

PROJECT ADMINISTRATION DATA SHEET

ORIGINAL REVISION NO. _____

Project No. G-37-609 GTRI/GIT DATE 7/25/84

Project Director: Dr. W. F. Ames School/~~XXX~~ Mathematics

Sponsor: U. S. Army Research Office, Research Triangle Park, NC 27709

Type Agreement: SFRC DAAG29-84-K-0083

Award Period: From 7/1/84 To 10-15-86 (~~6/30/85~~) (Performance) 8/30/85 (Reports)

| | | |
|-----------------|--------------------|----------------------|
| Sponsor Amount: | <u>This Change</u> | <u>Total to Date</u> |
| Estimated: | \$ <u>24,986</u> | \$ <u>24,986</u> |
| Funded: | \$ <u>24,986</u> | \$ <u>24,986</u> |

Cost Sharing Amount: \$ 12,418 Cost Sharing No: G-37-317

Title: Linear or Nonlinear Wave Problems with Input Sets - II

ADMINISTRATIVE DATA

OCA Contact William F. Brown x-4820

1) Sponsor Technical Contact:

Dr. Julian J. Wu
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709

2) Sponsor Admin/Contractual Matters:

T. A. Bryant
ONR RR
Georgia Tech

Defense Priority Rating: None shown Military Security Classification: None
(or) Company/Industrial Proprietary: N/A

RESTRICTIONS

See Attached Gov't. Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval - Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of \$500 or 125% of approved proposal budget category.

Equipment: Title vests with GIT (Requires prior approval to purchase equipment not included in the proposal budget.

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Date 4-23-87

Project No. G-37-609 School XXX Math

Includes Subproject No.(s) N/A

Project Director(s) W.F. Ames GTRC XXX

Sponsor U.S. Army Research Office, Research Triangle Park, NC 27709

Title Linear or Nonlinear Wave Problems with Input Sets - II

Effective Completion Date: 10/15/86 (Performance) 12/15/86 (Reports)

Grant/Contract Closeout Actions Remaining:

- None
- Final Invoice or Final Fiscal Report
- Closing Documents
- Final Report of Inventions
- Govt. Property Inventory & Related Certificate
- Classified Material Certificate
- Other _____

Continues Project No. G-37-629 Continued by Project No. _____

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 Other Duane H.
Angela DuBose
Russ Embry

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PROGRESS REPORT

TWENTY COPIES REQUIRED

1. ARO PROPOSAL NUMBER: 20602-MA
2. PERIOD COVERED BY REPORT: 1 July 1984 - 31 December 1984
3. TITLE OF PROPOSAL: Linear or Nonlinear Wave Problems with Input Sets-II
4. CONTRACT OR GRANT NUMBER: DAAG29-84-K-0083
5. NAME OF INSTITUTION: Georgia Institute of Technology
6. AUTHORS OF REPORT: W. F. Ames

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES:
 - a) "Nonlinear Waves in the Pellet Fusion Process," in Wave Phenomena: Modern Theory and Applications (C. Rogers, T. B. Moodie, eds.) Elsevier 1984, pp. 199-210 (V. J. Ervin, W. F. Ames and E. Adams).
 - b) "Linear or Nonlinear Hyperbolic Wave Problems with Input Sets II," in press Int. Journal of Computers and Mathematics with Applications (E. Adams and W. F. Ames).
8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:
 - a) V. J. Ervin - 1 month
W. F. Ames - 1 month
E. Adams - 3 weeks
 - b) V. J. Ervin - Ph.D. awarded substantial support was contributed by this project

William F. Ames
 Department of Mathematics
 Georgia Institute of Technology
 Atlanta, GA 30332

BRIEF OUTLINE OF RESEARCH FINDINGS

I. E. Adams (Army Support) and J. V. Herod

"Bounds for Spatially Nonhomogeneous Model Boltzmann Energy Equations,"
(To be given at World Congress of IMACS, OSLO, August 1985).

