GEORGIA INSTITUTE OF TECHNOLOGY — OFFICE OF CONTRACT ADMINISTRATION

PROJECT ADMINISTRATION DATA SHEET

[X] ORIGINAL [ ] REVISION NO. ______

Project No. G-37-616 (R6160-0A0) GTRC/~~XXX~~ DATE 7 / 15 / 86

Project Director: R. P. Kertz School/~~XXX~~ Math

Sponsor: National Science Foundation

Type Agreement: Grant No. DMS-8601153

Award Period: From 7/1/86 To 12/31/88 * (Performance) 3/31/89 (Reports)

Sponsor Amount:

	This Change	Total to Date
Estimated:	$	$ 29,300
Funded:	$	$ 29,300

Cost Sharing Amount: $ ______ Cost Sharing No: G-37-328 (F6160-0A0)

Title: Optional Stopping and Decision Making in Discrete Parameter Stochastic Processes

ADMINISTRATIVE DATA

OCA Contact John B. Schonk X4820

1) Sponsor Technical Contact:

~~Peter Purdue~~ Dr. Y. Mittal

National Science Foundation

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Washington, DC 20550

202-357-9764

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Stephen G. Burnisky

National Science Foundation

DGC/MPS

Washington, DC 20550

202-357-9671

Defense Priority Rating: N/A

Military Security Classification: N/A

(or) Company/Industrial Proprietary: N/A

RESTRICTIONS

See Attached NSF Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval – Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with GIT

COMMENTS:

*Includes a 6 month unfunded flexibility period.

No funds may be expended after 12/31/88.

SPONSOR'S I. D. NO. ______

COPIES TO:

Project Director	Procurement/GTRI Supply Services	GTRC
Research Administrative Network	Research Security Services	Library
Research Property Management	Reports Coordinator (OCA)	Project File
Accounting	Research Communications (2)	Other A. Jones

12/88

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Date 7/27/89

roject No. G-37-616 Center No. R6160-0A0

roject Director R. P. Kertz School/Lab Math

ponsor NSF

ontract/Grant No. DMS-8601153 GTRC XX GIT ___

rime Contract No. ___

itle Optional Stopping and Decision Making in Discrete Parameter Stochastic Processes.

ffective Completion Date 12/31/88 (Performance) 3/31/89 (Reports)

loseout Actions Required:

X None
___ Final Invoice or Copy of Last Invoice
___ Final Report of Inventions and/or Subcontracts
___ Government Property Inventory & Related Certificate
___ Classified Material Certificate
___ Release and Assignment
___ Other ___

ncludes Subproject No(s). ___

ubproject Under Main Project No. ___

ontinues Project No. ___ Continued by Project No. ___

istribution:

X Project Director
X Administrative Network
X Accounting
X Procurement/GTRI Supply Services
X Research Property Management
___ Research Security Services
X Reports Coordinator (OCA)
X GTRC
X Project File
X Contract Support Division (OCA)
___ Other ___

PROGRESS REPORT: NSF GRANT NO. 8601153
ROBERT P. KERTZ

Progress has been made in the two main areas of the grant. Measuring the Effect of Increase or Decrese of Information in Optimal Stopping.

The survey [1] of results, techniques, and variations of prophet vs. statistician comparisons includes some of the new results obtained in this research in the setting of martingale transforms. Optimally-stopped random variables from the optimal stopping of i.i.d. r.v.'s were found to exhibit asymptotic distributional behavior analogous to that of the partial maxima of the i.i.d. r.v.'s in [2]; applications were made to optimal stopping. A distribution-based comparison based on fixing terminating r.v. or supremum r.v. distributions for martingales led to additional characterizations and applications of known results, and new comparisons in a converse setting [3]. The prophet vs. statistician comparison was placed in the unknown distribution setting of multi-armed Bernoulli bandits in [4]; sharp inequalities and some complete comparison regions were found, along with connections to the notion of regret.

Decision-Making in Some Specially-Structured Classes of Multidimensional Markov Processes.

A convexity-based, duality approach was used to find optimal plans of (pure or randomized) Gittins-index-type structure for multi-armed bandit processes under constraints with geometric discounting [5]. Results were obtained [6] on weak convergence of simulated annealing-type Markov chains to diffusions and reflected diffusions; these connections were used to initiate research on using simulated annealing algorithms for finding global optima of functions in nonlinear programming type settings. For the problem of maximizing the probability of exiting from an interval in a limited playing time, it was shown that bold play is optimal for both random walk and diffusion type settings [7]. Optimal stopping and stochastic control is being used to investigate models of proofreading and of computer program debugging in [8].

