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Abstract

A shared belief in the financial industry is that markets are driven by two types of regimes. Bull markets would be characterized by high returns and low volatility whereas bear markets would display low returns coupled with high volatility. Modelling the dynamics of different asset classes (stocks, bonds, commodities and currencies) with a Markov-Switching model and using a density-based test, we reject the hypothesis that two regimes are enough to capture asset returns' evolutions. Once the accuracy of our test methodology has been assessed through Monte Carlo experiments, our empirical results point out that between three and five regimes are required to capture the features of each asset's distribution. A probit multinomial regression highlights that only a part of the underlying number of regimes is partially explained by the absolute average yearly risk premium and by distributional charateristics of the returns such as the kurtosis.

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1 Introduction

Financial asset prices fluctuate in a tick-by-tick fashion to a globalized news flow. The nature and frequency of this flow has a tremendous impact on the dynamics of financial markets, as reflected by the evolutions of the major indices usually used by financial data providers and newspaper to summarize the flavor of the financial week. In this summarizing process, two kinds of episodes are usually identified and used as labels: a growing valuation of risky assets in a low volatility environment is referred to as "bull market". On the contrary, "bear market" is the wording used to describe a period during which government bonds are used as safe havens whereas risky assets deliver strongly negative returns and volatility goes through a sharp increase. Useful though these approximations may be, this article questions the existence of these two regimes in financial markets. Given the complexity and deepness of the information reflected in asset prices, there are great chances that there are more than just bulls and bears in financial markets.

Simplifications are commonplace when facing complex mechanism: this bull/bear distinction nourished a large financial literature - part of which is academic. A selection of these academic articles turned to Hamilton (1989)'s Markov Switching model as with such a model, bull and bear days can be measured through probabilities. This combination led to strong improvements in our understanding of the behavior of financial markets: see for example what is presented in Chauvet and Potter (2000) or Ang and Bekaert (2002a,2002b). Beyond the insight regarding the times series dynamics of returns, this approach still suffers from one drawback: the number of market regimes is assumed before the estimation and this hypothesis was hardly checked in the literature. However and more recently Maheu et al. (2010) describe how expanding the assumed number of regimes - adding bullish bear and bearish bull regimes to the sole bull/bear usual assumption - is essential to portfolio managers.

This article intends to fill a gap: we present a test to determine the number of regimes implicit in returns based on a conditional density argument when the underlying model is a standard Markov Switching model with n regimes. This test is inspired by the likelihood ratio tests for possibly non nested models presented in Vuong (1989) and Amisano and Giacomini (2007). A Monte Carlo test shows that our approach allows us to consistently estimate the number of regimes. We turn our attention to a large dataset of weekly returns on several indices: we show that only two foreign exchange rates - namely the Swiss Franc and the Yen versus Dollar - can be modeled by a two-regime MS model. For most of the assets covered here, the number of regimes is equal to three. This additional regime is of various natures and asset dependent. For selected cases, the number of regimes can be equal up to 6, as it is the case for the European High Yield index. We discuss the persistence and performances under each regime, underlining that the bull/bear specification may be oversimplifying a complex reality. Finally, we show that the number of states estimated has only a weak link with the distributional properties of returns.

This article is structured as follows: Section 2 presents the underlying modeling methodology, jointly with the Monte Carlo evaluation of the test used to estimate the number of regimes. Section 3 discusses the elements involved in our dataset. Section 4 presents a detailed discussion of the results. Section 5 concludes.

2 Testing for the number of regimes implied by financial returns' dynamics

This section is devoted to the presentation of the material needed for the test used in this article. We first briefly review the basics of Hamilton (1989)'s switching model before turning to the presentation of specification test inspired by Vuong (1989) and Amisano and Giacomini (2007).

2.1 A brief presentation of the Markov Switching model

We provide the reader with a short presentation of Hamilton (1989)'s markov switching model. This model has initially been introduced in the literature by focusing on the US business cycle. Its use to estimate the regimes in financial markets has been since developped in various articles such as Chauvet and Potter (2000), Ang and Bekaert (2002a,2002b) or more recently Maheu et a. (2010). This time series model aims at modelling and estimating the changes in regimes that affect economic and market series. It relies on the assumption that the probability to move from one state to the other is time varying, while the transition probabilities are constant.

We present the basic intuitions using a two-regimes MS model before turning to a general case. Let r_t be the logarithmic return on a given asset at time t, for the holding period between $t-1$ and t. Let s_t be an integer value variable that is equal to 1 (respectively 2) at time t if regime 1 (respectively 2) prevails in the economy. Given that the regime i prevails, the conditional distribution of returns is as follows:

$$r_t \sim N(\mu_i, \sigma_i). \tag{1}$$

The probability to be in regime 1 at time t writes:

$$P(s_t = 1) = P(s_t = 1|s_{t-1} = 1) \times P(s_{t-1} = 1) + P(s_t = 1|s_{t-1} = 2) \times P(s_{t-1} = 2). \tag{2}$$

$P(s_t = 1|s_{t-1} = 1)$ is assumed to be constant and equal to p, and $P(s_t = 2|s_{t-1} = 1) = 1-p$. With a similar argument, $P(s_t = 2|s_{t-1} = 2) = q$ and $P(s_t = 1|s_{t-1} = 2) = 1-q$. These transition probabilities can be gathered into a transition matrix as follows:

$$\Pi = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}, \tag{3}$$

such that

$$\mathbf{P}_t = \Pi \mathbf{P}_{t-1}, \tag{4}$$

with $\mathbf{P}_t = (P(s_t = 1), P(s_t = 2))^{>}$. The parameters driving the model are thus the moment associated to asset returns for each state and the matrix Π. The usual estimation strategy is a maximum likelihood one, based on the filtering approach developed in Hamilton (1989).

This two-regime case can be generalised to a n-regime one: in such a case, s_t can take integer values ranging from 1 to n, and the Π matrix become a $n \times n$ matrix.

2.2 Testing for the number of regimes in a MS model

As presented in the introduction, little attention has been devoted in the literature to testing the optimal number of regimes that are actually driving financial returns.

The approach that we propose here is inspired by the LR tests presented in Vuong (1989) and Amisano and Giacomini (2007) and aims at comparing two models in terms of goodness of fit of the returns's distribution.

Let $f_{n_1}(r_t;\hat{\theta}_{n_1})$ be the likelihood function assciated to an estimated Markov-Switching model with n_1 states. Let $f_{n_2}(r_t;\hat{\theta}_{n_2})$ be a similar quantity in the case of a MS model with n_2 regimes. θ_{n_i} is the vector of the parameters to be estimated by maximum likelihood in the n_i-regime case. the two specifications are compared through their associated log density computed with the estimated sample. Let $z_t^{n_1,n_2}$ be the following quantity:

$$z_t^{n_1,n_2} = \log f_{n_1}(r_t;\hat{\theta}_{n_1}) - \log f_{n_2}(r_t;\hat{\theta}_{n_2}). \tag{5}$$

The approach proposed here is based on the following test statistics:

$$t_{n_1,n_2} = \frac{\frac{1}{T}\sum_{t=1}^{T} z_t^{n_1,n_2}}{\hat{\sigma}_{n_1,n_2}\sqrt{T}}, \tag{6}$$

where T is the total number of available observations in the sample used to estimate the parameters, and $\hat{\sigma}_{n_1,n_2}$ a properly selected estimator of the standard deviation of $\frac{1}{T}\sum_{t=1}^{T} z_t^{n_1,n_2}$. We propose to estimate this standard deviation using a Newey West (1987) estimator.