Nonlinear model Boltzmann equations for particle distribution functions $f(t,r,v)$ are notoriously complicated. Such equations describe the evolution in time t of distributions f of particles located at position r and having velocity v . The equations are often described by two types of evolution mechanism: a dispersion term and a collision term. It is necessary that reasonable assumptions be made about these two terms if the model is to remain physically realistic and if the model is to yield the mathematical analysis.

A variety of model Boltzmann equations have been studied: for some, it is hoped that more complicated models can be reduced to the one being studied; for others, it is hoped that techniques developed to study the simple models will provide general techniques that work in other settings.

Because spatially inhomogeneous distributions of the Boltzmann equations are of considerable interest, we suggest a more general Boltzmann energy equation which retains a dependence on the spatial variable r , as well as the energy variable x . The two particle collision term remains independent of space and time; it depends only on the energy of the colliding particles. We shall use a general collision kernel which includes the Tjon-Wu model as an example.

It is shown that disregarding the direction of the velocity vectors and using just its magnitude may lead to an uncountable family of model Boltzmann energy equations. Solutions of any of these equations will be dominated by bounds that are obtained through appropriate differential inequalities.

II. W. F. Ames and E. Adams (Both with support from the Army)

"Nonlinear Constitutive Equations and Uniform Boundedness of Perturbed Solutions of Evolution-Type."

This work considers the influences of nonlinear terms in the constitutive equations in the case of evolution type mathematical models, making use of a quantitative sufficient criterion ensuring stability. The application of this criterion rests on the construction of a very sharp componentwise enclosure of the set of perturbed solutions of the given or approximating system of differential equations, where these perturbations are due to initial data or parameters in the equations. If this criterion is satisfied, the following well-known difficulties of standard mathematical or numerical methods have been resolved in the problem at hand:

(A) The influence of nonlinearities has been accounted for almost precisely and geared to the problem under investigation, i.e., not only by use of generally valid and, consequently, crude estimates. Consequently, a stabilizing influence of a nonlinearity can be ascertained.

(B) Stability is confirmed quantitatively and with respect to known finite initial or parameter perturbations.

(C) Finite sets of initial or parameter perturbations are admitted and their nonlinear influences are fully taken into account, without making use of any linearization.

BRIEF OUTLINE OF RESEARCH FINDINGS (Continued)

(D) Whereas the standard practical methods for nonlinear problems yield approximations without a quantitative and sharp error estimate, the subsequent treatment of ordinary differential equations accounts for every possible error, i.e., input deviations discretization errors, rounding errors, procedural errors in the determination of the values of standard functions, etc.

(E) A sufficient criterion asserts stability for all positive t , even though its practical execution is confined to a finite time interval.

(F) The execution of the test, leading to the fulfillment of the sufficient criterion, provides a rigorous sensitivity and/or safety analysis with respect to the non-enumerably many combinations of admissible initial and parameter perturbations.

III. V. J. Ervin and W. F. Ames (Both supported by the Army)

"Nonlinear Waves in the Pellet Fusion Process."

A Thesis submitted to the Georgia Institute of Technology for the Ph.D. in Applied Mathematics (September 1984).

A gas dynamic model of the pellet fusion process having a time-invariant source term is studied by means of group analysis. Some exact solutions of this nonlinear system are constructed for specific (physical) values of parameters. The development of multiple shock waves is demonstrated in several cases analytically. Additional numerical results illustrate the evolution of singularities.

PROGRESS REPORT

TWENTY COPIES REQUIRED

1. ARO PROPOSAL NUMBER: 20602-MA
2. PERIOD COVERED BY REPORT: 1 January 1985 - 30 June 1985
3. TITLE OF PROPOSAL: Linear or Nonlinear Wave Problems with Input Sets-II
4. CONTRACT OR GRANT NUMBER: DAAG29-84-K-0083
5. NAME OF INSTITUTION: Georgia Institute of Technology
6. AUTHORS OF REPORT: W. F. Ames