[1] "Prophet Problems in Optimal Stopping: Results, Techniques, and Variations," Technical Report, School of Mathematics, Georgia Institute of Technology, Atlanta, (1986).

[2] "Limit Theorems for Threshold-Stopped Random Variables," co-authored with D. P. Kennedy, submitted for publication (1987).

[3] "Martingales with Fixed Supremum or Terminating Random Variables," co-authored with Uwe Rosler. Technical Report, School of Mathematics, Georgia Institute of Technology, Atlanta (1987).

[4] "Advantage of Perfect Information-Based Strategies in Bernoulli Bandits," co-authored with D. A. Berry. Technical Report, School of Mathematics, Georgia Institute of Technology, Atlanta (1987).

[5] "Decision Processes Under Total Expected Concomitant Contraints, with Applications to Bandit Processes," Technical Report, School of Mathematics, Georgia Institute of Technology, Atlanta (1986).

[6] "Simulating Annealing, Diffusion Approximations, and Large Deviations for Nonlinear Programming," co-authored with C. Tovey, Technical Report, School of Mathematics, GeorgiaInstitute of Technology, Atlanta (1987).

[7] "Leaving an Interval in Limited Playing Time," co-authored with David Heath, to appear in Advances in Applied Probability (1987).

[8] "Stochastic Control for Problems of Proofreading and Debugging," joint with K. Siegrist. In preparation.

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
NSF FORM 98A

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I–PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Tech Research Corporation Georgia Institute of Technology Atlanta, GA 30332	Statistics & Probability	DMS-8601153
	4. Award Period From 7/1/86 To 12/31/88	5. Cumulative Award Amount $29,300

6. Project Title
Mathematical Sciences: Optimal Stopping and Decision Making in Discrete Parameter Stochastic Processes

PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

Research was carried out in the mathematical theory of probability, focusing chiefly on problems in sequential decision theory and related areas. The primary objectives were to determine: (i) for various classes of sequences of random variables and information structures, how much better a person with specified information concerning the process can do than one without that information; (ii) connections between sequential decision processes and related extreme value processes; and (iii) optimal actions for some specific problems in decision-making.

The project involved one faculty researcher over a two-year period, with input from various collaborators and one graduate student; nearly all the work was theoretical and did not involve computers.

The primary findings included universal bounds for comparisons and limit theorems within various classes of discrete parameter stochastic processes including martingales, independent random variables, and exchangeable random variables, and identification of best possible actions in several specific stochastic control settings. Applications of these results were given to applied areas such as operations research and statistics, as well as to theoretical areas such as classical probability theory.

PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (✓)	Approx. Date
a. Abstracts of Theses	X				
b. Publication Citations		X			
c. Data on Scientific Collaborators		X			
d. Information on Inventions	X				
e. Technical Description of Project and Results		X			
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
Robert P. Kertz		4/3/89

b. Publication Citations

List of papers by Robert P. Kertz which were partially supported by NSF Grant DMS-8601153.

1. "Leaving an interval in Limited Playing Time," co-authored with David Heath, (1988). Advances in Applied Probability 20, 635-645.

2. "Comparison of Stop Rule and Maximum Expectations for Finite Sequences of Exchangeable Random Variables," co-authored with J. Elton, (1986), submitted for publication.

3. "Prophet Problems in Optimal Stopping: Results, Techniques, and Variations," (1986). Technical Report, School of Mathematics, Georgia Institute of Technology, Atlanta.

4. "Decision Processes Under Total Expected Concomitant Constraints, with Applications to Bandit Processes," (1987), submitted for publication.

5. "The Asymptotic Behavior of the Reward Sequence in the Optimal Stopping of i.i.d. Random Variables," co-authored with D. P. Kennedy, (1987), submitted for publication.

6. "Limit Theorems for Threshold-Stopped Random Variables, with Applications to Optimal Stopping," co-authored with D. P. Kennedy, (1988), submitted for publciation.

7. "Value of Perfect Information in Bernoulli Bandits," co-authored with Donald A. Berry, (1988), submitted for publication.

8. "Martingales with Given Maxima and Terminal Distribution," co-authored with Uwe Rösler, (1989), submitted for publication.

9. "Stochastic Control for Problems of Proofreading and Debugging," co-authored with Kyle Siegrist, (1989), in preparation.

