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SPONSORED PROJECT INITIATION

Date: 2/9/81

Project Title: Multi-Dimensional and Interactive Phenomena in the Dynamic Response of Nonlinear Acoustical Systems

Project No: E-25-640

Project Director: Dr. Jerry H. Ginsberg

Sponsor: National Science Foundation

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Sept. 30, 1982 (Reports)

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SPONSORED PROJECT TERMINATION SHEET

Date November 22, 1982

Project Title: Multi-Dimensional and Interactive Phenomena in the Dynamic Response of Nonlinear Acoustical Systems

Project No: E-25-640

Project Director: Dr. Jerry H. Ginsberg

Sponsor: National Science Foundation

Effective Termination Date: 6/30/82

Clearance of Accounting Charges: 6/30/82

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FINAL PROJECT REPORT
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PART I–PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Institute of Technology School of Mech. Eng. Atlanta, GA 30332	Mechanical Systems	CME 8026496
	4. Award Period From 1-81 To 9-30-82	5. Cumulative Award Amount $22,997

6. Project Title: Multi-Dimensional and Interactive Phenomena in the Dynamic Response of Nonlinear Acoustical Systems.

PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

The overall effort of the research project was to improve analytical capabilities for predicting the nonlinear acoustic signal radiating from realistic vibrating structures. The nonlinearity is associated with very intense sound levels which result when the peak structural velocity is large fraction of the speed of sound. A general procedure for treating nonlinear acoustic waves, known as the "direct method," was developed earlier by the Principal Investigator with NSF support. The earlier work had treated spatially periodic structures, which are associated with the method of separation of variables for solving partial differential equations.

The current project focused on introducing integral transforms into the direct method, in order to extend the generality of the method. This research was pursued by studying a problem that is closely associated with underwater sonar. In it, a strip transducer on an otherwise inert flat boundary vibrates harmonically at a very high frequency. This results in radiation of an acoustic signal away from the boundary. The research goal was to obtain an analytical representation of the signal which simultaneously accounts for spreading loss, diffraction, and nonlinear distortion. This representation was required to be descriptive of the nearfield around the transducer, as well as the far field, where the linearized signal is essentially a cylindrical wave.

The research was successfully completed by utilizing a Fourier integral transform to represent the signal. Nonlinear effects were evaluated by employing the method of stationary phase, which was a unique innovation for nonlinear acoustics. The analytical result was an integral whose evaluation by numerical methods gives all features of the signal.

This representation was particularly important because it provided, for the first time an analytical description of an experimentally observed phenomenon that the compression and rarefaction phases of the signal behave quite differently. At the time that the project closed, it had begun to apply these new capabilities to the more realistic situation of a circular piston, in which the signal is that of an axisymmetric beam of sound which tends to a spherical wave in the far field.

PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (✓)	Approx. Date
a. Abstracts of Theses	X				
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Jerry H. Ginsberg		11-2-82

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1. INTRODUCTION

This project has had a dual purpose. From an abstract viewpoint, it had the academic goal of improving the capability to analyze mathematically key questions in nonlinear wave propagation. The practical purpose of the project, which was at least equally important, was to apply any newly developed techniques to practical problems in acoustics. The systems in which these problems arise share the common feature that the intensity of their signal is very high, e.g. 130 dB in air or 210 dB in water. As a consequence, the usual results of linearized theories are invalidated. As will be revealed in the following, the project has been very successful in achieving its dual goal.

2. PREVIOUS STATUS OF THE PROJECT

Some basic analytical concepts were developed with National Science Foundation support while the Principal Investigator was a faculty member at Purdue University. Because the research project covered by this report is a continuation of that effort, it is appropriate to summarize the earlier accomplishments. Greater detail than presented here may be found in the overviews reported in References [1] and [2].

The general approach for studying the propagation of acoustic waves having finite amplitude involves formulating the problem in terms of a nonlinear (partial differential) wave equation.

$$c_o^2\nabla^2\phi - \frac{\partial^2\phi}{\partial t^2} = (\gamma-1)\,\frac{\partial\phi}{\partial t}\,\nabla^2\phi + c_o^2\,\frac{\partial}{\partial t}(\nabla\phi\cdot\nabla\phi) + O(\phi^3) \tag{1}$$

This equation, which governs the velocity potential ϕ, is solved by expanding the potential in a perturbation series.

$$\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 \tag{2}$$

The first order term ϕ_1 is the one obtained from linear theory. It is used to represent the linear terms, which then become source terms for the second order potential ϕ_2. It is only necessary to solve for the portion of ϕ_2 which grows as the signal propagates, for it is such behavior which represents the cumulative nature of nonlinear distortion. At this stage, the potential will have the form

$$\phi = \varepsilon f_1(x_1,x_2,x_3,t) + \varepsilon^2 x_1 f_2(x_1,x_2,x_3,t) + O(\varepsilon^2) \tag{3}$$

where f_1 and f_2 represent bounded functions of the spatial coordinates x_1 and time t, with x_1 denoting the propagation direction.

The next step after the potential function is determined is to use it to obtain the velocity components and pressure. As was true for the potential function, these state variables will also contain second order terms which grow. However, such behavior is not acceptable in state variables. Correcting this situation involves the introduction of a coordinate straining transformation. In essence, this step amounts to changing variables, under the condition that the change should remove any unacceptable terms. The general form of such a transformation may be described by letting α_i denote the variable corresponding to the position coordinates x_i. Then

$$x_i = \alpha_i + \varepsilon x_i^{\beta} g_i(\alpha_1, \alpha_2, \alpha_3, t) \qquad (4)$$

where g_i are functions to be determined.

