09:21:53 OCA PAD INITIATION - PROJECT HEADER INFORMATION 06/21/88

Active

Project #: [redacted]
Center # : R6515-0A0
Cost share #: G-37-341
Center shr #: F6515-0A0
Rev #: 0
OCA file #:
Work type : RES
Document : GRANT
Contract entity: GTRC

Contract#: [redacted]
Prime #:
Mod #:

Subprojects ? : N
Main project #:

Project unit: MATH
Unit code: 02.010.144
Project director(s):
[redacted]

Sponsor/division names: [redacted]
Sponsor/division codes: [redacted]

Award period: [redacted]

Sponsor amount	New this change	Total to date
Contract value	28,355.00	28,355.00
Funded	28,355.00	28,355.00
Cost sharing amount		8,861.00

Does subcontracting plan apply ?: N

Title: [redacted]

PROJECT ADMINISTRATION DATA

OCA contact: [redacted] 894-4820

Sponsor technical contact

YASHASWINI MITTAL
(202)357-3693
NATIONAL SCIENCE FOUNDATION
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WASHINGTON, D.C. 20550

Sponsor issuing office

PAULETTE L. GREEN
(202)357-9843
NATIONAL SCIENCE FOUNDATION
DGC/MPS
WASHINGTON, D.C. 20550

Security class (U,C,S,TS): U
Defense priority rating : N/A
Equipment title vests with: Sponsor
NONE PROPOSED.

ONR resident rep. is ACO (Y/N): [redacted]
NSF supplemental sheet
GIT X

Administrative comments -
INITIATION. NSF PROVIDING FUNDING FOR THE FIRST YEAR. FUNDING FOR THE
SECOND YEAR WILL COME FROM THE AIR FORCE AT A LATER TIME.

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 03/05/91

Project No. G-37-639 Center No. R6515-0A0

Project Director TONG Y L School/Lab MATH

Sponsor NATL SCIENCE FOUNDATION/GENERAL

Contract/Grant No. DMS-8801327 Contract Entity GTRC

Prime Contract No.

Title INEQUALITIES IN MULTIVARIATE ANALYSIS AND RELIABILITY THEORY

Effective Completion Date 901130 (Performance) 910228 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	
Final Report of Inventions and/or Subcontracts	Y	910228
Government Property Inventory & Related Certificate	Y	
Classified Material Certificate	N	
Release and Assignment	N	
Other	N	

CommentsREQUIREMENT FOR A PATENT REPORT IS SATISFIED BY THE SUBMISSION OF THE 98A._PER ACCTG, OUTSTANDING FINANCIAL OBLIGATIONS (3/19)

Subproject Under Main Project No.

Continues Project No.

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other	N
	N

NOTE: Final Patent Questionnaire sent to PDPI.

NSF Grant No. DMS8801327 Progress Report

Principal Investigator: Yung L. Tong

School of Mathematics

Geogia Institute of Technology

Atlanta, Georgia 30332

During the period of June 16-October 15, 1988, Professor Yung L. Tong continued his research in the general area of inequalities in multivariate analysis and reliability theory. The following is a description of the work completed during that period:

(1) The revision of "Optimal Arrangement of Components Via Pairwise Rearrangements". This is a paper on optimal assembly of systems in reliability theory using probability inequalities. It is joint with Philip Boland and Frank Proschan, and has been submitted for publication.

(2) The revision of "Optimal-Partitioning Inequalities in Classification and Multi-Hypotheses Testing" (joint with Ted Hill). This paper concerns probability inequalities in multi-hypotheses testing and classification theory using Levy concentration function and the Karlin-Rubin theorem. A complete revision of the paper was made, a shorter proof for one of the main results was found, and a counterexample to a conjecture proposed by the associate editor was obtained. The paper has been recently accepted for publication in the Annals of Statistics.

(3) A technical report entitled "Crossing properties of Mixture Distributions", joint with Philip Boland and Frank Proschan, is in preparation. It will be revised and submitted for publication soon.

(4) The manuscript of a research book entitled The Multivariate Normal Distribution authored by the principal investigator has been partially completed, and it has been scheduled for publication by Springer-Verlag in the Springer Series in Statistics. The book contains important results reported recently in the literature but cannot be found in most books on multivariate analysis, and the main themes are dependence, inequalities, and their roles in theory and applications.

A copy of items (1), (2), and one chapter of item (4) are being enclosed with this report.

