GEORGIA INSTITUTE OF TECHNOLOGY — OFFICE OF CONTRACT ADMINISTRATION

PROJECT ADMINISTRATION DATA SHEET

[X] ORIGINAL [ ] REVISION NO. ____

Project No. G 37-635 DATE 6-21-82

Project Director: W. J. Layton School/Lab Mathematics

Sponsor: National Science Foundation, Washington, D.C.

Type Agreement: Grant No. MCS-8202058

Award Period: From 6-15-82 To 11-30-84 (Performance) 2-28-85 (Reports)

Sponsor Amount: $20,900 Contracted through:

Cost Sharing: $5,421 (G-37-339) GTRI/GIT

Title: Finite Element Methods for First Order Systems With Applications to Mixed Equations (Mathematical Sciences)

ADMINISTRATIVE DATA OCA Contact Don Hasty

1) Sponsor Technical Contact:

James M. Greenberg

(202) 357-9764

2) Sponsor Admin/Contractual Matters:

Patrick A. Welch

(202) 357-9671

Defense Priority Rating: N/A Security Classification: N/A

RESTRICTIONS

See Attached NSF Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval – Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with N/A - none proposed

COMMENTS:

A six (6) months unfunded flexibility period is included in the grant - to 11/30/84.

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GEORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 11/8/85

Project No. G-37-635 School/~~Lab~~ XXXX Math

Includes Subproject No.(s) N/A

Project Director(s) W. J. Layton GTRC / ~~GIT~~ XXX

Sponsor National Science Foundation, Washington D.C.

Title Finite Element Methods for First Order Systems with Applications to Mixed Equations (Mathematical Sciences)

Effective Completion Date: 11/30/84 (Performance) 2/28/85 (Reports)

Grant/Contract Closeout Actions Remaining:

- [ ] None
- [ ] Final Invoice or Final Fiscal Report
- [ ] Closing Documents
- [X] Final Report of Inventions Questionnaire note to Project Director (For our file)
- [ ] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other ____________

Continues Project No. ____________ Continued by Project No. ____________

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G-37-635

Georgia Institute of Technology

A UNIT OF THE UNIVERSITY SYSTEM OF GEORGIA

ATLANTA, GEORGIA 30332

SCHOOL OF MATHEMATICS

(404) 894-2700

December 1, 1983

Dr. P. A. Welsh
National Science Foundation
Washington, DC 20550

Dear Dr. Welsh:

It has recently come to my attention that the annual project report for my N.S.F. grant MCS-8202025 (Georgia Tech account number G-37-635) is due. The grant was to develop a class of non-standard finite element methods to approximate nonelliptic problems such as the equations governing transonic flow.

The project has been successful in studying these non-coercive problems from a variety of points of view. A brief description of the research to date is attached. Further, I am happy to report that I have recently had a research break-through on the motivating problem of approximating mixed-type equations by nonstandard least squares methods.

This research also led peripherally to investigations in the related area of the resonance/nonresonance behavior of certain nonlinear differential equations. This work is also described briefly.

Enclosed with this letter, you will find a brief description of the research completed and under way, a list of the resulting publication citations, and data on my scientific collaborators in this period.

Let me take this opportunity to thank you and the N.S.F. for supporting my research projects in basic science.

Sincerely yours,

William Layton

WL+ar

Enclosures

Finite Element Methods for First Order Systems with Applications to Mixed Equations

National Science Foundation
Grant MCS-8202025
G-37-635

Annual Progress Report
June 15, 1983 to June 14, 1983

William J. Layton
Assistant Professor
School of Mathematics
Georgia Institute of Technology
Atlanta, GA 30332

December 1, 1983

TABLE OF CONTENTS

1. Abstract

The principal investigator conducted research in numerical analysis for partial differential equations that do not satisfy the usual coercivity conditions required by the finite element method. These types of equations are motivated by problems in fluid dynamics, e.g., the mixed elliptic-hyperbolic equations governing transonic flow and the singularly perturbed equations governing flow at high Reynold's number.

Substantial progress was made in the study of the time evolution of the error in these types of problems, the study of boundary layers and the effects of degeneracies. Recently, the original problem of finding accurate numerical methods for mixed-type equations has been solved.

In addition, peripheral results in nonlinear analysis have been obtained.

2. Objectives of Research Programs

The objective of this research is to study the behavior of classical and nonstandard numerical methods on problems in P.D.E.'s that are not positive definite. Thus, first order systems of partial differential equations, equations of mixed type, degenerate or singular equations, etc. are studied. Using a careful description of the behavior of, e.g., finite element and finite difference methods, on these problems new "nonstandard" methods can be proposed and analyzed. These nonstandard methods promise to give more accurate, stable etc. solutions to these problems than methods borrowed from elliptic equations.