The research project began in 1974 by considering only systems fitting a formulation in terms of rectangular Cartesian coordinates. First, planar waves [3] and waves from plates [4-6] were studied. Each of these involved a single acoustic mode. The extension to multiple modes was achieved in a study of two dimensional wave propagation in a duct [7]. Each mode j in the small signal (linear) case had a propagation speed c_j, and the spatial coordinates x_i for each mode were replaced by distinct sets of strained coordinates $\alpha_i j$. These qualities are described by the following general form for the pressure signal.

$$p = \sum_{j=1}^{N} p_j(\alpha_{1j} - c_j t, \alpha_{2j}, \alpha_{3j}) \qquad (5)$$

where the relationship between the strained and physical coordinates must be evaluated for each mode j.

Obviously, not all systems of engineering significance have a rectangular geometry. Thus a major emphasis was placed on extending the analytical method to curvilinear coordinates. This goal was achieved in the studies of cylindrical waves [8-9]. The difficulty in studying curvilinear geometries is that the source terms in the velocity potential become products of higher transcendental functions, such as Bessel functions. It does not seem possible to treat such terms directly. Instead the derived method infers the overall response by examing the response in the far field (the region far from the active boundary). This permits replacing complicated mathematical functions by their simpler asymptotic representations. After the coordinate straining is ascertained in the far field, returning to the near field involves concepts drawn from the method of matched asymptotic expansions.

Although this technique was originally developed for a single cylindrical wave mode, it was extended to several modes [10] just before the principal investigator resigned from the faculty of Purdue University. At that time, another major expansion in the analytical technique was initiated.

Integral transforms, such as those of Laplace and Fourier, are widely employed in linear acoustics. The same techniques cannot be applied directly to nonlinear systems because the principal of superposition is not valid. However, it was recognized that the perturbation equations that arise in the formulation of the velocity potential equations (1) and (2), were linear at each step. It seemed appropriate to employ integral transforms to solve those equations.

The correctness of this approach was demonstrated in the case of planar waves [11], where a Laplace transform was used to treat an arbitrary time dependence. The last accomplishment at Purdue University was to begin to apply the foregoing concepts to a more complicated problem than any that had been explored previously.

The ultimate goal was to describe the finite amplitude signal that radiates from a transducer, such as a loudspeaker, which is embedded in a wall. (This is called the "infinite baffle problem"). The complications in this system arise even in the linear case because the signal makes a transition from a quasi-planar wave near the transducer to a quasi-spherical wave in the far field, with diffraction effects playing a strong role everywhere. Nonlinearity in this case had been found experimentally to have a strange effect, in which the compression and rarefaction phases of the signal distort in different ways.

The classical infinite baffle problem leads to an axisymmetric beam of sound, for which a formulation in terms of cylindrical coordinates is appropriate. Rather than addressing this complication simultaneously with developing the overall procedure, it was decided to study a simpler geometry. The transducer was considered to be situated on an infinitely long strip; this configuration could be treated by using only two Cartesian spatial coordinates, x_1 oriented normal to the boundary and x_2 transferse to the strip. The excitation driving the x_1 component of particle velocity was then described as

$$v_1|_{x_1=0} = \begin{cases} \varepsilon f(x_2)\sin[wt-g(x_2)]; & |x_2| \leq a \\ 0; & |x_2| > a \end{cases} \tag{6}$$

where $f(x_2)$ represents an arbitrary spatial dependence of the amplitude and $g(x_2)$ is a comparable phase lag.

The first portion of the analysis of this problem [12-13] was performed before the project at Purdue University was terminated. That work was devoted to the evaluation of the velocity potential. The first order terms were obtained by using a Fourier cosine transform to describe the arbitrary spatial variation transverse to the transducer strip. Determining the second order potential proved to be a far more difficult task than in any previous study using the direct method.

The first order term was a transform inversion featuring a continuous spectrum of transverse wave numbers k.

$$\phi_1 = \int \frac{V_k}{\lambda_k} \cos(t - \lambda_k x_1 - \theta_k) \cos(k x_2)\, dt$$
$$\lambda_k = c_k (1 - k^2)^{1/2} \tag{7}$$

where V_k and θ_k are transformed amplitudes and phase lags, respectively.

Using such a representation to form the quadratic source terms which excite the second order potential leads to a double spectrum over wave number k and n. It was found that the portion of the latter which causes cumulative growth (the major nonlinear effect) was situated in the narrow band where $n \approx k$. The evaluation of this contribution was achieved by means of an integration using the method of stationary phase. A subsidiary of that analysis was the solution of a pair of coupled second order ordinary differential equation which are singular in the region of interest.

Thus, at the closure of the effort at Purdue University, a major advance in the analytical technique had been achieved. However, the strip transducer problem was not yet solved, because all that had been obtained was the velocity potential -- it still remained to evaluate the coordinate straining, the particle velocity, and the pressure.

3. RESEARCH AT THE GEORGIA INSTITUTE OF TECHNOLOGY

The first priority for the Principal Investigator in making the transition to his new institution was to become familiar with the new computer facility. Once that was achieved the remaining questions regarding the infinite strip problem could be addressed.

The expression for the velocity potential that had been obtained could be differentiated to find the physical response variables. The second order terms in these quantities all exhibited a tendency to grow without bound. When u_j is used to denote any of these variables, their mathematical form was found to be

$$\begin{aligned} u_j = \int [\varepsilon\, u_{j1}(t-\lambda_k x_1, x_2) \\ + \varepsilon^2 x_1^{1/2}\, u_{j2}(t-\lambda_k x_1, x_2)]\,dk \end{aligned} \tag{8}$$

The coordinate straining plays the role of correcting the growth tendency indicated by the $x_1^{1/2}$ factor. This task was complicated by the fact that equation (8) is in the form of an integral. After several attempts, the appropriate choice was found by recognizing an analogy with earlier developments.