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES:
 - a) Bounds for Spatially Nonhomogeneous Model Boltzmann Energy Equations, in Proceedings of the 11th IMACS World Congress, August 5-9, 1985, Oslo, Norway (E. Adams, J. V. Herod and H. Spreuer), in press.
 - b) Analysis of Fluid Equations by Group Methods, in Proceedings of the Third U. S. Army Conference on Applied Mathematics and Computing, Atlanta, GA (W. F. Ames and M. C. Nucci), in press.
8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:
 - W. F. Ames, 2 Months (Including cost-share time).
 - J. F. Froehlich, 6 Months (1/3 time)

William F. Ames
Department of Mathematics
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Atlanta, GA 30332

BRIEF OUTLINE OF RESEARCH FINDINGS

I. E. Adams (Army Support) and J. V. Herod

The Boltzmann energy equation is generalized by means of an additional convection term. The energy density F solving this equation is considered as a function of time t , vectors r , and energy x in o, L . An upper bound is verified by use of the theory of differential and integral inequalities. (Two copies of this manuscript are enclosed).

II. W. F. Ames (Army Support) and M. C. Nucci (NATO Support)

Using the machinery of Lie group analysis several equations arising in fluid mechanics are studied. In particular, the Burgers' equation, the KdV equation, the Hopf equation, the two dimensional KdV equation and the Lin-Tsien equation are analyzed. In all cases the particular group includes arbitrary functions of time which permit the transformation of time dependent equations into the corresponding time independent ones. Infinitely many time dependent solutions are associated with each steady solution. Some solutions are constructed. (Two copies of this manuscript are enclosed).

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1. ARO PROPOSAL NUMBER: 20602-MA
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3. TITLE OF PROPOSAL: Linear or Nonlinear Wave Problems with Input Sets-II
4. CONTRACT OR GRANT NUMBER: DAAG29-84-K-0082
5. NAME OF INSTITUTION: Georgia Institute of Technology
6. AUTHORS OF REPORT: William F. Ames
7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES:
 - (a) Analysis of Fluid Equations by Group Methods, to appear in the Journal of Engineering Mathematics, (with M. C. Nucci)
 - (b) Untersuchung der praktischen Stabilitat von Losunger Nichtlinearer hyperbolischer Anfangsrandwertaufgaben, A. Angew.Math. u.Mech, 65, 1985, pp. T 76- T 78 (with Lohner, R. and Adams, E.) (copy enclosed)
 - (c) Approximate Practical Stability for Nonlinear Evolution Partial Differential Equations, Preprint enclosed (with E. Adams and R. Lang), to appear in Eleventh IMACS World Congress Proceedings, Oslo, Norway, Volume 1, Edited by R. Vichnevetsky and J. Vignes, North-Holland.
 - (d) ~~Probability Distributions over Output Intervals for nonlinear Ordinary differential equations with given distributions over input intervals.~~ In 11th IMACS Proceedings (see (c)). Preprint enclosed/
8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

W. F. Ames, One Month
 Armin Holzmuller, 4 months (1/6 time)

Note: The following authors in 7 (above) have been supported in the past: R. Lohner, E. Adams, R. Lang

William F. Ames
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BRIEF OUTLINE OF RESEARCH FINDINGS

PAPER C

The computer-aided execution of a verification algorithm ensures the uniform boundedness of all perturbed solutions with initial values and parameters from finite neighborhoods. This test is applied with respect to the system of ordinary differential equations as following from a Fourier-approximation and a collocation method. Applications are concerned with a system of three PDEs (of first order each) from continuum mechanics. The decisive importance of the computer-aided execution of suitable enclosure methods is shown.

PAPER D

componentwise output enclosure is determined for the set of solutions of a linear or nonlinear ordinary initial value problem with input intervals of parameters or initial values. Distributions are chosen over these input intervals. In the present paper, bounds of the distribution function over an output interval are constructed. Numerous examples are presented.

Uncertainties in real world problems generate mathematical models with deterministic or stochastic sets of input parameters. Subsequently these parameters will be confined to given intervals such that (i) every number in an interval is simultaneously admitted with a (non)constant density function or (ii) the precise value of a parameter is not known in an interval. Whereas Case (ii) is typical for empirical data, case (i) appears in the simultaneous analysis of (non)enumerable different load conditions of a system.

