant to a continuation decision. For a bargained outsider wage $w_i^0(\epsilon)$, the expected value $J_i^0(\epsilon)$ of a filled job by a worker of age i is defined, $\forall i \in [1, T-1]$, by:

$$J_i^0(\epsilon) = \epsilon - w_i^0(\epsilon) + \beta \int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) + \beta G(R_{i+1}) \left(V - F_{i+1} \right) \tag{3}$$

whereas for a bargained insider wage $w_i(\epsilon)$, the expected value $J_i(\epsilon)$ of a filled job by a worker of age i is defined, $\forall i \in [1, T-1]$, by:

$$J_i(\epsilon) = \epsilon - w_i(\epsilon) + \beta \int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) + \beta G(R_{i+1}) \left(V - F_{i+1} \right) \tag{4}$$

The productivity thresholds R_i^0 and R_i are solutions of $J_i^0(R_i^0) = -H_i$ and $J_i(R_i) = -F_i$ respectively. Adding the free entry condition, $V = 0$, it is straightforward to derive the following equations:

$$R_i = w(R_i) - F_i - \beta \left[\int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) - G(R_{i+1}) F_{i+1} \right] \tag{5}$$

$$R_i^0 = R_i + F_i - H_i + w(R_i^0) - w(R_i) \tag{6}$$

[4]The recruiting costs could have been considered as age-dependent, as older workers have more experience in the search activity. But this effect could be more than compensated for by their lower search intensity due to the horizon effect. Their incentives to search more intensively for job offers decline as they get closer to retirement (Hairault, Langot and Sopraseuth [2010]). Overall, we consider in this paper that the search activity of unemployed workers does not matter.

Equation (5) gives the lowest level of productivity (R_i) necessary to avoid a separation and the equation (6) determines the lowest productivity level (R_i^0) for a newly created job. Because the labour hoarding value is the same for a newly created job as for an existing one, the link between R_i^0 and R_i is static (see equation (6)). The higher the wage, the higher the reservation productivity R_i (R_i^0), and hence the higher (lower) the job destruction (creation) flows. But firms also expect profits in the case of labour hoarding. The higher the option value of filled jobs, the weaker the job destructions and the greater the job creations. Because the job value vanishes at the end of the working life, labour hoarding of older workers is less profitable. It is worth determining the terminal age conditions: $R_{T-1} = w_{T-1}(R_{T-1}) - F_{T-1}$ and $R_{T-1}^0 = R_{T-1} + F_{T-1} - H_{T-1} + w(R_{T-1}^0) - w(R_{T-1})$.

2.3 Nash bargaining

The values of insiders, outsiders and unemployed workers of any age i, $\forall i < T$, are respectively given by:

$$\mathcal{W}_i^0(\epsilon) = w_i^0(\epsilon) - t_i + \beta\left[\int_{R_{i+1}}^1 \mathcal{W}_{i+1}(x)dG(x) + G(R_{i+1})\mathcal{U}_{i+1}\right] \quad (7)$$

$$\mathcal{W}_i(\epsilon) = w_i(\epsilon) - t_i + \beta\left[\int_{R_{i+1}}^1 \mathcal{W}_{i+1}(x)dG(x) + G(R_{i+1})\mathcal{U}_{i+1}\right] \quad (8)$$

$$\mathcal{U}_i = b - t_i + \beta\left[p(\theta)\int_{R_{i+1}^0}^1 \mathcal{W}_{i+1}^0(x)dG(x) + p(\theta)G(R_{i+1}^0)\mathcal{U}_{i+1} + (1-p(\theta))\mathcal{U}_{i+1}\right] \quad (9)$$

with b and t_i denoting the domestic production and the lump-sum tax respectively. As this tax, which allows the government budget to be balanced, is assumed to be paid equally by employed and unemployed workers, it is neutral with respect to the labour market equilibrium. Let us note that we assume that $\mathcal{W}_T = \mathcal{U}_T$, so that the social security provisions do not affect wage bargaining and the labour market equilibrium.

For a given bargaining power of the workers, γ, considered as constant across ages, the global surplus generated by a job is divided according to the following two sharing rules which are the solution of the conventional Nash bargaining problems in the context of two-tier contracts:

$$\mathcal{W}_i^0(\epsilon) - \mathcal{U}_i = \gamma\left[J_i^0(\epsilon) + H_i + \mathcal{W}_i^0(\epsilon) - \mathcal{U}_i\right] \quad (10)$$

$$\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma\left[J_i(\epsilon) + F_i + \mathcal{W}_i(\epsilon) - \mathcal{U}_i\right] \quad (11)$$

so that the equations for the initial and subsequent wage bargaining are (see Appendix B.2 for details on derivation):

$$w_i^0(\epsilon) = \gamma\left(\epsilon + c\theta\tau_i + H_i - \beta F_{i+1}\right) + (1-\gamma)b \quad (12)$$

$$w_i(\epsilon) = \gamma\left(\epsilon + c\theta\tau_i + F_i - \beta F_{i+1}\right) + (1-\gamma)b \quad (13)$$

where τ_i, for $i \in [1; T-2]$, is defined by[5]

$$\tau_i \equiv \frac{\int_{R^0_{i+1}}^1 [J^0_{i+1}(x) + H_{i+1}]dG(x)}{\sum_{i=1}^{T-2} \frac{u_i}{u} \int_{R^0_{i+1}}^1 [J^0_{i+1}(x) + H_{i+1}]dG(x)} = \frac{\int_{R^0_{i+1}}^1 [1 - G(x)]dx}{\sum_{i=1}^{T-2} \frac{u_i}{u} \int_{R^0_{i+1}}^1 [1 - G(x)]dx} \tag{14}$$

τ_i gives the hiring value of an unemployed worker of age i relative to the average hiring value according to the age distribution of unemployed workers. Despite the existence of a homogenous search cost for each match, wages are age-specific. Contrary to Mortensen and Pissarides' framework, the equilibrium is no longer symmetrical. Due to the finite-lived assumption, the way turn-over costs interact with the wage bargaining process depends on the age of the workers through the variable τ_i. Indeed, the ex-post bargaining process induces a hold-up problem as discussed in Acemoglu [1996]: at the time they invest in the search process on the basis of the expected return on a vacancy, firms do not know the age of the worker they will meet. These investments are sunk before agents meet and workers are rewarded for the saving of hiring costs that the firms enjoy when a job is created. Workers with a higher hiring value than average can capture a larger fraction of these hiring costs than workers with a lower hiring value. Due to ex-post wage bargaining, the worker with a higher hiring value has to be rewarded for more than the saving of the search costs[6].

On the other hand, as in Mortensen and Pissarides [1999a], the difference between the initial wage and subsequent renegotiation arises because hiring subsidies are sunk in the latter case but on-the-table in the former, and termination costs are not incurred if no match is formed initially but must be paid if an existing match is destroyed. From that point of view, the effects of firing taxes and hiring subsidies on these wage rules are conventional.

[5] To derive this expression, notice that ${J^0_{i+1}}'(\epsilon) = 1 - \gamma$ and $J^0_i(R^0_i) = -H_i$ implies that $J^0_i(\epsilon) = (1 - \gamma)\left(\epsilon - R^0_i\right) - H_i$. Furthermore, integrating by parts yields that $\int_{R^0_{i+1}}^1 \left(x - R^0_{i+1}\right) dG(x) = \int_{R^0_{i+1}}^1 [1 - G(x)]dx$.

[6] Acemoglu and Shimer [1999] show that this inefficiency can be solved if firms can post wages and workers direct their search toward different firms. In our case, this direct search is not an equilibrium as jobs are homogenous.

2.4 The labour market equilibrium

Proposition 1. *The labour market equilibrium with labour market policies in a finite-life economy is characterized by*[7]*:*

$$\frac{c}{q(\theta)} = \beta(1-\gamma)\sum_{i=1}^{T-2}\frac{u_i}{u}\int_{R_{i+1}^0}^{1}[1-G(x)]dx \tag{15}$$

$$R_i + \beta\int_{R_{i+1}}^{1}[1-G(x)]dx + F_i - \beta F_{i+1} = b + \beta\gamma p(\theta)\int_{R_{i+1}^0}^{1}[1-G(x)]dx \tag{16}$$

$$R_i^0 = R_i + F_i - H_i \tag{17}$$

$$u_{i+1} = u_i\left[1-p(\theta)(1-G(R_{i+1}^0))\right] + G(R_{i+1})(1-u_i) \tag{18}$$

$$\sum_{i=1}^{T-2} p(\theta)(1-G(R_{i+1}^0))u_i H_{i+1} = \sum_{i=1}^{T-2} G(R_{i+1})(1-u_i)F_{i+1} + \sum_{i=1}^{T-1} t_i \tag{19}$$

with terminal conditions $R_{T-1} = b - F_{T-1}$, $R_{T-1}^0 = b - H_{T-1}$ and a given initial condition u_1.

Proof. Combining (2), (5), (6), (7) (13) and noticing that ${J_{i+1}^0}'(\epsilon) = 1-\gamma$ and $J_i^0(R_i^0) = -H_i$ implies that $J_i^0(\epsilon) = (1-\gamma)\left(\epsilon - R_i^0\right) - H_i$, as well as $J_{i+1}'(\epsilon) = 1-\gamma$ and $J_i(R_i) = -F_i$ implies $J_i(\epsilon) = (1-\gamma)(\epsilon - R_i) - F_i$. Furthermore, integrating by parts yields that $\int_{R_{i+1}^0}^1 \left(x - R_{i+1}^0\right) dG(x) = \int_{R_{i+1}^0}^1 [1-G(x)]dx$ and $\int_{R_{i+1}}^1 (x - R_{i+1})\, dG(x) = \int_{R_{i+1}}^1 [1-G(x)]dx$. □

In particular, equation (16) shows that a job is destroyed when the expected profit from the marginal job, which is equal to the current product plus the option value from expected productivity shocks (left-hand side of (16)) fails to cover the worker's reservation wage which is formed by the domestic production and the expected return on unemployment (right-hand side). Equation (19) is the budgetary constraint of the government.

The age-profile of job creations and job destructions depends on the age design of employment protection. Most OECD countries have experimented with simultaneously active labour market policies, modeled as $H_i > 0$, and employment protection, modeled as $F_i > 0$. These policies have both ambiguous effects on unemployment and opposite effects on job flows: $H_i > 0$ ($F_i > 0$) reduces (increases) unemployment duration but increases (reduces) unemployment incidence. Imposing $H_i = F_i$ hereafter ensures that the simultaneous introduction of these policies decreases the equilibrium unemployment rate, which is in accordance with empirical evidence on OECD data (Bassani and Duval [2009]). Moreover, this case implies that the condition $R_i^0 = R_i$ holds (equation (17)), which will be ultimately consistent with the optimal policy design: the role of the hiring subsidies is to compensate for the negative effect of the firing costs on the hiring decisions.

[7] See Appendix A for a discussion on the existence and the uniqueness of this equilibrium.

2.5 The age-profile of job creations and job destructions in a *laissez-faire* economy

As a benchmark case, we first consider the equilibrium age-profile of job creations and job destructions without any labour market policies. This allows us to show that older workers face a higher probability of exit from employment.