An integral is essentially a summation, which suggested that equation (8) consists of an infinite number of modes having infinitesimal amplitudes; a different coordinate straining is usually required for each mode. For the infinite strip, such a requirement meant that the coordinate straining in equation (4) should be dependent on the value of the transverse wave number k for the mode forming the integrand in equation (8).

Even with this basic detail identified, a major question still remained. It was found that not all of the growth terms could be removed. The appropriate choice for the coordinate straining could only be ascertained by developing numerical algorithms for digital computer evaluation of the results. Subsequently, it was proven analytically that the remaining growth term is required for satisfaction of the condition that the energy raditing from the boundary be finite.

Upon conclusion of the analysis, the computer algorithm was refined to provide generality and convenience in viewing the predicted responses as spatial profiles and temporal waveforms. One difficulty that was encountered in the numerical work pertains to diffraction integrals whose integrand oscillates rapidly. In order to expedite obtaining quantitative results a simple Simpson's rule (equal interval) integration scheme was implemented. This required a large number of integration points. Also, the coordinate transformation, which consisted of a pair of simultaneous transcendental equations, had to be solved at each integration point by a Newton-Raphson (method of tangents) algorithm. In total, this led to a complicated computer program which was rather slow. Specifically, some of the spatial profiles that were evaluated consumed two minutes of CPU time on a CDC 7600 computer. NSF computer funds were quickly consumed in evaluating examples of a variety of excitation patterns. The analytical details of the coordinate straining, as well as the results for one example were described by the Principal Investigator in References [14-16].

The major effort in the last part of the project was to extend the method for an infinite strip to that of a circular transducer. As was mentioned earlier, this involved a conversion from two dimensional Cartesian coordinates to cylindrical coordinates. At the point where project funds were fully expended, the basic approach had been developed. This involved a combination of the integral transform techniques for the strip problem, and the inner and outer expansion ideas for curvilinear coordinates. The latter was necessary because the Hankel transform, whose kernel is a Bessel function, was required to treat the axisymmetric geometry. The prospective analysis was outlined in a presentation [17]; the details are only now being finalized. Final completion and validation of the results could prove to be one of the most significant developments in the realm of nonlinear acoustics.

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*17. Ginsberg, J.H., "Quantitative Response Patterns for Two-Dimensional Finite Amplitude Sound Beams," 9th International Symposium on Nonlinear Acoustics, University of Leeds, England, July 20-24, 1981.

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S C I E N T I F I C   C O L L A B O R A T O R S

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RECENT INVESTIGATIONS OF THE PROPAGATION OF FINITE AMPLITUDE MULTI-DIMENSIONAL ACOUSTIC WAVES

J.H. Ginsberg*

Abstract. Phenomena are described that arise in multi-dimensional systems in which wave intensity is not uniform transverse to the direction of propagation. Use of the direct method and applications of integral transforms in the analysis of wave propagation are described.

Earlier review articles [1, 2] described a variety of interesting phenomena that arise when acoustical waves have reasonably large amplitudes; e.g., in excess of 120 dB (re 20 μPa) in air or 210 dB (re 1 μPa) in water. The phenomena of wave steepening and formation of shocks in planar waves are well-documented.

The effects that arise in multi-dimensional systems, in which wave intensity is not uniform transverse to the direction of propagation, are less familiar but equally dramatic. Various techniques have been employed to evaluate these effects. Multi-dimensional systems have resisted exact analytical solutions. As a result analytical solutions using perturbation methods have received a great deal of attention.

GENERAL CONCEPTS

Consider the case of linear waves when the signal strength is infinitesimal. In general, the acoustic medium is nondispersive; that is, a one-dimensional wave having infinitesimal amplitude will propagate at the speed of sound regardless of its wavelength. However, the speed at which nonuniform waves propagate is dependent on the rates of variation in the propagating and transverse directions, as might be measured by a ratio of wavelengths. In fact, waves having comparatively small transverse wavelengths decay exponentially rather than propagate [3]. Because the phase speed for propagating waves is dependent on a ratio of wavelengths in orthogonal directions, individual waves are often found to propagate jointly. Thus an acoustic wave can be considered to be comprised of groups of waves having different wave speeds.

Let p denote the acoustic pressure and v_1, v_2, v_3 the particle velocity components. Then

$$\begin{Bmatrix} p \\ v_1 \\ v_2 \\ v_3 \end{Bmatrix} = \sum_{i=1}^{M} \begin{Bmatrix} p \\ v_1 \\ v_2 \\ v_3 \end{Bmatrix}_i \qquad (1)$$

where i represents the group number. Each group has a unique phase speed, which is denoted as c_i.

Let x_1 be the direction of propagation. Then each group can be written as

$$\begin{Bmatrix} p \\ v_1 \\ v_2 \\ v_3 \end{Bmatrix}_i = \sum_{j=1}^{M} \begin{Bmatrix} (p)_{ij} \\ (v_1)_{ij} \\ (v_2)_{ij} \\ (v_3)_{ij} \end{Bmatrix} \qquad (2)$$

The terms to the right of the equality sign are functions of $t-x_1/c_i$, x_2, and x_3.

The lack of dispersion for the individual contributors to a specific group i has a profound nonlinear effect. For discussion purposes nonlinearity can be considered to create source terms that excite a second order linear signal. Because these source terms propagate at the speed of the group from which they are formed, a resonant-like condition is established. This leads to a cumulative growth of distortion effects as the group propagates.

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PERIODIC GEOMETRIES

The direct method described in preceding surveys [1, 2] has been particularly successful in evaluating nonlinear effects. The first step in the direct method is to formulate the problem in terms of the nonlinear hyperbolic wave equation governing the velocity potential [4]. This partial differential equation is solved asymptotically using a singular perturbation method [5]. Most investigators have used the method of renormalization, in which coordinate straining transformations are introduced to describe the distortion associated with nonlinearity.

