# FUTURE AIRCRAFT NETWORKS AND SCHEDULES

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# FUTURE AIRCRAFT NETWORKS AND SCHEDULES

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*To myself,*

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# SUMMARY

Because of the importance of air transportation scheduling, the emergence of small aircraft and the vision of future fuel-efficient aircraft, this thesis has focused on the study of aircraft scheduling and network design involving multiple types of aircraft and flight services. It develops models and solution algorithms for the schedule design problem and analyzes the computational results.

First, based on the current development of small aircraft and on-demand flight services, this thesis expands a business model for integrating on-demand flight services with the traditional scheduled flight services. This thesis proposes a three-step approach to the design of aircraft schedules and networks from scratch under the model. In the first step, both a frequency assignment model for scheduled flights that incorporates a passenger path choice model and a frequency assignment model for on-demand flights that incorporates a passenger mode choice model are created. In the second step, a rough fleet assignment model that determines a set of flight legs, each of which is assigned an aircraft type and a rough departure time is constructed. In the third step, a timetable model that determines an exact departure time for each flight leg is developed.

Based on the models proposed in the three steps, this thesis creates schedule design instances that involve almost all the major airports and markets in the United States. The instances of the frequency assignment model created in this thesis are large-scale non-convex mixed-integer programming problems, and this dissertation develops an overall network structure and proposes iterative algorithms for solving these instances. The instances of both the rough fleet assignment model and the timetable model created in this thesis are large-scale mixed-integer programming problems, and this dissertation develops subproblem schemes for solving these instances. Based on these solution algorithms, this dissertation also presents computational results of these large-scale instances.

To validate the models and solution algorithms developed, this thesis also compares the daily fight schedules that it designs with the schedules of the existing airlines. Furthermore, it creates instances that represent different economic and fuel-prices conditions and derives schedules under these different conditions. In addition, it discusses the implication of using new aircraft in the future flight schedules. Finally, future research in three areas–model, computational method, and simulation for validation–is proposed.

# CHAPTER I

# INTRODUCTION

"Real world problems come first. Mathematical modeling comes second. Theory and algorithms follows as needed."—George Dantzig

In modern society, a very important industry is the air transportation industry. It provides passengers with the fastest transportation services, which greatly shortens passengers' travel time, especially in long-haul travel. Therefore, it leads to substantial time cost savings for the entire society. Furthermore, it makes long-haul trips more convenient and more comfortable for people than other transportation services. Therefore, it facilitates the movement of human resources among different places. As a result, it promotes the productivity of the entire society, enhances the exchange of culture, education, and information among different people, and facilitates business meetings and academic conferences. Thus, it enhances social development. In addition, as it bridges the distance between different countries and continents, it integrates every part of the world into a globalized economy. According to statistical data from the Bureau of Transportation Statistics[21], in 2009, the air transport industry in the United States delivered over 600 million passengers and 20 million tons of cargos. It is also an important part of the economy by itself, providing over five million jobs and contributing over 400 billion dollars to global GDP in 2006 [25].

In spite of its an important role in the development of our economy and society, the air transportation industry also faces a challenging scheduling problem with an unmanageable size and intractable complexity, involving a broad range and a large quantity of expensive resources. Each day, a typical large airline must operate thousands of flights, fly hundreds of aircraft, and manage hundreds of crews. Furthermore, these resources are correlated. As a consequence, the solutions to the scheduling problem are complex and challenging. However, generating a better schedule plan will not only provide passengers with a more convenient transportation system, but also improve the overall revenue of the air transportation industry. Therefore, the air transportation scheduling problem deserves extensive study.

The remainder of this chapter is organized as follows. Section 1.1 will present the background and an overview of the current practice in air transport schedule planning. Section 1.2 will describe the research problem and its scope. Section 1.3 will state the contributions and present an outline of this thesis.

## 1.1 Overview of Air Transportation Schedule Planning

This section will present an overview of the current practice in air transportation schedule planning. Before presenting an overview, this thesis will provide some background and introduce some terminology. These terms will also be used throughout this thesis.

### 1.1.1 Background and terminology

Since the flight schedule is the primary product of the transportation industry, this section will first introduce terminology about flight schedule. A *flight schedule* is a sequence of *flight legs*, each of which is a non-stop flight from an origin to a destination with a specified departure time. A *daily flight schedule* is the schedule that each flight leg repeats each day, a *weekly flight schedule* is the schedule that each flight leg repeats each week, and a *monthly flight schedule* is the schedule that each flight leg repeats each month.

In reality, the flight schedule of a commercial airline is relatively stable, but it may change seasonally due to the seasonal changes in passenger demand. However, small changes in the schedule are made monthly. Furthermore, during each week, most flight legs fly every day. Table 1 illustrates a part of an airline schedule. For example,

**Table 1:** An example of an airline flight schedule

| Flight Number | Origin | Destination | Departure Time | Arrival Time | Frequency |
|---|---|---|---|---|---|
| 1 | A | B | 9:00 am | 11:00 am | 12345 |
| 2 | A | C | 9:10 am | 11:40 am | 135 |
| 3 | A | D | 9:20 am | 11:20 am | 12345 |
| 4 | B | C | 11:30 am | 12:30 pm | 12345 |

it shows that Flight 1 is from A to B, with a departure time at 9:00 am and an arrival time at 11:00 am. Furthermore, this flight operates Monday through Friday. Utilizing the relative stability of their schedules, commercial airlines decompose their flight schedule problems into a daily flight schedule problem, a weekly flight schedule problem, and a monthly flight schedule problem, which greatly reduces computational complexity. To be more clear, after solving a daily problem, a commercial airline will solve a weekly and a monthly problem with exemptions, that is, some flights will not fly on certain days of a week or a month.

Usually, airlines publish their flight schedule three to six month in advance so that passengers can book their flight tickets ahead of their actual flight times. A published schedule normally includes an aircraft type used in the flight leg such as a Boeing 737 or a McDonnell Douglas DC-9. For simplicity, an aircraft type is often referred to as a *fleet*. Different fleets often have different characteristics, such as capacity and a fuel consumption rate, which influence their usage in a flight schedule.

The primary consumers of a flight schedule are air passengers. Therefore, considerable terminology pertains to a passenger. A passenger's *trip* is a route from an origin to a destination. The origin usually refers to a place close to a passenger's home or workplaces. From there, the passenger can either take ground transit or drive to a departure airport. Passengers take trips because they have a purpose for travel. The purpose could be a business meeting or a vacation. The term that describes how many people have a desire to travel is *passenger demand*. In contrast, the term that describes the number of enplaned passengers is *passenger traffic*. Passenger demand includes not only enplaned passengers but also those who had a desire to fly but could not be accommodated due to insufficient capacity. Both passenger demand and passenger traffic are typically measured in terms of flow per time period.

In reality, gathering the data describing the truly origin and destination of passengers' trips is not practical. Therefore, some degree of aggregation of passenger demand is necessary. In public data, passenger demand is often aggregated around airports or cities. For example, the Transportation Bureau of Statistics provides passenger data for airport pairs (DB1B). A *market* is an airport pair. For example, A-B is a market, and B-A is the reverse market of A-B. Usually, a round trip is classified into two markets.

A flight *itinerary* is a path of flights connecting a departure airport to an arrival airport. Usually, serval itineraries serve the same market. Passengers make choices of an itinerary for their trips based on a balance of flight cost, flight time, convenience and so on. The fare paid by passengers to travel by air varies according to distance and the characteristics of the fare product purchased. In Figure 1, Lewe et. al. [64] illustrated two types of flight itineraries in the current air transportation system, one consists of purely scheduled flights, and one consists of purely on-demand flights.

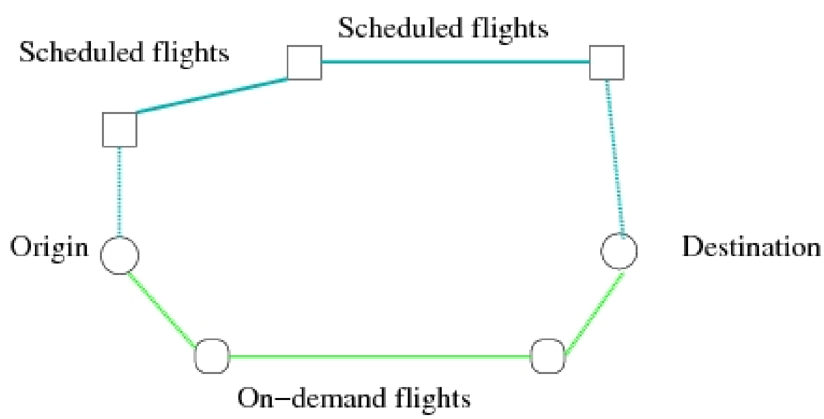



**Figure 1:** Illustration of two types of flight itineraries (based on a graph in Lewe et al. [64])

One important decision in the schedule development is to adopt an appropriate network structure. In reality, carriers may use two network structures, the hub-and-spoke network and the point-to-point network. A *hub-and-spoke* network is a network in which airports are divided into hubs and spokes; hubs have non-stop flights to many other airports, and flight itineraries to or from a spoke need to connect through a hub. A *point-to-point* network is a network that links each airport pair by non-stop flights. A hub-and-spoke structure enables carriers to have many itineraries but operate few flights. Furthermore, it enables airlines to serve small markets through aggregation of the demand in different markets and lower itinerary prices through economy of scale. Figure 2 illustrates both a hub-and-spoke network and a point-to-point network. In the illustration, the hub-and-spoke network can serve fifteen markets altogether but operate only six flights. Furthermore, even if the demand in these markets is small, the demand for each flight leg can still be large through aggregation.

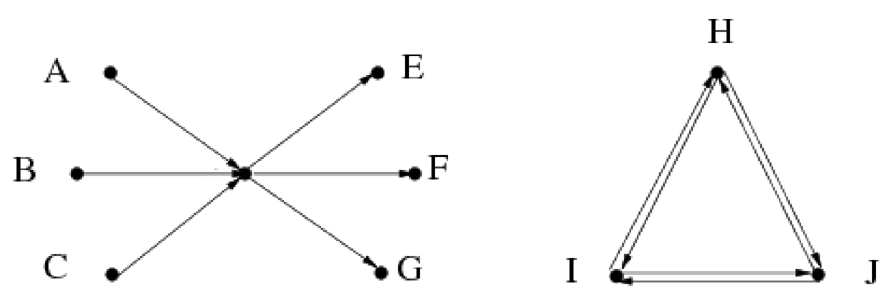



**Figure 2:** A hub-and-spoke network and a spoke-to-spoke network

In reality, the network of each air carrier is a combination of the hub-and-spoke and point-to-point networks. To make a proper decision about what network to use, schedule planners should emphasize the composition of passengers in each market. According to how passengers make choices, passengers are divided into two groups: leisure passengers and business passengers. Leisure passengers, who are price-sensitive, choose less expensive itineraries. In contrast, business passengers, who are time-sensitive, are willing to pay more if the flight schedule is more convenient. Therefore, in a market composed heavily of business passengers, planners should schedule higher flight frequencies and more non-stop flights, while in a market with a majority of leisure passengers, they should schedule more connection flights through hubbing to lower the itinerary prices.

### 1.1.2 Overview of commercial airline schedule planning

Schedule planning involves a number of decisions such as where to fly, when to fly, which aircraft to use and which crew to assign, and so on. Due to the size and complexity of the entire problem, in practice, schedule planning is usually decomposed into four steps—schedule design, fleet assignment, aircraft routing, and crew scheduling—and solved sequentially, which is illustrated in Figure 3.

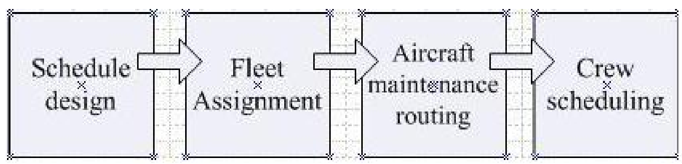



**Figure 3:** Scheduling planning process

#### *1.1.2.1 Schedule design*

The first step of the schedule planning is schedule design. Traditionally, schedule design is further decomposed into two sequential steps: frequency planning and timetable planning. Frequency planning refers to a decision about which market to serve and with what frequency. Timetable planning is the generation of a set of flight legs that meets the frequency requirements determined during frequency planning.

To make a good decision about frequency, planners need to balance operating cost and passenger revenue. On the one hand, higher frequencies provide more convenience to passengers and increasing itinerary frequencies can stimulate more passenger demand. Typically, passengers desire a particular departure time. With higher itinerary frequencies, the difference between a passenger's desired departure time and actual departure times, called *schedule displacement*, could be smaller. In particular, in a short-haul trip, increasing its frequency can promote the advantage of air transportation over the other transportation modes. Therefore, passengers can shift from other modes of transportation to air transportation because of the increasing frequency. In addition, increasing itinerary frequencies can also reduce inconvenience when passengers miss their original flights. On the other hand, increasing itinerary frequencies could lead to increasing operating costs. Therefore, a proper itinerary frequency depends on a good balance between the operating costs and passenger revenue.

After completing frequency planning, planners need to develop a flight schedule such that the requirement of itinerary frequency can be met. In reality, a large number of passengers prefer departure times around 9:00 am and 6:00 pm, which are also called *peak departure time*. Therefore, another goal of timetable planning is to schedule as many flights around the peak times as possible.

The most strategic step of schedule planning is schedule design. For one, it greatly influences decisions made in the following steps since the schedule that it outputs serves as an input to the following steps. In addition, because it largely determines the passenger demand that can be stimulated by the final schedule made after the entire planning process, it largely determines the profitability of the final schedule.

Although schedule design is very important, it is a daunting task. First of all, to make a good schedule, planners should consider the interaction between passenger demand and transportation supply. However, data on passenger demand is difficult to collect. In addition, the relationship between passenger demand and transportation supply is hard to analyze. Furthermore, a schedule model including the interaction between passenger demand and transportation supply is extremely challenging. However, relatively little work has been done in this area. To attack the schedule design problem, some researchers have presented an incremental approach. That is, based on an existing schedule, they search for more profitable schedules by adding some new flights and / or by deleting or adjusting some existing flights.

#### *1.1.2.2 Fleet assignment*

The step following schedule design is fleet assignment, which refers to the assignment of a fleet on each flight leg. Different fleets have different seat capacity and different operating costs per seat per hour. Typically, fleets with larger seat capacity have higher hourly operating costs but lower hourly operating costs per seat. Choosing the right fleet for each leg is important. On the one hand, fleet operating cost accounts for a major portion of the total operating costs of a schedule. On the other hand, the capacity of a fleet on a flight leg influences the passenger revenue achieved on this leg. For example, if passenger demand for a flight is high but a small aircraft is assigned to that flight, then many passengers will be reassigned or spilled to other flights or even to other transportation modes, leading to revenue losses. The revenue losses attributed to the spilled passenger demand is referred to as *passenger spill cost*.

A good fleet assignment should minimize the sum of its fleet operating costs and passenger spill costs. Furthermore, it should also satisfy some feasibility constraint. For one, the flight schedule for each fleet should satisfy the flow balance constraint at each station so that aircraft can circulate. In addition, the number of aircraft for each fleet needed in the schedule should be less than the number of available aircraft.

Fleet assignment model (FAM) is a good example of application of operations research in practice. In a three-year study of using FAM at Delta, Subramanian et al. [83] reported savings of $300 million. Rushmeier and Kontogiorgis [76] reported a $15 million savings of using the fleet assignment model at US Airways.

#### *1.1.2.3 Aircraft maintenance routing*

Once a fleet assignment solution is determined, the next step is to assign a specific aircraft on each flight leg so that each aircraft satisfies its minimum maintenance requirements, which is referred to as aircraft maintenance routing step. For safety purpose, the Federal Aviation Administration (FAA) mandates safety checks during a period of time for each aircraft. Indeed, each carrier has its own maintenance requirements, which is usually more stringent than the FAA mandatory checks. Furthermore, some major airlines may require that each aircraft in its fleet fly all the legs assigned to that fleet for equal utilization of each aircraft. With this additional requirement, the aircraft routing problem becomes an Euler tour problem.

##### *1.1.2.4 Crew scheduling*

A crew scheduling problem is assigning crews to cover each flight leg at a minimum cost, which is usually modeled as a set partition problem. The assignment of each crew consists of *crew pairings*, a sequence of flight legs that starts and ends at the same crew base. A legal crew pairing should follow numerous rules defined by the FAA. Furthermore, it has complicated cost structures, which are defined by the FAA and contractual restrictions. Because of the complicated cost structures and legality issues, a crew scheduling problem is usually decomposed into two subproblems: a crew pairing problem and a crew assignment problem. A crew pairing problem is to generate a set of pairings at minimal cost covering all the flight legs, while a crew assignment problem is assigning a set of pairings to each crew at minimal cost.

### 1.1.3 Overview of On-demand Schedule Planning

Nowadays, as small aircraft technology continues to develop, the value of time and convenience to passengers increases. In addition, the United States has over 5,000 public-use airports capable of operating on-demand service ([28],[49]). Therefore, on-demand air transportation has gradually became a reality, and the demand for door-to-door, on-demand services also increases.

Currently, the three major types of on-demand flight service providers are fractional airlines, charter airlines, and air taxi companies. Although these providers use different business models, they face similar scheduling problems. Typically, a customer calls an on-demand service provider one to three days in advance and reserves a flight. Then, the on-demand service provider either accepts or rejects the request and determines its flight schedules according to its accepted flight requests and its demand forecasting.

As business grows, on-demand operators face more and more challenging scheduling problems. In general, they also need to solve the four problems that commercial airlines must solve: schedule design, fleet assignment, aircraft routing, and crew assignment problems. Different from the flight schedule of commercial airlines, the schedule of on-demand operators varies from day to day due to the strongly stochastic property of passenger demand. Furthermore, aircraft repositioning costs are a very important factor in the total operating cost [94]. Because of this dynamic and stochastic feature, the scheduling problem of on-demand flight services is very difficult to solve. However, researchers have conducted numerous studies ([94], [49]) on the scheduling problems of on-demand flight services. In addition, a good forecast of passenger demand in on-demand flight service can help on-demand operators design efficient schedules and distribute aircraft effectively.

## *1.2 Statement of Problem*

A field worthy of intensive study is air transportation scheduling. In reality, the solution of an air transportation scheduling problem is typically decomposed into four steps: schedule design, fleet assignment, aircraft maintenance routing, and crew assignment. Among the four steps, schedule design and fleet assignment are the two most important steps because the intermediate schedule built after these two steps, which consists of a sequence of flight legs with an assigned fleet, determines the overall profitability of the final schedule. On the one hand, the intermediate schedule determines the final flight itineraries offered to passengers and hence overall passenger revenue. On the other hand, it determines the fleet operating cost, which is the biggest component of total operating costs.

Due to its importance, the fleet assignment problem has been extensively analyzed. Early work on this problem dates back to 1954. However, Abara [31] and Hane et al. [56] laid foundational work for the fleet assignment model. Abara [31] built a connection network and solved practical-sized fleet assignment problems. Hane et al. [56] studies the fleet assignment problem based on a time-space network and discussed several computational methods to solve fleet assignment problems. Following these seminal studies on FAM, researchers have examined a variety of extensions of the basic FAM. Specifically, researchers have investigated incorporating FAM with maintenance, routing, and crew considerations ([41], [37], [73], [75],[80]). In addition, researchers have also studied FAM with enhanced passenger considerations such as spill, recapture, and the supply-demand interaction ([38],[62]).

In contrast to the fleet assignment problem, the schedule design problem has not yet been well studied because of its complexity. Yan and Tseng [93] analyzed the problem of simultaneously scheduling and routing flights. They built a model that routes passengers through the network at minimum cost. They applied their method to eleven cities, one hundred seventy flights, and two fleets. They pointed out the need for further study of cases of larger size. Lohatepanont and Barnhart [66] addressed the problem of selecting flight legs from an initial schedule that comprises mandatory legs and optional legs as well as assigning a fleet to each selected leg. They used demand correction terms to capture the supply-demand interaction. They used both column generation and row generation to deal with a large number of demand correction terms and demand spill constraints. Nitika [70] analyzed integrating schedule design with fleet assignment along with incorporating demand-supply interaction. In particular, the author examined the schedule design problem of selecting a subsect of flight legs to operate, given a set of candidate flight legs. The author employed a logit model that distributes of passenger demand in each market to each itinerary. However, the problem size that the author addressed consisted of no more than two markets, two airports, and two fleets. Vaze and Barnhart [89] addressed the problem of designing a rough flight schedule that satisfies passenger demand while minimizing airport congestion. However, one assumption of their model—passengers in each market would choose any available itinerary—showed that they did not examine the effect of passengers' itinerary choices. Furthermore, due to computational difficulties, the authors ignored flow balance constraints. Their results showed that their rough schedules generally does not satisfy these constants.

In view of previous research on fleet assignment and schedule design, more studies are needed. For one, studies that incorporate passenger demand forecasting models into schedule design models are needed. In schedule design, understanding where passengers want to fly, when they want to fly, how they want to get there, is critical to the profitability of the air transport industry. Currently, many articles discuss the factors that influence passenger demand. However, very few articles on network design analyzes the effect of passenger demand on the flight network, particularly the effect of passengers' itinerary choices.

Furthermore, studies pertaining to the integration of scheduled and on-demand flight service are needed. Recently, as small aircraft technology continues to develops, and the value of convenience and time to passengers increases, the demand for door-to-door, on-demand services has also risen. Furthermore, incorporating on-demand services into air transport scheduling can improve the quality of flight services provided to passengers. Although many articles discuss scheduled flight planning and some discuss on-demand service scheduling, none integrate scheduled with on-demand service.

In addition, studies pertaining to the inclusion of multiple aircraft types in schedule planning are needed. Recently, engineers have developed many new aircraft types, including very large transports (VLTs), very light jets (VLJs), short takeoff and landing vehicles (SOLs), and many others. These aircraft types have different characteristic such as different seating capacity, speed, and runway occupancy. Therefore, a new direction of air transport scheduling is to design schedules that effectively utilize different aircraft. To develop the next generation air transport system, researchers must address this topic.

To provide understanding on these aspects, this thesis focuses on an aircraft network and schedule design problem. To clarify the problem, it is to design an aircraft network and schedules that uses multi-fleets and integrates both scheduled services and on-demand services by analyzing supply-demand interactions. Because of its complexity, this problem will be simplified. Existing commercial airlines have a very complicated fare system, different passengers may pay a different price for the same itinerary. However, in the schedule design problem discussed in this thesis, passengers are divided into two types: business passengers and leisure passengers. For each itinerary, two fares are included, one for leisure passengers and one for business passengers. Passengers in each type are assumed to be homogeneous, that is, they have the same itinerary choice behavior.

## *1.3 Statement of Purpose*

The purpose of the thesis is to develop models and solution algorithms for designing an aircraft network and schedule from scratch. The designing process will analyze the interaction between passenger demand for each itinerary and the flight schedule, integrate both scheduled and on-demand flight service, and examine multiple fleets.

## *1.4 Organization of the Thesis*

The remainder of the thesis is organized as follows. Chapter II reviews the literature related to the schedule design problem. Chapter III briefly summarizes new aircraft technology, lists types of new small aircraft, and then expands a business model of using small aircraft. Chapter IV first illustrates the decomposition scheme for a three-step approach to building aircraft networks and schedules and then presents the models in each of the three steps. Chapter V describes related data and parameters and then develops algorithms for solving the models in each step. Chapter VI presents computational results, and Chapter VII summarizes the results of the thesis and proposes directions for further study.

# CHAPTER II

# LITERATURE REVIEW

This thesis develops a three-step approach to address a schedule design problem. Because a passenger demand model, a frequency assignment model, a fleet assignment model, and a schedule design model are all used in the three-step approach, this section reviews the literature on these models. For a more comprehensive review on models in air transportation, readers can refer to Gopalan and Talluri [53], Barnhart et al. [36], and Clarke and Smith [42].

## *2.1 Review of the Literature on Passenger Demand Models*

Because of its importance in transportation planning, passenger demand forecasting has received researchers' attention for a long time. Researchers have analyzed passenger demand on a high-level of aggregation such the as air transportation system level ([27], [48], [95]), the country-pair level ([61]), and the city-pair level ([54], [90]). In the high level, the explanatory factors are mainly variables of economic activities and geographical characteristics. Researchers have developed several types of models for forecasting demand on this level, such as the gravity model ([54]), the grey model ([61]), the time series model ([95]), and so on.

Given forecasts of passenger volume on a high-level, researchers have studied its allocation to the itinerary level, which greatly helps air carriers build their schedules. Some researchers (e.g., [46], [44]) have developed multinomial and GEV models to study the influence of independent variables on passengers' itinerary choices such as the level of service, the quality of connections, and the time of day. However, these studies do not investigate the influence of basic independent variables on passenger's itinerary choices such as flight times and ticket prices, which are addressed by Seshadri et al. [78] and Adler et al. [32].

To study passengers' itinerary choice, Seshadri et. al [78] built a logit model with frequency, relative fare, and relative flight time as explanatory variables. The author used the *Official Airline Guide* to extract flight schedules and used the *Airline Origin and Destination Survey* to extract the fares of and the number of passengers on each itinerary. Based on an estimation of the parameters of the explanatory variables and passenger demand on each market, they predicted passenger demand on each itinerary. However, the authors did not segment passengers according to their trip purposes.

Adler et. al [32] developed a logit model to analyze trade-offs between service variables such as flight time, the number of connections, one-way fares, and schedule displacement in passengers' itinerary choices based on a stated-preference survey data. In contrast to Seshadri et al [78], Adler et. al segmented passengers into business passengers and nonbusiness passengers. Furthermore, their results show that the trade-offs between these service variables to business passengers are different from those to nonbusiness passengers. In addition, fare substitution values for those service variables are within a normal range.

Recently, along with the development of small aircraft technology, small aircraft have gradually been incorporated into the air transportation system and utilized in on-demand, door-to-door services. Recognizing this transition, researchers have started studying passengers' choice behavior in the transportation system incorporated with on-demand flight services. Ashiabor et al. [34] developed logit models to study the influence of travel time and travel cost on passengers' choices among automobile, commercial air transportation, and small aircraft transportation based on a National Travel Survey. However, their parameter estimates are counterintuitive. For example, their results imply that the value of time to business passengers is lower than that to leisure passengers. Baik et al. [35] developed a mode choice model to study passengers' choices among automobiles, commercial airlines, and air taxis based on the *American Travel Survey*. They segmented passengers according to their trip purposes and household income. They validated their results on some historical passenger boarding data.

This thesis focuses on building aircraft networks and schedules, but it does not focus on research on passenger demand models. Therefore, it builds passenger demand model based on models in the literature. With respect to scheduled services, it uses the parameter estimates of Ashiabor et al. [34]. With respect to on-demand services, it uses the parameter estimates of Baik et al. [35]. The reasons for choosing the results of these researchers are that they segmented business and leisure passengers and the elasticity of their parameter estimates are fell within a normal range.

### 2.1.1 Overview of the Discrete Choice Model

Efficient scheduling depends on a thorough understanding of passenger demand. Because of the importance of passenger demand, researchers develop several theories to study passenger demand. One of the theories attracting more and more interest is discrete choice theory. This thesis will apply discrete choice model to estimate passenger demand. Therefore, this section will briefly introduce the discrete choice model. A comprehensive discussion of discrete choice model can be found in [40], [51], and [86].

Discrete choice theory has provided a framework for analyzing the choice process and predicting the choice behavior of decision makers. It views the choice process as follows: Decision makers first determine their available alternatives; then they evaluate attributes of these alternatives; and finally, they make decisions according to their decision rules.

One of the most studied decision rules is the utility maximization rule. Utility is a numerical way of representing the value of the alternatives of a decision maker.

Formally, the utility maximization rule can be presented as follows. Let $C$ denote the set of alternatives and $X_i$ the attribute vector of the ith alternative. For decision maker $t$, let $S_t$ denote the attribute vector of decision maker $t$ and $U(X_i, S_t)$ the utility of the ith alternative to this decision maker. Under the utility maximization rule, decision maker $t$ chooses alternative $i$ if and only if $U(X_i, S_t) \geq U(X_j, S_t)$ for any $j \in C$.

In fact, utility functions consist of several underlying random factors. First, decision makers may have incomplete information about their alternatives and evaluate them incorrectly. Second, analysts may have incomplete information about decision makers and their alternatives. Furthermore, analysts may not completely understand the relationship between the attributes of the alternatives and the choices of the decision maker. To capture these observational errors and analytical uncertainties, researchers develop probabilistic choice theory, in which the utility function is represented by $U(X_i, S_t) = V(X_i, S_t) + \epsilon_{i,t}$, where $V(X_i, S_t)$ denotes the utility observed by analysts and $\epsilon_{i,t}$ denotes the random error.

Based on different assumptions of the distribution of these random errors, researchers have established different discrete choice models. Probit models and logit models are two mainstream models. Probit models, using normal distributions for random errors, have no closed-form and are hard to compute. On the other hand, logit models, using Gumbel distributions for random errors, have a closed form and are relatively easy to compute. Furthermore, using logit models, analysts can more easily include their intuitions of explanatory variables in the choice process and derive more detailed knowledge about these variables. Therefore, the logit model is widely used in practice. One branch of the logit model is the multinomial logit model, which is described as follows.

Let $C$ denote the set of alternatives, $S$ the set of decision makers, and $U_{ni} = V_{ni} + \epsilon_{ni}$ the utility of alternative $i$ to decision maker $n$. The probability that decision maker $n$ will choose $i$ is represented as

$$\begin{aligned} Pr(U_{ni} \geq U_{nj}, \forall j \neq i) &= Pr(U_{ni} \geq \max_{j \neq i} U_{nj}) \\ &= Pr(\epsilon_{ni} + V_{ni} \geq \max_{j \neq i}(\epsilon_{nj} + V_{nj})). \end{aligned}$$

The multinomial logit model is based on the assumption that the error terms, $\epsilon_{ni}s$, are independent and identically Gumbel-distributed across decision makers, which greatly simplifies the calculation of the probability $Pr(U_{ni} \geq U_{nj}, \forall j \neq i)$. Now, assume that $\epsilon_{ni}s$ have Gumbel distribution $G(\eta, \gamma)$, where $\eta$ is the location parameter, and $\gamma$ is the scale parameter. Using two properties of the Gumbel distribution, $\max_{j \neq i}(\epsilon_{nj} + V_{nj})$ has Gumbel distribution $G(\frac{\ln \sum_{j \neq i} e^{\gamma V_{nj}}}{\gamma}, \gamma)$, and $\epsilon_{ni} + V_{ni} - \max_{j \neq i}(\epsilon_{nj} + V_{nj})$ follows the logistic distribution. In particular, $Pr(\epsilon_{ni} + V_{ni} \geq \max_{j \neq i}(\epsilon_{nj} + V_{nj})) = \frac{e^{\gamma V_i}}{\sum_j e^{\gamma V_j}}$. In this formula, $\gamma V_j$ can be regarded as the normalized utility. To clarify the notion of normalized utility, let $U^*_{nj} = \gamma U_{nj}$, $V^*_{nj} = \gamma V_{nj}$, and $\epsilon^*_{nj} = \gamma \epsilon_{nj}$, then

$$\begin{aligned} Pr(U^*_{ni} \geq U^*_{nj}, \forall j \neq i) &= Pr(U_{ni} \geq U_{nj}, \forall j \neq i) \\ &= \frac{e^{\gamma V_{ni}}}{\sum_j e^{\gamma V_{nj}}} \\ &= \frac{e^{V^*_{ni}}}{\sum_j e^{V^*_{nj}}}. \end{aligned}$$

Note that $\epsilon^*_{nj}$ has Gumbel distribution $G(\gamma\eta, 1)$. From now on, for simplicity, the scale parameter of the error terms are assumed to be normalized to 1 in the multinomial logit model. Under this assumption, $P_{ni} = \frac{e^{V_{ni}}}{\sum_j e^{V_{nj}}}$, where $P_{ni}$ denotes the probability that alternative $i$ is chosen by individual $n$.

As shown above, the multinomial logit model has a very simple closed form. In addition to its simplicity, it has several other good properties. First of all, the probability $P_{ni} = \frac{e^{V_{ni}}}{\sum_j e^{V_{nj}}}$ is a logistic function of $V_{ni}$ for fixed $V_{nj}$, $j \neq i$. The logistic function has application in many areas, including economics, mathematical psychology, and statistics. It models the S-curve of growth of some set: Initially, growth is approximately exponential; then, at a critical point, it slows, and finally, stops. In the context of the multinomial logit model, this growth pattern has the following implications:

1. Let $V_{no}$ denote the nominal utility of the combination of all the other alternatives. A critical point occurs when utility $V_{ni}$ of alternative $i$ is equal to the utility $V_{no}$.

2. When $V_{ni} \preccurlyeq V_{no}$ or $V_{ni} \succcurlyeq V_{no}$, a small change in $V_{ni}$ will not incur a big change in the probability $P_{ni}$. In order words, if $i$ is much better or much worse than the other alternatives, a small change in $i$ will not change the probability of $i$ being chosen.

3. When $V_{ni}$ is close to $V_{no}$, a small change in $V_{ni}$ will greatly change the probability $P_{ni}$.

## 2.2 Review of the literature on frequency assignment model

In his book, Teodorovic [85] presented a basic frequency assignment model. Given passenger demand and a lower bound of the itinerary frequency between any city pair, the model determined flight frequencies that satisfied passenger demand in each market and minimized the total fleet operating cost. In addition, the model addressed the frequency assignment problem with only one fleet. The notations and the formulation of the model are presented in the following paragraphs.

$Notations:$

$I:$ Set of flight legs indexed by $(i, j)$

$M:$ Set of markets indexed by $(o, d)$

$Freq_{ij}:$ Daily flight frequency from airport $i$ to airport $j$.

$c_{ij}:$ The cost of one flight from airport $i$ to airport $j$.

$D_{odp}:$ Number of passengers on path $p$ from airport $o$ to airport $d$.

$D_{od}:$ Total number of passengers from airport $o$ to airport $d$.

$Seat:$ Capacity of an aircraft.

$Freq_{odp}:$ Daily frequency of path $p$ from airport $o$ to airport $d$.

$L_{od}:$ Lower bound of itinerary frequency from airport $o$ to airport $d$.

$Formulation:$

$$\max \sum_{i,j} c_{i,j} Freq_{i,j}$$

$$s.t. \quad Seat \cdot Freq_{ij} - \sum_{odp \in ij} D_{odp} \geq 0, \forall (i, j) \in I \tag{1}$$

$$\sum_{p} D_{odp} = D_{od}, \forall (o, d) \in M \tag{2}$$

$$Freq_{i,j} \geq Freq_{odp} \forall (i, j) \in I, \forall (o, d) \in M, odp \in ij \tag{3}$$

$$\sum_{p} Freq_{odp} \geq L_{od}, \forall (o, d) \in M \tag{4}$$

The objective of the model is to minimize the total fleet operating costs. Constraint (1) ensures that the total number of passengers on a flight leg is less than the capacity of that leg. Constraint (2) guarantees that passenger demand in each market is satisfied. Constraint (3) ensures that the frequency of an itinerary is lower than the frequency of any leg on that itinerary. Constraint (4) imposes the lower bound of the itinerary frequency in each market.

Hsu and Wen [59] determined the shape of a network and the frequencies of flights simultaneously. To determine the shape of a network, they first searched $k$-shortest paths and then used the Grey clustering algorithm to reduce the number of candidate routes for each city pair. Based on the basic model of Teodorovic [85], they built a multi-objective programming model with the objective of minimizing both operating costs and passenger itinerary costs. They developed a Grey model that forecasted passenger demand between any city pair, which served as the input to their frequency model. To solve their multi-objective programming problems, they discussed the notion of Pareto optimality and applied the constraint method to obtain optimal Pareto solutions.

Hsu and Wen [60] determined flight frequencies through the supply-demand interaction. Their demand model consisted of two parts: a Grey model that determines passenger demand between each city pair and an analytical model that estimates passenger demand on each route with flight frequencies as input. Their supply model was based on the model developed by Teodorovic [85], mentioned above. To determine optimal flight frequencies, they iterated between the supply model and the demand model until it reached demand-supply equilibrium.

This thesis builds a frequency assignment model that incorporates a passenger path choice model for scheduled flights and a frequency assignment model that incorporates a passenger mode choice model for on-demand flights. Different from previous studies, this thesis addresses large-scale instances of the frequency assignment model. To solve large-scale nonlinear programming problems, this thesis first discusses the overall network structure and simplifies the problems, and then it develops iterative algorithms for solving the problems.

## 2.3 Review of the Literature on the Fleet Assignment Model

One of the most prominent applications of operations research in the air transportation industry is the fleet assignment model (FAM) [81], which has attracted considerable attention due to its importance in airline scheduling. Early research on FAM dates back to 1954 with the work of Dantzig and Ferguson[65]. Later, among numerous operations researchers, Abara [31] and Hane et al. [56] laid the foundation for subsequent work in this field.

The objective of a basic FAM is to maximize profits, and the problem comprises flight cover, plane count, and flow balance constraints. Flight cover constraints ensure that a fleet is assigned to each flight leg. Plane count constraints impose the upper bound of the number of aircraft in each fleet needed in the assignment. Flow balance constraints guarantee that no aircraft disappears in the network. To develop their FAM, researchers have used two types of networks: a connection network and a time-space network.

In one of the first significant studies in FAM, Abara [31] treated fleet assignment problems of a practical size. To formulate his FAM, Abara adopted a connection network. The following explains two major components of a connection network, nodes and arcs. A connection network has four types of nodes: departure nodes, arrival nodes, source nodes, and sink nodes. Each station has two timelines. The nodes in one timeline are arrival nodes, which denote the incoming events while the nodes in the other timeline are departure nodes, which denote the outgoing events. The source node and the sink node, not related to any station, represent the start and end times of the fleet assignment. A connection network has two types of arcs: flight arcs and connection arcs. A flight arc, representing a flight leg, is an arc from a departure node to an arrival node. At each station, a connection arc is an arc from an arrival node to a departure node, representing a potential connection between two flight arcs. Figure 4 illustrates a small example of a connection network.

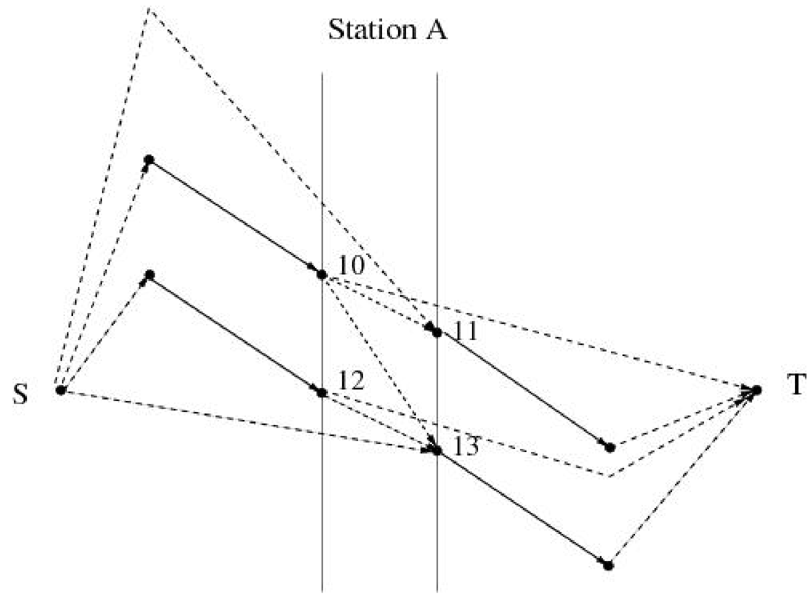



**Figure 4:** A connection network

Hane et al. were one of the first researchers that build a FAM based on times-pace network [79]. In a times-pace network, each station has a timeline that describes the arrival and departure events of the station. A time-space network contains three types of arcs: flight arcs, ground arcs, and wrap around arcs. At each station, the arc between any two consecutive nodes is a ground arc, which means that aircraft can hold at this station; the arc from the last node to the first node in the timeline is a wrap around arc, which means that the schedule is repeated in a period of time. In their paper, Hane et al. [56] discussed plenty of computational methods in solving FAM, which is seminal work in the computation of FAM. Figure 5 illustrates a small example of a time-space network.

