A mathematical resurgence of risk management: an extreme modeling of expert opinions

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Abstract

The Operational Risk Advanced Measurement Approach requires financial institutions to use scenarios to model these risks and to evaluate the pertaining capital charges. Considering that a banking group is composed of numerous entities (branches and subsidiaries), and that each one of them is represented by an Operational Risk Manager (ORM), we propose a novel scenario approach based on ORM expertise to collect information and create new data sets focusing on large losses, and the use of the Extreme Value Theory (EVT) to evaluate the corresponding capital allocation. In this paper, we highlight the importance to consider an *a priori* knowledge of the experts associated to a *a posteriori* backtesting based on collected incidents.

Keywords: Operational risks - EVT - AMA - Expert - Value-at-Risk - Expected Shortfall

1 Introduction

The Advanced Measurement Approach (BCBS (2001; 2009)) requires banks to carry out scenario analysis to compute the capital allocation pertaining to operational risks (Cruz (2004), Chernobai et al. (2007) and Shevchenko (2011)). Scenarios may have multiple forms depending on the kind of risk modeled. For example, exogenous extremal risks such as a flood, an earthquake or a pandemic may be modeled using Bayesian networks, or disasters and ruin theory, etc. For some other scenarios modeling endogenous risks, for example, frauds, execution failures etc., one may use expert opinions. Indeed, such experts exist in banks and insurance companies and have a very good knowledge of the incidents that may occur in their specific work segment.

We have multiple motivations to use expert opinions. First, considering local operational risk managers as experts, they are the tip of sword and the guardian of the system efficiency, and they represent an important link with the permanent control system. Some of them collect the loss incidents, others have in charge deploying some plans to prevent operational risks, therefore they have a real experience of the operational risks and are able to anticipate them accurately. Their opinions incorporate different types of information such as what behaviors are important, permanent, cyclic...; how strong is the activity in a particular entity in a particular period; how efficient are the measures taken to prevent these risks, etc. We have a real opportunity to use their expertise several times a year either to understand the evolution of the operational risks, either to estimate a capital allocation or to evaluate prospective amounts.

For obvious reasons, working with historical data sets bias our vision of extremal events as their frequency is much lower than for regular events (small and medium sized). Therefore large losses are difficult to analyze and model. A solution stands in modeling extremal events in a specific framework, for instance considering the Generalized Pareto distribution to model the severities (Pickands (1975), Coles (2004) and Guégan et al. (2010)), nevertheless, this method requires large enough data sets to ensure the robustness of the estimations. Using historical data, if we cannot correctly fit these distributions whose information is contained in the tails, a possibility is to use experts opinions to build new data sets that we will analyze. Indeed, the analysis capacity of these experts and their anticipation analysis regarding operational risks large amounts incidents can be profitable to create reliable information sets to model operational risks.

The information obtained from the experts may be heterogeneous as they have not the same experience, the same information quality or the same location, thus in order to reduce the impact of heterogeneous information sets, we are only going to ask them the maximum value a bank could lose if a particular event type occurs on a particular business unit in a specific period (a week, a month, etc.). Therefore, each expert is going to provide several maxima per cell of the Basel matrix and also for different levels of granularity and for a defined horizon.

Our objective is to provide capital charges associated to the different cells of the Basel matrix[1] built with these data sets, and as soon as we work with sequences of maxima, we will use the Extreme Value Theory (EVT) (Leadbetter et al. (1983), Resnick (1987), Embrechts et al. (1997) and de Haan and Ferreira (2010)) to compute them. This theoretical framework tells that under regularity conditions, a series of maxima follows a Generalized Extreme Value (GEV) distribution given in (2.1). Using the maxima series, the GEV distributions' parameters are estimated by MLE[2] (Hoel (1962)) and for each cell a capital charge is provided considering two risk measures: the Value-at-Risk and the Expected Shortfall.