Under the null hypothesis that both models provide and equivalent fit of the returns' distribution, Amisano and Giacomini (2007)'s Theorem 1 provides the asymptotic distribution of this test statistics:

$$t_{n_1,n_2} \sim N(0,1). \tag{7}$$

This test statistics allow us to compare non nested and nested models as well. We assume that the conditions stated in the theorem 1 of Amisano and Giacomini are granted in our case.

The main issue with this kind of test is that when comparing the in-sample fit of the distribution, the bigger the number of parameters and the better the fit of the distribution obtained. Hence, by dwelling our analysis on a in-sample test of MS models with a number of regimes ranging between 1 and 10, we are very likely to decide that the model with 10 regimes should be retained for it provides us with the best fit possible. To circumvent this problem, we propose to retain the number of regimes that delivers the best fit from the previous statistics point of view, while being as parsimonious as possible. Hence, our approach combines the previous test inspired by Vuong (1987) and Amisano and Giacomini (2007) to the following number of regime selection procedure:

Step 1: Start with a number of regime equal to 1. Estimate the parameters.

Step 2: Estimate the parameters of a two-regime model. Compare the model with two regimes to the model with one regime: if $t_{2,1} > 1.96$, go to Step 3. If not, select the model with 1 regime.

Step 3: Estimate the parameters of a three-regime model. Compare the model with three regimes to the model with two regimes: if $t_{3,2} > 1.96$, go to Step 4. If not, select the model with 2 regimes.

Step 4: ... Carry on this procedure until $t_{i+1,i} < 1.96$.

With this selection procedure, we select the most parsimonious specification for the number of regimes that provides a statistical superior fit to any specification using a lower number of regimes while being equivalent to a model with one more regime. By doing so, we are due to select the number of regimes that does not overfit the returns' distribution, while providing us with an approximate measure of the structural number of regimes driving the returns.

2.3 A Monte Carlo investigation of our test methodology

Before applying the previous methodology to a real dataset of financial returns, we ran two different Monte Carlo tests. These tests aim at gauging the ability of the test to estimate the number of underlying regimes when this number is known and the model is a Markov Swithching model.

We use the two different specifications: a MS model with three regimes and another one with five regimes, as they are two of the common cases found in the empirical results presented in the next Section. These models present a special interest for our work, as they differ from the usual bull-bear dyptic: the test would behave very badly if it was to diagnose two underlying regimes when there are indeed more.

The parameters used in this Monte Carlo exercise are obtained by estimating a MS(3) using the Eurostoxx index's returns, and the parameters for the MS(5) are obtained from the US High Yield index's returns. The parameters are the following:

- The 3 regime model is characterized by two types of bear episodes and a single type of bull regime. The moderate and very volatile bear regime is the dominant one as 47% of the trading weeks for the Eurostoxx deliver a negative return to the investor. What is more, around 63% of the dataset is made of weekly returns between -2.5% and 2.5% which is a rather average low weekly return for an equity index.

$$\mu_1 = -0.63\%, \sigma_1 = 5.69\%, \tag{8}$$

$$\mu_2 = 1.23\%, \sigma_2 = 1.74\%, \tag{9}$$

$$\mu_3 = -2.06\%, \sigma_3 = 1.82\%, \tag{10}$$

$$P = \begin{pmatrix} 0.93 & 0.07 & 0.00 \\ 0.00 & 0.62 & 0.38 \\ 0.06 & 0.70 & 0.25 \end{pmatrix} \tag{11}$$

- The 5 regime model is characterized by four types of bear regimes, involving different various scales of expected returns and their associated volatility. The most persistent regime is regime 1: for this bullish regime, investors holding high yield bonds benefit from a 0.09% (=4.82%/52) weekly return on average. Amongst of the bearish regimes, one should differenciate strongly negative expected returns with medium volatility from moderatly negative returns coming alongside a large volatility. This latter case corresponds to a large uncertainty mixed a progressive widening of risk premia in the high yield market. On a yearly basis, we get:

$$\mu_1 = 4.82\%, \sigma_1 = 2.30\%, \tag{12}$$
$$\mu_2 = -10.78\%, \sigma_2 = 25.04\%, \tag{13}$$
$$\mu_3 = 50.55\%, \sigma_3 = 5.80\%, \tag{14}$$
$$\mu_4 = -28.58\%, \sigma_4 = 5.82\%, \tag{15}$$
$$\mu_5 = 21.62\%, \sigma_5 = 2.67\%, \tag{16}$$
$$P = \begin{pmatrix} 0.96 & 0.00 & 0.00 & 0.04 & 0.00 \\ 0.00 & 0.94 & 0.00 & 0.06 & 0.00 \\ 0.06 & 0.00 & 0.63 & 0.00 & 0.32 \\ 0.00 & 0.02 & 0.14 & 0.80 & 0.04 \\ 0.02 & 0.00 & 0.06 & 0.12 & 0.80 \end{pmatrix} \tag{17}$$

We believe these two specifications provide two stylized and realistic settings with which we should try to assess the quality of the test presented above.

For each of these specifications we sample 650 weeks of returns, as the samples used in the empirical section are of that length. That is 12 years and a half of returns, which involves in our view enough market episodes to provide us with a rather structural view about the regimes affecting asset returns. For each of the simulated samples, we reestimate the MS(i) parameters, for i ranging from 1 to 10. Once the parameters are estimated, we run the test to select the optimal number of regime.

The Table 2 provides descriptive statistics regarding the optimal number of regimes. Given the discrete nature of the support of the number of regimes, we are mostly interested in the median of the selected stats, that is the state that has the highest probability to be selected. Both in the 3- and the 5-regime cases, the median matches the real number of regimes. This is essential to our article. We nonetheless discuss the average number of state: when in the 5-regime case the sample average is also 5, in the 3-regime one, we get a slightly different story. In this case, the average is equal to 3.5, underlying the fact that one average the test seems to over-estimate the number of regimes. The skewness presented in this table corroborate this fact. The frequencies of for each state are presented in Figure 1. The figure presents the frequency for which the state i has been selected out of the 10 000 replications. The test does not seem to have a perfect precision, given the non-zero frequencies at which it selects a number of state different from the true one. This will be something to be kept in mind when investigating the real dataset results. Finally, we find that when the true model is either a MS(3) or a MS(5), the frequency at which a model with 2 regimes is selected is fairly close to zero. Again, this point will be essential in the forthcoming section.

3 Dataset

The dataset used here jointly with the statistical test presented earlier is made of a wide scope of financial assets. The dataset encompasses four types of assets:

- Equities: we consider different type of equity indices covering different regions of the world. First, we consider both large and small capitalization indices for the developped world stock market. In the US case, the large caps index is chosen to be the SP500 and the small caps one is the Russel 3000. Similar roles are played in the EMU case with the Eurostoxx 50 and the MSCI small caps EMU. In the emerging case, we focus our attention on broad indices that are segregated depending on geographic arguments: we use the MSCI EM Asia, Latam and Europe.