Compared with a timeline network, a connection network has both disadvantages and advantages. One disadvantage of a connection network is that the large number of connection arcs can make the size of the network huge. One advantage of a connection network is that the fleet assignment of a connection network specifies the route of each aircraft, but the fleet assignment of a time-space network specifies only the assignment of each fleet.

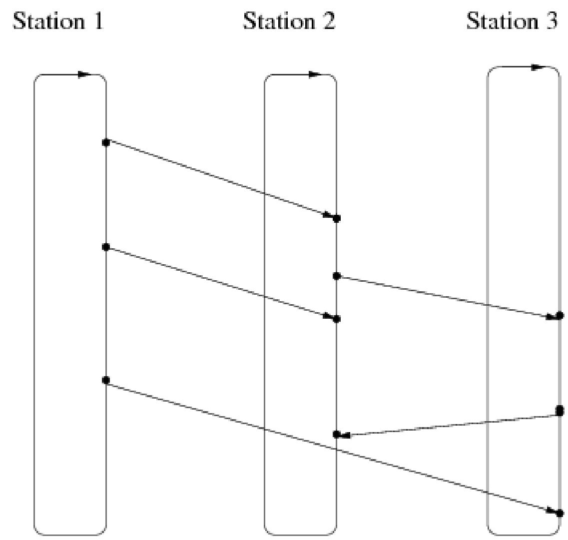



**Figure 5:** A timeline network

Following these seminal studies on FAM, researchers have also studied a variety of extensions of the basic FAM. In particular, researchers have investigated incorporating FAM with maintenance, routing, and crew considerations ([41], [37], [73], [75],[80]). In addition, they have also studied FAM with enhanced passenger consideration such as spill, recapture, and the demand-supply interaction ([38],[62]).

In the three-step approaches developed in this thesis, the second step builds a rough fleet assignment model, a slight variation of the fleet assignment model. The only difference lies in the timeline of each station. The timeline of each station in the rough fleet assignment model consists of only few nodes, which reduces the size of the problems but maintains the key feature of the problem. Because this thesis does not examine aircraft routing but instead needs to address instances of large size, the rough fleet assignment model uses the time-space network.

## *2.4 Review of the Literature on Schedule Design*

Lohatepanont [65] summarized some early studies on schedule design in his master thesis. Most of these early studies concentrated on schedule design with fixed transportation demand or incremental schedule design. Recently, researchers started integrating schedule design with other planning steps such as fleet assignment and aircraft routing. Furthermore, researchers also started incorporating demand-supply interaction in their schedule design models.

However, very few studies pertain to schedule design. Furthermore, the specific schedule design problems discussed in the studies differ from one another. Therefore, the following paragraphs thoroughly review related studies on schedule design in chronological order. In particular, these paragraphs review the model used and the computational method developed in each study.

Yan and Tseng [93] analyzed the problem of simultaneously scheduling and routing flights. They built a model based on an integer multiple commodity network, which combined a fleet-flow time-space network with a passenger-flow time-space network. The objective of their model was to minimize system cost, namely, the total fleet assignment costs and passenger cost. In their model, passenger demand was fixed between any pair of airports. Furthermore, their model aimed at routing passenger through the network at minimum cost, which suggested that they did not examine the impact of a schedule except for that of cost on passengers' itinerary choice. To solve the problem, they used the Lagrangian relaxation method, the network simplex method, and the sub-gradient method. Furthermore, they used a flow decomposition algorithm to generate the route of each airplane. Finally, they applied their method in an instance of eleven cities, one hundred seventy flights, and two fleets. For instances involving a large number of cities and flights, they recommended further study.

Lohatepanont and Barnhart [66] addressed the problem of selecting flight legs from an initial schedule that comprised mandatory legs and optional legs as well as assigning a fleet to each selected leg. In their model, they used demand recapture terms and demand correction terms to capture the demand-supply interaction. Demand recapture was the accommodation of passengers with alternative itineraries when they were spilled from their desired itineraries, while demand correction was referred to the adjustment of passenger demand on each itinerary along with the change in schedule. Because the size of the problem was very large, they simplified their demand correction terms by including only a first-order degree correction term. For example, if itineraries $q, r$ were deleted simultaneously, then the demand change $\Delta^p_{q,r}$ on path $p$ was approximated by the sum of $\Delta^p_q$ (passenger demand change in $p$ when only $q$ was deleted) and $\Delta^p_r$ (passenger demand change in $p$ when only $r$ is deleted).

Overall, the integrated schedule design and fleet assignment model (ISD-FAM) developed by Lohatepanont and Barnhart was an itinerary-based FAM. As their model had a large number of demand correction constraints and spill variables, they used both column generation and row generation. The authors first formulated a restricted master problem by excluding demand correction constraints and most of the spill variables from their ISD-FAM. Then, they used column generation to price out cost-reduced spill variables and used row generation to detect violated demand correction constraints. After determining a schedule from the restricted master problem, they used a passenger mix model (PMM) to calculate the revenue associated with the schedule. Their algorithm terminated when the revenue determined in the PMM did not differ greatly from the approximate revenue in their ISD-FAM.

Nitika [70] considered integrating schedule design with fleet assignment along with incorporating demand-supply interaction. In particular, the author examined the schedule design problem of selecting a subset of flight legs to operate, given a set of candidate flight legs. This problem was much simpler than the schedule design problem addressed in this thesis. Based on the basic FAM model, the author included both recapture and spill constraints. Furthermore, the author employed a logit model to the distribution of passenger demand in each market to each itinerary. The logit model introduced non-linear terms in the form of $\frac{e^{B_i}}{\sum_j e^{B_j} Z_j}$, in which $B_j$ was constant, representing the utility of itinerary $i$, and $Z_j$ is 0-1 variable denoting whether itinerary

$i$ was operating or not. In order to solve the non-linear model, the author used a technique given by Wu [92] to remove nonlinearity. However, due to the complicacy of his model, the problem size that the author addressed consisted of no more than two markets, two airports, and two fleets, which was much smaller than any realistic airline schedule design.

Vaze and Barnhart [89] considered the problem of designing a schedule that satisfies passenger demand while minimizing airport congestion. They divided their schedule development into three stages. In the first stage, they determined the overall structure of their network such as hub locations and allowable connection airports. In the second stage, they determined the flight frequency of using each fleet in each segment while satisfying the daily demand and the minimum daily frequency requirements. Their daily passenger demand was derived from data on the website of BTS [24]. For each leg, the minimum daily flight frequency was defined as the total number of non-stop flights offered by all the existing carriers as long as the flight departure times were not too close to each other. It was also referred to as an effective non-stop frequency, which was suggested by Cohas et al. [43]. One assumption of their model was that passenger demand in each market will be the same as that found in the data as long as the minimum daily frequency is satisfied. Another assumption of their model was that passengers in each market would choose any available itinerary, which showed that Vaze and Barnhart did not examine the effect of passengers' itinerary choices, and the authors used leg frequency instead of itinerary frequency as indices of service quality. As the size of the model in this stage was very large, they solved LP relaxation and rounded up to the nearest integer solution.

In the third stage, in order to satisfy the daily frequency of using each fleet in each segment, vaze and Barnhart generated a set of flight legs with departure times and arrival times shifted to the nearest hour. Their objective was not to build an operational schedule but to build a rough schedule so that they could estimate delays in the schedule. Furthermore, they ignored the aircraft flow balance constraints, which were important constraints in a scheduling model. Because the number of markets with small demand was huge, the LP solution to their model yielded too many fractions. To deal with this problem, they first included only markets with a demand of more than 250 and rounded up the fractional to a nearest integer. After that, they added flight legs heuristically to meet passenger demand in the small markets.

Following the review of the schedule design literature is a summary of the previous research on schedule design as well as a comparison of previous research and the research in this thesis. First of all, all the studies reviewed above except that by Vaze and Barnhart focus on incremental schedule design, namely, selecting a good subset of flight legs out of a set of candidate flight legs. Moreover, in their research, Vaze and Barnhart build a rough flight schedule from scratch. They do not include important constraints such as flow balance constraints. Their results show that their rough schedules generally do not satisfy these constraints. Thus, currently, no work pertains to building an exact schedule from scratch. Furthermore, passengers' itinerary choices are not examined very thoroughly in the current models of schedule design. One common assumption of these studies (except the study by Nitika) is that passengers would choose any available itineraries. In addition, none of the research incorporates on-demand flight service into the scheduled flight service. In contrast to previous studies, this thesis discusses building an exact flight schedule, that combines scheduled flights and on-demand flights from scratch. In addition, it integrates passengers itinerary choices in the schedule design.

# CHAPTER III

# THE AIR TRANSPORTATION SYSTEM WITH SMALL AIRCRAFT

## *3.1 Introduction*

Most commercial airlines in the United States have hub-and-spoke networks that utilize only commercial airports. This hub-and-spoke model of operating scheduled flights can reduce flight costs and increase itinerary frequencies by consolidating the passenger demand of different itineraries, which greatly benefits passengers, especially passengers in hub cities and spoke communities [58]. However, this model does not promote passenger convenience or door-to-door time efficiencies [58]. Statistics show that only around 40% of the population lives within a half hour of commercial airports and over 90% of domestic air passengers have to fly through fewer than 500 commercial airports [91]. Furthermore, because the hub-and-spoke flight network utilizes only the commercial airports, it “does not serve rural, regional, and intra-urban travel well” for trip distances less than 500 miles [69]. Furthermore, passengers choose automobiles more often than air transportation for trip distances of less than 500 miles [69]. Therefore, the existing transportation system, dominated by the hub-and-spoke network, must be enhanced if rural, regional, and intra-urban air service is to be improved.

One way of enhancing the existing transportation system is to promote the utilization of small community airports and small aircraft. Currently, the United States has over 3,000 underutilized small community airports with a paved runway of lengths not shorter than 3,000 ft [91]. Proper utilization of these small community airports can not only relieve the congestion in the current transportation system but also help to improve the quality of the entire air transportation service. Statistics show that over 90% of the population lives within a half hour of small community airports ([91],[58]). By utilizing these small community airports, small aircraft operating point-to-point, on-demand services can increase the utility of air transportation relative to the automobile in short-haul travel. Although recent developments in small aircraft technology will promote the utilization of small aircraft and small community airports, studies on the efficient utilization of these air transportation resources are still needed as the operation modes of small aircraft and large aircraft differ considerably.

To efficiently utilize small aircraft and community airports to increase the capacity and diversify the structure of existing transportation systems, the FAA and the National Aeronautics and Space Administration (NASA) have launched research on the Small Aircraft Transportation System (SATS) ([77], [58], [74]). The SATS project is comprised of research in three areas: new aircraft technology, new service modes, and the supporting airspace infrastructure [77].

Recently, NASA has also initiated research on the Next Generation Air Transportation System (NextGen). Studies focus on the impact of new vehicle concepts and operations on the NextGen [63]. New aircraft concepts include cruise-efficient short takeoff and landing (CESTOL) transport, very light jets (VLJ), unmanned aircraft systems (UAS), superSonic transport (SST), and large civil tiltrotor (LCTR). The NextGen project also emphasizes studies on the integration of these new aircraft concepts into the air transportation system [63].

Following these concepts about the air transportation system, this thesis proposes an integrated air transportation concept that emphasizes the integration of small aircraft into the hub-and-spoke, commercial air transportation system. In other words, it emphasizes the integration of on-demand, door-to-door flight services with traditional scheduled flight services. The integration aims at not only enhancing the profitability of on-demand, door-to-door flight services but also promoting the quality of the entire air transportation services. Delaurentis and Fry [47] pointed out that a good air transportation concept relies on the development of three areas: new aircraft technology, new business model of operating aircraft, and proper policies of managing and operating air resources. Therefore, to support this integration concept, the following sections of this chapter will first present current new aircraft technology and discuss a business model of operating aircraft.

## *3.2 Development of Aircraft*

### 3.2.1 New aircraft concepts

As airframe and propulsion technology is developing, researchers are proposing many new aircraft concepts that aim to reduce aircraft noise and air pollution in community, improve aircraft fuel-efficiency and comfortability, and enable aircraft to access more airports [23].

Because the development of short/vertical take-off and landing (S/VTOL) aircraft can enhance the operations of small aircraft around urban areas and in point-to-point flight service, researchers have proposed many concepts about S/VTOL such as ideal rotor, tilt-nacelle, and PETA V/STOL concepts [69]. The ideal rotor, a low-speed concept developed under a partnership between Georgia Tech Aerospace Design Lab, Tom Hanson, and NSSA Langely, utilizes “a unique, simplified, rigid, auto-trimming rotor hub” [69]. Its unique rotor system is much cheaper than existing systems and requires significantly less maintenance [69]. Furthermore, this system is stable because of its handling qualities [69]. A prototype of the ideal rotor system has been built and tested [69], but the claims of this concept have never been successfully verified [69].

The tilt-nacelle concept, a high-speed concept explored by Mdot Aerospace, Sharpery

Gyronautics, the Georgia Tech Research Institute, and NASA Langley, involves a relatively simple vehicle control in transition and hover [69]. Typically, to avoid crashing in the flight path with single engine failure, V/STOL-capable aircraft have a much stronger propulsion system requirements than conventional aircraft. To deal with this requirement, tilt-nacelle uses a multi-gas generator Fan (MGGF) that loses only 20% thrust in a five gas generator arrangement in case of single gas engine failure. Because of its capability of using a lower pressure ratio and lower peak temperature turbines, Tilt-Nacelle permits less expensive turbo-machinery.

The PETA V/STOL concept, a high-speed concept examined by Boeing and NASA Langley, involves a " true distributed propulsion concept that exhibits extreme redundancy and robustness" ([69],[74]). The PETA concept reduces the engine-out penalty to a negligible level by using many small-pulsed engines. However, one disadvantage of this concept is the incredible level of noise produced by the large number of engines, which is a great disadvantage of this concept because the main goal of S/VTOL concepts is to be able to operate in close proximity to business, homes, and people that have strict noise restrictions ([69],[74]).

In view of the ideal rotor, tilt-nacelle, and PETA V/STOL concepts, S/VTOL-capable aircraft will be more likely utilized as air taxis in the near future. Although it has the potential to achieve a low cost, the ideal concept has not been validated yet. The PETA concept can not pass the regulations for noise restriction because of the excessive noise produced by its pulsed engines. Titl-Nacelle is not likely to reach individual ownership cost goals owing to the requirements of higher power and advanced technology ([69],[74]). In fact, higher power requirements are very common in V/STOL aircraft. Therefore, the vehicle acquisition costs of S/VTOL aircraft will still be high in the short term. Hence, to amortize the high acquisition costs, use of the S/VTOL capable aircraft in air taxi service will be a good choice in the near future ([69],[74]).

In 2008, NASA set goals for aircraft in 2030 and beyond: reducing noise and nitrogen oxide emissions, improving fuel efficiency, and expanding the capacity of the entire air transportation [23]. To achieve these goals, the teams, directed by General Electric (GE), the Massachusetts Institute of Technology (MIT), Northrop Grumman, and the Boeing Company, developed several new aircraft concepts.

In view of flying small aircraft between community airports as a way to mitigate congestion at big commercial airports, the GE Aviation team proposed a 20-passenger aircraft concept [23]. To reduce noise, the new 20-passenger aircraft will employ advanced turboprop engines that support low-noise propellers [23]. Similarly, in viewing of utilization small airports as a way to increase the capacity of the air transportation system, the Northrop Grumman team developed a smaller 120-passenger aircraft concept. The new smaller 120-passenger aircraft, capable of using small airports with short runways, aims to be silent, efficient, low-emission commercial transport [23].

To improve fuel efficiency, the MIT team proposed the 180-passenger aircraft concept, which involves combining two aircraft bodies and mounting three turbofan jet engines on the tail [23]. The new aircraft uses composite materials to lower its weight and turbofan engines with a high bypass ratio to improve its thrust efficiency [23]. These techniques allow the 180-aircraft to reduce fuel-burn by 50 to 70 percent [30]. To reduce air pollution, the Boeing Company developed the Subsonic Ultra Green Aircraft Research Volt concept. Owing to progress in battery technology, they proposed a hybrid propulsion system that could use both an engine by burning fuel and a turbofan by using electricity in cases of power-down of the engine [23].

The concepts proposed by the four teams have ideas in common: flying aircraft at higher altitudes and slower cruising speed to improve fuel efficiency; flying smaller aircraft in more direct routes with shorter length to improve cost-efficiency, using engines that require less power on takeoff to reduce noise, and utilizing shorter runways to increase the capacity of the air transportation system [23].

In addition to the goals set by NASA, the European Commission created a project named "Clean Sky," which aimed at reducing the impact of the air transportation industry on the environment. In fact, the main goals are to reduce fuel consumption, mono-nitrogen oxides emissions, and sensed noise, and to minimize the impact of aircraft on the environment in their entire life cycle [2]. To accomplish these goals, they proposed six concepts: an SMART fixed-wing aircraft concept, a green regional aircraft concept, a green rotorcraft concept, a sustainable and green engine concept, a system for the green operation concept, and an eco-design concept [2].

To improve fuel efficiency, the SMART fixed-wing aircraft concept focuses on both designing a "smart wing" that involves small drag in the cruise and integrating innovative engines that may require substantial modifications in the aircraft architecture [2]; to reduce the pollution and noise of regional aircraft, the green regional aircraft concept focuses on using advanced materials and structures that lower weight and developing configurations that reduce aerodynamic noise [2]; to reduce noise and improve fuel efficiency, the Rotorcraft concept focuses on developing novel rotor blades, integrating diesel engines, minimizing airframe drag, and developing advanced electrical systems [2].

The sustainable and green engine concept concentrates on developing engines with reduction in noise and NOx emissions and real-time diagnosis [2]; the system for green operations concept concentrates on optimizing aircraft energy, green trajectories, and green missions [2]; the eco-design concept concentrates on carrying out a strategy for protecting the environment throughout the entire life cycle of aircraft [2].

### 3.2.2 Small aircraft

#### *3.2.2.1 Very light jet*

Very light jets (VLJs) are small jet aircraft with weights up to 4,540 kg (10,000 lb) and a seat capacity ranging from 4 to 8. VLJs are authorized for single-pilot operation and capable of taking off and landing on runways as short as 914 m (3,000 feet) [3].

Because the costs of VLJs are much lower than those of conventional jets, they are also labeled as entry-level jets [3].

As the technology of VLJs continuously develops, aircraft manufacturers have developed several models of VLJs. Eclipse Aviation Corporation has developed Eclipse 500, a six-seat VLJ, which received the FAA VLJ certification in late 2006 [74]. In the Eclipse 500, the skin and the underlying structures are welded, which reduces the aircraft weight, and techniques from automotive industry for building more robust cabin are applied [74]. Owing to the popularity of the Eclipse 500, the corporation, in 2006, received about 2,800 orders, of which 2000 are firm [74]. Furthermore, the corporation delivered around 260 Eclipse 500's in total from 2006 to 2008 [4]. However, due to lack of funding, the corporation went bankrupt in 2008.

In 2002, Cessna Aircraft Company declared its entry into the market of VLJ with the Cessna Citation Mustang, Model 510 ([74], [88]). In 2006, the FAA granted full-type certification to the Mustang 510 and approved it for flying into "known icing conditions" ([74], [5]). The first delivery of the Citation Mustang 510 was in 2006, and the company had delivered around 300 by the end of 2009 [6].

In 2005, Embreaer, a Brazilian aircraft manufacturer, developed Embraer Phenonm 100, a VLJ with a capacity ranging from 4 to 7 passengers [7]. One security feature of the aircraft is that it can boost engine output to about 1,700 lbs in case of engine failure on take-off. In 2008, the Embreaer received its certification from the FAA and delivered its first aircraft. From 2008 to 2010, the Embreaer delivered a total of 166 Embraer Phenonm 100's [7].

In addition to the VLJs that have been certified by the FAA, several models of VLJs are in development and waiting for certification from the FAA. Diamond Aircraft Industries has developed a five-seat VLJ, the Diamond D-jet, that targets the personal aircraft market [74]. To be safer in case of a failure in pressurization, it restricts its altitude up to 25,000 feet [8]. Currently, the D-jet is still undergoing flight testing certification [8]. Cirrus Aircraft Corporation has developed a seven-seat VLJ, Cirrus Vision SF50, which aims to compete with the Diamond D-jet in the personal aircraft market [9]. The building of a prototype is anticipated by the end of 2010, and its certification is expected in 2013 [9].

Honda Motor Company developed a 6-seat VLJ, the HondaJet, which launched its first flight in 2003 [74]. It applies an unusual engine configuration that enlarges space within the fuselage and reduces drag at higher speeds, and it uses lightweight composites for its fuselage ([74], [10]). With these techniques, the Hondajet is claimed to be 30 to 35% more fuel efficient than similar aircraft ([74],[10]). The FAA is expected to certify it in 2012 [10].

In 2006, Piper Aircraft, Inc announced the PiperJet, a VLJ with up to seven passengers. The PipterJet mounts its engine above the center of gravity, which enables the aircraft to be highly stabilized, and it uses a "straight duct air intake" design for the engine that is mounted in the vertical stabilizer [11]. In 2010, Piper announced the future production of the Piper PiperJet Altaire, a larger re-designed aircraft based on the PA-47 PiperJet prototype.

The following paragraphs in the subsection intend to present a brief overview of current VLJs. Information about the VLJs discussed above is collected in Table 2, which is based on the tables shown in [1]. As shown in the table, the prices of VLJs range from \$1.72 to \$3.6 million; the maximum cruise speeds of VLJs range from 556 km/h to 778 km/h; the seat capacities range from 4 to 8. Given a fuel cost of \$4.50 per gallon, Trani et al. [88] estimated that the minimum operating costs of VLJs should fluctuate between \$1.85 to \$2.25 per seat mile for an airport network structure with 10 to 20% repositioning flights. In fact, according to its specifications, provided online, the operating cost of the Citation Mustang is \$2.31 per mile given a fuel cost of \$4.50 per gallon [12].

According to Table 2, from 2008 to 2010, VLJ industry delivered a total of about

**Table 2:** List of selected VLJs [1]

| Aircraft | Seats | Costs | Delivery | Orders | Maximum Cruise Speed |
|---|---|---|---|---|---|
| Eclipse 500 | 6 | $2.15m | 260 | | 695 km/h |
| Citation Mustang | 6 | $2.65m | 300 | $\geq 500$ | 630 km/h |
| Embraer Phenom 100 | 6–8 | $3.6m | 160 | $\geq 500$ | 704 km/h |
| Cirrus Vision SF50 | 4–7 | $1.72m | 0 | $\geq 500$ | 556 km/h |
| Diamond D-Jet | 5 | $1.89m | 0 | $\approx 300$ | 583 km/h |
| HA-420 HondaJet | 6–8 | $3.65m | 0 | $\geq 130$ | 778 km/h |
| PiperJet | 7 | $2.2m | 0 | $\approx 180$ | 667 km/h |

820 Eclipse 500's, Citation Mustang's, and Embraer Phenom 100's, which indicates an annual production rate of about 270. Before 2008, researchers [88] were optimistic about the VLJ industry and predicted an annual production rate of at least 500 yearly production rate. However, the optimistic predictions of the VLJ industry has been challenged by several factors, including the bankruptcy of Eclipse, the discontinued services of several air taxi operators, and the current economic downturn. Consequently, FAA forecasts an annual rate of increase in VLJs of 270 to 300 and a total of 4875 by 2025 [26].

#### *3.2.2.2 Vertical takeoff and landing aircraft*

Vertical takeoff and landing (VTOL) aircraft are aircraft that can take off and land vertically. With this feature, VTOL aircraft are able to land at small community airports, and in isolated areas, congested areas, and restricted-size areas. Therefore, VTOL aircraft can be used in emergency services and on-demand services, which can promote the service level of air transportation and expand the capacity of the air transportation system. However, VTOL aircraft also have several constraints such as higher fuel consumption and controllability issues. Therefore, VTOL aircraft necessitate further study.

Currently, two main types of civil aircraft in the category of VTOL aircraft are helicopters and tiltrotors. A helicopter is a type of rotorcraft whose rotors enable it to take off and land vertically and fly horizontally [13]. Helicopters are commonly used in emergency medical services, fire fighting, and search and rescue. In addition, they are used in tourism and on-demand services. However, they also have many drawbacks such as low speed, safety and comfort issues, excessive noise, and high operating and maintenance costs ([72], [71]). In fact, Moore [68] pointed out that a four-seat light helicopter costs \$4.25 per mile and a two-seat light helicopter costs \$2.7 per mile. To overcome these shortcomings, researchers are studying safe and comfortable helicopter concepts [71]. In addition, researchers are studying smart helicopter ([29], [72]) and electric-powered helicopter concepts [55].

As a big helicopter producer, the United States have many helicopter manufacturers. According to statistics, until 2001, there were around 26,000 helicopters in the world, and around 40 percent of them were in North America [14]. Bell Helicopter Company, an American rotorcraft manufacturer, has introduced a series of Bell helicopters. Notably, Bell 47, introduced in 1946, was the first helicopter that received certification for civilian use [15]. In 1967, the company introduced a five-seat helicopter with a maximum speed of 224 km/h, the Bell 206, which is one of the most popular helicopters [16]. In fact, 7,300 Bell 206's, each costing \$700,000 to \$1.2 million dollars, have been built so far [16]. In 1995, the company introduced Bell 407, a seven-seat helicopter with a maximum speed of 260 km/h. By 2010, about 1,000 Bell 407's, each costing 2.54 million dollars, have been built. In 2007, the company introduced the Bell 429 with a cruise speed of 273 km/h, which was certified by the FAA in 2009. The company has received around 300 orders for the Bell 429.

A tiltrotor is an aircraft that combines both the good hover performance of helicopters and the high-speed capability of fixed-wing aircraft [50]. Its rotors provides the initial lift, and then as the aircraft gains speed, the wing takes over, providing the lift, and the rotors provide thrust instead [17]. Because of the greater efficiency of the wings, tiltrotor is more higher fuel-efficient and faster than helicopters [17].

Although tiltrotor aircraft are the most effective VTOL aircraft up to now, the design of tiltrotor aircraft is very difficult [33]. Until now, only a few models of tiltrotor aircraft have been developed. The Bell XV-15, an experimental VTOL aircraft designed by Bell Helicopter Textron, was the first to demonstrate high-speed performance relative to conventional helicopters [18]. In 2007, the V-22 Osprey, a $67 million tiltrotor, was introduced by Bell Helicopter and Boeing Rotorcraft Systems. It has primarily been used by the United States Marine Corps and the United States Air Force [19]. The Agusta BA 609, a VTOL with a seat capacity from 6 to 9, developed by Bell/Agusta Aerospace Company, launched its first flight in 2003 [20]. Each Agusta BA 609 costs at least $10 million.

The tiltrotor concept is still under development. Currently, tiltrotor aircraft are expensive and most of them are used in the military. However, researchers are envisioning next generation civil tiltrotor aircraft. For example, Bell Helicopter Textron has been studying civil titlrotor aircraft with seat capacities of 10, 30, 90, and 120 [96]. Young et al. [96] pointed out that these tiltrotor aircraft could be used differently: small ones in air-taxi type services, mid-size ones in regional airports or suburban vertiports, and large ones at congested airports. In addition, Price [72] proposed the quad tiltrotor and advanced tiltrotor concepts.

## *3.3 Business Model of Operating Small Aircraft*

Throughout the history of the United States, the development of transportation technology has led to the evolution of the transportation system, which promotes the continuous development of the economy. In the 17th century, as ships sailed on the seas, the economy around seaports boomed; in the 18th century, as steamboats shuttled between the canals and rivers, the economy around canal and river ports boomed; in the 19th century, as trains criss-crossed across the land, the economy around train stops boomed; and in the 20th century, as cars moved along the interstate highways, the economy at on/off-ramps boomed ([77], [58]).

In the 1950s, jet aircraft were introduced into commercial use [39], and in the 1970s, wide-body aircraft entered into service. Particularly, since airline deregulation in 1970, most airlines have adapted their flight network to hub-and-spoke networks, and the economy around hubs and spokes has thrived. By consolidating passenger demand of different itineraries, the hub-and-spoke network can increase the utilization of aircraft capacity and reduce flight costs. Furthermore, through consolidation, it can provide more flight itineraries at less costs, and therefore it enhances the service level of the air transportation. However, after 40 years of development, as passengers value time and convenience more and more, defects in the air transportation system focusing on hub-and-spoke networks have also been exacerbated.

One disadvantage of the hub-and-spoke network is its time-efficiency with respect to door-to-door transit time. In order to consolidate passengers with different itineraries, the itineraries in the hub-and-spoke system are routed through a hub, which are usually much longer than the non-stop flights. Furthermore, commercial airports used in hub-and-spoke networks are within a half-hour distance to only 40% of passengers [58]. In fact, Moore [69] pointed out one study that showed that gate-to-gate flight time occupies less than 30% of the total door-to-door transit time for trips under 500 miles. In addition to its time inefficiency, commercial airports used in the hub-and-spoke network do not serve rural and regional areas well.

Recently, the development of small aircraft technology has been accompanied by the emergence of all types of small aircraft. These small aircraft are light-weight with high operating capabilities that enable them to access community airports with short runways and even helipads under nearly any weather condition [58]. Furthermore, they can fly in underutilized altitude strata [57], which promotes the utilization of airspace resources. In addition, these small aircraft cost much less than airplanes used in commercial airlines, which enables their usage for on-demand flight services such as air taxi services by amortizing their acquisition costs.

Promoting the utilization of small aircraft and small community airports will be one way to remedy the disadvantages of a transportation system relying on a hub-and-spoke network. In particular, these small aircraft could be used in air-taxi service. First, this approach will improve the time efficiency of the air transportation system. According to their collected travel data, Holmes and Durham [58] pointed out that about half of travel time might be saved if the air-taxi mode is used instead. Furthermore, it enhances the air transportation service in rural and regional areas. Small aircraft could land on and take off from community airports, which are widespread spread throughout rural and regional areas in the United States. In addition, it provides on-demand service, which does not incur any schedule displacement. Therefore, the development of small aircraft technology and the utilization of small aircraft will be critical to the air transportation system in the 21th century and beyond.

In addition to its importance, promoting the utilization of small aircraft and small community airports in on-demand services will likely be profitable. First, due to the high door-to-door speed of on-demand flight services, the demand for such services will increase. Studies showed that the average time that each person spent on travel each day stayed at about 1.2 hours during the last 100 years although travel speeds kept increasing ([58], [69]). Based on studies on travel time, the increasing value of time will drive passengers to choose faster transportation. Analysis showed that by about 2020, the demand for higher-speed travel modes would increase to twice of the amount in 1990 ([82] [58]), and it would be larger than the total demand for automobiles in 1990 [82]. On the other hand, studies showed that the door-to-door speed of the hub-and-spoke network is unsatisfactory. Data showed that for less than 600-mile trips, the average doorstep-to-destination speed of of a hub-and-spoke trip was about 80 miles; for trips less than 1,100 miles, the fastest average speed could be only 160 miles while the speed of a trip using air-taxi could be more than 320 miles [58]. Therefore, owing to their high door-to-door speed, on-demand flight services will attract more and more passengers. Furthermore, due to their services coverage in rural and regional areas and their short-haul travel services, demand for such services will also increase. Data show that air transportation neither occupies a big portion of the total travel market [69] nor has share of the travel market with trip lengths from 25 to 100 miles [69]. Promoting on-demand flight service will reinforce the competition between air and automobile transportation and therefore increases the share of the air travel market.

Currently, in the United States, while commercial airlines mainly provide scheduled flight services, fractional airlines, charter airlines, and air taxi companies provide on-demand fight services. The current vision of air transportation services is that air transportation suppliers provide itineraries either consisting of purely scheduled flights or purely on-demand flights, which is illustrated in Figure 1 by Lewe et al. [64]. However, according to some analysts in the aviation business, because air taxi can not capture a sufficient share of the market, they can not be profitable [67]. Mane and Crossley [67] pointed out that the load factor is very critical to the its profitability. Furthermore, their studies showed that the yearly utilization of aircraft is also very important to the profitability of air taxi providers [67]. Related to these concerns about profitability, some air taxi companies are also facing hard times in the current economic down term. For example, Dayjet, an air taxi company, started flying passengers in 2007, but discontinued operations in 2008.

To increase the load factor and aircraft utilization of air taxi services, this thesis proposes creating itineraries that integrate both the scheduled and on-demand services. In particular, on-demand flights serve as add-on services to scheduled flights in these itineraries. This new type of itinerary is illustrated in Figure 6, which is based on an illustration in Lewe et al. [64]. With this new type of itinerary, passengers can fly from community airports near their origins to commercial airports, then take scheduled flights, and finally fly to community airports close to their destinations.

These new itineraries have the advantages of both scheduled flight services and on-demand services. On the one hand, they can serve passengers in regional and rural areas well; on the other hand, they can offer lower prices than purely on-demand services on an entire trip. In the current economy down term, including these itineraries would stimulate demand for air transportation because it would provide passengers with more flight choices and enhance the service level of air transportation. Currently, some airlines are also starting such services. For example, Delta has fleets ranging from big commercial airplanes to small jets, which allow Delta to operate both scheduled and on-demand services. In addition, CitationAir cooperates with commercial airlines in Europe: the commercial airlines bring passengers from Europe to the United States on scheduled flights, and CitationAir serves these passengers in the United States with on-demand services.

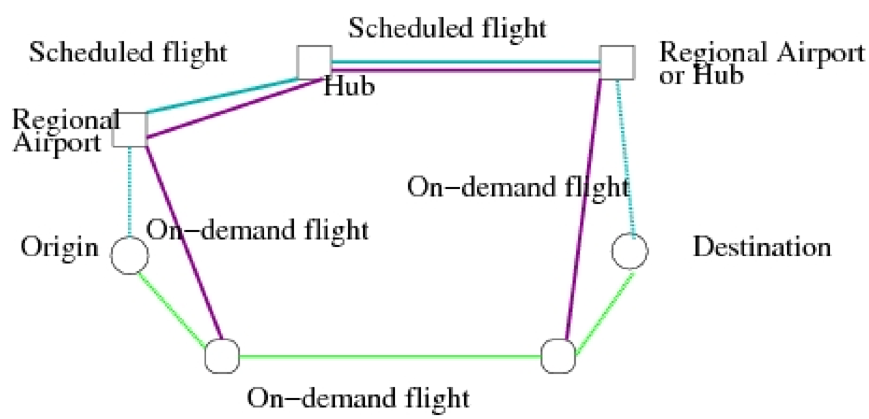



**Figure 6:** Illustration of a new type of flight itineraries (based on a graph in Lewe et al. [64])

# CHAPTER IV

# MODELS IN FUTURE AIRCRAFT NETWORK AND SCHEDULE DESIGN

Overall, this thesis addresses an aircraft network and schedule design problem that is closely related to the schedule design and the fleet assignment problem. However, the schedule design problem itself is a very complex problem in schedule planning. In addition, this thesis proposes to build an aircraft network and flight schedule from scratch, which exacerbates the solving of the problem. Tackling these difficulties necessitates the decomposition of the entire problem into some small problems and the sequential solution of these smaller problems. Therefore, this thesis decomposes the entire network design problem into three subproblems: the frequency assignment problem, the rough fleet assignment problem, and the timetable problem. Based on this decomposition, this thesis proposes a three-step approach for building an aircraft network and schedule from scratch.

This chapter is organized as follows. Section 4.1 introduces the decomposition scheme of the entire problem. Section 4.2 and Section 4.3 present a frequency assignment model for scheduled service and on-demand service, respectively. Section 4.4 describes a rough fleet assignment model. Section 4.5 gives a formulation of a timetable model.

## 4.1 Decomposition Scheme

Building an aircraft network and schedule from scratch implies that the schedule can not be built simply by making minor changes in any existing schedule. Thus, formulating the problem with just one model will involve too many decision variables that indicate when and where to put the flight legs. Furthermore, the model will be very difficult to solve. Therefore, building an aircraft network and schedule from scratch requires a decomposition scheme. The following paragraphs will explain the decomposition scheme in detail.

The first level of the decomposition is developed for integrating scheduled service and on-demand service in a network. Because the scheduled flight service has advantages over other transportation modes in long-haul travel, and in the model, on-demand service is used as an add-on service to the scheduled service, taking passengers from airports to community airports close to their final destination, in the model, this thesis determines the overall structure of the flight network by analyzing the itinerary choices of passengers over itineraries consisting of purely scheduled flights. Furthermore, because several transportation modes can take passengers from airports to places close to their final destination, this thesis uses a mode choice model to determine the frequency of on-demand service.

The second level of the decomposition scheme is developed for building schedule for scheduled service. For scheduled service, each flight requires four pieces of information: an origin station, a destination station, departure time, and a fleet. The determination of the departure time of flights makes the schedule design problem intractable. To deal with the complexity of time determination, the solution to the problem is divided into three steps. The first step determines a sequence of flights without a specified departure time. In other words, it determines the frequency of flights between each station pair. Thus, this step is referred to as the frequency assignment step. The second step generates a sequence of flights, each with a rough departure time, to meet the requirements of the flight frequencies determined in the previous step. To derive a rough departure time, the timeline in each station is divided into blocks, each of which represents one hour. A flight departure time is also required to be at the ends of the time blocks in this step, referred to as the rough fleet assignment step. The third step, which is referred to as the timetable model step, generates more detailed departure time for each flight.

This paragraph explains the treatment of passenger demand in the decomposition. As the volume of passenger demand for an itinerary in each market is strongly related to the itinerary frequencies in that market, the frequency assignment model incorporates an itinerary choice model that models how passenger demand distributes among different itineraries. In addition, the rough fleet assignment model addresses the effect of time on the demand because the itinerary choice of a passenger is also influenced by the departure time of different itineraries.

This paragraph explains the treatment of multiple fleets in the decomposition. The frequency assignment model is a complicated nonlinear optimization model. Furthermore, the model becomes more complicated when multiple fleets are included. In addition, this model needs an estimation on the fleet operating cost so that the flight frequencies are not much greater than the minimum flight frequencies that meet passenger demand. Therefore, the frequency assignment model uses a representative fleet and determines the flight frequencies with respect to this fleet. On the other hand, the rough fleet assignment model includes multiple fleets and determines a proper fleet for each selected flight. In this step, the seat capacity and fuel consumption rate of each fleet will influence its usage on each flight leg.