In a first section, we present our experimental process and in a second section, we provide and analyze the results we obtained. The last section concludes.

2 A Strategy based on risk managers

2.1 Maxima series construction

We assume a banking group as illustrated in Figure 1, which has several branches and subsidiaries all around a country or even all over the world. In each branch or subsidiary, the group has experts responsible for the operational risks on different business units such as those included in the Basel matrix. Note that regarding the recent compulsory takeovers we eyewitnessed more and more financial institutions present a similar shape.

[1] Tables 1 and 3 provide examples of the Basel Matrix

[2] Maximum Likelihood Estimation

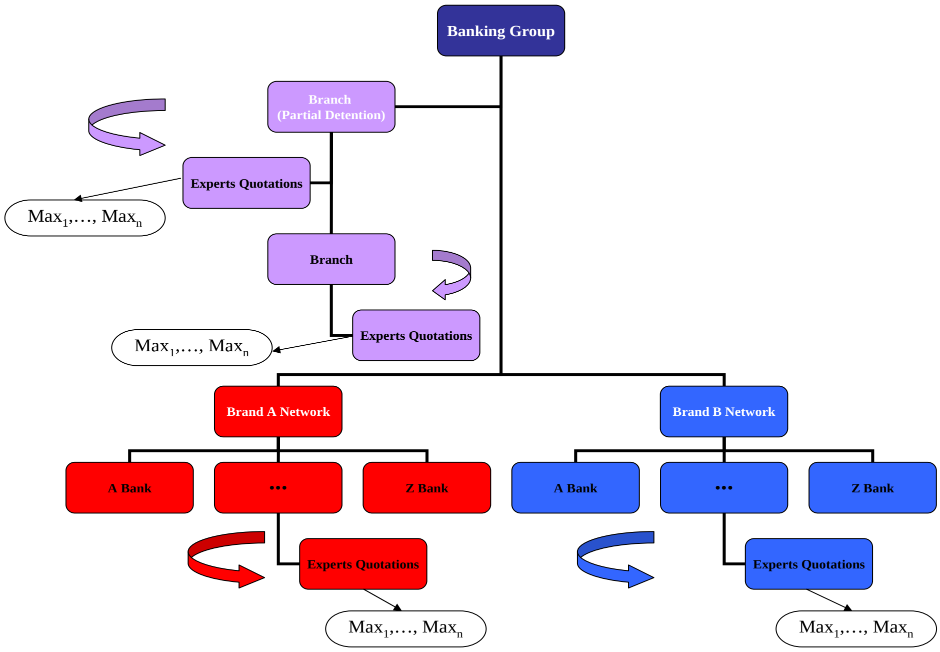

Figure 1: A typical financial group, with the headquarters on the top and its different subsidiaries.

Assuming that we have $i = 1, ..., p$ subsidiaries or branches, each one is represented by a operational risk manager (ORM). This manager can provide $j = 1, ..., n$ quotations per cell in a year (for instance). Thus, for a given date, we can have np quotations for a cell. Denoting q the level of granularity of the cell, if $q = 0$ it means we work on the first level of granularity. If $q \neq 0$[3] then we consider particular sub-event types, saying that we work on the second level of granularity. We denote e the event type and b the business unit in the Basel Matrix. For each cell, focusing on the maximum values, we denote $Max^{(j)}_{i;eb;q}$ an expert value. For example, looking at Table 1 or 3, $Max^{(1)}_{1;25;1}$ denotes the first quotation given by the ORM of the *Caisse d'Epargne Ile-de-France* (a group entity) on the cell ("Payment and Settlement";"External Fraud/Theft and Fraud") and $Max^{(3)}_{3;43;0}$ the third quotation given by the ORM of the *Caisse d'Epargne Rhône Alpes* on the cell ("Commercial Banking";"Clients, Products & Business Practices").