The systems treated in this section have repetitive geometric configurations. This permits the use of separation of variables to solve the linear differential equations.

The study of two-dimensional waves in a hard-walled duct [6] was significant in the development of the direct method because of the general nature of the excitation. An arbitrary periodic input excited a variety of duct modes that were grouped according to phase speeds. It was found that the distortion of each mode forming a nondispersive group is a consequence of all modes in that group. Furthermore, the distortion is not influenced by the responses in other groups. This conclusion resulted from the fact that a different coordinate straining was found for each group. Because the system was two dimensional, two strained coordinates were introduced. For group i, they had the form

$$\begin{aligned} t - x_1/c_j &= \alpha_i + F_i x_1 (v_1)_i \\ x_2 &= \beta_i + G_i x_1 (v_2)_i \end{aligned} \tag{3}$$

where F_i and G_i are constants. Each term $(p)_{ij}$, $(v_1)_{ij}$, and $(v_2)_{ij}$ ($j = 1, 2, \ldots, N$) depends on the values of α_i and β_i. A constant value of α_i defines a wave front for group i, and constant β_i is a ray. Thus, equations (3) define a phenomenon of self-refraction, in which the wave fronts and rays for a group are distorted by the response in the group.

A similar conclusion was obtained in an analysis of the acoustical waves that result when oppositely traveling waves propagate along an infinie plate [7]. The investigation followed the method of renormalization for an inviscid medium. However, the method of multiple scales, with its increased generality and complexity was required to develop the coordinate straining in the presence of dissipation.

The tendency for non-dispersive groups to interact only with themselves has been identified in general cylindrical [8, 9] and spherical [10, 11] waves. These analyses developed the solutions in terms of eigenmodes of the respective curvilinear coordinate system. The analytical techniques employed followed the grouping concepts developed for the duct problem. The method for evaluating the second order potential function in curvilinear coordinates was identical to that derived for excitation in a single cylinder harmonic [12, 13]. In that procedure the asymptotic expansion of the response in the far field is used to ascertain the coordinate straining. The result is then matched to the expansion of the response in the near field.

The primary differences in the analyses of cylindrical and spherical waves arise from differences in the nature of the dispersion. In the far field, infinitesimal spherical waves propagate at the speed of sound, regardless of the spherical harmonic to which they correspond. Thus, only a single wave group existed in the spherical geometry.

APPLICATIONS FEATURING INTEGRAL TRANSFORMS

A key element in the foregoing analyses was the identification of the portion of the second order potential that grows as the wave propagates. Identification was expedited by the fact that the analogous linearized system could be analyzed by separating variables in the governing partial differential equation. Many systems encountered in practice require more general solution techniques. The feasibility of using integral transforms in the context of the direct method was demonstrated in a study of planar waves [14]. The Laplace transform was used in the analysis to treat an arbitrary time dependence. The coordinate straining, which was performed in terms of the transform, was shown to reduce to the result obtained by more conventional methods.

Integral transforms have been implemented in the analysis of systems related to the infinite baffle problem. Such systems feature the oscillation of a flat boundary, which results in a confined beam of sound when the frequency is sufficiently high. The linear solution of these problems is usually formulated in terms of source theory, which leads to the Rayleigh integral [3]. However, al alternative formulation using integral transforms has proven to be more convenient for nonlinear problems.

The case of an infinitely long strip on the boundary was treated first. A Fourier cosine transform was used in an evaluation of the growth of harmonics [15] to treat the variation transverse to the axis of propagation. The propagating part of the first order signal was

$$\phi_1 = \int_0^1 \frac{V_k}{\lambda_k} \cos(t-\lambda_k x_1)\cos(kx_2)\,dk \qquad (4)$$

where $\lambda_k = (1 - k^2)^{1/2}$ and V_k is a transform parameter for the excitation. A comparison of this representation with equations (1) and (2) reveals that the sound beam is composed of a continuous spectrum of infinitesimal wave groups. Each group consists of a single harmonic that propagates at the nondimensional speed $1/\lambda_k$.

Equation (4) leads to source terms occupying a double spectrum of transverse wave numbers k. An analysis [15] reduced the double spectrum to a single one by means of an asymptotic integration (the method of stationary phase). In essence this procedure describes the cancellations resulting from destructive interference between higher harmonics. The part found in the integration represents the primary contribution to the far field.

The potential function that resulted from this analysis was the starting point for a companion study [16] to determine coordinate straining. It was found that the transformation is dependent on the transverse wave number, specifically

$$\begin{aligned} t-\lambda_k x_1 &= \psi_k - \epsilon s_k x_1^{1/2} \sin(\psi_k-\pi/4)\cos(\eta_k) \\ kx_2 &= \eta_k - \epsilon s_k x_1^{1/2} \cos(\psi_k-\pi/4)\sin(\eta_k) \end{aligned} \qquad (5)$$

where ψ_k and η_k are the strained coordinates, and s_k is a coefficient that depends on V_k and λ_k. The variables ψ_k and η_k replace $t-\lambda_k x_1$ and kx_2 respectively in equation (4).

This result is consistent with the earlier observation regarding wave groups. Each infinitesimal segment in the spectrum of transverse wavelets forms a wavelet. Because the phase speed of the wavelet is an analytical function of its wave number, it forms an individually propagating group. The distortion of each group is independent of the response in other groups.

The same type of analysis was employed to study the more realistic problem of axisymmetric sound beams [17]. The unique feature was the treatment of the variation of the signal transverse to the beam axis. Because it was convenient to use cylindrical coordinates to treat the spatial dependence, a Hankel (Fourier-Hankel) transform was used. The analysis of the second order potential was achieved off the beam axis in order to simplify the representation of the Bessel functions that appear in the integral transform. The method whereby the off-axis response was matched to the on-axis result is derived from the treatment of cylindrical waves that propagate in the radial direction [12].