The decomposition scheme is illustrated in the flowchart in Figure 7. The following sections will present the models in each step.

## *4.2 Frequency Assignment Model for Scheduled Flights*

This section first provides a background of the frequency assignment model for scheduled flights, which incorporates a passenger path choice model. Then, it introduces notations that will be used in the passenger path choice model and the frequency assignment model. After that, it presents a path choice model with price and time as

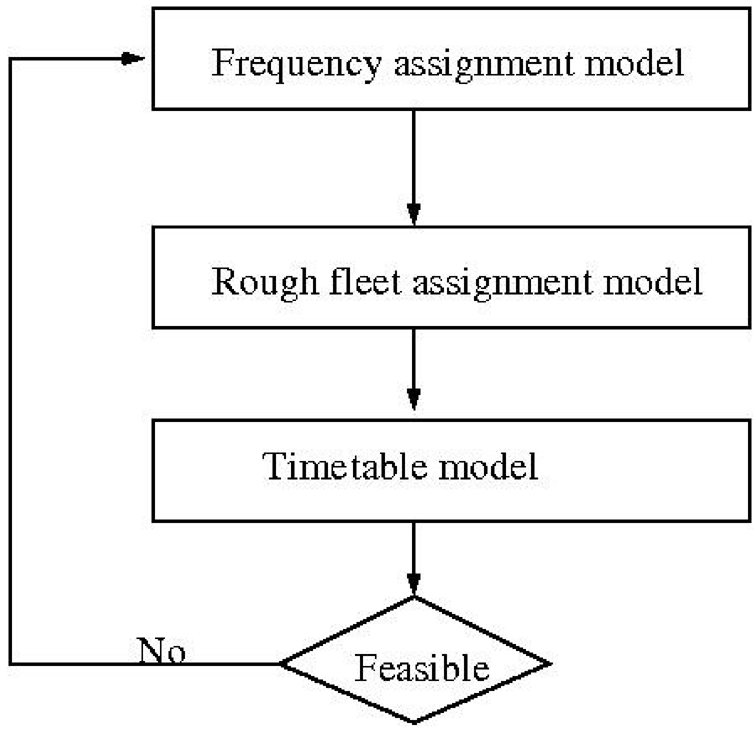



**Figure 7:** Flow chart

the main explanatory variables. Finally, it presents the frequency assignment model with constraints such as constraints of station capacity, flow balance, fleet capacity, and so on.

### 4.2.1 Introduction

The frequency assignment model is formulated according to the demand-supply interaction. With regard to demand, passengers' choices of itineraries depend on the flight schedule. For example, if an itinerary in a market has a much higher frequency than other itineraries in this market, then more passengers will probably choose this itinerary because some departure times of this itinerary is more likely to be close to passengers' desired departure times. To model passengers' itinerary choice behaviors, a discrete choice model has been created and incorporated into the frequency assignment model. With respect to transportation supply, operators build their flight network based on the analysis of passenger demand in different markets and passengers' itinerary choice behaviors. Their decisions about the frequencies of flight legs and flight itineraries follow a profit maximization principal. Thus, the objective of the frequency assignment model is to maximize passenger revenue minus frequency assignment costs.

The frequency assignment model uses the notion of a representative fleet, and it determines flight frequencies with respect to the representative fleet. One reason for creating a representative fleet is that the main goal of the frequency assignment model is the determination of the overall network structure, that is, where flight legs should be placed and with what frequency. Another reason is that multiple fleets will be addressed in the rough fleet assignment model. In addition, the frequency assignment model is a large-scale nonlinear programming problem that is very difficult to solve. Using a representative fleet can simplify the solution of the frequency assignment model. The characteristic of the representative fleet, which is related to fleets used in practice, can be influenced by the overall structure that a planner wants to enforce in the network. For example, if planners want to increase the number of small aircraft in the network, they could reduce the seat capacity of the representative fleet.

### 4.2.2 Notations

Overall, the network of the frequency assignment model is a directed graph. Nodes in the network represent stations. The directed arc from a node $A$ to a node $B$ represents a flight from a station $A$ to a station $B$. A path is a sequence of flight arcs. The frequency of each flight arc is determined by the frequency model, which does not determine departure times for each flight arc. Therefore, each path corresponds to several potential itineraries that will be determined in the rough fleet assignment model step.

#### *4.2.2.1 Sets*

$S$ : Set of airports indexed by $s$.

$A$ : Set of directed arcs indexed by $a$.

$L_{s-}$ : Set of outgoing arcs of airport $s$.

$L_{s+}$ : Set of incoming arcs of airport $s$.

$M$ : Set of markets indexed by $(o, d)$.

$D^b_{o,d}$ : Demand of business passengers in market $(o, d)$.

$D^l_{o,d}$ : Demand of leisure passengers in market $(o, d)$.

$P$ : Set of paths indexed by $p$.

$P_{o,d}$ : Set of paths $p \in P$ in market $(o, d)$.

$P_a$ : Set of paths $p \in P$ containing arc $a$.

*4.2.2.2 Parameters*

$Cap_s$ : Capacity of airport $s$.

$Cap$ : Quantity of airport capacity occupied by the representative fleet per equipment.

$Seat$ : Average seat capacity of the representative fleet.

$C$ : Cost of the representative fleet per hour.

$U_p^b$ : Utility of path $p$ to a business passenger.

$U_p^l$ : Utility of path $p$ to a leisure passenger.

$R_p^b$ : Revenue of path $p$ per business passenger.

$R_p^l$ : Revenue of path $p$ per leisure passenger.

$FT_p$ : Flight time of using the representative fleet on path $p$.

$FT_a$ : Flight time of using the representative fleet on arc $a$.

$n_p$ : Number of stops of a path $p$.

$MaxHour$ : Maximum block hours of using the representative fleet per day.

$\gamma$ : Parameter that links the frequency of a path and those of the arcs on the path .

*4.2.2.3 Decision variables*

$y_a$ : Frequency of arc $a$ of using the representative fleet

$y_p$ : Frequency of path $p$ of using the representative fleet

$x_p^l$ : Number of leisure passengers on path $p$

$x_p^b$ : Number of business passengers on path $p$

### 4.2.3 Passenger path choice model

This subsection presents a passenger path choice model. In the choice model, decision makers are the passengers in each market, and the paths in each market form the choice set for a passenger in that market. In addition, according to passengers' trip purposes, they are divided into two groups: leisure passengers and business passengers. Furthermore, passengers in each group are assumed to be homogeneous and have the same choice behavior.

Passengers' path choices are influenced by many factors, the most important ones being the price, the flight time, the number of stops, and the schedule displacement of a path. A thorough understanding of the tradeoff between these factors helps schedule planners to build efficient schedules. For example, if most passengers prefer to take nonstop flights rather than pay less but take connection flights, then a schedule planner should consider building a flight schedule network with more non-stop flights. Therefore, a good passenger path choice model should include these factors as explanatory variables. To capture these explanatory variables, the choice model in this subsection relates each path $p$ to a four-dimensional vector $(C_p, T_p, n_p, s_p)$ that measures the price, the flight time, the number of stops, and the schedule displacement of path $p$.

Numerically, each leisure passenger is assumed to value these four dimensions by $(a_1^l, a_2^l, a_3^l, a_4^l)$ and with an observation random error $\epsilon_p^l$, where $\epsilon_p^l$ follows the Gumbell distribution $G(1,1)$. Similarly, each business passenger is assumed to value these four dimensions by $(a_1^b, a_2^b, a_3^b, a_4^b)$ and with an observation error $\epsilon_p^b$, where $\epsilon_p^b$ follows the Gumbell distribution $G(1,1)$.

For simplicity, the following paragraphs illustrate formulas for only a leisure passenger since the formulas for a business passenger are similar to those of a leisure passenger. For any path $p$, the utility of this path to a leisure passenger is represented as

$$U_p^l = V_p^l + \epsilon_p^l,$$

where $V_p^l = a_1^l C_p + a_2^l FT_p + a_3^l n_p + a_4^l s_p$, which is often referred to as an observational utility.

In the schedule design, a planner often decides the frequency of each path but not the schedule displacement of each path. Therefore, the choice model above should relate the frequency of a path frequency to the schedule displacement of the path. Obviously, when the frequency of a path increases, the expectation of the schedule displacement of that path decreases. In other words, these two are negatively related. Teodorović [85] proposes a method of relating the frequency of an itinerary to the displacement of the itinerary. The author assumes that the departure time of each itinerary is uniformly distributed from 00:00 am to 24:00 pm. Therefore, the schedule displacement is equal to the total time span $24 \cdot 60$ minutes divided by four times the itinerary frequency. In the frequency assignment model in this section, the departure times of the flights are assumed to range from 6:00 am to 10:00 pm. Therefore, the schedule displacement is represented as follows:

$$s_p = \frac{16 \cdot 60}{4 \cdot f_p} = \frac{240}{f_p},$$

where the unit is a minute. Thus, the observational utility of path $p$ to a leisure passenger can also be represented as

$$V_p^l = a_1^l C_p + a_2^l FT_p + a_3^l n_p + a_4^l \frac{240}{f_p}.$$

Under the utility maximization assumption, each leisure passenger compares the utility of the paths in his or her choice set and chooses the one with maximum utility to him or her. According to the logit model, the probability that a leisure passenger will choose a path $p$ is

$$Prob(p) = \frac{e^{V_p^l}}{\sum_{q \in P_{o,d}} e^{V_q^l}}.$$

Therefore, the expectation of the number of leisure passengers that will choose a path $p$ is

$$Exp(p) = \frac{e^{V_p^l}}{\sum_{q \in P_{o,d}} e^{V_q^l}} D_{o,d}^l.$$

Incorporating this formula into the frequency assignment model for scheduled flights derives the following constraint:

$$x_p^l \leq \frac{e^{V_p^l}}{\sum_{q \in P_{o,d}} e^{V_q^l}} D_{o,d}^l,$$

which means that the number of leisure passengers flying on path $p$ should be less than the number of leisure passengers who regard path $p$ as their most desirable path.

### 4.2.4 A formulation of the frequency assignment model

The following is a formulation of the frequency assignment model.

$$\max \sum_{(o,d)\in M} \Big( \sum_{p\in P_{o,d}} R_p^l \cdot x_p^l + \sum_{p\in P_{o,d}} R_p^b \cdot x_p^b \Big) - C \cdot \sum_{a\in A} FT_a \cdot y_a$$

$$s.t. \sum_{a\in L_{s-}} y_a = \sum_{a\in L_{s+}} y_a, \forall s \in S, \tag{5}$$

$$2 \sum_{a\in L_{s-}} Cap \cdot y_a \leq Cap_s, \forall s \in S, \tag{6}$$

$$\sum_{a\in A} FT_a \cdot y_a \leq MaxHour, \tag{7}$$

$$y_p \leq \gamma y_a, \forall a \in A, \forall p \in P_a, \tag{8}$$

$$\sum_{p\in P_a} (x_p^l + x_p^b) \leq Seat \cdot y_a, \forall a \in A, \tag{9}$$

$$x_p^l + x_p^b \leq Seat \cdot y_p, \forall p \in P, \tag{10}$$

$$x_p^l \leq \frac{e^{V_p^l}}{\sum_{q\in P_{o,d}} e^{V_q^l}} \cdot D_{o,d}^l, \forall (o,d) \in M, \forall p \in P_{o,d}, \tag{11}$$

$$x_p^b \leq \frac{e^{V_p^b}}{\sum_{q\in P_{o,d}} e^{V_q^b}} \cdot D_{o,d}^b, \forall (o,d) \in M, \forall p \in P_{o,d}, \tag{12}$$

$$V_p^l = (a_1^l \cdot C_p + a_2^l \cdot FT_p + a_3^l \cdot n_p) + a_4^l \cdot \frac{240}{y_p}, \forall p \in P, \tag{13}$$

$$V_p^b = (a_1^b \cdot C_p + a_2^b \cdot FT_p + a_3^b \cdot n_p) + a_4^b \cdot \frac{240}{y_p}, \forall p \in P, \tag{14}$$

$$x_p^l \geq 0, x_p^b \geq 0, y_a \geq 0, y_p \geq 0, x_p^l, x_p^b, y_a, y_p \text{ integer}. \tag{15}$$

The objective of the frequency model is to maximize the total revenue of both leisure and business passengers minus operating costs. Constraint (5) guarantees that the number of incoming fights to a station is equal to that of the outgoing flights. Constraint (6) ensures that the number of flights scheduled to a station does not exceed the capacity of that station. Constraint (7) imposes the upper bound of the number of operating hours of the representative fleet. In the frequency assignment step, the number of operating hours is used to estimate the number of aircraft needed in a schedule. Therefore, constraint (7) ensures that the number of aircraft needed in a schedule does not exceed an upper bound. Constraint (8) guarantees that the frequency of a path is less than the frequency of any flight arc on that path multiplied by $\gamma$. Constraint (9) guarantees that the total number of leisure and business passengers on an arc is less than the capacity of that arc while Constraint (10) guarantees that the total number of leisure and business passengers on a path is less than the capacity of that path. Constraint (11) ensures that the number of leisure passengers assigned to a flight arc is less than the demand of leisure passengers on that flight arc while constraint (12) ensures that the number of business passengers assigned on a flight arc is less than the demand of business passengers on that flight arc. Equalities (13) and (14) represent the utility of a path to a leisure passenger and a business passenger, respectively.

## 4.3 Frequency Assignment Model for On-demand Flights

### 4.3.1 Introduction

The flight network uses small aircraft such as very light jets and short takeoff and landing aircraft in on-demand flights. Furthermore, an on-demand service is used as an add-on service to a traditional scheduled service. With the new property, the flight network can provide two types of itineraries to passengers: one is the traditional itinerary, which consists of purely scheduled flights, and one is a new type of itinerary, which uses on-demand flights as an add-on service. In fact, traditional itineraries can also be viewed as a combination of flight services plus ground transportation services. In other words, in the traditional itineraries, the ground transportation services can be viewed as add-on services to the scheduled flights.

The new type of itineraries enlarges the choices of a passenger, which enhances the service level of the entire flight network. For example, if a passenger wants to fly from Atlanta, Georgia, to Denver, Colorado, for skiing, a new itinerary might consist of a flight from Hartsfield-Jackson Atlanta International Airport to Denver International Airport and an air taxi from Denver International Airport to a ski resort.

### 4.3.2 Notations

To simplify the notations for transportation modes, let mode 0 denote ground transportation and mode $k$ the on-demand service with fleet $k$, for $k \geq 1$.

#### *4.3.2.1 Sets*

$S$ : Set of airports indexed by $s$

$KO$ : Set of fleets used in on-demand flights indexed by $k$, $k \geq 1$

$M$ : Set of markets indexed by $(o, d)$

$P$ : Set of paths indexed by $p$

$D_p^b$ : Number of business passengers on path $p$

$D_p^l$ : Number of leisure passengers on path $p$

$P_{o,d}$ : Set of paths $p \in P$ in market $(o, d)$

$\bar{P}^k$ : Set of paths, indexed by $\bar{p}^k$, that use transportation mode $k$ in the add-on service

#### *4.3.2.2 Parameters*

$Cap_d$ : Capacity of airport $d$

$Cap_k$ : Capacity occupied by fleet $k$ per piece of equipment

$Seat_k$ : Average seat capacity of fleet $k$

$C_k$ : Cost of fleet $k$ per seat per hour

$U^b_{\bar{p}^k}$ : Utility of the add-on service of path $\bar{p}^k$ to a business passenger

$U^l_{\bar{p}^k}$ : Utility of the add-on service of path $\bar{p}^k$ to a leisure passenger

$R^b_{\bar{p}^k}$ : Revenue of the add-on service of path $\bar{p}^k$ per business passenger

$R^l_{\bar{p}^k}$ : Revenue of the add-on service of path $\bar{p}^k$ per leisure passenger

$FT_{\bar{p}^k}$ : Travel time of the add-on service of path $\bar{p}^k$

$MaxHour^k$ : Maximum blocked hours of fleet $k$ per day

#### *4.3.2.3 Decision variables*

$x^l_{\bar{p}^k}$ : Number of leisure passengers on path $\bar{p}^k$

$x^b_{\bar{p}^k}$ : Number of business passengers on path $\bar{p}^k$

$y^k_d$ : Frequency of using fleet $k$ at airport $d$

### 4.3.3 Passenger mode choice model

This subsection presents a passenger mode choice model. After passengers arrive at their destination airports, they choose to either take ground transportation or use on-demand flights for a short-haul travel to their final destinations. Therefore, in the mode choice model, the decision makers are the passengers at their destination airports. Furthermore, in the model, passengers are also divided into two groups: leisure passengers and business passengers. In addition, passengers in each group are assumed to be homogeneous and to have the same choice behavior.

So far, no data about air passengers' final destinations are available. In fact, even if data are available, the set of the true destinations would be huge. Therefore, it is necessary to make some assumptions about trips from passengers' destination commericial airports to their final destinations. For each airport $s$, let $\ell_s$ denote the average length of the trips from passengers' destination commericial airport $s$ to their final destination community airport. Furthermore, for such airport $s$, the mode choice model assumes that passengers whose destination airport is $s$ would take trips of length $\ell_s$ to their final destination community airports, and the transportation modes that are available in these trips form passengers' choice sets.

Passengers' mode choices are influenced by many factors, the most important ones being the price and the travel time of each mode in a trip. A thorough understanding of the tradeoff between these factors helps transportation planners to effectively distribute transportation resources. To capture these factors, the mode choice model in this subsection relates the add-on service in path $\bar{p}^k$ to a two-dimensional vector $(C_{\bar{p}^k}, T_{\bar{p}^k})$, which measures the price and time of the add-on service in path $\bar{p}^k$.

Numerically, each leisure passenger is assumed to value these two dimensions by $(b_1^l, b_2^l)$ and with an observational random error $\eta_{\bar{p}^k}^l$, where $\eta_{\bar{p}^k}^l$ follows Gumbell distribution $G(1,1)$. Similarly, each business passenger values these two dimensions by $(b_1^b, b_2^b)$ and with an observational random error $\eta_{\bar{p}^k}^b$, where $\eta_{\bar{p}^k}^b$ follows Gumbell distribution $G(1,1)$.

For simplicity, the following paragraphs illustrate formulas for only a leisure passenger since the formulas for a business passenger are similar to those of a leisure passenger. For path $\bar{p}^k$, the utility of the add-on service in this path to a leisure passenger is represented as

$$U_{\bar{p}^k}^l = V_{\bar{p}^k}^l + \eta_{\bar{p}^k}^l,$$

where $V_{\bar{p}^k}^l = b_1^l C_{\bar{p}^k} + b_2^l F T_{\bar{p}^k}$, which is often referred to as an observational utility.

Under the utility maximization assumption, each leisure passenger compares the utility of using different modes in the add-on service in his or her choice set and chooses the one with maximum utility. According to the logit model, the probability that a leisure passenger will choose fleet $k$ in the add-on service following path $p$ is

$$Prob(\bar{p}^k) = \frac{e^{V^l_{\bar{p}^k}}}{e^{V^l_{\bar{p}^0}} + \sum_{k \in KO} e^{V^l_{\bar{p}^k}}}.$$

Therefore, the expectation of the number of leisure passengers that will choose fleet $k$ in the add-on service following path $p$ is

$$Exp(\bar{p}^k) = \frac{e^{V^l_{\bar{p}^k}}}{e^{V^l_{\bar{p}^0}} + \sum_{k \in KO} e^{V^l_{\bar{p}^k}}} D^l_p.$$

Incorporating this formula into the frequency assignment model for on-demand flights derives the following constraint:

$$x^l_{\bar{p}^k} \leq \frac{e^{V^l_{\bar{p}^k}}}{e^{V^l_{\bar{p}^0}} + \sum_{k \in KO} e^{V^l_{\bar{p}^k}}} D^l_p,$$

which indicates that the number of leisure passengers choosing fleet $k$ in the add-on service following path $p$ should be lower than the number of leisure passengers who regard mode $k$ as their most desirable transportation mode in the add-on service.

### 4.3.4 Frequency assignment model

With all the previous preparations, the formulation of the frequency assignment model for on-demand services is given as follows.

$$\max \sum_{(o,d)\in M} \sum_{k\in KO} (x^l_{\bar{p}^k} R^l_{\bar{p}^k} + x^b_{\bar{p}^k} R^l_{\bar{p}^k}) - \sum_{d\in S} \sum_{k\in KO} C^k \cdot T^k_d \cdot y^k_d$$

$$s.t. \sum_{(o,d)\in M} \sum_{p\in P(o,d)} \sum_{k\in KO} Cap^k \cdot y_{\bar{p}^k} \leq Cap_d, \forall d \in S, \tag{16}$$

$$\sum_{d\in S} \sum_{k\in KO} T^k_d \cdot y^k_d \leq MaxHour^k, , \forall k \in KO, \tag{17}$$

$$x^l_{\bar{p}^k} \leq \frac{e^{V^l_{\bar{p}^k}}}{e^{V^l_{\bar{p}^0}} + \sum_{k\in KO} e^{V^l_{\bar{p}^k}}} D^l_p, \forall k \in KO, \forall p \in P, \tag{18}$$

$$x^b_{\bar{p}^k} \leq \frac{e^{V^b_{\bar{p}^k}}}{e^{V^b_{\bar{p}^0}} + \sum_{k\in KO} e^{V^b_{\bar{p}^k}}} D^b_p, \forall k \in KO, \forall p \in P, \tag{19}$$

$$\sum_{o\in S} \sum_{p\in P_{o,d}} (x^l_{\bar{p}^k} + x^b_{\bar{p}^k}) \leq Seat_k \cdot y^k_d, \forall k \in KO, \forall d \in S, \tag{20}$$

$$x^l_p \geq 0, x^b_p \geq 0, y_a \geq 0, y_p \geq 0, y_a, y_p \text{ integer}. \tag{21}$$

The main idea of this model is to determine an optimal way of distributing small aircraft. The objective of the model is to maximize passenger revenue minus the operating cost of on-demand flight services. Constraint (16) ensures that the capacity of each airport would not be exceeded. Constraint (17) imposes the upper bound of the number of operating hours of each fleet. Constraint (18) guarantees that the number of leisure passengers assigned to each path is less than the demand of leisure passengers on that path while constraint (19) guarantees that the number of business passengers assigned on each path is less than the demand of business passengers on that path. For each fleet $k$ and airport $d$, constraint (20) ensures that the total number of passengers that take on-demand flights using fleet $k$ at airport $d$ does not exceed the total capacity that could be provided by the on-demand flights using fleet$k$ at airport $d$.

## *4.4 Rough Fleet Assignment Model*

After the flight frequencies are determined in the frequency assignment model, a set of flight legs need to be generated to match the frequencies. However, the search space of the candidate flight legs is huge, which necessitates a certain degree of simplification. To include as many candidate flights as possible but ensure as small a search space as possible, the rough fleet assignment model adopts a rough timeline network, which is also discussed by Vaze and Barnhart [89].

### 4.4.1 Rough timeline network

In the fleet assignment model, each station has a timeline that describes its arrival and departure events. The basic idea of a rough timeline network is to aggregate events with close departure and arrival times so that the total number of events at each station is small. To clarify this idea, as most flights start from any time between 06:00 am and 10:00 pm, the timeline from 06:00 am to 10:00 pm at each station is divided into blocks, each of which is exactly one hour. Furthermore, each flight leg in the rough fleet assignment model is enforced to departs exactly at the end of a block on time between 06:00 am and 10:00 pm. In addition, the arrival time of each flight leg in the rough fleet assignment model is moved to the end of a block that is closest to and later than the true arrival time of that leg. Furthermore, the arc corresponding to the time span from 22:00 pm to 06:00 am on the timeline is a wrap around arc. For example, if a flight arrives at 5:00 am, the arrival node will be the node that represents 06:00 am for the purpose of aggregation. In this way, the size of the problem is reduced, but the main feature of the problem is captured.

A flight arc in the frequency assignment model corresponds to several flight legs with rough departure and arrival times in the rough timeline network. Furthermore, for each flight arc, the goal of the fleet assignment is to select a set of flight legs that matches the frequency of that flight arc. Furthermore, each path in the frequency model corresponds to several itineraries in the timeline network, and passenger demand on the path is distributed over these corresponding itineraries. If the capacity of these itineraries is insufficient, then some passengers would be spilled.

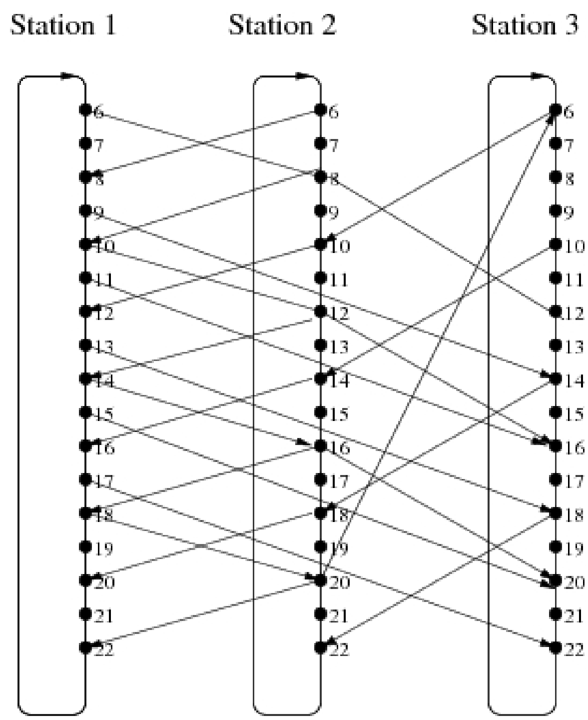



**Figure 8:** Rough timeline network

### 4.4.2 Notations

#### *4.4.2.1 Sets*

$T$ : Set of time nodes indexed by $t$

$K$ : Set of fleet indexed by $k$

$A$ : Set of flight arcs indexed by $a$

$L$ : Set of flight legs indexed by $a_t$

$a_t^k$ : Copy of flight arc $a$ departure at time $t$ with fleet $k$

$S$ : Set of stations indexed by $s$

$P$ : Set of paths indexed by $p$

$P_{a^k}$ : Path containing flight arc $a$ and using fleet $k$ on arc $a$

$I$ : Set of itineraries

$I_p$ : Set of itineraries corresponding to path $p$

$I_{a_t^k}$ : Set of itineraries containing flight leg $a_t^k$

$L_{s,t-}^k$ : Set of flight legs departing from station $s$ at time $t$

$L_{s,t+}^k$ : Set of flight legs arriving at station $s$ at time $t$

#### *4.4.2.2 Parameters*

$C_{a_t^k}$ : Operating cost of fleet $k$ on flight leg $a_t$

$R_i$ : Revenue of itinerary $i$

$Cap_{s,t}$ : Capacity of airport $s$ at time t

$Cap^k$ : Capacity occupied by fleet $k$ per aircraft

$D_p$ : Passenger demand on path $p$

$Freq(a)$ : Frequency of flight arc $a$

*4.4.2.3 Decision variables*

$x_{a_t^k}$ : Index variable of assigning an aircraft in fleet $k$ to flight leg $a_t$

$y_{s,t}^k$ : Number of aircraft in fleet $k$ at station $s$ immediately before time $t$

$z_i$ : Number of passengers on itinerary $i$

### 4.4.3 Itinerary-based rough fleet assignment model

A formulation of the itinerary-based rough fleet assignment model is given as follows.

$$\max \quad \sum_{i \in I} R_i z_i - \sum_{a \in A} \sum_{t \in T} \sum_{k \in K} C_{a_t^k} \cdot x_{a_t^k}$$

$$s.t. \quad \sum_{k \in K} \sum_{a_t^k \in L_{s,t-}^k} Cap^k \cdot x_{a_t^k} + \sum_{k \in K} \sum_{a_t^k \in L_{s,t+}^k} Cap^k \cdot x_{a_t^k} \le Cap_{s,t}, \forall s \in S, \forall t \in T, \tag{22}$$

$$\sum_{a_t^k \in L_{a,t}^k} x_{a_t^k} + y_{s,t}^k = \sum_{a_{t+1}^k \in L_{a,t+1}^k} x_{a_{t+1}^k} + y_{s,t+1}^k, \forall s \in S, \forall t \in T, \tag{23}$$

$$\sum_{t \in T} \sum_{k \in K} x_{a_t^k} \ge Freq(a), \forall a \in A, \tag{24}$$

$$\sum_{i \in I_{a_t}} z_i \le \sum_{k \in K} Cap_k x_{a_t^k}, \forall a \in A, \forall t \in T, \tag{25}$$

$$\sum_{i \in I_p} z_i \le D_p, \forall p \in P, \tag{26}$$

$$x_{a_t^k} \in \{0, 1\}, \forall a \in A, t \in T, \tag{27}$$

$$y_{s,t}^k \ge 0, \text{integer}, \forall s \in S, t \in T, \tag{28}$$

$$z_i \ge 0, \text{integer}, \forall i \in I. \tag{29}$$

The objective of the model is to maximize itinerary-based passenger revenue minus operating costs. Constraint (22) imposes an upper bound of the total number of arrival and departure flights at each station during each hour. Constraint (23) is a flow balance constraint that ensures that the number of incoming flights equals that of the outgoing flights. Constraint (24) ensures that the arc frequency requirement is satisfied. Constraint (25) guarantees that the capacity of a leg is greater than the total number of passengers on all the itineraries that use this leg. Constraint (26) ensures that passenger demand on a path is greater than the total number of passengers on the itineraries corresponding to the path.

## *4.5 Timetable Model with Time Windows*

Given the rough schedule constructed in the rough fleet assignment model step, the goal of the timetable model in this section is to build an exact schedule by adjusting each flight in the rough flight schedule within certain time windows. The following subsections will explain the objective of the timetable model and the network created for the model in detail, introduce the notations used in the model, and finally present a timetable model with time windows.

### 4.5.1 Network for a timetable model with time windows

Adjusting the departure times of the flights in a network mainly influences the quality of the connecting flights and the number of aircraft used in the network. In fact, for each flight, changing its departure time would not only influence the original connecting flights that contain this flight but also create some new connecting flights. Because the hubs of each airline have many incoming and outgoing flights, adjusting the departure times of these flights of a hub would strongly influence its connecting flights. Hence, to maximize the revenue brought by these connecting flights, schedule planners put great effort in adjusting the departure times of the incoming and outgoing flights of each hub. Usually, at each hub, planners schedule a sequence of incoming flights followed by a sequence of outgoing flights, called a "connecting bank." In fact, a connecting bank can form network of many connecting flights. Figure 9 illustrates a connecting bank at a hub.

Besides its influence on connecting flights, adjusting the departure times of incoming and outgoing flights could influence the number of aircraft needed in the network. For example, in Figure 10, both stations A and B have exactly one incoming flight

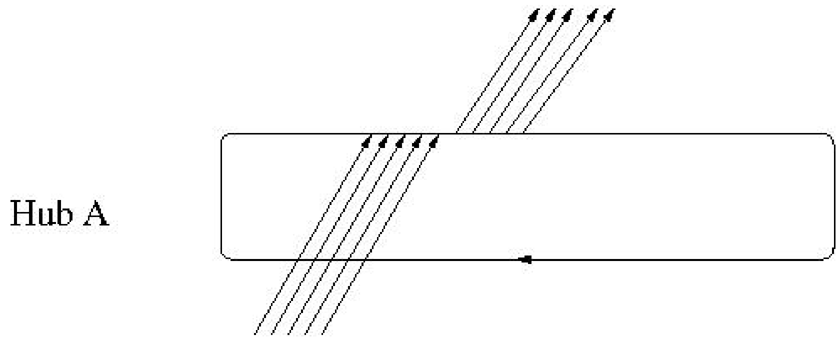
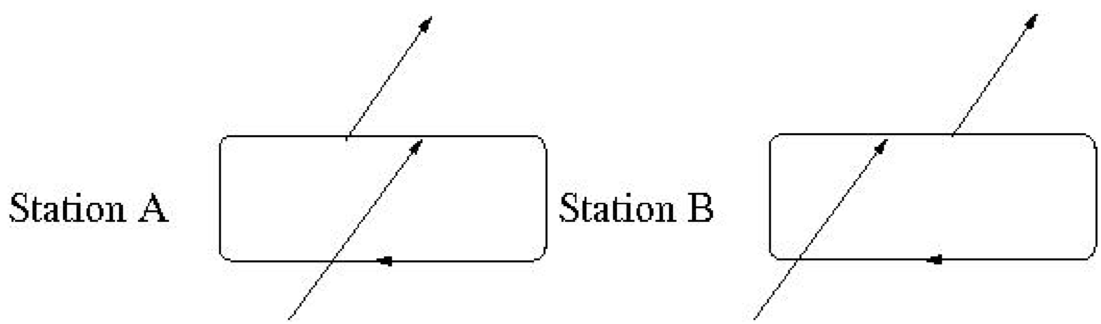



**Figure 9:** Illustration of a connecting bank at a hub

and one outgoing flight. The only difference is that at station A, the arrival time of the incoming flight is later than the departure time of the outgoing flight, and at station B, the arrival time of the incoming flight is earlier than the departure time of the outgoing flight. Because of this difference, station A requires at least one aircraft on the ground overnight, but station B does not have such a requirement. To capture the two effects that accompany adjusting the flight departure times, the objective of the timetable model in this section is to maximize the difference between the revenue related to the connecting flights and the costs of the aircraft needed in the network.




(a) Timeline of Station A (b) Timeline of Station B

**Figure 10:** Comparison of the timelines of Stations A and B

As discussed in Chapter II, both the timeline network and the connection network are widely used in flight scheduling. Because the timetable model in this section needs to capture both the number of fleets needed and the connection flights in the network, a network that is based on the timeline network and that contains connection arcs is created. Figure 11 illustrates a network for the timetable model. The network has three candidate flights, $\ell_1, \ell_2$ and $\ell_3$, from station A to station B, three candidate flights, $\ell_4, \ell_5$ and $\ell_6$, from station B to station C, and three connections arcs, $\ell_1\ell_5, \ell_1\ell_6$ and $\ell_2\ell_6$. Each flight $\ell$ is associated with an indicator variable $x_\ell$, which indicates whether the flight is used in the flight network, and each connection arc $\ell_i\ell_j$ is associated with an indicator variable $x_{\ell_i\ell_j}$, which indicates whether the connection $\ell_i\ell_j$ exists in the flight network. Because the connection $\ell_i\ell_j$ exists in the flight network if and only both of the flights $\ell_i$ and $\ell_j$ that it links with are used in the flight network, its indicator variable $x_{\ell_i\ell_j}$ equals 1 if and only if both of the indicator variables, $x_{\ell_i}$ and $x_{\ell_j}$ equal 1.

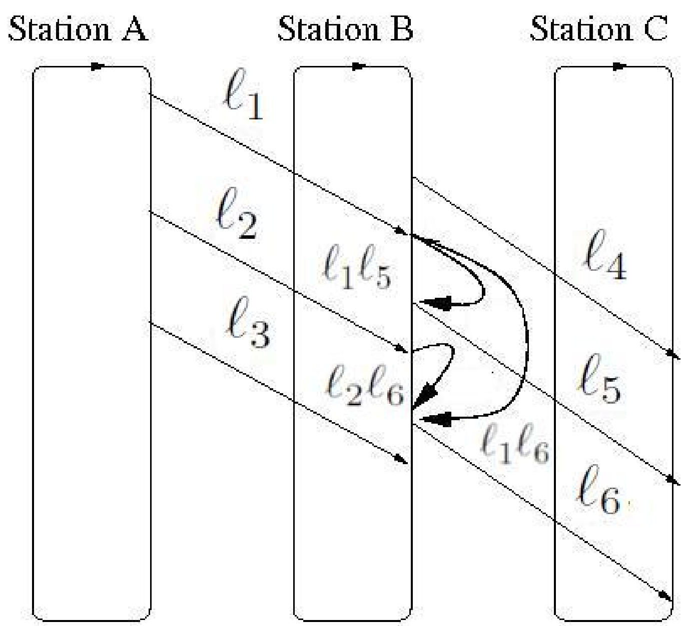



**Figure 11:** Illustration of a timeline network with connection arcs

### 4.5.2 Notations

#### *4.5.2.1 Sets*

$S$ : Set of stations indexed by $s$

$K$ : Set of fleets indexed by $k$

$L$ : Set of flight legs indexed by $\ell$

$TW$ : Set of time windows indexed by j

$\ell^{(j)}$ : $j$-th copy of flight leg $\ell$

$T_s$ : Timeline of station $s$

$L^k_{s,t-}$ : Set of flight legs using an aircraft in fleet $k$ and arriving at station $s$ at time $t$

$L^k_{s,t+}$ : Set of flight legs using an aircraft in fleet $k$ and departing from station $s$ at time $t$

$CL^k$ : Set of flight legs using fleet $k$ and passing the count time

$W$ : Set of connection arcs indexed by $\ell_1^{(i)}\ell_2^{(j)}$

$t_c$ : Count time

#### *4.5.2.2 Parameters*

$R_{\ell_1^{(i)}\ell_2^{(j)}}$ : Revenue related to connection flights $\ell_1^{(i)}\ell_2^{(j)}$

$C^k$ : Cost per aircraft in fleet $k$

$U^k$ : Upper bound of the number of aircraft in fleet $k$

$\beta$ : Adjusting factor for connection revenue

#### *4.5.2.3 Decision variables*

$x_{\ell^{(j)}}$ : Indicator variable of using the $j$-copy of flight leg $\ell$ in the flight network

$y^k_{a,t-}$ : Number of aircraft in fleet $k$ at station $s$ at the time immediately before $t$

$y^k_{a,t+}$ : Number of aircraft in fleet $k$ at station $s$ at the time immediately after $t$

$u^k$ : Number of aircraft in fleet $k$

$x_{\ell_1^{(i)}\ell_2^{(j)}}$ : Indicator variable that indicates whether connection $\ell_1^{(i)}\ell_2^{(j)}$ exists in the flight network

### 4.5.3 Mathematical formulation

$$\min \sum_{k \in K} C^k u^k - \beta \sum_{\ell_1^{(i)} \ell_2^{(j)} \in W} R_{\ell_1^{(i)} \ell_2^{(j)}} x_{\ell_1^{(i)} \ell_2^{(j)}}$$

$$s.t. \sum_{j \in TW} x_{\ell^{(j)}} = 1, \forall \ell \in L, \tag{30}$$

$$\sum_{j \in TW} \sum_{\ell^{(j)} \in L_{s,t-}^k} x_{\ell^{(j)}} + y_{s,t-}^k = \sum_{j \in TW} \sum_{\ell^{(j)} \in L_{s,t+}^k} x_{\ell^{(j)}} + y_{s,t+}^k, \forall s \in S, \forall t \in T_s, \forall k \in K, \tag{31}$$

$$\sum_{s \in S} y_{s,t_c}^k + \sum_{j \in TW} \sum_{\ell^{(j)} \in CL^k} x_{\ell^{(j)}} \leq u^k, \forall k, \tag{32}$$

$$u^k \leq U^k, \forall k, \tag{33}$$

$$x_{\ell_1^{(i)} \ell_2^{(j)}} \leq x_{\ell_1^{(i)}}, \forall \ell_1^{(i)} \ell_2^{(j)} \in W, \tag{34}$$

$$x_{\ell_1^{(i)} \ell_2^{(j)}} \leq x_{\ell_2^{(j)}}, \forall \ell_1^{(i)} \ell_2^{(j)} \in W, \tag{35}$$

$$x_{\ell^{(j)}} \in \{0,1\}, \forall j \in TW, \forall \ell \in L, \tag{36}$$

$$x_{\ell_1^{(i)} \ell_2^{(j)}} \in \{0,1\}, \forall i, j \in TW, \forall \ell_1, \ell_2 \in L, \tag{37}$$

$$u^k \geq 0, \text{integer}, \forall k \in K, \tag{38}$$

$$y_{s,t-}^k, y_{s,t+} \geq 0, \text{integer}, \forall s \in S, \forall t \in T_s, \forall k \in K. \tag{39}$$

The objective of the model is to minimize the difference between the total cost of the aircraft and the revenue related to the connection arcs. Constraint (30) guarantees that each flight leg is assigned to exactly one time window. Constraint (31) is a flow balance constraint, which ensures that no aircraft is missing in the network. Constraint (32) counts the minimum number of aircraft needed in the network. Constraint (33) imposes the upper bound of the number of aircraft in each fleet. Constraints (34) and (35) ensure that a connection exists in the flight network if and only if both of the flight legs that it links with exist in the flight network.