[3] In our example, $q = 1, ..., 6$

Then, these np quotations per cell provide a data set which corresponds to a sequence we refer as a Maxima Data Set (MDS). In the following, we analyze such data sets obtained from branches of BPCE to evaluate capital requirements corresponding to these large losses.

2.2 Methodology

Based on these MDS, we use the extreme value theory and mainly the Fisher-Tippet theorem (Fisher and Tippett (1928), Gnedenko (1943), Appendix A.1) which states that under regular conditions the distribution of a sequence of maxima converges asymptotically to a GEV distribution H_ξ whose density is equal to,

$$h(x; u, \beta, \xi) = \frac{1}{\beta}\left[1 + \xi\left(\frac{x-u}{\beta}\right)\right]^{\left(\frac{1}{\xi}\right)-1} e^{-\left[1+\xi\left(\frac{x-u}{\beta}\right)^{\frac{-1}{\xi}}\right]}, \tag{2.1}$$

for $1 + \xi\frac{x-u}{\beta} > 0$, where $u \in \mathcal{R}$ is the location parameter, $\beta \in \mathcal{R}^{+*}$ is the scale parameter and $\xi \in \mathcal{R}$ is the shape parameter. This distribution contains the Fréchet ($\xi > 0$), the Gumbel ($\xi = 0$) and the Weibull ($\xi < 0$) distributions.

Remark 2.1. *It is interesting to note that if $\xi > 1$ in (2.1), then the distribution has no first moment. This property is fundamental in the applications, because in this latter case we cannot use the GEV distribution otherwise the capital charges would be infinite. Therefore, we have to pay attention to the value of shape parameter (ξ).*

In order to obtain capital charges for the banks or the insurance companies pertaining to operational risks, we directly use this distribution to compute the corresponding capital charges through the two following risk measures:

Definition 2.1. *Given a confidence level $\alpha \in [0, 1]$, the Value-at-Risk (VaR) associated to a random variable X is given by the smallest number x such that the probability that X exceeds x is not larger than $(1 - \alpha)$*

$$VaR_{(1-\alpha)\%} = \inf(x \in \mathbb{R} : P(X > x) \leq (1 - \alpha)), \tag{2.2}$$

and,

Definition 2.2. *Let η be the $VaR_{(1-\alpha)\%}$, and X a random variable which represents losses during a prespecified period (such as a day, a week, or some other chosen time period) then, the Expected Shortfall (ES) is equal to:*

$$ES_{(1-\alpha)\%} = E(X|X > \eta) \tag{2.3}$$

Using the previous definitions in our example, the random variable X will follow the GEV distribution adjusted on the MDS built with experts opinions. As soon as these information sets are known for each cell and assuming that the data sets can be characterized by the distribution (2.1), the parameters of this distribution will be estimated by MLE.

3 In the reality...

A company such as BPCE is a compound of numerous entities: 17 Caisse d'Epargne, 20 Banques Populaires, Natixis plus all its own subsidiaries, the Credit Foncier de France etc. Thus this group has almost 250 operational risk managers[4]. Therefore, we build the Basel Matrix made up of 56 cases - 8 business lines ("b") $\times$ 7 event types ("e")[5] in the first level of granularity, and 152 in the second level of granularity using the information provided by these experts. We observe almost 200 quotations per cell every year, but this number can attain 3000.

In Tables 2 and 4 we provide the values of the estimated parameters, using the MDS, for each cell, at the first level of granularity ($q = 0$) in Table 2, and at the second level of granularity ($q \neq 0$) in Table 4. We do not provide the standard deviations to keep the result readable. Nevertheless all the standard deviations enable validating parameters estimations[6]. The parameter of interest is ξ because it characterizes the shape of the distribution. We observe that its value decreases as the level of granularity increases. This remark is important because when $\xi > 1$ at the first level of granularity, its value can be less than 1 as soon as $q \neq 0$.This is fundamental to interpret our results because in that latter case, it has a sense using the GEV to compute the capital

[4]To draw a parallel, the Société Générale has several thousands ORM.