Proposition 2. *The equilibrium without any labour policies ($H_i = F_i = 0, \quad \forall i$) is characterized by $R_{i+1} > R_i$, so that $jd_{i+1} > jd_i$ and $jc_{i+1} < jc_i$, $\forall i \in [2, T-1]$.*

Proof. The age-profile of job creations and job destructions is governed by the sequence $\{R_i\}_{i=2}^{T-1}$ which solves:

$$R_i \quad = \quad b - \beta[1 - \gamma p(\theta)] \int_{R_{i+1}}^{1} [1 - G(x)]dx \tag{20}$$

with terminal conditions $R_{T-1} = b$. Let us define $\Psi(y) \equiv \beta(1 - \gamma p(\theta)) \int_y^1 [1 - G(x))\, dx$, so that $\Psi'(y) < 0$. By definition $R_i = b - \Psi(R_{i+1})$, so that $R_i - R_{i+1} = \Psi(R_{i+2}) - \Psi(R_{i+1})$. Accordingly, as $R_{T-2} < R_{T-1}$ and $\Psi'(y) < 0$, then $R_i < R_{i+1}, \ \forall i$. □

Without any persistent shocks, the arrival rate of productivity draws is higher than the arrival rate of job offers. New opportunities are higher within the firm. As shown by equation (20), with a probability equal to 1, the worker-firm pair has access to the expected value of a new draw. Outside the firm, this lottery is accessible with a probability $p(\theta)$ and only a fraction γ of the search surplus is given to the workers. So the labour hoarding has a higher value than the search activity $(1 - \gamma p(\theta) > 0)$. Because the horizon of older workers is shorter, the sum of the profit flows they can generate in the future is particularly low: the labour hoarding value is weaker for older workers. Indeed, a longer horizon generates more productivity draws within the firm. In that sense, even without persistence, a longer horizon somehow generates a capitalisation effect, not on the existing productivity draw (which, by assumption, lasts only one period), but on the number of new opportunity draws until retirement. In particular, the labour hoarding value is nil for the close-to-retirement worker. This makes this worker more vulnerable to idiosyncratic shocks than the worker of the next generation: her reservation productivity is then higher. This terminal condition on the reservation productivity differential ($R_{T-1} > R_{T-2}$) feeds back on all reservation productivity differentials between two successive ages ($R_{i+1} > R_i$) through a dynamic selection effect: as higher expected reservation productivity at the next age makes more productivity levels unprofitable tomorrow, this reduces even more the labour hoarding value of the older worker and then makes her present reservation productivity higher than that of the worker of the previous generation, and so on.

Overall, older workers suffer from a lower labour hoarding value, because their shorter horizon reduces the number of new opportunities within the firm and these new opportunities are less likely to be profitable through the dynamic selection effect. This increasing (decreasing) age-profile of job destructions (creations) is, at least qualitatively, able to account for the observed low employment rate of older workers[8].

3 The Impact of Firing Taxes Revisited

Faced with the low employment rate of older workers, most developed countries have experimented with higher employment protection combined with subsidies targeted at these workers. The main objective of this section is to question, from a positive point of view, the impact of such policies on job separations.

3.1 On the age-differentiated effect of firing taxes

As a preliminary step, our objective is first to examine the age-differentiated effect of a constant firing tax on the age-profile of job flows. In a finite horizon setting, there is a specific intertemporal trade-off related to the introduction of a firing tax.

Proposition 3. *Considering $F_i = F > 0, \ \ \forall i$, the labour market equilibrium is characterized by:*

$$0 > \frac{dR_2}{dF} \quad > \frac{dR_i}{dF} \dots > \quad \frac{dR_{T-1}}{dF} \qquad \forall i \in [2, T-2]$$

Proof. Considering $F_{i+1} = F_i \equiv F$, from Proposition 1, it yields that $\frac{dR_i}{dF} = -(1-\beta) + \frac{dR_{i+1}}{dF}\beta(1-\gamma p(\theta))\,[1 - G(R_{i+1})] + \frac{d\theta}{dF}\gamma p'(\theta)\beta \int_{R_{i+1}}^{1}[1 - G(x)]dx$ with $\frac{dR_{T-1}}{dF} = -1$. Then, it remains to iterate backward from $T - 1$ to $i = 2$, having in mind that $\beta \leq 1$, $p'(\theta) \geq 0$, $\frac{d\theta}{dF} \geq 0$ and $\lim_{T\to\infty} \frac{dR_2}{dF} = \frac{dR_\infty}{dF} < 0$. □

A given firing tax is thus found to reduce the job destruction rate of older workers more than that of younger ones. At the end of the working cycle, introducing a firing tax increases the present firing cost without any future consequences on the job value as the worker will be retired in the next period. In other words, the value of avoiding punishment today is not canceled out by the expected future loss. On the other hand, for all other workers, the option value of keeping the job is decreased by the expected firing tax. In particular, this explains why the separation rate for the workers two years from retirement is less decreased by the firing tax[9]. The terminal condition ($\frac{dR_{T-1}}{dF} < \frac{dR_{T-2}}{dF}$) feeds back on all reservation productivity differentials between two successive ages ($\frac{dR_{i+1}}{dF} < \frac{dR_i}{dF}$) by the same dynamic selection effect described in the previous

[8] See Chéron, Hairault and Langot [2008] for a more quantitative exercise.

[9] Let us note that assuming $F_i = H_i$, which implies $\frac{d\theta}{dF} \geq 0$, guarantees this result. Otherwise, the reduction in the outside option value could decrease more the reservation productivity of younger workers. This allows us to focus on the direct impact of any firing taxes, leaving aside the controversial effect on the labour market tightness.

section. Overall, these results suggest that evaluating employment protection in an infinite-lived agent context understates the potential employment gains for older workers.

As the impact of the firing tax is particularly high at the end of the life cycle, older workers could face a lower probability of job destruction than younger ones if the tax is high enough.

Proposition 4. *There exists $\hat{F} > 0$ such that if $F > \hat{F}$, then $R_{i+1} < R_i$, $\forall i \in [2, T-2]$.*

Proof. Let us consider Proposition 1 with $F_{i+1} = F_i \equiv F$, $\forall i$. As established in the proof of Proposition 2, $Sign(R_{i+1} - R_i) = Sign(R_{T-1} - R_{T-2})$. On the other hand, $R_{T-2} > R_{T-1} \iff F > (1-\gamma p(\theta)) \int_{b-F}^{1} [1-G(x)]dx$. There exists $\hat{F}$ solving $\hat{F} = (1-\gamma p(\theta)) \int_{b-\hat{F}}^{1} [1-G(x)]dx$, such that $F > \hat{F}$ implies $R_i > R_{i+1}$, $\forall i$. $\square$

To conclude, at this stage Propositions 3 and 4 emphasize that the laying off of older workers is, in relative terms with respect to younger workers, very sensitive to the firing tax. Employment protection has an age-differentiated impact. It may make age-dependent employment protection unnecessary if the only objective is to stimulate the employment of older workers.

3.2 The impact of age-increasing firing taxes

It is obvious that not only the level of firing taxes, but also their shape, depending on age, play a key role in the age-profile of job flows. A quick look at Proposition 1 shows that the current tax F_i tends to push down the productivity threshold by increasing the current cost of firing, while the future tax F_{i+1} increases this threshold by reducing the value of labour-hoarding. This suggests that the shape of the discounted firing costs is crucial for the separation decisions. As emphasized in Introduction, many European countries have experimented with age-increasing firing taxes. Typically, firing taxes on older workers have been increased. What are the implications for the younger workers?

Proposition 5. *If $dF_i = dF > 0$, $\forall i \in [T-n, T-1]$, and $dF_i = 0$ otherwise, then $\frac{dR_i}{dF} < 0$, $\forall i \in [T-n, T-1]$ and $\frac{dR_i}{dF} > 0$ otherwise.*

Proof. $\frac{dR_i}{dF} < 0, \forall i \in [T-n, T-1]$ is part of Proposition 3. At age $T-n-1$, as $\frac{dR_{T-n-1}}{dF} = \beta + \frac{dR_{T-n}}{dF}\beta(1-\gamma p(\theta))\,[1-G(R_{T-n})] + \frac{d\theta}{dF}\gamma p'(\theta)\beta \int_{R_{T-n}}^{1} [1-G(x)]dx$, it is straightforward to show that $\frac{dR_{T-n-1}}{dF} > 0$, as $\frac{dR_{T-n}}{dF} > -1$ (Proposition 3). $\forall i \in [2, T-n-2]$, $\frac{dR_i}{dF} = \frac{dR_{i+1}}{dF}\beta(1-\gamma p(\theta))\,[1-G(R_{i+1})] + \frac{d\theta}{dF}\gamma p'(\theta)\beta \int_{R_{i+1}}^{1} [1-G(x)]dx > 0$. Iterating backward from $i = T-n-1$, as $\frac{dR_{T-n-1}}{dF} > 0$, then $\frac{dR_i}{dF} > 0$, $\forall i \in [2, T-n-1]$. $\square$

At the age just before the increase in the firing tax, the dominant effect is precisely the expected increase of the tax in the future, which decreases the value of labour hoarding. This raises the reservation productivity at this age. It turns out to raise, through a more stringent selection effect, the productivity thresholds for all younger workers, as their expected reservation productivity is increased.

Overall, Proposition 5 highlights the facts that higher firing taxes for older workers increase the job destruction rates for younger workers. It is worth emphasizing that these results can give theoretical support to the empirical findings of Behaghel, Crépon and Sédillot [2008] who use microeconometric estimates to assess the French experiment with a higher firing tax for workers of 55 years old or more. The estimates mainly show that firings of people under 55 have increased. However, it is important to go beyond this positive approach by proposing a welfare analysis of age-dependent employment protection in order to evaluate the real perversity of higher firing taxes for older workers.

4 Optimal age-dependent employment protection

Traditionally, the equilibrium unemployment framework is known to generate congestion effects which take the decentralized equilibrium away from the efficient allocation (search externality). However, when the elasticity relative to vacancies in the matching function is equal to the bargaining power of firms (Hosios condition), social optimality can be reached. As demonstrated hereafter, this result no longer holds here, because there is a specific intergenerational externality, which leads to imperfectly internalizing the search costs through the Nash bargaining process.

4.1 The optimal allocation

The problem of the planner is to determine the optimal allocation of each worker between the production and the search sectors and the optimal investment in the search sector. The per-unemployed worker social value in the search sector and the per-employed worker social value in the good sector are respectively given by:

$$Y_i^s = b - c\theta^\star + \beta\left[p(\theta^\star)\int_0^1 \max\{Y_{i+1}(x); Y_{i+1}^s\}dG(x) + (1-p(\theta^\star))Y_{i+1}^s\right] \quad (21)$$

$$Y_i(\epsilon) = \epsilon + \beta\int_0^1 \max\{Y_{i+1}(x); Y_{i+1}^s\}dG(x) \quad (22)$$

where $c\theta^\star \equiv c\frac{v^\star}{u^\star}$ represents the total cost of vacancies ($cv^\star$) per-unemployed worker ($u^\star$). The planner's decisions $R_i^\star$, $\forall i$ and $\theta^\star$ are solution to[10]:

$$\begin{cases} S(R_i^\star) \equiv Y_i(R_i^\star) - Y_i^s &= 0 \\ \theta^\star &= Sup(\sum_i u_i^\star Y_i^s) \end{cases}$$

[10]This intertemporal planner problem is strictly equivalent, provided that $\beta = 1$, to maximizing the production net of vacancy costs averaged over age class, as a worker of age $i+j$ is representative of the future of any worker of age i. When shocks are persistent, this equivalence is no longer true and the social allocation must be derived from the intertemporal problem.

At each age, the social allocation between the two sectors is governed by the arbitration condition: $S(R_i^\star) = 0$, with S the job surplus. On the other hand, the non-segmented search process implies choosing the labour market tightness that maximizes the social value of search per-unemployed worker averaged over age groups.

Proposition 6. *Let $\eta = 1 - \frac{\theta^\star p'(\theta^\star)}{p(\theta^\star)}$, the social optimum allocation is characterized by:*

$$\frac{c}{q(\theta^\star)} = (1-\eta)\beta \sum_{i=1}^{T-2} \frac{u_i^\star}{u^\star} \int_{R_{i+1}^\star}^1 [1-G(x)]dx \tag{23}$$

$$R_i^\star + \beta \int_{R_{i+1}^\star}^1 [1-G(x)]dx = b + \beta p(\theta^\star) \int_{R_{i+1}^\star}^1 [1-G(x)]dx - c\theta^\star \tag{24}$$

$$u_{i+1}^\star = u_i^\star \left[1 - p(\theta^\star)(1 - G(R_{i+1}^\star))\right] + G(R_{i+1}^\star)(1-u_i^\star) \tag{25}$$

Proof. See Appendix C.1. □

Equation (23) is similar to equation (15) obtained in the decentralized equilibrium, as the planner faces the same non-segmented search technology. Equation (24) shows the optimal allocation of the age i labour force: the expected profit from the marginal employed worker (the current product plus the option value for expected productivity shocks) must be equal to the social return on unemployment, formed by the domestic production and the expected return on search activity net of the vacancy costs (per employed worker).