# CHAPTER V

# IMPLEMENTATION OF FUTURE AIRCRAFT NETWORK AND SCHEDULE DESIGN

The objective of this chapter is to illustrate the data used in the models and to develop algorithms for solving the models. The overall structure of this chapter is described as follows. Section 1 presents the sources and some analysis of the data. Section 2 develops solutions for the frequency assignment model of scheduled flights, Section 3 for the frequency assignment model of on-demand flights, Section 4 for the rough fleet assignment model, and Section 5 for the timetable model.

## *5.1 Data Analysis*

This section extracts both the characteristic parameters of fleets from the financial reports of major airlines and passenger demand in each market from DB1B data on the BTS website; it extracts airport capacity parameters from the airport capacity benchmark report of the FAA; and it obtains parameter estimates of the explanatory variables in the passengers' path choice model and mode choice model from related literature.

### 5.1.1 Fleet

#### *5.1.1.1 Fleets for scheduled services*

The BTS website [24] contains the financial reports of several airlines, referred to as Form 41 Financial Data. The tables of Schedules P51 and P52 in Form 41 contain the aircraft operating expenses of several air carriers, which include flying expenses, equipment maintenance expenses, and equipment depreciation costs. Through a comparison of the operating costs of the fleets of major air carriers, five fleets—wide-body, narrow-body, and regional jets—are extracted and listed in Table 3.

**Table 3:** Characteristics of fleets used in scheduled flights

| Fleet | Capacity | Cruise Speed (mile/hr) | Fuel Burn Rate (gallon/(passenger · mile)) | Operating Cost Rate (dollar/(passenger·mile)) |
|---|---|---|---|---|
| Fleet 1 | 350 | 560 | 0.011397959 | 0.057015306 |
| Fleet 2 | 250 | 530 | 0.012430189 | 0.064158491 |
| Fleet 3 | 180 | 530 | 0.012620545 | 0.073616352 |
| Fleet 4 | 120 | 480 | 0.014166667 | 0.083038194 |
| Fleet 5 | 70 | 410 | 0.043275261 | 0.100278746 |

The five fleets in Table 3 thoroughly represent the fleets used in real scheduled services of the domestic markets. As mentioned in Chapter 3, the Boeing Company, GE Aviation, the MIT Group, and the Northrop Grumman Corporation have proposed new models of fuel-efficient, noise-reduced aircraft for the next generation air transportation. According to the features of the proposed aircraft and the characteristics of the current most efficient aircraft, six fleets that represent future fuel-efficient aircraft are created. Because Fleet 1 is an efficient fleet in its class, it is renamed Fleet 1* in Table 4. Therefore, Table 4 illustrates the characteristics of six fuel-efficient fleets. The frequency assignment model uses a representative fleet. To cover both big and small markets effectively, the frequency assignment model uses either Fleet 3 or Fleet 3* as the representative fleet. On the other hand, the rough fleet assignment model uses either the six fleets in Table 3 or the six fleets in Table 4.

**Table 4:** Characteristics of new fleets for scheduled flights

| Fleet | Capacity | Cruise Speed (mile/hr) | Fuel Burn Rate (gallon/(passenger · mile)) | Cost Rate (dollar/(passenger · mile)) |
|---|---|---|---|---|
| Fleet 1* | 350 | 560 | 0.011397959 | 0.057015306 |
| Fleet 2* | 250 | 560 | 0.009942857 | 0.044935714 |
| Fleet 3* | 180 | 476 | 0.009418768 | 0.067787115 |
| Fleet 4* | 120 | 496 | 0.009593414 | 0.067741935 |
| Fleet 5* | 70 | 410 | 0.030313589 | 0.087282230 |

#### 5.1.1.2 *Fleets for on-demand services*

The fleets used in on-demand services will be 20-passenger aircraft proposed by GE aviation, very light jets, and helicopters. Using these aircraft in small airports promotes the usage of small under-utilized airports and the service level of air transportation. Based on the specification files of several brands of very light jets and helicopters, two fleets are created, one representing a very light jet and one representing a helicopter. Table 5 lists the characteristics of the fleets in on-demand services. In fact, Fleets 6* aircraft represent 20-passenger aircraft, Fleet 7* aircraft represent very light jets, and Fleet 8* aircraft represent helicopters. As shown in Table 5, Fleet 6* and Fleet 7* aircraft are more fuel-efficient than Fleet 8* aircraft. On the other hand, because of their vertical takeoff and landing capability, Fleet 8* aircraft have wider usage than Fleet 6* and Fleet 7* aircraft. For example, they can be used at both vertiports and congested airports.

**Table 5:** Characteristics of fleets used in on-demand services

| Fleet | Capacity | Cruise Speed (mile/hr) | Fuel Burn Rate (gallon/(passenger · mile)) | Operation Cost Rate (dollar/(passenger · mile)) |
|---|---|---|---|---|
| Fleet 6* | 20 | 360 | 0.010555556 | 0.170972222 |
| Fleet 7* | 10 | 400 | 0.045408163 | 0.369897959 |
| Fleet 8* | 5 | 150 | 0.060526316 | 0.955263158 |

### 5.1.2 Passenger demand

The BTS website contains the Airline Origin and Destination Survey (DB1B), which is a “10% sample of airline tickets from reporting carriers” [24]. It describes the origin, the destination, and the ticket price of an itinerary chosen by a passenger. However, it does not specify whether a passenger is a business passenger or a leisure passenger. In practice, researchers apply heuristic methods to segment passengers. Therefore, this thesis heuristically segments leisure passengers and business passengers from the data. First, the average fare of an itinerary is calculated. Then, if a passenger pays more than 1.2 times the average fare, he or she is regarded as a business passenger. Otherwise, he or she is regarded as a leisure passenger.

Because the DB1B data represent a quarterly sample, and this thesis focuses on building daily schedules, daily passenger demand is extracted from the DB1B data. To illustrate simple statistics of daily passenger demand, this section uses the DB1B data of a specific quarter in a specific year. Figure 12 shows that the daily passenger demand in each market ranges from 0 to 2,600. Numerically, the passenger demand of 361 markets ranges between 600 and 2,600; of 1,055 markets between 200 and 600; of 3,978 markets between 30 and 200; of 4,286 markets between 10 and 30; and of 16,487 markets between 0 and 10. In addition, Figures 13 and 14 show the range of passenger demand of leisure passengers and business passengers, respectively. Numerically, the leisure passenger demand of 361 markets range between 600 and 2,100; of 906 markets between 200 and 600; of 3,517 between 30 and 200; of 3,769 markets between 10 and 30; and of 17,670 markets between 0 and 10. In addition, the business passenger demand of 191 markets between 200 and 800; of 1,791 markets between 30 and 200; of 2,323 markets between 10 and 30; of 19,187 markets between 0 and 10.

### 5.1.3 Airports

According to the passenger demand of each airport, about 200 airports, which will be included in the models built in this thesis, are selected. Figure 15 presents the range of the daily passenger volume of the selected airports. Numerically, the daily passenger volume of 39 airports range between 20,000 and 87,000; of 51 airports between 4,000 and 20,000; of 65 airports between 1,000 and 4,000; and of 45 airports between 70 and 1,000.

In an airport capacity benchmark report [22], the FAA determined the current and future capacity of thirty-five airports of the United States that have very high daily passenger volumes. In particular, the report defines capacity of each airport

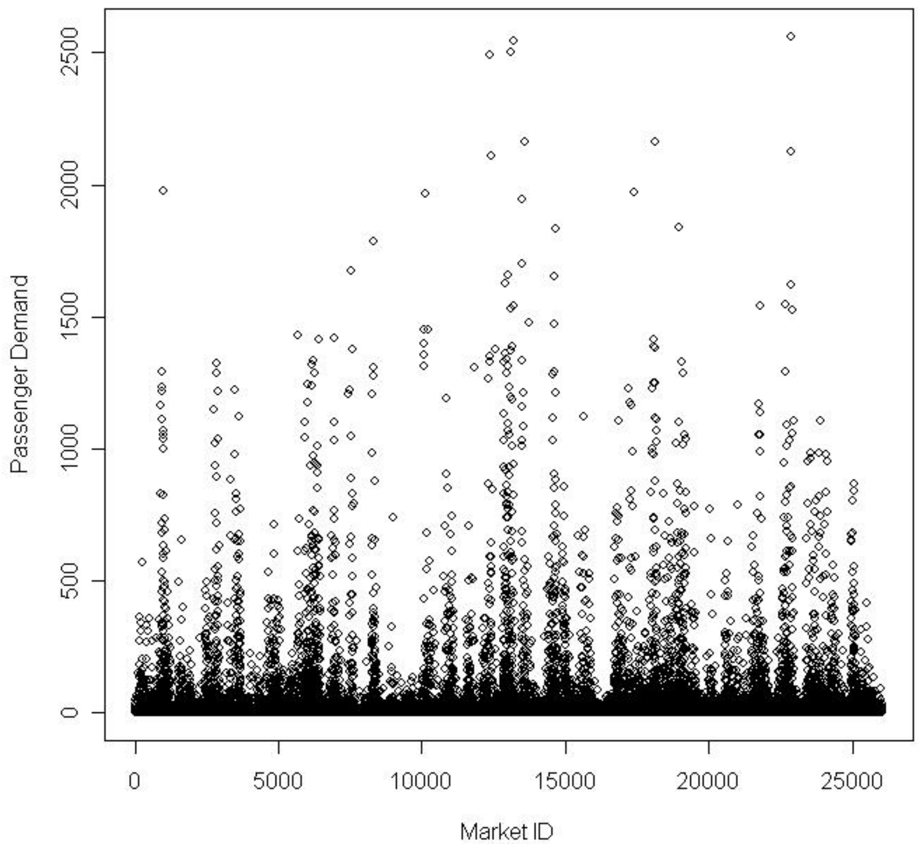



**Figure 12:** Illustration of one sample of passenger demand

as the maximum number of departures and arrivals per hour, and it estimates the capacity of these airports under different weather conditions. Because the capacity of these airports are limited, optimizing the flight resources in these airports is necessary. Based on the report [22], Figure 16 presents the capacity ranges of these airports under good weather conditions. Numerically, among the thirty-five airports, the capacity of four airports range between 180 and 280; of 23 airports between 100 and 180; and 10 between 60 and 100.

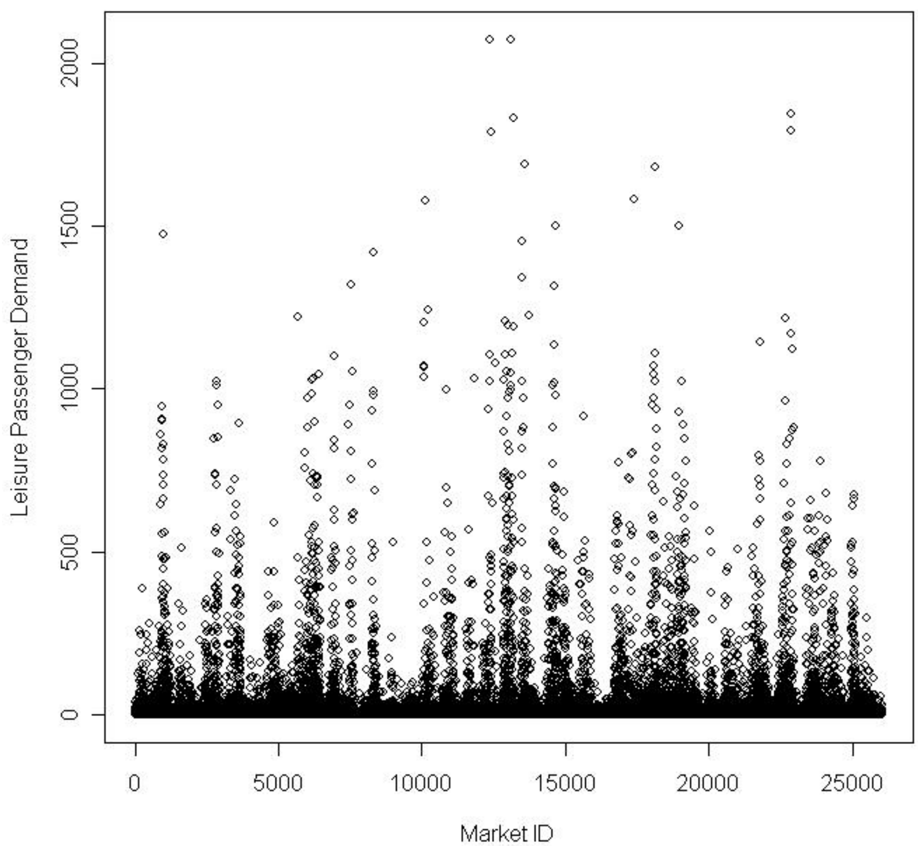



**Figure 13:** Illustration of one sample of leisure passenger demand

### 5.1.4 Parameters for the path choice model

Adler et al. [32] built a logit model to study the effect of explanatory variables such as flight time, one-way fares, the number of connections, and schedule displacement on the itinerary choices of passengers. Furthermore, the substitution values of their service variables are within a reasonable range. Because this thesis focuses on network and schedule design, it applies the parameters estimated by Adler et al. [32] in their passenger choice model. Tables 6 lists the parameters from their results that will be used in the passenger choice model.

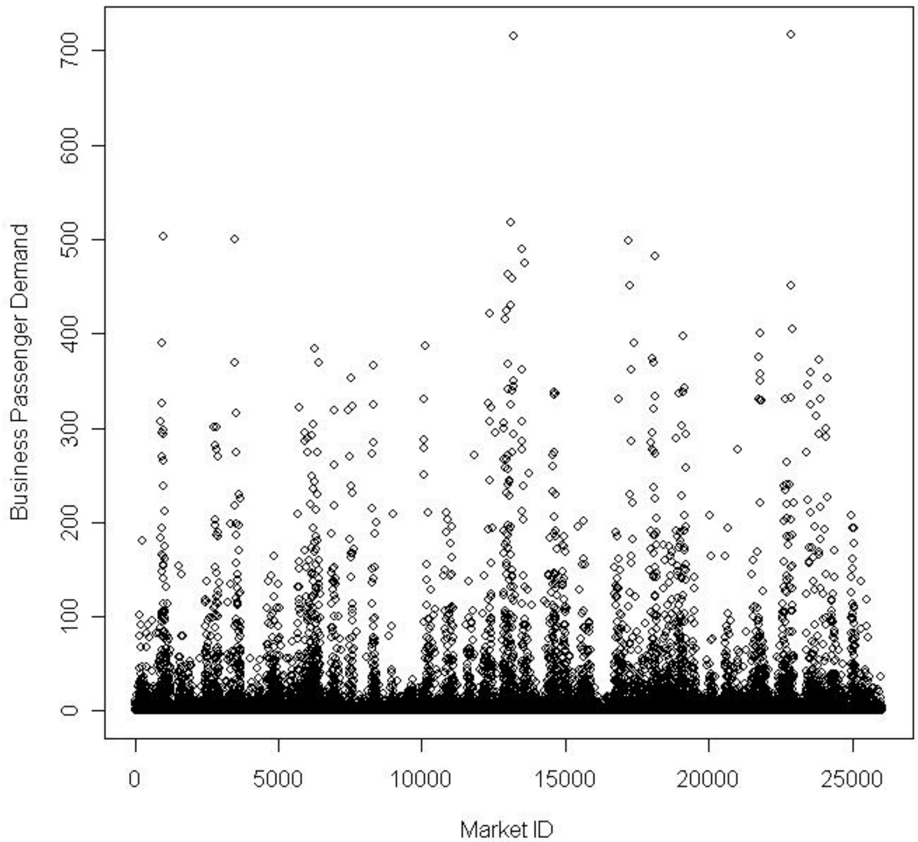



**Figure 14:** Illustration of one sample of business passenger demand

**Table 6:** Parameter estimation of the travel logit model (from Adler et al. [32])

| | Business | | Leisure | |
|---|---|---|---|---|
| Service Variables | Coefficient | T-Stat | Coefficient | T-Stat |
| One-way fare ($) | -0.00556 | -10.9 | -0.0125 | -22.7 |
| Flight time (min) | -0.00883 | -7.1 | -0.00734 | -11.0 |
| Number of connections | -0.368 | -3.0 | -0.303 | -4.8 |
| Schedule time difference (min) | -0.00200 | -2.3 | -0.00126 | -3.5 |

### 5.1.5 Parameters for the mode choice model

Baik et al. [35] built mode choice models to study the effect of travel time and travel cost in passengers' choices of automobiles, commercial airlines, or air taxis. They segmented passengers according to their trip purposes and household incomes. Because

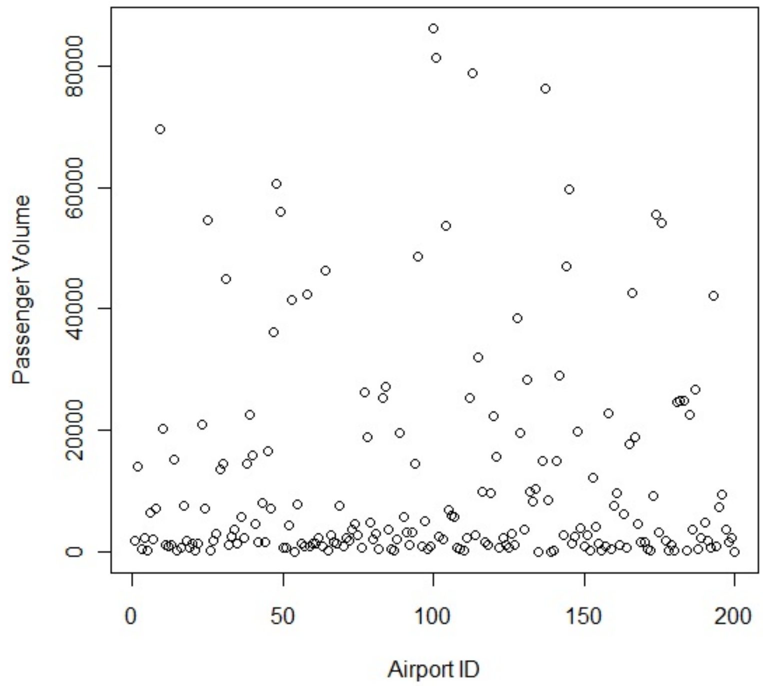



**Figure 15:** Illustration of one sample of passenger volume at each airport

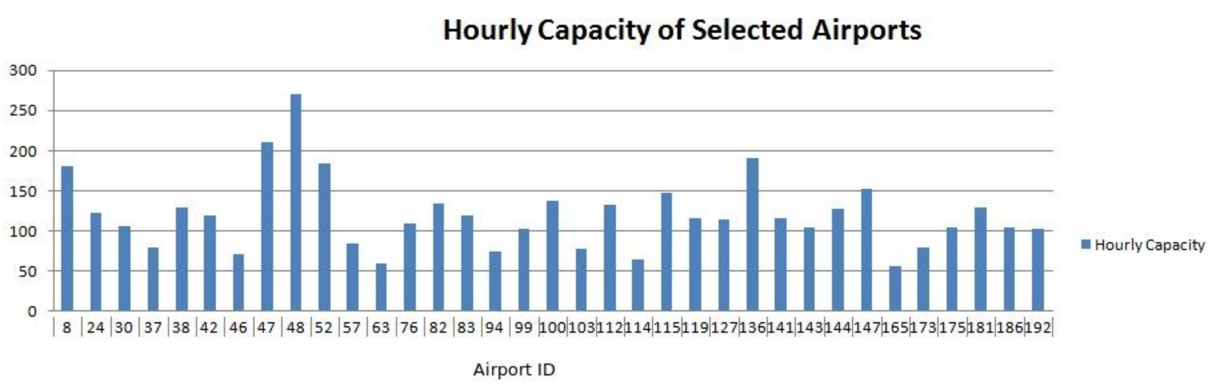



**Figure 16:** Illustration of the capacity of 35 selected airports

this thesis focuses on network and schedule design, it applies the parameters estimated by Baik et al. [32] in their mode choice models. Tables 7 lists the parameters from their results that will be used in the mode choice model.

**Table 7:** Parameter estimation in the mode choice model (from Baik et al. [35])

| | Business | Leisure |
|---|---|---|
| | Coefficient | Coefficient |
| Travel cost (\$) | -0.0117 | -0.0275 |
| Travel time (hour) | -0.2087 | -0.1329 |

## 5.2 Implementation of the Frequency Assignment Model for Scheduled Flights

This section develops algorithms for solving the following frequency assignment model for scheduled flights.

$$\max \sum_{(o,d)\in M} \left( \sum_{p\in P_{o,d}} R_p^l \cdot x_p^l + \sum_{p\in P_{o,d}} R_p^b \cdot x_p^b \right) - C \cdot \sum_{a\in A} FT_a \cdot y_a$$

$$s.t. \sum_{a\in L_{s-}} y_a = \sum_{a\in L_{s+}} y_a, \forall s \in S, \tag{40}$$

$$2 \sum_{a\in L_{s-}} Cap \cdot y_a \leq Cap_s, \forall s \in S, \tag{41}$$

$$\sum_{a\in A} FT_a \cdot y_a \leq MaxHour, \tag{42}$$

$$y_p \leq \gamma y_a, \forall a \in A, \forall p \in P_a, \tag{43}$$

$$\sum_{p\in P_a} (x_p^l + x_p^b) \leq Seat \cdot y_a, \forall a \in A, \tag{44}$$

$$x_p^l + x_p^b \leq Seat \cdot y_p, \forall p \in P, \tag{45}$$

$$x_p^l \leq \frac{e^{V_p^l}}{\sum_{q\in P_{o,d}} e^{V_q^l}} \cdot D_{o,d}^l, \forall (o,d) \in M, \forall p \in P_{o,d}, \tag{46}$$

$$x_p^b \leq \frac{e^{V_p^b}}{\sum_{q\in P_{o,d}} e^{V_q^b}} \cdot D_{o,d}^b, \forall (o,d) \in M, \forall p \in P_{o,d}, \tag{47}$$

$$V_p^l = (a_1^l \cdot C_p + a_2^l \cdot FT_p + a_3^l \cdot n_p) + a_4^l \cdot \frac{240}{y_p}, \forall p \in P, \tag{48}$$

$$V_p^b = (a_1^b \cdot C_p + a_2^b \cdot FT_p + a_3^b \cdot n_p) + a_4^b \cdot \frac{240}{y_p}, \forall p \in P, \tag{49}$$

$$x_p^l \geq 0, x_p^b \geq 0, y_a \geq 0, y_p \geq 0, x_p^l, x_p^b, y_a, y_p \text{ integer} \tag{50}$$

The frequency assignment model for scheduled flights is a large-scale nonlinear programming problem. It is not even a convex programming problem, which would make it more difficult to solve. The difficulty in solving the frequency assignment problem necessitates an analysis of the structure of the problem and a simplification of the entire problem. The following subsections will first determine the overall structure of the network and then discuss iterative algorithms for solving the problem.

### 5.2.1 Overall network structure

This subsection determines the overall structure of the network. In particular, it segments airports into hubs, medium airports, and spokes, and it segments markets into big, medium, and small markets. Furthermore, this subsection generates candidate itineraries for each market.

#### *5.2.1.1 Airport segmentation*

Among the selected airports, certain airports, usually located in big metropolitan areas or a tourist sites, have a high daily volume of incoming and outgoing passengers. Among the airports with high volume, 24 airports are selected as hubs in the network. All the other airports except these hubs are defined as a medium or small airport, mainly depending on whether the number of incoming and outgoing passengers is large and whether it is in a big metropolitan area or not. For each medium airport or spoke, its hub neighbor is defined as the hub closest to it. These hub neighbors can be used as connection airports for medium or small airports, which reflects the geological structure behind these airports located all across the United States. Figure 17 illustrates median airports and spokes and their hub neighbors.

#### *5.2.1.2 Market segmentation*

The volume of the passenger demand of each market varies greatly. According to the volume of their daily passenger demand, markets are divided into big, medium, and

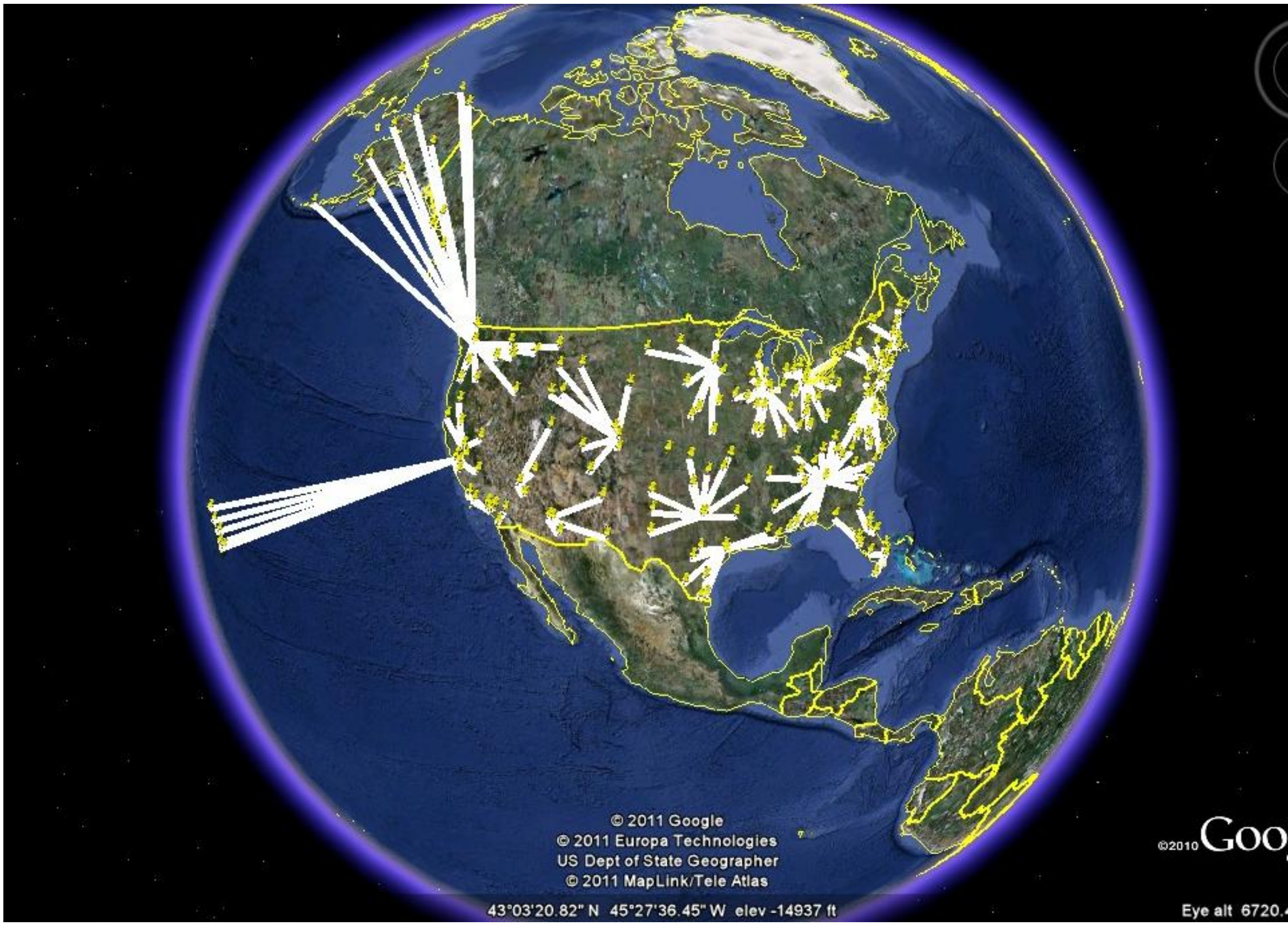



**Figure 17:** Illustration of the geological structure of the 200 selected airport

small markets. In the data, since passengers usually book round trips, the volume of the passenger demand of different markets is almost symmetrical across the entire network. To maintain this symmetry, a market and its reverse market are categorized in the same group. Overall, in our segmentation, a market is defined as a big market if it or its reverse market has a passenger demand volume of over 600. Similarly, a market is defined as a small market if it or its reverse market has a passenger demand volume of less than 200. All the other markets are medium markets. Our division consists of about 370 big, 1,060 medium, and 24,737 small markets. Figure 18 illustrates one sample of passenger demand in each market.

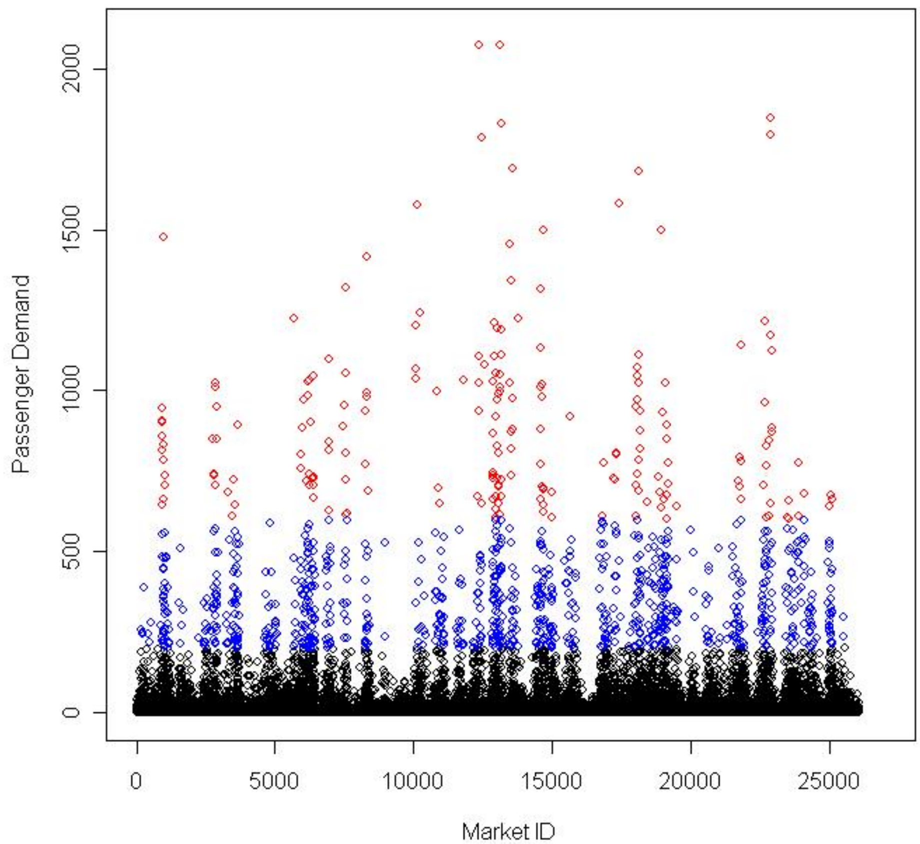



**Figure 18:** Illustration of one sample of passenger demand in each market

#### *5.2.1.3 Candidate itineraries*

In general, big markets are profitable. Therefore, in the network, the set of candidate itineraries for each big market consists of only a direct flight. However, creating itineraries that connect through a hub for small markets would generally be more profitable. Therefore, in the network, the set of candidate itineraries for each small market consists of itineraries with one or two legs that connect through a hub. In addition, the set of candidate itineraries for each medium market is a mixture of a direct flight and itineraries with two legs. According to these principles, a set of itineraries is generated for each market. Furthermore, these itineraries for each market are ordered by their length, and the ones with shorter lengths have higher priorities. To limit the size of the frequency assignment problem, an upper bound of the number of candidate itineraries for each market is imposed, and up to the upper bound, itineraries with shorter lengths for each market are selected.

### 5.2.2 Iterative algorithm for frequency assignment problem

Because of the total number of markets included, the instances of the frequency assignment problem created for the scheduled flights involve a large number of passenger variables, $x_p^l$'s and $x_p^b$'s, and itinerary frequency variables $y_p$'s. Furthermore, with these integer variables, the frequency assignment problem is a very large-scale integer programming problem that is very difficult to solve. Furthermore, it includes a large number of passenger itinerary choice constraints that are nonlinear. Including these nonlinear constraints makes the frequency assignment model even more difficult to solve. Table 8 summarizes the instances that are created for the frequency assignment problem in this thesis.

**Table 8:** A summary of instances of the frequency assignment problem created in this thesis

| Instances of frequency assignment problem | |
|---|---|
| The number of integer variables $x_p^l$'s,$x_p^b$'s | [150,000, 170,000] |
| The number of integer variables $y_p$'s | [84,000, 96,000] |
| The number of integer variables $y_a$'s | [19,000, 22,000] |
| The number of linear inequality constraints | [250,000, 290,000] |
| The number of nonlinear constraints | [150,000, 170,000] |
| The number of equality constraints | 200 |

To deal with the difficulties of solving these instances, an iterative algorithm is developed. First of all, the main decision variables in the frequency assignment model are the arc frequency variables $y_a$'s, the values of which are inputs to the rough fleet assignment model. On the other hand, variables $x_p^l$'s and $x_p^b$'s are created for estimating only the overall passenger revenue related to a flight network, and the frequency assignment problem includes a large number of variables, $x_p^l$'s and $x_p^b$'s. Therefore, variables $x_p^l$'s and $x_p^b$'s are relaxed into continuous variables.

The iterative algorithm can be described as follows: It first creates an initial subproblem and solves this initial subproblem to derive an initial assignment of arc frequencies and itinerary frequencies, and starting from the initial assignment, it will keep generating new assignments of arc frequencies and itinerary frequencies based on the assignment derived in the previous step, and finally, it creates a final subproblem and selects the best assignment generated so far by solving the final subproblem. The iterative algorithm can also be viewed as the following process: Initially, air transportation suppliers do not have any information about passengers' itinerary choice behavior. Therefore, to determine the initial leg frequencies and itinerary frequencies, they solve the initial subproblem, which does not contain any passenger itinerary choice constraints. Based on the frequencies provided by the transportation suppliers, passengers choose their preferred itineraries. After gathering new information about passengers' itinerary choice, transportation suppliers solve a new assignment problem based on their estimation of passenger demand in different itineraries. The process repeats until it reaches equilibrium.

The initial subproblem is formed by relaxing the passenger itinerary choice constraints. Furthermore, it includes constraints that guarantee that the total number of seats allocated to leisure passengers on itineraries in each market is less than the leisure passenger demand in that market and the similar conditions for business passengers. In fact, these constraints are valid constraints of the frequency assignment model that are implied by the passenger itinerary choice constraints. The notations introduced for the frequency assignment model on pages 50 and 51 are also valid in the formulations in this section. Based on these previous notations, the formulation of the initial subproblem is presented as follows.

$$\max \sum_{(o,d)\in M} (\sum_{p\in P_{o,d}} R_p^l \cdot x_p^l + \sum_{p\in P_{o,d}} R_p^b \cdot x_p^b) - C \cdot \sum_{a\in A} FT_a \cdot y_a$$

$$s.t. \sum_{a\in L_{s-}} y_a = \sum_{a\in L_{s+}} y_a, \forall s \in S, \tag{51}$$

$$2\sum_{a\in L_{s-}} Cap \cdot y_a \leq Cap_s, \forall s \in S, \tag{52}$$

$$\sum_{a\in A} FT_a \cdot y_a \leq MaxHour, \tag{53}$$

$$y_p \leq \gamma y_a, \forall a \in A, \forall p \in P_a, \tag{54}$$

$$\sum_{p\in P_a} (x_p^l + x_p^b) \leq Seat \cdot y_a, \forall a \in A, \tag{55}$$

$$x_p^l + x_p^b \leq Seat \cdot y_p, \forall p \in P, \tag{56}$$

$$\sum_{p\in P_a} x_p^b \leq D_p^b, \forall p \in P \tag{57}$$

$$\sum_{p\in P_a} x_p^l \leq D_p^l, \forall p \in P \tag{58}$$

$$x_p^l \geq 0, x_p^b \geq 0, y_a \geq 0, y_p \geq 0, y_a \text{ integer}, y_p \text{ integer} \tag{59}$$

In the formulation of the initial subproblem, for each market, constraint 57 ensures that the total capacity allocated to business passengers in all the candidate itineraries in that market does not exceed the total business passenger demand in that market, and constraint 58 ensures a similar condition for leisure passenger demand.

To test the computation property of this subproblem, two sets of instances are created, and Tables 9 and 10 summarize the computational results of these instances. The computations in this thesis are all carried out on the Unix servers of the Industrial System and Engineering department of Georgia Institute of Technology. In fact, these two sets of instances of the initial subproblem are still very hard to solve. Therefore, two relaxations of the initial subproblem are calculated. In Tables 9 and

10, the relaxed problem 1 of the initial subproblem represents relaxing the arc frequency variables to be continuous variables, and the relaxed problem 2 of the initial subproblem represents relaxing the itinerary frequency variables to be continuous variables. Tables 9, 10, 11, 12, and 13 show the computational result when $\gamma = 1$.

As shown in Tables 9 and 10, it takes quite a long time to solve these instances. Furthermore, the objectives of these two relaxed subproblems are very close to each other. In fact, the objective of relaxed subproblem 2 is always a little bit bigger than that of the relaxed subproblem 1, except only the instance 6_3_1. Using the objective of relaxed subproblems 1 and 2, an upper bound of the objective of the initial subproblem are also derived and listed in Tables 9 and 10. Because the values of the itinerary frequency variables will be inputs to the following subproblems, only the solution to the relaxed subproblem 2 are used as inputs to the following subproblems.

