GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION
SPONSORED PROJECT INITIATION

Date: 8/26/80

Project Title: NON LINEAR PROGRAMMING AND CUTTING-PLANE THEORY

Project No: M-50-638

Project Director ROBERT G. JEROSLOW

Sponsor: NATIONAL SCIENCE FOUNDATION, WASHINGTON, D.C.

Agreement Period: From 7/1/80 Until ~~12/31/81*~~ ~~82~~ 12/31/83
(Grant period includes 6 month unfunded flexibility period)

Type Agreement: GRANT NO. ECS-8001763

Amount:
$25,751 NSF*
2,575 GIT (M-50-324) (First years portion only)
$28,326 TOTAL

Reports Required: PROGRESS REPORT CONTAINING REQUEST FOR CONTINUED SUPPORT, FINAL REPORT

Sponsor Contact Person (s):

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*THIS IS A CONTINUING GRANT WHICH HAS BEEN APPROVED FOR APPROXIMATELY THREE (3) YEARS. CONTINGENT ON AVAILABILITY OF FUNDS AND SCIENTIFIC PROGRESS, NSF EXPECTS TO CONTINUE SUPPORT AT $28,416 FOR YEAR TWO AND $32,229 FOR YEAR THREE.

Defense Priority Rating: NONE

Assigned to: COLLEGE OF MANAGEMENT (School/Laboratory)

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SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 3/29/84

Project No. M-50-638 School/Lab Management

Includes Subproject No.(s) __________

Project Director(s) Robert G. Jeroslow GTRI / GIT

Sponsor National Science Foundation

Title Non Linear Programming and Cutting-Plane Theory

Effective Completion Date: 12/31/83 (Performance) __________ (Reports)

Grant/Contract Closeout Actions Remaining:

- [ ] None
- [X] Final Invoice or Final Fiscal Report
- [ ] Closing Documents
- [X] Final Report of Inventions
- [ ] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other __________

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Form OCA 60:1028

NSF Grant ECS-8001763: Progress Report for the Period July, 1980 to February, 1981

We divide this progress report into two sections, and discuss separately our recent work in cutting-plane theory and in nonlinear programming. Most of our recent research has been in the theory of cutting-planes.

1. Recent results in cutting-plane theory.

At the conclusion of our previous grant (NSF grant ENG7900284) in the spring of 1980, an unexpected and favorable development occurred, which is treated in our joint paper [3]. During the present grant period, we have been continuing that earlier research.

To be specific, we found it possible to give a simple, inductive characterization of the optimal value function of the pure integer program in rational data:

$$(IP)_b \qquad \text{subject to} \quad \begin{array}{l} \min cx \\ Ax = b \\ x \geq 0 \\ x \text{ integer} \end{array}$$

(In this section of the report, all quantities discussed are rational). The value function of $(IP)_b$ is defined by:

(1) $z(b) = \min \{cx \mid Ax = b,\ x \geq 0 \text{ and integer}\}$

$z(b)$ provides the optimal value of the integer program parametrically in its right-hand-side b. This value function $z(b)$ embodies much of the "sensitivity analysis" information of importance in the applications.

Value functions are, moreover, directly related to cutting-planes, as shown in [7]. If $A = [a^j]$ (cols) in $(IP)_b$, then a valid cutting-plane for $(IP)_b$ is:

(CP) $$\sum_{j=1}^{n} \theta_j x_j \geq z(b)$$

In (CP), the θ_j are arbitrary scalars satisfying $\theta_j \geq z(a^j)$ for j = 1,..., n. Moreover, as c varies in $(IP)_b$, we obtain all valid cutting-planes for $(IP)_b$ via (CP).

We found that the value functions of pure integer programs $(IP)_b$ are exactly those functions built up from linear functions (such as $2b_1 - b_2$, or $b_1 + b_2$) by rounding them up to the nearest integer, taking non-negative multiples of the round-ups (such as $\frac{1}{2} \lceil 2b_1 - b_2 \rceil + 3 \lceil b_1 + b_2 \rceil$, where $\lceil x \rceil$ is the least integer not less than x), taking maxima of the previous results, and iterating this process. For example, the function $F(b_1, b_2) = -b_1 + \max \{\frac{1}{2} \lceil 2b_1 - b_2 \rceil + 3 \lceil b_1 + b_2 \rceil, \lceil b_2 - b_1 \rceil + 3b_1 \}$ is the value function of some pure integer program. Moreover, for any pure integer program for which $z(0) > -\infty$ (i.e., which is not unbounded in value), there is a function built up from linear functions inductively by non-negative addition, round-up, and maxima, which agrees with the value function z(b) of (IP) where z(b) is defined (i.e., for those r.h.s. b for which (IP) is consistent). We call the functions which are inductively built-up in this way, <u>Gomory functions</u>.

As to the domain of definition of z(b), there is another Gomory function H which is a <u>consistency tester</u> for $(IP)_b$, i.e.:

(2) $H(b) \leq 0 \leftrightarrow (IP)_b$ is consistent

A much simpler inductive characterization of value functions z(b) seems unlikely, since the value functions of linear programs are those functions inductively built-up from linear functions by (non-negative) addition and maxima - only the round-up operation needs to be added to account for the optimal value of integer programs. Moreover, the relationship between the integer programming value function, and the value function of its linear relaxation, is that the latter is obtained from the former by erasing round-up operations and collecting terms. For example, if the function F given earlier is the value function for $(IP)_b$, then the linear program $(LP)_b$ obtained when "x integer" is deleted as a constraint of $(IP)_b$, is $\tilde{F}(b) = -b_1 + \max\{\frac{1}{2}(2b_1 - b_2) + 3(b_1 + b_2), (b_2 - b_1) + 3b_1\} = -b_1 + \max\{4b_1 + \frac{5}{2}b_2, b_2 + 2b_1\}$.

In [3], we also obtained results regarding simultaneous variation in the r.h.s. b and the criterion function c of $(IP)_b$, and identified an inductive class of functions in which the components of an optimal solution to $(IP)_b$ lie. See the discussion of "Integer analogues" in [6, pages 6-10] for an alternate perspective on these results.

In earlier papers [1], [2] we had derived certain properties of optimal value functions of mixed-integer programs in rationals. These are more general optimizations which permit continuous as well as integer variables:

$$(MIP)_b \qquad \begin{array}{rl} \min & cx + dy \\ \text{subject to} & Ax + By = b \\ & x, y \geq 0 \\ & x \text{ integer} \end{array}$$

However, our earlier work did not characterize mixed-integer value functions. Nor did our earlier work provide any inductive description of these value functions, and for good reason: this class of functions is not closed under the operations of non-negative addition, or maximization, or the taking of round-ups. For example, the fractional parts function $F(b_1)$ = fractional part of $b_1 = b_1 + \lceil -b_1 \rceil$ is a value function (for $\min\{y \mid x_1 - x_2 + y = b_1;\ x_1, x_2 \text{ integer};\ x_1, x_2, y \geq 0\}$), and $G(b_1) = \lceil b_1 \rceil$ is a value function (for $\min\{x_1 - x_2 \mid x_1 - x_2 - y =$

b_1; x_1, x_2 integer; x_1, x_2, $y \geq 0\}$), and $H(b_1) = -b_1$ is a value function, but their sum $K(b_1) = F(b_1) + G(b_1) + H(b_1) = \lceil b_1 \rceil + \lceil -b_1 \rceil$ can be shown not to be a value function of any mixed integer program (as mixed-integer programs have directional derivatives at $b_1 = 0$ by results in [7], and $K(b_1)$ does not have such derivatives).

While the fact that mixed-integer value functions are not inductively constructed is indeed the primary barrier to a treatment of mixed-integer sensitivity analysis, at least one other complication rises, for we must enlarge the class of Gomory functions. For example, it can be shown that the value function of the mixed integer program $\min\{y_1 + y_2 | x_1 - x_2 + y_1 - y_2 = b_1$, x_1, x_2 integer; x_1, x_2, y_1, $y_2 \geq 0\}$, which is in fact the distance from b_1 to the nearest integer, is not a Gomory function.

We now describe some recent results we have obtained during the current grant period (since July 1980), regarding value functions and consistency testers for $(MIP)_b$.

First, since the Gomory functions are closed under the inductive construction, we sought to imbed the class of value functions inside another inductively-closed class. If $F(v)$ is a Gomory function and C a rational matrix, then $F(Cb)$ is trivially a Gomory function; hence the inductively-closed class must also have this property with respect to matrix multiplication. It therefore seems natural to study the "pre-multiplied constraint sets" of the form:

$$(PMIP)_b \qquad \begin{aligned} Ax + By &= Cb \\ x, y &\geq 0 \\ x &\text{ integer} \end{aligned}$$

and their "pre-multiplied" value functions $z(b) = \inf\{cx + dy | (PMIP)_b$ holds$\}$. Problems of the type $(PMIP)_b$ would arise in practice when, for example, the r.h.s. of certain constraints depended linearly on other right-hand-sides.

It is easy to show that the consistency testers and value functions for $(PMIP)_b$ are inductively-closed classes of functions. In fact, the Gomory functions are precisely the class of consistency testers for constraints of the form $(PMIP)_b$. In other words, G(b) is a Gomory function if and only if there are rational matrices A, B, and C such that:

(3) $G(b) \leq 0 \leftrightarrow (PMIP)_b$ is consistent

Furthermore, we obtained a computational procedure which determines, given a Gomory function G, whether or not it is a consistency tester for a problem $(MIP)_b$ (i.e., whether or not we can take C = identity matrix in $(PMIP)_b$, for suitable A and B). Thus the class of consistency testers for $(MIP)_b$ have been identified within the inductively-closed class of functions under study.

It follows easily from the results cited in the previous paragraph, that z(b) is a pre-multiplied value function if and only if its epigraph is detemined by a Gomory function, i.e., if and only if there is a Gomory function G(w, b) with:

(4) $w \geq z(b)$ if and only if $G(w, b) \leq 0$

Therefore the class of functions F of the form

(5) F(b) = the least w such that $G(w, b) \leq 0$

is exactly the class of pre-multiplied value functions, and F is a value function for an $(MIP)_b$ precisely if G(w, b) is a consistency tester for $(MIP)_b$.

We have also established that, if the continuous part dy of the criterian function of $(PMIP)_b$ actually does not occur (i.e., if d = 0), then the value function z(b) is a Gomory function. Moreover, z(b) is always a minimum of finitely many Gomory functions.

We desire to have an inductive class of functions, which extend the Gomory functions by some further closure conditions, and which are identical with the value functions of $(PMIP_b)$. We are now very close to proving, we believe, that the closure of the Gomory functions under infimal convolution (see [9]) gives exactly these value functions.

A paper containing these results is currently in preparation. We are also working on certain computability issues in connection with value functions and we recently obtained a procedure for determining when two Gomory functions F and G are the same (i.e., F(b) = G(b) for all b). More generally, given a vector $\vec{F}(b)$ of Gomory functions, rational matrices A_1 and A_2 with rational r.h.s. d_1 and d_2, we have a procedure for determining whether this mixed inequality system has a solution:

(6) $A_1\vec{F}(b) < d_1, \ A_2\vec{F}(b) \leq d_2.$

The procedure is in the class NP, i.e., is equivalent to solving an integer program.

We believe that the refinements of earlier results, as described above, will sharpen many results of cutting-plane theory and will, in particular, prove valuable as we proceed to study constraint systems more complex than $(MIP)_b$ or $(PMIP)_b$. We plan to continue the research as described in the grant proposal.

2. Recent results in nonlinear programming.

We are currently preparing a paper [5] on duality in semi-infinite linear programming, in which we study the linear optimization

$$\text{(SI)} \qquad \begin{array}{l} \min cx \\ \text{subject to } a^i x \geq b_i,\ i \varepsilon I \end{array}$$

and its formal dual program

$$\text{(D)} \qquad \begin{array}{ll} \sup & \sum_{i \varepsilon I} \lambda_i b_i \\ \text{subject to} & \sum_{i \varepsilon I} \lambda_i a^i = c \\ & \lambda_i \geq 0,\ i \varepsilon I \end{array}$$

(In (SI) and (D), $I \neq \phi$ is a possibly infinite index set, and any summation in the dual involves only finitely non-zero vectors $\lambda = (\lambda_i | i \varepsilon I)$).

We say that the constraint system in (SI) yields uniform LP duality if, for every $c \varepsilon R^n$, exactly one of these cases hold: (a) Both programs are inconsistent; (b) One program is unbounded in value, and the other is inconsistent; (c) Both programs are consistent, have equal finite values, and the value in the dual (D) is attained. When (SI) is consistent, clearly only cases (b) and (c) can apply.

The "uniformity" of the definition refers to the arbitrary choice of criterion function $c \varepsilon R^n$. Insofar as the usual "constraint qualifications" refer only to the constraints, they provide sufficient conditions for uniform LP duality (but see [8] for a sufficient condition which also involves the objective function). However, we are now studying constraint qualifications which are necessary as well as sufficient.

In [5], we show that the following condition is both necessary and sufficient, for uniform LP duality in (SI), when (SI) is consistent: there are sets S and T in R^{n+1}, which generate the same cone as that generated by the set $\{(a^i, -b_i) | i \epsilon I\} \cup \{(0, 1)\}$, where S is finite and T is compact, and there exists $\bar{x} \epsilon R^n$ with:

(7a) $s\bar{x} = s_{n+1}$ for every $(s, - s_{n+1}) \epsilon S$;

(7b) $t\bar{x} > t_{n+1}$ for every $(t, - t_{n+1}) \epsilon T$.

We also provide several other necessary and sufficient conditions. (In the preceeding, the cone generated by a set is obtained only by non-negative multiples, and hence need not be closed; but it is a consequence of our results that the specific cones cited above are closed).

We are currently using the results on uniform duality for semi-infinite linear programs to obtain results for semi-infinite convex programs. We are studying constraint sets of the form:

(C) $$f_h(x) \leq 0, \ h \epsilon H$$
$$x \epsilon L$$

where each f_h is a closed, convex function and L is a closed, convex set. The index set $H \neq \phi$ may be of arbitrary cardinality.

We say that (C) possesses <u>uniform convex duality</u> if, for all closed, convex functions f defined on R^n, whenever the value

(8) $v^* = \inf\{f(x) | f_h(x) \leq 0, \ h \epsilon H \text{ and } x \epsilon L\}$

is finite, there exists a finite $\phi \subsetneq H' \subseteq H$ and non-negative scalars $\lambda_h \geq 0, \ h \epsilon H'$, satisfying:

(9) $f(x) + \sum_{h \epsilon H'} \lambda_h f_h(x) \geq v^*$ for all $x \epsilon L$.