– Bonds: we cover three types of bond indices in the developped market case. The first one is Government bond indices both in the US and in the EMU case. The second and thirs ones are investment grade and high yield bond indices for both US and EMU. These indices are Bank of America Merrill Lynch indices.

– Foreign Exchange rates: a important part of the financial transactions around the world come from the FX investment universe. We retained four key exchange rates for their ample liquidity and well known economic interest: the Euro, the Yen, the Swiss Franc and the British Pound against Dollar.

– Commodities: we use two additional series of returns from the commodity universe. We focus on the NYMEX crude oil index and on the Dow Jones broad commodity index. The second index is a diversified index representing the whole commoditiy markets, as it includes both hard and soft commodities.

The dataset starts on the 04/03/1998 and ends on the 12/31/2010. This period is selelected for it stands a good chance to be as stationary as possible : this period is at the end of the disinflationnary period that starts in 1979 with the Volcker era – which is essential in terms of bonds behavior – and it covers the moment when China joined the World Trade Organization – which is due to have a tremendous impact in terms of emerging world related assets.

The data frequency is weekly: as we are interested in focusing on regimes, we need to find in returns enough persistence to obtain reliable estimates of the MS parameters. The weekly frequency offers a balanced mix between non Gaussian returns and a stronger persistence than daily data. Descriptive statistics are presented in Table 1 that underline the non-Gaussian behavior of these returns.

4 Empirical Results

4.1 Main Results

The main results of our methodology are presented in Figure (2). We show that contrary to a shared belief, more than two states generally explain asset return dynamics. In addition to that, the results of a multivariate probit regression of the number of regimes implicit in asset returns on the four lowest moments are provided by Table (3). They allow to point out that the number of relevant states does not depend on the statistical properties of the underlying asset.

Nevertheless, some parallels may be drawn between asset classes : bonds are characterized by the same number of states than exchange rates, equities having an additional state and high yield bonds presenting the highest number of states. More precisely, even if a lot studies and practitionners assume that financial markets are characterized by a two-state regime, Figure (2) shows clearly that this assumption may only be considered as valid for two assets. Only the Swiss Franc against US Dollar and the Yen against US Dollar are characterized by a two-state regime. The other exchange rates of our sample, the bonds – high yield bonds excepted – the commodities and one equity index – the MSCI EM Asia – are driven by a three-state regime. The other equity indices are characterized by a four-state regime and at least five states have to be taken into account to catch high yield bonds returns.

4.2 Detailed results

Tables (4) and (5) provide the detailed results relative to the foreign exchange rates between the Yen and Dollar. The market is characterized by a two-state regime : the return in the bull regime is equal to -1.08% whereas the volatility is equal to 10.503%. In the bear regime, the statistics are respectively -90.563% and 38.599%. Following the transition matrix, the bull regime appears to be the more persistent: the probability to be in regime 1 at time t if regime 1 prevails in the economy at time $t-1$ is equal to 0.989. On the contrary, the state 2 is more volatil : the probability to move from regime 2 to regime 1 is closed to 0.43. Accordingly, the foreign exchange rates between the Yen against Dollar is characterized by a strong stability in the regime 1. Some crisis may induce a jump to the other regime but the market switches quickly to its initial status. Figure (3) illustrates this phenomenon.

Tables (6) and (7) provide the results for the US government debt. US debt market is characterized by a three-state regime. States 1 and 3 correspond to the bear market with a negative return whereas the state 2 presents a return equals to 12.880%. For each state, the volatility is quite low, closed to 4%. We have to note that the volatility in the bull regime is higher than the volatility in the bear states. State 1 is the more persistent state with a transition probability equals to 0.989. When a crisis occurs, the market may switches to state 2 but this state appears to be a transitory state, the probability to be in the same state a time $t+1$ being only equal to 0.85. The state 3 is quite curious. It appears unfeasible to stay in this state more than one period : $p_{33} = 0$. State 2 seems to be the more probable step after this state. Historically, Figure (4) shows that state 3 has not occured.

Crude Oil is also characterized by a three-state regime, two states corresponding to the bear market – negative average return – and one bullish regime – average return equals to 37.967% and volatility equals to 20.244%. Curiously, the state two appears to be the more persistent regime, $p_{22} = 0.95$, in spite of of the fact it is the more volatil – the variance of the state 2 return is equal to 0.81%. State 3 is the less persistent regime and if the state 3 prevails at time t in the economy, state 1 will the more probable state which will occur at time $t+1$: $p_{31} = 0.73$. According to Figure (5), state 1 is the more frequent state which prevails in the economy. Even if $p_{11} < p_{22}$, this phenomenon may be explain by the fact that to move from state 1 to state 2, the market has to transit by state 3. On the contrary, if market is in the state 2, it may directly switch to state 1.

In terms of equities, Tables (10) and (11) show that the S&P index is characterized by a four-state regime. Two states correspond to the bullish regime (states 3 and 4) and two states stand for the bear market. The volatility is the highest for the state having the lowest average return. Accordingly with Figure (6) this state – state 1 – corresponds to a pure crisis regime. For example, this state prevailed at the beginning of the 2008 financial crisis. This state is quite persistent $p_{11} = 0.83$. But, the more persistent state is the state 3, with $p_{33} = 0.98$. On the contrary, Table (11) shows that the states 2 and 4 are not persistent : the highest transition probability

of the state 2 corresponds to the probability to move to the state 4, (p_{24} = 0.99). Similarly, p_{42} = 0.56 and p_{44} = 0.43.
But, in addition to that, we have to consider two pairs of regimes, on the one hand the states 1 and 3, on the other hand the states 2 and 4. Indeed, when a state prevails in the economy, for example the state 2 or the state 4, it is very difficult to jump in the other block : p_{21} = 0.000, p_{23} = 0.003, p_{41} = 0.000 and p_{43} = 0.013. On the contrary, p_{24} = 0.992, p_{42} = 0.560 and p_{44} = 0.427. Similar results are got if the state
1 or 3 prevails. This results may be interpreted as follows : in "standard" conditions, returns fluctuates between two regimes. When a crisis occurs, the market switches from one block to the other. Anoter shock will be required to go back to the previous market conditions. This phenomenon is clearly illustrated in the Figure (6) which shows that the states 1 and 3 prevailed in the economy on the one hand between 1998 and 2004 and on the other hand beetween 2008 and 2010. Between 2004 and 2008, the states 2 and 4 drove the market.

4.3 Comparisons

The detailed results for the other assets of our sample are presented from table (12), page (18), to table (19), page (18). Parallels may be drawn between some asset classes. For example, the foreign exchange rate Swiss Franc against Dollar is characterized by a two-state regime, like the Yen. Moreover, characteristics of each state are very similar, both in terms of average return, volatility and transition probability. One state appears to be very persistent, with a probability to stay in the same state closed to 0.99. The other state corresponds to crisis phenomenons and is absolutly not persistent, the probability to switch being above 0.4.