637 - 639

OMB Number 345-0058

NATIONAL SCIENCE FOUNDATION
1800 G STREET, NW
WASHINGTON, DC 20550

BULK RATE
POSTAGE & FEES PAID
National Science Foundation
Permit No. G-69

PI/PD Name and Address

Yung L. Tong
School of Mathematics
Georgia Tech [redacted]
Atlanta GA 30332

NATIONAL SCIENCE FOUNDATION
FINAL PROJECT REPORT

PART I - PROJECT IDENTIFICATION INFORMATION

1. Program Official/Org. Dr. Nell Sedransk - DMS

2. Program Name STATISTICS & PROBABILITY

3. Award Dates (MM/YY) From: 06/88 To: 11/90

4. Institution and Address
Georgia Tech Research Corp
Administration Building
Atlanta GA 30332

5. Award Number 8801327

6. Project Title
Mathematical Sciences: Inequalities in Multivariate Analysis and Reliability

This Packet Contains
NSF Form 98A
And 1 Return Envelope

NSF Grant Conditions (Article 17, GC-1, and Article 9, FDP-II) require submission of a Final Project Report (NSF Form 98A) to the NSF program officer no later than 90 days after the expiration of the award. Final Project Reports for expired awards must be received before new awards can be made (NSF Grants Policy Manual Section 677).

Below, or on a separate page, provide a summary of the completed projects and technical information and attach it to this form. Be sure to include your name and award number on each separate page. See below for more instructions.

PART II - SUMMARY OF COMPLETED PROJECT (for public use)

The summary (about 200 words) must be self-contained and intelligible to a scientifically literate reader. Without restating the project title, it should begin with a topic sentence starting the project's major thesis. The summary should include, if pertinent to the project being described, the following items:

- **The primary objectives and scope of the project**
- **The techniques or approaches used only to the degree necessary for comprehension**
- **The findings and implications stated as concisely and informatively as possible**

During the period of June 1988 - November 1990, Professor Yung L. Tong had conducted research in the general area of stochastic inequalities, with special emphasis on convexity-related inequalities and dependence-related inequalities in statistics. The research deals with derivation of new results with applications in multivariate statistical analysis, reliability theory, and statistical inference; and the basic mathematical tools used include convexity, majorization theory, conditioning, and mixture of distributions. The completed work consists of seven research papers, one technical report, and one book. In addition to their intrinsic interest, the results also have direct applications in obtaining bounds in multivariate statistics, optimal assembly of systems, comparison of experiments, and other related areas.

PART III - TECHNICAL INFORMATION (for program management use)

List references to publications resulting from this award and briefly describe primary data, samples, physical collections, inventions, software, etc. created or gathered in the course of the research and, if appropriate, how they are being made available to the research community.

Please see attached sheets.

	January 28, 1991
Principal Investigator/Project Director Signature	**Date**

IMPORTANT:
MAILING INSTRUCTIONS

Return this *entire* packet plus all attachments in the envelope attached to the back of this form. Please copy the information from Part I, Block I to the *Attention line* on the envelope.

PART III - TECHNICAL INFORMATION

During the period of June 1988 – November 1990, Professor Yung L. Tong had been the principal investigator for NSF Grants DMS-8801327 (1988-1989) and DMS-8801327, A01 (1989-1990). The primary area of his research was probability inequalities and stochastic orderings in multivariate analysis and reliability theory.

(a) He has coauthored seven papers and one technical report. The contents of the papers are described below:

(a1) The paper Hill and Tong (1989) contains results on optimal-partitioning and minimax risk inequalities for the classification and multi-hypotheses testing problems. Best possible bounds are derived for the minimax risk for location parameter families, based on the tail concentrations and Lévy concentrations of the distributions. Special attention is given to continuous distributions with the maximum likelihood ratio property and to symmetric unimodal continuous distributions, and bounds for general (including discontinuous) distributions are also given. The key mathematical tools used are Lyapounov's convexity theorem and its generalization given by Dvoretzky, Wald and Wolfowitz (1951).

(a2) The paper Boland, Proschan and Tong (1989a) deals with crossing properties of mixtures of density functions from a one-parameter exponential family. Using the results for totally positive functions and the variation diminishing property of such, the effect of sign crossing properties of two mixing densities λ_1 and λ_2 on the resulting mixture distributions f_1 and f_2 is investigated. The new results enable us to make stochastic variability comparisons for binomial-beta, mixed Weibull, and mixed Gamma distributions.

(a3) In Boland, Proschan and Tong (1989b) a notion of criticality of two nodes in a coherent system is introduced, and a monotonicity result concerning the reliability function under component pairwise rearrangements is obtained. That result is then used to develop a method for finding the optimal arrangement of components in a coherent system. As a special application, it yields the result in Theorem 2 of Tong (1985) without evaluating the reliability function of the system.