In other words, (C) possesses uniform convex duality if and only if it exhibits no duality gap, in Lagrangean duality, for all closed, convex objective functions.

To give an idea of the kind of results which can be established, we shall assume that (C) is consistent, that $L = R^n$ and that all functions f_h, $h \varepsilon H$, are finite-valued on R^n.

We shall also introduce some terminology. Specifically, we shall say that the closed, convex function g is a <u>dominated implication</u> of the functions $\{f_h | h \varepsilon H\}$ if, either g(x) is identically a non positive constant, or if there exists a finite set H', $\phi \subsetneq H' \subseteq H$ such that

(10) $g(x) \leq \max_{h \varepsilon H'} f_h(x)$ for all $x \varepsilon R^n$

Finally, a set of functions $\{g_k | k \varepsilon K\}$ is called <u>compact</u> if, given a sequence $g_{k(n)}$ of functions drawn from the set (i.e., $k(n) \varepsilon K$ for $n = 1, 2, \ldots$), there exists a sequence $n_1 < n_2 < n_3 \ldots$. and a $k^* \varepsilon K$ such that

(11) $g_{k^*}(x) = \lim_{t \to +\infty} g_{k(n_t)}(x)$ for all $x \varepsilon R^n$

With this terminology and these assumptions, it can be shown that (C) possesses uniform convex duality if and only if there exist a set $\{g_k | k \varepsilon K_1 \cup K_2\}$ of dominated implications, a point x^o, and strictly positive scalars β_k for $k \varepsilon K_1$, such that:

(12a) K_2 is finite, g_k is linear affine and $g_k(x^o) = 0$ for $k \varepsilon K_2$;

(12b) The collection of functions $\{\beta_k \, g_k | k \varepsilon K_1\}$ is compact, and $g_k(x^o) < 0$ for $k \varepsilon K_1$.

(12c) The set equality holds: $\{x | g_k(x) \leq 0, \; k \varepsilon K_1 \cup K_2\} = \{x | f_h(x) \leq 0, \; h \varepsilon H\}$

Since any finite collection of functions is trivially compact, the result cited above establishes the sufficiency of a common constraint qualification in the case that H is finite. However, here H may be infinite, and necessity of these suitably-generalized conditions is established, up to the concept of dominated implications.

We were led to explore convex Lagrangean duality because of its connection with exact penalties, through the "sectioning" approach discussed in our grant proposal. In future research, we shall bring our results and techniques from semi-infinite programming to the study of these penalty methods for nonconvex optimization.

References for Progress Report

1. C. E. Blair and R. G. Jeroslow, "The Value Function of a Mixed-integer Program: I", Discrete Mathematics 19 (1977), pp. 121-138.

2. C. E. Blair and R. G. Jeroslow, "The Value Function of a Mixed-integer Program: II", Discrete Mathematics 25 (1979), pp. 7-19.

3. C. E. Blair and R. G. Jeroslow, "The Value Function of an Integer Program", to appear in Mathametical Programming.

4. C. E. Blair and R. G. Jeroslow, "Sensitivity Analysis for Mixed-Integer Programs", in preparation.

5. R. J. Duffin, R. G. Jeroslow, and L. A. Karlovitz, "Duality in Semi-Infinite Linear Programming", in preparation.

6. R. G. Jeroslow, "Some Influences of Generalized and Ordinary Convexity in Disjunctive and Integer Programming", August 1980.

7. R. G. Jeroslow, "Minimal Inequalities", Mathematical Programming 17 (1979), pp. 1-15.

8. D. F. Karney, "Duality Gaps in Semi-Infinite Linear Programming - An Approximation Problem", to appear in Mathematical Programming.

9. R. T. Rockafellar, Convex Analysis, Princeton University Press, Princeton, N. J., 1970.

Summary of Work to be Performed from the Present through June, 1982

By the end of the current grant period, in June of 1981, we shall have written the papers [1] and [3], which are currently in preparation, and we expect to have completed [2], for which we have extensive notes. During the summer 1981, we shall complete a paper on facial and nonfacial constraints, and related issues of finite convergence and deep cuts, which includes the results announced in our grant proposal and further work. Much of the latter work exists now in note form.

In terms of active, current research, which will continue into the second year of the grant period (July 1981 to June 1982), we have two ongoing projects:

(1) Investigations of exact norm penalties and limiting norm penalties, as discussed in our grant proposal. This will extend, to the nonconvex case, the lagrangean results we have obtained for the convex case, using primarily the "sectioning" method of the grant proposal;

(2) Further work on the value funtion of mixed-integer programs, with an emphasis on, but not limited to, computational complexity issues in connection with the value function. At present, very little is known about the computational complexity of discrete programming problems as the right-hand-side parameters vary. It appears likely some new complexity heirarchies may be needed for certain of the

phenomena (although not always - for example, it is not hard to show that, given two pure integer programs with the same dimension for their right-hand-sides, it is only NP-complete to determine if they give the same value for all r.h.s.).

Depending upon how the work on items (1) and (2) above progresses, we may address the following issues during the second year of the grant period:

(3) The algorithmic utilization of our results on the value functions of mixed-integer programs, to improve the sensitivity and ranging analysis features of integer programming algorithms;

(4) The extension of our results on the value function to more complex constraint sets, including complementarity constraints and the handling of (implicit) upper bounds on the variables. This work will include cut-strengthening procedures for these constraint sets, and will involve functions of a generalized subadditive type.

During the current year, we saw two unexpected developments that will aid in this research. We did not expect to obtain an explicit, inductive description of pure-integer value functions, nor the extensive information on closed-form expressions for value functions for mixed-integer and premultiplied mixed-integer constraint sets. It has been clearly both necessary and worthwhile to pursue these new leads, for they are certain to have far-reaching consequences for practical implementation and for other constraint sets (as e.g. items (2), (3), and (4) above). Nor did we expect to find constraint qualifications for convex programs which (up to transformations we can exactly describe) are necessary as well as sufficient. We expect this latter work to sharpen our results on norm penalties (item (1) above).

Papers in Preparation

1. "Duality in Semi-Infinite Linear Programming", with R. J. Duffin and L. A. Karlovitz.

2. "Duality in Semi-Infinite Convex Programming".

3. "Sensitivity Analysis for Mixed-integer Programs", with C. E. Blair.

Other Support

The Principal Investigator has no other grant support, and no grant proposals are current or contemplated.

Funds Remaining at the End of the Current Grant Period (June 1981)

We expect to essentially use all funds of the current grant increment by the end of June. There may be some small amounts remaining in the minor categories (e.g., travel and supplies).

Publication Activity, July 1979 to February 1981

Papers Published

1. "Representations of Unbounded Optimizations as Integer Programs", Journal on Optimization Theory and Its Applications (30), 1980, pp. 339-351.

2. "Lagrange Dual Problems with Linear Constraints on the Multipliers", with C. E. Blair, Constructive Approaches to Mathematical Models, C. V. Coffman and G. Fix (eds.), Academic Press, 1979, pp. 137-152.

3. "An Introduction to the Theory of Cutting-Planes", Annals of Discrete Mathematics (5), 1979, pp. 71-95.

4. "A Cutting-Plane Game for Facial Disjunctive Programs", SIAM Journal on Control and Optimization (18), 1980, pp. 264-281.

5. "Strengthening Cuts for Mixed-integer Programs", with E. Balas, European Journal of Operations Research (4), 1980, pp. 224-234.

Papers Accepted for Publication

1. "A Limiting Lagrangean for Infinitely-constrained Convex Optimization in R^n", Journal of Optimization and Theory Applications.

2. "Lagrangean Functions and Affine Minorants", with R. J. Duffin, Mathematical Programming.

3. "An Exact Penalty Method for Mixed-integer Programs", with C. E. Blair, Mathematics of Operations Research.

4. "The Value Function of an Integer Program", with C. E. Blair, Mathematical Programming.

Other Papers Submitted for Publication

1. "The Limiting Lagrangean", with R. J. Duffin, June 1979.

2. "A Limiting Infisup Theorem", with C. E. Blair and R. J. Duffin, August 1979.

3. "Some Influences of Generalized and Ordinary Convexity in Disjunctive and Integer Programming", August 1980.

DR. ROBERT G. JEROSLOW

February 1981

Personal

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Education:

B. S. 1964	Columbia University School of Engineering Department of Industrial Engineering
1964 - 1966	Cornell University School of Engineering Department of Operations Research (completed Comprehensive Examination)
Ph. D. 1969	Cornell University Department of Mathematics Professor Anil Nerode, Advisor

Experience:

University of Minnesota
School of Mathematics
September 1969 - August 1972
Assistant Professor

Carnegie-Mellon University
Graduate School of Industrial Administration and
Department of Mathematics
Associate Professor
September 1972 - August 1976
Professor
September 1976 - June 1980

Georgia Institute of Technology
College of Management
Professor
September 1978 to date

Research Interests

Mathematical programming, with emphasis on cutting-plane theory and its uses in integer programming and linear complementarity; nonlinear programming; integer programming; programming aspects of computational complexity; multicriteria optimization. Strategic planning systems, and the utilization of quantitative models to assist in nonquantitative decision making.

Teaching Interests

Research interests, plus production and applications of Operations Research techniques; management strategy.

Journals

Member of the Editorial Board (Associate Editor), Discrete Applied Mathematics, Mathematical Programming and Mathematical Programming Studies.

Referee for Operations Research, SIAM Journal on Applied Mathematics, Management Science, Mathematical Programming, and Discrete Mathematics.

Reviewer for Bulletin of the American Mathematical Society.

Grants and Fellowships

Ford A Fellowship 1964-1966
NSF Graduate Fellowship 1966-1969
NSF Research Grant GP 21067, Principle Investigator, 1971-1972
(Grant Awarded 1970)
NSF Research Grant GP-37510X, Associate Investigator, 1973-1975
NSF Research Grant MCS76-12026, Co-Principle Investigator, 1976-1978
Research Fellowship, January - June 1977, from the Center for
Operations Research and Econometrics, Belgium
NSF Research Grant ENG-79000284, Principle Investigator, 1979-1980
NSF Research Grant ECS-8001763, Principle Investigator, 1980-1982

Organizational Responsibilities

Representative to the Faculty Senate from the business school, 1977-1978
Organizer of the Operations Research Seminar for the business school, 1977-1978

Representative of the management college to the Seminar on Operations Research, 1978-1981 (co-sponsored with the School of Industrial and Systems Engineering and the School of Mathematics)
Member of the Program Committee of the symposium in honor of R. J. Duffin, Constructive Approaches to Mathematical Models, July 10-15, 1978
Co-organizer (with Cedric Suzman) of the Colloquiua on Strategic Planning, September 28, 1979, and October 10, 1980 (third Colloquium projected for Fall 1981)
Member of the Organizing Committee of the 1981 International Symposium on Semi-infinite Programming and Applications
Member of the Teaching Evaluation Committee in the College of Management
Member of the Personnel Committee in the College of Management

Hobbies and Personal Interests

Light hiking; swimming; jogging; weight training; reading in literature, archaeology and history.

Published Articles

1. "Consistency Statements in Formal Theories", Fundamentae Mathematicae, LXXXII (1971), pp. 17-40.

2. "Non-effectiveness in S. Orey's Arithematical Compactness Theorem", Zeithschrift f. math. Logik and Grundlagen d. Math., Bd. 17, 1971, pp. 285-289.

3. "On Semi-infinite Systems of Linear Inequalities", with K. O. Kortanek, Israel Journal of Mathematics, vol. 10, no. 2, 1971, pp. 252-258.

4. "A Note on Some Classical Methods in Constrained Optimization and Positively Bounded Jacobians", with K. O. Kortanek, Operations Research, vol. 15, no. 5, 1967, pp. 964-969.

5. "Comments on Integer Hulls of Two Linear Constraints", Operations Research, vol. 19, no. 4, July-August 1971, pp. 1061-1069.

6. "On an Algorithm of Gomory", with K. O. Kortanek, SIAM Journal on Applied Mathematics, vol. 21, no. 1, July 1971, pp. 55-59.

7. "Canonical Cuts on the Unit Hypercube", with E. Balas, SIAM Journal on Applied Mathematics, vol. 23, no. 1, July 1972, pp. 61-69.

8. "There Cannot be any Algorithm for Integer Programming with Quadratic Constraints", Operations Research, Programming Volume, vol. 21, no. 1, January-February 1973, pp. 221-224.

9. "Redundancies in the Hilbert-Bernays Derivability Conditions for Godel's Second Incompleteness Theorem", Journal of Symbolic Logic, vol. 38, no. 3, September 1973, pp. 359-367.

10. "Linear Programs Dependent on a Single Parameter", Discrete Mathematics (6), 1973, pp. 119-140.

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34. "A Cutting-plane Game for Facial Disjunctive Programs", SIAM Journal on Control and Optimization (18), 1980, pp. 264-281.

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M. R. Hestenes' Optimization Theory: The Finite-dimensional Case, reviewer in Bulletin of the American Mathematical Society (83), May 1977, pp. 324-334.

Accepted for Publication

1. "A Limiting Lagrangean for Infinitely-constrained Convex Optimization in R^n", Journal of Optimization Theory and Applications.

2. "Lagrangean Functions and Affine Minorants", with R. J. Duffin, Mathematical Programming.

3. "An Exact Penalty Method for Mixed Integer Programs", with C. E. Blair, Mathematics of Operations Research.

4. "The Value Function of an Integer Program", with C. E. Blair, Mathematical Programming.

Submitted for Publication

1. "The Limiting Lagrangean", with R. J. Duffin, June 1979.

2. "A Limiting Infisup Theorem", with C. E. Blair and R. J. Duffin, August 1979.

3. "Some Influences of Generalized and Ordinary Convexity in Disjunctive and Integer Programming", August 1980.

Current Research (Papers in Preparation)

1. "Duality in Semi-infinite Linear Programming", with R. J. Duffin and L. A. Karlovitz.

2. "Duality in Semi-infinite Convex Programming".

3. "Sensitivity Analysis for Mixed Integer Programs", with C. E. Blair.

4. "Proceedings of the Second Annual Georgia Tech Colloquium on Strategic Planning", C. Suzman, co-editor.

Invited Talks

1. "On Godel's Consistency Theorem", University of Texas at Austin, October 1971.

2. "K-descriptions", Lakehead University, Thunder Bay, Ontario, Canada, January 1972.

3. "Asymptotic Linear Programming", Carnegie-Mellon University, February 1972.