The waveform in both the strip and axisymmetric beam configurations displayed a type of distortion that is not observed in periodic geometries. Wave steepening due to amplitude dispersion occurs even in one-dimensional waves [3]. The figure displays some typical waveforms for the axisymmetric sound beam; it exhibits such distortion. (The dotted curves are the predictions of linear theory.) It can be seen that the shape of the compression phase is high and narrow; the rarefaction phase is broad. This prediction is consistent with experimental observations [18, 19].

Figure to be inserted.

Waveforms on Axis in an Intense Sound Beam in Water. ka = 20; maximum pressure, 240 dB

Comparable predictions have been reported in the Soviet literature [20-25]. In these analyses finite difference techniques were used to solve a modified version of Burger [26] that is often used as a prototypical equation for nonlinear waves. The modification [27, 28] is intended to account for spreading transverse to the beam axis. However, the assumptions made in the derivation of the modified equation are prone to error when small scale diffraction effects are involved.

Analyses and experimental observations of sound beams indicate that the asymmetry in the profile occurs when the excitation is sufficiently strong to cause significant distortion in the near field, where the beam resembles a quasi-planar wave [19]. Less strong excitations result in a transition of the sound beam to a quasi-spherical in the far field [3]. The mechanism underlying this change in distortion type is a current research topic.

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A Singular Perturbation Analysis of Axisymmetric Finite Amplitude Sound Beams *

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NINTH INTERNATIONAL SYMPOSIUM ON NONLINEAR ACOUSTICS

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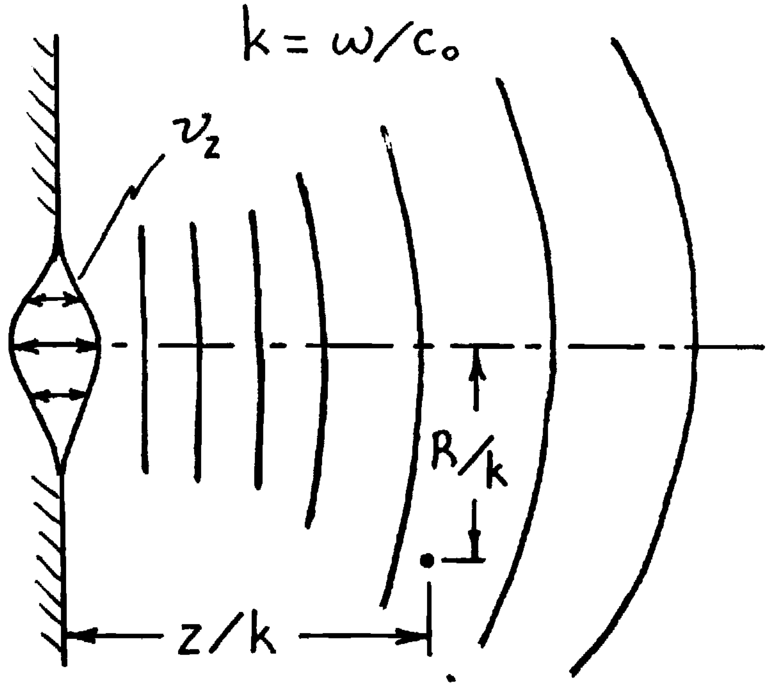
Grant No. CME-8026496

Infinite Baffle Problem

CW Source



Nondimensional

$$z = k\hat{z}$$

$$R = k\hat{R}$$

$$t = \omega\hat{t}$$

On $z = 0$:

$$v_z = \varepsilon c_o f(R)\sin[t + g(R)]$$

1. $f(R)$ & $g(R)$ arbitrary
2. $|\varepsilon| << 1$ but $O(\varepsilon^2)$ not negligible

Note that $\dfrac{\text{n.d. dist.}}{ka}$ = units of piston radii

$\dfrac{\text{n.d. dist.}}{2\pi}$ = units of a wavelength

Linear Solution

King Integral

Non-uniform planar waves – spectrum of transverse wave numbers

Fourier - Bessel transform

$$\left.\begin{aligned} V_n \cos \theta_n &= \int_0^\infty f(R) \cos[g(R)]\, J_0(nR)\, R\, dR \\ V_n \sin \theta_n &= \int_0^\infty f(R) \sin[g(R)]\, J_0(nR)\, R\, dR \end{aligned}\right\} \text{excitation on } z = 0$$

Result

$$P/\rho_0 c_0^2 = \varepsilon \int_0^1 \frac{n V_n}{\lambda_n} \sin(t + \theta_n - \lambda_n z) J_0(nR)\, dn$$ propagate in the z direction

$$+ \varepsilon \int_1^\infty \frac{n V_n}{\bar{\lambda}_n} \cos(t + \theta_n) \times \exp(-\bar{\lambda}_n z) J_0(nR)\, dn$$ evanesce

where

$$\lambda_n = (1 - n^2)^{1/2}, \quad \bar{\lambda}_n = (n^2 - 1)^{1/2}$$

Fluid medium is an infinite wave guide

Discuss advantages of formulation for near field i.e. one integral

Nonlinear King Integral

1. Perturbation analysis
   a. Matched expansions — far field & off axis
   b. Coordinate straining

$$\left.\begin{aligned} \lambda_n z &\rightarrow \xi_n \\ nR &\rightarrow \eta_n \end{aligned}\right\} \text{coupled transcendental equations}$$