(a4) In Olkin and Tong (1989) partial orderings of positive dependence of a large class of multivariate exponential distributions are studied, with special emphasis on its effect on the reliability function of a system and the survival probability function of a shock model. A majorization inequality and a dependence-related inequality are given, monotonicity properties of the reliability function and of the survival probability function are established. The proof of the second inequality involves a similar (but more general) argument given in the Marshall and Olkin (1967) paper.

(a5) In Boland, Proschan and Tong (1990) the effect of a certain type of positive dependence among the components of a consecutive k-out-of-n: F system on its reliability function is investigated. Examples of such systems exist in telecommunications, oil pipelines, and integrated circuitry, and most of the existing research on the reliability of such a system assumes that the components function independently. In this paper a model incorporating positive dependence via dependence between adjacent components is introduced, and it is shown that, for $k \geq (n+1)/2$, the system reliability is a decreasing function of this dependence.

(a6) In Shaked and Tong (1990a) the effect of positive dependence on the information provided by the data, under the framework of comparison of experiments introduced by Blackwell (1951, 1953), is studied. It is shown that some sets of positively dependent random variables are less informative than similar sets of independent random variables. It is also shown that the information content (on the common mean μ) of permutation-symmetric multivariate normal random vectors with a common known variance increases as the common correlation coefficient ρ decreases. (When $\rho \geq 0$, two

extreme cases are $\rho = 0$ and $\rho = 1$.) Some results, which compare members of two-parameter exponential families, are also included.

(a7) The paper Shaked and Tong (1990b) contains a survey of recent results on partial orderings of positive dependence of random variables with a common marginal distribution. The paper was presented at the Florida State Statistics Days conference, and will appear in the special issue of *Communications in Statistics.*

(a8) Boland, Proschan and Tong (1991) contains inequalities for the expectations of permutation-invariant concave functions of partial sums of nonnegative exchangeable random variables. Two majorization inequalities are obtained, and an application in reliability theory is given.

(b) A book, entitled *The Multivariate Normal Distribution*, was written by Yung L. Tong during 1987-1989 and published by Springer-Verlag in 1990. The work was partially supported by NSF grants DMS-8502346 and DMS-8801327. The book consists of the following chapters:

Chapter 1. Introduction
2. The Bivariate Normal Distribution
3. Fundamental Properties and Sampling Distribution of the Multivariate Normal Distribution
4. Other Related Properties
5. Positively Dependent and Exchangeable Normal Variables
6. Order Statistics of Normal Variables
7. Related Inequalities
8. Statistical Computing Related to the Multivariate Normal Distribution
9. The Multivariate t Distribution.

As stated in the preface, this volume represents a comprehensive and coherent treatment of the classical and new results related to the multivariate normal distribution, and the main themes are dependence, probability inequalities, and their roles in theory and applications. In particular, Chapters 4, 5, and 7 are directly related to stochastic inequalities, and many of the results contained in those chapters have become available after the publications of Marshall and Olkin (1979) and Tong (1980).

REFERENCES

Blackwell, D. (1951). Comparison of experiments. In *Proc. Second Berkeley Symp. Math. Statist. Probab.*, University of California Press, Berkeley, California, pp. 93-102.

Blackwell, D. (1953). Equivalent comparison of experiments. *Ann. Math. Statist.* 24, 265-272.

Boland, P. J., Proschan, F. and Tong, Y. L. (1989a). Crossing properties of mixture distributions. *Probab. Eng. Inf. Sci.* 3, 355-366.

Boland, P. J., Proschan, F. and Tong, Y. L. (1989b). Optimal arrangement of components via pairwise rearrangements. *Naval Res. Logistics*, 36, 807-815.

Boland, P. J., Proschan, F. and Tong, Y. L. (1990). Linear dependence in consecutive k-out-of-n: F systems. *Probab. Eng. Inf. Sci.*, 4, 391-397.

Boland, P. J., Proschan, F. and Tong, Y. L. (1991). Some majorization inequalities for functions of exchangeable random variables. To appear in *Topics in Statistical Dependence*, H. W. Block, A. R. Sampson and T. H. Savits, eds., Institute of Mathematical Statistics, Hayward, California.

Dvoretzky, A., Wald, A. and Wolfowitz, J. (1951). Relations among certain ranges of vector measures. *Pacific J. Math.* 1, 59-74.