The output set of solutions generally possesses a concentration of its values in a neighborhood of a certain point of an output interval. A quantitative investigation of this concentration is the predominant purpose of this practically oriented paper. Applications of the subsequent analysis require the mutual independence of the input parameters. This property perhaps holds only for a suitably identified deduced version of the problem to be investigated.

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- 4. CONTRACT OR GRANT NUMBER: DAAG-29-84-K-0083 G 37-609
DAAG-29-84-K-0083
- 5. NAME OF INSTITUTION: Georgia Institute of Technology
- 6. AUTHORS OF REPORT: William F. Ames

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES:

"Analysis of Fluid Equations by Group Methods," J. Eng. Math. 20, 181-187, 1986. An expanded version of this work will be presented to the IMACS Symposium on Computational Acoustics (Yale University August 6, 7, 8, 1986). A preprint is enclosed with the original of this report.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

W. F. Ames, one month
Armin Holzmuller, five months (1/6 time)

William F. Ames
Department of Mathematics
Georgia Institute of Technology
Atlanta, GA 30332

1. Group Analysis for Wave Propagation in Viscoelastic and Viscoplastic Materials. (Ames, Holzmuller)

The full admitted group has been generated, together with determination of the arbitrary element (equation of state) for the equations describing wave propagation in nonlinear viscoelastic and viscoplastic one dimensional materials. The generators of the group are being used to study specific exact solutions. This work is an extension of results published in the International Journal of Nonlinear Mechanics 17, pp. 223-230, 1982 by Ames and Suliciu and sponsored by an earlier Army Project.

2. Group Analysis for Wave Propagation in Acoustics. (Ames, Richards)

The full group admitted by the Helmholtz equation in circular cylindrical coordinates has been generated. In the infinitesimal form the coefficients of the r (radial) and z (longitudinal) satisfy Laplace's equation while that for θ (angle) is constant. An infinite number of transformations are therefore made available for the generation of large classes of new solutions. Several have been explored.

3. Nonlinear Membrane Tire Model. (Adams, Ames, Weiler)

This generalization of literature models includes a geometric nonlinearity. The numerical results show that a periodic solution exists for every reasonable choice of initial tire deformation. Nonlinear elasticity and dissipation will also be considered.

4. Numerical Upper and Lower Bounds. (Adams, Ames, Lang)

In paper (C) of the previous report, "Approximate Practical Stability for Nonlinear Evolution PDE's," considerable computation has been accomplished. The system is third order with a nonlinear constitutive relation.

6-37-609

LINEAR OR NONLINEAR WAVE
PROBLEMS WITH INPUT SETS

FINAL REPORT

PROF. WILLIAM F. AMES
SCHOOL OF MATHEMATICS

MARCH 6, 1987

U.S. ARMY RESEARCH OFFICE
CONTRACT NO. DAAG 29-84-K-0083

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| | | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s) PROF. WILLIAM F. AMES | | 8. CONTRACT OR GRANT NUMBER(s) DAAG29-84-K-0083 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS SCHOOL OF MATHEMATICS GEORGIA INSTITUTE OF TECHNOLOGY ATLANTA GA 30332 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park NC 27709 | | 12. REPORT DATE MARCH 5, 1987 |
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LINEAR OR NONLINEAR WAVE
PROBLEMS WITH INPUT SETS

FINAL REPORT

PROF. WILLIAM F. AMES
SCHOOL OF MATHEMATICS

MARCH 6, 1987

U.S. ARMY RESEARCH OFFICE
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A. Research Program

This research has three main parts. The first part consists of a systematic search, compilation and generation of exact explicit solutions for classes of practical nonlinear wave problems. These solutions, to be used in the second and third parts, will be developed, or have already been generated, using a wide variety of ad-hoc methods.

The second part of the research concerns the construction of approximate solutions, with quantitative error estimates, which are generalizations of the explicit exact solutions. The approximate solutions are obtained by a variety of analytical methods.

The third part of the project concerns input sets for both data and coefficients in wave problems. The purpose here is to develop a quantitative sensitivity analysis admitting finite deviations of inputs. The method consists chiefly in the application of truncated Taylor expansions with respect to input parameters followed by a quantitative estimate of the remainder using a Neumann series. Numerical implementation using deferred interval analysis will be carried out.