Proposition 7. *Efficient allocation is characterized by $R_{i+1}^\star > R_i^\star$, so that $jd_{i+1}^\star > jd_i^\star$ and $jc_{i+1}^\star < jc_i^\star$, $\forall i \in [2, T-2]$.*

Proof. If $b > c\theta^\star$, ensuring that $R_{T-1}^\star > R_{T-2}^\star$, the proof is straightforward by solving backward the equation $R_i^\star = b - c\theta^\star - \beta[1-p(\theta^\star)] \int_{R_{i+1}^\star}^1 [1-G(x)]dx$ and by noticing that $0 < [1-p(\theta^\star)] < 1$ and $c\theta^\star$ is not age-dependent. If the condition $b > c\theta^\star$ does not hold, then $R_{T-1}^\star$ is bounded by zero, and it unambiguously appears that $R_i^\star = 0$, $\forall i \in [2, T-1]$. □

Proposition 7 emphasizes that higher job destruction rates for older workers may be an efficient age-pattern of labour market flows, as in the decentralized equilibrium, for the same basic reason that these workers have a shorter horizon. This does not imply that the equilibrium job destruction and job creation rates are consistent with their efficient counterparts, due to the existence of an intergenerational externality.

Proposition 8. *The relative hiring value $\tau_i^\star$ of the different generations of workers is decreasing with age at the social optimum.*

Proof. Straightforward by considering equation (14) and the age-decreasing dynamics of the reservation productivity (Proposition 7). □

Proposition 8 shows that the older workers have a lower hiring value than younger workers at the social optimum for the same basic reason as at the equilibrium: the shorter horizon reduces the number of new opportunities and these new opportunities are less likely to be profitable.

4.2 The intergenerational externality

Compared to Pissarides [2000], our life-cycle framework introduces another externality, namely an intergenerational externality, besides the traditional search externalities related to the transition probabilities. This intergenerational externality arises from the impact of each generation of unemployed workers on the average return on firms' vacancies. This intergenerational externality is the key implication of the horizon effect in a non-directed search framework. As the wage is bargained with more information than used by the firms for their vacancy decisions, this leads to a distortion of both job creations and job destructions in our economy.

In order to illustrate this fundamental property, let us focus on both the equilibrium and optimal conditions underlying the firing decisions by using the free entry conditions (15) and (23) to substitute the vacancy costs for the expected value of search:

$$R_i + \beta \int_{R_{i+1}}^{1} [1 - G(x)]dx = b + \frac{1}{1-\gamma} c\theta\tau_i - c\theta\tau_i - F_i + \beta F_{i+1} \tag{26}$$

$$R_i^\star + \beta \int_{R_{i+1}^\star}^{1} [1 - G(x)]dx = b + \frac{1}{1-\eta} c\theta^\star\tau_i^\star - c\theta^\star \tag{27}$$

At the equilibrium, the firing decision is not necessarily based on a proper evaluation of the social return on unemployment (right-hand side of equation (27)). Notwithstanding the presence of the firing tax, the gap between the equilibrium and the social optimum allocations lies on two externalities, both relying on the return on unemployment.

The first externality, captured by the gap between the second terms of the right-hand side of equations (26) and (27) is the traditional intra-generational search externality. When the Hosios condition ($\gamma = \eta$) holds, the Nash wage bargaining eventually leads to fully internalizing the search externalities created by the number of traders in the search sector on the transition probabilities. The outside opportunities of the insider workers properly account for the individual hiring value of the outsider unemployed worker. This is why the first two terms in the left-hand side of equations (27) and (26) are the same provided that $\gamma = \eta$. In a homogenous environment ($\tau_i = \tau_i^\star = 1$), this would lead to an efficient allocation.

The second externality, captured by the gap between the third terms in the right-hand side of equations (26) and (27) shows the existence of another source of inefficiency, the intergenerational externality.

Proposition 9. *The Hosios condition, $\eta = \gamma$, does not achieve efficiency in our economy without an age-segmented search due to the existence of the intergenerational externality.*

Proof. Straightforward by comparing the expression of $R_i^\star$ in Proposition 6 and equation (23) and R_i in Proposition 1 and equation (15). $\square$

When there is worker heterogeneity and the search process is not segmented, only the average hiring value gives a proper evaluation of the search costs, as shown by equation (2). As the wage is bargained with more information than used by the firms for their vacancy decisions, both job creation and job destruction decisions are distorted in our economy. More precisely, the ex-post bargaining process leads the wage to depend on the worker's individual hiring value, whereas firms decide their vacancies on the basis of the average hiring value across the pool of unemployed workers. As each individual hiring value differs from the average hiring value ($\tau_i \neq 1$), the social search costs $c\theta^\star$, independent of age, are not properly internalized at the equilibrium where they are estimated at the level $c\theta\tau_i$ for the worker of age i. In the case of older workers, as their hiring value is lower than on average ($\tau_i < 1$), this leads to underestimating the search costs. There are more separations for older workers than there should be. On the other hand, as the productivity thresholds are distorted, the averaged hiring value over all ages on which firms' vacancy decisions rely does not coincide with the expected hiring value used by the planner to invest in job vacancies: the job creation decisions are also distorted[11].

Overall, the inefficiency gap implied by the intergenerational externality in the evaluation of the search costs is given by $c\theta(1-\tau_i)$: the search costs are either under-estimated for older workers ($\tau_i < 1$) or over-estimated for younger workers ($\tau_i > 1$). Focusing on the oldest worker makes the intergenerational externality understandings straightforward. At the end of the life-cycle, the social return on a worker occupied in the search process is at its lowest value: the oldest workers can be contacted by a firm whereas the surplus associated with this match is nil. As there are no search costs internalized at the equilibrium, the size of the intergenerational inefficiency gap takes its maximum value $c\theta$: the social value of unemployment for workers of age $T-1$ turns out to be $b - c\theta$, whereas the private return is b.

The existence of the intergenerational externality suggests that age-specific labour market policies are needed to improve the outcome of the decentralized equilibrium. This could give some theoretical foundation to the age-specific firing taxes and hiring subsidies in place in a large set of OECD countries.

4.3 The optimal age-profile of firing costs and hiring subsidies

Labour market policies designed by age may allow firms and workers to internalize the true value of the search costs. To focus on the impact of intergenerational externalities on the age design

[11] The Hosios condition is not sufficient to restore the first best allocation even if the separation rate is exogenous. Due to the horizon heterogeneity, the averaged hiring value over all ages still differs from the expected hiring value used by the planner to invest in job vacancies.

of labour market policies, we assume throughout this section that the Hosios condition holds.

Proposition 10. *Assuming $\eta = \gamma$, an optimal age-sequence for firing taxes and hiring subsidies $\{F_i^\star, H_i^\star\}_{i=1}^{T-1}$, implying lump-sump taxes $\{t_i\}_{i=1}^{T-1}$, solves:*

$$\begin{aligned} \beta F_{i+1}^\star - F_i^\star &= c\theta^\star \left(\tau_i^\star - 1\right) \quad \forall i \in [2, T-2] \\ F_{T-1} &= c\theta^\star \\ H_i^\star &= F_i^\star \quad \forall i \in [2, T-1] \end{aligned}$$

where $\{R_i^\star\}_{i=2}^{T-1}$ and $\theta^\star$ are defined in Proposition 6 and $\{t_i\}_{i=2}^{T-1}$ in equation (19) evaluated at the social optimum.

Proof. Straightforward by comparing equations (27) and (26) when assuming $\eta = \gamma$. □

Overall, the firing tax policy makes the search cost internalized at the equilibrium. Note that it implies that the wages for any level of productivity are no longer age-dependent when the optimal policy is implemented, since the search costs are independent of age. From equation (13), it comes that:

$$\overline{w}_i(\epsilon) \quad = \quad \gamma\left(\epsilon + c\theta\right) + (1-\gamma)b$$

This wage level $\overline{w}_i(\epsilon)$ is also the averaged wage over all ages in the case of no firing taxes. The policy corrects for the fact that the individual wage is bargained ex-post with more information, implying an informational rent. Note that replacing the ex-post bargained wage by the average wage would make the job creation decisions at the market equilibrium consistent with that of the planner. However, let us emphasize that forcing firms and workers to split any match surplus according to the average wage rule is not enough to reach the social optimum allocation in our economy: when separations are endogenous, this wage rule does not allow the market decisions on separations to internalize the social return on searching for each age, since the individual hiring value is no longer present in the wage value[12]. Despite this wage rule, firing taxes would still be needed to signal that older workers have a lower return on searching. Finally, the optimal firing tax policy allows the market equilibrium with ex-post wage bargaining to internalize both the homogenous search costs and the differentiated individual hiring values.

Proposition 11. *There exists an age $\tilde{i}$ defined by $\tau_{\tilde{i}} = 1$ such that $\beta F_{i+1}^\star - F_i^\star > 0 \;\; \forall i < \tilde{i}$ and $\beta F_{i+1}^\star - F_i^\star \leq 0 \;\; \forall i \geq \tilde{i}$.*

[12]Note that implementing only hiring subsidies in order to equalize the individual hiring values cannot succeed in coping with the intergenerational externality for the same reason. Each individual unemployment value in the wage bargaining process would allow private decisions to properly evaluate the social search costs in that case. But it would be at the expense of a proper evaluation of the different social surplus generated by each unemployed worker.

Proof. Straightforward from Proposition 10 by recalling that $\tau_1^\star > 1$ and $\tau_{T-1}^\star = 0$. $\square$

Let us first consider the oldest worker: as explained above, the size of the intergenerational inefficiency gap takes its maximum value $c\theta$. The firing tax F_{T-1} must then be equal to $c\theta$ at the latest age, as there is no expected tax in the future just before retirement. At age $T-2$, the social search costs are still underestimated (provided that $\tau_{T-2} < 1$) and the present net value of the firing taxes $(F_{T-2}^\star - \beta F_{T-1}^\star)$ must be positive at this age, in order to restore the social optimality. An age-constant tax would be consistent with this requirement, but this age-profile has no reason to be enough with respect to the size of the inefficiency gap. There is indeed a huge discontinuity between age $T-2$ and $T-1$ as far as the impact of a constant tax is considered (see Proposition 3), whereas the age-profile of $\tau_i^\star$, and then that of the inefficiency gap, are smoother. So, it is optimal to implement an age-decreasing path of firing taxes. As emphasized, the expectation of lower taxes in the future gives the right incentives for firms to postpone job destruction, as long as the social search costs are underestimated at the market equilibrium $(i > \tilde{i})$.

On the other hand, considering workers at age $i < \tilde{i}$, for which the search costs are over-estimated, implies that an age-increasing dynamic of firing taxes turns out to be the optimal age-dependent employment protection. The threat of higher firing taxes in the near future leads to more job destructions today. Overall, the age-profile of the optimal firing taxes then displays a hump-shaped profile. Quite paradoxically, the firing tax is near its highest level F_{max} in the case of the workers of age $\tilde{i}$ for whom the search costs are perfectly internalized $(c\theta^\star(\tau_{\tilde{i}}^\star - 1) = 0)$: $F_{\tilde{i}} = \beta F_{\tilde{i}+1} = \beta F_{max}$.