**Table 9:** Computational result 1 of instances of two relaxation problems of the initial subproblem

| Instance Name | Relaxed Problem 1 Objective | Relaxed Problem 1 Solution Time (CPU Seconds) | Relaxed Problem 2 Objective | Relaxed Problem 2 Solution Time (CPU Seconds) | Objective Upper Bound |
|---|---|---|---|---|---|
| 2_1_0 | 1.72E+08 | 16089 | 1.74E+08 | 25706 | 1.72E+08 |
| 2_2_0 | 1.98E+08 | 33053 | 1.99E+08 | 70625 | 1.98E+08 |
| 2_3_0 | 1.96E+08 | 37147 | 1.97E+08 | 47753 | 1.96E+08 |
| 2_4_0 | 1.93E+08 | 334731 | 1.94E+08 | 15506 | 1.93E+08 |
| 3_1_0 | 1.75E+08 | 42790 | 1.76E+08 | 18179 | 1.75E+08 |
| 3_2_0 | 2.04E+08 | 32267 | 2.05E+08 | 16809 | 2.04E+08 |
| 3_3_0 | 2.04E+08 | 62617 | 2.05E+08 | 29338 | 2.04E+08 |
| 3_4_0 | 2.06E+08 | 55359 | 2.06E+08 | 30082 | 2.06E+08 |
| 4_1_0 | 1.95E+08 | 35175 | 1.95E+08 | 18006 | 1.95E+08 |
| 4_2_0 | 2.29E+08 | 49743 | 2.30E+08 | 17503 | 2.29E+08 |
| 4_3_0 | 2.26E+08 | 66129 | 2.27E+08 | 16180 | 2.26E+08 |
| 4_4_0 | 2.26E+08 | 37814 | 2.27E+08 | 22648 | 2.26E+08 |
| 5_1_0 | 2.13E+08 | 45687 | 2.14E+08 | 18001 | 2.13E+08 |
| 5_2_0 | 2.48E+08 | 117604 | 2.50E+08 | 11031 | 2.48E+08 |
| 5_3_0 | 2.42E+08 | 40930 | 2.44E+08 | 44339 | 2.42E+08 |
| 5_4_0 | 2.34E+08 | 56651 | 2.35E+08 | 41383 | 2.34E+08 |
| 6_1_0 | 2.21E+08 | 25277 | 2.23E+08 | 11579 | 2.21E+08 |
| 6_2_0 | 2.54E+08 | 54934 | 2.55E+08 | 13846 | 2.54E+08 |
| 6_3_0 | 2.41E+08 | 67914 | 2.42E+08 | 23560 | 2.41E+08 |
| 6_4_0 | 2.40E+08 | 45495 | 2.41E+08 | 96142 | 2.40E+08 |
| 7_1_0 | 2.25E+08 | 43159 | 2.27E+08 | 14721 | 2.25E+08 |
| 7_2_0 | 2.58E+08 | 53379 | 2.60E+08 | 23582 | 2.58E+08 |
| 7_3_0 | 2.49E+08 | 87476 | 2.50E+08 | 48398 | 2.49E+08 |
| 7_4_0 | 2.38E+08 | 47472 | 2.39E+08 | 10867 | 2.38E+08 |
| 8_1_0 | 2.22E+08 | 46314 | 2.22E+08 | 19695 | 2.22E+08 |
| 8_2_0 | 2.47E+08 | 23487 | 2.49E+08 | 33737 | 2.47E+08 |
| 8_3_0 | 2.28E+08 | 49631 | 2.30E+08 | 42991 | 2.28E+08 |
| 8_4_0 | 2.14E+08 | 22120 | 2.19E+08 | 33635 | 2.14E+08 |
| 9_1_0 | 1.98E+08 | 20772 | 2.00E+08 | 24692 | 1.98E+08 |
| 9_2_0 | 2.32E+08 | 57617 | 2.33E+08 | 28518 | 2.32E+08 |
| 9_3_0 | 2.26E+08 | 55271 | 2.28E+08 | 49624 | 2.26E+08 |
| 9_4_0 | 2.16E+08 | 47159 | 2.16E+08 | 21923 | 2.16E+08 |

After solving the relaxed subproblem 2 of the initial problem, the values for the itinerary frequency variables $y_p$'s are determined. After plugging these new values of $y_p$'s into the formulation of $V_p^l$'s and $V_p^b$'s, the utility $V_p^l$ and $V_p^b$ of each path $p$ are calculated. Plugging the values of $V_p^l$ and $V_p^b$ into the frequency assignment model

**Table 10:** Computational result 2 of instances of two relaxation problems of the initial subproblem

| Instance Name | Relaxed Problem 1 | | Relaxed Problem 2 | | Objective Upper Bound |
|---|---|---|---|---|---|
| | Objective | Solution Time (CPU Seconds) | Objective | Solution Time (CPU Seconds) | |
| 2_1_1 | 1.78E+08 | 89905 | 1.79E+08 | 12092 | 1.78E+08 |
| 2_2_1 | 2.04E+08 | 133242 | 2.05E+08 | 15823 | 2.04E+08 |
| 2_3_1 | 2.02E+08 | 157965 | 2.03E+08 | 91528 | 2.02E+08 |
| 2_4_1 | 1.98E+08 | 119686 | 1.99E+08 | 38473 | 1.98E+08 |
| 3_1_1 | 1.80E+08 | 76053 | 1.81E+08 | 51023 | 1.80E+08 |
| 3_2_1 | 2.10E+08 | 168520 | 2.10E+08 | 56007 | 2.10E+08 |
| 3_3_1 | 2.10E+08 | 176099 | 2.11E+08 | 24411 | 2.10E+08 |
| 3_4_1 | 2.12E+08 | 71214 | 2.13E+08 | 40880 | 2.12E+08 |
| 4_1_1 | 2.00E+08 | 61613 | 2.01E+08 | 17503 | 2.00E+08 |
| 4_2_1 | 2.36E+08 | 135512 | 2.38E+08 | 16098 | 2.36E+08 |
| 4_3_1 | 2.32E+08 | 134978 | 2.34E+08 | 77666 | 2.32E+08 |
| 4_4_1 | 2.32E+08 | 120076 | 2.33E+08 | 45201 | 2.32E+08 |
| 5_1_1 | 2.20E+08 | 66044 | 2.20E+08 | 48775 | 2.20E+08 |
| 5_2_1 | 2.56E+08 | 167052 | 2.57E+08 | 17754 | 2.56E+08 |
| 5_3_1 | 2.50E+08 | 171689 | 2.52E+08 | 50584 | 2.50E+08 |
| 5_4_1 | 2.41E+08 | 135425 | 2.42E+08 | 97740 | 2.41E+08 |
| 6_1_1 | 2.28E+08 | 37123 | 2.29E+08 | 44484 | 2.28E+08 |
| 6_2_1 | 2.61E+08 | 78875 | 2.62E+08 | 38184 | 2.61E+08 |
| 6_3_1 | 2.47E+08 | 102729 | 2.42E+08 | 39997 | 2.42E+08 |
| 6_4_1 | 2.47E+08 | 98215 | 2.49E+08 | 44779 | 2.47E+08 |
| 7_1_1 | 2.32E+08 | 69806 | 2.33E+08 | 35409 | 2.32E+08 |
| 7_2_1 | 2.66E+08 | 104478 | 2.67E+08 | 11241 | 2.66E+08 |
| 7_3_1 | 2.56E+08 | 94138 | 2.57E+08 | 45669 | 2.56E+08 |
| 7_4_1 | 2.45E+08 | 100421 | 2.46E+08 | 39400 | 2.45E+08 |
| 8_1_1 | 2.28E+08 | 57272 | 2.30E+08 | 18513 | 2.28E+08 |
| 8_2_1 | 2.54E+08 | 107542 | 2.56E+08 | 41542 | 2.54E+08 |
| 8_3_1 | 2.37E+08 | 106486 | 2.38E+08 | 34703 | 2.37E+08 |
| 8_4_1 | 2.24E+08 | 121846 | 2.25E+08 | 77023 | 2.24E+08 |
| 9_1_1 | 2.05E+08 | 70673 | 2.06E+08 | 49175 | 2.05E+08 |
| 9_2_1 | 2.38E+08 | 216306 | 2.41E+08 | 22228 | 2.38E+08 |
| 9_3_1 | 2.34E+08 | 173213 | 2.35E+08 | 25932 | 2.34E+08 |
| 9_4_1 | 2.22E+08 | 213090 | 2.23E+08 | 18932 | 2.22E+08 |

creates the following mixed integer problem, which does not contain any nonlinear constraints. For simplicity, it is called an "iterative subproblem". Notice that, due to the tremendous time in solving this subproblem, the arc frequency variables are also relaxed to be continuous variables.

$$\max \sum_{(o,d)\in M} (\sum_{p\in P_{o,d}} R_p^l \cdot x_p^l + \sum_{p\in P_{o,d}} R_p^b \cdot x_p^b) - C \cdot \sum_{a\in A} FT_a \cdot y_a$$

$$s.t. \sum_{a\in L_{s-}} y_a = \sum_{a\in L_{s+}} y_a, \forall s \in S, \tag{60}$$

$$2 \sum_{a\in L_{s-}} Cap \cdot y_a \leq Cap_s, \forall s \in S, \tag{61}$$

$$\sum_{a\in A} FT_a \cdot y_a \leq MaxHour, \tag{62}$$

$$y_p \leq \gamma y_a, \forall a \in A, \forall p \in P_a, \tag{63}$$

$$\sum_{p\in P_a} (x_p^l + x_p^b) \leq Seat \cdot y_a, \forall a \in A, \tag{64}$$

$$x_p^l + x_p^b \leq Seat \cdot y_p, \forall p \in P, \tag{65}$$

$$x_p^l \leq \frac{e^{V_p^l}}{\sum_{q\in P_{o,d}} e^{V_q^l}} \cdot D_{o,d}^l, \forall (o,d) \in M, \forall p \in P_{o,d}, \tag{66}$$

$$x_p^b \leq \frac{e^{V_p^b}}{\sum_{q\in P_{o,d}} e^{V_q^b}} \cdot D_{o,d}^b, \forall (o,d) \in M, \forall p \in P_{o,d}, \tag{67}$$

$$x_p^l \geq 0, x_p^b \geq 0, y_a \geq 0, y_p \geq 0, y_p \text{ integer} \tag{68}$$

A sequence of assignment solutions are derived by iteratively solving the "iterative subproblem". For each assignment, plugging the values of itinerary frequency $y_p$'s into the frequency assignment problem creates the following subproblem, called the "final subproblem", in which $y_p$'s are constant. An assignment with the maximum objective is selected.

$$\max \sum_{(o,d)\in M} (\sum_{p\in P_{o,d}} R_p^l \cdot x_p^l + \sum_{p\in P_{o,d}} R_p^b \cdot x_p^b) - C \cdot \sum_{a\in A} FT_a \cdot y_a$$

$$s.t. \sum_{a\in L_{s-}} y_a = \sum_{a\in L_{s+}} y_a, \forall s \in S, \tag{69}$$

$$2 \sum_{a\in L_{s-}} Cap \cdot y_a \leq Cap_s, \forall s \in S, \tag{70}$$

$$\sum_{a\in A} FT_a \cdot y_a \leq MaxHour, \tag{71}$$

$$\sum_{p\in P_a} (x_p^l + x_p^b) \leq Seat \cdot y_a, \forall a \in A, \tag{72}$$

$$x_p^l + x_p^b \leq Seat \cdot y_p, \forall p \in P, \tag{73}$$

$$x_p^l \leq \frac{e^{V_p^l}}{\sum_{q\in P_{o,d}} e^{V_q^l}} \cdot D_{o,d}^l, \forall (o,d) \in M, \forall p \in P_{o,d}, \tag{74}$$

$$x_p^b \leq \frac{e^{V_p^b}}{\sum_{q\in P_{o,d}} e^{V_q^b}} \cdot D_{o,d}^b, \forall (o,d) \in M, \forall p \in P_{o,d}, \tag{75}$$

$$y_p \leq \gamma y_a, \forall a \in A, \forall p \in P_a, \tag{76}$$

$$x_p^l \geq 0, x_p^b \geq 0, y_a \geq 0, y_a \text{ integer} \tag{77}$$

Table 11 presents the computation results of the iterative algorithm on the two sets of instances of the frequency assignment model. Using the upper bound of the objective shown in Tables 9 and 10, the optimality gaps of the solutions are calculated and listed in Table 11. As show in these two tables, the iterative algorithms can achieve good solutions but it may take a very long time.

**Table 11:** Computational result of the iterative algorithm on instances of the frequency assignment problem

| Instance | Objective | Optimality Gap | Instance | Objective | Optimality Gap |
|---|---|---|---|---|---|
| 2_1_0 | 1.63E+08 | 5.16 % | 2_1_1 | 1.70E+08 | 4.73 % |
| 2_2_0 | 1.87E+08 | 5.38 % | 2_2_1 | 1.95E+08 | 4.62 % |
| 2_3_0 | 1.86E+08 | 5.17 % | 2_3_1 | 1.92E+08 | 5.05 % |
| 2_4_0 | 1.82E+08 | 5.67 % | 2_4_1 | 1.89E+08 | 4.56 % |
| 3_1_0 | 1.65E+08 | 5.53 % | 3_1_1 | 1.71E+08 | 5.01 % |
| 3_2_0 | 1.93E+08 | 5.45 % | 3_2_1 | 1.99E+08 | 5.06 % |
| 3_3_0 | 1.94E+08 | 5.13 % | 3_3_1 | 2.01E+08 | 4.52 % |
| 3_4_0 | 1.95E+08 | 5.35 % | 3_4_1 | 2.02E+08 | 4.85 % |
| 4_1_0 | 1.84E+08 | 5.85 % | 4_1_1 | 1.87E+08 | 6.34 % |
| 4_2_0 | 2.18E+08 | 4.69 % | 4_2_1 | 2.26E+08 | 4.23 % |
| 4_3_0 | 2.15E+08 | 4.71 % | 4_3_1 | 2.22E+08 | 4.29 % |
| 4_4_0 | 2.15E+08 | 4.66 % | 4_4_1 | 2.22E+08 | 4.46 % |
| 5_1_0 | 2.03E+08 | 4.64 % | 5_1_1 | 2.09E+08 | 4.78 % |
| 5_2_0 | 2.37E+08 | 4.56 % | 5_2_1 | 2.45E+08 | 4.2 % |
| 5_3_0 | 2.31E+08 | 4.64 % | 5_3_1 | 2.39E+08 | 4.49 % |
| 5_4_0 | 2.23E+08 | 4.86 % | 5_4_1 | 2.31E+08 | 4.07 % |
| 6_1_0 | 2.11E+08 | 4.57 % | 6_1_1 | 2.19E+08 | 4.1 % |
| 6_2_0 | 2.42E+08 | 4.57 % | 6_2_1 | 2.50E+08 | 4.21 % |
| 6_3_0 | 2.29E+08 | 4.88 % | 6_3_1 | 2.37E+08 | 2.05 % |
| 6_4_0 | 2.28E+08 | 4.81 % | 6_4_1 | 2.38E+08 | 3.52 % |
| 7_1_0 | 2.14E+08 | 4.85 % | 7_1_1 | 2.22E+08 | 4.36 % |
| 7_2_0 | 2.48E+08 | 4.02 % | 7_2_1 | 2.55E+08 | 4.2 % |
| 7_3_0 | 2.37E+08 | 4.73 % | 7_3_1 | 2.46E+08 | 4.06 % |
| 7_4_0 | 2.26E+08 | 4.83 % | 7_4_1 | 2.35E+08 | 4.26 % |
| 8_1_0 | 2.11E+08 | 4.98 % | 8_1_1 | 2.18E+08 | 4.5 % |
| 8_2_0 | 2.35E+08 | 4.67 % | 8_2_1 | 2.44E+08 | 3.87 % |
| 8_3_0 | 2.18E+08 | 4.36 % | 8_3_1 | 2.25E+08 | 4.97 % |
| 8_4_0 | 2.07E+08 | 3.37 % | 8_4_1 | 2.14E+08 | 4.63 % |
| 9_1_0 | 1.89E+08 | 4.78 % | 9_1_1 | 1.94E+08 | 5.18 % |
| 9_2_0 | 2.20E+08 | 5.2 % | 9_2_1 | 2.28E+08 | 4.07 % |
| 9_3_0 | 2.15E+08 | 4.78 % | 9_3_1 | 2.23E+08 | 4.61 % |
| 9_4_0 | 2.05E+08 | 5.11 % | 9_4_1 | 2.12E+08 | 4.48 % |

Because the great difference in solving a linear relaxation of the initial subproblem and its relaxation problems 1 and 2, a randomized iterative algorithm is also developed. In the randomized iterative algorithm, it first generates a sequence of random numbers to perturb the revenue of each itinerary in the objective function so that it can start from different initial itinerary frequency assignments. Then, it solves the linear relaxation of the perturbed initial subproblem, and the values of the itinerary frequency variables are round up to an integer number. After that, it also solves the "iterative subproblem" up to a certain number of iterations or until it can not find better solution. Then, it perturbs the objective function again, and it runs the previous process again until it runs up to a certain mount of time. In Tables 12 and 13, the iterative algorithm 1 represents the previous iterative algorithm, and iterative algorithm 2 represents the randomized iterative algorithm. In addition, Tables 12 and 13 compare the computational results of the iterative algorithms 1 and 2.

Table 14 presents a comparison of the computational results of instances of frequency assignment problem that differ only in the value of link parameters $\gamma$. As shown Table 14, when $\gamma$ increases from 1 to 2, the total number of distinct origin-destionation arcs increases, the total number of distinct origin-destination arcs with frequency that equals to 1 increases, and the total sum of origin-destination arc frequencies increases.

**Table 12:** Comparison 1 of the two iterative algorithms on the instances of the frequency assignment problem

| Instance Name | Iterative Algorithm 1 | | Iterative Algorithm 2 | | Comparison | |
|---|---|---|---|---|---|---|
| | Objective | Solution Time (CPU Seconds) | Objective | Solution Time (CPU Seconds) | Objective | Solution Time |
| 2_1_0 | 1.63E+08 | 26909 | 1.32E+08 | 4112 | -19.30 % | -84.72 % |
| 2_2_0 | 1.87E+08 | 71993 | 1.53E+08 | 4320 | -18.21 % | -94.00 % |
| 2_3_0 | 1.86E+08 | 49218 | 1.52E+08 | 4827 | -18.37 % | -90.19 % |
| 2_4_0 | 1.82E+08 | 15506 | 1.49E+08 | 2029 | -18.30 % | -86.91 % |
| 3_1_0 | 1.65E+08 | 19781 | 1.34E+08 | 2015 | -18.92 % | -89.81 % |
| 3_2_0 | 1.93E+08 | 18413 | 1.58E+08 | 2153 | -17.83 % | -88.31 % |
| 3_3_0 | 1.94E+08 | 30791 | 1.58E+08 | 2398 | -18.33 % | -92.21 % |
| 3_4_0 | 1.95E+08 | 30791 | 1.59E+08 | 2151 | -18.42 % | -93.01 % |
| 4_1_0 | 1.84E+08 | 19628 | 1.51E+08 | 1757 | -18.00 % | -91.05 % |
| 4_2_0 | 2.18E+08 | 17646 | 1.80E+08 | 2282 | -17.59 % | -87.07 % |
| 4_3_0 | 2.15E+08 | 19694 | 1.77E+08 | 2121 | -17.67 % | -89.23 % |
| 4_4_0 | 2.15E+08 | 23712 | 1.77E+08 | 1893 | -18.05 % | -92.02 % |
| 5_1_0 | 2.03E+08 | 19523 | 1.66E+08 | 1947 | -18.42 % | -90.03 % |
| 5_2_0 | 2.37E+08 | 12249 | 1.97E+08 | 2064 | -16.90 % | -83.15 % |
| 5_3_0 | 2.31E+08 | 45414 | 1.92E+08 | 2042 | -16.93 % | -95.50 % |
| 5_4_0 | 2.23E+08 | 42835 | 1.85E+08 | 2180 | -16.90 % | -94.91 % |
| 6_1_0 | 2.11E+08 | 12663 | 1.74E+08 | 6202 | -17.28 % | -51.02 % |
| 6_2_0 | 2.42E+08 | 15062 | 2.03E+08 | 4953 | -16.31 % | -67.12 % |
| 6_3_0 | 2.29E+08 | 25885 | 1.90E+08 | 6952 | -16.91 % | -73.14 % |
| 6_4_0 | 2.28E+08 | 98055 | 1.91E+08 | 4995 | -16.58 % | -94.91 % |
| 7_1_0 | 2.14E+08 | 16082 | 1.78E+08 | 1807 | -16.92 % | -88.76 % |
| 7_2_0 | 2.48E+08 | 24295 | 2.06E+08 | 2279 | -16.90 % | -90.62 % |
| 7_3_0 | 2.37E+08 | 49999 | 1.97E+08 | 2553 | -16.90 % | -94.89 % |
| 7_4_0 | 2.26E+08 | 11940 | 1.88E+08 | 2309 | -17.02 % | -80.66 % |
| 8_1_0 | 2.11E+08 | 20849 | 1.75E+08 | 2062 | -17.11 % | -90.11 % |
| 8_2_0 | 2.35E+08 | 37604 | 1.96E+08 | 2401 | -16.86 % | -93.62 % |
| 8_3_0 | 2.18E+08 | 44321 | 1.80E+08 | 2323 | -17.56 % | -94.76 % |
| 8_4_0 | 2.07E+08 | 34759 | 1.70E+08 | 2383 | -17.76 % | -93.14 % |
| 9_1_0 | 1.89E+08 | 26889 | 1.54E+08 | 2020 | -18.08 % | -92.49 % |
| 9_2_0 | 2.20E+08 | 31036 | 1.82E+08 | 2151 | -17.12 % | -93.07 % |
| 9_3_0 | 2.15E+08 | 50999 | 1.77E+08 | 2068 | -17.60 % | -95.95 % |
| 9_4_0 | 2.05E+08 | 23558 | 1.69E+08 | 2062 | -17.76 % | -91.25 % |

## 5.3 *Implementation of the Frequency Assignment Model for On-Demand Flights*

This section discusses the implementation of the frequency assignment model for on-demand flights, which is presented as follows. Because the frequency assignment

**Table 13:** Comparison 2 of the two iterative algorithms on the instances of the frequency assignment problem

| | Iterative algorithm 1 | | Iterative algorithm 2 | | Comparison | |
|---|---|---|---|---|---|---|
| Instance Name | Objective | Solution Time (CPU Seconds) | Objective | Solution Time (CPU Seconds) | Objective | Solution Time |
| 2_1_1 | 1.70E+08 | 13435 | 1.37E+08 | 7023 | -19.42 % | -47.73 % |
| 2_2_1 | 1.95E+08 | 17202 | 1.59E+08 | 2369 | -18.49 % | -86.23 % |
| 2_3_1 | 1.92E+08 | 92958 | 1.57E+08 | 2465 | -18.06 % | -97.35 % |
| 2_4_1 | 1.89E+08 | 39798 | 1.54E+08 | 2064 | -18.65 % | -94.81 % |
| 3_1_1 | 1.71E+08 | 52154 | 1.39E+08 | 2160 | -18.83 % | -95.86 % |
| 3_2_1 | 1.99E+08 | 57162 | 1.64E+08 | 2528 | -17.71 % | -95.58 % |
| 3_3_1 | 2.01E+08 | 25323 | 1.63E+08 | 2544 | -18.55 % | -89.95 % |
| 3_4_1 | 2.02E+08 | 42127 | 1.64E+08 | 2190 | -18.63 % | -94.80 % |
| 4_1_1 | 1.87E+08 | 18179 | 1.56E+08 | 2148 | -16.95 % | -88.18 % |
| 4_2_1 | 2.26E+08 | 17217 | 1.86E+08 | 2195 | -17.49 % | -87.25 % |
| 4_3_1 | 2.22E+08 | 79194 | 1.84E+08 | 2104 | -17.33 % | -97.34 % |
| 4_4_1 | 2.22E+08 | 46301 | 1.83E+08 | 1877 | -17.62 % | -95.95 % |
| 5_1_1 | 2.09E+08 | 49238 | 1.71E+08 | 1968 | -18.32 % | -96.00 % |
| 5_2_1 | 2.45E+08 | 19461 | 2.03E+08 | 2131 | -17.05 % | -89.05 % |
| 5_3_1 | 2.39E+08 | 51201 | 1.99E+08 | 2201 | -16.78 % | -95.70 % |
| 5_4_1 | 2.31E+08 | 101058 | 1.91E+08 | 2030 | -17.17 % | -97.99 % |
| 6_1_1 | 2.19E+08 | 48114 | 1.80E+08 | 6370 | -17.57 % | -86.76 % |
| 6_2_1 | 2.50E+08 | 39823 | 2.10E+08 | 2882 | -16.11 % | -92.76 % |
| 6_3_1 | 2.37E+08 | 42796 | 1.97E+08 | 2568 | -16.89 % | -94.00 % |
| 6_4_1 | 2.38E+08 | 45715 | 1.97E+08 | 2343 | -17.20 % | -94.87 % |
| 7_1_1 | 2.22E+08 | 36836 | 1.84E+08 | 2158 | -17.06 % | -94.14 % |
| 7_2_1 | 2.55E+08 | 12202 | 2.13E+08 | 2580 | -16.34 % | -78.86 % |
| 7_3_1 | 2.46E+08 | 46343 | 2.04E+08 | 2083 | -16.99 % | -95.51 % |
| 7_4_1 | 2.35E+08 | 40441 | 1.95E+08 | 2496 | -17.04 % | -93.83 % |
| 8_1_1 | 2.18E+08 | 20006 | 1.81E+08 | 2253 | -16.84 % | -88.74 % |
| 8_2_1 | 2.44E+08 | 42594 | 2.03E+08 | 2546 | -16.76 % | -94.02 % |
| 8_3_1 | 2.25E+08 | 35616 | 1.86E+08 | 2520 | -17.38 % | -92.92 % |
| 8_4_1 | 2.14E+08 | 78462 | 1.76E+08 | 2414 | -17.42 % | -96.92 % |
| 9_1_1 | 1.94E+08 | 50693 | 1.60E+08 | 2150 | -17.60 % | -95.76 % |
| 9_2_1 | 2.28E+08 | 23870 | 1.89E+08 | 2122 | -17.13 % | -91.11 % |
| 9_3_1 | 2.23E+08 | 26992 | 1.84E+08 | 2123 | -17.68 % | -92.13 % |
| 9_4_1 | 2.12E+08 | 19555 | 1.75E+08 | 2016 | -17.56 % | -89.69 % |

model incorporates a passenger mode choice and data about passenger demand in on-demand flights are not available, this section introduces some additional assumptions that follow the formulation of the model.

**Table 14:** Comparison of the computational results of the frequency assignment problem with different link parameters $\gamma$

| Instance Name | $\gamma = 1$ Number of arcs | Number of arcs with frequency =1 | Total frequency | $\gamma = 2$ Number of arcs | Number of arcs with frequency =1 | Total frequency |
|---|---|---|---|---|---|---|
| 2_1_0 | 3083 | 1597 | 7205 | 3900 | 2640 | 7756 |
| 2_2_0 | 3319 | 1675 | 8028 | 4197 | 2688 | 8827 |
| 2_3_0 | 3297 | 1708 | 8040 | 4151 | 2696 | 8762 |
| 2_4_0 | 3267 | 1641 | 7963 | 4117 | 2663 | 8725 |
| 3_1_0 | 3106 | 1596 | 7293 | 3948 | 2632 | 8086 |
| 3_2_0 | 3385 | 1741 | 8227 | 4230 | 2777 | 8818 |
| 3_3_0 | 3335 | 1715 | 8114 | 4214 | 2753 | 8918 |
| 3_4_0 | 3384 | 1711 | 8335 | 4237 | 2782 | 8846 |
| 4_1_0 | 3264 | 1641 | 7751 | 4119 | 2758 | 8346 |
| 4_2_0 | 3496 | 1732 | 8871 | 4417 | 2819 | 9727 |
| 4_3_0 | 3493 | 1751 | 8792 | 4389 | 2848 | 9609 |
| 4_4_0 | 3423 | 1727 | 8702 | 4274 | 2757 | 9336 |
| 5_1_0 | 3374 | 1691 | 8327 | 4269 | 2793 | 9112 |
| 5_2_0 | 3678 | 1795 | 9613 | 4595 | 2908 | 10219 |
| 5_3_0 | 3625 | 1755 | 9251 | 4488 | 2908 | 9889 |
| 5_4_0 | 3503 | 1676 | 8956 | 4484 | 2900 | 9950 |
| 6_1_0 | 3451 | 1715 | 8540 | 4345 | 2840 | 9239 |
| 6_2_0 | 3709 | 1826 | 9672 | 4646 | 2996 | 10134 |
| 6_3_0 | 3608 | 1758 | 9169 | 4549 | 2919 | 10129 |
| 6_4_0 | 3605 | 1760 | 9512 | 4458 | 2895 | 9898 |
| 7_1_0 | 3514 | 1769 | 8846 | 4311 | 2761 | 9162 |
| 7_2_0 | 3733 | 1880 | 9775 | 4630 | 2938 | 10218 |
| 7_3_0 | 3677 | 1786 | 9479 | 4492 | 2862 | 10002 |
| 7_4_0 | 3610 | 1744 | 9218 | 4452 | 2904 | 9799 |
| 8_1_0 | 3444 | 1721 | 8532 | 4336 | 2793 | 9353 |
| 8_2_0 | 3718 | 1818 | 9625 | 4669 | 2969 | 10472 |
| 8_3_0 | 3565 | 1738 | 9062 | 4431 | 2909 | 9574 |
| 8_4_0 | 3515 | 1749 | 8686 | 4411 | 2917 | 9269 |
| 9_1_0 | 3348 | 1699 | 8009 | 4229 | 2767 | 8748 |
| 9_2_0 | 3644 | 1805 | 9026 | 4554 | 2969 | 9727 |
| 9_3_0 | 3546 | 1778 | 8905 | 4422 | 2905 | 9424 |
| 9_4_0 | 3475 | 1742 | 8568 | 4370 | 2843 | 9366 |

$$\max \sum_{(o,d)\in M} \sum_{k\in KO} (x^l_{\bar{p}^k} R^l_{\bar{p}^k} + x^b_{\bar{p}^k} R^b_{\bar{p}^k}) - \sum_{d\in S} \sum_{k\in KO} C^k \cdot T^k_d \cdot y^k_d$$

$$s.t. \sum_{(o,d)\in M} \sum_{p\in P(o,d)} \sum_{k\in KO} Cap^k \cdot y_{\bar{p}^k} \le Cap_d, \forall d \in S, \tag{78}$$

$$\sum_{d\in S} \sum_{k\in KO} T^k_d \cdot y^k_d \le MaxHour^k, \forall k \in KO, \tag{79}$$

$$x^l_{\bar{p}^k} \le \frac{e^{V^l_{\bar{p}^k}}}{e^{V^l_{\bar{p}^0}} + \sum_{k\in KO} e^{V^l_{\bar{p}^k}}} D^l_p, \forall k \in KO, \forall p \in P, \tag{80}$$

$$x^b_{\bar{p}^k} \le \frac{e^{V^b_{\bar{p}^k}}}{e^{V^b_{\bar{p}^0}} + \sum_{k\in KO} e^{V^b_{\bar{p}^k}}} D^b_p, \forall k \in KO, \forall p \in P, \tag{81}$$

$$\sum_{o\in S} \sum_{p\in P_{o,d}} x^l_{\bar{p}^k} + x^b_{\bar{p}^k} \le Seat_k \cdot y^k_d, \forall k, \forall d \in S. \tag{82}$$

The main goal of this model is to determine an optimal distribution of vehicles for on-demand services among the selected airports. In particular, the vehicles for on-demand services in this thesis are 20-passenger aircraft, VLJs, and helicopters. 20-passenger aircraft and VLJs are faster and more comfortable than helicopters. On the other hand, because of their capability of vertical takeoff and landing, helicopters can access more places and use fewer airport resources. Therefore, the model assumes that helicopters occupy much less airport capacity than VLJs and VLJs occupy much less airport capacity than 20-passengers aircraft. Thus, constraint (78) limits the number of 20-passenger aircraft, VLJs, and helicopters at each airport.

The frequency assignment model integrates a passenger mode choice model. The mode choice model assumes that after passengers arrive at their destination airports, they can choose an automobile, a 20-passenger aircraft, a very light jet, or a helicopter as a transportation mode to reach their final destination. Due to the lack of trip data, at their destination airports, passengers are assumed to take a trip, the length of which is related to the destination commercial airports. Furthermore, the data on the TBS website of small commuter air carriers would be used to process the average lengths of the trips that passengers would take from their destination airports. Under these assumption, the mode choice model would calculate the proportion of business and leisure passengers that choose each transportation mode at each airport.

## *5.4 Implementation of the Rough Fleet Assignment Model*

This section illustrates the necessity of formulating a leg-based fleet assignment model and using a decomposition scheme. Furthermore, it discusses passengers' preference for departure times in the context of the fleet assignment model. Finally, it presents leg-based fleet assignment models that utilize passengers' time preference.

### 5.4.1 Leg-based rough fleet assignment model

In the itinerary-based rough fleet assignment model, because the number of paths is large, and each path corresponds to several itineraries, the number of itineraries is huge. Making the problem tractable necessitates formulating a leg-based rough fleet assignment model. In contrast to the itinerary-based rough fleet assignment model, the leg-based rough fleet assignment model approximates passenger demand and passenger revenue by allocating passenger demand on each itinerary to each flight leg of that itinerary and distributing the passenger revenue of each itinerary to each of its flight legs. To clarify such an allocation, Figure 19 illustrates the transition from an itinerary-based FAM to a leg-based FAM. In Figure 19, flight leg $AB$ appears in itineraries $ABC$, $ABE$, and $ABF$. Therefore, the demand $D_{AB}$ and the revenue $R_{AB}$ of flight leg $AB$ is related to the demand $D_{ABC}$ and the revenue $R_{ABC}$ of itinerary $ABC$, the demand $D_{ABE}$ and the revenue $R_{ABE}$ of itinerary $ABE$, and the demand $D_{ABF}$ and the revenue $R_{ABF}$ of itinerary $ABF$.

### 5.4.2 Decomposition scheme

The leg-based rough fleet assignment problem is still a large-scale integer programming problem that contains numerous equality constraints. Because it is extremely

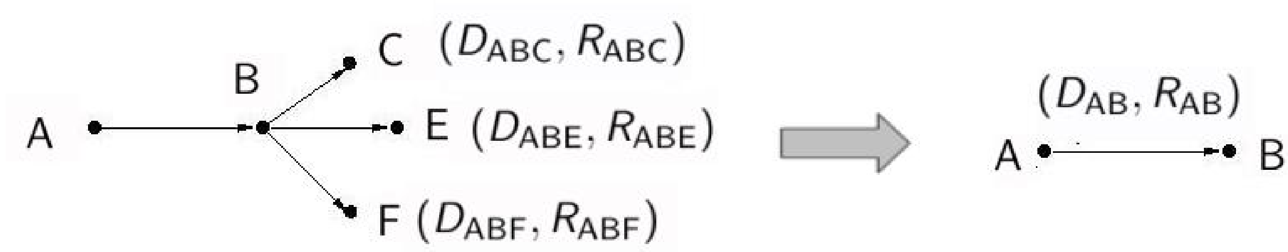



**Figure 19:** An illustration of the transition from an itinerary-based FAM to a leg-based FAM

difficult to solve, the leg-based rough fleet assignment problem is further decomposed into two subproblems. In the first subproblem, the candidate fleets consist of only the representative fleet, which is introduced for the frequency assignment model as the only candidate fleet for each arc. Furthermore, it determines a set of flight legs to which the representative fleet will be assigned such that the passenger revenue is maximized and flight frequencies are satisfied. Therefore, the first subproblem can be regarded as a time-slot assignment problem, illustrated in Figure 20, which shows that given the demand $D_{AB}$ and the frequency $F_{AB}$ of flight arc $AB$, the time-slot assignment subproblem will determine rough departure times for this flight arc such that frequency $F_{AB}$ is achieved and the revenue related to demand $R_{AB}$ is maximized. In the second subproblem, its candidate fleets consist of the original fleets. Furthermore, it determines a proper fleet for each flight leg. Therefore, the second subproblem can be regarded as a normal fleet assignment problem, illustrated in Figure 21, in which, given the rough departure times for flight arcs from station $A$ to station $B$, the fleet assignment subproblem determines a fleet for each flight leg, represented by legs with difference colors in the graph. Thus, these two subproblems decompose the entire rough fleet assignment problem in two dimensions, namely, time and fleet.

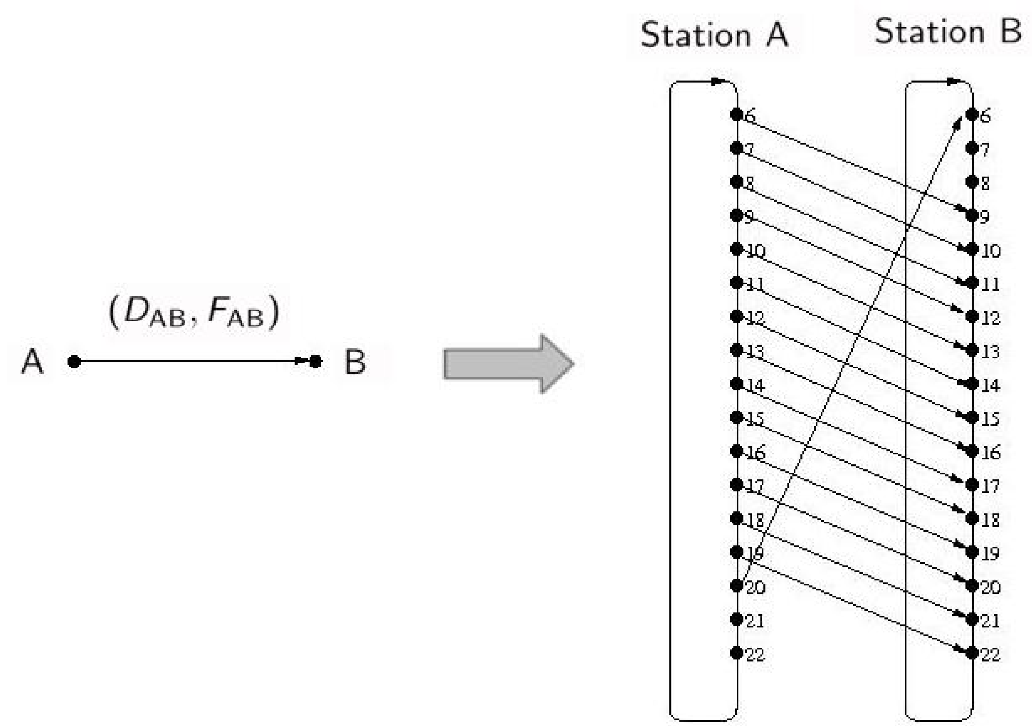



**Figure 20:** An illustration of time-slot assignment subproblem

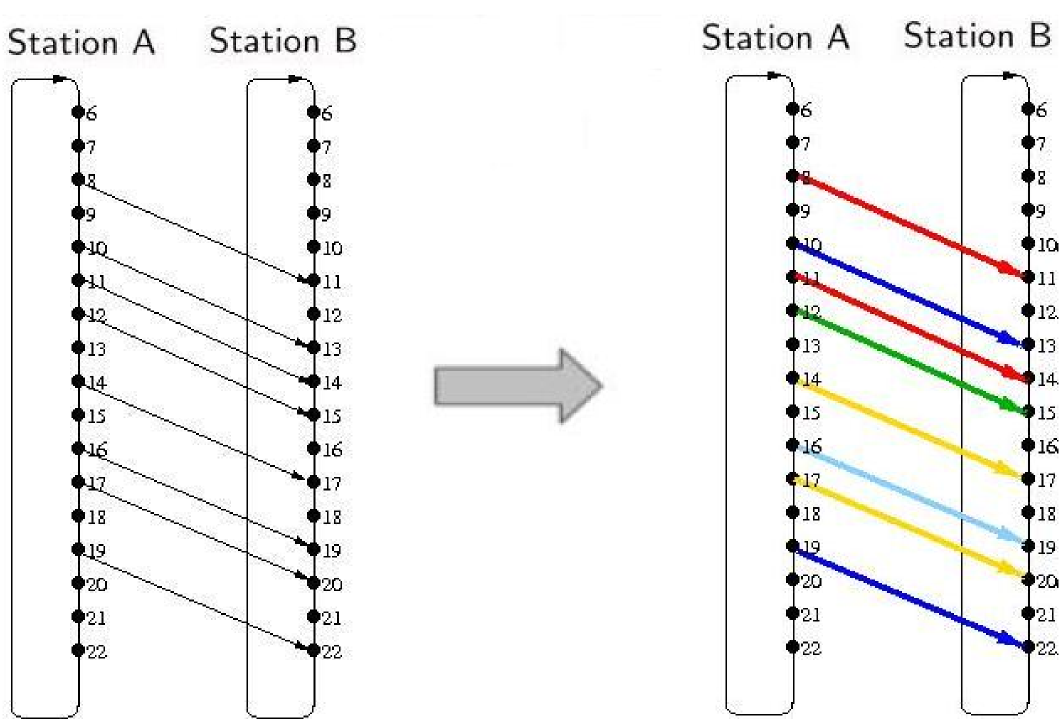



**Figure 21:** An illustration of fleet-assignment subproblem

### 5.4.3 Formulation of leg-based rough fleet assignment models

As mentioned in Chapter I, passengers have preferences for their departure times, which greatly influences planners' decisions regarding the departure times of their flight legs. Many researchers have extensively studied passengers' preference of time. For example, based on a survey of passengers' preference in different markets across the United States, Garrow et al. [52] analyzed the percentages of passengers that prefer different departure times. In addition, based on passenger booking data and airline schedule data, Coldren and Koppelman [45] formulated discrete choice models to analyze the influence of the time of day on passengers' itinerary choices. Overall, passengers' preferences for particular times follow a certain pattern. Its distribution has two distinct peaks, one in the morning and one in the evening. Figure 22, which is based on figures in Garrow et al. [52], illustrates one shape of the pattern.