Applications have been made to several physical nonlinear wave problems. These are discussed in the sequel insofar as they are completed. The unfortunate interruption of funding for the third year means that some of these results are incomplete.

B. Summary of Results C. Publications (Given Together)

I. Analytical Studies

1. Nonlinear Waves in the Pellet Fusion Process

(V. J. Ervin, W. F. Ames and E. Adams)

A gas dynamic model of the pellet fusion process having a time-invariant source term is studied by means of group analysis. Some exact solutions of this nonlinear system are constructed for specific (physical) values of parameters. The development of multiple shock waves is demonstrated in several cases analytically. Additional numerical results illustrate the evolution of singularities.

Published as follows:

- i) Transactions of the Third Army Conference on Applied Mathematics and Computing, ARO Rept. 86-1, pp. 605-618, 1986.
- ii) Wave Phenomena: Modern Theory and Applications (Ed. T. B. Moodie and C. Rogers) North Holland, pp. 199-210, 1984.
- iii) Ph.D. Thesis for V. J. Ervin - Sept. 1984, Georgia Institute of Technology.

2. Analysis of the Von Karman Equations by Group Methods

(K. A. Ames and W. F. Ames)

One of the systems of equations approximating the large deflection of plates consists of two coupled nonlinear fourth order partial differential equations, known as the von Karman equations. The full symmetry group for the steady equations is a finitely generated Lie group with ten parameters. For the time-dependent system the full symmetry group is an infinite parameter Lie group. Several subgroups of the full group are used to generate exact solutions of the time-independent and the time-dependent systems. These include the dilatation group (similar solutions), rotation group, screw group and others. Physical implications and applications are discussed.

Published as follows:

- i) Transactions of the First Army Conference on Applied Mathematics and Computing, ARO Rept. 84-1, pp. 289-300, 1985.
- ii) International Journal of Nonlinear Mechanics, 20, 201-209, 1985.

3. Analysis of Fluid Equations by Group Methods

(W. F. Ames and M. C. Nucci)

Using the machinery of Lie-group analysis several equations arising in fluid mechanics are studied. In particular, the Burgers' equation, the KdV equation, and the Lin-Tsien equation are analyzed. In all cases the particular group includes arbitrary functions of time which

permit the transformation of time-dependent equations into the corresponding time-independent ones. Infinitely many time-dependent solutions are associated with each steady solution. Some solutions are constructed.

Published as follows:

- i) Transactions of the Third Army Conference on Applied Mathematics and Computing, ARO Rept. 86-1, pp. 589-596, 1986.
 - ii) Journal of Engineering Mathematics, 20, pp. 181-187, 1985.
 - iii) An expanded version is to be published in the Proceedings of the Yale International Symposium on Computational Acoustics (1987).
4. Nonlinear Wave Propagation in Viscoplastic, Viscoelastic and Electrical Transmission Lines
(W. F. Ames, A. Holzmuller, E. Adams)

The exploration of the three equations for wave propagation in one-dimensional viscoplastic, viscoelastic and electrical transmission lines, described in the paper by W. F. Ames and I. Suliciu ("Some Exact Solutions for Wave Propagation in Viscoelastic, Viscoplastic and Electrical Transmission Lines," Int. J. Nonlinear Mechanics, 17, pp. 223-230, 1982), demonstrated that certain classes of realistic nonlinear media possessed exact invariant solutions. For others a solution algorithm was developed.

In an incomplete work (enclosed) this system is studied in general to discover what restrictions need be made on the physical constitutive relations in order for invariant solutions to exist. This work continues. It is related to Problem II - 2 of the Approximate Studies Section.

II. Construction of Approximate Solutions

- 1. Bounds for Spatially Nonhomogeneous Model Boltzmann Energy Equations
(E. Adams, J. Herod, H. Spreuer)

Nonlinear model Boltzmann equations for particle distribution functions $f(t,r,v)$ are notoriously complicated.