The ultimate goal is the development of highly accurate, type independent methods that can be used on mixed problems, such as the equations governing transonic flow.

3. Status of Research Program

The error in finite element and finite difference approximations to nonlinear hyperbolic systems was studied in detail in the sequence of papers [1]-[5]. Spectral methods were considered in [6]. A major topic studied is the asymptotic in time behavior of the error. In [1], [3] this was considered in the dissipative case and the conservative case. A notion of nonlinear dissipation was introduced in [1] under which it was shown that the accumulation of local errors is not catastrophic. In [3] the implications of local energy decay in the continuous equation are investigated for difference approximations in the borderline case. In [4] a complete analysis is performed and the long term evolution of the error in the stable, conservative and unstable cases categorized. Note that the question of the long term behavior of the error is extremely important in many applied areas. In these areas, long term forecasts are important or the scale of the observation is large compared to the scales on which the physical interactions occur.

The trade off between accuracy and regularity in hyperbolic problems is considered in [2], [6]. This analysis is carried out on a finite time interval $0 \le t \le T^* < \infty$, where T^* is the time required for the equation to blow-up or produce shocks.

Singularly perturbed and degenerate equations are considered in [7]-[10]. The treatment of problems with severe boundary layers arising from elliptic-hyperbolic singular perturbations is considered in [8]. Such problems arise in conduction-convection phenomena and high Reynold's number flow. Convergence of the F.E.M. for degenerate hyperbolic equations is addressed in [7], [9], [10]. The evolution of the error in dicrete systems is studied in [10]. The effects of numerical integration on this in [9] and superconvergence for such equations investigated in [7].

The investigations in the related but peripheral area of nonlinear analysis are summarized in [11]-[14]. These papers study in detail the resonance-nonresonance possibilities of nonlinear differential equations describing phenomena with memory. The categorization of various types of solutions, e.g., almost periodic, quasiperiodic, limiting behavior at $\pm\infty$, causal properties, etc. is accomplished in [11]-[14].

4. Ongoing Research Program

The research summarized is a part of an ongoing research program. The principal investigator is currently involved in extending the defect correction method, considered in [8], to the incompressible Navier-Stokes equations at high Reynold's number. This work is cooperative with researchers in the numerical analysis institute in the Univ. of Nijmegen, the Netherlands. The approximate solution of the Navier-Stokes equations at high Reynold's number is a tremendously important area of current research in applied mathematics.

A breakthrough has also recently been made in the use of nonstandard types of least squares methods for mixed equations modeling transonic flow. When the research is completed manuscripts will be sent to the N.S.F.

In addition to the above, a number of minor research projects are underway. These will be summarized in the next project report.

5. Publication Citations

Evolution of Error in Non-Coercive Problems

1. On the behavior over long time intervals of finite difference and finite element approximations to hyperbolic equations, to appear in Comp. and Math w. Appls.

2. The accuracy of finite element approximations to semilinear, first order, hyperbolic systems, submitted for publication, currently under review.

3. Error estimates for finite difference approximations to hyperbolic equations for large time, submitted for publication, currently under review.

4. (with R. Mattheij), Estimates over infinite intervals of approximations to initial value problems, Tech. Report 8307, Univ. of Nijmegen, the Netherlands; submitted, currently under review.

5. Effects of artificial viscosity in Galerkin methods, submitted, currently under review.

6. On the convergence of spectral methods for semilinear hyperbolic equations, submitted, currently under review.

Degenerate and Singularly Perturbed Problems

7. The F.E.M. for a degenerate hyperbolic partial differential equation, B.I.T., V. 23 (1983), pp. 231-238.

8. (with O. Axelsson), Defect correction methods for convection dominated, convection-diffusion equations, Tech. Report, Univ. of Nijmegen, the Netherlands, submitted, currently under review.

9. Some effects of numerical integration in the F.E.M. for degenerate evolution equations, to appear in Calcolo.

10. An energy analysis of a degenerate hyperbolic partial differential equation, submitted, currently under review.

Theory of Nonlinear Differential Equations

11. Existence of almost periodic solutions to delay differential equations with Lipschitz nonlinearities, to appear in J. Diff. Eqns.

12. (with L. Drager), On nonlinear difference approximations to nonlinear F.D.E.'s, to appear in Libertas Math.

13. (with L. Drager), Nonresonance in F.D.E.'s with small time lag, to appear in Proc. Int. Conf. on F. D. Systems III, Banach Center Publications, 1983.

14. (with L. Drager), Nonlinear delay equations and function algebras, to appear in Proc. Int. Conf. on Diff. Eqns., North Holland, 1983.