4. "Trial-and-error Logics", State University of New York at Buffalo, February 1973.

5. "On a Theorem of Chvatal and Gomory", SIGMAP-UW Symposium on Nonlinear Programming, University of Wisconsin, April 1974.

6. "Proof Theory and Hilbert's Finitism", a series of nine lectures, Universita di Siena, Instituto di Matematica, Siena, Italy, May 1974.

7. "Cutting-planes for Relaxations of Integer Programs", ORSA/TIMS meeting in San Juan, P. R., October 1974.

8. "Some Results and Constructions of Cutting-plane Theory", NSF Regional Conference on Convex Polytypes and Mathematical Programming, Tuscaloosa, Alabama, June 9-13, 1975.

9. "Algebraic Methods, Disjunctive Methods", for the Workshop in Integer Programming, Bonn, Germany, September 8-11, 1975.

10. "Completeness Theorems for Cutting-planes", seminar at the University of North Carolina, November 13, 1975.

11. "Minimal Inequalities", ORSA/TIMS meeting in Las Vegas, November 17-19, 1975.

12. "Completeness Results in Cutting-plane Theory", Centre de Recherches Mathematiques, Montreal, January 1976.

13. "Cutting-planes for Complementary Constraints", IX International Symposium on Mathematical Programming, Budapest, August 1976.

14. "Bracketing Discrete Problems by Two Problems of Linear Optimization", Symposium on Operations Research, Heidelberg, September 1976.

15. "Treeless Searches", ORSA/TIMS Joint National Meeting (joint with C. E. Blair), Miami Beach, November 1976.

16. "The Complexity of Certain Linear and Integer Programming Algorithms", University of Bonn, February 1977.

17. "Subadditivity and Value Functions of Mixed-Integer Programs", University of Cologne, May 1977.

18. "Linear Programs Dependent on a Single Parameter", University of Aachen, May 1977.

19. "An Introduction to the Theory of Cutting-planes", Summer School on Combinatorial Optimization at Sogesta, in Urbino, Italy, June 1977.

20. "An Introduction to the Theory of Cutting-planes", NATO Advanced Research Institute on Discrete Optimization and Systems Applications, Vancouver, August 1977.

21. "Cutting-planes and Cutting-plane Algorithms for Complementary Constraints", International Symposium on Extremal Methods and Systems Analysis, Austin, September 1977.

22. "A Cutting-plane Game and Its Algorithms", Georgia Institute of Technology, Atlanta, February 1978.

23. "A Limiting Lagrangean for Infinitely Constrained Convex Optimization in R^n", at Constructive Approaches to Mathematical Models, Pittsburgh, July 1978.

24. "Representations of Unbounded Optimizations as Integer Programs", ORSA/TIMS meeting in New Orleans, April 30 - May 2, 1979.

25. "Recent Results in Nonlinear and Integer Programming", at the meeting on Mathematical Programming, at Mathematisches Forschungsinstitut in Obersolfach, Germany, May 6-12, 1979.

26. "A Limiting Infisup Theorem", at the Tenth International Symposium on Mathematical Programming, Montreal, August 27-31, 1979.

27. "Nonlinear Optimization Treated by Linear Inequalities", ORSA/TIMS meeting in Milwaukee, October 15-17, 1979.

28. "Sensitivity Analysis for Integer Programs", ORSA/TIMS National Meeting at Colorado Springs, November 10-12, 1980.

29. "Integer Analogues", Mathematical Sciences Department, University of Delaware, November 1980.

30. "Sensitivity Analysis for Mixed Integer Programs", CORS/ORSA/TIMS Joint National Meeting, Toronto, May 3-6, 1981.

E INSTRUCTIONS ON
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MPLETING)

SUMMARY PROPOSAL BUDGET

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (MONTHS)	
	Proposed	Granted
AWARD NO.		

RGANIZATION
eorgia Tech Research Institute

RINCIPAL INVESTIGATOR/PROJECT DIRECTOR
obert G. Jeroslow

SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.6. show number in brackets)	NSF FUNDED PERSON-MOS CAL.	ACAD	SUMR	FUNDS REQUESTED BY PROPOSER	FUNDS GRANTED BY NSF (IF DIFFERENT)
Robert G. Jeroslow		1	2	$ 15,253	$
none					
( ) OTHERS (LIST INDIVIDUALLY ON BUDGET EXPLANATION PAGE)					
( ) TOTAL SENIOR PERSONNEL (1-5)				15,253	
OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
( ) POST DOCTORAL ASSOCIATES					
( ) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)					
( ) GRADUATE STUDENTS					
( ) UNDERGRADUATE STUDENTS					
( ) SECRETARIAL CLERICAL					
( ) OTHER					
TOTAL SALARIES AND WAGES (A+B)				15,253	
FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS) at 19.5% of full-time staff				2,974	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A+B+C)				18,227	
PERMANENT EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $1,000; ITEMS OVER $10,000 REQUIRE CERTIFICATION)					
TOTAL PERMANENT EQUIPMENT				0	
TRAVEL 1. DOMESTIC (INCL. CANADA AND U.S. POSSESSIONS)				900	
2. FOREIGN				0	
PARTICIPANT SUPPORT COSTS					
1. STIPENDS $					
2. TRAVEL					
3. SUBSISTENCE					
4. OTHER					
TOTAL PARTICIPANT COSTS				0	
OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				100	
2. PUBLICATION COSTS/PAGE CHARGES				300	
3. CONSULTANT SERVICES					
4. COMPUTER (ADPE) SERVICES				100	
5. SUBCONTRACTS					
6. OTHER					
TOTAL OTHER DIRECT COSTS				1,400	
TOTAL DIRECT COSTS (A THROUGH G)				19,627	
INDIRECT COSTS (SPECIFY) On campus: 46.5% of modified total direct costs					
TOTAL INDIRECT COSTS				9,127	
TOTAL DIRECT AND INDIRECT COSTS (H + I)				28,754	
RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS GPM 252 AND 253)				0	
AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$28,754	$

I/PD TYPED NAME & SIGNATURE*	DATE	FOR NSF USE ONLY		
Robert G. Jeroslow [signature]	Feb. 19, 1981	INDIRECT COST RATE VERIFICATION		
NST. REP. TYPED NAME & SIGNATURE*	DATE	Date Checked	Date of Rate Sheet	Initials - DGC Program

*SIGNATURES REQUIRED ONLY FOR REVISED

Robert G. Jeroslow

NSF Grant ECS-8001763:
Progress Report for the Period
March, 1981 to February, 1982

This section is a continuation of our "Progress Report for the Period July, 1980 to February, 1981," which we sent to NSF last year at this time.

The research of the grant is on schedule with the section, "Summary of Work to be Performed from the Present through June, 1982," of our earlier report. Of the work cited there, the promised references 1., 2., and 3. have been written and submitted to publication. These occur below as items 1., 2., and 3. of "Submitted for Publication," in our section below, "Publication Activity, 1980 to Date." The paper on facial and nonfacial constraints is item 4. of "Submitted for Publication." The research item (2) under "ongoing projects," on the computational complexity of value function questions, is currently being written up. Following that, research item (1), on norm penalties and limiting norm penalties, will be written. We anticipate the completion of all this work on time, by June 1982.

In addition, a new direction has opened up, in our modelling work joint with J. Lowe (see item 5. of "Submitted for Publication").

The remainder of this progress report is divided into five subsections, A through F, as follows.

In subsections A and B, we continue our discussions of some research which was current at the time of our earlier progress report. Basically, the final results went furthur than what we knew at the time of that report, and we briefly summarize these extensions. Subsections A and B assume familiarity with our earlier progress report.

In sections C through F, we discuss more recent developments. Some of these new results were known in part earlier, but most were not.

A. Improvements of February 1981 Results on Uniform Convex Duality

Consider the set of constraints:

$$\text{(C)} \qquad f_h(x) \leq 0 \,, \; h \in H \qquad x \in L$$

where each f_h is a closed, convex function and L is a closed, convex set; each f_h, $h \in H$, is defined on L; and there is a point x^o in the relative interior of L with $f_h(x^o) < 0$ for $h \in H$. The index set $H \neq \emptyset$ may be of arbitrary cardinality.

We say that (C) possesses uniform convex duality (u.c.d.), if these constraints exhibit no duality gap, in finite Lagrangian duality, for all closed, convex objective functions. Equivalently, (C) possesses u.c.d. if, whenever the program value

$$\text{(1)} \qquad v^* = \inf \{f(x) \mid f_h(x) \leq 0, \; h \in H\}$$

is finite, there exists a finite subset H' of H, and scalars $\lambda_h \geq 0$ for $h \in H'$, with:

$$\text{(2)} \qquad f(x) + \sum_{h \in H'} \lambda_h f_h(x) \geq v^* \quad \text{for all } x \in L.$$

In our earlier progress report, we provided necessary and sufficient conditions for (C) to possess u.c.d. when $L = R^n$. This case requires, in particular, that all f_h, $h \in H$, are finite-valued on R^n. In the process of completing this research, we were able to completely treat the case of a general closed, convex set L. The final result is very much like the one given for $L = R^n$, and we refer the reader to our paper (which is Appendix A).

B. Improvements in February 1981 Results on the Value functions of Mixed Integer Programs

In our previous progress report, we were studying the value functions of mixed integer programs:

$$(3)\qquad \begin{aligned} &\inf cx + dy \\ \text{subject to } &Ax + By = b \\ &x,y \geq 0 \text{ and } x \text{ integer} \end{aligned}$$

(We always assume "rational data" - i.e. A, B, c, d, b rational). We were encountering difficulties, in part because the class of such value functions is not closed under the inductive formation operations of addition or maximum. We had, therefore, imbedded the problem (3) into the larger class of problems

$$(4)\qquad \begin{aligned} &\min cx + dy \\ \text{subject to } &Ax + By = Cb \\ &x,y \geq 0 \text{ and } x \text{ integer} \end{aligned}$$

of "premultiplied constraint sets." Problems of the type (4) arise if there is even one material balance equation, with right-hand-side zero. We had ascertained that the class of value functions for (4) is closed under addition and maximum.

(The term "premultiplied," refers to the rational matrix C which premultiplies b. There is no generality in value functions for premultiplied pure integer programs - these are the same as value functions for ordinary integer programs. Similarly, there is no generality in premultiplied linear programs. However, there is significant generality in (4) beyond (3), i.e. when both continuous and discrete variables are present).

When we reported last, we had a conjecture on the value functions for (4), and it is correct. Specifically, it is true, that the class of value functions for (4) consists of the infimal convolution of Gomory functions restricted to the domain of definition of the value function. (We had earlier determined, that this domain of definition consists of those b for which an associated Gomory function is nonpositive). Once again, concepts from convexity - specifically, infimal convolution - have been useful in developing the algebraic theory of discrete programs.

Our proof of this result revealed a second characterization of the value functions of (4). Specifically, this class of functions (where defined) is identical to those finite minima of Gomory functions, which yield a subadditive minimum.

We have also made progress on identifying the value functions of ordinary mixed-integer programs (3) within the class of premultiplied ones (4). Specifically, we have provided a constructive proceedure such that, given a Gomory function G, it determines if there are rational matrices A and B with:

$$\text{(5)} \quad \begin{array}{l}\text{For some } x,y \geq 0 \text{ with } x \text{ integer} \iff G(b) \leq 0 \\ \qquad Ax + By = b\end{array}$$

for all b.

In the language of our previous report, the proceedure determines if G is the "consistency tester" for some mixed integer program (3). From earlier results, Gomory functions are excactly the consistency testers for some program (4). By combining this proceedure with our earlier characterization of value functions for (4), we can obtain a characterization of the value functions for (3) within the class of value functions for (4).

We have also determined an upper bound on the complexity of our proceedure - it is no more difficult than solving one pure integer program.

This work is reported in Appendix B.

C. New Research on Semi-infinite Duals

In the usual semi-infinite dual, we consider the optimization

$$\text{(6)} \quad \begin{array}{l}\sup \sum_{i=1}^{\infty} \lambda_i b_i \\ \text{subject to } \sum_{i=1}^{\infty} \lambda_i a_i = c \\ \lambda_i \geq 0 \quad \text{for } i=1, 2, \ldots\end{array}$$

where all $b_i \in R$, $a^i \in R^n$, and there is the further restriction:

(7) "At most finitely many λ_i are positive"

Our colleague Professor Dennis F. Karney brought to our attention an application of (6) in which (7) cold no longer be assumed. In joint research, we determined that (7) can be entirely dropped, in favor of the hypothesis that all sums indicated converge: the optimal value of (6) does not change (nor does attainment of this value).

We are currently extending this result to more complex constraint sets, and also weakening the hypothesis of convergence.

D. Extensions of Balas' Theorem

In Appendix C, we report on several extensions of a theorem due to Balas, which provides a characterization of the convex hull of feasible solutions to large classes of programming problems.

To present this result, let P be a convex set; let $I = \bigcup_{j=1}^{t} I_j$ be a union of disjoint finite index sets I_j (and t may be finite or infinite), let Q_i for $i \in I$ be a convex set, and define inductively:

(8a) $$P_0 = P$$

(8b) $$P_j = \text{clconv}\ \{\bigcup_{i \in I_j} (P_{j-1} \cap Q_i)\}$$

(8c) $$P_\infty = \bigcap_{j=1}^{\infty} P_j \quad , \quad \text{if } t = +\infty.$$

In (8b), clconv(S) denotes the closure of the convex hull conv(S) of a set $S \subseteq R^n$. Let P_j' be defined as is P_j, but with "conv" replacing "clconv" in (8b).

Balas showed the following in [1]: if P is a polytope, t is finite, and each Q_i for $i \in I$ is a face of P, then P_t' is the convex hull conv(F) of the feasible points F, where:

(9) $F = \{x \in P \mid \text{for each } j=1,\ldots,t \text{ there is an } i \in I, \text{ with } x \in Q_i\}$

This result applies to pure zero-one integer programs and bounded generalized linear complementarity problems. With furthur analysis, it can be shown to yield the "deduction rule" of Blair [2] and generalizations of that rule [4], and the principle involved can be used to obtain finite primal or dual cutting-plane algorithms for the problems cited [5].

We drop the hypotheses of boundedness and polyhedrality, and assuming only that each $P \cap Q_i$ is an extreme subset of P, we verify Balas' result that $P_t' = \text{conv}(F)$. Also, when P is closed and bounded, $P_t = P_t'$.