[5]The business lines are corporate finance, trading & sales, retail banking, commercial banking, payment and settlement, agency services, asset management and retail brokerage. The event types are internal fraud, external fraud, employment practices & workplace safety, clients, products & business practices, damage to physical assets, business disruption & system failures and execution, delivery & process management.

[6]These values may be provide on request

requirement, as the mean of this distribution is no more infinite. In a recent paper, this fact has already been highlighted in Guégan et al. (2010), Guégan and Hassani (2011) using a GPD. Pointing at this point is important: indeed, mixing different nature of incidents in a same cell (for example, the "system security" breaches and the "theft and fraud" in the "external fraud" event type) may induce distortions in the estimation procedures.

In our example, we observe in the cell "Payment and Settlement"/"Internal Fraud" that the estimated value for ξ is 4.30 when $q = 0$, thus this estimated GEV distribution cannot be kept. Working on the second level of granularity, even if the ξ value decreases, we cannot use this estimated GEV distribution to compute capital charges. Thus, one may try to work on a third level, nevertheless, in this case in our example the data were not available.

In the particular case of the "Retail Banking"/"Clients, Products & Business Practices/Improper Business or Market Practice" cell, disaggregating the data set from $q = 0$ to $q \neq 0$, the value of ξ increases from $\xi = 0.02657934$ to $\xi = 3.013321$. In our opinion, the explanation of this fact stands in the aggregation of many different risk natures - the definition behind this sub-event covers many kinds of incidents - in a single cell. The fact that for $q = 0$, the estimation for ξ was lower than 1 is explained by the fact that this information set was overwhelm by the other data.

Otherwise, for the cell "Payment and Settlement"/"Execution, Delivery and Process Management", $\xi = 2.08$ for $q = 0$, and $\xi = 0.23$ for $q \neq 0$ only for the disaggregated cell "Payment and Settlement"/"Vendors and Suppliers". Note that some cells are empty, because BPCE's top risk managers dealt with these risks differently and did not ask quotations to the ORM. We also note that the shape parameter ξ is positive in every cell of Tables 1 and 3, thus the quotations' distributions follow Fréchet distributions (Figure 2) (Gnedenko (1943)) given in (2.1).

With this approach (the use of expert opinions), we are able to anticipate the losses and the corresponding capital requirement, and by the way we have confirmed the influence of the Basel matrix construction. To convince risk managers of the interest of this last methodology which is not based on a collected incidents investigated with the classical loss distribution approach (LDA) (Lundberg (1903), Frachot et al. (2001) and Guégan and Hassani (2009)), we have compared the capital amounts obtained using the experts opinions with the ones obtained from the collected losses. These results are provided in Table 5. We observe that even focusing on extreme losses, this methodology does not always provides a superior capital than the LDA. Therefore, we think risk managers should be aware that using the EVT does not always mean that we have extreme capital charges.

On the other hand, comparing both approaches (Experts Vs LDA), even if the amounts may vary, the ranking of these ones with respect to the class of incidents is globally maintained. Regarding the volatility between the results obtained from the two methods (Tables 5), we can mention that the experts tend to provide quotations embedding the entire information available at the moment they are filling the forms, whereas using historical information sets, due to the impact of the data processing we observe a long delay between the moment an incident have been detected and the moment it has been entered in the collecting device. Another reason explaining the differences between the two procedures can also be interpreted by the fact that the experts anticipate the losses max values with respect to the internal policy of risk management, such as the efficiency of the operational risk control system, the quality of the communication from the top management or the lack of hindsight regarding a particular risk. For example a result such as the one provided on the first line of Table 5, corresponding to the capital charges estimated on the "Retail Banking" business line for the "Internal Fraud" event type, we obtain 7 203 175 € using experts opinions against 190 193 051 € with the LDA. The difference between these two amounts may be interpreted as a failure of the operational risk control system to prevent these frauds[7] We definitively highlighted the importance to consider an *a priori* knowledge of the experts associated to *a posteriori* backtesting based on collected incidents.