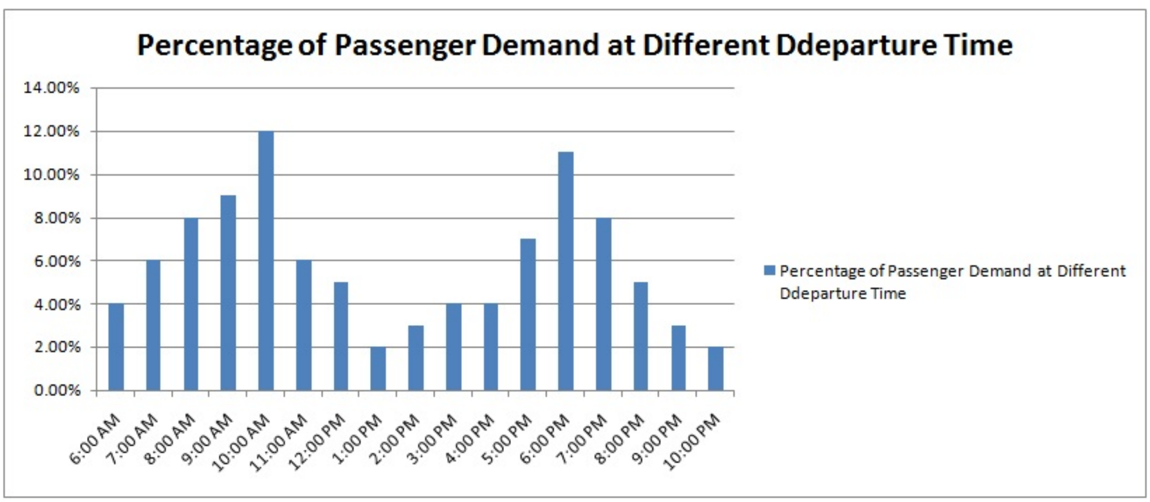



**Figure 22:** An illustration of passengers preference of departure time

In reality, understanding passengers' preference of time greatly helps schedule planners to make profitable schedules. To take advantage of passengers' preference of time, schedule planners have two strategies to use. One is from revenue management, that is, the schedule planners can set the itineraries preferred by more passengers at higher prices. The other one is from schedule planning, that is, they can schedule their fleets better such that the capacity of the fleet assigned to each leg matches the passenger demand of that leg better. For example, the larger fleet is assigned to itineraries preferred by more passengers. Since the rough fleet assignment model is a model that helps a schedule planner to assign a proper time and a proper fleet to a flight leg, these two strategies should be incorporated into the model.

Because the rough fleet assignment problem is decomposed into a time-slot assignment subproblem and a normal fleet assignment subproblem, the optimal solutions to these two subproblems should incorporates the two strategies, which necessitates some special design of the cost parameters in the objectives of the time-slot assignment and the fleet assignment models. To the time-slot assignment model, its optimal solutions could only incorporate the idea of the pricing strategy because the model includes exactly one fleet, namely, the representative fleet. To the fleet assignment model, its optimal solutions could incorporate both of the strategies. In both of these models, the flight legs between each city pair are set at different prices corresponding to their different departure times. For example, the flights depart at a time that is preferred by more passengers have higher prices. With this design, the optimal solution to the time-slot assignment model would choose more flights that depart at passengers' preferred times. In contrast with the time-slot assignment model, for each flight leg, the fleet assignment model also introduces a penalty term that is related to the difference between the fleet capacity and passenger demand of that leg. Thus, on the one hand, pricing differently will make the optimal solutions tend to assign bigger fleets for the flight legs that depart at passengers' preferred times. On the other hand, the penalty terms will ensure that not too big fleets are assigned. Balancing these two effects, the optimal solutions to the fleet assignment model will be a profitable schedule that is also reasonable in reality.

The following paragraphs will present the mathematical formulation of a leg-based fleet assignment model. In addition to the notations introduced before, the leg-based rough fleet assignment model requires some new notations, which are listed as below.

$R_{a_t}$ : Parameter that denotes passenger revenue of flight arc $a$ at time $t$.

$D_{a_t}$ : Parameter that estimates passenger demand of flight arc $a$ at time $t$.

$\alpha$ : Parameter that adjusts penalty that is related to the difference between passenger demand and fleet capacity of each flight arc.

$z_{a_t}$ : Variable that denotes the number of passengers that fly arc $a$ at time $t$.

$v_{a_t}$ : Variable that denotes the difference between passenger demand and fleet capacity of flight leg $v_{a_t}$.

With the new notations, the formulation for the leg-based rough fleet assignment model is presented as follows.

$$\max \quad \sum_{a\in A}\sum_{t\in T} R_{a_t} z_{a_t} - \sum_{a\in A}\sum_{t\in T}\sum_{k\in K} C_{a_t^k} x_{a_t^k} - \alpha\Big(\sum_{a\in A}\sum_{t\in T} R_{a_t} v_{a_t}\Big)$$

$$s.t. \quad \sum_{k\in K}\sum_{a_t^k\in L_{s,t-}^k} Cap^k \cdot x_{a_t^k} + \sum_{k\in K}\sum_{a_t^k\in L_{s,t+}^k} Cap^k \cdot x_{a_t^k} \leq Cap_{s,t}, \forall s\in S, \forall t\in T, \tag{83}$$

$$\sum_{a_t^k\in L_{a,t}^k} x_\ell^k + y_{s,t}^k = \sum_{a_t^k\in L_{a,t+1}^k} x_\ell^k + y_{s,t+1}^k, \forall s\in S, \forall t\in T, \tag{84}$$

$$\sum_{t\in T}\sum_{k\in K} x_{a_t^k} \geq Freq(a), \forall a\in A, \tag{85}$$

$$z_{a_t} \leq \sum_k Cap_k \cdot x_{a_t^k}, \forall t\in T, \forall a\in A, \tag{86}$$

$$\sum_{t\in T} z_{a_t} \leq D_a, \forall a\in A, \tag{87}$$

$$\Big|\sum_{k\in K} Cap^k x_{a_t^k} - D_{a_t}\Big| \leq v_{a_t}, \forall a\in A, \forall t\in T, \tag{88}$$

$$x_{a_t^k} \in \{0,1\}, \forall a\in A, t\in T, \tag{89}$$

$$y_{s,t}^k \geq 0, \text{integer}, \forall s\in S, t\in T, \tag{90}$$

$$z_{a_t} \geq 0, \text{integer}, \forall a\in A, t\in T. \tag{91}$$

The objective of the model is to maximize passenger revenue minus the sum of operating costs and penalty costs. Constraint (83) imposes an upper bound of the total number of arrival and departure flights at each station during each hour. Constraint (84) is a flow balance constraint that ensures that the number of incoming flights equals that of the outgoing flights. Constraint (85) ensures that the leg frequency requirement is satisfied. Constraint (86) guarantees that the capacity of a leg is greater than the number of passengers assigned to that leg. Constraint (87) ensures that passenger demand of a flight arc is greater than the total number of passengers assigned to flight legs that corresponding to that flight arc. Constraint

(88) guarantees that for each flight leg $a_t$, $v_{a_t}$ is not less than the absolute difference between passenger demand and fleet capacity of that leg.

This section decomposes the leg-based FAM problem into a time-slot assignment subproblem and a fleet assignment subproblem. Given the frequency of using the representative fleet between each city pair that is determined in the frequency assignment problem, the time-slot subproblem determines the proper number of time slots so that the frequencies are satisfies. In fact, different from the leg-based FAM, the time-slot assignment model presented below does not include the penalty costs, and it uses exactly one fleet. The mathematical formulation of the time-slot assignment model is presented as follows.

$$\begin{aligned}
\max \quad & \sum_{a\in A}\sum_{t\in T} R_{a_t} z_{a_t} - \sum_{a\in A}\sum_{t\in T} C_{a_t} x_{a_t} \\
s.t. \quad & \sum_{a_t \in L_{a,t-}} Cap \cdot x_{a_t} + \sum_{a_t^k \in L_{a,t+}} Cap \cdot x_{a_t} \leq Cap_{s,t}, \forall s \in S, \forall t \in T, && (92)\\
& \sum_{a_t \in L_{a,t}} x_\ell + y_{s,t} = \sum_{a_t \in L_{a,t+1}} x_\ell + y_{s,t+1}, \forall s \in S, \forall t \in T, && (93)\\
& \sum_{t\in T}\sum_{k\in K} x_{a_t} = Freq(a), \forall a \in A, && (94)\\
& z_{a_t} \leq Seat \cdot x_{a_t}, \forall t \in T, \forall a \in A, && (95)\\
& \sum_{t\in T} z_{a_t} \leq D_a, \forall a \in A, && (96)\\
& x_{a_t} \in 0, 1, \forall a \in A, t \in T, && (97)\\
& y_{s,t} \in Z^+, \forall s \in S, t \in T, && (98)\\
& z_{a_t} \in Z^+, \forall a \in A, t \in T. && (99)
\end{aligned}$$

The time-slot subproblem determines the time slots for flight leg. In other words, it determines the value of each variable $x_{a_t}$, for all $a \in A$ and for all $t \in T$. Given the values of these $x_{a_t}$'s, the fleet assignment subproblem determines the fleet of each flight leg. The fleet assignment subproblem is represented as follows.

$$\max \quad \sum_{a \in A} \sum_{t \in T} R_{a_t} z_{a_t} - \sum_{a \in A} \sum_{t \in T} \sum_{k \in K} C_{a_t^k} x_{a_t^k} - \alpha (\sum_{a \in A} \sum_{t \in T} R_{a_t} v_{a_t})$$

$$s.t. \quad \sum_{k \in K} \sum_{a_t^k \in L_{a,t-}^k} Cap^k \cdot x_{a_t^k} + \sum_{k \in K} \sum_{a_t^k \in L_{a,t+}^k} Cap^k \cdot x_{a_t^k} \leq Cap_{s,t}, \forall s \in S, \forall t \in T, \quad (100)$$

$$\sum_{a_t^k \in L_{a,t}^k} x_\ell^k + y_{s,t}^k = \sum_{a_t^k \in L_{a,t+1}^k} x_\ell^k + y_{s,t+1}^k, \forall s \in S, \forall t \in T, \quad (101)$$

$$\sum_{k \in K} x_{a_t^k} = x_a, \forall a, \forall t \in T \quad (102)$$

$$z_{a_t} \leq \sum_k Cap_k \cdot x_{a_t^k}, \forall t \in T, \forall a \in A, \quad (103)$$

$$\sum_{t \in T} z_{a_t} \leq D_a, \forall a \in A, \quad (104)$$

$$|\sum_{k \in K} C_{a_t^k} x_{a_t^k} - D_{a_t}| \leq v_{a_t}, \forall a \in A, \forall t \in T, \quad (105)$$

$$x_{a_t^k} \in 0, 1, \forall a \in A, t \in T, \quad (106)$$

$$y_{s,t}^k \in Z^+, \forall s \in S, t \in T, \quad (107)$$

$$z_{a_t} \in Z^+, \forall a \in A, t \in T. \quad (108)$$

The previous paragraphs presents a subproblem scheme in which a rough fleet assignment problem is decomposed into a time-slot assignment subproblem and a fleet assignment subproblem. However, many instances of the fleet assignment subproblems created in this chapter still take tremendous time to solve. To reduce the solution time of the fleet assignment subproblem, the following paragraphs explain a further subproblem scheme.

The instances of the fleet assignment subproblems created in this chapter include a large number of flight variables that makes the instance very hard to solve. On the other hand, the number of flight variables in the subproblem is proportional to the number of fleets used in the subproblem. The instances of the fleet assignment subproblem addressed in this chapter include five fleets. Therefore, reducing the number of fleets used in the fleet assignment subproblem will make the subproblems easier to solve. However, the instances involving five fleets still need to be addressed. To address the difficulties, this thesis extends the idea of using representative fleets in the frequency assignment subproblem and the time-slot assignment subproblem to solve the fleet assignment problem. The five fleets are aggregated into three fleet categories, a big fleet, a medium fleet, and a small fleet categories. Furthermore, a fleet assignment subproblem is decomposed into a fleet-category assignment subsubproblem and a normal fleet assignment subsubproblem. In the fleet-category assignment subsubproblem, each flight leg is first assigned with a fleet category, and in the normal fleet assignment subsubproblem, it is assigned with a fleet in that category. Figure 23 illustrates the entire subproblem scheme of the rough fleet assignment problem discussed in this section. In addition, Table 15 summarizes the instances of the fleet assignment problem that are created in this thesis.

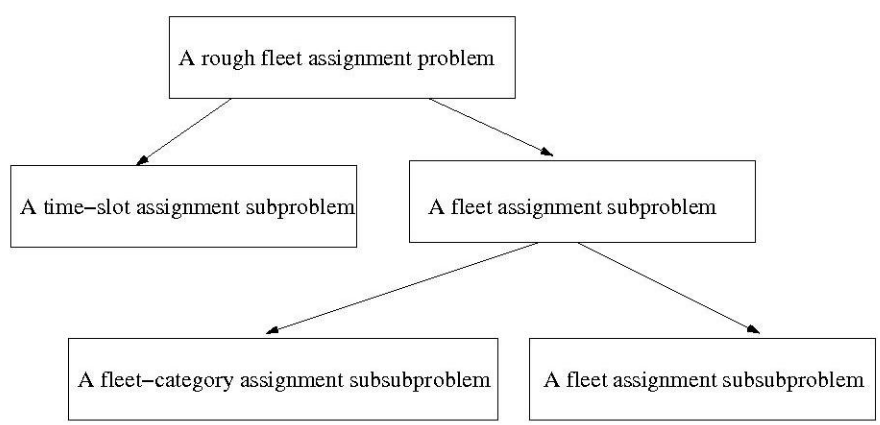



**Figure 23:** An illustration of an entire subproblem scheme of the rough fleet assignment problem

**Table 15:** A summary of instances of the rough fleet assignment problem created in this section

| Instances of rough fleet assignment problem | |
|---|---|
| The number of flights variables $x_{a_t^k}$'s | [298,000, 634,000] |
| The number of balance constraints | 17,000 |
| The number of all the variables | [494,000, 634,000] |
| The number of all the constraints | [206,000, 235,000] |
| **Instances of time-slot assignment subproblem** | |
| The number of flights variables $x_{a_t}$'s | [59,000, 69,000] |
| The number of balance constraints | 3,400 |
| The number of all the variables | [182,000, 209,000] |
| The number of all the constraints | [74,000, 85,000] |
| **Instances of fleet assignment subproblem** | |
| The number of flights variables $x_{a_t^k}$'s | [30,000, 44,000] |
| The number of penalty variables $x_{a_t^k}$'s | [6,000, 9,000] |
| The number of balance constraints | [1,800, 2,100] |
| The number of all the variables | [44,000, 64,000] |
| The number of all the constraints | [27,000, 37,000] |

Table 16 presents computational results of the instances created for the time-slot assignment subproblem in this section. As show in Table 16, it takes less than 3 hours to get a solution with optimality gap within 4%. However, it takes much longer time to reduce the optimality gap to 3%. In fact, for some instances, it takes more than 20 hours to reduce the optimality gap to 3%.

Tables 17 and 18 present computational results of two sets of instances created for the fleet assignment subproblem in this section. In fact, this thesis also creates instances that are larger than the instances in this section and are used for deriving flight schedules in Chapter VI. In these tables, the computation results of the fleet assignment subproblem without using further subproblem scheme is used as a base line.

In Table 17, without using subsubproblem approache, the solution times of the instances of the fleet assignment subproblem range between 6 and 50 hours. However, for each instance, using subsubproblem approach can reduce the solution time by at least 50%. To most of the instances, the reduction could be more than 80%. For

Table 16: Computational result of instances of the time-slot assignment subproblem

| | Optimality Gap Within 4% | Optimality Gap Within 3% |
|---|---|---|
| Instance Name | Solution Time (CPU Seconds) | Solution Time (CPU Seconds) |
| 6_1_0 | 3391 | 25444 |
| 6_2_0 | 9372 | 54898 |
| 6_3_0 | 2575 | 37700 |
| 6_4_0 | 312 | 85526 |
| 7_1_0 | 6405 | 63558 |
| 7_2_0 | 1159 | 62579 |
| 7_3_0 | 8945 | 63132 |
| 7_4_0 | 599 | 7546 |
| 8_1_0 | 2174 | 60201 |
| 8_2_0 | 6863 | 59776 |
| 8_3_0 | 4283 | 95551 |
| 8_4_0 | 5378 | 69895 |
| 9_1_0 | 3266 | 35424 |
| 9_2_0 | 4747 | 73830 |
| 9_3_0 | 3299 | 67291 |
| 9_4_0 | 211 | 2788 |
| 6_1_1 | 413 | 4486 |
| 6_2_1 | 579 | 8150 |
| 6_3_1 | 327 | 5761 |
| 6_4_1 | 489 | 7115 |
| 7_1_1 | 374 | 3865 |
| 7_2_1 | 719 | 8892 |
| 7_3_1 | 234 | 4229 |
| 7_4_1 | 185 | 3849 |
| 8_1_1 | 221 | 2492 |
| 8_2_1 | 682 | 6461 |
| 8_3_1 | 449 | 4135 |
| 8_4_1 | 379 | 31340 |
| 9_1_1 | 388 | 3514 |
| 9_2_1 | 368 | 13140 |
| 9_3_1 | 3695 | 3695 |
| 9_4_1 | 183 | 1730 |

example, for instance 3_2_1_3, without using subsubproblem approach, solving the instance takes more than 324 hours, but using subsubproblem approach, solving the instance takes less than 5 hours. Furthermore, for each instance, using subsubproblem approach does not decrease the objective by more than 3.27%.

In Table 18, without using subsubproblem apparoach, the solution times of the

**Table 17:** Computational result 1 of instances of the fleet assignment subproblem

| | Not using subsubproblems | | Using subsubproblems | | Improvement | |
|---|---|---|---|---|---|---|
| Instance Name | Objective ($10^8$) | Solution Time (CPU Seconds) | Objective ($10^8$) | Solution Time (CPU Seconds) | Objective | Solution Time |
| 2_1_0_3 | 1.61383 | 39950 | 1.60038 | 8031 | -0.83 % | -79.90 % |
| 2_2_0_3 | 1.85014 | 84179 | 1.83708 | 17519 | -0.71 % | -79.19 % |
| 2_3_0_3 | 1.83588 | 68873 | 1.82161 | 13624 | -0.78 % | -80.22 % |
| 2_4_0_3 | 1.80641 | 59082 | 1.79088 | 13416 | -0.86 % | -77.29 % |
| 3_1_0_3 | 1.63702 | 44849 | 1.62342 | 9798 | -0.83 % | -78.15 % |
| 3_2_0_3 | 1.91484 | 82801 | 1.89968 | 17779 | -0.79 % | -78.53 % |
| 3_3_0_3 | 1.91032 | 104059 | 1.89675 | 25025 | -0.71 % | -75.95 % |
| 3_4_0_3 | 1.93044 | 107368 | 1.91702 | 19992 | -0.70 % | -81.38 % |
| 4_1_0_3 | 1.83767 | 23593 | 1.77766 | 5610 | -3.27 % | -76.22 % |
| 4_2_0_3 | 2.16662 | 66840 | 2.15037 | 14514 | -0.75 % | -78.29 % |
| 4_3_0_3 | 2.13880 | 42584 | 2.12239 | 13927 | -0.77 % | -67.30 % |
| 4_4_0_3 | 2.13316 | 43302 | 2.11532 | 12351 | -0.84 % | -71.48 % |
| 5_1_0_3 | 2.01220 | 28345 | 1.99184 | 10620 | -1.01 % | -62.53 % |
| 5_2_0_3 | 2.35028 | 95714 | 2.33169 | 20189 | -0.79 % | -78.91 % |
| 5_3_0_3 | 2.30506 | 99087 | 2.28876 | 14648 | -0.71 % | -85.22 % |
| 5_4_0_3 | 2.22484 | 50605 | 2.20525 | 18272 | -0.88 % | -63.89 % |
| 2_1_1_3 | 1.73357 | 89905 | 1.71345 | 25520 | -1.16 % | -71.61 % |
| 2_2_1_3 | 1.98419 | 133242 | 1.96103 | 36981 | -1.17 % | -72.25 % |
| 2_3_1_3 | 1.96466 | 157965 | 1.94352 | 36469 | -1.08 % | -76.91 % |
| 2_4_1_3 | 1.94117 | 119686 | 1.91601 | 34296 | -1.30 % | -71.35 % |
| 3_1_1_3 | 1.75894 | 76053 | 1.73608 | 21543 | -1.30 % | -71.67 % |
| 3_2_1_3 | 2.05730 | 168520 | 2.03160 | 15247 | -1.25 % | -90.95 % |
| 3_3_1_3 | 2.05005 | 176099 | 2.02308 | 18133 | -1.32 % | -89.70 % |
| 3_4_1_3 | 2.07022 | 71214 | 2.04502 | 22144 | -1.22 % | -68.90 % |
| 4_1_1_3 | 1.96570 | 61613 | 1.93722 | 18510 | -1.45 % | -69.96 % |
| 4_2_1_3 | 2.32363 | 135512 | 2.29647 | 28948 | -1.17 % | -78.64 % |
| 4_3_1_3 | 2.28574 | 134978 | 2.25680 | 32633 | -1.27 % | -75.82 % |
| 4_4_1_3 | 2.28212 | 120076 | 2.25537 | 31696 | -1.17 % | -73.60 % |
| 5_1_1_3 | 2.15718 | 66044 | 2.13079 | 15118 | -1.22 % | -77.11 % |
| 5_2_1_3 | 2.51469 | 167052 | 2.48812 | 41025 | -1.06 % | -75.44 % |
| 5_3_1_3 | 2.46204 | 171689 | 2.43308 | 81701 | -1.18 % | -52.41 % |
| 5_4_1_3 | 2.38303 | 135425 | 2.35635 | 49666 | -1.12 % | -63.33 % |

instances of the fleet assignment subproblem range between 5 and 60 hours, and using subsubproblem approach, the solution times of the instances range between 1 and 16 hours. For all the instances except only three of the instances, the run time performance does improve greatly. Indeed, for all the instances except only three

Table 18: Computational result 2 of instances of the fleet assignment subproblem

| Instance Name | Not using subsubproblems Objective ($10^8$) | Solution Time (CPU Seconds) | Using subsubproblems Objective ($10^8$) | Solution Time (CPU Seconds) | Improvement Objective | Solution Time |
|---|---|---|---|---|---|---|
| 2_1_0_4 | 1.61796 | 18683 | 1.60705 | 4901 | -0.67 % | -73.77 % |
| 2_2_0_4 | 1.85106 | 28798 | 1.83609 | 10725 | -0.81 % | -62.76 % |
| 2_3_0_4 | 1.83491 | 27531 | 1.82197 | 6860 | -0.71 % | -75.08 % |
| 2_4_0_4 | 1.80604 | 22982 | 1.78930 | 6742 | -0.93 % | -70.66 % |
| 3_1_0_4 | 1.63647 | 21297 | 1.62197 | 13126 | -0.89 % | -38.37 % |
| 3_2_0_4 | 1.91566 | 30289 | 1.90050 | 21203 | -0.79 % | -30.00 % |
| 3_3_0_4 | 1.91058 | 56311 | 1.89518 | 6585 | -0.81 % | -88.31 % |
| 3_4_0_4 | 1.93118 | 61958 | 1.91641 | 7835 | -0.76 % | -87.35 % |
| 4_1_0_4 | 1.83772 | 47409 | 1.82068 | 5411 | -0.93 % | -88.59 % |
| 4_2_0_4 | 2.16441 | 84079 | 2.15037 | 17799 | -0.65 % | -78.83 % |
| 4_3_0_4 | 2.14285 | 65606 | 2.12239 | 12752 | -0.95 % | -80.56 % |
| 4_4_0_4 | 2.12762 | 77293 | 2.11670 | 33092 | -0.51 % | -57.19 % |
| 5_1_0_4 | 2.01560 | 27551 | 2.00059 | 27332 | -0.74 % | -0.79 % |
| 5_2_0_4 | 2.35102 | 119483 | 2.33575 | 44404 | -0.65 % | -62.84 % |
| 5_3_0_4 | 2.30285 | 68565 | 2.28045 | 35481 | -0.97 % | -48.25 % |
| 5_4_0_4 | 2.22364 | 106230 | 2.20925 | 37514 | -0.65 % | -64.69 % |
| 2_1_1_4 | 1.73357 | 37123 | 1.71353 | 9140 | -1.16 % | -75.38 % |
| 2_2_1_4 | 1.98188 | 78875 | 1.96170 | 19061 | -1.02 % | -75.83 % |
| 2_3_1_4 | 1.96466 | 102729 | 1.94475 | 13699 | -1.01 % | -86.66 % |
| 2_4_1_4 | 1.93955 | 98215 | 1.91449 | 14395 | -1.29 % | -85.34 % |
| 3_1_1_4 | 1.75746 | 69806 | 1.73916 | 56445 | -1.04 % | -19.14 % |
| 3_2_1_4 | 2.05456 | 104478 | 2.03379 | 19017 | -1.01 % | -81.80 % |
| 3_3_1_4 | 2.04667 | 94138 | 2.02308 | 56316 | -1.15 % | -40.18 % |
| 3_4_1_4 | 2.06988 | 100421 | 2.04412 | 21753 | -1.24 % | -78.34 % |
| 4_1_1_4 | 1.96479 | 57272 | 1.93789 | 27830 | -1.37 % | -51.41 % |
| 4_2_1_4 | 2.31952 | 107542 | 2.29536 | 50219 | -1.04 % | -53.30 % |
| 4_3_1_4 | 2.28457 | 106486 | 2.25787 | 28357 | -1.17 % | -73.37 % |
| 4_4_1_4 | 2.27851 | 121846 | 2.25412 | 43529 | -1.07 % | -64.28 % |
| 5_1_1_4 | 2.15355 | 70673 | 2.12846 | 17750 | -1.17 % | -74.88 % |
| 5_2_1_4 | 2.51226 | 216306 | 2.48867 | 38865 | -0.94 % | -82.03 % |
| 5_3_1_4 | 2.45718 | 173213 | 2.43524 | 37063 | -0.89 % | -78.60 % |
| 5_4_1_4 | 2.37758 | 213090 | 2.35421 | 30315 | -0.98 % | -85.77 % |

of the instances, using subsubproblem approach reduces the run time by more than 40%. On the other hand, for each instance, using subsubproblem approach does not decrease the objective by more than 1.37%.

The fleet assignment model contains a penalty parameter $\alpha$, which adjusts the penalty that is related to the difference between passenger demand and fleet capacity of each flight leg. For testing the impact of the penalty parameters on the fleet assignment model, two sets of instances are created, and Tables 19 and 20 present related computational results. In fact, the instances of the frequency assignment problem in Table 19 use only fleets 1,2,3,4, and 5, and the instances of the frequency assignment problem in Table 20 use only fleets $1^*, 2^*, 3^*, 4^*$, and $5^*$. Table 19 shows that both the proportion of flights using fleets 1 and 2 and the proportion of flights using fleets 4 and 5 increases when the penalty parameter $\alpha$ increases from 0.1 to 0.3, Table 20 shows that both the proportion of flights using fleets $1^*$ and $2^*$ and the proportion of flights using fleets $4^*$ and $5^*$ increases when the penalty parameter $\alpha$ increases from 0.1 to 0.3. In other words, Tables 19 and 20 show that when the penalty parameter $\alpha$ increases, the proportion of flights using either big fleet or small fleet increase. One explanation is that passenger demand changes with respect to time, in peak times, passenger demand is very high, while other times, passenger demand is very low, and that the fleet assignment model determines fleet assignments that match passenger demand better when the penalty parameter $\alpha$ is higher.

**Table 19:** Computational result 1 of instances of the fleet assignment problem

| Instance Name | $\alpha = 0.1$ | | | $\alpha = 0.3$ | | |
|---|---|---|---|---|---|---|
| | Flights of fleets 1 and 2 | Flights of fleet 3 | Flights of fleets 4 and 5 | Flights of fleets 1 and 2 | Flights of fleet 3 | Flights of fleets 4 and 5 |
| 2_1_0 | 22.29 % | 34.13 % | 43.58 % | 23.19 % | 28.20 % | 48.62 % |
| 2_2_0 | 22.68 % | 32.32 % | 45.00 % | 23.91 % | 26.41 % | 49.69 % |
| 2_3_0 | 24.39 % | 31.30 % | 44.31 % | 25.44 % | 25.95 % | 48.61 % |
| 2_4_0 | 24.32 % | 32.71 % | 42.98 % | 24.75 % | 27.11 % | 48.14 % |
| 3_1_0 | 23.88 % | 33.05 % | 43.07 % | 24.52 % | 26.47 % | 49.01 % |
| 3_2_0 | 24.05 % | 31.82 % | 44.13 % | 24.79 % | 26.30 % | 48.91 % |
| 3_3_0 | 24.20 % | 31.84 % | 43.96 % | 24.72 % | 26.36 % | 48.92 % |
| 3_4_0 | 24.26 % | 31.88 % | 43.86 % | 24.95 % | 26.30 % | 48.75 % |
| 4_1_0 | 23.55 % | 33.14 % | 43.31 % | 24.43 % | 27.12 % | 48.45 % |
| 4_2_0 | 24.08 % | 31.70 % | 44.22 % | 25.38 % | 26.38 % | 48.24 % |
| 4_3_0 | 23.90 % | 31.88 % | 44.22 % | 25.08 % | 26.11 % | 48.81 % |
| 4_4_0 | 22.61 % | 31.72 % | 45.67 % | 24.31 % | 26.08 % | 49.61 % |
| 5_1_0 | 22.14 % | 32.03 % | 45.83 % | 23.55 % | 26.90 % | 49.54 % |
| 5_2_0 | 23.22 % | 31.41 % | 45.37 % | 24.88 % | 26.41 % | 48.71 % |
| 5_3_0 | 23.59 % | 32.39 % | 44.02 % | 25.49 % | 26.20 % | 48.32 % |
| 5_4_0 | 22.33 % | 32.18 % | 45.49 % | 24.32 % | 26.32 % | 49.36 % |
| 6_1_0 | 21.59 % | 32.95 % | 45.46 % | 23.54 % | 27.09 % | 49.37 % |
| 6_2_0 | 23.34 % | 31.35 % | 45.31 % | 25.35 % | 25.91 % | 48.73 % |
| 6_3_0 | 23.50 % | 32.03 % | 44.47 % | 24.81 % | 26.69 % | 48.50 % |
| 6_4_0 | 22.39 % | 33.24 % | 44.37 % | 24.48 % | 26.56 % | 48.96 % |
| 7_1_0 | 23.41 % | 32.93 % | 43.66 % | 24.96 % | 26.72 % | 48.32 % |
| 7_2_0 | 24.08 % | 31.79 % | 44.13 % | 26.03 % | 26.06 % | 47.91 % |
| 7_3_0 | 23.97 % | 32.09 % | 43.94 % | 25.41 % | 26.73 % | 47.86 % |
| 7_4_0 | 23.26 % | 31.95 % | 44.79 % | 25.10 % | 26.30 % | 48.60 % |
| 8_1_0 | 23.60 % | 33.61 % | 42.79 % | 25.11 % | 26.84 % | 48.05 % |
| 8_2_0 | 22.73 % | 31.84 % | 45.43 % | 24.84 % | 26.16 % | 49.00 % |
| 8_3_0 | 23.42 % | 31.42 % | 45.16 % | 25.21 % | 25.74 % | 49.05 % |
| 8_4_0 | 22.31 % | 32.11 % | 45.58 % | 24.59 % | 26.02 % | 49.39 % |
| 9_1_0 | 22.38 % | 32.48 % | 45.14 % | 23.86 % | 26.63 % | 49.51 % |
| 9_2_0 | 23.76 % | 31.93 % | 44.31 % | 25.40 % | 26.19 % | 48.41 % |
| 9_3_0 | 24.03 % | 31.53 % | 44.44 % | 25.39 % | 26.10 % | 48.51 % |
| 9_4_0 | 22.33 % | 32.44 % | 45.23 % | 24.21 % | 26.20 % | 49.59 % |

**Table 20:** Computational result 2 of instances of the fleet assignment problem

| | $\alpha = 0.1$ | | | $\alpha = 0.3$ | | |
|---|---|---|---|---|---|---|
| Instance Name | Flights of fleets 1*, 2* | Flights of fleet 3* | Flights of fleets 4*, 5* | Flights of fleets 1*, 2* | Flights of fleet 3* | Flights of fleets 4*, 5* |
| 2_1_1 | 22.08 % | 32.68 % | 45.24 % | 23.64 % | 25.97 % | 50.39 % |
| 2_2_1 | 22.18 % | 32.28 % | 45.53 % | 24.54 % | 24.91 % | 50.55 % |
| 2_3_1 | 22.44 % | 31.45 % | 46.1 % | 24.54 % | 24.96 % | 50.5 % |
| 2_4_1 | 22.21 % | 32.76 % | 45.03 % | 24.39 % | 25.76 % | 49.85 % |
| 3_1_1 | 21.91 % | 32.63 % | 45.46 % | 23.82 % | 25.62 % | 50.56 % |
| 3_2_1 | 22.01 % | 32.33 % | 45.66 % | 24.76 % | 24.83 % | 50.4 % |
| 3_3_1 | 22.43 % | 31.93 % | 45.64 % | 24.69 % | 24.74 % | 50.57 % |
| 3_4_1 | 21.77 % | 32.64 % | 45.59 % | 24.39 % | 25.16 % | 50.45 % |
| 4_1_1 | 21.57 % | 33.32 % | 45.11 % | 23.91 % | 26.85 % | 49.25 % |
| 4_2_1 | 22.71 % | 31.79 % | 45.49 % | 25.18 % | 24.92 % | 49.9 % |
| 4_3_1 | 22.65 % | 31.30 % | 46.05 % | 25.67 % | 23.91 % | 50.42 % |
| 4_4_1 | 22.11 % | 32.24 % | 45.65 % | 25.27 % | 24.72 % | 50.01 % |
| 5_1_1 | 21.82 % | 32.98 % | 45.21 % | 24.02 % | 26 % | 49.98 % |
| 5_2_1 | 22.89 % | 31.85 % | 45.26 % | 25.60 % | 25.13 % | 49.27 % |
| 5_3_1 | 23.08 % | 31.64 % | 45.28 % | 25.97 % | 24.95 % | 49.09 % |
| 5_4_1 | 22.61 % | 32.06 % | 45.33 % | 25.37 % | 24.79 % | 49.84 % |
| 6_1_1 | 22.36 % | 32.20 % | 45.44 % | 25.14 % | 24.71 % | 50.16 % |
| 6_2_1 | 23.16 % | 32.01 % | 44.82 % | 26.08 % | 24.48 % | 49.43 % |
| 6_3_1 | 22.76 % | 31.76 % | 45.48 % | 25.66 % | 24.79 % | 49.55 % |
| 6_4_1 | 22.94 % | 32.58 % | 44.48 % | 26.03 % | 25.29 % | 48.68 % |
| 7_1_1 | 22.52 % | 33.13 % | 44.35 % | 25.30 % | 25.52 % | 49.17 % |
| 7_2_1 | 23.56 % | 31.59 % | 44.84 % | 26.68 % | 24.36 % | 48.96 % |
| 7_3_1 | 23.21 % | 32.07 % | 44.72 % | 26.35 % | 24.87 % | 48.79 % |
| 7_4_1 | 23.27 % | 32.36 % | 44.37 % | 25.94 % | 24.99 % | 49.06 % |
| 8_1_1 | 22.18 % | 33.29 % | 44.53 % | 25.38 % | 25.3 % | 49.33 % |
| 8_2_1 | 23.25 % | 31.79 % | 44.96 % | 26.24 % | 24.63 % | 49.13 % |
| 8_3_1 | 22.30 % | 32.08 % | 45.63 % | 25.67 % | 24.46 % | 49.86 % |
| 8_4_1 | 22.08 % | 32.83 % | 45.09 % | 24.85 % | 25.71 % | 49.44 % |
| 9_1_1 | 21.83 % | 32.71 % | 45.46 % | 24.30 % | 25.77 % | 49.93 % |
| 9_2_1 | 22.81 % | 31.68 % | 45.5 % | 25.72 % | 24.63 % | 49.65 % |
| 9_3_1 | 22.62 % | 31.07 % | 46.31 % | 25.13 % | 24.1 % | 50.77 % |
| 9_4_1 | 22.10 % | 32.52 % | 45.38 % | 24.86 % | 25.01 % | 50.13 % |

## 5.5 Implementation of the timetable model

This section discusses the implementation and solution of the timetable model, which is formulated as follows.

$$\min \sum_{k\in K} C^k u^k - \beta \sum_{\ell_1^{(i)}\ell_2^{(j)}\in W} R_{\ell_1^{(i)}\ell_2^{(j)}} x_{\ell_1^{(i)}\ell_2^{(j)}}$$

$$s.t. \sum_{j\in TW} x_{\ell^{(j)}} = 1, \forall \ell \in L, \tag{109}$$

$$\sum_{j\in TW} \sum_{\ell^{(j)}\in L_{s,t-}^k} x_{\ell^{(j)}} + y_{s,t-}^k = \sum_{j\in TW} \sum_{\ell^{(j)}\in L_{s,t+}^k} x_{\ell^{(j)}} + y_{s,t+}^k, \forall s\in S, \forall t\in T_s, \forall k\in K, \tag{110}$$

$$\sum_{s\in S} y_{s,t_c}^k + \sum_{j\in TW} \sum_{\ell^{(j)}\in CL^k} x_{\ell^{(j)}} \le u^k, \forall k\in K, \tag{111}$$

$$u^k \le U^k, \forall k \in K, \tag{112}$$

$$x_{\ell_1^{(i)}\ell_2^{(j)}} \le x_{\ell_1^{(i)}}, \forall \ell_1^{(i)}\ell_2^{(j)} \in W, \tag{113}$$

$$x_{\ell_1^{(i)}\ell_2^{(j)}} \le x_{\ell_2^{(j)}}, \forall \ell_1^{(i)}\ell_2^{(j)} \in W, \tag{114}$$

$$x_{\ell^{(j)}} \in \{0,1\}, \forall j \in TW, \forall \ell \in L, \tag{115}$$

$$x_{\ell_1^{(i)}\ell_2^{(j)}} \in \{0,1\}, \forall i,j \in TW, \forall \ell_1, \ell_2 \in L, \tag{116}$$

$$u^k \ge 0, \text{integer}, \forall k \in K, \tag{117}$$

$$y_{s,t-}^k, y_{s,t+} \ge 0, \text{integer}, \forall s \in S, \forall t \in T_s, \forall k \in K. \tag{118}$$

After solution are found for the rough fleet assignment model, a set of flight legs with rough departure and flight times is determined. To derive an exact schedule, the timetable model creates five copies for each flight leg that has a rough departure time. In fact, these five copies represent an adjustment of the rough departure time of a flight leg by $0, \pm 12$, and $\pm 24$ minutes. With these five copies, the timetable model can move a flight leg forward and backward, which implies great opportunities of switching two consecutive flights. Furthermore, because the rough fleet assignment model determines a fleet for each flight leg, an accurate flight time is calculate for each flight leg in the timetable model.