Actually, when P is not bounded, we feel that natural interest lies in P_t, since the convex span alone may not be closed. Accordingly, we explore the set P_j defined above - rather than the sets P_j' originally defined - and for P and Q_i polyhedral, we achieve a characterization of P_t when all $P \cap Q_i$ are faces of P (see Theorem 3.4 of Appendix C). Essentially, we determine that $P_t = \text{clconv}(F) + K$, where K is a closed cone of recession directions, which is precisely characterized.

We also depart entirely from the faciality hypothesis, and provide results on P_t for this very general case. (The case is relevant to general integer programming, which is not facial). Essentially, we show that "convergence" to the convex hull conv(F) of feasible points occurs when P is compact and convex and all Q_i are closed, i.e. that $P_\infty = \text{conv}(F)$. On the other hand, convergence may fail for any finite j (i.e. all $P_j \supsetneq \text{conv}(F)$ can occur).

In contrast, "finite convergence" does occur when $F = \emptyset$, and the previous hypotheses hold, for then we have $P_j = \emptyset$ for large enough j. This convergence is then applied to interpret the finite convergence properties of certain pure cutting-plane algorithms.

E. The Computational Complexity of Some Questions of Parametric Programming

In our forthcoming joint paper [3], we begin the study of the computational complexity of certain question concerning value functions. For example:

Data: Matrices A, A′ and vectors c, c′ (rational)

Question: Does the following hold for all right-hand-size b:

$$\min\{cx \mid Ax = b,\ x \geq 0\} =$$

$$\min\{c'x' \mid A'x' = b,\ x' \geq 0\}. \tag{10}$$

Using Khachian's result, it is easy to show that the question posed in (10) is solvable in polynomial time, i.e. is no harder than its nonparametric counterpart. However, it is less obvious that this question "jumps" in complexity:

Data: Matrices A, A′, C and vectors c, c′.

Question: Does the following hold for all right-hand-sides b:

$$\min\{cx \mid Ax = Cb,\ x \geq 0\}$$

$$= \min\{c'x' \mid A'x' = b,\ x' \geq 0\} \tag{11}$$

In fact, the question posed in (11) is NP-complete.

In (10), if the variables x and x′ are constrained to be integer, the question becomes NP-complete, again no harder than its non-parametric counterpart. An alternative way of asking parametric integer programming questions is by use of Gomory functions, since (by our earlier work on function classes) these provide the value functions for pure integer programs. Indeed we can show that the following question is NP-complete:

Data: Gomory functions F and G

Question: Does there exist a vector b

(12) with $F(b) \neq G(b)$?

Actually, NP-completeness holds if F and G are only Chvatal functions.

Nevertheless, it now seems that certain other related questions of parametric integer programming will be harder than NP. Indeed, it is not hard to show that merely writing the optimum to a subadditive dual can require exponential space - an interesting counterpart to examples where subadditive duality is exponentially faster than branch-and-bound. We plan to focus on the "harder" parametric questions in our next paper on this topic.

F. Modelling with Integer Variables

Appendix D reports some joint work, where progress was unexpectedly made on basic questions, of modelling real-world problems by the use of integer variables.

R. R. Meyer initiated the study of such modelling, originally in terms of the existence or nonexistence of a modelling, and more recently, with emphasis on "relaxation optimal" modellings. "Relaxation optimal" modelling are the function version of what we call "sharp" modellings; our treatment includes sets and functions. We have recovered his results, both in the case of a general-integer and a bounded-integer modelling, and extended these from functions of one variables to those of several variables. Similarly, we have extended the results on "sharp" modellings, by showing that these always exist, and - for bounded-integer modellings - by giving two general forms of a "sharp" modellings. When applied in standard, known cases, as in seperable programming, we recover the most efficient modellings known. In other cases, we find modellings not known to exist before, or new modellings with superior properties.

We illustrate these points by three examples.

First, consider the case of two products or transmission lines in a network, at quantities x_1 and x_2, which have both individual fixed costs f_1 and f_2, and a joint fixed cost f_b:

$$(13) \qquad g(x_1, x_2) = \begin{cases} 0, & \text{if } x_1 = x_2 = 0; \\ f_1, & \text{if } 0 < x_1 \leq M_1,\ x_2 = 0; \\ f_2, & \text{if } x_1 = 0,\quad 0 < x_2 \leq M_2; \\ f_b, & \text{if } 0 < x_1 \leq M_1,\ 0 < x_2 \leq M_2. \end{cases}$$

The function g describes the fixed cost in terms of x_1 and x_2. (This example is drawn from Section 4.4, pp. 38-42, of Appendix D).

A joint fixed charge, as in (13), can arise when two products share common facilities. If the two products are made completely independently, then $f_b = f_1 + f_2$, and actually (13) is the sum of two one-dimensioanl fixed charges. If the products share a facility, and one can favorably employ some of the "set up" for the other, we have $f_b < f_1 + f_2$. If they share a facility but interact unfavorably (e.g. some machines have to be cleaned after one product is run, in order to run the second), then we can have $f_b > f_1 + f_2$.

In order for any integer representation (in rationals) of the joint fixed charge (13) to exist, it is necessary that:

$$(14) \qquad 0 \leq f_1, \quad 0 \leq f_2 \quad , \quad \max \{f_1, f_2\} \leq f_b.$$

Once (14) holds, actually a representation with zero-one variables is possible, and our methods supply two such representations, one of which is worked in detail in Appendix D. (Also, the upper bounds M_1 and M_2 in (13) are needed for a modelling to exist in rationals; this is a result of Meyer).

The modellings provided by our methods are "sharp," i.e., their linear relaxation gives the convex span of the epigraph of g, and no modelling can do better than this. This linear relaxation is used by branch-and-bound algorithms.

By working out the details of this model, we can obtain simplifications of our general results for this case, which aids in implementation. As one example of such simplifications, consider the case $f_1 = f_2 = f_b$ of "two set-ups for the price of one." Just by simple algebraic manipulation, we learn that the linear relaxation is equal to the best (maximum) of the two independent relaxations, with an explicit formula that can be inserted in the formulation, if desired.

Moreover, an interesting insight arises: when $f_b > f_1 + f_2$, then the linear relaxation is that of independent fixed costs f_1 and f_2, in the range of (x_1, x_2) with $x_1/M_1 + x_2/M_2 \leq 1$. The fact that the joint fixed cost f_b does not show up in the linear relaxation of the problem for (x_1,x_2) in this range, indicates that branch-and-bound will not pick up very much useful information on such problems until deep into the tree, when many variables are arbitrated (set to values). I.e., branch-and-bound will have particular difficulty in solving such problems - beyond the normal difficulty with independent fixed charges. However, this kind of difficulty does not arise if $f_b \leq f_1 + f_2$.

For our second example, we consider the fixed benefit function:

$$(15) \qquad h(x) = \begin{cases} 0 \ , & \text{if } x = 0 \\ -b \ , & \text{if } x \geq L \end{cases}$$

where b, L > 0 are positive (rationals). For example, a vendor may offer a one-time cost reduction of b > 0 if at least L units are purchased. (More complex vendor offers result in more complex modellings). The benefit b is entered negatively (-b) since our framework is minimization.

As regards the existence of modellings, the function of (15) does have an integer modelling, in fact one with a zero-one variable (this can be seen either by direct construction, or as a direct consequence of our work).

$L > 0$ is necessary for a modelling; i.e. there is no modelling for $L = 0$. On the other hand, an upper bound on x is not needed to have a modelling.

As regards efficient representations, our theory provides one which is sharp and compact. Next, a valuable insight also is available, by comparison of the convex span of the epigraph of h in (15) with that of any one-variable fixed charge. The convex span for (15) agrees with h for $x = 0$ and $x \geq L$, and differs only in the interval $0 < x < L$, where L is usually small. In contrast, the convex span for a fixed charge has error, usually substantial error, except at zero and the upper bound. Therefore, branch-and-bound will far more easily solve fixed benefit problems, than fixed charge problems.

For our third example, we have explored the conventional modelling of situations where some one of several set of linear constraints is to hold. This is a common instance of modelling a set (rather than a function), and we compared the modelling from our theory with the conventional one. The new modelling is about twice the size, involving more variables and constraints; however, the new modelling is sharp, and the previous is not sharp and its linear relaxation itroduces large errors. The conventional modelling, while a correct integer modelling, can be close to useless in branch-and-bound codes. The new modelling is the best which can be achieved.

The examples above are only illustrations of our results, for Appendix D contains complete characterizations of integer and bounded integer modelling.

References

[1] E. Balas, "Disjunctive Programming," Annals of Discrete Mathematics (5) 1979, pp. 3-51.

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FINAL PROJECT REPORT

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PART I—PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Institute of Technology North Avenue, Atlanta, GA 30332	Engineering	ECS-8001763
	4. Award Period From 7/1/80 To 12/31/83	5. Cumulative Award Amount

6. Project Title

Nonlinear Programming and Cutting-Plane Theory

PART II—SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

We have obtained necessary and sufficient conditions for zero duality gap in semi-infinite convex optimization, in terms primarily of a compactness condition on the functions of an equivalent problem formulation.

The process of analogy from linear programming duality and value functions to integer programming duality and value functions has been extended, allowing characterizations of pre-multiplied mixed-integer constraint sets and value functions. An algorithmic identification of mixed-integer programs within the premultiplied class was given.

We have shown that the complexity, of certain simple sequenced-move games with perfect information, ascends the polynomial hierarchy and so exhibits apparently far greater complexity than that of optimization. This affects views on possible decision support for models in areas like finance and marketing.

We have identified certain problems in parametric discrete programming which remain low in the polynomial hierarchy.

In our study of modeling with integer variables, we have obtained necessary and sufficient conditions for representability, together with general techniques for constructing representations when these exist. Our representations are "best possible" in terms of the linear relaxation. Implications for decision support have been studied.

PART III—TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (✓)	Approx. Date
a. Abstracts of Theses	X				
b. Publication Citations		X			
c. Data on Scientific Collaborators	X				
d. Information on Inventions	X				
e. Technical Description of Project and Results		X			
f. Other *(specify)* See "Table of Contents" below					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
Robert G. Jeroslow	[signature]	March 17 1984

NSF Form 98A (5-78) Supersedes All Previous Editions

Form Approved OMB No. 99R0013

NSF Grant ECS-8001763

FINAL REPORT

Part III-Table of Contents

Robert G. Jeroslow

Papers Written Under This Grant

A. Published

1. "Some Influences of Generalized and Ordinary Convexity in Disjunctive and Integer Programming," Generalized Concavity in Optimization and Economics, S. Schaible and W. T. Ziemba, eds., Academic Press 1981, pp. 689-699 (ISBN-0-12-621120-5).

2. "Duality in Semi-infinite Linear Programming," with R. J. Duffin and L. A. Karlovitz, Semi-infinite Programming and Optimization, edited by A. V. Fiacco and K. O. Kortanek, Springer-Verlag, 1983.

3. "Uniform Duality in Semi-infinite Convex Optimization," Mathematical Programming 27 (1983), pp. 144-154.

B. Accepted for Publication

4. "Cluster Sets of Vector Series," with D. F. Karney, to appear in Advances in Applied Mathematics.

5. "Constructive Characterizations of Value Functions of a Mixed-Integer Program, I" with C. E. Blair, accepted for Discrete Applied Mathematics.

6. "Extensions of a Theorem of Balas," with C. E. Blair, accepted for Discrete Applied Mathematics.

C. Submitted for Publication

7. "Constructive Characterizations of the Value Function of a Mixed Integer Program, II," with C. E. Blair.

8. "Modelling with Integer Variables," with J. Lowe.

9. "Computational Complexity of Some Problems in Parametric Discrete Programming, I" with C. E. Blair.

10. "The Polynomial Hierarchy and a Simple Model for Competitive Analysis."

11. "Experimental Results on the New Techniques for Integer Programming Formulations," with J. Lowe.

D. Preliminary Report

12. "Sensitivity Analysis in Mixed- Integer Programming via Subadditive Families."

Technical Description of Project and Results

During the grant period, we have pursued research activities in these four topics: semi-infinite programming (items 2, 3 and 4 of "Papers Written Under This Grant," page 3 above); value functions (items 1, 5, 7 and 12); computational complexity (items 9 and 10); and modelling problems as mixed-integer programs (items 6, 8 and 11).

In this Technical Description we will summarize our work in these four topics. We shall recapitulate main results in specific papers, add general comments and perspectives, and relate some more recent work which was in progress as of the end of the grant period (December 1983) and which continues to date.

In the last sixteen to eighteen months, and primarily in the last eight months, we have experienced several experimental successes using the new techniques for modelling with integer variables. At the same time, we have been reading in the literature on decision support, including the techniques in artificial intelligence, and see approaches to combining our modelling work with the other approaches. We will relate some of these ideas in Section IV below, when we discuss modelling.

It now seems likely that methodological work on decision support will become a major theme for my work in the next few years, although it was not anticipated as such in my grant proposal of Fall 1982.

I. Semi-infinite Programming

My work on semi-infinite programming has involved sharpening some basic knowledge about duality in the linear case (item 2 of "Papers Written Under This Grant) and about the dual for the linear case (item 4), as well as a substantively new type of result on duality in the convex case (item 3). We also have generalized the linear primal-dual pair in our personal notes (see below), but this work has been temporarily put aside.

In "Duality in Semi-infinite Linear Programming" (item 2), we provide a basic principle for semi-infinite linear programming. We used this principle heavily in item 3.

The paper is concerned with conditions, which are necessary as well as sufficient, for there to be no duality gap between a consistent semi-infinite linear program

$$\begin{aligned} &\inf\ cx \\ \text{s.t.}\ \ &a^i x \geq b_i,\ i \in I \end{aligned}$$

and its dual

$$\begin{aligned} &\max \sum_{i \in I} \lambda_i b_i \\ \text{s.t.}\ \ &\sum_{i \in I} \lambda_i a^i = c \\ &\lambda_i \geq 0,\ i \in I \\ &\{i \mid \lambda_i > 0\} \text{ is finite} \end{aligned}$$

for <u>all</u> objective functions $c = (c_1, \ldots, c_n) \in R^n$. In the above, the $a^i \in R^n$, the λ_i and b_i are scalars, and I is an index set of arbitrary cardinality. (Of course, when $|I|$ is finite, the above becomes a linear program and its dual).

The need for special hypotheses to insure no duality gap (for $|I|$ infinite) has been known for some time (Duffin and Karlovitz [1965]; Charnes, Cooper and Kortanek [1965]. The content of this paper is that a small extension of commonly-used sufficient conditions are also necessary, when all possible objective functions are of concern.