Such equations describe the evolution in time t of distributions f of particles located at position r and having velocity v . The equations are often described by two types of evolution mechanism: a dispersion term and a collision term. It is necessary that reasonable assumptions be made about these two terms if the model is to remain physically realistic and if the model is to yield the mathematical analysis. Because spatially inhomogeneous distributions of the Boltzmann equations are of considerable interest, we suggest a more general Boltzmann energy equation which retains a dependence on the spatial variable r , as well as the energy variable x . The two particle collision term remains independent of space and time; it depends only on the energy of the colliding particles. We shall use a general collision kernel which includes the Tjon-Wu model as an example.

It is shown that disregarding the direction of the velocity vectors and using just its magnitude may lead to an uncountable family of model Boltzmann energy equations. Solutions of any of these equations will be dominated by bounds that are obtained through appropriate differential inequalities.

Published as follows:

- i) Proceedings of the 11th IMACS World Congress (Oslo, 1985), Volume 1, pp. 123-126, 1985.
 - ii) Submitted to the Journal of Nonlinear Analysis and Applications.
2. Nonlinear Constitutive Equations and Uniform Boundedness of Perturbed Solutions of Evolution-Type
(E. Adams, W. F. Ames and R. Lang)

This work considers the influences of nonlinear terms in the constitutive equations in the case of evolution type mathematical models, making use of a quantitative sufficient criterion ensuring stability. The application of this criterion rests on the construction of a very sharp

componentwise enclosure of the set of perturbed solutions of the given or approximating system of differential equations, where these perturbations are due to initial data or parameters in the equations. If this criterion is satisfied, the following well-known difficulties of standard mathematical or numerical methods have been resolved in the problem at hand:

- a) The influence of nonlinearities has been accounted for almost precisely and geared to the problem under investigation, i.e., not only by use of generally valid and, consequently, crude estimates. Consequently, a stabilizing influence of a nonlinearity can be ascertained.
- b) Stability is confirmed quantitatively and with respect to known finite initial or parameter perturbations.
- c) Finite sets of initial or parameter perturbations are admitted and their nonlinear influences are fully taken into account, without making use of any linearization.
- d) Whereas the standard practical methods for nonlinear problems yield approximations without a quantitative and sharp error estimate, the subsequent treatment of ordinary differential equations accounts for every possible error, i.e., input deviations discretization errors, rounding errors, procedural errors in the determination of the values of standard functions, etc.
- e) A sufficient criterion asserts stability for all positive t , even though its practical execution is confined to a finite time interval.
- f) The execution of the test, leading to the fulfillment of the sufficient criterion, provides a rigorous sensitivity and/or safety analysis with respect to the non-enumerably many combinations of admissible initial and parameter perturbations.

Published as follows:

- i) Approximate Practical Stability for Nonlinear Evolution Partial Differential Equations, Transactions of the 11th IMACS World Congress (Oslo, 1985). To appear in Journal Form, 1987.
- ii) Untersuchung der praktischen Stabilität von Losunger Nichtlinearer hyperbolischer Anfangsrandwertaufgaben, A. Angew. Math. u. Mech, 65, 1985, pp. T 76-T 78.

3. Nonlinear Membrane Models for Waves in Tires

(E. Adams, W. F. Ames, D. Weiler and R. Lohner) (see item III-1)

The numerical enclosure method of III-1 is being applied to study wave propagation in nonlinear membrane models of tires. The absence of standing waves indicated that the adopted models have to include a better simulation for the contact area with the road. Work initiated.

4. Nonlinear Ordinary Initial Value Problems

(E. Adams, R. Lang)

Applications of "mapping the initial intervals into themselves," a theory developed before this project is being applied to nonlinear ordinary initial value problems generated by a collocation method applied to nonlinear wave equations. Completed applications are concerned with the nonlinear transverse vibrations of beams under compression and with the one-dimensional wave propagation in media governed by classes of nonlinear constitutive equations. Here, free parameters in these equations are identified such that all perturbations of the trivial solutions are uniformly bounded provided the initial data are confined to prescribed intervals. Preliminary results have appeared and work continues.