The existing policies rather implement uniformly higher firing taxes (and hiring subsidies) from an arbitrary threshold age until retirement age (one-step age-profile): the firing taxes never continuously decrease as retirement gets closer, because this is somewhat paradoxical if the objective is to sustain the oldest worker's employment. We show that it is the natural implication of our finite working life-time environment, which implies that the impact of the firing tax is particularly high for the oldest workers (Proposition 3). However, the existing policies are not totally at odds with this optimal profile as their age-profile is increasing over the life cycle. The increase in the firing costs after a given age is an efficient tool to fire more workers below this age (Proposition 5) and an uniformly higher tax after this age has more impact on the workers closer to the retirement age (Proposition 3), for whom the inefficiency gap is at its highest level.

Another important source of inefficiency conveyed by the existing policies relies on the age threshold triggering the increase in the firing tax. Should it be too high or rather too close to the retirement age, some workers would suffer from the expected increase in the firing tax whereas they have to be protected. This situation typically occurs when this age threshold is far above the age $\tilde{i}$ onwards from which the search costs are under-estimated by private decisions. Let us notice that this threshold must be indexed on the retirement age. The lower the retirement age, the lower the threshold. However, a precise statement on this point would require us to quantify a more general model in order to give an empirically relevant estimate[13] of the age $\tilde{i}$. For instance, let us emphasize that taking into account the existence of unemployment benefits would imply implementing more age-decreasing firing taxes, as uniform unemployment benefits increase the reservation productivity at each age. Age-decreasing firing taxes are again the right tool to sustain employment: the higher the unemployment benefits, the sooner the age-decreasing path of firing taxes and hiring subsidies. Ultimately, unemployment benefits could be large enough to imply a monotonously age-decreasing dynamic of optimal firing taxes and hiring subsidies[14].

5 Human capital accumulation

In our benchmark model without human capital, the employment age-profile is exclusively led by the horizon of the workers. Nevertheless, conventional life-cycle analysis also takes into account human capital accumulation, which can be general (related to experience) or specific (related to job tenure). In this section, for didactic reasons, we consider in turn the analysis of these two components, even though experience and tenure interact together in the worker-job efficiency process.

5.1 General human capital

Let h_i^g denote the general human capital at age i. It is transferable (it can be used in all jobs and in home production) and is interpreted as experience: individuals accumulate this capital inside and outside the firms. The productivity of the job is now given by $h_i^g \epsilon$ and it is assumed that the instantaneous opportunity cost of employment follows the same process as individual productivity: $b_i \equiv b h_i^g$. The equilibrium productivity thresholds with general human capital are characterized by the following equations[15]:

$$\frac{c}{q(\theta)} = \beta(1-\gamma)\sum_{i=1}^{T-2}\frac{u_i}{u}h_{i+1}^g\int_{R_{i+1}^0}^1[1-G(x)]dx \tag{28}$$

$$R_i h_i^g + \beta h_{i+1}^g\int_{R_{i+1}}^1[1-G(x)]dx = b h_i^g + \beta h_{i+1}^g \gamma p(\theta)\int_{R_{i+1}}^1[1-G(x)]dx - F_i + \beta F_{i+1} \tag{29}$$

Equation (29) is the same as equation (20) at the equilibrium without human capital accumulation, except for the actualization term, which now takes into account the rate of growth of human

[13]This quantification is left for further research.

[14]With age-uniform unemployment benefits z, it is straightforward to show that $F_i^\star - \beta F_{i+1}^\star = c\theta^\star(1-\tau_i^\star) + z$. See Chéron, Hairault and Langot [2009].

[15]The presentation and the solving of the model with general human capital accumulation is available upon request.

capital accumulation. This implies that the age-profile of hirings and separations is unchanged when there are no firing taxes. General human capital raises the gains inside and outside the firms simultaneously, leaving the arbitration on the reservation productivity unchanged. As the firing tax is not indexed on the level of human capital, its impact depends on this level. However, the impact of the firing tax is still differentiated by age and affects the older workers more for the same basic reason that the oldest workers do not suffer from the tax in the next period.

On the other hand, as general human capital accumulation impacts individual hiring values, it may alter the optimal policy recommendations. The counterparts of equations (28) and (29) at the social optimum are:

$$\frac{c}{q(\theta^\star)} = \beta(1-\eta)\sum_{i=1}^{T-2}\frac{u_i^\star}{u^\star}h_{i+1}^g\int_{R_{i+1}^\star}^1[1-G(x)]dx \qquad (30)$$

$$R_i^\star h_i^g + \beta h_{i+1}^g\int_{R_{i+1}^\star}^1[1-G(x)]dx = b+\beta h_{i+1}^g p(\theta^\star)\int_{R_{i+1}^\star}^1[1-G(x)]dx - c\theta^\star \qquad (31)$$

Following the same reasoning as in Section 4.2, it is then straightforward to show that the optimal rule for the firing tax is then unchanged: $\beta F_{i+1} - F_i = c\theta^\star(\tau_i^\star - 1)$. But now the relative hiring value takes into account the human capital accumulation:

$$\tau_i \equiv \frac{h_{i+1}^g\int_{R_{i+1}}^1[1-G(x)]dx}{\sum_{i=1}^{T-2}\frac{u_i}{u}h_{i+1}^g\int_{R_{i+1}}^1[1-G(x)]dx},\ \forall i\in[1;T-2]$$

The age-profile of τ_i is now indeterminate. Higher general human capital for older workers could overcompensate for their shorter horizon effect: the expected hiring value of older workers could then be higher than that of younger workers. In that case, there would be too many destruction of younger worker job at the equilibrium and these jobs should be protected. But this case is quite unrealistic. The human capital of older workers is considered as hardly transferable to other firms and other sectors: it is viewed as more specific than general, as recently claimed by Saint-Paul [2009]. The age-profile of general human capital accumulation is typically considered as hump-shaped[16], with a peak about forty (Kotlikoff and Gokhale [1992]). In this case, the social hiring value of the older workers which combines both a short horizon and a low general human capital is unambiguously lower than that of the younger workers. Actually, the introduction of the general human capital rather magnifies our policy recommendations.

Concerning the wage age-profile, it is obvious that the human capital accumulation introduces into our model a force that increases individual wages throughout the working life, leading to potential overcompensation for the decreasing force implied by the horizon effect.

[16] A still age-increasing general human capital accumulation for older workers would be hardly consistent with the flat pattern of the *mincerian* wage-equation at the end of the working life (Willis [1986], Mincer [1991], Heckman, Lochner and Todd [2003], Altonji, Smith and Vidangos [2009]).

5.2 Specific human capital

Let us now consider specific human capital accumulation, which relies only on the duration of the employer-worker pair, the job tenure. It is not transferable to the next job. For simplicity, the instantaneous opportunity cost of employment b is not indexed on human capital[17].

Let h_j denote the deterministic component of the productivity value of the job where j stands for the job tenure (we normalize h_0 to 1). The overall productivity of the job is $h_j\epsilon$. The firms decide to close down any jobs whose productivity is below R_i^j which depends both on the age of the worker (i) and the job tenure (j). The equilibrium productivity thresholds with general human capital are characterized by the following equations[18]:

$$\frac{c}{q(\theta)} = \beta(1-\gamma)\sum_{i=1}^{T-2}\frac{u_i}{u}\int_{R_{i+1}^0}^1[1-G(x)]dx \tag{32}$$

$$h_jR_i^j = b - F_i + \beta F_{i+1} - \beta h_{j+1}\int_{R_{i+1}^{j+1}}^1[1-G(x)]dx + \beta\gamma p(\theta)\int_{R_{i+1}^0}^1[1-G(x)]dx \tag{33}$$

In contrast with general human capital, workers improve their ability only inside the firm. Specific human capital leads to increasing the value of the labour hoarding relative to the search activity. This interacts with the horizon effect: the older workers benefit less from this productivity prospect. The shorter horizon of the older workers explains why, for a given tenure, the separation rate of the older workers is still higher than that of the younger workers. Nevertheless, as older workers display longer tenure on average, they benefit from a higher productivity on average. Finally, nothing general can be claimed: the horizon effect diminishes the higher profitability of the seniors given by their longer tenure, possibly compensating for the higher specific human capital of the older workers[19].

On the other hand, it is straightforward that the results relative to the employment protection are left unchanged by specific human capital. Contrary to the case with general human capital, the relative hiring values across ages are not affected by the specific human capital accumulation because the knowledge is not transferable. This is why the hiring values still display an age-decreasing profile as only the horizon effect matters. The welfare analysis is then kept unchanged. It allows us to emphasize that older worker employment protection is desirable because of the lower value of these workers in the search sector, not on their lower value in the production sector.

[17] Note that our result does not crucially depend on the indexation of the opportunity cost of employment on the human capital accumulation. If we introduced unemployment benefits, as in Ljungqvist and Sargent [2007] and Ljungqvist and Sargent [2008], depending on the human capital used in the last job, our results would even be reinforced, as the unemployment benefits would be higher for older workers on average.

[18] The presentation of the model with specific human capital accumulation is also available upon request.

[19] As for the case with general human capital, the individual wage can be increasing with the duration of the employment spell if the impact of seniority on productivity is high enough. If the accumulation rate is high enough, it can be the case that $w_{i+1}^{j+1} > w_i^j$ for some i, j.

Overall, taking human capital accumulation into account shows that the horizon effect and the human capital may interact to make the employment protection of older workers an optimal device in a non-segmented search process provided that younger (older) workers have more (less) general capital than specific capital, which is quite realistic. Moreover, it must be emphasized that we have restricted our analysis to an exogenous accumulation process, like learning-by-doing. Taking into account the individual choices underlying human capital accumulation and job technology updating would reinforce the distance to retirement argument. The shorter horizon reduces the incentives to update the technology of the jobs occupied by older workers or to invest in human capital at the end of the life cycle. Then, the learning-by-doing assumption is clearly a conservative choice.

6 Persistent idiosyncratic shocks

Until now, we have used a simplifying assumption: there was no persistence at all in match-specific productivity shocks. In this section, we assume that the productivity shocks display some persistence, as in Mortensen and Pissarides [1994]: a new productivity level is drawn on the distribution $G(\epsilon)$ with probability $\lambda \leq 1$.

We firstly show that all our results remain valid when some degree of persistence is considered, provided that the arrival rate of productivity draws remains higher than the arrival rate of job offers. In this case, the search is still too slow, relative to the probability of having a new opportunity within the firm: the longer horizon of younger workers still gives them a higher labour hoarding value. But, secondly, it can be the case that the persistence is so high that the search strategy becomes a better strategy than labour hoarding. This reverses the implication of the horizon differentials in the production sector. Older workers are less likely to leave their jobs, simply because there is less time for them to reap the benefit of finding a better job in the search sector. However, we finally show that this high persistence case does not necessarily invalidate our age-dependent employment protection analysis, as the older workers can keep a lower value in the search sector. This last result is crucial as employment protection for older worker jobs is still based on their lower hiring value. It allows us to emphasize that older worker employment protection relies on their lower value in the search sector, not necessarily on their lower value in the production sector.

6.1 Productivity draws, job offers and the horizon effect

Assuming persistence implies that the existing level of productivity matters tomorrow for the firm value. Whether the reservation productivity remains profitable or not depends on the age-profile of the reservation productivity. Let us consider the reservation productivity at age i:

$$R_i = b - \underbrace{\beta \left((\lambda - \gamma p(\theta)) \int_{R_{i+1}}^{1} \frac{J_{i+1}(x)}{1-\gamma} dG(x) \right)}_{\textit{new opportunities}} - \underbrace{(1-\lambda) \max \left(\frac{J_{i+1}(R_i)}{1-\gamma}, 0 \right)}_{\textit{current productivity}} \tag{34}$$

Restricting ourself to monotonic age-profiles, there is now a two-regime solution, one for an age-decreasing profile and one for an age-increasing profile[20]. Without persistence, solving the model did not imply being in a particular regime, as the continuation values were independent of the actual productivity level, which was a valuable simplification. When the age-profile is increasing, it means that the current reservation productivity will become unprofitable at the next age: $\max(J_{i+1}(R_i), 0) = 0$. In that case, as in the limit case without persistence, the marginal job productivity is only influenced by the set of new opportunities in the future, which differs across workers according to their horizon, but not to their job experience: the only state variable is age. On the other hand, when the reservation productivity profile is age-decreasing, there is an additional labour hoarding motive as ageing makes some currently unprofitable levels of productivity profitable in the future. Because productivity persists into the future, this creates another channel for the horizon effect through a capitalization effect on the current reservation productivity, which becomes another state variable in addition to age.