The semi-infinite programming format has been used as one approach toward convex optimization in numerous theoretical and numerical settings. The volume in which 2. appear illustrates this and discusses applications.

In "Cluster Sets of Vector Series," we explored whether or not the finiteness restriction in the dual semi-infinite program, on the set $\{i\varepsilon I \mid \lambda_i > 0\}$ of positive multipliers, was really significant. I.e., do these two programs have the same value?:

$$\begin{array}{ll} \max \sum_{i\varepsilon I} \lambda_i b_i & \\ \text{s.t. } \sum_{i\varepsilon I} \lambda_i a^i = c & \\ \qquad \lambda_i \geq 0,\ i\varepsilon I & \\ \quad \{i \mid \lambda_i > 0\} \text{ is finite} & \end{array} \quad \text{and} \quad (D_\infty) \quad \begin{array}{l} \max \sum_{i\varepsilon I} \lambda_i b_i \\ \text{s. t. } \sum_{i\varepsilon I} \lambda_i a^i = c \\ \qquad \lambda_i \geq 0,\ i\varepsilon I \end{array}$$

In fact, they do, and so the finiteness restriction is not of substance.

There is some earlier related work in which the finiteness restriction is relaxed - see, for example, the discussion of "ideal convexity" in Holmes [1975] - but then other irrelevant conditions are added (which often amount to $\sum_{i\varepsilon I} |\lambda_i| < +\infty$). Moreover, we did our analysis of this question in a much broader setting, that of "semi-convergence" of a formal series $\sum_{i\varepsilon I} \lambda_i a^i$ to a set over an index set I with a net structure. For I = {1,2,3,...} the integers with the usual net structure, we would say that $\sum_{i\varepsilon I} \lambda_i a^i$ converges to S if, for every integer n and $\varepsilon > 0$, there is $m > n$ with $\| \sum_{i=1}^{m} \lambda_i a^i - v \| < \varepsilon$ for some $v\varepsilon S$. The case S = {c} of a singleton set gave our results on (D) and (D_∞). Note that, with this definition of semi-convergence, the sum of two semi-convergent multipliers $\{\lambda_i \mid i\varepsilon I\}$ need not be semi-convergent, so a direct line of argument is needed.

Our main result in 4 states this, for S a compact set: if there exists a formal series $\sum_{i\epsilon I} \lambda_i a^i$ (all $\lambda_i \geq 0$) which semi-converges to S, then there are some other multipliers $\{\lambda_i' | i\epsilon F\}$ for some finite subset $F \subseteq I$ such that $\sum_{i\epsilon F} \lambda_i' a^i \epsilon S$. As mentioned in 4, the result is still true if $S = C \times P$ where C is compact and P a polyhedron, and the result is not true for arbitrary sets S.

In "Uniform Duality in Semi-infinite Convex Optimization" (item 3), we are concerned with the consistent semi-infinite program:

(CP)
$$\begin{aligned} &\min f_0(x) \\ &\text{s.t. } f_h(x) \leq 0, \quad h\epsilon H \\ &\qquad x\epsilon K \end{aligned}$$

where H is an index set of finite or of infinite cardinality, f_h for $h\epsilon\{0\} \cup H$ is a closed convex function, and K is a closed convex set, and some other mild hypotheses (which amount simply to consistency, if $K=R^n$) are made which are weaker than the "Slater point" condition.

The Lagrangean dual for (CP) we consider is:

(LD)
$$\sup_{x\epsilon\Lambda} \inf_{x\epsilon K} \{f_0(x) + \sum_{h\epsilon H} \lambda_h f_h(x)\}$$

where $\Lambda = \{\lambda \geq 0 \mid \lambda_h$ is positive only for finitely many $h\epsilon H\}$, and the paper focuses on necessary and sufficient conditions for there to be no gap in duality between (CP) and (LD) for <u>all</u> convex objectives $f_0(x)$ on R^n. The lack of a gap for all f_0 is called <u>uniform convex duality</u>.

To illustrate the idea of the main result, we consider the case that $K = R^n$ and there is a Slater point (i.e. an x^0 with $f_h(x^0) < 0$ for all $h\epsilon H$), although such assumptions are not made in the main result.

A set of functions $\{g_p \mid p \varepsilon P\}$ is called a <u>positive</u> <u>derivant</u> for the constraints of (CP) if two conditions hold: (i) The set of feasible solutions is the same (i.e. $g_p(x) \leq 0$ for all $p \varepsilon P$ if $f_h(x) \leq 0$ for all $h \varepsilon H$); and (ii) For each function $g = g_p$, $p \varepsilon P$, <u>either</u>

(ii.1) $$g(x) \leq 0 \text{ for all } x \varepsilon R^n$$

or

(ii.2) $$g(x) \leq \max_{h \varepsilon H'} f_h(x) \quad \text{for all } x \varepsilon R^n,$$

where in (ii.2) H' is some <u>finite</u> subset of H.

A set of functions $\{h_p \mid p \varepsilon P\}$ is called <u>compact</u> if it is compact in the topology of pointwise convergence. I.e. it is compact if, for any sequence of functions h_{p1}, h_{p2}, ... drawn from $\{h_p \mid p \varepsilon P\}$, there is an index $p^* \varepsilon P$ and a subsequence $h_{pn(1)}$, $h_{pn(2)}$ ... with:

(C) $$h_{p*}(x) = \lim_{k \to +\infty} h_{pn(k)}(x) \quad \text{for all } x \varepsilon R^n$$

Any finite set of functions ($|P|$ finite) is clearly compact.

Here is our necessary and sufficient condition for uniform convex duality in the case considered ($K=R^n$, a Slater point exists): there is a positive derivant of closed, convex functions $\{g_p \mid p \varepsilon P\}$ for the constraints of (CP), a point $\bar{x} \varepsilon R^n$, and positive scalars $\beta_p > 0$ for $p \varepsilon P$, with $\{\beta_p g_p \mid p \varepsilon P\}$ compact. Also we show that, when uniform convex duality holds, the positive derivant can be taken with all g_p, $p \varepsilon P$, linear affine.

For $|H|$ finite in this case, it is a basic result of convex optimization that uniform convex duality holds. That result follows directly from our main result (take $P = H$, $g_p = f_h$, $\bar{x} = x^0$, and all $\beta_p = 1$).

The essential <u>novelty</u> here is that a compactness condition on functions replaces finiteness in the case of $|H|$ infinite, and for general K and simple consistency (i.e. no Slater point) we have necessary, as well as sufficient, hypotheses. The essential complication for $|H|$ infinite, when necessary and sufficient conditions are desired, is that we have to consider possible "problem reformulations" - i.e. positive derivants - and this is a somewhat technical idea.

Examples are given in 3. to illustrate the need for both clauses ii.1) and ii.2) above. Moreover, the precise results become further obscured by technical details in the case of a general closed, convex set K. See 3. for details.

In our notes, we have explored a "symmetric dual" for semi-infinite linear programming. Here we are embarked upon "remedying" the fact that the usual primal-dual pair are not symmetric (i.e. the dual of the dual is not even defined - so certainly it cannot be the primal!) Our main decision thus far is that the semi-infinite dual is the more fundamental object. We have sought symmetry by generalizing the dual.

Let two closed, convex sets U land V be explicitly given representations $U = \{x \in R^n \mid a^i x \geq b_i,\ i \in I\}$ and $V = \{x \in R^n \mid \alpha^j x \leq \beta_j, j \in J\}$ for index sets I and J (with a^i, $\alpha^j \in R^n$ and all b_i, $\beta_j \in R$). The U-V program involves the V-representation, and it is:

$$\text{(U-V)} \qquad \begin{aligned} &\inf \sum_{j \in J} \Theta_j \beta_j \\ &\text{s.t.} \sum_{j \in J} \Theta_j \alpha_j \in U \\ &\Theta_j \geq 0,\ j \in J \\ &\{j \in J \mid \Theta_j > 0\} \text{ is finite} \end{aligned}$$

Its dual is the V-U program, which involves the U-representation, and is:

$$\text{(V-U)} \qquad \begin{aligned} &\sup \sum_{i\varepsilon I} \lambda_i b_i \\ &\text{s.t.} \sum_{i\varepsilon I} \lambda_i a^i \ \varepsilon\ V \\ &\qquad \lambda_i \geq 0,\ i\varepsilon I \\ &\qquad \{i \mid \lambda_i > 0\} \text{ is finite} \end{aligned}$$

The dual of the U-V program is then the V-U program, and conversely.

For $V = \{c\}$, with explicit representation $V = \{x \ \varepsilon\ R^n \mid x_k \leq c_k$ and $-x_k \leq -c_k$ for $k=1, \ldots, n\}$ we recover the usual semi-infinite primal in (U-V). In fact, if either one of the index sets I or J are finite, there are easy algebraic manipulations which reduce the dual pair above to the usual linear semi-infinite study. However, for general (i.e. non-polyhedral) U and V, this appears to be a new construction. It can be shown that the optimal value of (V-U) never exceeds that of (U-V).

To date, we have a number of results on equality of optimal values in (U-V) and (V-U) which tend to confirm the appropriateness of our construction, but furthur work is needed for full confirmation.

II. Value Functions

During the previous grant period, we explored several issues regarding the optimal value function $z(b) = \min \{cx \mid Ax = b,\ x \geq 0$ and integer$\}$ of a pure integer program in Blair and Jeroslow - [1982]. The work continues into this grant and to the present. The work reported here is concerned primarily with the issue of characterizing the class of functions involved by an inductive definition insofar as possible. Our most recent work (now underway) is directly at practical implementation of right-hand-side sensitivity analysis in mixed-integer programming; however, some theoretical issues also remain.

In "Some Influences of Generalized and Ordinary Convexity in Disjunctive and Integer Programming," which is item 1 of "Papers Written Under This Grant" (page 3 above), we are concerned with an analogy between linear and integer programming. This analogy is primarily in terms of results on value functions and duality, and the intuitive process behind much of the work in Blair and Jeroslow [1982]. The process of analogy has been since carried furthur; see below.

Recall that a Chvatal function is obtained from starting with linear functions, and then iteratively applying the process of rounding-up to the nearest integer ($\lceil x \rceil$ = round-up of x) and taking nonnegative combinations. For example, $f(b_1, b_2) = \lceil 2 \lceil b_1 - 3b_2 \rceil + \lceil b_2 \rceil \rceil + b_1 - b_2$ is a Chvatal function, and it has a carrier $\bar{f}(b) = 2(b_1 - 3b_2) + b_2 + b_1 - b_2 = 3b_1 - 6b_2$ obtained by erasing round-ups. The carrier is of course a linear function, and there will be a bound $k > 0$ such that $0 \leq \bar{f}(b) - f(b) \leq k$ for all $b = (b_1, \ldots, b_m)$ (here $m = 2$). A Chvatal function is a discrete analogue of a linear function. It is a "linear function with bumps," but the "bumps" must be strategically placed to make what follows be valid.

Now just as the ordinary linear program

$$\begin{array}{lcl} \min\ cx & & \max\ \lambda b \\ \text{s.t. } Ax = b & \text{has the dual} & \text{s.t. } \lambda a^{(j)} \leq c_j \\ x \geq 0 & & \qquad j = 1, \ldots, n \end{array}$$

where $c = (c_1, \ldots, c_n)$ and $A = [a^{(j)}]$ (cols), the linear integer program

$$\begin{array}{lcl} \min\ cx & & \max\ f(b) \\ \text{s.t. } Ax = b & \text{has the dual} & \text{s.t. } f(a^{(j)}) \leq c_j \\ x \geq 0 & & \qquad j = 1, \ldots, n \\ x \text{ integer} & & \end{array}$$

with f Chvatal. When the integer program is consistent, it has the same value as its dual. Just as the ordinary linear program has a value function $F(b) = \min \{cx | Ax = b, x \geq 0\}$ which is a maximum of linear functions, the linear integer program has a value function $G(b) = \min \{cx | Ax = b, x \geq 0, x \text{ integer}\}$ which is a maximum of Chvatal functions, called a Gomory function.

There are other 'integer analogues' of linear programming results given in 1., and also some limitations of the analogue process are cited there. Our interest in the analogue process, is that it provides an intuitive way of conjecturing results, although it does not provide proofs and some analogues must be refined in order to be valid. We have been interested in how far the analogy process can be taken, for we believe that pursuing it furthur will lead to new insights about mixed-integer programs.

Here is one instance where the analogy process encountered a "snag" in terms of some unexpected technical conditions. If F(b) is a polyhedral function (a maximum of linear forms in b), there will be a matrix A such that:

$$Ax = b, \ x \geq 0 \quad \Longleftrightarrow \quad F(b) \leq 0$$

is consistent.

However, the 'integer analogue' of this result had the technical condition that b be an integer vector. I.e., if G is a Gomory function then there is a matrix A such that:

$$Ax = b, \ x \geq 0 \text{ integer} \quad \Longleftrightarrow \quad G(b) \leq 0 \text{ and } b \in Z^m$$

is consistent.

When we dropped the integrality condition "$b \in Z^m$" we found that it described certain mixed-integer constraint sets $Ax + By = Cb$. I.e., if G is a Gomory function then there are rational matrices A, B, C such that

(*)

$$\begin{array}{l} Ax + By = Cb; \\ x, y \geq 0; \ x \text{ integer} \quad \Longleftrightarrow \quad G(b) \leq 0 \\ \text{is consistent.} \end{array}$$

A converse also holds: for rational A, B, and C a Gomory function G exists with the above bi-conditional satisfied. This suggests the study of such "premultiplied mixed integer programs" or PMIP's - the adjective "premultiplied" noting the occurrence of the matrix C. Unlike pure integer programming or ordinary linear program, a (generally noninvertible) matric C must occur. Such constraints are familiar from practice: generally the row of a material balance equation must have a right-hand-side of zero, although other r.h.s. may be permitted to vary.

At the same time, in trying to inductively characterize the optimal value functions $z(b) = \min\{cx + dy \mid Ax + By = b;\ x,\ y \geq 0;\ \text{integer}\}$ of mixed-integer programs we had encountered the difficulty that this class of functions is not closed under any of the operations involved in Gomory functions - the class is not closed under addition, or maximum, or the taking of integer round-ups. For example, the distance to the nearest integer is an MIP value function

$z(b) = \min\{y_1 + y_2 \mid x_1 - x_2 + y_1 - y_2 = b;\ x_1,\ x_2,\ y_1,\ y_2 \geq 0;\ x_1$ and x_2 integer$\}$ but its round-up $\lceil\ z(b)\ \rceil$, which is zero for $b \in Z$ and one for $b \notin Z$, can easily be seen not to be the value function of a mixed-integer program (as value functions have finite directional derivatives at the origin). Of course, $\lceil\ z(b)\ \rceil$ is the value function of the PMIP $\min\{x_3 \mid x_3 - y_1 - y_2 \geq 0,\ x_1 - x_2 + y_1 - y_2 = b;\ x_1,\ x_2,\ x_3,\ y_1,$ $y_2 \geq 0;\ x_1,\ x_2,\ x_3$ integer$\}$ in which one r.h.s. is to be fixed at zero.