The goal of the timetable model is to determine exactly one copy of each flight leg so that the difference between the revenue related to the connection flights and the total cost of the aircraft needed in the network is maximized. Because numerous connection arcs exist in the network, the timetable model becomes very hard to solve, which necessitates some simplification. Because hubs have many incoming and outgoing flights, changing the departure and arrival times of these flights greatly influences the connection arcs of the entire network. Therefore, to limit the number of connection arcs, only profitable connection flights that are connected at hubs are considered.

This following paragraphs are going to discuss the computational issues of the timetable model. First, Table 21 summarizes the instances of the timetable problem that are created in this section. In fact, this thesis also creates instances that are larger then the instances in this section and are used for deriving fight schedules in Chapter VI. Although in the timetable model, the connection variables and the ground arc variables could be relaxed to continues variables, and the relaxation does not change the problem itself, given the total number of flight copy variables and the total number of constraints, the instances created in this thesis are still not easy to solve. Furthermore, in using Cplex solve these instances, a lot of time spends on branching while the objective does not improve. Therefore, to derive a good solution in a short period of time, a decomposition of the timetable model is proposed, which will be explained in the following paragraphs.

The instances of the timetable problem created in this chapter includes a large number of flight copy variables that make the instance very hard to solve. On the other hand, the number of flight copy variables in the subproblem is proportional to the number of copies that is created for each flight arc. The instances of the timetable problem addressed in this chapter include five copies for each flight arc. Therefore,

**Table 21:** A summary of instances of the timetable problem created in this section

| Instances of timetable problem | |
|---|---|
| The number of flight copy variables $x_l^{(j)}$'s | [30,000, 43,000] |
| The number of connection variables $x_{\ell_1^{(i)}\ell_2^{(j)}}$'s | [48,000, 119,000] |
| The number of ground arc variables $y^k_{s,t-}$'s and $y^k_{s,t+}$'s | [6,000, 11,000] |
| The number of flow balance constraints | [6,000, 11,000] |
| The number of flight cover constraints | [6,000, 9,000] |
| The number of all the variables | [85,000, 174,000] |
| The number of all the constraints | [110,000, 258,000] |

reducing the number of copies for each flight arc will make the subproblems easier to solve. However, the instances involving five copies, adjusting 0, $\pm 12, \pm 24$ minutes, still need to be addressed.

To address the difficulties of solving the instances of the timetable model created in this thesis, the five copies for each flight arc are aggregated into three time adjustment categories, one adjusting the flight arc by 0 minutes, one by +18 minutes, and one by 18 minutes. Adjusting a flight arc by +12 or +24 minutes belongs to the category of adjusting by +18 minutes, and adjusting a flight arc by -12 or -24 minutes belongs to the category of adjusting by -18 minutes. Furthermore, a fleet assignment subproblem is decomposed into a time-category subproblem and a timetable subproblem. In the time-category subproblem, three time adjustment categories are created for each flight leg, and in the timetable subproblem, each flight leg is assigned with a time adjustment in that category. Using the subproblem approach to the timetable problem not only reduces the size of the problem, but it also addresses the computational issue of exhaustive branching but not improving the objective. Table 22 summarizes the instances of the time-category subproblem created in this section.

The following paragraphs will present and analyze the computational results of the timetable problem. In the timetable model, parameter $\beta$ is created for adjusting the impact of the aircraft cost and the connection revenue on the objective. When $\beta$ is very small or even close to 0, the timetable model mainly determines the set

**Table 22:** A summary of instances of the time-category subproblem created in this section

| Instances of time-category subproblem | |
|---|---|
| The number of flight copy variables $x_l^{(j)}$'s | [18,000, 27,000] |
| The number of connection variables $x_{\ell_1^{(i)}\ell_2^{(j)}}$'s | [34,000, 87,000] |
| The number of ground arc variables $y_{s,t-}^k$'s and $y_{s,t+}^k$'s | [4,000, 7,000] |
| The number of flow balance constraints | [4,000, 7,000] |
| The number of flight cover constraints | [6,000,9,000] |
| The number of all the variables | [39,000, 76,000] |
| The number of all the constraints | [45,000, 102,000] |

of flight copies such that the cost of the aircraft is minimized. Furthermore, under this situation, the value of the plane count variables, $u^k$'s, are very important to the objective. Tables 23 and 24 present two sets of instances that are created for the timetable problem with very small parameter $\beta$ such that the overall connection revenue is much less than the cost of the aircraft. Table 23 shows that using the subproblem approach will cause the objective to increase less than 10% but reduce the solution time by more than 91%, and Table 24 shows that using the subproblem approach will cause the objective to increase less than 12% but reduce the solution time by more than 94%.

When $\beta$ is very large, the timetable mainly determines the set of flight copies such that the connection revenue is maximized. However, in this situation, without using the subproblem approach, the instances of the timetable problem created in this thesis become very difficult to solve. It is because the values of the flight copy variables influence the connection arc variables, and the number of connection arc variables is very huge. Table 25 summarizes the computational results of the instances that are created for the timetable problem with a very large parameter $\beta$ such that the overall connection revenue is much greater than the cost of the aircraft. Without using decomposition, the Cplex solver takes an exhaustive amount of time in branching the tree of the candidate solution, and the memory required for storing the tree becomes very large as the solver continues to penetrate the tree. Therefore, in a limited time, without using the subproblem approach, the Cplex solver cannot find an optimal solution of the instances that are created, and it cannot even find feasible solutions for several instances. Therefore, Table 25 includes only those instances that can derive feasible solution. Furthermore, to gauge the quality of the solution found by using the subproblem approach, Table 25 lists the objective achieved and the upper bound of the objective found by the Cplex solver when the subproblem approach was not used. Table 25 shows that for these hard instances, using the subproblem approach can reduce the solution time greatly. Furthermore, comparing with the upper bounds of the objective found without using the subproblem approach, the objective of the optimal solution found by using the subproblem approach has a less than 52% optimality gap.

**Table 23:** Computational result 1 of instances of the timetable problem

| | Not using subproblem approach | | Using subproblem approach | | Improvement | |
|---|---|---|---|---|---|---|
| Instance Name | Objective | Solution Time (CPU Seconds) | Objective | Solution Time (CPU Seconds) | Objective | Solution Time |
| 2_1_0 | 2.28E+11 | 16976 | 2.47E+11 | 179 | 8.50 % | -98.95 % |
| 2_2_0 | 2.38E+11 | 23783 | 2.56E+11 | 754 | 7.32 % | -96.83 % |
| 2_3_0 | 2.37E+11 | 24098 | 2.59E+11 | 171 | 9.16 % | -99.29 % |
| 2_4_0 | 2.43E+11 | 22494 | 2.60E+11 | 211 | 6.81 % | -99.06 % |
| 3_1_0 | 2.37E+11 | 14281 | 2.55E+11 | 233 | 7.53 % | -98.37 % |
| 3_2_0 | 2.42E+11 | 25057 | 2.68E+11 | 526 | 10.74 % | -97.90 % |
| 3_3_0 | 2.42E+11 | 21477 | 2.64E+11 | 220 | 9.26 % | -98.98 % |
| 3_4_0 | 2.50E+11 | 18505 | 2.65E+11 | 1011 | 6.13 % | -94.54 % |
| 4_1_0 | 2.45E+11 | 7846 | 2.60E+11 | 209 | 6.03 % | -97.34 % |
| 4_2_0 | 2.50E+11 | 23384 | 2.65E+11 | 215 | 6.08 % | -99.08 % |
| 4_3_0 | 2.42E+11 | 15702 | 2.62E+11 | 821 | 8.21 % | -94.77 % |
| 4_4_0 | 2.46E+11 | 19771 | 2.59E+11 | 426 | 5.02 % | -97.85 % |
| 5_1_0 | 2.45E+11 | 46946 | 2.59E+11 | 390 | 6.06 % | -99.17 % |
| 5_2_0 | 2.39E+11 | 61701 | 2.57E+11 | 1452 | 7.62 % | -97.65 % |
| 5_3_0 | 2.31E+11 | 52006 | 2.50E+11 | 464 | 8.39 % | -99.11 % |
| 5_4_0 | 2.36E+11 | 31686 | 2.59E+11 | 212 | 9.81 % | -99.33 % |
| 6_1_0 | 2.43E+11 | 21512 | 2.54E+11 | 1915 | 4.70 % | -91.10 % |
| 6_2_0 | 2.37E+11 | 84811 | 2.56E+11 | 594 | 7.87 % | -99.30 % |
| 6_3_0 | 2.47E+11 | 49369 | 2.61E+11 | 407 | 6.04 % | -99.18 % |
| 6_4_0 | 2.37E+11 | 97813 | 2.52E+11 | 575 | 5.97 % | -99.41 % |
| 7_1_0 | 2.46E+11 | 28361 | 2.66E+11 | 276 | 8.32 % | -99.03 % |
| 7_2_0 | 2.44E+11 | 23002 | 2.66E+11 | 310 | 8.77 % | -98.65 % |
| 7_3_0 | 2.43E+11 | 15159 | 2.62E+11 | 1024 | 8.11 % | -93.24 % |
| 7_4_0 | 2.49E+11 | 22696 | 2.65E+11 | 254 | 6.15 % | -98.88 % |
| 8_1_0 | 2.54E+11 | 22514 | 2.70E+11 | 193 | 6.02 % | -99.14 % |
| 8_2_0 | 2.40E+11 | 18173 | 2.54E+11 | 597 | 5.76 % | -96.71 % |
| 8_3_0 | 2.48E+11 | 15251 | 2.61E+11 | 258 | 5.05 % | -98.31 % |
| 8_4_0 | 2.37E+11 | 21234 | 2.58E+11 | 181 | 9.03 % | -99.15 % |
| 9_1_0 | 2.39E+11 | 29899 | 2.59E+11 | 359 | 8.26 % | -98.80 % |
| 9_2_0 | 2.48E+11 | 54181 | 2.65E+11 | 458 | 6.77 % | -99.15 % |
| 9_3_0 | 2.46E+11 | 40752 | 2.70E+11 | 1099 | 9.52 % | -97.30 % |
| 9_4_0 | 2.37E+11 | 52219 | 2.58E+11 | 561 | 8.65 % | -98.93 % |

**Table 24:** Computational result 2 of instances of the timetable problem

| | Not using subproblem approach | | Using subproblem approach | | Improvement | |
|---|---|---|---|---|---|---|
| Instance Name | Objective | Solution Time (CPU Seconds) | Objective | Solution Time (CPU Seconds) | Objective | Solution Time |
| 2_1_1 | 2.52E+11 | 16976 | 2.85E+11 | 114 | 13.25 % | -99.33 % |
| 2_2_1 | 2.71E+11 | 23783 | 2.97E+11 | 143 | 9.28 % | -99.40 % |
| 2_3_1 | 2.71E+11 | 24098 | 2.91E+11 | 138 | 7.63 % | -99.43 % |
| 2_4_1 | 2.69E+11 | 22494 | 2.86E+11 | 112 | 6.21 % | -99.50 % |
| 3_1_1 | 2.53E+11 | 14281 | 2.81E+11 | 206 | 11.32 % | -98.56 % |
| 3_2_1 | 2.67E+11 | 25057 | 2.98E+11 | 148 | 11.48 % | -99.41 % |
| 3_3_1 | 2.70E+11 | 21477 | 2.96E+11 | 149 | 9.82 % | -99.31 % |
| 3_4_1 | 2.71E+11 | 18505 | 2.95E+11 | 190 | 8.72 % | -98.97 % |
| 4_1_1 | 2.62E+11 | 7846 | 2.88E+11 | 241 | 10.07 % | -96.93 % |
| 4_2_1 | 2.83E+11 | 23384 | 3.04E+11 | 378 | 7.42 % | -98.38 % |
| 4_3_1 | 2.73E+11 | 15702 | 2.93E+11 | 229 | 7.39 % | -98.54 % |
| 4_4_1 | 2.78E+11 | 19771 | 2.98E+11 | 1098 | 7.05 % | -94.45 % |
| 5_1_1 | 2.72E+11 | 46946 | 2.94E+11 | 350 | 8.16 % | -99.25 % |
| 5_2_1 | 2.87E+11 | 61701 | 3.10E+11 | 519 | 8.05 % | -99.16 % |
| 5_3_1 | 2.80E+11 | 52006 | 3.04E+11 | 288 | 8.89 % | -99.45 % |
| 5_4_1 | 2.86E+11 | 31686 | 3.05E+11 | 222 | 6.87 % | -99.30 % |
| 6_1_1 | 2.86E+11 | 21512 | 3.02E+11 | 359 | 5.83 % | -98.33 % |
| 6_2_1 | 2.76E+11 | 84811 | 3.00E+11 | 195 | 8.56 % | -99.77 % |
| 6_3_1 | 2.72E+11 | 49369 | 2.92E+11 | 840 | 7.34 % | -98.30 % |
| 6_4_1 | 2.76E+11 | 97813 | 3.02E+11 | 280 | 9.52 % | -99.71 % |
| 7_1_1 | 2.79E+11 | 28361 | 3.00E+11 | 543 | 7.79 % | -98.09 % |
| 7_2_1 | 2.82E+11 | 23002 | 3.05E+11 | 219 | 7.99 % | -99.05 % |
| 7_3_1 | 2.77E+11 | 15159 | 3.09E+11 | 201 | 11.66 % | -98.67 % |
| 7_4_1 | 2.81E+11 | 22696 | 3.07E+11 | 969 | 9.10 % | -95.73 % |
| 8_1_1 | 2.83E+11 | 22514 | 2.99E+11 | 139 | 5.61 % | -99.38 % |
| 8_2_1 | 2.82E+11 | 18173 | 3.04E+11 | 150 | 8.10 % | -99.17 % |
| 8_3_1 | 2.80E+11 | 15251 | 3.03E+11 | 654 | 8.07 % | -95.71 % |
| 8_4_1 | 2.75E+11 | 21234 | 3.02E+11 | 211 | 10.07 % | -99.01 % |
| 9_1_1 | 2.69E+11 | 29899 | 2.93E+11 | 926 | 8.96 % | -96.90 % |
| 9_2_1 | 2.75E+11 | 54181 | 2.99E+11 | 299 | 8.59 % | -99.45 % |
| 9_3_1 | 2.73E+11 | 40752 | 2.97E+11 | 257 | 8.80 % | -99.37 % |
| 9_4_1 | 2.78E+11 | 52219 | 2.99E+11 | 457 | 7.49 % | -99.12 % |

**Table 25:** Computational result 3 of instances of the timetable problem

| Instance Name | Not using subproblem approach | | | Using subproblem approach | |
|---|---|---|---|---|---|
| | Objective | Objective Lower bound | Solution Time (CPU Seconds) | Objective | Solution Time (CPU Seconds) |
| 3_1_0 | -2.35E+14 | -4.29E+14 | 75508 | -2.04E+14 | 1059 |
| 5_2_0 | -1.99E+14 | -6.60E+14 | 75796 | -3.46E+14 | 2288 |
| 6_2_0 | -2.31E+14 | -7.61E+14 | 246012 | -4.73E+14 | 13675 |
| 6_3_0 | -3.29E+14 | -9.76E+14 | 188043 | -3.74E+14 | 21036 |
| 7_1_0 | -2.07E+14 | -8.49E+14 | 43436 | -3.38E+14 | 11905 |
| 8_1_0 | -1.81E+14 | -6.38E+14 | 74292 | -3.71E+14 | 1740 |
| 9_1_0 | -1.55E+14 | -4.73E+14 | 76214 | -2.61E+14 | 1623 |
| 9_2_0 | -1.63E+14 | -6.55E+14 | 22966 | -3.60E+14 | 15746 |
| 9_3_0 | -1.94E+14 | -6.41E+14 | 49059 | -3.50E+14 | 4928 |
| 9_4_0 | -1.86E+14 | -6.07E+14 | 66949 | -2.92E+14 | 987 |
| 2_1_1 | -2.39E+14 | -4.32E+14 | 52149 | -2.50E+14 | 4559 |
| 2_2_1 | -1.38E+14 | -5.19E+14 | 76767 | -2.85E+14 | 1365 |
| 2_3_1 | -1.21E+14 | -4.40E+14 | 52251 | -2.51E+14 | 976 |
| 2_4_1 | -1.29E+14 | -4.91E+14 | 10777 | -2.70E+14 | 1583 |
| 3_2_1 | -1.18E+14 | -4.45E+14 | 11164 | -2.61E+14 | 813 |
| 3_3_1 | -2.85E+14 | -5.06E+14 | 248670 | -2.79E+14 | 5035 |
| 3_4_1 | -3.22E+14 | -5.55E+14 | 94023 | -3.12E+14 | 5913 |
| 4_1_1 | -1.57E+14 | -4.98E+14 | 47336 | -2.72E+14 | 2163 |
| 4_2_1 | -1.52E+14 | -8.49E+14 | 13192 | -3.26E+14 | 8976 |
| 4_3_1 | -2.59E+14 | -5.55E+14 | 41742 | -3.09E+14 | 17760 |
| 4_4_1 | -1.75E+14 | -5.50E+14 | 75368 | -3.16E+14 | 1471 |
| 5_1_1 | -1.65E+14 | -4.94E+14 | 165317 | -2.87E+08 | 919 |
| 5_2_1 | -1.87E+14 | -7.56E+14 | 45780 | -4.04E+14 | 19821 |
| 5_3_1 | -2.09E+14 | -6.54E+14 | 42598 | -3.57E+14 | 5127 |
| 5_4_1 | -1.78E+14 | -6.70E+14 | 54251 | -3.71E+14 | 2390 |
| 6_1_1 | -1.60E+14 | -6.27E+14 | 84657 | -3.51E+14 | 12389 |
| 6_2_1 | -3.59E+14 | -8.64E+14 | 98159 | -4.73E+14 | 13675 |
| 6_3_1 | -2.09E+14 | -7.52E+14 | 247247 | -4.13E+14 | 2719 |
| 6_4_1 | -3.48E+14 | -7.70E+14 | 110018 | -4.18E+14 | 40120 |
| 7_1_1 | -1.61E+14 | -5.89E+14 | 49153 | -3.46E+14 | 8931 |
| 7_2_1 | -4.89E+14 | -8.80E+14 | 150073 | -4.83E+14 | 37721 |
| 7_3_1 | -3.71E+14 | -8.38E+14 | 62163 | -4.24E+14 | 12731 |
| 7_4_1 | -4.79E+14 | -8.76E+14 | 250193 | -4.77E+08 | 1119 |
| 8_2_1 | -2.18E+14 | -8.87E+14 | 64491 | -4.87E+14 | 3725 |
| 8_3_1 | -2.14E+14 | -8.41E+14 | 59857 | -4.46E+14 | 9711 |
| 8_4_1 | -2.05E+14 | -6.73E+14 | 59002 | -3.75E+14 | 3203 |
| 9_3_1 | -2.15E+14 | -7.29E+14 | 75931 | -3.91E+14 | 2571 |
| 9_4_1 | -2.10E+14 | -6.45E+14 | 77524 | -3.59E+14 | 3749 |

# CHAPTER VI

# ANALYSIS OF FUTURE AIRCRAFT NETWORKS AND SCHEDULES

"Joy in looking and comprehending is nature's most beautiful gift."—Albert Einstein
The overall goal of this chapter is to analyze the computational results of the aircraft schedule and the network design problem studied in this thesis. In fact, this chapter uses the historical seasonal passenger demand data as the main input and the models developed in Chapter IV to create numerous instances of the design problem, and it solves these instances by using the solution algorithms developed in Chapter V to derive schedules corresponding to the input demand. To validate the models and solution algorithms developed in this thesis, this chapter compares the daily schedules that it designs for four seasons in selected years with daily schedules of the existing airlines during the same time. Moreover, this chapter creates instances that represent different economic and fuel-price conditions, and it derives schedules under these different conditions by solving these instances. Finally, it briefly discusses the implications of the computational results.

## *6.1 Comparison with the Schedules and the Networks of the Airlines in the United States*

The goal of this section is to compare the schedule of existing airlines with schedules that are found by solving the models in the three-step approach. In fact, schedules of the existing airlines from 2002 to 2010 are selected as baseline schedules because they represent the most recent trends of the flight schedules in the United States. Because the models in this thesis include only 200 selected airports, only the passenger demand and the flights among these 200 airports are studied.

The following paragraphs present figures that illustrate the overall characteristics of the passenger demand of the 200 selected airports in the United States from 2002 to 2010. Figure 24 shows the yearly passenger demand among these 200 selected airports from 2002 to 2010. As shown in Figure 24, passenger demand kept increasing between 2002 and 2007 and attained its maximum in 2007, which was also the beginning of the economic recession, and then it decreased between 2007 and 2009. From 2009 to 2010, passenger demand stayed at almost the same level.

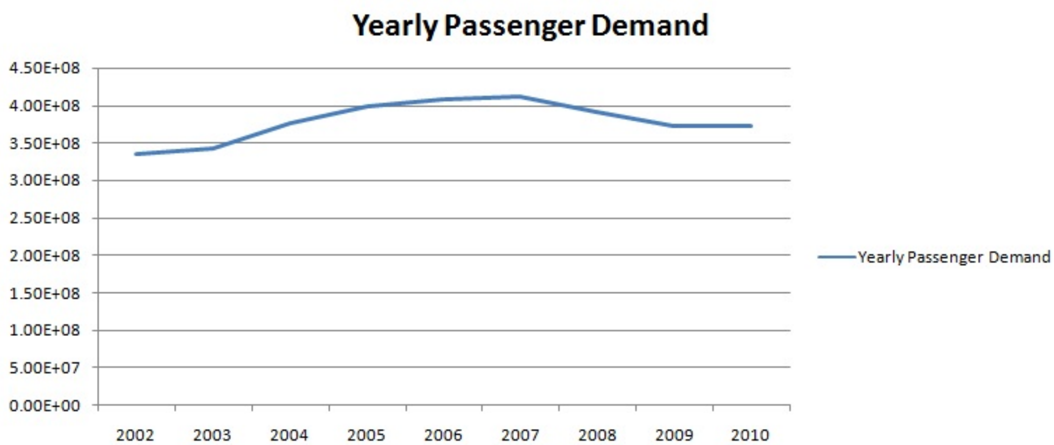



**Figure 24:** Yearly passenger demand among the 200 selected airports

In addition to the yearly trends of air passenger demand shown in Figure 24, Figure 25 illustrates the average daily passenger demand from the first season in 2002 to the fourth season in 2010. As show in Figure 25, the patterns of the average daily passenger demand from 2002 to 2010 are similar: The daily passenger demand in the first season fell to a minimum during each year with the exception of 2008, and the daily passenger demand in the second season attained the maximum in each year with the exception of 2003. Figure 26 presents the average travel distance of air passengers from the first season in 2002 to the fourth season in 2010. As shown in Figure 26, the average travel distance by each air passenger remains around 1,110 miles, and the average travel distance always attains the maximum in the third season of each year.

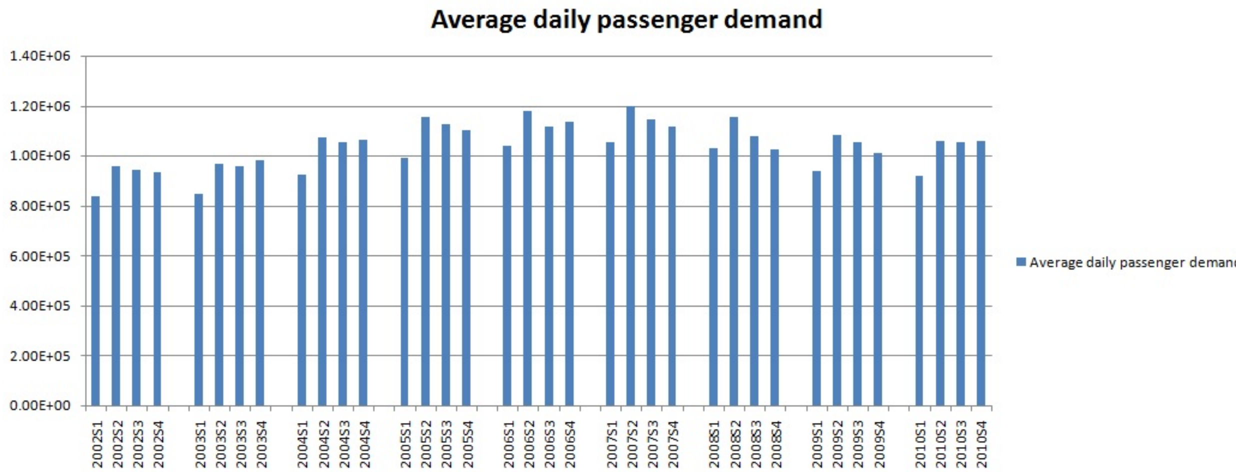



**Figure 25:** Average daily passenger demand among the 200 selected airports

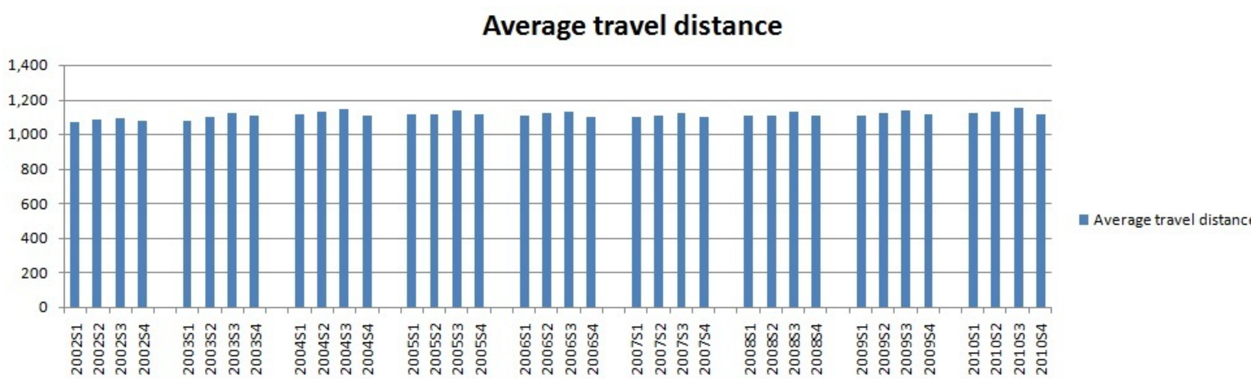



**Figure 26:** Average travel distance in miles of each air passenger

Figures 25 and 26 show the relative stableness of passenger travel behavior. The following paragraphs will present an overview of the flight schedules of the airlines in the United States from 2002 to 2010. First, Figure 27 shows the total number of yearly flights among the 200 selected airports in the United States. As shown in the graph, the number of yearly flights grows very fast from 2002 to 2004, and then it grows slowly from 2004 to 2007 and then decreases from 2007 to 2009. Finally, the total number of yearly flights stays almost the same from 2009 to 2010. Figure 28 presents the average number of daily flights among the 200 selected airports from the first season in 2002 to the fourth season in 2010. As shown in Figure 28, the range of the average number of daily flights is between 13,000 and 19,000. In addition,

Figure 29 presents the number of distinct flight arcs that are operated among the 200 selected airports from the first season in 2002 to the last season in 2010. As shown in Figure 29, the total number of distinct origin-destination flights arcs ranges from 2,700 to 4,300.

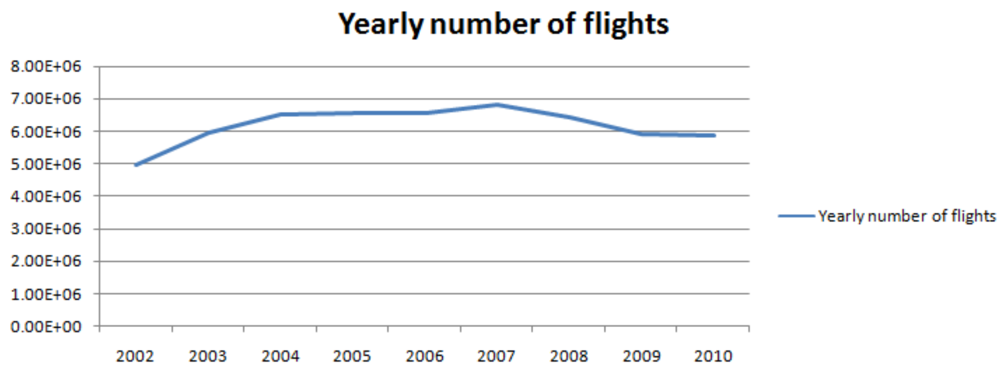



**Figure 27:** Average number of daily flights among the 200 selected airports

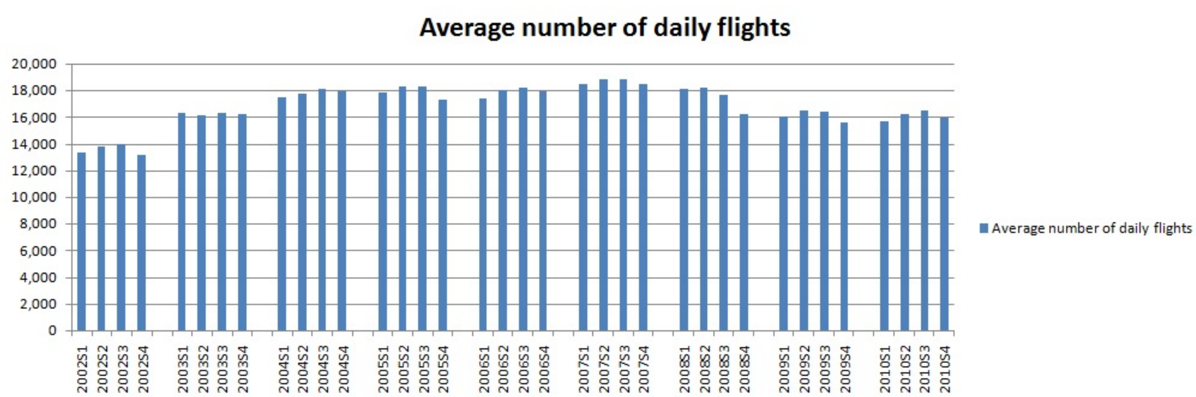



**Figure 28:** Total number of daily flights among the 200 selected airports

To give another view of the flight schedule in the United States, this section also selects six major airlines that use a hub-and-spoke network and presents the flight schedules of these major airlines altogether. The selected major airlines are American Airlines, Continental Airlines, United Airlines, Delta Air Lines, Northwest Airlines, and US Airways. Delta Air Lines and Northwest Airlines merged in 2008, and Continental Airlines and United Airlines merged in 2010. Furthermore, the mergers of these airlines form two of the largest commercial airlines in the world.

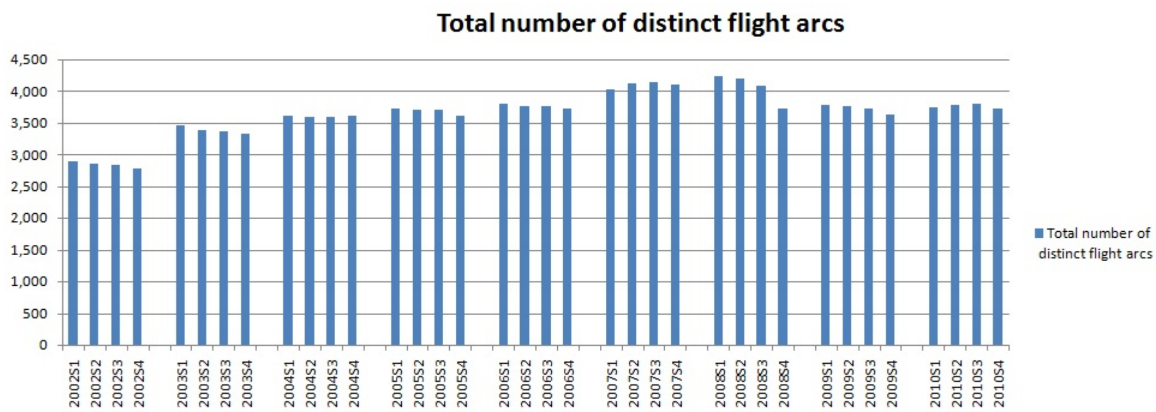



**Figure 29:** Total number of distinct origin-destination flight arcs among the 200 selected airports

First, Figure 30 shows that the total market share of the six airlines was more than 56%. On the other hand, Figure 31 shows that the total number of yearly flights of these airlines decreased between 2002 and 2010. Figure 32 shows the total average number of daily flights of the selected airlines from the first season in 2002 to the fourth season in 2010. In addition, Figure 33 shows that the total number of distinct origin-destination flight arcs operated by these airlines ranged from 1,400 to 2,100.

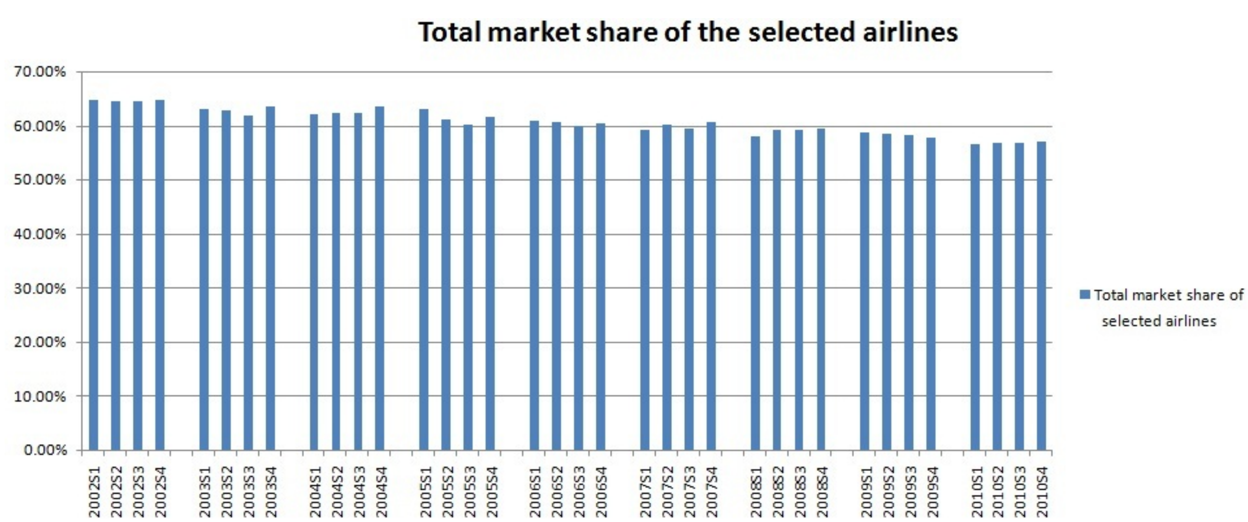



**Figure 30:** Total number of daily flights of the six airlines

Using the passenger demand extracted from the DB1B data, the models created in the three-step approach, and the solution algorithms developed for each of the

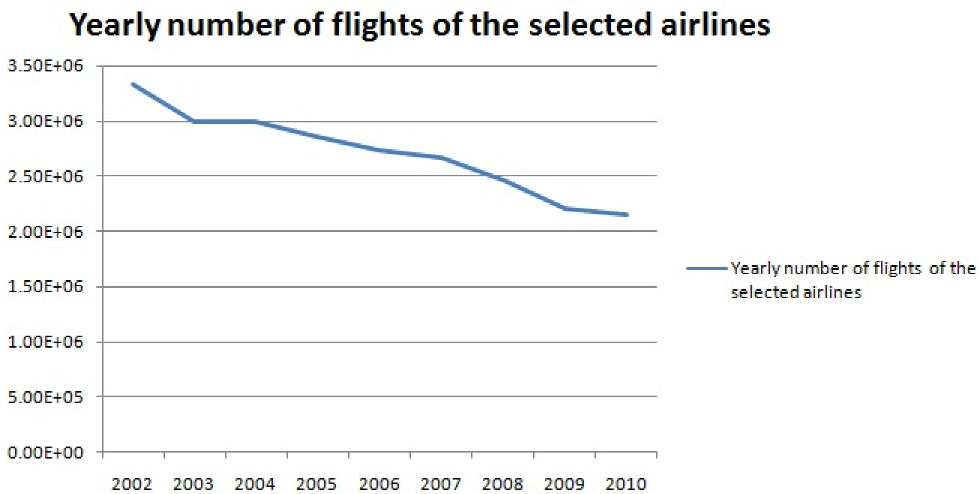



**Figure 31:** Total number of daily flights of the six airlines

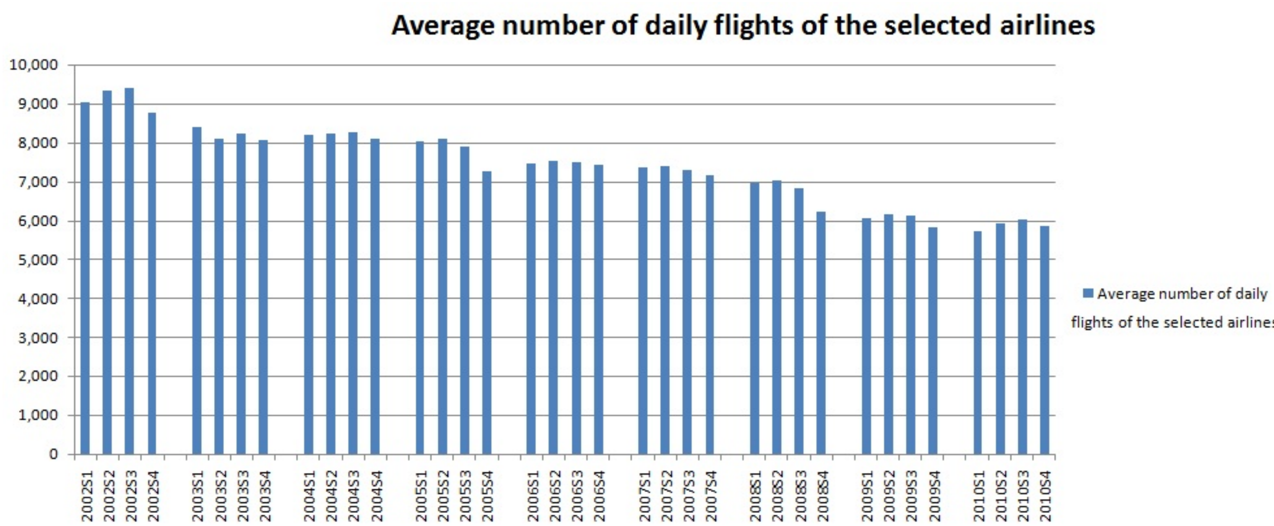



**Figure 32:** Total number of daily flights of the six airlines

models, this section develops daily flight schedules for the first season in 2002 until the last season in 2010 and compares them with the real flight schedules. In fact, fleets 1,2,3,4, and 5, representing fleets used by the current airlines, are used in these daily flight schedules. First, Figure 34 compares the total number of flights in the designed flight schedules and in the real flight schedules. Figure 34 shows that the number of flights in the designed schedules is about half that in the real schedules. One reason behind this phenomenon follows. This thesis assumes that there is only one airline

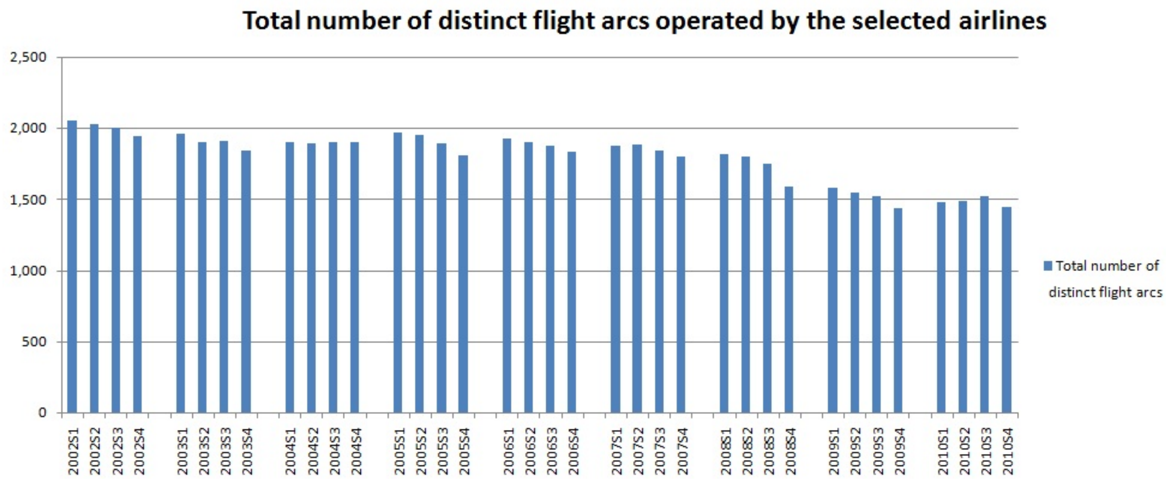



**Figure 33:** Total number of distinct flight arcs operated by the six airlines

in the market and designs flight schedules for this airline. Therefore, it does not consider competition. However, in reality, airlines schedule flights and compete with each other to obtain a larger market share. Therefore, in some profitable markets, much more flights are scheduled to satisfy passenger demand. In fact, Figure 38 shows that in the real schedule, the daily frequencies of some flight arcs are larger than 30. To reduce the number of flights scheduled by airlines, researchers such as Vaze and Barnhart [89] have proposed several management strategies.