4 Conclusion

In this paper, we have developed a new methodology based on experts opinions and extreme value theory to evaluate operational risks capital charges. This method does not suffer from numerical methods and provide an analytic capital charge.

[7]Theoretically, the two approaches (Experts Vs LDA) are different, therefore this way of thinking may be easily challenged, nevertheless it might lead practitioners to question their system of control.

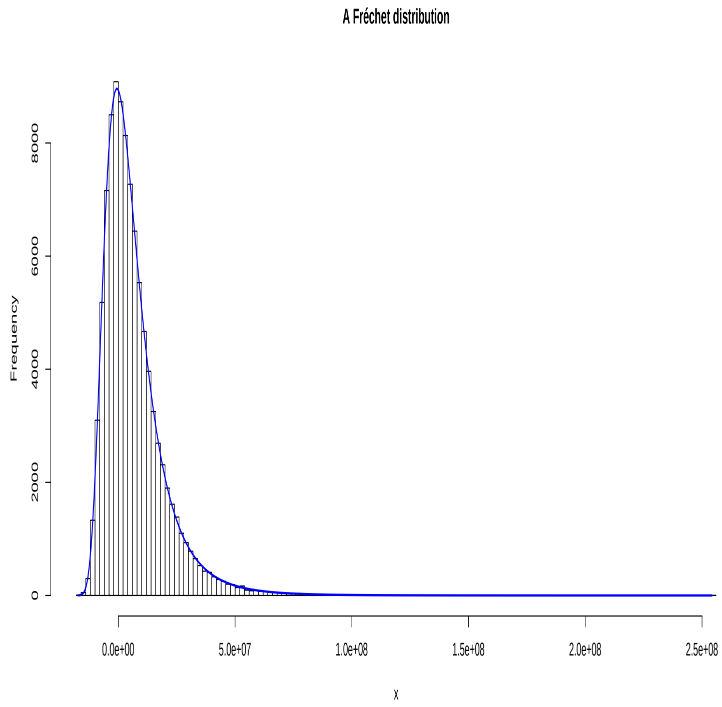

Figure 2: The Fréchet distribution.

With this method, we transformed practitioners judgments into computational values and final capital allocations. We also illustrated the fact that the data are note contaminated. Their potential unexploitability ($\xi > 1$) is just caused by the fact that we mix risks natures for example "Theft and Fraud" and "System Security" in a same event, here the "External Fraud" one.

Nevertheless, the reliability of the results depends on the risk management quality and particularly on the aptitude for ORM to work together.

EVENT TYPES	INVESTMENT BANKING			BANKING									OTHERS		
	Trading & Sales (2)			Retail Banking (3)			Commercial Banking (4)			Payment and Settlement (5)			Retail Brokerage (8)		
	u	β	ξ	u	β	ξ	u	β	ξ	u	β	ξ	u	β	ξ
Internal Fraud (1)	-542681.1	7530499.2	0.1406130	126706.2	446906.3	0.2142564	306675.7	1089039.7	0.1771072	166392.2	714520.8	4.3033391			
External Fraud (2)				-58764.18	364626	0.06920223	-301871.62	1628811	0.05140783						
Employment Practices and Workplace Safety (3)															
Clients, Products & Business Practices (4)				-907365.494	2309660.7	0.02657934	-2456868.694	12048503.7	0.06902022	-9005.202	132807.1	0.14745615	-252553.964	1125568.3	0.12959489
Damage to Physical Assets (5)															
Business Disruption and System Failures (6)															
Execution, Delivery and Process Management (7)	299643.163	1283535.63	4.30529387	-494098.678	1481653.81	0.04103866	-5456628.940	14556609.34	0.04533764	37237.017	81221.14	2.08854875	-9833.143	83283.79	0.09838064