These facts suggested to us that, instead of seeking an inductive structure for MIP value functions, we seek one for PMIP value functions. Also, we knew that we must look in a larger class of functions than the Gomory functions, since the distance function z(b) above is not Gomory (a nontrivial fact). Nevertheless, we did not seem to need a function class which was far in <u>distance</u> from Gomory functions.

Since the analogy process with just linear programs appeared to have ran out, we sought analogies with more general convex programs. In convex programming the <u>infimal</u> <u>convolution</u> operation occurs, and indeed it did turn out to be the case that the infimal convolution of Gomory functions provides precisely the class of value functions of PMIP's (including the value function $G(b) \equiv -\infty$ which is identically $-\infty$), and issues regarding the domain of definition can be treated by Gomory functions (see 7. for precise results).

We call these infimal convolutions "mixed-Gomory functions," and they are an inductively-defined class: i.e., the infimal convolution of mixed-Gomory functions is mixed-Gomory. Also, in terms of distance, these functions are "nearly Gomory functions." I.e., if G is mixed Gomory and G(b) is finite for some b (i.e. we rule out only the $G(b) \equiv -\infty$ function), then for any integer D there exists a Gomory function H(b) such that $0 \leq H(b) - G(b) \leq 1/D$ for all b.

While our results do provide a characterization of PMIP value functions, they are not entirely satisfactory from the perspective of the analogue process. For example, they do not given an "adequate" analogue <u>dual</u> for a PMIP or an MIP, since mixed-Gomory functions are <u>not</u> "linear with bumps": they can get very far from any linear "approximation."

In the pure integer case, Chvatal functions, which can be thought of as "subgradients" to Gomory functions, provide an adequate dual. What serves as "subgradients" to mixed-Gomory functions? - as judged by duality, for example. While this question is admittedly imprecise, it illustrates how the analogy process continues to guide our work in value funtions.

We also found a second characterization of PMIP value functions: they are the finite minima of Gomory functions which turn out to be subadditive. This characterization is interesting, but not inductive.

We also algorithmically identified that (non-inductive) class of Gomory functions G which in (*) allow one to take C = I for some A, B - i.e. which test for the consistency of MIP's, as opposed to PMIP's. While our algorithm is finite, it is fairly complex. The details are in 5.

Toward the end of the grant period, we initiated work on the uses of subadditive concepts and constructions for practical sensitivity analysis in mixed-integer programs. In item 12. we showed how a subadditive function construction allowed the generation and retention of information gained at one node of branch-and-bound tree, for use at other nodes and in other search trees for different right-hand-sides. This infomation will improve the fathoming capabilities in later runs for other r.h.s. and reduce the size of later search trees. The work on this topic continues.

III. Computational Complexity

Our papers in computational complexity address different issues. In 9. of "Papers Written Under This Grant," we are concered with the P versus NP classification as in Cook [1971], Garey and Johnson [1979] and Karp [1972] in a discrete programming setting. In contrast, 10. is focused primarily on the polynomial hierarch above NP as a means of gaining insight into certain types of competitive (game-theoretic) behavior and related modelling issues.

In "Computational Complexity of Some Problems in Parametric Discrete Programming, I," we initiated our study of the computational complexity associated with questions that concern families of programs, rather than specific programs. Our main discoveries were certain questions whose complexity remained as low as P or NP, together with one example (concerning subadditive duality) to show that this low complexity is not usually to be expected. In a follow-up paper with C. E. Blair, we plan to explore some higher-complexity questions. We chose parametric programming issues to provide the different program families, in part because the "what if?" questions in mixed-integer programming practice are often of a parametric type.

Among the results of 9. are these: the question, as to whether or not two linear programs have the same optimal value for <u>all</u> right-hand-sides, remains in P; the question, as to whether two linear integer programs have the same optimal value for <u>all</u> right-hand-sides, remains in NP and is in fact NP-complete. Yet we also saw that complexity could jump suddenly by a relatively small change of the question: if certain right-hand-sides were to be fixed at zero, and we wished to know if the two <u>linear</u> programs had the same optimal value for all other r.h.s. variation, the complexity became NP-complete. In fact, NP-completeness is the complexity for r.h.s. restricted to cones which are described by their generators (a class which includes zero settings for specific r.h.s.).

In 9. we also asked the optimal value questions in function form (as well as the matrix form above). For example, we discovered that the question, as to whether two Gomory functions had equal value for all arguments, is NP-complete; and the same is true for two Chvatal functions. In the process of these proofs, we developed lemmas on the computational complexity of systems of inequalities in Gomory functions, and of programs with a bilinear constraint.

In contrast to the low complexity results cited above, we showed that the size of a Chvatal function which is optimal in the subadditive dual of a consistent and bounded integer program, is not necessarily polynomial in the size of the primal program. The primal programs used in 9. for our proof are very simple, so that the result indicates a very serious obstacle to the numerical use of subaddtive methods as the sole solution approach for general integer programs. However, for practitioners this new obstacle only furthur confirms the current tendency toward hybrid algorithms, as some integer programs are crucially aided by subadditive cuts (see Jeroslow [1974a]). However, our result does serve to show that some questions of integer programming - not of a parametric nature - can jump completely out of the polynomial hierarchy (as PSPACE is the union of the hierarchy, see Stockmeyer [1977]).

In "The Polynomial Hierarchy and a Simple Model for Competitive Analysis," we addressed a fundamental issue in the modelling of game-theoretic situations. For a number of years, we have sought to understand why advances in competitive analysis by quantitative methods, particularly those related to numerical solutions and decision support, were so few, rare and specialized.

Chvatal [1978] had shown that a very simple co-operative game was already NP-complete. NP-completeness, while challenging to deal with numerically, is of the same order of magnitude as the usual optimization models, such as mixed-integer programming, which do not involve other intelligent players. We had suspected that the intelligence of the other players would raise complexity beyond that seen before in quantitative method settings. Since the P=NP? question remains unresolved, as well as many other questions dealing with polynomial hierarchy of Meyer, Stockmeyer and Karp (see Stockmeyer [1977]), we cannot be sure that the distinctions on computational difficulty will not someday disappear. However, that hierarchy and other complexity measures now seem to be among the most fundamental constructions available to us, and minimally these constructions can relate our concerns in competitive analysis to basic questions of concern in the deepest investigations currently underway.

In 10. we studied a sequenced-move game with perfect information, so that from a game perspective it has a simple structure. The interaction between the players consists of a common set of linear constraints which is imposed on the total set of continuous variables, and linear objective functions in which a later-moving player can influence an earlier one's payoff. (The case of p=1 player is simply linear programming). We showed that the problem, of determining only approximately the optimal value to the first-moving player, is at least at level p of the polynomial hierarchy when there are (p+1) players. For two players, we placed the game at exactly level one, which of course is NP-completeness.

The two-player case had arisen in the practical setting of policy setting by a government agency (first-moving player), in view of the anticipated reaction of the firms affected by the regulation (aggregated together for simplicity, and treated as a second-moving player). It was observed empirically that one gets dramatically different policy prescriptions if one does not treat the firms as having an intelligent "rational reaction" to regulations (see Candler and Townsley [1982] and their references). In this light, it is a good development that the two-player case is only NP-complete. However, it seems to us to be one of the harder NP-complete problems; our work continues for this case with a graduate assistant who serves as a part-time programmer.

Of course, the enormous complexity of even these simple competitive situations brings into question the practical utility of basic solution concepts, such as Stackleberg equilibria (see Stackleberg [1934]). After all, a player cannot implement a move s/he does not know (numerically). Neither will other players implement such strategies, so the imperative to do so disappears.

Particularly the specializations in the business areas - such as marketing, finance, and production - are affected by the limitation to models which cannot realistically treat competitive responses. This issue becomes quite serious in those business problems where the firm's actions are quickly visible externally and there is a short lead time on competitive reaction. This unfortunate situation can, we feel, be remedied in some ways by a substantial re-thinking of solution frameworks and concepts, as well as new results.

IV. Modelling Using Integer Variables

It is generally viewed that there are standard formulation techniques for modelling various practical problems as mixed-integer programs, which have been known for over twenty years and have long been in textbooks. Many believe that, while important representation issues arise here and there in practice, they are largely heuristic and don't seem to yield to methodological study. During this grant, we discovered that the conventional view above is mistaken and that the implications of correcting it are very substantial when one needs numerical solutions. While many results about the properties of the new representation techniques may require advanced mathematical background, sophisticated use of these techniques can quickly be learned by technically-trained individuals at the masters' level (as in a course now underway in our School of Industrial and Systems Engineering).

The importance of tight linear relaxations for mixed-integer formulations had been noted in Geoffrion and Graves [1974] and H.P. Williams [1974], and is stressed again in more recent work (e.g. Johnson, Kostreva and Suhl [1982]). However, a systematic theory for obtaining many of them was lacking, which we now can provide. Also, we have been able to clarify when a condition has an MIP-representation, and we have been able to make furthur distinctions which seem necessry, such as sharpness, hereditary sharpness, variable co-ordination, and the value of distributivity in the lattice of MIP representations (see below). We are in the process of taking up furthur matters in the representation theory (see below). In our work, we have heavily used concepts and results of the disjunctive methods (see Balas [1974], [1979], [1983] and Jeroslow [1974b], [1977]).

The first question concerning representability, is whether a given (generally nonlinear) function or (generally nonconvex) set can be represented using linear constraints in continuous and integer variables. Actually, until the research of R. R. Meyer [1975], [1976] it was not widely appreciated that there were substantial issues in connection with the existence of representations.

For simplicity in our discussion below, we will observe three restrictions: (1) We will consider only representations in rational data, since that covers the implementation needs; (2) We will consider only the use of binary (equivalently, bounded) integer variables in a representation, as the case of a truly general integer variable (with no bound) is quite rare in practice; (3) When functions are discussed, we shall assume that they appear only in a minimizing objective function, and not in the constraints.

If a single variable x is known to be restricted to a bounded interval $[0,B]$ and a fixed charge of $c > 0$ is assessed when $x > 0$, this is viewed as modelling the function

$$f(x) = \begin{cases} 0 , & x = 0; \\ c , & 0 < x \leq B. \end{cases}$$

As is well known, this function f - appearing only in a (minimizing) criterion in an occurrence "+f(x)" - can be modelled by introducing a binary variable $z \in \{0, 1\}$, putting the term "+cz" in place of "+f(x)" in the criterion, and adding this new constraint:

$$0 \leq x \leq Bz$$

Now suppose that, in place of a charge, a benefit of $c > 0$ is obtained if $x > 0$. Then we must model this function:

$$f(x) = \begin{cases} 0 , & \text{if } x = 0; \\ -c , & \text{if } 0 < x \end{cases}$$

(E.g., a manufacturer offers a one-time rebate for trying his product). This function cannot be modelled even if a bound $x \leq B$ is added. The fact that it cannot be modelled is seen from these necessary and sufficient conditions for an (bounded) integer modelling, from item 8 of "Papers Written Under This Grant": the epigraph $\text{epi}(f) = \{z,x) \mid z \geq f(x)\}$ of f is to be a finite union of polyhedra $\text{epi}(f) = P_1 \cup \ldots \cup P_t$ with the same recession directions ($\text{rec}(P_i)$ is independent of i, $1 \leq i \leq t$). In fact, for the fixed benefit function above, epi(f) is not closed (as $(-c/2, 0) \in \text{cl(epi)} \setminus \text{epi}(f)$) so it cannot be a finite union of polyhedra.

If we change the fixed benefit function to have strictly positive minimum usage level $\delta > 0$ (e.g. the rebate requires that at least δ be purchased), we can consider this function:

$$f(x) = \begin{cases} 0\ , & \text{if}\ \ 0 \leq x < \delta; \\ -c\ , & \text{if}\ \ x \geq \delta. \end{cases}$$

Then $\text{epi}(f) = P_1 \cup P_2$, where $P_1 = \{(z,x) \mid x \geq 0,\ z \geq 0\}$, $P_2 = \{(z,x) \mid x \geq \delta,\ z \geq -c\}$. Moreover, the two polyhedra have P_1 as common recession directions. Hence, there is a model for f - we will give one below.

In yet another setting of a multi-period production problem, a fixed charge is to be assessed for set-up if a manufacturing process is to be run this period ($y > 0$) and it was idle last period ($x=0$). However, if it was run last period ($x > 0$) then no charge is assessed. For simplicity, let a known bound B apply when the process is run. We can conceptualize this as representing this function

$$f(x,y) = \begin{cases} 0\ , & \text{if}\ \ x > 0,\ x \leq B,\ 0 \leq y \leq B; \\ 0\ , & \text{if}\ \ x = 0 \text{ and } y = 0; \\ c\ , & \text{if}\ \ x = 0 \text{ and } 0 < y \leq B \end{cases}$$

If we do so, there is no representation, since epi(f) is not closed (for $c > 0$, $B > 0$, $(z,x,y) = (0,0,B) \in \text{cl}(\text{epi}(f))$ yet $(0,0,B) \notin \text{epi}(f)$). On the other hand, if we recognize a minimum level $\delta > 0$ for just x, we can consider this function:

$$g(x,y) = \begin{cases} 0\ , & \text{if } x \geq \delta,\ x \leq B, \text{ and } 0 \leq y \leq B; \\ 0\ , & \text{if } x = 0 \text{ and } y = 0; \\ c\ , & \text{if } x = 0 \text{ and } 0 < y \leq B \end{cases}$$

(Here values of x between 0 and δ are not permitted). This latter function is representable, since $\text{epi}(g) = P_1 \cup P_2 \cup P_3$, where $P_1 = \{(z,x,y) \mid x = 0,\ y = 0,\ z \geq 0\}$, $P_2 = \{(z,x,y) \mid x = 0,\ 0 \leq y \leq B,\ z \geq c\}$, $P_3 = \{(z,x,y) \mid x \geq \delta,\ x \leq B,\ 0 \leq y \leq B,\ z \geq 0\}$ and all recession sets $\text{rec}(P_i) = \{(z,x,y) \mid x = y = 0,\ z \geq 0\}$ are the same.