On the contrary, FX Euro and GBP against Dollar present an additional state. That may be explained by the fact that Swiss Franc or Yen are ofter considered as safe haven by investors. Both EUR/USD and GBP/USD markets present two bear states and one bull regime. Volatility are similar. Nevertheless, EUR/USD presents only one very persistent state – state 3, p_{33} = 0.98, p_{11} and p_{22} being closed to 0.5 – whereas GBP/USD is characterized by two persistent states – p_{22} = 0.91 and p_{33} =
0.96.

In terms of bonds, two types of behaviours may be highlighted. First of all, both government and investment bonds are characterized by a three-states model whereas 5 or 6 states have to be considered for high yield bonds. Two bull regimes drive the returns of EU Government Debt and of the Investment Grade Debt US. On the contrary, two bull regimes may be noted for US Government and EU Investment Grade Debt. In each case, the volatility is very low, the variance being minus that 0.1%. Nevertheless, compared to the EU Investment Grade Debt, the US Investment Grade Debt is much more persistent. Chronology may be presented as follows. First of all, state 2 is very persistent : p_{22} = 0.94. But... it seems to be very difficult to reach this state, because p_{12} = 0.013 and p_{32} = 0.000. Thus, market switches from state 1 to state 3 and inversely, p_{13} = 0.23 and p_{31} = 0.73. Only a crisis may induce a jump to the state 2. If similar results are got for the EU Investment Grade Debt, the transition probability are less closed to one or zero and then the regime changes are much more frequent. Figures (7) and (8) illustrate this point.

Equity indices are very homogeneous. Each index is characterized by a four-state regime, like S&P Index, excepted MSCI Asia Index. Moreover, each index presents two or three bear regimes. In addition to that, one very persistent state may be

highlighted. For example, for the Eurostoxx Index, the probability to stay in state two prevails in the economy, p_{22}, is 0.91. We get $p_{33} = 0.93$ for the MSCI Europe, $p_{0.93}$ for the MSCI Latin Amarica and $p_{0.89} = 0.89$ for the MSCI Europe Small Caps. The same figures than for the S&P Index have to be highlighted. Two blocks of bull and bear regimes may be considered, the market sitching from one to the other through some major shocks. For example, Figures (9) and (10) provide the following chronology :

- in 1999, between 2001 and 2002, and from 2003 to 2008, state 1 prevailed in the Eurostoxx market.
- state 2 was mainly driving in the others periods.
- for the Small Caps, states 1 and 2 were mainly prevailing until 2008, the states 3 and 4 in 1998, in January 2002 and from 2008 to 2010.

5 Conclusion

In this paper, we show that contrary to a shared belief, more than two states generally explain asset return dynamics. In addition to that, the number of relevant states does not depend on the statistical properties of the underlying asset. Nevertheless, some parallels may be drawn between asset classes : bonds are characterized by the same number of states than exchange rates, equities having an additional state and high yield bonds presenting the highest number of states.

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A Tables and Figures

Asset	Volatility	Average return	Sharpe ratio	Skewness	Kurtosis
Govt. Debt US	5.99%	-36.41%	-0.844	-0.01	0.05
SP500	19.89%	-0.05%	0.000	-0.76	5.93
Russell 2000	24.83%	2.35%	0.013	-0.64	3.56
Investment Grade US	6.90%	-35.89%	-0.721	-0.24	2.28
High Yield US	9.43%	-35.50%	-0.522	-1.12	14.37
Govt. Debt EMU	4.52%	-33.78%	-1.036	-0.44	2.32
Eurostoxx	24.10%	-1.39%	-0.008	-0.77	5.83
MSCI Small Cap EMU	22.08%	2.68%	0.017	-1.47	7.41
Investment Grade EMU	4.06%	-33.68%	-1.150	-0.66	3.36
High Yield EMU	12.18%	-34.61%	-0.394	-1.52	11.35
Crude Oil	26.77%	7.94%	0.041	-0.70	2.83
DJ Commodity	17.53%	1.93%	0.015	-0.87	3.28
MSCI EM Asia	23.10%	5.96%	0.036	-0.45	2.03
MSCI EM Latam	25.14%	12.26%	0.068	-0.52	4.16
MSCI EM Europe	31.77%	9.62%	0.042	-0.31	7.90
USDCHF	10.61%	-3.21%	-0.042	-0.19	0.26
EURUSD	10.17%	1.45%	0.020	-0.23	0.90
USDJPY	12.26%	-3.41%	-0.039	-1.39	10.32
GBPUSD	9.75%	-0.63%	-0.009	-0.61	4.21

Table 1: **Descriptive statistics of the returns on the assets considerend in the dataset**

The statistics presented in the table are computed using logarithmic returns over the period that starts on the 04/03/1998 and ends on the 12/31/2010. The data frequency is weekly. Both the standard deviation and the average returns are scaled into yearly quantities for the ease of their reading.

	Mean	Median	Std. Dev.	Skewness
3 States	3.50	3	0.619	0.800
5 States	5.09	5	0.997	-0.129

Table 2: **Monte Carlo test results**

This table presents the results of the Monte Carlo experiments presented in Section 2.3. Two different specifications are used, one using 3 different regimes and another using 5 regimes. For each simulation, a sample of size of 650 weeks of trading is sampled. Using each of the 10 000 replications, we estimate the number of regime using the test presented in Section 2.2. The parameters used for these different models are presented in Section 2.3. The table presents the resulting average, median, standard deviation and skewness obtained from the 10 000 replications.

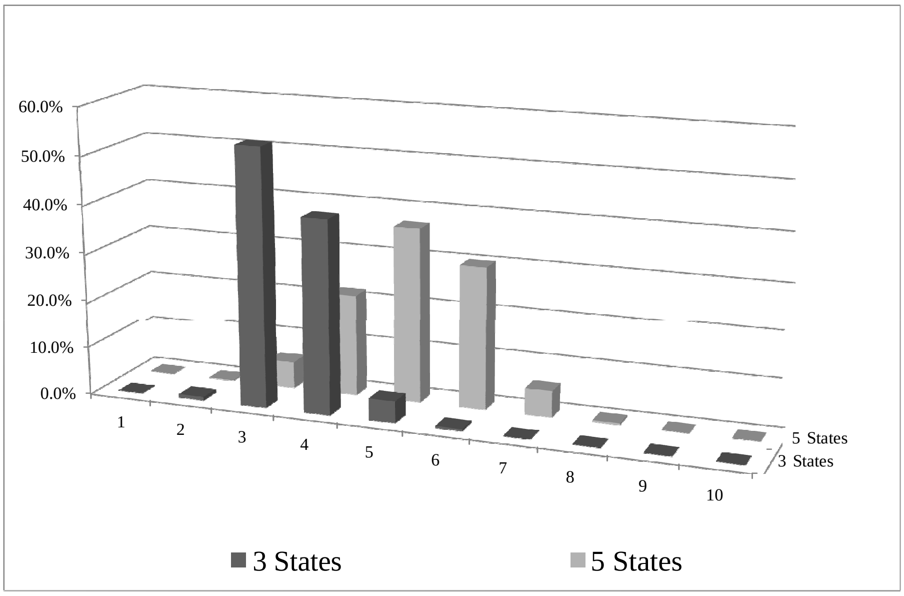

Figure 1: **Frequencies for each possible number of regime obtained with the Monte Carlo simulations**

This figure presents the results of the Monte Carlo experiments presented in Section 2.3. Two different specifications are used, one using 3 different regimes and another using 5 regimes. For each simulation, a sample of size of 650 weeks of trading is sampled. Using each of the 10 000 replications, we estimate the number of regime using the test presented in Section 2.2. The parameters used for these different models are presented in Section 2.3. The figure presents the resulting empirical probabilities of estimating a given number of regimes, given that the underlying model has either 3 or 5 regimes.