Table 1: Estimated parameters of the GEV distribution (2.1) using the MDS when $q = 0$ (first degree of granularity) for each cell of the Basel matrix for which we had enough data. The estimations are obtained using MLE. Regarding the standard deviation (s.d.), the results are workable. The s.d. can be provided on request

EVENT TYPES	INVESTMENT BANKING		BANKING						OTHERS	
	Trading & Sales (2)		Retail Banking (3)		Commercial Banking (4)		Payment and Settlement (5)		Retail Brokerage (8)	
	VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES
Internal Fraud (1)	87 353 770	110 466 700	7 203 175	9 827 801	15 054 950	19 685 000	1.466686e+18	1.85577e+27		
External Fraud (2)			3 170 292	3 805 896	13 205 207	15 754 417				
Employment Practices and Workplace Safety (3)										
Clients, Products & Business Practices (4)			16 604 385	19 492 837	104 169 954	124 708 703	1 584 330	2 011 734	12 321 049	15 453 953
Damage to Physical Assets (5)										
Business Disruption and System Failures (6)										
Execution, Delivery and Process Management (7)	2.208458e+18	7.944526e+25	11 337 920	13 394 360	112 613 600	133 277 400	7.603561e+10	1.862236e+14	813 831	986 422

Table 2: Capital charges (in €) using the VaR and the ES measures at a 99.9% confidence level, computed with the parameters estimates for the corresponding GEV distribution provided in Table 1.

EVENT TYPES LEVEL 1	EVENT TYPES LEVEL 2	Trading & Sales (2)			Retail Banking (3)			Commercial Banking (4)			Payment and Settlement (5)			Retail Brokerage (8)		
		u	β	ξ	u	β	ξ	u	β	ξ	u	β	ξ	u	β	ξ
Internal Fraud (1)	Unauthorized Activity (1)	498407.4	8289573	0.1456669	53636.38	297713.4	0.1985132	417293.2	924079.7	0.2158522						
	Theft and Fraud (2)	-1563951	6621163	0.1119371	184711.5	499772.5	0.2214736	74479.75	1429723	0.157062	331191.4	772850	2.342449			
External Fraud (2)	Theft and Fraud (1)															
	Systems Security (2)															
Employment Practices and Workplace Safety (3)	Employee Relations (1)															
	Diversity (2)															
Clients, Products & Business Practices (4)	Suitability, Disclosure & Fiduciary (1)				-1454972	3712415	0.04175584				3435.737	48382.38	0.2099116			
	Improper Business or Market Practices (2)				271224.2	823168.5	3.013321									
	Product Flaws (3)															
	Selection, Sponsorship & Exposure (4)				-45251.44	291165.7	0.06523997	-207718.3	3101592	0.08644248						
	Advisory Activities (5)				-283041.3	1283606	0.08904748	2182888	25090131	0.1698491				-252554	1125568	0.1295949
Damage to Physical Assets (5)	Disasters and other events (1)															
Business Disruption and System Failures (6)	Systems (1)															
Execution, Delivery & Process Management (7)	Transaction Capture, Execution & Maintenance (1)				-11401.59	299892.9	0.08265402	-5539912	15032658	0.04719648				-11676.5	106997.2	0.1270485
	Monitoring and Reporting (2)				297216.9	5600027	0.2087871									
	Customer Intake and Documentation (3)				-657699.3	2971715	0.1158047							5213.254	29558.82	0.2099193
	Customer / Client Account Management (4)															
	Trade Counterparties (5)	224175.4	770588	3.451007												
	Vendors & Suppliers (6)				23749	47260.4	1.927688				63650	69280	0.2314611			

Table 3: Estimated parameters of the GEV distribution (2.1) using the MDS when $q \neq 0$ (second degree of granularity) corresponding to each cell of the Basel matrix for which we had enough data. The estimations are obtained using MLE. Regarding the standard deviation (s.d.), the results are workable. The s.d. can be provided on request. The "External Fraud" line in Table 1 is only composed of the "Theft and Fraud" sub-event in our data base therefore we would have identical results in the current Table.