In the practical applications, of course, the variables involved generally have both upper bounds and minimum levels. However, it takes a knowledge of representability to know if these data are to be used (and how). A lack of such knowledge has sometimes led to erroneous formulations which may or may not be later detected. Typically, such errors model the smallest representable set or function containing the nonrepresentable one, which in the case of the nonrepresentable multi-period function f above, would be the identically zero function (with no fixed charge).

After one has treated the question of existence of a representation, one encounters the second issue of providing a representation when one exists. We have several methods for doing so. We will next illustrate one of these methods - the "extreme point formulation" - for the fixed benefit function f above with upper bound B on x and with minimum usage level $\delta > 0$.

We have $\text{epi}(f) = P_1 \cup P_2$, and this method proceeds (for functions on bounded domains) by writing down the polyhedra and then listing their extreme points, each of which is then assigned a multiplier:

Polyhedron	Extreme Points	Multiplier
$P_1 = \{(z,x) \mid x \geq 0,\ z \geq 0,\ x \leq B\}$	(0,0)	λ_{11}
	(0,B)	λ_{12}
$P_2 = \{(z,x) \mid x \geq \delta,\ z \geq -c,\ x \leq B\}$	$(-c,\delta)$	λ_{21}
	(−c,B)	λ_{22}

Multiplier $\lambda_{ij} \geq 0$ is assigned to the j-th extreme point of the i-th polyhedron P_i denoted v_{ij}. The polyhedron P_i is assigned a binary 'control variable' λ_i ($\lambda_i = 1$ indicates that $(z,x) \in P_i$). One writes equations $\lambda_i = \sum_j \lambda_{ij}$ so that, when $\lambda_i = 0$ all $\lambda_{ij} = 0$; while when $\lambda_i = 1$ the λ_{ij} are coefficients from a convex combination. Then the equation $(z,x) = \sum_i \sum_j \lambda_{ij} v_{ij}$ assures that (z,x) is the convex combination for the i with P_i 'activated' (i.e. with $\lambda_i = 1$). In this example, we obtain this representation:

$$\lambda_1 = \lambda_{11} + \lambda_{12} \qquad \lambda_1 + \lambda_2 = 1$$

$$\lambda_2 = \lambda_{21} + \lambda_{22} \qquad \lambda_1\ ,\ \lambda_2 \text{ binary}$$

$$\text{all } \lambda_{ij} \geq 0$$

$$(z,x) = \lambda_{11}(0,0) + \lambda_{12}(0,B) + \lambda_{21}(-c,\delta) + \lambda_{22}(-c,B).$$

After we take components in the last equality, we would enter $z = \lambda_{11}\cdot 0 + \lambda_{12}\cdot 0 + \lambda_{21}(-c) + \lambda_{22}(-c)$ or $-\lambda_2 c$ for f(x) in the objective function, while retaining as a constraint $x = \lambda_{12} B + \lambda_{21}\delta + \lambda_{22}B$.

When a function is on an unbounded domain, one must lastly add in recession directions of the epigraph of f in the (z,x) equation.

For example, if there is no upper bound B on x in the representable fixed benefit problem, the extreme points (0,B) and (−c,B) do not occur, but the recession direction (0,1) occurs (the recession direction (1,0) can be omitted in a minimizing objective function). One then obtains this representation, which greatly simplifies:

$$\lambda_1 = \lambda_{11} \qquad \lambda_1 + \lambda_2 = 1$$

$$\lambda_2 = \lambda_{21} \qquad \lambda_1 , \lambda_2 \text{ binary}$$

$$\lambda_{11}, \lambda_{12}, \sigma \geq 0$$

$$(z,x) = \lambda_{11}(0,0) + \lambda_{21}(-c,\delta) + \sigma\,(0,1)$$

Here we can disregard λ_1 and λ_{11}, call $\lambda_2 \equiv \lambda$, and remove λ_{21} in favor of λ. We get the constraint $x = \lambda\,\delta + \sigma$ with $\sigma \geq 0$, λ binary and with $(z=) - \lambda\,c$ to be entered in the objective function for $f(x)$. (If $x \geq \delta$, it is true that these constraints allow a feasible solution with $\lambda = 0$, $x = \sigma$. However, as $c > 0$, in a minimizing objective we would have at optimality $\lambda = 1$, $\sigma = x - \delta$ if $x \geq \delta$).

A third issue now arises, of developing standards for choosing among various representations. One such standard is what we call 'sharpness': a representation is sharp, provided it has as tight (small) as possible a linear relaxation (LR). (The linear relaxation is the linear program arising, when all binary variables are relaxed to be in [0,1]). The linear relaxation of the representation for a function f must at least contain its epigraph, since that arises for binary values alone; as it is linear, it must at least contain the closed, convex span of epi(f). This issue was first taken up in Meyer [1981], and in item 8. above we showed that all our formulation techniques - such as the 'extreme point formulation' above - are sharp. We also showed that sharp representations give exactly the convex span of the epigraph, which for representable functions will be closed.

Sharp representations are desirable, since most MIPs from industrial problems are solved by branch-and-bound codes, which use the LR as a relaxation and as a guide for the search procedure. Most of the textbook formulations of the 'standard' one-dimensional functions, such as simple fixed charges or continuous piecewise-linear functions, are sharp. But in several important cases, the usual treatments recommend very poor formulations.

Consider, for example, either/or constraints. Suppose two variables x_1 and x_2 are always contrained by $0 \leq x_1, x_2 \leq 200$ and that at least one of these two contraints are to hold:

$$\text{either } -x_1 + x_2 \geq 100 \quad \text{or} \quad x_1 - x_2 \geq 100$$

Since a lower bound on $-x_1 + x_2$ and $x_1 - x_2$ is -200, the standard textbook treatments would model the above via a binary variable z, as follows:

$$-x_1 + x_2 \geq -200 + 300z$$

$$x_1 - x_2 \geq 100 - 300z$$

This is an accurate modelling for z binary: if z = 1 the first inequality $-x_1 + x_2 \geq 100$ holds, while if z = 0 the second inequality $x_1 - x_2 \geq 100$ holds.

However, if z is relaxed to simply be in [0,1], all (x_1,x_2) are possible in the square $0 \leq x_1, x_2 \leq 200$. (Just put $y = x_1 + x_2$ and $z = \max\{0, (100 + y)/300\}$ and $z \in [0,1]$ if (x_1, x_2) is in the square). However, whichever of the two inequalities hold, we will have $x_1 + x_2 \geq 100$, and this latter information is lost in the LR of this recommended formulation (even though best possible bounds were used). This information will <u>not</u> be lost in the relaxation of the sharp formulations.

The importance of a tight LR has been empirically demonstrated in the context of this fixed-charge function, which occurs in the setting of flows of commodities from a potential warehouse. The fact that the warehouse must be built, at a cost $c > 0$, if any flows from it are to be positive, leads to the issue of modelling this fixed-charge function, where the $\mu_k > 0$ are upper bounds on the flows x_k:

$$f(x_1, \dots, x_t) = \begin{cases} 0, & \text{if all } x_k = 0; \\ c, & \text{if any } x_k > 0, \text{ where} \\ & 0 \leq x_k \leq \mu_k \quad \text{for } k = 1, \dots, t \end{cases}$$

Many textbooks recommend that this modelling be used, where λ is a binary varible and "$+c\lambda$" has been entered into the objective function for "$+f(x_1, \dots, x_t)$":

$$x_1 + \dots + x_t \leq (\mu_1 + \mu_2 + \dots + \mu_t)\lambda$$

Some textbooks also note that the following is a modelling:

$$x_k \leq \mu_k \lambda \qquad k=1,\dots,t$$

Despite the fact that the second modelling requires many more constraints than the first, experimenters concur that the second is much superior. In fact, some programs which do not run in an hour with the first representation, run in several minutes with the second. This outcome is attributed in Geoffrion and Graves [1974] to a better linear relaxation for the second, where the issue of easily producing such LR is first raised.

Let us model the function f above by giving one of the sharp representations in item 8 for epi(f). We will use the 'polyhedral representation,' since there are exponentially many extreme points in epi(f).

We write $\text{epi}(f) = P_1 \cup P_2$, where

$P_1 = \{(z, x_1,\ldots,x_t) \mid z \geq 0, x_1 = \ldots = x_t = 0\}$ and

$P_2 = \{(z, x_1,\ldots, x_t) \mid z \geq c, \quad 0 \leq x_k \leq \mu_k$ for $k=1,\ldots,t\}$.

The 'polyhedral representation' arises first by writing down the inequalities for P_i with variables superscripted by 'i,' and with a binary control variable λ_i used to 'homogenize the right-hand-side'. Then we write that each variable from the main program is the sum of the corresponding variables with superscripts. In this case we obtain:

$$x_k^{(1)} = 0 \cdot \lambda_1 \text{ all } k \qquad\qquad 0 \leq x_k^{(2)} \leq \mu_k \lambda_2 \text{ all } k$$

$$z^{(1)} \geq 0 \cdot \lambda_1 \qquad\qquad z^{(2)} \geq c\,\lambda_2$$

$$x_k = x_k^{(1)} + x_k^{(2)} \quad \text{all } k = 1,\ldots,t$$

$$z = z^{(1)} + z^{(2)}$$

$$\lambda_1 + \lambda_2 = 1$$

$$\lambda_1, \lambda_2 \text{ binary}$$

Upon simplification, we have $x_k = 0+x_k^{(2)} = x_k^{(2)}$, hence we obtain $0 \leq x_k \leq \mu_k\lambda_2$. At a minimum $z = c\lambda_2$(as $z \geq 0 + c\lambda_2$). Upon renaming $\lambda=\lambda_2$, our formulation above is identical to the one found to be preferable in experimental work.

In item 11, we conducted two series of experiments on the new formulations, for both either/or constraints and single-variable piecewise linear functions with fixed charges. These experiments also confirmed the advantage of the new formulations. We leave the details to item 11, which also contains other examples worked by the 'extreme point' and 'polyhedral' methods.

A _fourth_ issue arises, as to whether or not sharp representations can retain their sharpness - i.e. can be _hereditarily sharp_ - as a branch-and-bound algorithm proceeds to fix certain values of certain variables. Possibly, a reformulation might be needed at a node lower in the search tree: such reformulations can be needed if one obtains formulations from facets of the convex hull, for example (as a facet of the hull may not even touch the linear relaxation for the setting e.g. of $x_3 = 0$ at some lower node). However, our sharp formulations are hereditarily sharp, and this is true whether or not one uses ordinary branching or (the preferred) special-ordered-set type 1 branching on the constraint $\sum_i \lambda_i = 1$ that links control variables λ_i.

Yet a _fifth_ issue arises when one considers _sums_ of representable functions. One can develop a modelling for two functions f_1 and f_2 _separately_, and enter the two appropriate terms in the objective function, to model $f = f_1 + f_2$. Alternatively, on can _directly_ model f and enter one term for f in the objective; and this latter method is always better in terms of the LR. In fact, by use of the lattice of polyhedra ('meet' being intersection, 'join' being the closed, convex span of the union) one can exactly describe what is achieved by both modelling approaches, and one can prove the second to be better. (We will elaborate on such issues in a forthcoming paper.)

We do not wish to furthur belabor the fact, that the subject of MIP modelling is rich in issues and subleties, and in mathematical structure, some of which we have explored and are continuing to explore. It is also a subject of significance to practice - one in 'the methodology of decision support'.

We conclude this section with a brief discussion of some more recent work in modelling, now in progress.

Certain of the artificial intelligence techniques have been appearing in recent years in a decision support context (such as the MYCIN-like programs discussed in Davis, Buchanan, and Shortliffe [1977]). Whinston and other researchers (for a systematic treatment, see Bonczek, Holsapple and Whinston [1981]) advanced the view that AI techniques can be combined with earlier quantitative techniques of Operations Research in a fruitful way that will improve the technology for decision making. We concur with this view, and are looking for some unification in these two approaches in our current research. The fact that our Ph.D. degree and early research are in mathematical logic has helped our research, particularly since many AI approaches are logic-based. In the last year we have begun extensive reading in AI as well.

We ran some experiments last fall on using MIP to do consistency testing in propositional logic. For example, the clauses of a conjunctive normal form give rise to a generalized set covering problem, which is an IP (integer program). As it turns out, APEX IV solves such problems generally in several seconds to a half-minute, for randomly-generated logic problems in 100 propositional letters with 150 to 600 clauses. (We have also artificially generated harder problems which APEX does not solve in several minutes). Such result are, I believe, significantly better than one expects using AI techniques, such as resolution or other search methods - simply because so much information is in the linear relaxation of the generalized covering constraints (even though the logic representation we used is a textbook one and is not sharp). In any event, we are unaware of problem sets of this size in the literature.

We plan to soon begin work with E. Balas in improving the representation of propositional logic problems. At the same time, we are developing formulations of fragments of the predicate calculus as MIP's and studying some of the issues in this context. Such work appears to provide the kind of decision support developed by Whinston, where the predicate calculus is needed.

Other topics now under investigation include convex representability and hierarchical MIP approaches.

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Publication Activity
July 1980 to December 1983

Papers Published

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6. "A Limiting Infisup Theorem," with C. E. Blair and R. J. Duffin, Journal of Optimization Theory and Applications 37 (1982), pp. 163-175.

7. "Duality in Semi-infinite Linear Programming," with R. J. Duffin and L. A. Karlovitz, Semi-Infinite Programming and Optimization, edited by A. V. Fiacco and K. O. Kortanek, Springer-Verlag, 1983.

8. "Uniform Duality in Semi-infinite Convex Optimization," Mathematical Programming, 27 (1983), pp. 144-154.

Papers Submitted for Publication

1. "Constructive Characterizations of the Value Function of a Mixed Integer Program, II," with C. E. Blair.

2. "Modelling with Integer Variables," with J. Lowe.

3. "Computational Complexity of Some Problems in Parametric Discrete Programming," with C. E. Blair.

4. "The Polynomial Hierarchy and a Simple Model for Competitive Analysis."