PARAMETERS ESTIMATES			
	Estimate	**Std. Dev.**	**t-value**
Expectation	45.5362	26.1348	1.7424
Standard Deviation	-2.3466	11.064	-0.2121
Sharpe ratio	-0.37017	2.6202	-0.1413
Skewness	-4.4045*	2.1936*	-2.0079*
Kurtosis	0.08024	0.2163	0.3709
THRESHOLDS ESTIMATES			
	Estimate	**Std. Dev.**	**t-value**
2 | 3	1.4711	1.9966	0.7368
3 | 4	4.1991	2.2619	1.8565
4 | 5	6.8112	2.6375	2.5824
5 | 6	8.6196	2.8335	3.042

Table 3: **Multivariate probit estimates of the factors explaining the number of regimes in asset returns**

This table presents the results of a multivariate probit regression trying to explain the number of regimes implicit in asset returns. The top panel presents the estimates and their standard deviations whereas the bottom panel presents the estimated thresholds. * indicates a significantly different from zero estimate up to a 5% risk level.

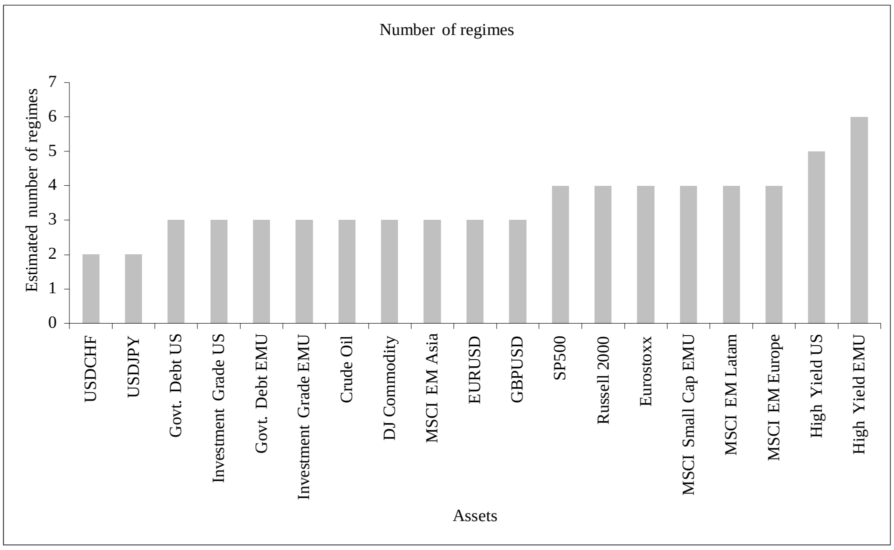

Figure 2: **Estimated number of regimes in asset returns**

This figure presents the estimated number of regimes for each asset.

	Mean	Volatility
State 1	−1.076	10.503
State 2	−90.563	38.599

Table 4: **JPY/USD market characteristics – in %**

	State 1	State 2
State 1	0.98859	0.01141
State 2	0.42627	0.57373

Table 5: **JPY/USD transition matrix**

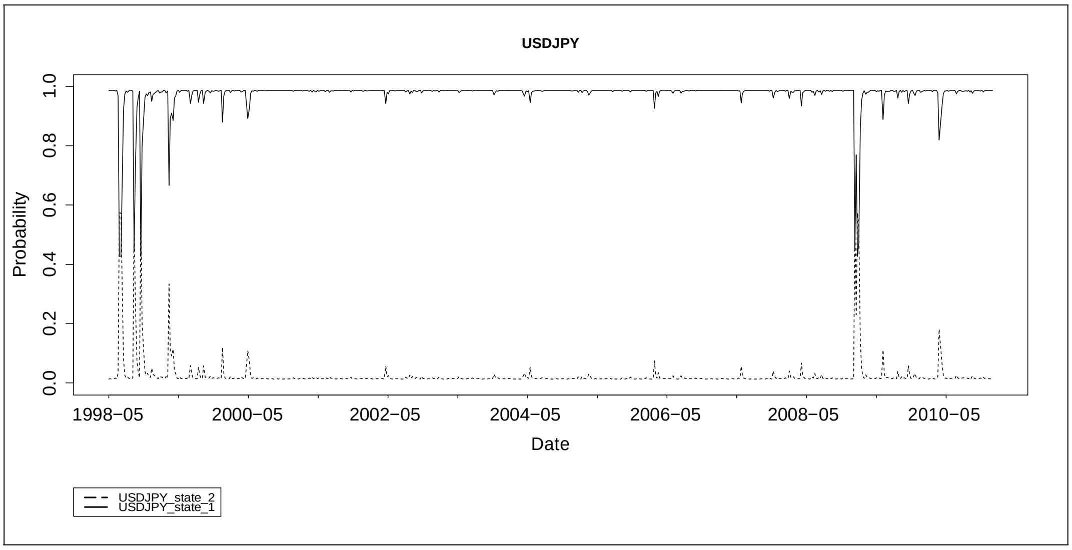

Figure 3: **State probability for JPY/USD exchange rate from 1998 to 2010**

	Mean	Volatility
State 1	−0.796	3.639
State 2	12.880	4.469
State 3	−72.137	3.990

Table 6: **US Government Debt market characteristics – in %**

	State 1	State 2	State 3
State 1	0.98857	0.01143	0.00000
State 2	0.00755	0.85724	0.13521
State 3	0.04630	0.95370	0.00000