Proposition 12. *Consider $F_i = H_i = 0$. If $\lambda > \gamma p(\theta)$, the productivity thresholds at the market equilibrium are characterized by an age-increasing dynamic ($R_i < R_{i+1}$). $\lambda < \gamma p(\theta)$ is a necessary condition for an age-decreasing dynamic ($R_i > R_{i+1}$).*

Proof. See Appendix B.5.1. □

The intuition behind Proposition 12 is straightforward. When the arrival rate of new draws is still higher than that of new jobs weighted by the worker share in the match surplus ($\lambda > \gamma p(\theta)$), the labour hoarding strategy still has a higher value than the search activity: the older workers are fired more than younger workers. On the other hand, for high enough persistence ($\lambda < \gamma p(\theta)$), the labour hoarding strategy is dominated by the search activity as far as new opportunities are considered. As the younger workers bear the consequences of a bad draw within the firms for a longer time, older workers are less fired than younger workers. This is a clear-cut difference from the case with low persistence. When the persistence is sufficiently high, the reservation productivity may be age-decreasing ($R_i > R_{i+1}$). However, the search value can remain age-increasing. In order to go to detail on this latter point, let us examine the age-profile of the

[20] Appendix B proposes a detailed presentation of the model with $\lambda \leq 1$.

productivity thresholds[21] when $\lambda < \gamma p(\theta)$:

$$(1+\beta(1-\lambda)P(i+1))(R_i - R_{i+1}) = \underbrace{\beta(1-\lambda)P(i+2)(R_{i+1}-R_{i+2})}_{LH_{i,i+1}(+)}$$
$$+ \underbrace{[\gamma p(\theta)-\lambda]\beta\left(P(i+1)I(R_{i+1}) - P(i+2)I(R_{i+2})\right)}_{SS_{i,i+1}(?)} \tag{35}$$

with $P(i+1) = \sum_{j=0}^{T-i-2}\beta^j(1-\lambda)^j$ and $I(R) = \int_R^1 1 - G(x)dx$. Workers of age $T-2$ are unambiguously fired more than workers of age $T-1$. But this terminal condition ($R_{T-2} > R_{T-1}$) is not sufficient to ensure a monotonic path for the productivity thresholds over the life cycle. The ambiguity comes from the age-profile of the search value: the term SS on the right-hand side of equation (35), which is the variation in the expected search value, is not *a priori* signed, as the result of two opposite effects. Firstly, if the reservation productivity is lower and lower as workers age, the selection process is less and less stringent at the end of the working life, and the new opportunities in the search sector are easier and easier to reach for older workers ($I(R_{i+1}) < I(R_{i+2})$). This dynamic selection effect, ceteris paribus, implies that the older workers are more fired, which is not internally consistent. Secondly, as the horizon over which the next reservation productivity will be capitalized is longer in the case of younger workers ($P(i+1) > P(i+2)$), the age-profile of the search value may be increasing, the capitalization effect compensating for the dynamic selection effect. In this case, the search value of older workers is lower whereas there are less job separations for older workers. The former condition $SS_{i,i+1} > 0$ even ensures that the latter condition $R_i > R_{i+1}$ holds. In the nutshell, workers accept a lower wage at the end of the working life if ageing is viewed as a reduction in the value of outside opportunities, which is consistent with the domination of the capitalization effect.

Proposition 13. *If $\lambda < \gamma p(\theta)$ and if the expected search value is age-decreasing ($SS_{i,i+1} > 0, \forall i$), the dynamic of the productivity thresholds is age-decreasing ($R_i > R_{i+1}, \forall i$).*

Proof. Straightforward when considering equation (35). □

When persistence is high, the reservation productivity path is unambiguously age-decreasing if the expected value in the search sector is also age-decreasing. This point deserves to be emphasized, as its counterpart at the social optimum will play a great role in the welfare analysis.

6.2 Reconsidering the impact of firing taxes

When there is some persistence, but still low enough to have $\lambda > \gamma p(\theta)$, Proposition 4 is modified as follows:

[21] See Appendix B.5.1 for a complete derivation of this equation.

Proposition 14. *If $\lambda > \gamma p(\theta)$, considering an age-uniform firing tax, the higher the persistence (the lower λ), the lower the firing tax threshold $\widehat{F}$ below which the reservation thresholds are still age-increasing ($R_i < R_{i+1}$).*

Proof. See Appendix B.5.2. □

As the persistence decreases the labour hoarding value relative to the outside option, the competitive disadvantage of the older workers is reduced. In this case, the firing tax can more easily reverse the age-increasing profile of the productivity thresholds. Proposition 3 is not modified as far as $F < \hat{F}$.

On the other hand, when $\lambda < \gamma p(\theta)$, provided that the condition that ensures the age-decreasing dynamics for the productivity thresholds holds, the firing tax still has the same differentiated effect on workers, according to their distance to retirement. Higher firing taxes make the reservation productivity path decrease even more with age[22].

6.3 Reconsidering the optimal policy

In Appendix C.2, the age-profile of the reservation productivity at the social optimum is fully characterized.

Proposition 15. *If $\lambda > p(\theta^\star)$, the social optimum allocation is characterized by $R_i^\star < R_{i+1}^\star$, $\forall i$. If $\lambda < p(\theta^\star)$ and if the expected social search value is age-decreasing ($SS_{i,i+1}^\star > 0$, $\forall i$), $R_i^\star > R_{i+1}^\star$, $\forall i$.*

Proof. See Appendix C.2 □

Proposition 15 is nothing more than the counterpart of Proposition 13 at the social optimum. The only minor difference is that the decisions of the planner are based on the total surplus generated by a new job, which implies that the condition $\lambda > \gamma p(\theta)$ is substituted out by the condition $\lambda > p(\theta)$.

Proposition 16. *Whatever the persistence level, the optimal policy rule is $\beta F_{i+1} - F_i = c\theta^\star(\tau_i^\star - 1)$. If $\lambda > p(\theta^\star)$, $\tau_i^\star$ is age-decreasing. If $\lambda < p(\theta^\star)$ and if $SS_{i,i+1}^\star > 0$, $\forall i$, $\tau_i^\star$ is still age-decreasing.*

Proof. See Appendix C.3 □

[22] Using equation (43) in Appendix B.4, $R_{T-2} > R_{T-1}$ is satisfied if $F > (\lambda - \gamma p(\theta))I(b - F)$. If $\lambda < \gamma p(\theta)$, this condition is satisfied $\forall F \geq 0$.

It is noticeable that the optimal policy rule is independent of the degree of persistence: it still relies on the age-profile of the relative social hiring value $\tau_i^\star$. However, is $\tau_i^\star$ still lower than 1 for the older workers, whatever the level of persistence?

When the persistence is not too high ($\lambda > p(\theta^\star)$), the age-increasing path of the productivity thresholds unambiguously implies that the hiring value of unemployed older workers is lower than the average hiring value, due to the combination of a more stringent selection effect and of the horizon effect. $\tau_i^\star$ is age-decreasing and there are still too many older worker job destructions. This generalizes the results obtained in the case of purely transitory shocks.

When the persistence is high enough ($\lambda < p(\theta^\star)$), the age-decreasing optimal path for the productivity thresholds could make the hiring value of unemployed older workers higher than the average one, as their probability of being recruited is higher due to a lesser stringent selection in the job creation process at the end of the working life. But, as explained above, this selection effect can be overcompensated by a capitalization effect, which unambiguously reduces the hiring value of unemployed workers with a shorter horizon. The case with high persistence is then less clear-cut due to the age-increasing productivity thresholds. But it is worth to noticing that an age-decreasing hiring value ($SS^\star_{i,i+1} > 0$), far from being incompatible with this threshold profile, actually ensures its existence[23], as shown in Proposition 13. When the capitalization effect dominates, our results are robust to persistence as the older workers keep their comparative disadvantage in the search activity. Admittedly the probability for the older workers to be hired is higher due to a lower reservation productivity, but their shorter horizon once recruited still makes the return on their hirings lower than that of younger workers through a lower capitalization effect. The social return on unemployment is then still over-estimated in the case of older workers at the market equilibrium. The optimal features of the age-dependent employment protection are not altered by the degree of persistence and the horizon effect is still at work, even through a different mechanism, as in our benchmark case without any persistence.

This result restates a key property of the age-dependent employment protection: older worker employment protection is desirable because of the lower value of these workers in the search sector, not on their lower value in the production sector. The latter feature crucially depends on the degree of persistence, but not the former one. Highly persistent shocks in interaction with a lower horizon can be responsible for lower separations for older workers, both at the market equilibrium and the social optimum, but older workers can still be fired too often because of the lower hiring value they have in the search sector.

[23]In the opposite case, corresponding to $SS^\star_{i,i+1} < 0$, the social hiring value of the older workers would become higher than that of the younger workers, whose employment should then be protected. But as Proposition 13 shows, this setup is not necessarily internally consistent, as the condition $SS^\star_{i,i+1} < 0$ does not imply an age-decreasing dynamic.

7 Conclusion

Our theory of age-dependent employment protection relies on the failure of the market equilibrium to internalize the search costs implied by unemployed older workers. Due to the horizon effect, the hiring value of the older workers is lower than the hiring value averaged over all generations, which is used by firms to make their vacancy decision. This explains why the ex-post Nash wage bargaining leads the market equilibrium to understate the search costs as far as older workers are concerned.

We show that this implies implementing age-decreasing firing taxes for the older workers in order to cope with the excessive separations we observe at the market equilibrium. This is not totally inconsistent with the current practice in most OECD countries, which have implemented higher but age-uniform firing taxes for older workers. The close-to-retirement workers benefit from the high level of the firing tax as a firing tax increases the present firing cost without any future consequences on the job value. But all other older workers with a particularly low hiring value are badly protected by the age-uniform firing tax. Conversely, the expectation of lower taxes in the future would give the right incentive for firms to postpone job destructions. If firing costs were declining, firms would weigh the benefits of firing a worker today against the benefits of waiting until next year, when the firing costs of that worker will be lower. This policy would generalize the virtue of the proximity to retirement for the impact of firing taxes. This discrepancy between theory and practice would be even greater if the distortions created by the existence of unemployment benefits were taken into account. This last point is not anecdotal as the European countries which have implemented the highest employment protection for older workers also provide the most generous unemployment benefits. It is important to note that this reconsideration of the existing policies is not due to a conflict in terms of objectives: our framework is consistent with the idea that older workers' jobs must be protected. But reaching this objective implies adopting age-decreasing firing taxes for older workers' jobs. However, this approach implies precisely estimating the threshold age onwards from which the decrease in the firing taxes has to be triggered. It implies performing a careful quantitative evaluation of our life-cycle model, incorporating other real life features. This is left for future research.

References

[1] D. Acemoglu, A Microfoundation for Social Increasing Returns in Human Capital, *Quarterly Journal of Economics*, **111** (1996), 779-804.

[2] D. Acemoglu and R. Shimer, Holdup and Efficiency Search Frictions, *International Economic Review*, **40** (1999), 827-850.

[3] J. Albrecht and S. Vroman, A Matching Model with Endogenous Skill Requirements, *International Economic Review*, **43** (2002), 283-305.

[4] J. Albrecht, L. Navarro and S. Vroman, Efficiency in a search and mtching model with endogenous participation, *Economic Letters*, **106** (2010), 48-50.

[5] A. Alesina and A. Ichino and L. Karabarbounis, Gender Based Taxation and the division of Family Chores, *American Economic Journal: Economic Policy* (2010), forthcoming.

[6] J. Altonji, A. Smith and I. Vidangos, Modeling Earnings Dynamics, *NBER Working Paper*, **14743**, 2009.