10. "Simulated Annealing, Diffusions and Large Deviations for Nonlinear Programming," co-authored with Craig Tovey, (1989), in preparation.

c. Scientific Collaborators

The Principal Investigator for NSF Grant DMS-8601153 was Robert P. Kertz, professor in the School of Mathematics, Georgia Institute of Technology.

e. Technical Summary of Activities and Results

Results of research by R. P. Kertz which was supported in part by NSF Grant DMS-8601153 are summarized below. Abstracts of pertinent papers are given, followed by a summary of meetings, conferences, and workshops at which talks were given. Names of individuals taking part in the research are indicated as co-authors on the papers.

Abstracts of papers by R. P. Kertz which were partially supported by NSF Grant DMS 86-01153 are now given. For papers which determine, for various classes of sequences of random variables and information structures, how much better a person with specified information concerning the process can do than one without that information, see [2, 3, 5, 6, 7]. For papers drawing connections between sequential decision processes and extreme value processes, see [5, 6, 8]. For papers describing optimal actions in some specific problems in decision-making, see [1, 4, 9, 10].

1. "Leaving an interval in Limited Playing Time," co-authored with David Heath, (1988). Advances in Applied Probability 20, 635-645.

Abstract. A player starts at x in $(-G,G)$ and attempts to leave the interval in a limited playing time. In the discrete time problem, G is a positive integer and the position is described by a random walk starting at integer x, with mean increments zero, and variance increment chosen by the player from [0,1] at each integer playing time. In the continuous time problem, the player's position is described by an Ito diffusion process with infinitesimal mean parameter zero and infinitesimal diffusion parameter chosen by the player from [0,1] at each time instant of play. To maximize the probability of leaving the interval $(-G,G)$ in a limited playing time, the player should play boldly by always choosing largest possible variance increment in the discrete-time setting and largest possible diffusion parameter in the continuous-time setting, until the player leaves the interval. In the discrete time setting, this result affirms a conjecture of Spencer. In the continuous time setting, the value function of play is also identified.

2. "Comparison of Stop Rule and Maximum Expectations for Finite Sequences of Exchangeable Random Variables," co-authored with J. Elton, (1986), submitted for publication.

Abstract. For sequences of exchangeable random variables $X_1,\dots,X_n$ taking values in [0,1], results are given which compare $E(\max_{1\leq i\leq n} X_i)$ and $V(X_1,\dots,X_n) = \sup\{EX_t\text{: t is a stop rule for } X_1,\dots,X_n\}$. Complete results are given for the $n = 2$ and $n = \infty$ cases; results are also given for the $2 < n < \infty$ case. A constructive approach is taken, with extremal distributions given where applicable. For the $n = 2$ case, conjugate function theory is used in an essential way. The transformations and constructions of finite sequences of exchangeable random variables which are given should be of independent interest to those working with this class of processes.

3. "Prophet Problems in Optimal Stopping: Results, Techniques, and Variations," (1986). Technical Report, School of Mathematics, Georgia Institute of Technology, Atlanta.

Abstract. This paper presents background, statements, techniques, and variations of prophet problems in optimal stopping. The main formulation of the prophet problem and equivalent formulations are given in sections one and two. Techniques of proof are given in sections three and four; these include reduction procedures, use of verification lemmas, use of moment theory, and other techniques. Variations of the prophet problem in optimal stopping are given in section five. Prophet-type stochastic control problems for transforms of processes are discussed in section six. Examples of prophet inequalities and prophet regions for many classes of processes are given. Proofs in section 5b and in the appendix illustrate the techniques used to prove these results.

4. "Decision Processes Under Total Expected Concomitant Constraints, with Applications to Bandit Processes," (1987), submitted for publication.

Abstract. A problem of optimization under constraints is stated for general nonstationary stochastic decision models, and studied using convexity theory. For constrained multi-armed bandit processes with geometric discounting, optimal plans are characterized through dynamic allocation indices, and value functions are expressed in terms of single-arm value functions. Techniques of proof include randomizations, conjugate duality, and characterizations for the unconstrained problem. Explicit closed-form representations are given for some Bernoulli bandit processes.

5. "The Asymptotic Behavior of the Reward Sequence in the Optimal Stopping of i.i.d. Random Variables," co-authored with D. P. Kennedy, (1987), submitted for publication.