EVENT TYPES LEVEL 1	EVENT TYPES LEVEL 2	Trading & Sales (2)		Retail Banking (3)		Commercial Banking (4)		Payment and Settlement (5)		Retail Brokerage (8)	
		VaR	ES	VaR	ES	VaR	ES	VaR	ES	VaR	ES
Internal Fraud (1)	Unauthorized Activity (1)	99 237 528	125 236 784	4 462 805	5 971 142	15 149 745	20 299 741				
	Theft and Fraud (2)	67 443 730	83 607 260	8 347 063	11 246 330	17 907 540	22 934 990	3.509638e+12	1.783117e+17		
External Fraud (2)	Theft and Fraud (1)										
	Systems Security (2)										
Employment Practices and Workplace Safety (3)	Employee Relations (1)										
	Diversity (2)										
Clients, Products & Business Practices (4)	Suitability, Disclosure & Fiduciary (1)			28 268 221	33 506 920			755 463	1 001 857		
	Improper Business or Market Practices (2)			2.990547e+14	6.326994e+19						
	Product Flaws (3)										
	Selection, Sponsorship & Exposure (4)			2 495 528	3 012 018	29 099 639	35 094 170				
	Advisory Activities (5)			11 966 597	14 622 995	331 937 137	428 247 737			12 321 049	15 575 823
Damage to Physical Assets (5)	Disasters and other events (1)										
Business Disruption and System Failures (6)	Systems (1)										
Execution, Delivery & Process Management (7)	Transaction Capture, Execution & Maintenance (1)			2 781 952	3 397 026	117 218 782	138 237 729			1 171 594	1 456 133
	Monitoring and Reporting (2)			86 924 946	118 128 598						
	Customer Intake and Documentation (3)			30 785 309	37 833 492					464 673	615 942
	Customer / Client Account Management (4)										
	Trade Counterparties (5)	5.025132e+15	3.232162e+23								
	Vendors & Suppliers (6)			1.486297e+10	9.626941e+12			1 245 020	1 678 603		

Table 4: Capital charges (in €) using the VaR and the ES measures at a 99.9% confidence level, computed with the parameters estimates of the GEV distribution provided in Table 3.

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A Fisher-Tippett theorem

We denote X a random variable (r.v.) with a cumulative distribution function (c.d.f.) F. Let $X_1, ..., X_n$ be a sequence of independent and identically distributed (i.i.d.) r.v., and let $M_n = max(X_1, ..., X_n)$. Then, the Fisher and Tippett (1928) theorem says:

Theorem A.1. *If there exists constants $c_n > 0$ and $d_n \in \mathcal{R}$, then*

$$\mathbb{P}\setminus\left(\frac{M_n - d_n}{c_n} \leq x\right) = F^n(c_n x + d_n) \xrightarrow{d} H_\xi \tag{A.1}$$

for some non-degenerate distribution H_ξ. Then H_ξ belongs to the generalized extreme value distribution given in (2.1).

Business Lines	Event Type	VaR Experts	VaR LDA	ES Experts	ES LDA
Retail Banking	Internal Fraud	7 203 175	190 193 051	9 827 801	367 340 363
Commercial Banking	Clients, Products & Business Practices	104 169 954	5 683 804	124 708 703	10 189 398
Retail Brokerage	Clients, Products & Business Practices	12 321 049	8 161 387	15 453 953	11 717 631
Retail Brokerage	Execution, Delivery and Process Management	813 831	113 234	986 422	170 038

Table 5: Capital charges with VaR & ES measures computed for specific cells of the Basel matrix using experts opinions combined with the EVT approach (column 3 and 5) and historical data associated with the LDA (column 4 and 6).