Hill, T. P. and Tong, Y. L. (1989). Optimal-partitioning inequalities in classification and multi-hypotheses testing. *Ann. Statist.* 17, 1325-1334.

Marshall, A. W. and Olkin, I. (1967). A multivariate exponential distribution. *J. Amer. Statist. Assoc.* 62, 30-44.

Marshall, A. W. and Olkin, I. (1979). *Inequalities: Theory of Majorization and Its Applications.* Academic Press, New York.

Olkin, I. and Tong, Y. L. (1989). Positive dependence of a class of multivariate exponential distributions. *Technical Report*, Department of Statistics, Stanford University, Stanford, CA.

Shaked, M. and Tong, Y. L. (1990a). Comparison of experiments for a class of positively dependence random variables. *Canad. J. Statist.* 18, 79-86.

Shaked, M. and Tong, Y. L. (1990b). Positive dependence of random variables with a common marginal distribution. To appear in *Comm. Statist. Ser. A.*

Tong, Y. L. (1985). A rearrangement inequality for the longest run, with an application to network reliability. *J. Appl. Probab.* 22, 386-393.

Tong, Y. L. (1980). *Probability Inequalities in Multivariate Distributions.* Academic Press, New York.

Tong, Y. L. (1990). *The Multivariate Normal Distribution.* Springer-Verlag, New York and Berlin.

PART IV — FINAL PROJECT REPORT — SUMMARY DATA ON PROJECT PERSONNEL

(To be submitted to cognizant Program Officer upon completion of project)

The data requested below are important for the development of a statistical profile on the personnel supported by Federal grants. The information on this part is solicited in response to Public Law 99-383 and 42 USC 1885C. All information provided will be treated as confidential and will be safeguarded in accordance with the provisions of the Privacy Act of 1974. You should submit a single copy of this part with each final project report. However, submission of the requested information is not mandatory and is not a precondition of future award(s). Check the "Decline to Provide Information" box below if you do not wish to provide the information.

Please enter the numbers of individuals supported under this grant.
Do not enter information for individuals working less than 40 hours in any calendar year.

	Senior Staff		Post-Doctorals		Graduate Students		Under-Graduates		Other Participants[1]	
	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.	Male	Fem.
A. Total, U.S. Citizens	1									
B. Total, Permanent Residents										
U.S. Citizens or Permanent Residents[2]:										
American Indian or Alaskan Native										
Asian	1									
Black, Not of Hispanic Origin										
Hispanic										
Pacific Islander										
White, Not of Hispanic Origin										
C. Total, Other Non-U.S. Citizens										
Specify Country 1.										
2.										
3.										
D. Total, All participants (A + B + C)	1									
Disabled[3]										

☐ Decline to Provide Information: Check box if you do not wish to provide this information (you are still required to return this page along with Parts I-III).

[1]Category includes, for example, college and precollege teachers, conference and workshop participants.

[2]Use the category that best describes the ethnic/racial status for all U.S. Citizens and Non-citizens with Permanent Residency. *(If more than one category applies, use the one category that most closely reflects the person's recognition in the community.)*

[3]A person having a physical or mental impairment that substantially limits one or more major life activities; who has a record of such impairment; or who is regarded as having such impairment. *(Disabled individuals also should be counted under the appropriate ethnic/racial group unless they are classified as "Other Non-U.S. Citizens.")*

AMERICAN INDIAN OR ALASKAN NATIVE: A person having origins in any of the original peoples of North America, and who maintain cultural identification through tribal affiliation or community recognition.

ASIAN: A person having origins in any of the original peoples of East Asia, Southeast Asia and the Indian subcontinent. This area includes, for example, China, India, Indonesia, Japan, Korea and Vietnam.

BLACK, NOT OF HISPANIC ORIGIN: A person having origins in any of the black racial groups of Africa.

HISPANIC: A person of Mexican, Puerto Rican, Cuban, Central or South American or other Spanish culture or origin, regardless of race.

PACIFIC ISLANDER: A person having origins in any of the original peoples of Hawaii; the U.S. Pacific Territories of Guam, American Samoa, or the Northern Marianas; the U.S. Trust Territory of Palau; the islands of Micronesia or Melanesia; or the Philippines.

WHITE, NOT OF HISPANIC ORIGIN: A person having origins in any of the original peoples of Europe, North Africa, or the Middle East.

THIS PART WILL BE PHYSICALLY SEPARATED FROM THE FINAL PROJECT REPORT AND USED AS A COMPUTER SOURCE DOCUMENT. DO NOT DUPLICATE IT ON THE REVERSE OF ANY OTHER PART OF THE FINAL REPORT.