Table 7: **US Government Debt transition matrix**

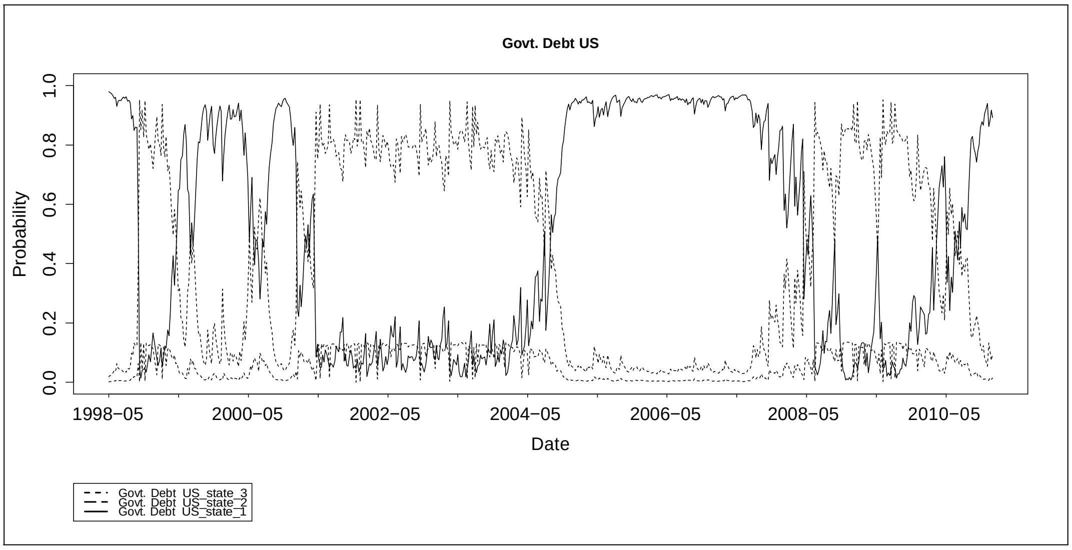

Figure 4: **State probability for US Government Debt from 1998 to 2010**

	Mean	Volatility
State 1	37.967	20.244
State 2	−160.332	64.849
State 3	−229.603	28.453

Table 8: **Crude Oil market characteristics – in %**

	State 1	State 2	State 3
State 1	0.92945	0.00000	0.07055
State 2	0.01446	0.95081	0.03474
State 3	0.73075	0.02252	0.24672

Table 9: **Crude Oil transition matrix**

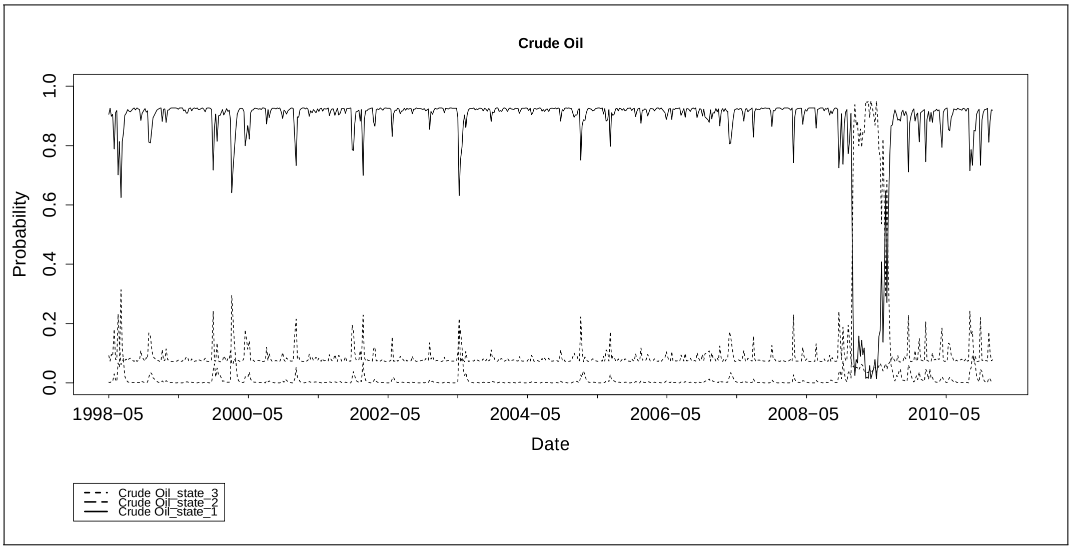

Figure 5: **State probability for Crude Oil from 1998 to 2010**

	Mean	Volatility
State 1	−89.054	48.720
State 2	−31.375	10.888
State 3	1.825	18.782
State 4	35.972	8.240

Table 10: **S&P market characteristics – in %**

	State 1	State 2	State 3	State 4
State 1	0.83453	0.00000	0.16547	0.00000
State 2	0.00000	0.00461	0.00336	0.99203
State 3	0.01657	0.00000	0.98034	0.00309
State 4	0.00000	0.56035	0.01306	0.42659

Table 11: **S&P transition matrix**

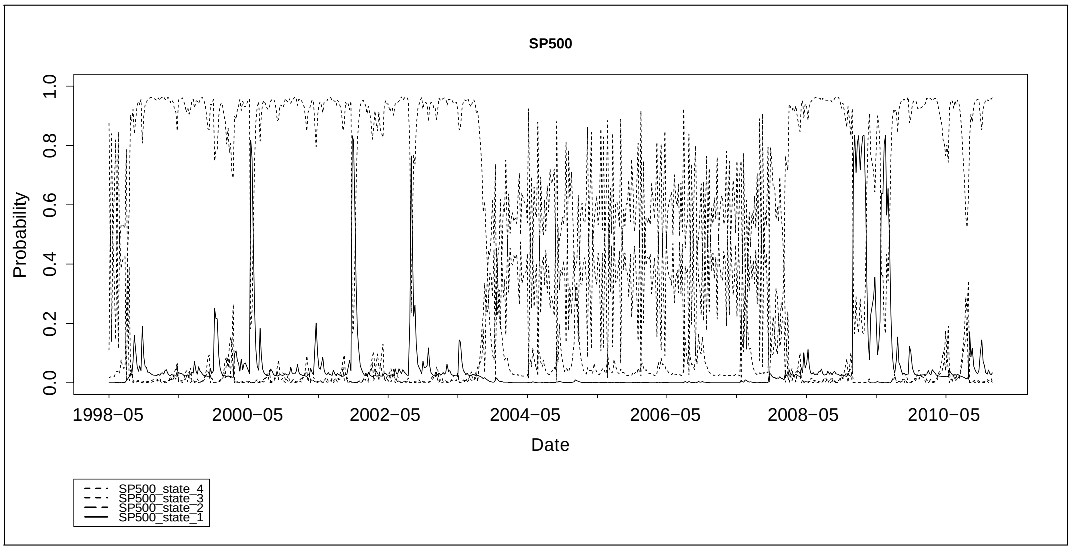

Figure 6: **State probability for S&P from 1998 to 2010**

	Mean	Volatility
State 1	−206.147	11.031
State 2	−1.661	10.305

Table 12: **CHF/USD exchange rates market characteristics – in %**

	State 1	State 2
State 1	0.42218	0.57782
State 2	0.00442	0.99558

Table 13: **CHF/USD transition matrix**

	Mean	Volatility
State 1	47.118	6.019
State 2	−34.735	7.175
State 3	−9.857	13.651

Table 14: **EUR/USD exchange rates market characteristics – in %**

	State 1	State 2	State 3
State 1	0.46658	0.53342	0.00000
State 2	0.49898	0.48578	0.01525
State 3	0.00003	0.01978	0.98019

Table 15: **EUR/USD transition matrix**

	Mean	Volatility
State 1	−73.497	6.340
State 2	8.855	7.418
State 3	−17.968	21.643

Table 16: **GBP/USD exchange rates market characteristics – in %**

	State 1	State 2	State 3
State 1	0.19627	0.77306	0.03067
State 2	0.08816	0.91184	0.00000
State 3	0.00000	0.03844	0.96156

Table 17: **GBP/USD transition matrix**

	Mean	Volatility
State 1	−76.936	33.583
State 2	21.342	13.721
State 3	−55.931	18.720