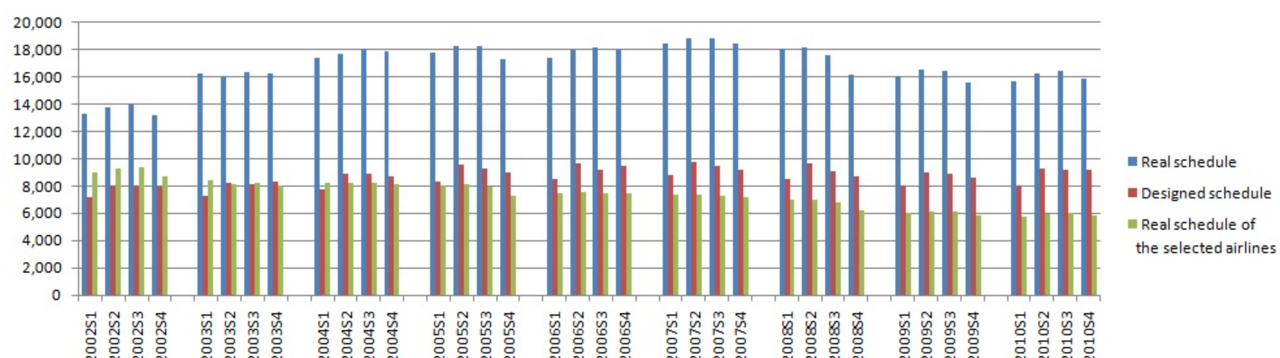



**Figure 34:** Comparison of the number of daily flights in real schedules and that in designed schedules

Figure 35 compares the total number of distinct flight arcs used in the designed flight schedules and that in the real flight schedules. As shown in Figure 35, the total number distinct flight arcs used in the designed fight schedules closely matches that in the real flight schedules. The range of the total number is from 2,700 to 4,300. In addition, Figure 35 shows that the total number of flight arcs shared by the designed and real schedules ranges from 2,000 to 2,800. One reason is that for each markets, the designed and real flight schedules may use a different set of itineraries.

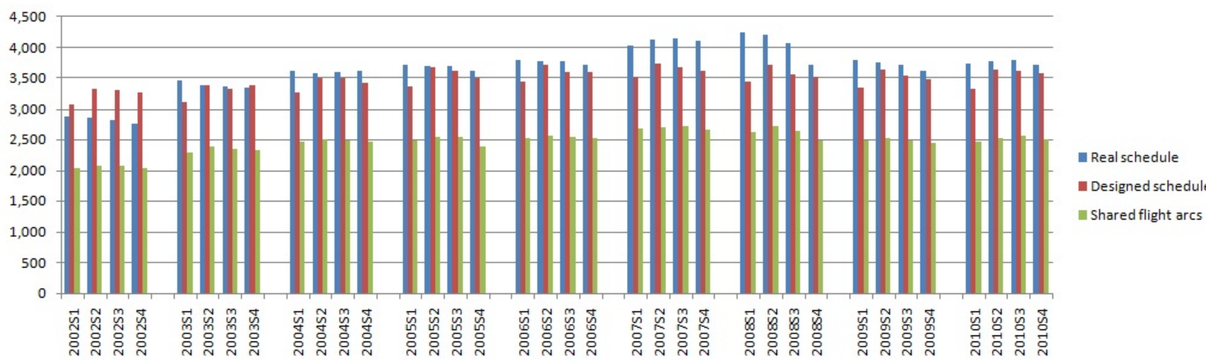



**Figure 35:** Comparison of the number of distinct flight arcs in real schedules and that in designed schedules

Figures 36 and 37 compare the frequencies of the distinct flight arcs in the designed and real flight schedules. Figure 36 presents the differences between the total number of flights of the real schedules and that of the designed schedules. On the other hand, Figure 37 shows that the average absolute frequency difference per flight arc of the real and that of the designed schedules ranges from 2.2 to 3.1. In particular, Figure 38 presents the frequencies of flight arcs in the real daily schedule of the second season in 2010 and in the daily schedule designed for the same period of time. Figures 34, 35, 36, 37, and 38 compare the real and the designed schedules and confirm the validity of the models and algorithms developed in the previous chapters.

## *6.2 New Aircraft Network and Schedules in Different Scenarios*

Figure 24 shows the yearly passenger demand between the 200 selected airports in the United States among 2002 and 2010. As shown in Figure 24, among the nine years,

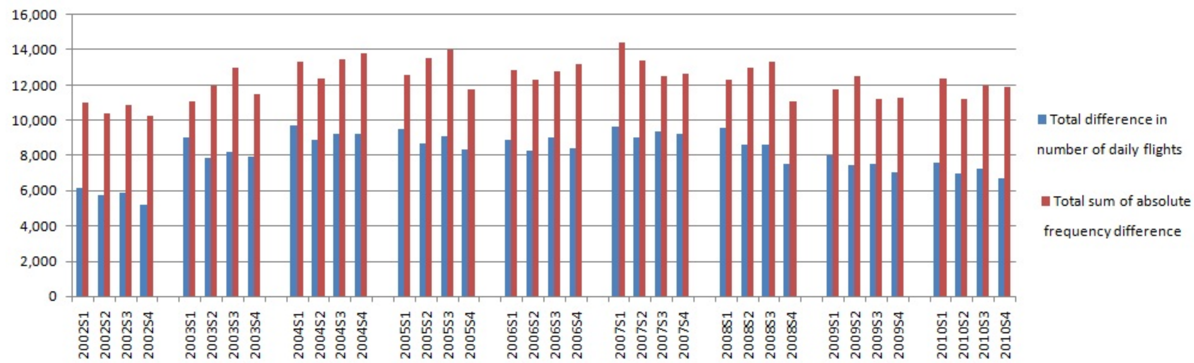



**Figure 36:** Total sum of the absolute frequency difference between the real and designed schedules

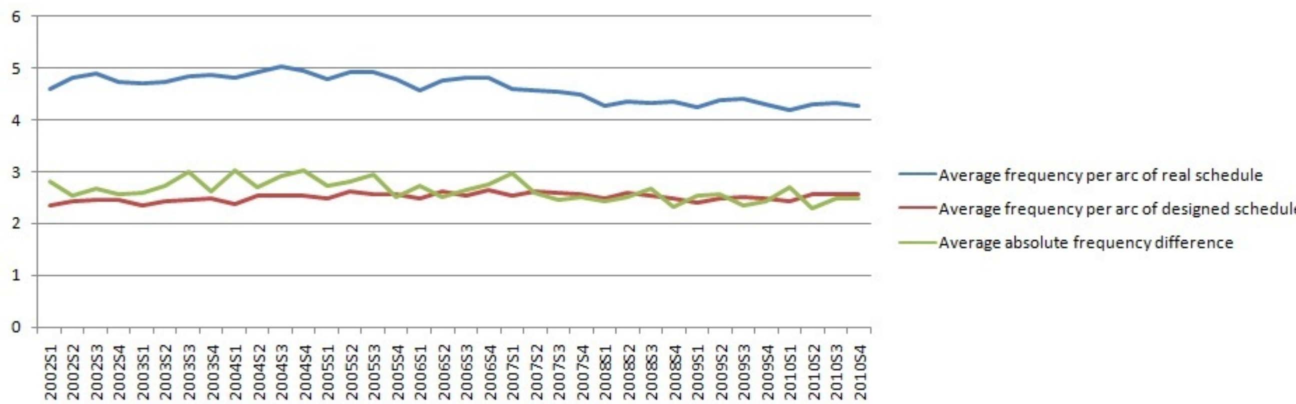
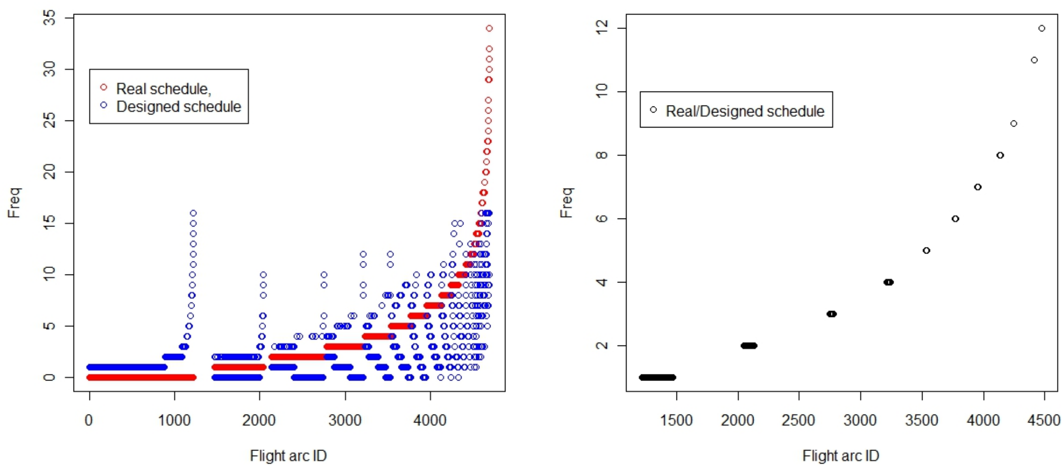



**Figure 37:** Average absolute frequency difference per flight arc between the real and designed schedules

yearly passenger demand decreases to a minimum in 2002 and reaches a maximum in 2007. In addition, yearly passenger demand in 2009 and 2010 are similiar. In addition, Figure 25 shows that average daily passenger demand follows a seasonal pattern. Therefore, for predicting air passenger demand in the future, this section uses the passenger demand in 2007 to represent air passenger demand in a strong economy and uses passenger demand in 2002 to represent air passenger demand in a weak economy. In other words, it assumes that in the future, if economic conditions improve, air passenger demand would increase to its level in 2007; and if economic conditions stay the same or become worse, air passenger demand would decrease to its level in 2002.



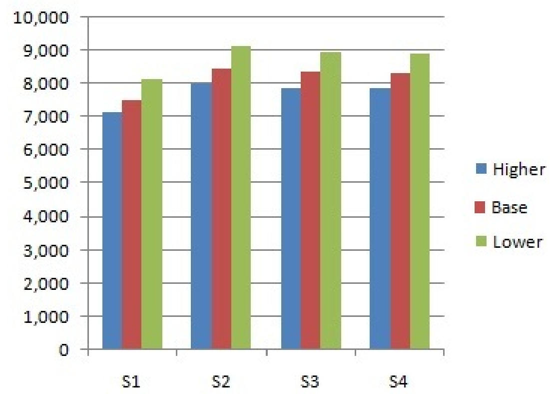

(a) Flight arcs with different frequencies

(b) Flight arcs with equal frequencies

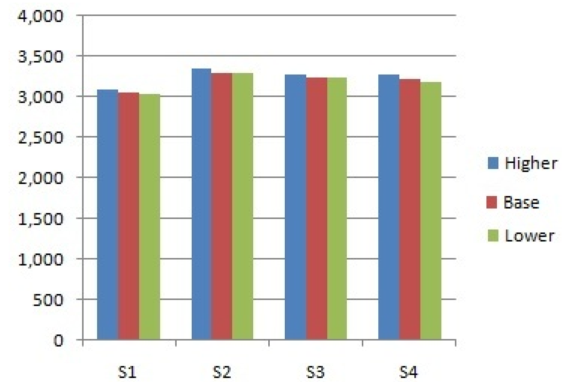
**Figure 38:** Illustration of the frequencies of flight arcs in the real daily schedule and the designed schedule

To compare with the designed schedule with those of the existing airlines, the previous section designs schedules that use fleets $1, 2, 3, 4$, and 5; and to predict the aircraft network and schedules in the future, this section designs schedules that use future fuel-efficient fleets, which are listed as fleets $1^*, 2^*, 3^*, 4^*$, and $5^*$ in Chapter V. In addition, because fuel prices greatly influence the airline industry, variations in fuel prices are also considered. Figure 39 illustrates the number of flights and the number of distinct flight arcs in the schedule designed for a weak economy, and Figure 40 illustrates them for a strong economy. Furthermore, in Figures 39 and 40, "base" represents the schedules of using the current fuel price, "higher" represents the schedules of using a price higher than the current fuel price, and "lower" represents the schedules of using a price lower than the current fuel price.

Figures 39 and 40 show that more flights and more distinct flight arcs are scheduled in a strong economy than in a weak economy. In a strong economy, the range of the total number of flights is between 8,500 and 10,500, and in a weak economy, the range of the total number of flights is between 7,200 and 9,200. In a strong economy, the

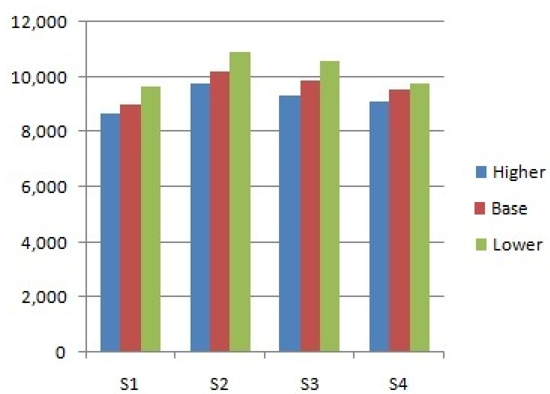



(a) Number of flights in designed schedule

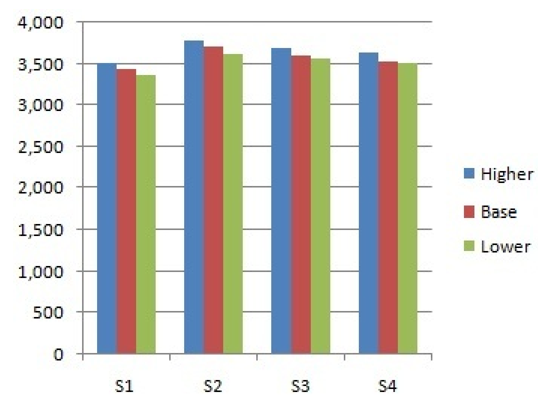



(b) Number of distinct flight arcs in designed schedule

**Figure 39:** Illustration of flight schedule designed for bad economy times



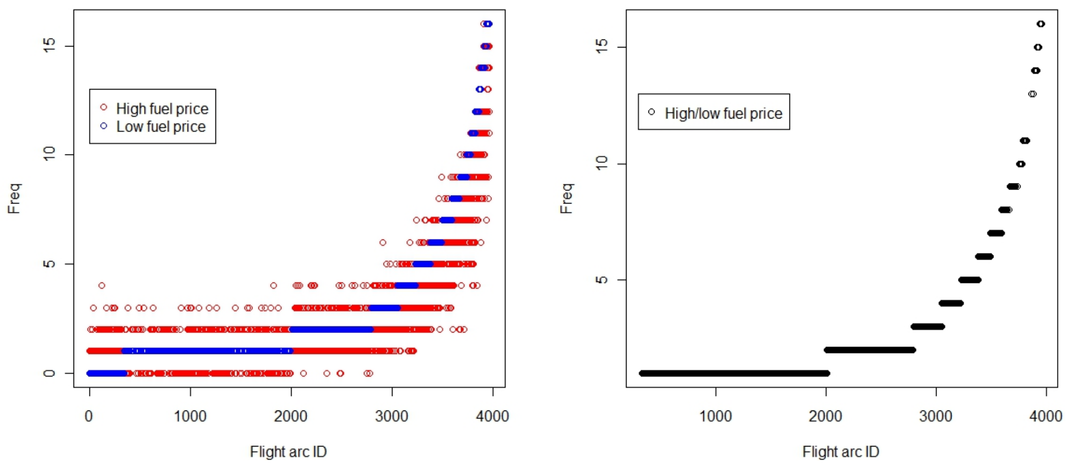

(a) Number of flights in designed schedule




(b) Number of distinct flight arcs in designed schedule

**Figure 40:** Illustration of flight schedule designed for good economy times

range of the total number of distinct origin-destination flight arcs is between 3,500 and 3,800, and in a weak economy, the range of the total number of distinct origin-destination flight arcs is between 3,000 and 3,400. Furthermore, they show that if fuel prices increase, the total number of flights in the designed schedule decreases, but the total number of distinct flight arcs increases. To explain this phenomenon, Figure 41 presents the frequencies of distinct flight arcs in schedules designed when fuel prices are high and low. As shown in Figure 41, when fuel prices decrease, the frequencies of many flight arcs increase, and when fuel prices increase, many new flight arcs with small frequency appear.




(a) Flight arcs with different frequencies (b) Flight arcs with the same frequencies

**Figure 41:** Illustration of the frequencies of distinct flight arcs in the designed schedules

## *6.3 Implication of Using New fleets in the Aircraft Network and Schedule*

This section discusses the implication of using new fleets in the aircraft network and schedule. First, using the passenger demand extracted from the DB1B data, the models created in the three-step approach, and the solution algorithms developed for each of the models, new flight schedules that uses fuel-efficient fleets $1^*, 2^*, 3^*, 4^*$, and $5^*$ are developed. Comparing the new flight schedules of fleets $1^*, 2^*, 3^*, 4^*$, and $5^*$ with those of fleets $1, 2, 3, 4$, and 5, Figure 42 shows that more flights would be scheduled in the new schedules. In fact, the percentage increase in the number of flights ranges from 1% to 6%. Figure 43 shows that less distinct fight arcs would be scheduled. In fact, the decrease in the percentage of the number of flights ranges from 0% to 2.5%. Because the objective calculated in the rough fleet assignment step represents roughly the overall profit of the final schedule, this section also calculates compares the objective of rough fleet assignment problem that using the old fleets with that of the new fleets. Figure 44 shows that improvement in profits ranges from 2.5% to 4.5%.

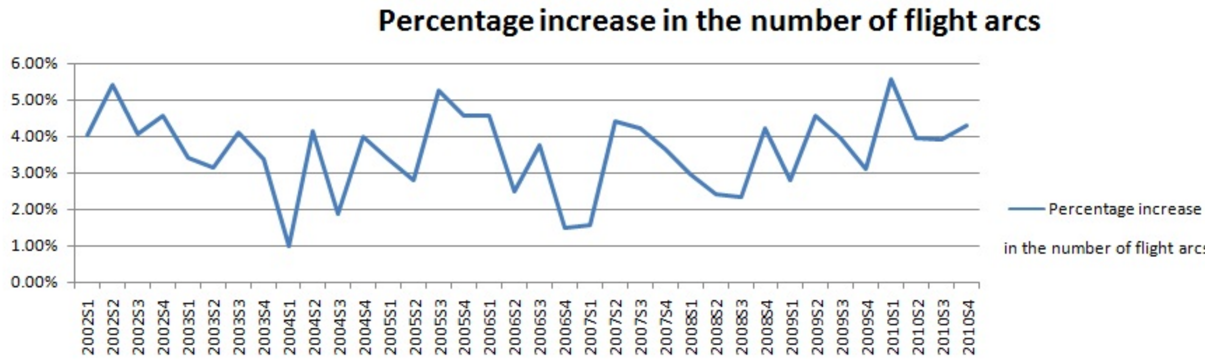



**Figure 42:** Illustration of change in the total number of flights in the designed schedules

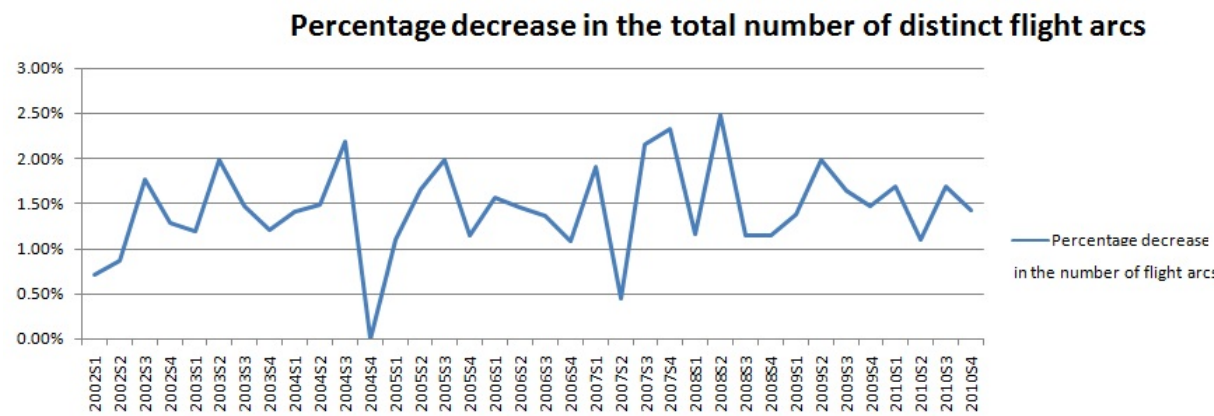



**Figure 43:** Illustration of change in the total number of distinct flight arcs in the designed schedules

For simplicity, this section selects the second season in 2007 and develops schedules that use old fleets, representing fleets $1, 2, 3, 4$, and $5$, and new fleets, representing fleets $1^*, 2^*, 3^*, 4^*$, and $5^*$. Figure 45 illustrates changes in the arc frequencies in the schedules corresponding to the changes in the fleets. In fact, Figure 45 shows that the schedules with new fleets have more flight arcs with frequencies greater than 6 than the schedules with old fleets.

Figure 46 compares the percentage of different fleets in the schedules. The comparison clearly shows a pattern of change in the fleet percentage. In the schedules

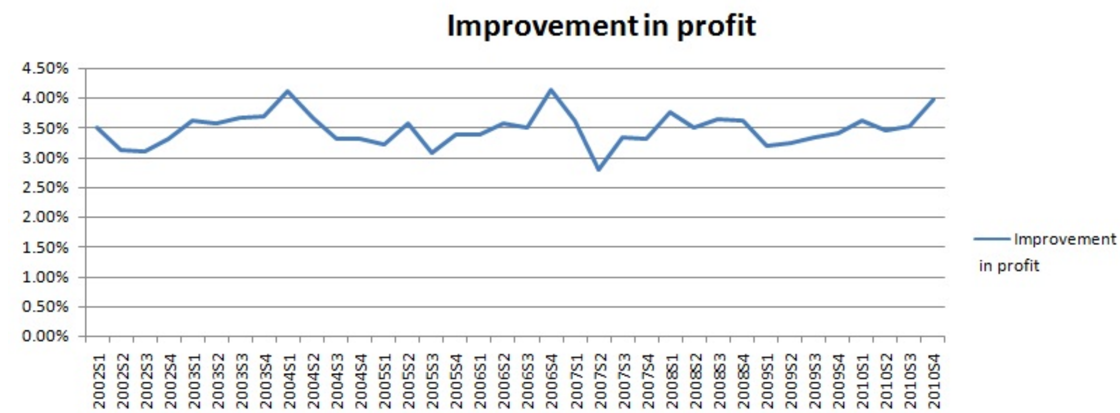
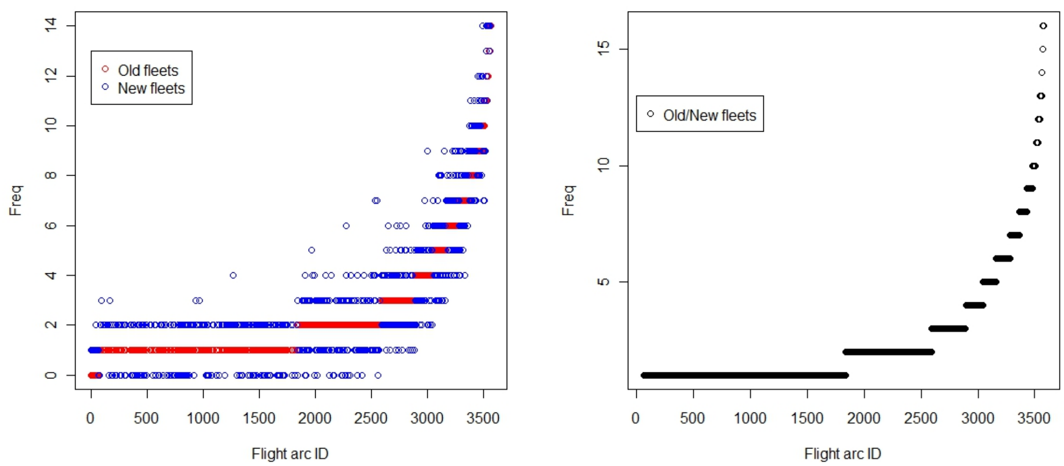



**Figure 44:** Illustration of improvement in the profit of the designed schedules

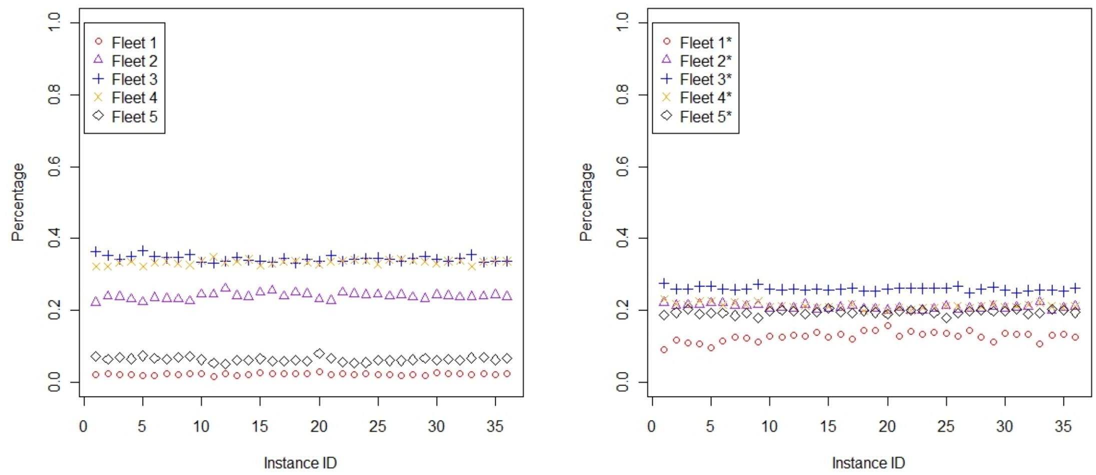



(a) Flight arcs with different frequencies (b) Flight arcs with equal frequencies

**Figure 45:** Illustration of frequencies of flight arcs in schedules using old fleets and new fleets

that use the old fleets, 1% – 3% of flights use fleet 1, 22% – 26% fleet 2, 33% – 37% fleet 3, 32% – 36% fleet 4, and 5% – 7% fleet 5. In the schedules that use the new fleets, 8% – 11% of flights use fleet 1*, 15% – 18% fleet 2*, 33% – 38% fleet 3*, 21% – 23% fleet 4*, and 16% – 19% fleet 5*.

The frequency model for the on-demand flights includes three fleets—GE 20-seat aircraft, VLJs, and helicopters. Chapter III mentioned that VLJ industry delivered 800 VLJs from 2008 to 2010, an annual production rate of 300 VLJs was estimated,




(a) Flight schedules with old fleets (b) Flight schedules with new fleets

**Figure 46:** Illustration of the percentage of different fleets

and more than 10,000 civil helicopters were in the United States. Based on the production rates of VLJs, this section assumes that the annual production rate of the GE-20-seat aircraft would be 100. Therefore, this section assumes that there would be 500 GE 20-seat aircraft and 2,300 VLJs in five years. Furthermore, because not all these aircraft would be used in on-demand services, this sections assumes that 250 GE 20-seat aircraft, 1,150 VLJs, and 4,000 Helicopters would be used in on-demand services. In addition, it assumes that each aircraft can run up to 8 hours every 24 hours, and up to 20% operating time is used in add-on service. Therefore, the maximum daily operating hours of these fleets used in add-on service are 400 hours, 1,840 hours, and 6,400 hours, respectively. Using the frequency model for the on-demand flights developed in Chapter IV and the schedules that are designed for the first season in 2002 to the last season in 2010, the frequency model for the on-demand flights determines that the total time of on-demand flights using GE 20-seat aircraft is 400 hours, VLJs 1,840 hours, and helicopters [1,600, 2,200] hours, which implies that GE 20-seat aircraft and VLJs would be fleets good for being used in the add-on service.

# CHAPTER VII

# CONCLUSION AND FUTURE RESEARCH

Nowadays, the air transportation system is experiencing great changes. Although still in its initial stage, small aircraft technology has also undergone dramatic change. For example, numerous VLJs have been produced and used in on-demand services. Furthermore, researchers working for NASA's Fundamental Aeronautics Program [23] have proposed several future aircraft that save on fuel and reduce noise. These innovations raise new questions about the efficient scheduling of air transportation resources. New aircraft technology and flight services will also influence the travel behaviors of air passengers and in turn influence the distribution and scheduling of air transportation resources. In addition, NASA has initiated research that focuses on the impact of new aircraft concepts and operations on the next generation air transportation system [63]. All of these changes, innovations, and programs have motivated the research in this thesis.

To better utilize new small aircraft, this thesis expanded a business model in which on-demand flight services are used as add-on services to traditional scheduled services and itineraries that combine traditional scheduled flights and on-demand flights are created. Under this business model, this thesis developed an approach to the design of flight schedules from scratch, consisting of three steps: a frequency assignment step, a rough fleet assignment step, and a timetable model step. In the first step, a frequency assignment model that incorporates a passenger path choice model was created for scheduled flights, and a frequency assignment model that incorporates a passenger mode choice model was developed for on-demand flights.

Based on the models proposed in the three steps, this thesis developed flight schedules involving 200 major airports and more than 25,000 markets, almost all the major airports and markets in the United States. To derive parameters for the models, this thesis first processed aviation data from the BTS website and an airport capacity report from the FAA and some literature about the discrete choice model. After that, it created large-scale instances of the frequency assignment model, the rough fleet assignment model, and the timetable model.

The instances of the frequency assignment model for scheduled flights created in this thesis were large-scale non-convex mixed-integer programming problems. Table 8 summarized the size of the variables and constraints of these instances. To solve these instances, this thesis developed two iterative approximation algorithms. For the created instances, Tables 11 and 12 showed that iterative algorithm 1 could derive solutions with optimality gaps smaller than 6% although it took more than 3 hours to solve them. On the other hand, iterative algorithm 2 could solve these instances in less than two hours. However, for each instance, using iterative algorithm 2 would result in a decrease in the objective by less than 20% but more than 16%. In addition, Tables 14 showed that increasing linking parameter $\gamma$ from 1 to 2 would result in an increase in the number of flight arcs with frequency equal to 1.

The instances of the rough fleet assignment model were large-scale mixed-integer programming problem, and Table 15 summarized the size of the variables and constraints of these instances. To solve these instances, this thesis first decomposed it into a time-slot assignment subproblem and a fleet assignment subproblem. Furthermore, because of the difficulties in solving the fleet assignment subproblem, this thesis further decomposed the fleet assignment subproblem into a fleet-category assignment subsubproblem and a fleet assignment subsubproblem. Figure 23 showed the entire subproblem scheme. For each instance created for the time-slot assignment subproblem, Table 16 showed that a solution with an optimality gap of less than 4% could be found in less than 3 hours. However, for some instances finding a solution with an optimality gap of less than 3% took more than 23 hours. For one set of instances created for the fleet assignment subproblem, Table 17 showed that using the subsubproblem approach could reduce the run time by more than 50%, but it did not decrease the objective by more than 4%. For another set of instances for the fleet assignment subproblem, Table 18 showed that using the subsubproblem approach did not decrease the objective by more than 2%, and for most of the instances it could reduce the run time by more than 50% but for some instances decomposition did not greatly reduce the run time. In addition, Tables 19 and 20 showed that when the penalty parameter $\alpha$ in the fleet assignment problem increased, more flights would be assigned with either big or small fleets. One explanations for this phenomenon was the variation in passenger demand corresponding to different departure times: during peak times, passenger demand was very high, and during other times, passenger demand was very low.

The instances of the timetable model were very large-scale integer programming problem, and Table 21 summarized the size of the variables and constraints of these instances. To reduce the solution time of these instances, this thesis decomposed the timetable problem into a time-category subproblem and a timetable subproblem. Tables 23 and 24 showed that when the connection parameter $\beta$ was small, using the subproblem approach could reduce the solution time by more than 90%, but it did not increase the objective by more than 14%. On the other hand, when the connection parameter $\beta$ was large, without using the subproblem approach, many instances could not be solved within a reasonable optimality gap and within a reasonable time. Therefore, Table 25 compared the objectives achieved by using the subproblem approach, and the objectives and lower bounds of the objective found by without using the subproblem approach. The computational results showed that using the subproblem approach could find solutions with a less than 52% optimality gap.

From the passenger demand extracted from the DB1B data, models created in the three-step approach, and solution algorithms developed for each of the models, this thesis developed daily flight schedules that used old fleets, namely, fleets $1, 2, 3, 4$, and 5 and compared them with the real flight schedules. The comparison presented in Figures 34, 35, 36, 37, and 38 confirmed the validity of the models and algorithms developed in this thesis. In addition, this thesis used passenger demand in 2002 and 2007 and the characteristics of the new fleets, fleets $1^*, 2^*, 3^*, 4^*$ and $5^*$, to predict future schedules under different economic conditions and different fuel prices. Furthermore, this thesis showed that using new fleets in the schedule design would lead to an increase in the total number of flights and a decrease in the total number of distinct flight arcs. In addition, this thesis compared the percentage of different fleets in the schedules that used the old fleets and the new fleets. Figure 46 showed that in the schedules that use the old fleets, $1\% - 3\%$ of flights use fleet 1, $22\% - 26\%$ fleet 2, $33\% - 37\%$ fleet 3, $32\% - 36\%$ fleet 4, and $5\% - 7\%$ fleet 5, and in the schedules that use the new fleets, $8\% - 11\%$ of flights use fleet $1^*$, $15\% - 18\%$ fleet $2^*$, $33\% - 38\%$ fleet $3^*$, $21\% - 23\%$ fleet $4^*$, and $16\% - 19\%$ fleet $5^*$.

Following the research in the thesis, future research in three areas—model, computational method, and simulation for validation—deserves studies. First, currently, both public data about on-demand flights and research on passengers' choice behavior of on-demand flights are very rare. Therefore, collecting data about on-demand flights and passengers' preference for on-demand flights is needed. Furthermore, more research using mode choice model to analyze passengers' choices of a transportation mode from ground transportation, traditional transportation, and on-demand transportation is needed.

Second, the schedule design problem discussed in this thesis is a very large-scale problem. To deal with the difficulty in solving the large-scale problem, this thesis extensively decomposes the large-scale schedule design problem into several smaller subproblems and subsubproblems. Therefore, both research on integrated solution approach and computational results of integrated approach are needed. Furthermore, because each of the subproblems and subsubproblems can take a very long time to solve, research on developing efficient algorithms of solving these subproblems and subsubproblems is needed. Furthermore, due to the great difficulty in solving itinerary-based frequency assignment model, this thesis solves a leg-based frequency assignment model. Therefore, research on developing efficient algorithm in solving large-scale itinerary-based frequency assignment model is needed.

Third, using simulation models to analyze the properties of new schedules and the influence of new schedules on the existing air transportation infrastructure deserves attention. In addition, because the operating characteristics of aircraft influence its usage in a schedule, researchers could also study the impact of aircraft scheduling on aircraft design [87].

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