Miscellaneous Numerical Analysis

15. Convergence of the F.E.M. for semilinear elliptic equations, submitted, currently under review.

16. On the uniform convergence of Euler's method, Tech. Report, Univ. of Nijmegen, the Netherlands.

Miscellaneous Remarks

In addition to the above papers, other scientific activity included numerous conferences attended, invited and contributed talks, seminars, and discussions of research interests with European and American scientists.

6. Data on Scientific Collaborators

O. Axelsson
Professor of Mathematics
Mathematics Institute
University of Nijmegen
Nijmegen, the Netherlands

R. Mattheij
Professor of Mathematics
Mathematics Institute
University of Nijmegen
Nijmegen, the Netherlands

L. Drager
Assistant Professor
Mathematics Department
Texas Technological University
Lubbock, Texas, U.S.A.

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
NSF FORM 98A

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I—PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
William Layton/School of Math. Georgia Institute of Tech. Atlanta, GA 30332	Apl. Math.	MCS-820-2025
	4. Award Period From 8/15/82 To 11/30/84	5. Cumulative Award Amount $ 20, ~~870~~ 900

6. Project Title

Finite element methods for first order systems with applications to mixed equations

PART II—SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

Research was conducted in numerical analysis for differential equations that do not satisfy the conditions required for most numerical methods to be successful. These types of equations are motivated by problems in transonic and supersonic aerodynamics, e.g., the mixed elliptic-hyperbolic equations governing transonic flow and the singularly perturbed equations governing flow at high Reynold's number.

Substantial progress was made in the study of the time evolution of the error in numerical methods for these types of problems, and the study of boundary layers and the effects of degeneracies. The original problem of finding accurate numerical methods for mixed-type equations has also been solved.

In addition very interesting peripheral results in nonlinear analysis have been obtained.

PART III—TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (✓)	Approx. Date
a. Abstracts of Theses	✓				
b. Publication Citations		✓			
c. Data on Scientific Collaborators		✓			
d. Information on Inventions	✓				
e. Technical Description of Project and Results		✓			
f. Other *(specify)* preprints and reprints				✓	10/85

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
WILLIAM J. LAYTON		9/6/85

NSF Form 98A (3-83) Supersedes All Previous Editions

Form Approved OMB No. 3145-0058

Carnegie-Mellon University

Department of Mathematics
Schenley Park
Pittsburgh, Pennsylvania 15213
(412) 578-2545

September 6, 1985

National Science Foundation
Washington, DC 20550

Dear Sir/Madam:

Enclosed is the final project report for my N.S.F. grant MCS-8202025. I was notified by the office of contract administration at Georgia Tech on 8/22/85 that it was overdue. Unfortunately, I was out of the country at the time and I have just now returned. Please excuse the delay.

I am collecting copies of all the reprints that arose from the grant and I will send these to the N.S.F. by late October.

Thank you for your patience.

Sincerely yours

William Layton

WL/sd

Enclosure

Finite Element Methods for First Order Systems with Applications to Mixed Equations

National Science Foundation
Grant MCS-8202025
G-37-635

Final Report
August, 1985

William J. Layton[(1)]
Associate Professor
Mathematics Department
Carnegie-Mellon University
Pittsburgh, PA 15213

September, 1985

[(1)] On leave from: School of Mathematics, Georgia Institute of Technology, Atlanta GA 30332

TABLE OF CONTENTS

1. Abstract

The principal investigator conducted research in numerical analysis for partial differential equations that do not satisfy the usual coercivity conditions required by the finite element method. These types of equations are motivated by problems in fluid dynamics, e.g., the mixed elliptic-hyperbolic equations governing transonic flow and the singularly perturbed equations governing flow at high Reynold's number.

Substantial progress was made in the study of the time evolution of the error in these types of problems, and the study of boundary layers and the effects of degeneracies. Recently, the original problem of finding accurate numerical methods for mixed-type equations has been solved. Specifically, a class of finite element methods has been devised and analyzed for (possibly nonpositive) symmetric systems of partial differential equations. Work is currently underway to study if the restriction of symmetry can also be dropped for specific systems, e.g., the primitive variable formulation of transonic flow.

In addition, very interesting peripheral results in nonlinear analysis have been obtained.

2. Objectives of Research Programs

The objective of this research is to study the behavior of classical and nonstandard numerical methods on problems in P.D.E.'s that are not positive definite. Thus, first order systems of partial differential equations, equations of mixed type, degenerate or singular equations, etc. are studied. Using a careful description of the behavior of, e.g., finite element and finite difference methods, on these problems new "nonstandard" methods have been proposed and analyzed. These nonstandard methods promise to give more accurate, stable etc. solutions to these problems than classical methods borrowed from elliptic equations.

The ultimate goal is the development of highly accurate, type independent methods that can be used on a wide class of problems in mathematical physics.