Abstract. Let $X_1, X_2, \ldots$ be integrable, i.i.d. r.v.'s with common distribution function F and let $\{v_n\}_{n\geq 1}$ be the sequence of optimal rewards or values in the associated optimal stopping problem, i.e., $v_n = \sup\{E(X_T)$: T is a stopping time for $\{X_m\}_{m\geq 1}$ and $T \leq n\}$ for $n \geq 1$. For distribution functions F in the domain of attraction of one of the three classical extreme-value laws G_I, G_{II}^{α} or G_{III}^{α}, it is shown that $\lim_n n(1-F(v_n)) = 1$, $1-\alpha^{-1}$, or $1+\alpha^{-1}$ if $F \in \mathfrak{D}(G_I)$, $F \in \mathfrak{D}(G_{II}^{\alpha})$ and $\alpha > 1$, or $F \in \mathfrak{D}(G_{III}^{\alpha})$ and $\alpha > 0$, respectively. From this result, the growth rate of $\{v_n\}_{n\geq 1}$ is obtained and compared to the growth rate of the expected maximum sequence; and the limit distribution of the optimal reward r.v.'s $\{X_{T_n^*}\}_{n\geq 1}$ is derived, where $\{T_n^*\}_{n\geq 1}$ are the optimal stopping times defined by $T_n^* \equiv 1$ if $n = 1$, and for $n = 2,3,\ldots$, by $T_n^* = \min\{1 \leq k < n: X_k > v_{n-k}\}$ if this set is $\neq \emptyset$, and $=$ n otherwise. This tail-distribution growth rate is shown to be sufficient for *any* threshold sequence to be asymptotically optimal.

6. "Limit Theorems for Threshold-Stopped Random Variables, with Applications to Optimal Stopping," co-authored with D. P. Kennedy, (1988), submitted for publciation.

Abstract. The Extremal Types Theorem identifies asymptotic behaviour for the maxima of sequences of i.i.d. r.v.'s. A parallel theorem is given which identifies the asymptotic behaviour of sequences of threshold-stopped r.v.'s. Three new types of limit distributions arise, but normalizing constants remain the same as in the maxima case. Limiting joint distributions are also given for maxima and threshold-stopped r.v.'s. Applications to the optimal stopping of i.i.d. r.v.'s are given.

7. "Value of Perfect Information in Bernoulli Bandits," co-authored with Donald A. Berry, (1988), submitted for publication.

Abstract. For k-armed Bernoulli bandits with discounting, sharp comparisons are given between average optimal rewards for a gambler and for a 'perfectly informed' gambler, over natural collections of prior distributions. Some of these comparisons are proved under general discounting, and

others under nonincreasing discount sequences. Connections are made between these comparisons and the concept of 'regret' in the minimax approach to bandit processes. Identification of extremal cases in the sharp comparisons is emphasized.

8. "Martingales with Given Maxima and Terminal Distribution," co-authored with Uwe Rösler, (1989), submitted for publication.

Abstract. Let μ be any probability measure on $\mathbb{R}$ with $\int|x|d\mu(x) < \infty$, and let μ^* denote its associated Hardy and Littlewood maximal p.m. It is shown that for any p.m. ν for which $\mu < \nu < \mu^*$ in the usual stochastic order, there is a martingale $(X_t)_{0\leq t\leq 1}$ for which $\sup_{0\leq t\leq 1}X_t$ and X_1 have respective p.m.'s ν and μ. The proof uses induction and weak convergence arguments; in special cases, explicit martingale constructions are given. These results provide a converse to results of Dubins and Gilat [6]; applications are made to give sharp martingale and 'prophet' inequalities.

9. "Stochastic Control for Problems of Proofreading and Debugging," co-authored with Kyle Siegrist, (1989), in preparation.

10. "Simulated Annealing, Diffusions and Large Deviations for Nonlinear Programming," co-authored with Craig Tovey, (1989), in preparation.

Talks were given at the following meetings, conferences, and universities by R. P. Kertz.

1. June-July, 1986. Conference on Optimal Stopping and Gambling, Oberwolfach; Workshop on Probabilistic Problems Concerning Strategies, Göttingen (6 lectures based on [3]); and Statistical Laboratory, Cambridge.

2. April, 1987. MSI Workshop on Deterministic and Stochastic Control, Cornell.

3. August, 1987. 16th Conference on Stochastic Processes and Their Applications, Stanford.

4. January, 1988. Co-organizer of Special Session on Optimal Stopping at the Annual Meeting (#839) of the American Mathematical Society, Atlanta. Co-authors Kennedy and Siegrist spoke on joint results [5] and [9] respectively.

5. August, 1988. 207th Meeting of the Institute of Mathematical Statistics, Colorado State Univ., Fort Collins.