2. Stationary phase integration

Result

distortion effects

$$p/\rho_0 c_0^2 = \varepsilon \int_0^1 \frac{n V_n}{\lambda_n} \Big\{ \sin(t + \theta_n - \xi_n)\, J_0(\eta_n)$$

$$+ \varepsilon s_n \left[J_0(\eta_n)^2 + J_1(\eta_n)^2 \right] \Big\}\, dn$$

produces zero mean value

$$+ \varepsilon \int_1^\infty \frac{n V_n}{\bar{\lambda}_n} \cos(t + \theta_n) \exp(-\bar{\lambda}_n z)\, J_0(nR)\, dn$$

$$+ O(\varepsilon^2) \quad \text{for all } R \;\&\; z$$

linear evanescent term

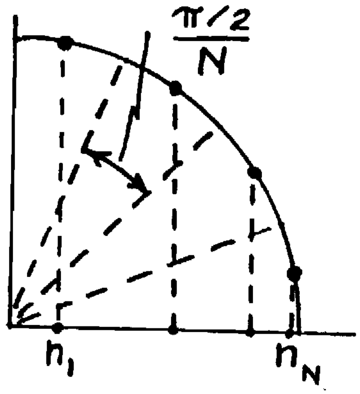
where

$$s_n = \beta_0 \left(\frac{\pi z}{2 \lambda_n} \right)^{1/2} n V_n$$

coefficient of nonlinearity

$$\beta_0 = 1 + \frac{1}{2} \frac{B}{A}$$

Numerical Algorithm



Gauss-Chebyshef quadrature

$$\int_0^1 \frac{F(n)}{(1-n^2)^{1/2}}\, dn \approx \frac{\pi}{2N} \sum_{i=1}^{N} F(n_i)$$

Station points

$$n_i = \frac{\pi}{4N}(2i-1)$$

For evanescent term: $\bar{n} = 1/n$

$$\int_1^{\infty} \frac{G(n)}{(n^2-1)^{1/2}}\, dn \equiv \int_0^1 \frac{G(1/\bar{n})}{\bar{n}(1-\bar{n}^2)^{1/2}}\, d\bar{n}$$

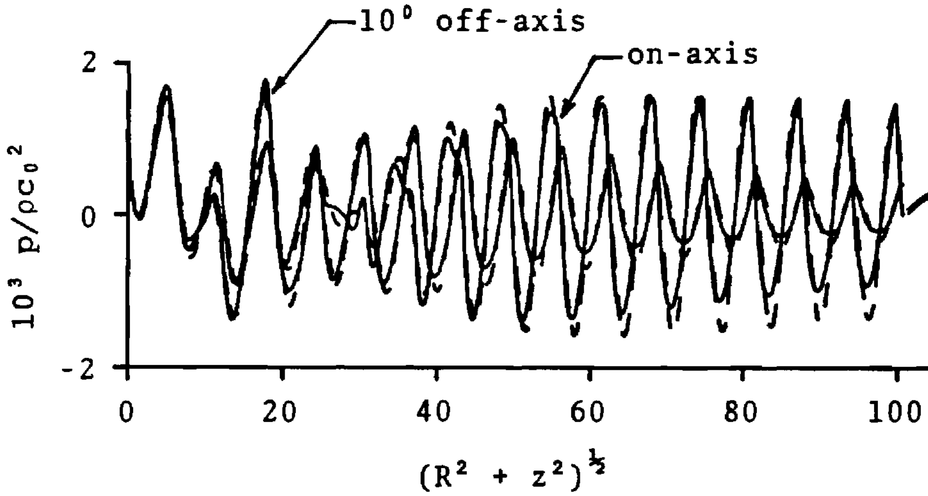
Newton-Raphson to solve for (ξ_n, η_n)
corresponding to specified (z, R, t) & n

along
radial line
intersecting
center of piston

Rayleigh length
≃ 200 in
nondim
units

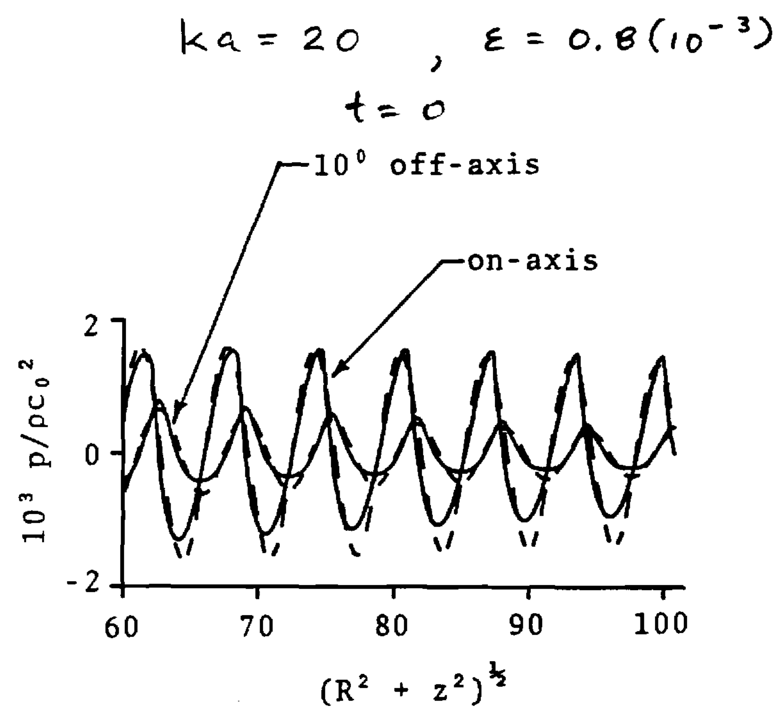
$ka = 20$, $\varepsilon = 0.8(10^{-3})$

$t = 0$



shock
at a
specific n

Figure 1



ka = 20 , ε = 0.8(10⁻³)
t = 0
10° off-axis
on-axis
10³ p/ρc₀²
2
0
-2
60
70
80
90
100
(R² + z²)^½