[7] A. Alesina and A. Ichino and L. Karabarbounis, Gender Based Taxation and the division of Family Chores, *American Economic Journal: Economic Policy* (2010), forthcoming.

[8] L. Behaghel and B. Crépon and B. Sédillot, The perverse effects of partial employment protection reform: The case of French older workers, *Journal of Public Economics*, **92** (2008), 696-721.

[9] M. Blázquez and M. Jansen, Search, mismatch and unemployment, *European Economic Review*, **52** (2008), 498-526.

[10] L. Borghans and B. TerWeel, Do Older Workers Have More Trouble Using Computers Than Younger Workers?, *Research in Labor Economics*, **21**, (2002), 139-173.

[11] A. Chéron, J.O. Hairault and F. Langot, Life-Cycle Equilibrium Unemployment, *IZA working paper*, **3396** (2008).

[12] A. Chéron, J.O. Hairault and F. Langot, Age-Dependent Employment Protection, *IZA working paper*, **3851** (2009).

[13] J. Conesa, S. Kitao and D. Krueger, Taxing Capital? Not a Bad Idea After All!, *American Economic Review* **99** (2009), 25-48.

[14] B. Crépon, N. Deniau and S. Perez-Duarte, Wages, Productivity and Workers Characteristics: A French Perspective, *Working Paper* (2002).

[15] S. Davis, The Quality Distribution of Jobs and the Structure of Wages in Search Equilibrium, *NBER Working Papers 8434*, (2001).

[16] E. Farhi and I. Werning, Insurance and Taxation over the Life Cycle, mimeo Harvard University, (2010).

[17] L. Friedberg, The Impact of Technological Change on OlderWorkers: Evidence from Data on Computer Use, *Industrial and Labor Relations Review*, **56** (2003), 511-529.

[18] J-O. Hairault, F. Langot, and T. Sopraseuth, Distance to Retirement and Older Workers' Employment: The Case For Delaying The Retirement Age. *Journal of European Economic Association*, (2010), Forthcoming.

[19] J. Heckman, L. Lochner and P. Todd, Fifty Years of Mincer Earnings Regressions, *Working Paper NBER*, **9732**, 2003.

[20] J.K. Hellerstein, D. Neumark and K.R. Troske, Wages, productivity and worker characteristics: evidence from plant-level production functions and wage equations, *Journal of Labor Economics*, **17** (1999), 409-446.

[21] A.J. Hosios, On the efficiency of matching and related models of search and unemployment, *Review of Economic Studies* **57** (1990), 279-298.

[22] G. Kambourov and I. Manovskii, Accounting for the Changing Life-Cycle Profile of Earnings, mimeo, 2009.

[23] M. Kremer, Should Taxes be Independent of Age? *Working Paper*, (1999).

[24] L. Jacobson, R. LaLonde and D. Sullivan, Earnings Losses of Displaced Workers, *American Economic Review*, **83** (1993), 685- 709.

[25] L. Kotlikoff and J. Gokhale, Estimating The Age-Productivity Profile Using the Present Value of Lifetime Earnings, *Quarterly Journal of Economics*, **107** (1992), 1215-42.

[26] L. Ljungqvist and T. Sargent, Understanding European Unemployment with Matching and Search-Island Models, *Journal of Monetary Economics*, **54** (2007), 2139-2179.

[27] L. Ljungqvist and T. Sargent, Two Questions about European Unemployment, *Econometrica*, **76**, (2008) 1-29.

[28] R. Marimon and F. Zilibotti, Unemployment versus Mismatch of Talents: Reconsidering Unemployment Benefits, *Economic Journal*, **109** (1999), 266-291.

[29] J. Mincer, Education and Unemployment, *NBER Working Paper*, **3838** (1991).

[30] E. Moen, Competitive search equilibrium, *Journal of Political Economy*, **105** (1997), 385-411.

[31] D.T. Mortensen and C. Pissarides, Job creation and job destruction in the theory of unemployment, *Review of Economic Studies*, **61** (1994), 397-415.

[32] D.T. Mortensen and C. Pissarides, New developments in models of search in the labor market, *Handbook of Labor Economics*,North-Holland:Amsterdam (1999a).

[33] D. Mortensen and C. Pissarides, Unemployment Responses to Skill-biased Technology Shocks: The Role of Labor Market Policy, *Economic Journal*, **109** (1999b), 242-265.

[34] D. Neumark, Age discrimination legislation in the United States, *NBER working paper*, **8152** (2001).

[35] OECD publishing, Live longer, work longer, *Ageing and Employment Policies* (2006).

[36] C. Pissarides, Short-run Equilibrium Dynamics of Unemployment, Vacancies and Real Wages, *American Economic Review*, **75** (1985), 676-690.

[37] C. Pissarides, Equilibrium unemployment, MIT Press (2000).

[38] C. Pissarides, The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?, *Econometrica*, **77** (2009), 1339-1369.

[39] A. Rosolia and G. Saint-Paul, The effect of unemployment spells on subsequent wages in Spain, *CPER working paper* (1998).

[40] G. Saint-Paul, Does the Welfare State Make Older Workers Unemployable ?, *IZA working paper*, **4440** (2009).

[41] M. Sattinger, Search and the Efficient Assignment of Workers to Jobs, *International Economic Review*, **36** (1995), 283-302.

[42] S. Shi, Frictional Assignment. I. Efficiency, *Journal of Economic Theory*, **98** (2001), 232-260.

[43] R. Shimer, The cyclical behavior of equilibrium unemployment and vacancies, *American Economic Review*, **95** (2005), 25-49.

[44] R. Shimer, The Assignment of Workers to Jobs in an Economy with Coordination Frictions, *Journal of Political Economy*, **113 (5)** (2005b), 996-1025.

[45] R. Shimer and L. Smith, Matching, Search and Heterogeneity, *Advances-in-Macroeconomics (BE Journal in Macroeconomics)*, **1 (1)** (2001), article 5.

[46] M. Weinzierl, The Surprising Power of Age-Dependent Taxes, *working paper*, (2010), Harvard Business School.

[47] R. Willis, Wage Determinants: A Survey and Reinterpretation of Human Capital Earnings Functions, *Handbook of Labor Economics*, (1986), Elsevier.

A The existence and uniqueness of the equilibrium

Let us rewrite equation (15) as follows:

$$\frac{c}{\beta(1-\gamma)} = q(\theta)\sum_{i=1}^{T-2}\frac{u_i}{u}\int_{R_{i+1}^0}^{1}[1-G(x)]dx \equiv f(\theta)$$

Firstly, let us note that $f(\theta)$ is a continuous function, as it is the sum and the product of continuous functions. Secondly, let us notice that $\lim_{\theta\to\infty} q(\theta) = 0$. If $f(0) > \frac{c}{\beta(1-\gamma)}$, then there exists at least one equilibrium. As $R_i^0 < b$, $\forall i$, a sufficient condition for the existence of equilibrium is: $\int_b^1[1-G(x)]dx > \frac{c}{\beta(1-\gamma)}$. Considering an uniform distribution for $G(x)$, it is possible to derive an explicit condition ensuring the existence of the equilibrium characterized in Proposition 1:

$$\frac{1}{2}(1-b)^2\beta(1-\gamma) > c$$

On the other hand, it is not possible to show an explicit condition under which this equilibrium would be unique. Equation (15) can be rewritten as follows: $\Psi(\theta) = (1-\gamma)\beta\Gamma(\theta)$ with $\Psi(\theta) \equiv \frac{c}{q(\theta)}$ and $\Gamma(\theta) \equiv \sum_{i=1}^{T-2}\frac{u_i}{u}\int_{R_{i+1}^0}^{1}[1-G(x)]dx$. It is straightforward that $\Psi' > 0$. The equilibrium is then unique if the function Γ is continuously decreasing:

$$\Gamma'(\theta) = \underbrace{\left.\frac{\partial\Gamma(\theta)}{\partial\theta}\right|_{I(R_{i+1}^0)=Cst}}_{A(?)} + \underbrace{\left.\frac{\partial\Gamma(\theta)}{\partial\theta}\right|_{u_i=Cst}}_{B\leq 0}$$

with $I(R_{i+1}^0) = \int_{R_{i+1}^0}^{1}[1-G(x)]dx$.

First, let us note that in MP, we have $A = 0$ so that the condition $\Gamma'(\theta) < 0$ is always satisfied. Indeed, this first term of the derivative of Γ represents the impact of a variation in θ on the unemployment rates by age: because in MP the agents are homogenous, this term, related to the non-segmented search, does not exist. In our framework, $A \neq 0$, and it is no longer possible to show that the equilibrium is unique for an arbitrary value of T.

B The complete model with persistence

We present the model in the general case where idiosyncratic shocks are persistent. At each age, we then assume that a new productivity ϵ is drawn in the distribution $G(\epsilon)$ with a probability $\lambda \leq 1$. To derive the results without any persistence, $\lambda = 1$ must be considered in all equations.

B.1 The value functions of the agents

The value functions for an occupied firm ($J_i(\epsilon)$), a worker ($W_i(\epsilon)$), a new firm ($J_i^0(\epsilon)$), a new employed worker ($W_i^0(\epsilon)$) and an unemployed worker are respectively given by:

$$
\begin{aligned}
J_i(\epsilon) &= \epsilon - w_i(\epsilon) + \beta \left[\lambda \int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) - \lambda G(R_{i+1}) F_{i+1} + (1-\lambda) \max\{J_{i+1}(\epsilon); -F_{i+1}\} \right] \\
W_i(\epsilon) &= w_i(\epsilon) - t_i + \beta \left[\lambda \int_{R_{i+1}}^{1} W_{i+1}(x) dG(x) + (1-\lambda) \max\{W_{i+1}(\epsilon); U_{i+1}\} + \lambda G(R_{i+1}) U_{i+1} \right] \\
J_i^0(\epsilon) &= \epsilon - w_i^0(\epsilon) + \beta \left[\lambda \int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) - \lambda G(R_{i+1}) F_{i+1} + (1-\lambda) \max\{J_{i+1}(\epsilon); -F_{i+1}\} \right] \\
W_i^0(\epsilon) &= w_i^0(\epsilon) - t_i + \beta \left[\lambda \int_{R_{i+1}}^{1} W_{i+1}(x) dG(x) + (1-\lambda) \max\{W_{i+1}(\epsilon); U_{i+1}\} + \lambda G(R_{i+1}) U_{i+1} \right] \\
U_i &= b - t_i + \beta \left[p(\theta) \int_{R_{i+1}^0}^{1} W_{i+1}^0(x) dG(x) + p(\theta) G(R_{i+1}^0) U_{i+1} + (1-p(\theta)) U_{i+1} \right]
\end{aligned}
$$

The productivity thresholds are defined by:

$$
J_i(R_i) = -F_i \quad \text{and} \quad J_i^0(R_i^0) = H_i
$$

B.2 Wage equations under a two-tier structure

The sharing rules can be written as:

$$
\begin{aligned}
-\gamma H_i - (1-\gamma)\mathcal{U}_i &= \gamma \left[J_i^0(\epsilon) + \mathcal{W}_i^0(\epsilon) \right] - \mathcal{W}_i^0(\epsilon) && (36) \\
-\gamma F_i - (1-\gamma)\mathcal{U}_i &= \gamma \left[J_i(\epsilon) + \mathcal{W}_i(\epsilon) \right] - \mathcal{W}_i(\epsilon) && (37)
\end{aligned}
$$

From the value functions, it turns out that:

$$
\begin{aligned}
J_i(\epsilon) + W_i(\epsilon) &= \epsilon - t_i + \beta \left[\lambda \int_{R_{i+1}}^{1} [J_{i+1}(x) + W_{i+1}(x)] dG(x) \right. \\
&\quad + (1-\lambda) \left(\max\{J_{i+1}(\epsilon); -F_{i+1}\} + \max\{W_{i+1}(x); U_{i+1}\} \right) \\
&\quad \left. + \lambda G(R_{i+1}) U_{i+1} - \lambda G(R_{i+1}) F_{i+1} \right]
\end{aligned}
$$