Table 18: **DJ Commodity market characteristics – in %**

	State 1	State 2	State 3
State 1	0.97413	0.02587	0.00000
State 2	0.00246	0.81826	0.17929
State 3	0.00000	0.86652	0.13348

Table 19: **DJ Commodity transition matrix**

	Mean	Volatility
State 1	−139.382	2.044
State 2	0.387	3.657
State 3	76.610	2.579

Table 20: **EU Government Debt market characteristics – in %**

	State 1	State 2	State 3
State 1	0.0000	0.00000	1.00000
State 2	0.0047	0.99192	0.00337
State 3	0.0000	1.00000	0.00000

Table 21: **EU Government Debt transition matrix**

	Mean	Volatility
State 1	13.132	3.619
State 2	2.569	9.929
State 3	−29.487	3.812

Table 22: **Invesment Grade US Debt market characteristics – in %**

	State 1	State 2	State 3
State 1	0.75783	0.01338	0.22878
State 2	0.06064	0.93935	0.00001
State 3	0.73304	0.00000	0.26696

Table 23: **Invesment Grade US Debt transition matrix**

	Mean	Volatility
State 1	8.558	1.915
State 2	−1.636	4.900
State 3	−11.656	2.096

Table 24: **Invesment Grade Euro Debt market characteristics – in %**

	State 1	State 2	State 3
State 1	0.81570	0.01753	0.16677
State 2	0.02902	0.96520	0.00578
State 3	0.36433	0.00455	0.63112

Table 25: **Invesment Grade Euro Debt transition matrix**

	Mean	Volatility
State 1	4.821	2.296
State 2	−10.777	25.037
State 3	50.545	5.800
State 4	−28.583	5.816
State 5	21.617	2.667

Table 26: **High Yield US Debt market characteristics – in %**

	State 1	State 2	State 3	State 4	State 5
State 1	0.96306	0.00000	0.00000	0.03693	0.00000
State 2	0.00000	0.93609	0.00000	0.06391	0.00000
State 3	0.05744	0.00000	0.62743	0.00000	0.31513
State 4	0.00345	0.01817	0.14142	0.80017	0.03679
State 5	0.02094	0.00000	0.06259	0.11617	0.80030

Table 27: **High Yield US Debt transition matrix**

	Mean	Volatility
State 1	−31.417	31.773
State 2	1.173	2.983
State 3	−23.002	15.011
State 4	15.617	1.608
State 5	33.209	7.717
State 6	17.497	8.019

Table 28: **High Yield Euro Debt market characteristics – in %**

	State 1	State 2	State 3	State 4	State 5	State 6
State 1	0.29099	0.00000	0.00054	0.00000	0.70847	0.00000
State 2	0.00000	0.69959	0.04939	0.04253	0.00000	0.20849
State 3	0.44393	0.00000	0.46768	0.00000	0.00143	0.08696
State 4	0.00000	0.06210	0.00000	0.93789	0.00000	0.00001
State 5	0.00067	0.00000	0.87476	0.00000	0.12457	0.00000
State 6	0.00000	0.14786	0.00000	0.01937	0.00000	0.83277

Table 29: **High Yield Euro Debt transition matrix**

	Mean	Volatility
State 1	41.703	11.436
State 2	−7.397	24.787
State 3	−91.022	52.390

Table 30: **MSCI Asia market characteristics – in %**

	State 1	State 2	State 3
State 1	0.92791	0.07209	0.00000
State 2	0.03636	0.95909	0.00455
State 3	0.00005	0.11152	0.88844

Table 31: **MSCI Asia transition matrix**

	Mean	Volatility
State 1	80.417	11.814
State 2	−26.741	41.953
State 3	−68.462	14.331
State 4	−139.275	12.436

Table 32: **Eurostoxx market characteristics – in %**

	State 1	State 2	State 3	State 4
State 1	0.41609	0.00000	0.58341	0.00051
State 2	0.08459	0.91358	0.00184	0.00000
State 3	0.65235	0.00000	0.28291	0.06474
State 4	0.11022	0.35946	0.00000	0.53032

Table 33: **Eurostoxx transition matrix**

	Mean	Volatility
State 1	88.275	28.797
State 2	−217.898	90.957
State 3	23.163	19.686
State 4	−399.324	21.566

Table 34: **MSCI Europe market characteristics – in %**

	State 1	State 2	State 3	State 4
State 1	0.85402	0.00000	0.04132	0.10467
State 2	0.07924	0.85555	0.00000	0.06521
State 3	0.00000	0.00000	0.97435	0.02565
State 4	0.63463	0.11428	0.19584	0.05525

Table 35: **MSCI Europe transition matrix**

	Mean	Volatility
State 1	40.180	11.653
State 2	−72.676	17.906
State 3	−408.481	39.635
State 4	85.885	26.867

Table 36: **MSCI Europe Small Caps market characteristics – in %**

	State 1	State 2	State 3	State 4
State 1	0.89323	0.10677	0.00000	0.00000
State 2	0.20839	0.75297	0.03863	0.00000
State 3	0.00000	0.00000	0.32453	0.67547
State 4	0.06049	0.00000	0.10178	0.83773

Table 37: **MSCI Europe Small Caps transition matrix**

	Mean	Volatility
State 1	211.271	10.588
State 2	21.555	14.706
State 3	−87.325	57.301
State 4	−64.051	21.639

Table 38: **Russel 2000 market characteristics – in %**

	State 1	State 2	State 3	State 4
State 1	0.01758	0.36731	0.00000	0.61511
State 2	0.00002	0.93943	0.00000	0.06055
State 3	0.18875	0.00000	0.81125	0.00000
State 4	0.21557	0.00000	0.04144	0.74299

Table 39: **Russel 2000 transition matrix**

	Mean	Volatility
State 1	−30.251	18.167
State 2	−67.684	59.653
State 3	56.296	13.548
State 4	−2.634	29.193

Table 40: **Latin America market characteristics – in %**

	State 1	State 2	State 3	State 4
State 1	0.12727	0.01605	0.00007	0.85661
State 2	0.06544	0.93456	0.00000	0.00000
State 3	0.10064	0.00000	0.89366	0.00570
State 4	0.60458	0.00003	0.17410	0.22129