Figure 2

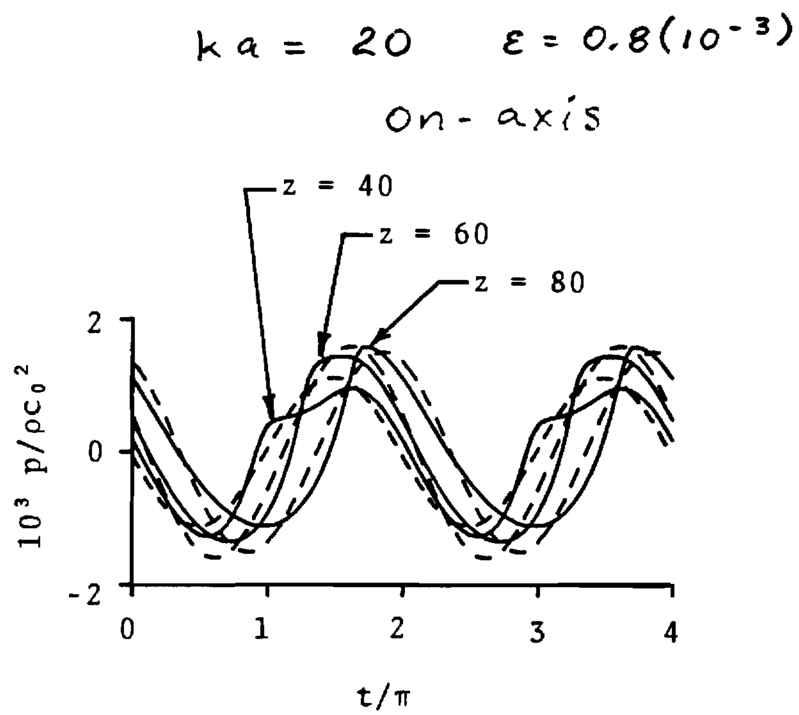


Figure 3

$ka = 20, \quad \varepsilon = 0.8(10^{-3})$

$(R^2 + z^2)^{1/2} = 70$

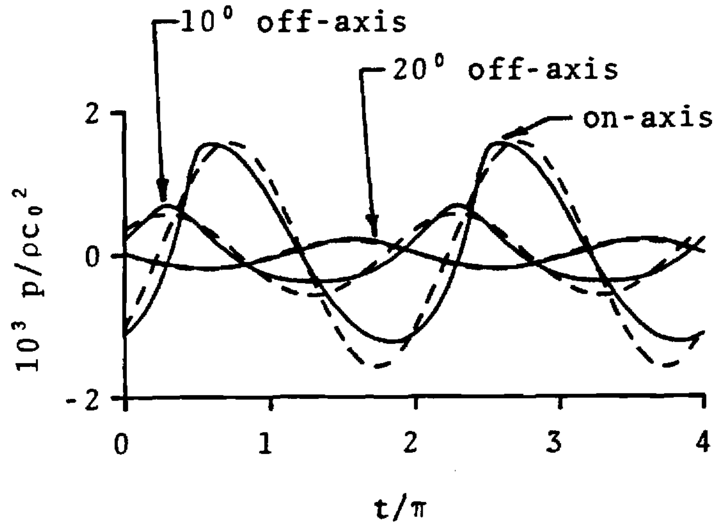


Figure 4

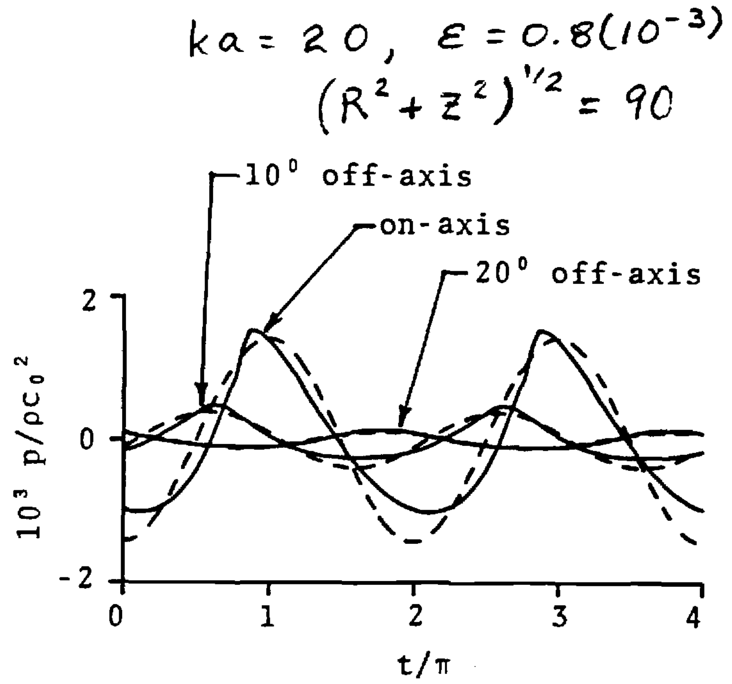

$ka = 20, \quad \varepsilon = 0.8(10^{-3})$
$(R^2 + z^2)^{1/2} = 90$
10° off-axis
on-axis
20° off-axis
$10^3\ p/\rho c_0^2$
2
0
-2
0
1
2
3
4
t/π

Figure 5

Fourier Series Data

$$p = \sum d_i \sin(it - \alpha_i)$$

$ka = 20$, $\varepsilon = 0.0008$, $\beta_0 = 3.625$.