Using the sharing rules, we deduce that:

$$
\begin{aligned}
\gamma \left[J_i(\epsilon) + W_i(\epsilon) \right] - W_i(\epsilon) &= \gamma\epsilon - w_i(\epsilon) + (1-\gamma) t_i - \lambda\gamma G(R_{i+1}) \beta F_{i+1} \\
&\quad - (1-\gamma)\lambda\beta U_{i+1} - \gamma\lambda[1 - G(R_{i+1})]\beta F_{i+1} \\
&\quad + \gamma\beta(1-\lambda) \max\{J_{i+1}(\epsilon); -F_{i+1}\} \\
&\quad - (1-\gamma)\beta(1-\lambda) \max\{W_{i+1}(\epsilon); U_{i+1}\}
\end{aligned}
$$

We then obtain the following wage equations:

$$\begin{aligned} w_i(\epsilon) &= \gamma\epsilon + (1-\gamma)t_i + (1-\gamma)\left[U_i - \beta U_{i+1}\right] + \gamma\left(F_i - \beta F_{i+1}\right) \\ &\quad + \gamma\beta(1-\lambda)\max\{J_{i+1}(\epsilon) + F_{i+1}; 0\} - (1-\gamma)\beta(1-\lambda)\max\{W_{i+1}(\epsilon) - U_{i+1}; 0\} \end{aligned}$$

Because we also have $\gamma\left(J_{i+1}(\epsilon) + F_{i+1}\right) = (1-\gamma)(W_{i+1}(\epsilon) - U_{i+1})$, and, as $\max\{J_{i+1}(\epsilon) + F_{i+1}; 0\} = J_{i+1}(\epsilon) + F_{i+1}$, then $\max\{W_{i+1}(\epsilon) - U_{i+1}; 0\} = W_{i+1}(\epsilon) - U_{i+1}$, we finally obtain:

$$\begin{aligned} w_i(\epsilon) &= \gamma\left(\epsilon + F_i - \beta F_{i+1}\right) + (1-\gamma)\left(U_i - \beta U_{i+1}\right) \\ &= \gamma\left(\epsilon + F_i - \beta F_{i+1} + c\theta\tau_i\right) + (1-\gamma)b \end{aligned}$$

given that

$$\begin{aligned} U_i &= b - t_i + \beta\left[p(\theta)\int_{R^0_{i+1}}^1 \left(W^0_{i+1}(x) - U_{i+1}\right)dG(x) + U_{i+1}\right] \\ &= b - t_i + \frac{\gamma}{1-\gamma}c\theta \underbrace{\frac{\int_{R^0_{i+1}}^1 J^0_{i+1}(x)dG(x)}{\sum_{i=1}^{T-2}\frac{u_i}{u}\int_{R^0_{i+1}}^1 J^0_{i+1}(x)dG(x)}}_{\tau_i} + \beta U_{i+1} \end{aligned}$$

We can also show, using the same computational method, that:

$$w^0_i(\epsilon) = \gamma\left(\epsilon + H_i - \beta F_{i+1} + c\theta\tau_i\right) + (1-\gamma)b$$

B.3 The firm's value

Given the solution for the wage, and the free entry condition ($V = 0$), the firm values are, $\forall i \in [1, T-1]$:

$$\begin{aligned} J_i(\epsilon) &= \underbrace{(1-\gamma)(\epsilon - b) - \gamma\left(F_i - \beta F_{i+1}\right) - \gamma p(\theta)\beta\int_{R^0_{i+1}}^1 [J^0_{i+1}(x) + H_{i+1}]dG(x)}_{\epsilon - w_i(\epsilon)} \qquad (38) \\ &\quad + \beta\left[-\lambda G(R_{i+1})F_{i+1} + \lambda\int_{R_{i+1}}^1 J_{i+1}(x)dG(x) + (1-\lambda)\max\{J_{i+1}(\epsilon); -F_{i+1}\}\right] \\ J^0_i(\epsilon) &= \underbrace{(1-\gamma)(\epsilon - b) - \gamma\left(H_i - \beta F_{i+1}\right) - \gamma p(\theta)\beta\int_{R^0_{i+1}}^1 [J^0_{i+1}(x) + H_{i+1}]dG(x)}_{\epsilon - w^0_i(\epsilon)} \qquad (39) \\ &\quad + \beta\left[-\lambda G(R_{i+1})F_{i+1} + \lambda\int_{R_{i+1}}^1 J_{i+1}(x)dG(x) + (1-\lambda)\max\{J_{i+1}(\epsilon); -F_{i+1}\}\right] \end{aligned}$$

We deduce that:

$$J^0_i(\epsilon) = J_i(\epsilon) + \gamma(F_i - H_i)$$

From equations (38) and (39), the reservation productivity, defined by $J_i(R_i) = -F_i$, is given by:

$$
\begin{aligned}
-F_i &= (1-\gamma)(R_i - b) - \gamma\left(F_i - \beta F_{i+1}\right) - \gamma p(\theta)\beta \int_{R^0_{i+1}}^{1} [J^0_{i+1}(x) + H_{i+1}]dG(x) \qquad (40)\\
&\quad +\beta\left[-\lambda G(R_{i+1})F_{i+1} + \lambda \int_{R_{i+1}}^{1} J_{i+1}(x)dG(x) + (1-\lambda)\max\{J_{i+1}(R_i); -F_{i+1}\}\right]
\end{aligned}
$$

and the reservation productivity for a new job, defined by $J^0_i(R^0_i) = -H_i$, is given by:

$$
\begin{aligned}
-H_i &= (1-\gamma)(R^0_i - b) - \gamma\left(H_i - \beta F_{i+1}\right) - \gamma p(\theta)\beta \int_{R^0_{i+1}}^{1} [J^0_{i+1}(x) + H_{i+1}]dG(x) \qquad (41)\\
&\quad +\beta\left[-\lambda G(R_{i+1})F_{i+1} + \lambda \int_{R_{i+1}}^{1} J_{i+1}(x)dG(x) + (1-\lambda)\max\{J_{i+1}(R_i); -F_{i+1}\}\right]
\end{aligned}
$$

From equations (40) and (41), we easily deduce that:

$$R^0_i = R_i + F_i - H_i$$

Using these first results, it is possible to express the value function as a function of the productivity reservation. Taking the difference between equations (38) and (40) gives

$$J_i(\epsilon) = (1-\gamma)(\epsilon - R_i) - F_i + \beta(1-\lambda)\left[\max\{J_{i+1}(\epsilon); -F_{i+1}\} - \max\{J_{i+1}(R_i); -F_{i+1}\}\right]$$

The solution to this equation depends on the age-pattern of the productivity thresholds, except for $\lambda = 1$. The strategy in the general case $\lambda \leq 1$ is to postulate a monotonic age-pattern and then to verify that the solution for the reservation productivity is consistent with the initial guess.

1. Assuming $R_i > R_{i+1}$, then $J_{i+1}(R_i) > -F_{i+1}$ and $J_{i+1}(\epsilon) > -F_{i+1}$. We deduce that:

$$
\begin{aligned}
J_i(\epsilon) &= (1-\gamma)(\epsilon - R_i) - F_i + \beta(1-\lambda)\left[J_{i+1}(\epsilon) - J_{i+1}(R_i)\right]\\
\Leftrightarrow J_i(\epsilon) &= -F_i + (1-\gamma)\left(\sum_{j=0}^{T-i-1} \beta^j (1-\lambda)^j\right)(\epsilon - R_i)
\end{aligned}
$$

2. Assuming $R_i < R_{i+1}$, then $J_{i+1}(R_i) < -F_{i+1}$. This implies that:

$$J_i(\epsilon) = (1-\gamma)(\epsilon - R_i) - F_i + \beta(1-\lambda)F_{i+1} + \beta(1-\lambda)\max\{J_{i+1}(\epsilon); -F_{i+1}\}$$

   Because the existing productivity level ϵ can be in the interval $]R_i, R_{i+1}[$, we have two subcases:

   (a) if $\epsilon \leq R_{i+1}$, then $J_{i+1}(\epsilon) \leq -F_{i+1}$, and $\max\{J_{i+1}(\epsilon); -F_{i+1}\} = -F_{i+1}$, implying that:

$$J_i(\epsilon) = (1-\gamma)(\epsilon - R_i) - F_i$$

(b) if $\epsilon > R_{i+1}$, then $J_{i+1}(\epsilon) > -F_{i+1}$, implying that:

$$J_i(\epsilon) = (1-\gamma)(\epsilon - R_i) - F_i + \beta(1-\lambda)F_{i+1} + \beta(1-\lambda)J_{i+1}(\epsilon)$$

By backward induction, we obtain:

$$J_i(\epsilon) = -F_i + (1-\gamma)\left(\sum_{j=0}^{T-i-1} \beta^j (1-\lambda)^j \max\{\epsilon - R_{i+j}; 0\}\right)$$

The value function can be rewritten as follows:

$$J_i(\epsilon) = -F_i + \mathcal{J}(\epsilon) \quad \text{with } \mathcal{J}(\epsilon) = \begin{cases} \Gamma\left(\epsilon, \{R_j\}_{j=i+1}^{T-1}\right) & \text{if} \quad R_i < R_{i+1} \\ \Psi(\epsilon, R_i) & \text{if} \quad R_i > R_{i+1} \end{cases} \tag{42}$$

Let us note that we have simply $J_i(\epsilon) = -F_i + (1-\gamma)(\epsilon - R_i)$ if $\lambda = 1$ whatever the shape of the reservation productivity sequence ($R_i \gtrless R_{i+1}$).

B.4 The reservation productivity

Using the general expression (equation (42)) of the value function for an occupied job, the reservation productivity is the solution to:

$$\begin{aligned} -F_i = {} & (1-\gamma)(R_i - b) - \gamma(F_i - \beta F_{i+1}) \\ & -\gamma p(\theta)\beta \int_{R_{i+1}+F_{i+1}-H_{i+1}}^{1} [-F_{i+1} + \mathcal{J}(\epsilon) + \gamma(F_{i+1} - H_{i+1}) + H_{i+1}] dG(x) \\ & +\beta\left[-\lambda G(R_{i+1})F_{i+1} + \lambda \int_{R_{i+1}}^{1} [-F_{i+1} + \mathcal{J}(\epsilon)] dG(x)\right. \\ & \left. +(1-\lambda)\max\{J_{i+1}(R_i); -F_{i+1}\}\right] \end{aligned}$$

Hereafter, we assume that $H_i = F_i$.

- If $R_i < R_{i+1}$, we have $\max\{J_{i+1}(R_i); -F_{i+1}\} = -F_{i+1}$ and then:

$$R_i = b - F_i + \beta F_{i+1} - [\lambda - \gamma p(\theta)]\beta \sum_{j=0}^{T-i-1} \beta^j (1-\lambda)^j I(R_{i+j}) \tag{43}$$

  where $I(R_{i+j}) = \int_{R_{i+j}}^{1} (x - R_{i+j}) dG(x) = \int_{R_{i+j}}^{1} (1 - G(x)) d(x) > 0$ with $I'(R_{i+j}) < 0$.

- If $R_i > R_{i+1}$, $\max\{J_{i+1}(R_i); -F_{i+1}\} = J_{i+1}(R_i)$. The solution of the value function leads to:

$$J_{i+1}(R_i) = -F_i + (1-\gamma)\left(\sum_{j=0}^{T-i-2} \beta^j (1-\lambda)^j\right)(R_i - R_{i+1})$$

We then deduce that

$$\begin{aligned} R_i &= b - F_i + \beta F_{i+1} - [\lambda - \gamma p(\theta)]\beta P(i+1) I(R_{i+1}) \\ &\quad + (1-\lambda)\beta P(i+1)(R_i - R_{i+1}) \end{aligned} \tag{44}$$

where $P(i+1) = \sum_{j=0}^{T-i-2} \beta^j (1-\lambda)^j$.