Published as follows:

- i) Mathematical Analysis of the Dependency of Dynamic Perturbations on Constitutive Equations, Proceedings of the 11th IMACS World Congress (Oslo, 1985), Volume 1, pp. 63-66, 1985.

III. Numerical Enclosure of Solutions

(E. Adams, W. F. Ames, R. Lohner, D. Cordes, A. Holzmuller)

Very modest

support by Army Grant

1. R. Lohner has completed his Ph.D. work on fully computer-implemented methods for the componentwise enclosure of solutions of nonlinear ordinary initial or boundary value problems with systems of explicit ODEs. Preliminary presentations of this work have been given.

Published as follows:

- i) Enclosure of solutions of ordinary initial value problems and applications, to appear in "Discretization in Differential Equations and Enclosures," Akademie-Verlag, Berlin, 1987.
- ii) (R. Lohner Only) Enclosing the solutions of ordinary initial and boundary value problems, Proceedings of the 11th IMACS World Congress (Oslo, 1985), Volume 1, pp. 99-102, 1985. An expanded version will appear in "Computer Arithmetic, Scientific Computation and Programming Languages," G. Teubner, 1987.

Currently, applications carried out by A. Holzmuller are concerned with problems in chemical kinetics, in particular, the enclosure and verification of the famous periodic solution of the Oregonator. Another application is concerned with a one-dimensional steady state analysis of thermochemical processes in rocket engines.

2. D. Cordes has completed his Ph.D. work on fully computer-implemented methods for the entirely safe verification of stability and the enclosure of quasistationary solutions of systems of linear ODEs with periodic coefficients. Applications are mainly concerned with gear drive vibrations. For preliminary presentations of this work see below. In the case of linear ODEs, Cordes employs a practical implementation of the Floquet theory or,

alternatively, a condition of mapping of initial intervals into themselves. In the case of nonlinear systems, only this mapping criterion is applicable and has been used for this purpose by Cordes.

Published as follows:

- i) Cordes, D., Adams, E., Test for Uniform Boundedness for Problems Admitting Parameter Resonance, Proceedings of the 11th IMACS World Congress (Oslo, 1985), Vol. 1, pp. 103-105, 1985.
- ii) Cordes, E., Adams, E., Test for Uniform Boundedness for Dynamic Problems Admitting Parameter and Combination Resonance, will appear in "Computer Arithmetic, Scientific Computation, and Programming Languages," G. Teubner, Stuttgart, 1987.

D. Participating Scientific Personnel

I) Funded

- a) Professor E. Adams, Professor of Applied Mathematics, University of Karlsruhe, 7500 Karlsruhe, Federal Republic of Germany (Funded One Month).
- b) Professor W. F. Ames, Regents' Professor of Mathematics and Director, School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332 (Funded Five Months).
- c) Dr. V. J. Ervin, Visiting Assistant Professor of Mathematics, School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332 (Funded One Month).
- d) Mr. J. F. Froehlich, Research Assistant, School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332 (Funded Six Months).
- e) Mr. Armin Holzmuller, Research Assistant, School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332 (Funded Nine Months).

II) Unfunded But Associated

- a) Professor K. A. Ames, Iowa State University, Ames, Iowa
- b) Dr. M. C. Nucci, University of Perugia, Italy
- c) Professor J. Herod, Georgia Tech, Atlanta, GA
- d) Dr. H. Spreuer, University of Karlsruhe, Germany
- e) Mr. R. Lang, University of Karlsruhe, Germany, Assistant to Dr. E. Adams
- f) Mr. R. Lohner, University of Karlsruhe, Germany, Assistant to Dr. E. Adams
- g) Mr. D. Weiler, University of Karlsruhe, Germany, Assistant to Dr. E. Adams
- h) Mr. D. Cordes, University of Karlsruhe, Germany, Assistant to Dr. E. Adams

E. Remarks:

The computations will continue apace in Germany under the auspices of my long time collaborator Prof. Dr. E. Adams of Karlsruhe. The preliminary results are very promising, but our association which has been modestly supported by the Army must cease. This international cooperation has been of enormous value far beyond the modest funds invested. A pity!