3. Status of Research Program

The error in finite element and finite difference approximations to nonlinear hyperbolic systems was studied in detail in the sequence of papers [1]-[7]. Spectral methods were considered in [5]. A major topic studied is the asymptotic in time behavior of the error. In [1], [3] this was considered in the dissipative case and the conservative case. A notion of nonlinear dissipation was introduced in [1] under which it was shown that the accumulation of local errors is not catastrophic. In [3] the implications of local energy decay in the continuous equation are investigated for difference approximations in the borderline case: In [4] a complete analysis is performed and the long term evolution of the error in the stable, conservative and unstable cases is categorized. Note that the question of the long term behavior of the error is extremely important in many applied areas. In these areas, long term forecasts are important or the scale of the observation is large compared to the scales on which the physical interactions occur.

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The investigations in the related but peripheral area of nonlinear analysis are summarized in [12]-[18]. These papers study in detail the resonance-nonresonance possibilities of nonlinear differential equations describing phenomena with memory. The categorization of various types of solutions, e.g., almost periodic, quasiperiodic, limiting behavior at $\pm\infty$, casual properties, etc. is accomplished in [12]-[18].

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In addition, the original (motivating) problem has been resolved. In [7] a class of high accuracy, stable numerical methods has been proposed for solving symmetric, possibly nonpositive, systems of first order partial differential equations. An analysis of the resulting methods was preformed. Currently the methods are being studied for nonsymmetric systems when the "nonsymmetries" have the specific structure of the ones occuring in the primitive variable formulation of transonic flow.

5. Publication Citations

Evolution of Error in Non-Coercive Problems

1. On the behavior over long time intervals of finite difference and finite element approximations to hyperbolic equations, Comp. and Math W. Appls., V. 11 (1985) 93-112

2. The accuracy of finite element approximations to semilinear, first order, hyperbolic systems, submitted for publication, currently under review.

3. Error estimates for finite difference approximations to hyperbolic equations for large time, PAMS, V. 92 (1984), 425-431.

4. (with R. Mattheij), Estimates over infinite intervals of approximations to initial value problems, Tech. Report 8307, Univ. of Nijmegen, the Netherlands.

5. On the convergence of spectral methods for semilinear hyperbolic equations, in: Num. Funct. Anal. w. Appls. (1985).

6. (with L. Drager and R.M.M. Mattheij), Asymptotics of numerical methods for nonlinear evolution equation, in: Proc. VI Int. Conf. on Thy, and Practice of Nonlinear Anal., North Holland, 1984.

7. High accuracy finite element methods for positive symmetric systems, to appear in Comp. and Math. w. Appls.

Degenerate and Singularly Perturbed Problems

8. The F.E.M. for a degenerate hyperbolic partial differential equation, B.I.T., V. 23 (1983), pp. 231-238.

9. (with O. Axelson), Defect correction methods for convection dominated, convection-diffusion equations, Tech. Report, Univ. of Nijmegen, the Netherlands, submitted, currently under review.

10. Some effects of numerical integration in the F.E.M. for degenerate evolution equations, Calcolo, V. 21 (1984), 425-431.

11. An energy analysis of a degenerate hyperbolic partial differential equation, Apl. Math., V. 29, (1984), 350-366.

Theory of Nonlinear Differential Equations

12. Existence of almost periodic solutions to delay differential equations with Lipschitz nonlinearities, in J. Diff. Eqns., V. 55 (1984), 151-164.

13. (with L. Drager), Qualitative properties of bounded solutions to nonresonant delay equations and algebras and ideals of bounded continuous functions, Tech. Report (1984).

14. (with L. Drager), Nonresonance in F.D.E.'s with small time lag, in Proc. Int. Conf. on F. D. Systems III, Banach Center Publications, 1983.

15. (with L. Drager), Nonlinear delay equations and function algebras, in Proc. Int. Conf. on F. Diff. Engs., North Holland, 1983.

16. (with L. Drager), On nonlinear difference approximation to nonlinear functional differential equations, Libertas Math. V. 3 (1983), 45-65.

17. (with L. Drager), Some results on non-resonant, nonlinear delay differential equation, in Proc. VI Int. Conf. on Thy and Practice of Nonlinear Anal., North Holland, 1984.

18. The Galerkin Method for the approximation of almost periodic solutions of functional differential equations, to appear in: Funk. Eval.

Miscellaneous Numerical Analysis

19. (with E. Harrell), convergence of the F.E. approximations, for semilinear elliptic equations, submitted, currently under review.

20. On the uniform convergence of Euler's method, Tech. Report, Univ. of Nijmegen, the Netherlands.

Miscellaneous Remarks

In addition to the above papers, other scientific activity included numerous conferences attended, invited and contributed talks, seminars, and discussions of research interests with European and American scientists.

6. Data on Scientific Collaborators

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