Table 41: **Latin America transition matrix**

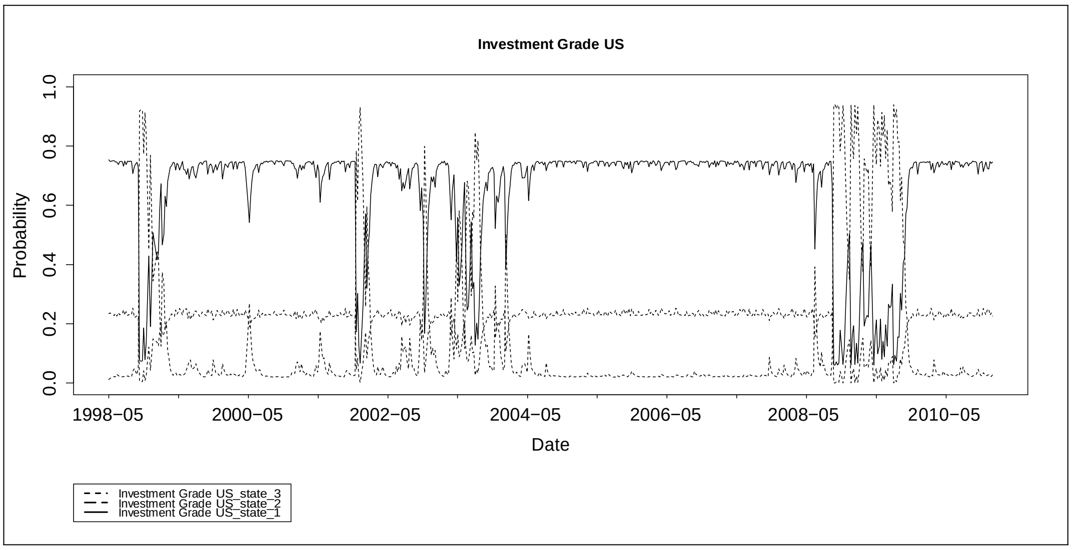

Figure 7: **State probability for Investment Grade US from 1998 to 2010**

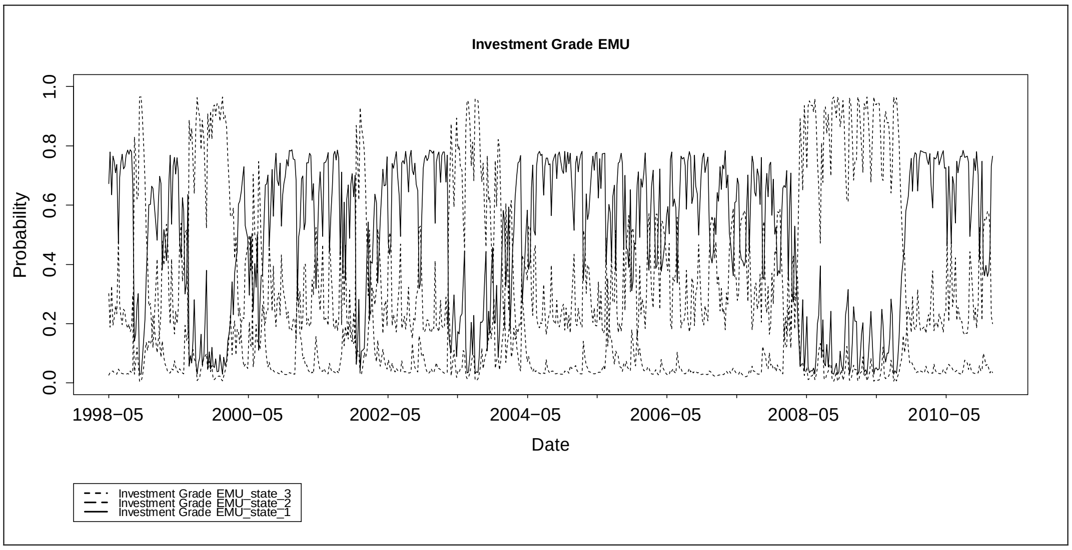

Figure 8: **State probability for Investment Grade EMU from 1998 to 2010**

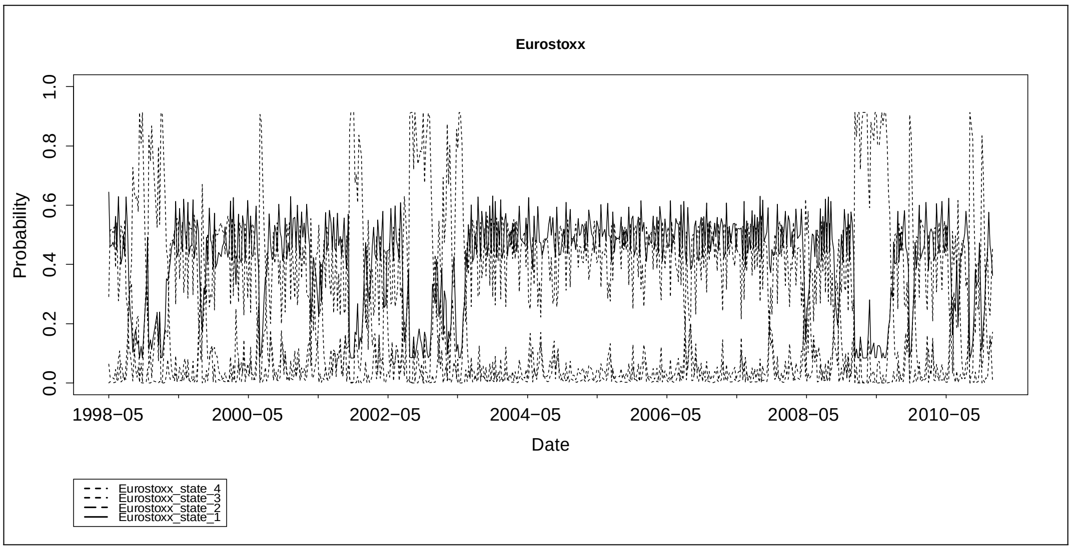

Figure 9: **State probability for Eurostoxx Index from 1998 to 2010**

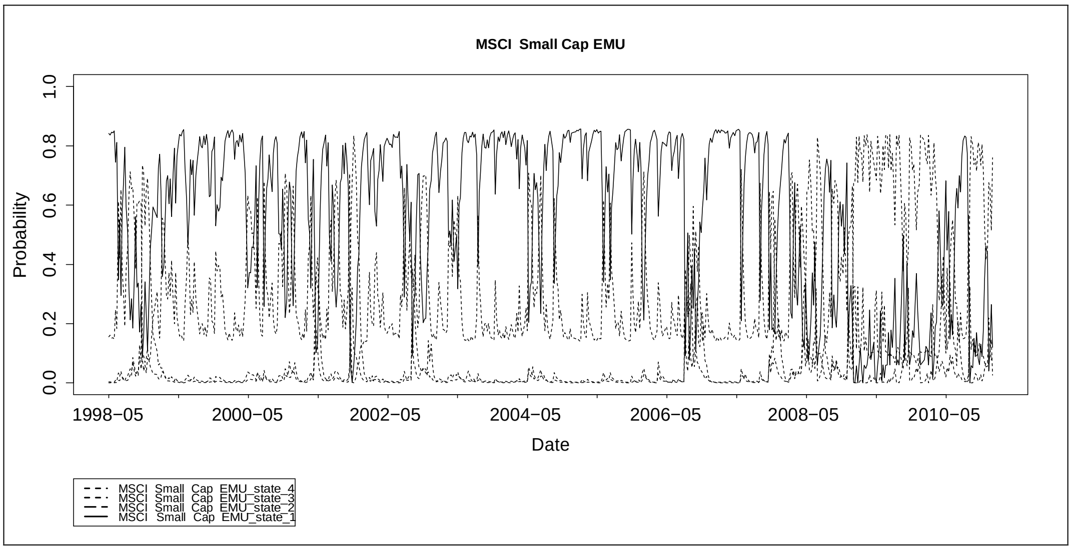

Figure 10: **State probability for MSCI Small Cap EMU from 1998 to 2010**