Location	Harmonic number	Amplitude $10^3 p/\rho_0 c_0^2$	Amplitude dB re 1 μ Pa	Phase lag re linear $-180° < \alpha_i \leq 180°$	Phase lag re linear Adjusted	Relative phase lag
r = 40 on axis	linear	1.1271	247.1	-	-	
	1	1.0841	246.8	5.4	5.4	
	2	0.2213	233.0	88.8	88.8	83
	3	0.1187	227.6	-82.6	277.4	189
	4	0.0555	220.9	93.2	453.2	176
	5	0.0298	215.3	-102.6	617.4	164
r = 50 on axis	linear	1.5014	249.6	-	-	
	1	1.3906	248.9	4.1	4.1	
	2	0.1579	230.0	-13.1	346.9	343
	3	0.1168	227.4	-5.5	714.5	368
	4	0.0672	222.6	23.8	1103.8	389
	5	0.0379	217.4	46.4	1486.4	383
r = 60 on axis	linear	1.5985	250.1	-	-	
	1	1.4493	249.3	4.8	4.8	
	2	0.2169	232.8	-129.5	230.5	226
	3	0.1052	226.6	78.7	438.7	208
	4	0.0645	222.3	-50.1	669.9	231
	5	0.0391	218.0	-174.4	905.6	236

Fourier Series Data

ka = 20, ε = 0.0008, β_0 = 3.625.

Location	Harmonic number	Amplitude $10^3 p/\rho_0 c_0^2$	Amplitude dB re 1 μ Pa	Phase lag re linear $-180° < \alpha_i \le 180°$	Phase lag re linear Adjusted	Relative phase lag
r = 90	linear	1.4235	249.1	-	-	
on axis	1	1.1631	247.4	5.9	5.9	
	2	0.2691	234.7	154.3	154.3	148
	3	0.1032	226.3	-57.1	302.9	149
	4	0.0525	220.5	94.7	454.7	152
	5	0.0322	216.2	-108.8	611.2	158
r = 90	linear	0.3900	237.9	-	-	
10°off axis	1	0.3326	236.5	6.6	6.6	
	2	0.0928	225.4	123.6	123.6	117
	3	0.0242	213.8	-122.4	237.6	114
	4	0.0124	208.0	-6.0	353.6	116
	5	0.0072	203.2	102.9	462.9	110
r = 90	linear	0.1186	227.5	-	-	
20°off axis	1	0.1231	227.9	1.2	1.2	
	2	0.0206	212.3	-49.1	310.9	310
	3	0.0028	194.9	-160.6	559.4	219
	4	0.0004	178.7	48.0	768.0	209
	5	0.0001	161.2	-91.6	988.4	220

$ka = 20, \quad \varepsilon = 0.8\,(10^{-3})$

$t = 0.5\,\pi$

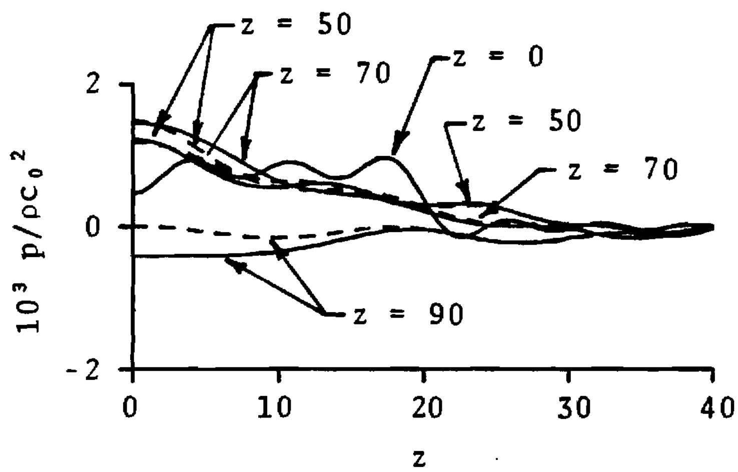


Figure 7

$ka = 20, \quad \varepsilon = 0.4(10^{-3})$

on-axis

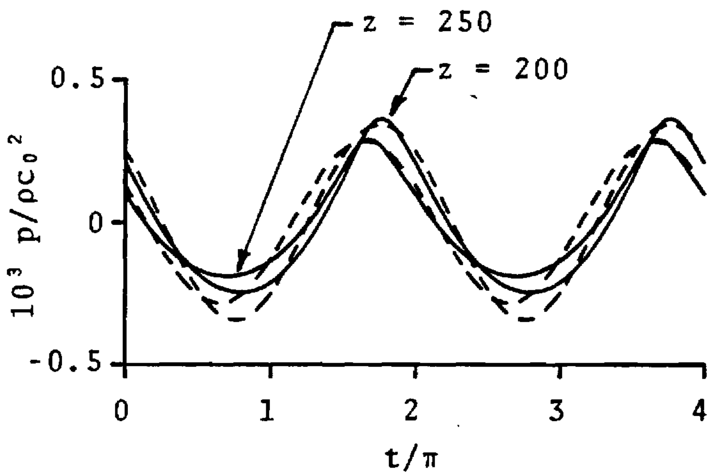


Figure 8

$ka = 10, \quad \varepsilon = 0.8(10^{-3})$
on-axis

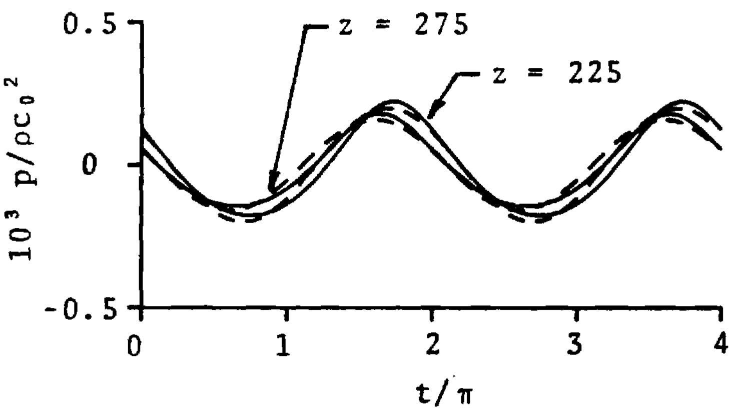


Figure 9