B.5 The dynamic of the reservation productivity: proofs of the Propositions 12, 13, 14

We determine the restrictions under which the reservation productivity increases or decreases with the worker's age. We assume that $H_i = F_i = F$, $\forall i$, implying that $R_i = R_i^0$.

B.5.1 Proofs of the Propositions 12 and 13: the case without employment protection $F_i = H_i = 0$

- When the conjecture is $R_i < R_{i+1}$, the sequence of the reservation productivity is given by equation (43). The restriction $R_{T-2} < R_{T-1}$ is satisfied if $\lambda > \gamma p(\theta)$. We deduce that if $R_{i+1} < R_{i+2}$, then $I(R_{i+1}) > I(R_{i+2})$ because $I'(x) < 0$. In this case,

$$R_{i+1} - R_i = -[\lambda - \gamma p(\theta)]\beta \underbrace{(I(R_{i+2}) - I(R_{i+1}))}_{-} + \beta(1-\lambda) \underbrace{(R_{i+2} - R_{i+1})}_{+}$$

  The condition $\lambda > \gamma p(\theta)$ is sufficient to ensure that $R_{i+1} > R_i$.

- When the conjecture is $R_i > R_{i+1}$, the sequence of the reservation productivity is given by equation (44). Then $R_{T-2} > R_{T-1}$ if $\lambda < \gamma p(\theta)$. Using backward iterations, we obtain:

$$\begin{aligned} &(1 + \beta(1-\lambda)P(i+1))(R_i - R_{i+1}) \\ &= \underbrace{[\gamma p(\theta) - \lambda]\beta \left(P(i+1)I(R_{i+1}) - P(i+2)I(R_{i+2})\right)}_{SS_{i,i+1}} \\ &+ \underbrace{\beta(1-\lambda)P(i+2)(R_{i+1} - R_{i+2})}_{LH_{i,i+1}} \end{aligned}$$

  Unambiguously, we have $LH_{i,i+1} > 0$ because from the previous iteration we have $R_{i+1} - R_{i+2} > 0$. It comes that $I(R_{i+1}) < I(R_{i+2})$: the strictness of the selection process is more important for the younger workers. On the other hand, $P(i+1) > P(i+2)$ due to the horizon effect. As these two effects go in opposite directions, the sign of $SS_{i,i+1}$ is indeterminate.

  As the condition $\lambda < \gamma p(\theta)$ must hold at the first iteration, we deduce that $SS_{i,i+1} > 0$ is a sufficient condition to ensure that $R_i > R_{i+1}$.

B.5.2 Proof of the Proposition 14: the case with constant employment protection $F = H$, $\forall i$

When the conjecture is $R_i < R_{i+1}$, using equation(43), $R_{T-2} < R_{T-1}$ if $F < [\lambda - \gamma p(\theta)] \int_{b-F}(1 - G(x))dx$. We then define $\widehat{F}(\lambda) = [\lambda - \gamma p(\theta)] \int_{b-\widehat{F}(\lambda)}(1 - G(x))dx$, implying that $\frac{d\widehat{F}}{d\lambda} > 0$.

C The efficient allocation

The problem of the planner is to determine the optimal allocation of any worker between the production and the search sectors ($R_i^\star$) and the optimal investment in the search sector ($\theta^\star$). For any $\lambda \leq 1$, the per-unemployed worker value in the search sector and the per-employed worker value in the goods sector are respectively given by:

$$Y_i^s = b - c\theta^\star + \beta \left[p(\theta^\star) \int_0^1 \max\{Y_{i+1}(x); Y_{i+1}^s\} dG(x) + (1 - p(\theta^\star)) Y_{i+1}^s \right] \tag{45}$$

$$Y_i(\epsilon) = \epsilon + \beta \left[\lambda \int_0^1 \max\{Y_{i+1}(x); Y_{i+1}^s\} dG(x) + (1 - \lambda) \max\{Y_{i+1}(\epsilon); Y_{i+1}^s\} \right] \tag{46}$$

where $c\theta^\star \equiv c\frac{v^\star}{u^\star}$ represents the total cost of vacancies ($cv^\star$) per unemployed worker ($u^\star$). The planner's decisions $R_i^\star$, $\forall i$ and $\theta^\star$ are solutions to:

$$\begin{cases} Y_i(R_i^\star) &= Y_i^s \\ \theta^\star &= Sup(\sum_i u_i^\star Y_i^u) \end{cases}$$

C.1 The efficient allocation in an economy without persistence: proof of Proposition 6

The optimal choice for θ is such that:

$$c \sum_i u_i^\star = cu^\star = p'(\theta^\star) \beta \sum_i u_i^\star \int_{R_{i+1}^\star}^1 S_{i+1}(x) dG(x) \tag{47}$$

Given that $\theta q(\theta) = p(\theta)$ and $p'(\theta) = q(\theta) \left[1 + \theta \frac{q'(\theta)}{q(\theta)} \right] = q(\theta)(1 - \eta)$, equation (47) can be rewritten as follows:

$$\frac{c}{q(\theta^\star)} = (1 - \eta) \beta \sum_i \frac{u_i^\star}{u^\star} \int_{R_{i+1}^\star}^1 S_{i+1}(x) dG(x)$$

The optimal reservation productivity at age i can be deduced from $Y_i(R_i^\star) = Y_i^u \Leftrightarrow S_i(R_i) = 0$:

$$0 = R_i^\star - b + c\theta^\star + [1 - p(\theta^\star)] \beta \int_{R_{i+1}^\star}^1 S_{i+1}(x) dG(x)$$

Since $S_i(\epsilon) = \epsilon - R_i^\star$, we have $\int_{R_{i+1}^\star}^1 S_{i+1}(x) dG(x) = \int_{R_{i+1}^\star}^1 (\epsilon - R_{i+1}^\star) dG(x) = \int_{R_{i+1}^\star}^1 [1 - G(x)] dx$. We then deduce equations (23) and (24).

C.2 The efficient allocation in an economy with persistence: proof of Proposition 15

The optimal choice for $\theta^\star$ does not depend directly on the persistence: the first order condition (47) holds for any $\lambda \leq 1$. On the other hand, the persistence changes the value of an occupied job. When $\lambda < 1$, the planner's surplus is:

$$S_i(\epsilon) \quad = \quad \epsilon - R_i^\star + (1-\lambda)\beta[\max\{S_{i+1}(\epsilon); 0\} - \max\{S_{i+1}(R_i^\star); 0\}]$$

- If $R_i^\star > R_{i+1}^\star$, then we have $\epsilon > R_i^\star > R_{i+1}^\star$. The surplus is:

$$S_i(\epsilon) \quad = \quad \left(\sum_{j=0}^{T-i-1} \beta^j (1-\lambda)^j \right) (\epsilon - R_i^\star) \equiv P(i)(\epsilon - R_i^\star)$$

- If $R_i^\star < R_{i+1}^\star$, then the surplus is

$$S_i(\epsilon) \quad = \quad \sum_{j=0}^{T-i-1} (1-\lambda)^j \beta^j \max\{\epsilon - R_{i+j}^\star; 0\}$$

The reservation productivity R_i of the efficient allocation then differs according to the age-profile that is assumed:

- If $R_i^\star < R_{i+1}^\star$, the reservation productivity is defined by:

$$R_i^\star \quad = \quad b - c\theta^\star - [\lambda - p(\theta^\star)]\beta \int_{R_{i+1}^\star}^1 \left[\sum_{j=0}^{T-i-2} (1-\lambda)^j \beta^j \max\{x - R_{i+j+1}^\star; 0\} \right] dG(x) \tag{48}$$

- If $R_i^\star > R_{i+1}^\star$, the reservation productivity is defined by:

$$\begin{aligned} R_i^\star \quad &= \quad b - c\theta^\star - [\lambda - p(\theta^\star)]\beta \int_{R_{i+1}^\star}^1 P(i+1)(x - R_{i+1}^\star) dG(x) \\ &- \quad (1-\lambda)\beta P(i+1)(R_i^\star - R_{i+1}^\star) \end{aligned} \tag{49}$$

In order to have an interior solution in each case, we assume that $b > c\theta^\star$.

- If $R_i^\star < R_{i+1}^\star$, we have at the first step of the backward iteration:

$$R_{T-2}^\star - R_{T-1}^\star \quad = \quad -[\lambda - p(\theta^\star)]\beta \int_{R_{T-1}^\star}^1 \max\{x - R_{T-1}^\star; 0\} dG(x)$$

  The condition $\lambda > p(\theta^\star)$ is sufficient to ensure that $R_{T-2}^\star < R_{T-1}^\star$. In the following iterations, we have:

$$R_{i+1}^\star - R_i^\star \quad = \quad -[\lambda - p(\theta^\star)]\beta \underbrace{\left(I(R_{i+2}^\star) - I(R_{i+1}^\star)\right)}_{-} + \beta(1-\lambda) \underbrace{\left(R_{i+2}^\star - R_{i+1}^\star\right)}_{+}$$

  The restriction $\lambda > p(\theta^\star)$ is sufficient to ensure that the age-increasing dynamics is internally consistent.

- If $R_i^\star > R_{i+1}^\star$, we have at the first step of the backward iteration:

$$(R_{T-2}^\star - R_{T-1}^\star)(1+(1-\lambda)\beta) = -[\lambda - p(\theta^\star)]\beta \int_{R_{T-1}^\star}^1 \max\{x - R_{T-1}^\star; 0\} dG(x)$$

We deduce that the restriction $\lambda < p(\theta^\star)$ is sufficient to ensure that $R_{T-2}^\star < R_{T-1}^\star$. In the following backward iterations, we obtain:

$$\begin{aligned}
&(1+\beta(1-\lambda)P(i+1))(R_i^\star - R_{i+1}^\star) \\
&= \underbrace{[p(\theta^\star) - \lambda]\beta\left(P(i+1)I(R_{i+1}^\star) - P(i+2)I(R_{i+2}^\star)\right)}_{SS_{i,i+1}^\star} \\
&+ \underbrace{\beta(1-\lambda)P(i+2)(R_{i+1}^\star - R_{i+2})^\star}_{LH_{i,i+1}^\star(+)}
\end{aligned}$$

$\lambda < p(\theta^\star)$ is no longer a sufficient condition to ensure that the $R_i^\star$ are monotonously decreasing. Adding the condition $SS_{i,i+1}^\star > 0,\ \forall i$ is sufficient to ensure that $R_i^\star > R_{i+1}^\star$.

C.3 Optimal age-dependent employment protection: proof of Proposition 16

Comparing on the one hand equation (48) with equation (44) and on the other hand equation (49) with equation (43), it is straightforward to show that $F_i - \beta F_{i+1} = c\theta^\star(\tau_i^\star - 1)$, $\forall \lambda \leq 1$ with $\tau_i^\star \equiv \frac{\int_{R_{i+1}^\star}^1 S_{i+1}(x)dG(x)}{\sum_i \frac{u_i^\star}{u^\star}\int_{R_{i+1}^\star}^1 S_{i+1}(x)dG(x)}$.

- If $R_i^* < R_{i+1}^*$, as

$$\left(R_{i+1}^* - R_i^*\right) \;=\; (\lambda - p(\theta))\,\beta\left\{\int_{R_{i+1}^*}^1 S_{i+1}(x)dG(x) - \int_{R_{i+2}^*}^1 S_{i+2}(x)dG(x)\right\}$$

  then $\tau_i^* > \tau_{i+1}^*$.

- If $R_i^* > R_{i+1}^*$, the sign of

$$\int_{R_{i+1}^*}^1 S_{i+1}(x)dG(x) - \int_{R_{i+2}^*}^1 S_{i+2}(x)dG(x) = (P(i+1)I(R_{i+1}^*) - P(i+2)I(R_{i+2}^*))$$

  is indeterminate. Proposition 15 shows that a sufficient condition for $R_i > R_{i+1}$ is that the sign of this expression is positive, which implies that $\tau_i^* > \tau_{i+1}^*,\ \forall i$.