5. "Experimental Results on the New Techniques for Integer Programming Formulation," with J. Lowe

DR. ROBERT G. JEROSLOW

January 1984

Personal

Birthdate:	September 13, 1942
Soc. Sec.:	151-32-1110
Citizenship:	USA
Business Address:	College of Management Georgia Institute of Technology Atlanta, Georgia 30332
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Marital Status:	Married to Sondra S. Jeroslow
Children:	David Avram Jeroslow, age 12

Education

B.S. 1964	Columbia University School of Engineering Department of Industrial Engineering
1964 - 1966	Cornell University School of Engineering Department of Operations Research (completed Comprehensive Examination)
Ph.D. 1969	Cornell University Department of Mathematics Professor Anil Nerode, Advisor

Experience

University of Minnesota
School of Mathematics
September 1969 - August 1972
Assistant Professor

Experience (continued)

Carnegie-Mellon University
Graduate School of Industrial Administration and
Department of Mathematics
Associate Professor
September 1972 - August 1976
Professor
September 1976 - June 1980

Georgia Institute of Technology
College of Management
Professor
September 1978 to date

Research Interests

1. Mathematical programming. Emphasis on integer programming model formulations and cutting-plane theory; nonlinear programming; programming aspects of computational complexity. I am interested in developing decision supports which are close to state-of-the-art developments.

2. Strategic Planning. From the perspective of industrial organization, with emphasis on the use of quantitative models as decision supports.

Teaching Interests

Research interests, plus production and applications of Operations Research techniques; management strategy.

Journals

Member of the Editorial Board (Associate Editor), Discrete Applied Mathematics, Mathematical Programming and Mathematical Programming Studies.

Referee for Operations Research, SIAM Journal on Applied Mathematics, Management Science, Mathematical Programming, and Discrete Mathematics.

Grants and Fellowships

Ford A Fellowship 1964-1966
NSF Graduate Fellowship 1966-1969
NSF Research Grant GP 21067, Principal Investigator, 1971-1972
(Grant Awarded 1970)
NSF Research Grant GP-37510X, Associate Investigator, 1973-1975

Published Articles (continued)

3. "On Semi-infinite Systems of Linear Inequalities", with K. O. Kortanek, Israel Journal of Mathematics 10 (1971), pp. 252-258.

4. "A Note on Some Classical Methods in Constrained Optimization and Positively Bounded Jacobians", with K. O. Kortanek, Operations Research 15 (1967), pp. 964-969.

5. "Comments on Integer Hulls of Two Linear Constraints", Operations Research 19 (1971), pp. 1061-1069.

6. "On an Algorithm of Gomory", with K. O. Kortanek, SIAM Journal on Applied Mathematics 21 (1971), pp. 55-59.

7. "Canonical Cuts on the Unit Hypercube", with E. Balas, SIAM Journal on Applied Mathematics 23 (1972), pp. 61-69.

8. "There Cannot be any Algorithm for Integer Programming with Quadratic Constraints", Operations Research 21 (1973), pp. 221-224.

9. "Redundancies in the Hilbert-Bernays Derivability Conditions for Godel's Second Incompleteness Theorem", Journal of Symbolic Logic 38 (1973), pp. 359-367.

10. "Linear Programs Dependent on a Single Parameter", Discrete Mathematics 6 (1973), pp. 119-140.

11. "The Simplex Algorithm with the Pivot Rule of Maximizing Criterion Improvement", Discrete Mathematics 4 (1973), pp. 367-378.

12. "An Exposition on the Constructive Decomposition of the Group of Gomory Cuts and Gomory's Round-off Algorithm", with K. O. Kortanek, Cahiers du Centre d'Etudes de Recherche Operationnelle 2 (1971), pp. 63-84.

13. "Asymptotic Linear Programming", Operations Research 21 (1973), pp. 1128-1141.

14. "On Algorithms for Discrete Problems", Discrete Mathematics 7 (1974), pp. 273-180.

15. "Trivial Integer Programs Unsolvable by Branch and Bound", Mathematical Programming 6 (1974), pp. 105-109.

16. "On Defining Sets of Vertices of the Hypercube by Linear Inequalities", Discrete Mathematics 11 (1975), pp. 119-124.

17. "A Generalization of a Theorem of Chvâtal and Gomory", pp. 313-332 in Nonlinear Programming 2, edited by O. L. Mangasarian, R. R. Meyer, and S. M. Robinson, Academic Press, New York, 1975.

Grants and Fellowships (continued)

NSF Research Grant MCS76-12026, Co-Principal Investigator, 1976-1978
Research Fellowship, January - June 1977, from the Center for Operations Research and Econometrics, Belgium
NSF Research Grant ENG-79000284, Principal Investigator, 1979-1980
NSF Research Grant ECS-8001763, Principal Investigator, 1980-1982
Senior U.S. Scientist Award of the Alexander von Humboldt Foundation, January to June, 1983
NSF Research Grant MCS-8304075, 1983-86

Organizational Responsibilities

Representative to the Faculty Senate from the business school, 1977-1978
Organizer of the Operations Research Seminar for the business school, 1977-1978
Representative of the management college to the Seminar on Operations Research, 1978-1981 (co-sponsored with the School of Industrial and Systems Engineering and the School of Mathematics)
Member of the Program Committee of the symposium in honor or R. J. Duffin, Constructive Approaches to Mathematical Models, July 10-15, 1978
Co-organizer (with Cedric Suzman) of the Colloquia on Strategic Planning, September 28, 1979, and October 10, 1980
Implementer and morning moderator, Third Colloquium on Strategic Planning, October 1981
Member of the Organizing Committee of the 1981 International Symposium on Semi-infinite Programming and Applications
Member of the International Program Committee for the XI International Symposium on Mathematical Programming, Bonn, August 23-27, 1982
Member of the Teaching Evaluation Committee in the College of Management, 1980-1981
Chairman and member of the Personnel Committee in the College of Management, 1980-1982

Hobbies and Personal Interests

Swimming; weight training; poetry and theater.

Published Articles

1. "Consistency Statements in Formal Theories", Fundamentae Mathematicae, LXXXII (1971), pp. 17-40.

2. "Non-effectiveness in S. Orey's Arithmetical Compactness Theorem", Zeithschrift f. math. Logik and Grundlagen d. Math., Bd. 17, 1971, pp. 285-289.

Published Articles (continued)

18. "Experimental Results on Hillier's Linear Search", with T. H. C. Smith, Mathematical Programming 9 (1975), pp. 371-376.

19. "Experimental Logics and Δ_2^0-Theories", Journal of Philosophical Logic 4 (1975), pp. 253-267.

20. "Cutting-plane Theory: Disjunctive Methods", Annals of Discrete Mathematics 1 (1977), pp. 293-330.

21. "Cutting-planes for Complementary Constraints", SIAM Journal on Control and Optimization 16 (1978), pp. 56-62.

22. "Bracketing Discrete Problems by Two Problems of Linear Optimization", in Operations Research Verfahren (Methods of Operations Research) XXV, 1977, pp. 205-216, Verlag Anton Hain, Meisenheim an Glan.

23. "The Value Function of a Mixed Integer Program: I", with C. E. Blair, Discrete Mathematics 19 (1977), pp. 121-138.

24. "Some Basis Theorems for Integral Monoids", Mathematics of Operations Research 3 (1978), pp. 145-154.

25. "Cutting-plane Theory: Algebraic Methods", Discrete Mathematics 23 (1978), pp. 121-150.

26. "A Converse for Disjunctive Constraints", with C. E. Blair, Journal of Optimization Theory and Its Applications (1978).

27. "Some Relaxation Methods for Linear Inequalities", Cahiers du Centre d'Etudes de Recherche Operationnelle 21 (1979), pp. 43-53.

28. "The Value Function of a Mixed-Integer Program: II", with C. E. Blair, Discrete Mathematics 25 (1979), pp. 7-19.

29. "Minimal Inequalities", Mathematical Programming 17 (1979), pp. 1-15.

30. "Two Lectures on the Theory of Cutting-planes", for Combinatorial Optimization, edited by N. Christofides et al., John Wiley and Sons, Ltd.

31. "Representations of Unbounded Optimizations as Integer Programs", Journal on Optimization Theory and Its Applications 30 (1980), pp. 339-351.

32. "Lagrange Dual Problems with Linear Constraints on the Multipliers", with C. E. Blair, Constructive Approaches to Mathematical Models, C. V. Coffman and G. Fix (eds.), Academic Press, 1979, pp. 137-152.

Published Articles (continued)

33. "An Introduction to the Theory of Cutting-planes", Annals of Discrete Mathematics 5 (1979), pp. 71-95.

34. "A Cutting-plane Game for Facial Disjunctive Programs", SIAM Journal on Control and Optimization 18 (1980), pp. 264-281.

35. "Strengthening Cuts for Mixed Integer Programs", with E. Balas, European Journal of Operations Research 4 (1980), pp. 224-234.

36. "Lagrangean Functions and Affine Minorants", with R. J. Duffin, Mathematical Programming, Study 14 (1981), pp. 48-60.

37. "An Exact Penalty Method for Mixed Integer Programs", with C. E. Blair, Mathematics of Operations Research 6 (1981), pp. 14-18.

38. "A Limiting Lagrangean for Infinitely-constrained Convex Optimization in R^n", Journal of Optimization Theory and Applications, 33 (1981), pp. 479-495.

39. "Some Influences of Generalized and Ordinary Convexity in Disjunctive and Integer Programming," Generalized Concavity in Optimization and Economics, Academic Press, 1981, pp. 689-699 (ISBN-0-12-621120-5).

40. "The Value Function of an Integer Program," with C. E. Blair, Mathematical Programming 23 (1982), pp. 237-273.

41. "A Limiting Infisup Theorem," with C. E. Blair and R. J. Duffin, Journal of Optimization Theory and Applications 37 (1982), pp. 163-175.

42. "Duality in Semi-infinite Programming," with R. J. Duffin and L. A. Karlovitz, Semi-Infinite Programming and Optimization, edited by A. V. Fiacco and K. O. Kortanek, Springer-Verlag, 1983.

43. "Uniform Duality in Semi-Infinite Convex Optimization," Mathematical Programming 27 (1983), pp. 144-154.

Book Review

M. R. Hestenes' Optimization Theory: The Finite-Dimensional Case, reviewed in Bulletin of the American Mathematical Society (83), May 1977, pp. 324-334.

Professional Newsletter

"Some Roads Hardly Taken," p. 1-3 of OPTIMA, July 1981, the newsletter of the Mathematical Programming Society.

Accepted for Publication

1. "Cluster Sets of Vector Series," with D. F. Karney, to appear in Advances in Applied Mathematics.

2. "Constructive Characterizations of the Value Function of a Mixed Integer Program, I" with C. E. Blair, to appear in Discrete Applied Mathematics.

Submitted for Publication

1. "Constructive Characterizations of the Value Function of a Mixed Integer Program, II" with C. E. Blair.

2. "Extensions of a Theorem of Balas," with C. E. Blair (under revision).

3. "Modelling with Integer Variables," with J. Lowe (under revision).

4. "Computational Complexity of Some Problems in Parametric Discrete Programming," with C. E. Blair.

5. "A Simple Model for Weight Loss Over Time," with P. Sparling (under revision).

6. "The Polynomial Heirarchy and a Simple Model for Competitive Analysis."

7. "Experimental Results on the New Techniques for Integer Programming Formulation," with J. Lowe.

Symposium Volume

"The Line Executive as Planner: Proceedings of the Second Annual Georgia Tech Colloquium on Strategic planning," co-edited with C. Suzman, available as a Georgia Tech report.

Research Projects in Progress

1. "Some Remarks on the Role of Applications Software in the Computer and Information Industries." A working draft has been circulated to academics and industry executives for their comments.

2. "Representations of Practical Problems as Mixed-Integer Programs."

3. "Value Functions and Computational Complexity: II," with C. E. Blair.

4. "Economies with Indivisibilities."

Research Projects in Progress (continued)

5. "Symmetric Duality in Semi-infinite Linear Programming." with D. F. Karney.

6. "Automata with Behavioral Strategies."

7. "Sensitivity Analysis in Mixed Integer Programming."

Invited Talks Since 1979

1. "Representations of Unbounded Optimizations as Integer Programs", ORSA/TIMES meeting in New Orleans, April 30 - May 2, 1979.

2. "Recent Results in Nonlinear and Integer Programming", at the meeting on Mathematical Programming, at Mathematisches Forschungsinstitut in Oberwolfach, Germany, May 6-12, 1979.

3. "A Limiting Infisup Theorem", at the Tenth International Symposium on Mathematical Programming, Montreal, August 27-31, 1979.

4. "Nonlinear Optimization Treated by Linear Inequalities", ORSA/TIMS meeting in Milwaukee, October 15-17, 1979.

5. "Sensitivity Analysis for Integer Programs", ORSA/TIMS National Meeting at Colorado Springs, November 10-12, 1980.

6. "Integer Analogues", Mathematical Sciences Department, University of Delaware, November 1980.

7. "Sensitivity Analysis for Mixed Integer Programs", CORS/ORSA/TIMS Joint National Meeting, Toronto, May 3-6, 1981.

8. "Some Influences of Generalized and Ordinary Convexity in Disjunctive and Integer Programs", NATO Advanced Study Institute on Generalized Concavity in Optimization and Economics, Vancouver, August 4-15, 1980.

9. "Computational Complexity and the Value Function", with C. E. Blair, ORSA/TIMS meeting in Houston, October 14-16, 1981.

10. "Necessary and Sufficient Constraint Qualifications", 1981 International Symposium on Semi-Infinite Programming and Its Applications, Austin, Texas, September 8-10, 1981.

11. "Duality in Semi-infinite Linear Programming", 1981 International Symposium on Semi-Infinite Programming and Its Applications, Austin, Texas, September 8-10, 1981.

12. "Modelling With Integer Variables", with J. Lowe, XI International Symposium on Mathematical Programming, Bonn, August 1982.

Invited Talks Since 1979 (continued)

13. "Experiments in Integer Modelling", Oberwolfach Conference on Mathematical Programming, January 1983.

14. "Some Recent Experimental Results on Integer Modelling", Mathematics Institute at Oberwolfach, Federal Republic of Germany, January 1983.

15. "Theory of Value Functions for Discrete Variable Problems: An Overview", University of Bonn, Federal Republic of Germany, February 1983.

16. "Some Recent Experimental Results on Integer Modelling", The Technion (Israel Institute of Technology), Israel, April 1983.

17. "A Problem in Competitive Analysis and the Polynomial Heirarchy", University of Bonn, Federal Republic of Germany, April 1983.

18. "The Value Function of Pure and of Mixed Integer Programs", Stichting Mathematics Center, Amsterdam, May 1983.

19. "Modelling with Integer Variables", Erasmus University, Rotterdam.

20. "Modelling with Integer Variables", Mathematical Programming Study Group, London, at the London School of Economics, June 1983.

21. "Computational Experiments with Integer Representations," ORSA/TIMS Orlando meeting, November 7-9, 1983.