GEORGIA INSTITUTE OF TECHNOLOGY — OFFICE OF CONTRACT ADMINISTRATION

PROJECT ADMINISTRATION DATA SHEET

[x] ORIGINAL [ ] REVISION NO. ______

Project No. G-37-634 DATE 11/4/81

Project Director: Dr. A. T. Bharucha-Reid School/~~Lab~~ Mathematics

Sponsor: U. S. Army Research Office; Research Triangle Park, NC 27709

Type Agreement: SFRC No. DAAG29-81-K-0174

Award Period: From 9/1/81 To 8/31/84 (Performance) 10/31/84 (Reports)

Sponsor Amount: $177,000 ($30,000 thru 2/28/82) Contracted through:

Cost Sharing: $50,904 (G-37-337) - To be budgeted with other increments GTRI/~~GIT~~

Title: Computational Solution of Random Equations

ADMINISTRATIVE DATA

OCA Contact Don S. Hasty x 4820

1) Sponsor Technical Contact:

Dr. Jagdish Chandra, Director
Mathematics Division
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709

2) Sponsor Admin/Contractual Matters:

Mr. Abram J. Van Hall*
Contracting Officer
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709

Defense Priority Rating: None

Security Classification: Unclassified

RESTRICTIONS

See Attached Government Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval — Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with GIT if specified in the proposal and not otherwise indicated in the Short Form Research Contract

COMMENTS:

* Mr. Thomas A. Bryant - Property Administration and Closeout duties.

ONR RR - Georgia Tech

COPIES TO:

Administrative Coordinator	Research Security Services	EES Public Relations (2)
Research Property Management	~~Reports Coordinator (OCA)~~	Computer Input
Accounting	Legal Services (OCA)	Project File
Procurement/EES Supply Services	Library	Other ______

FORM OCA 4:781

EORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 4/26/84

oject No. G-37-634 School/~~XXX~~ Mathematics

cludes Subproject No.(s) None

oject Director(s) ~~William F. Ames~~ A.T. Bharucha-Reid GTRI / ~~GIT~~

onsor U. S. Army Research Office, Research Triangle Park, N.C.

itle Computational Solution of Random Equations

ffective Completion Date: 11/1/83 (Performance) 12/31/83 (Reports)

rant/Contract Closeout Actions Remaining:

Termination based upon agreement documented by Mod P-00006.

- [ ] None
- [ ] Final Invoice or Final Fiscal Report
- [X] Closing Documents
- [X] Final Report of Inventions (OCA to submit)
- [X] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other

ontinues Project No. N/A Continued by Project No. N/A

OPIES TO:

roject Director
esearch Administrative Network
esearch Property Management
ccounting
rocurement/EES Supply Services
esearch Security Services
eports Coordinator (OCA)
egal Services

Library
GTRI
Research Communications (2)
Project File
Other

orm OCA 60:1028

PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: P-18328-M

2. PERIOD COVERED BY REPORT: 1 October 1981 - 30 June 1982

3. TITLE OF PROPOSAL: Computational Solution of Random Equations

4. CONTRACT OR GRANT NUMBER: DAAG29-81-K-0174

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: A. T. Bharucha-Reid

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

   (a) (with M. Christensen) Approximate solution of random integral equations; General methods, to appear in Proc. 10th IMACS World Congress on System Stimulation and Scientific Computation, August 1982.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

   M. Sambandham, Research Associate

Dr. A. T. Bharucha-Reid 18328-M
Georgia Institute of Technology
School of Mathematics
Atlanta, GA 30332

BRIEF OUTLINE OF RESEARCH FINDINGS

(1) M. Christensen and A. T. Bharucha-Reid are investigating the distribution of the eigenvalues of general random Fredholm integral equations, as well as the expected values of the eigenvalues.

(2) M. Christensen and A. T. Bharucha-Reid are investigating iterative methods for the computation of the inverse of a random matrix. We are concerned with this problem because many numerical methods for the solution of random operator equations leads to systems of random algebraic equations. Two areas are being explored: (1) the number of iterations required to obtain the inverse of a randomly perturbed deterministic matrix whose inverse is known, and (2) the formulation of an optimal stopping rule, which, using functional-analytic errors bounds and notions from statistical decision theory, will enable us to compute the probability of error if the iterative procedure is terminated at the m-th step and with the n-th generated sample function.

(3) M. Christensen and M. Sambandham are preparing a paper on the roots of random trigonometric polynomials and random orthogonal polynomials. These results are of interest because random polynomials of the above type are encountered in the computational solution of random equations.

PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: P-18328-M

2. PERIOD COVERED BY REPORT: 1 July 1982 - 31 December 1982

3. TITLE OF PROPOSAL: Computational Solution of Random Equations

4. CONTRACT OR GRANT NUMBER: DAAG29-81-K-0174

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: A. T. Bharucha-Reid

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:
   (a). (with M. Christensen) Approximate solution of random integral equations: General methods, Proc. 10th IMACS World Congress on System Stimulation and Scientific Computation, Vol. 4, pp. 299-304, 1982.

   (b). A revised version of (a) will appear in a special issue of the journal Mathematics and Computers in Simulation, 1983.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

   A. T. Bharucha-Reid, Principal Investigator

   M. J. Christensen, Faculty Associate

   M. Sambandham, Research Associate

Dr. A. T. Bharucha-Reid 18328-M
Georgia Institute of Technology
School of Mathematics
Atlanta, GA 30332

7(c). (with M. Sambandham and V. Thangaraj) On the variance of the number of real roots of random algebraic polynomials, <u>Stochastic Anal. Appl.</u> 1(2) (1983), in press.

Copies of 7(a) and (c) have been forwarded to the ARO.

(d). G. S. Ladde and M. Sambandham, Stochastic versus deterministic, <u>Math. Comp. Simulation</u> 24(1982), 507-514.

Reprints of 7(b) and (d) will be sent as soon as they are received.

BRIEF OUTLINE OF RESEARCH FINDINGS

(1) M. Christensen and A. T. Bharucha-Reid have completed their research on the eigenvalues of Fredholm integral equations with random degenerate kernels. Goodness-of-fit tests were utilized to determine the distribution of the random eigenvalues; and the "average problem" for the solutions was investigated. This paper is now being prepared for publication.

(2) M. Christensen and M. Sambandham have proved a very interesting theorem on the limiting distribution of the roots of random trigonometric and orthogonal polynomials. This result is required for our theoretical and numerical research on the application of random orthogonal polynomials to the numerical solution of random differential and integral equations. This paper is being prepared for publication.

(3) A. T. Bharucha-Reid and M. Sambandham are conducting research on the application of random Chebyshev polynomials to the numerical solution of Fredholm integral equations.

PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: P-18328-M

2. PERIOD COVERED BY REPORT: 1 January 1983 - 30 June 1983

3. TITLE OF PROPOSAL: Computational Solution of Random Equations

4. CONTRACT OR GRANT NUMBER: DAAG29-81-K-0174

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: A. T. Bharucha-Reid

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

   (a). M. Sambandham, V. Thangaraj and A. T. Bharucha-Reid, "On the variance of the number of real roots of random algebraic polynomials," Stochastic Anal. Appl. 1 (1983), 215-238.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

   M. Sambandham, Research Associate

Dr. A. T. Bharucha-Reid 18328-M
Georgia Institute of Technology
School of Mathematics
Atlanta, GA 30332

7 (b). A. T. Bharucha-Reid, "Systems of linear algebraic equations with random coefficients and the numerical solution of random integral equations," in Wissenschaftliche Beiträge Ingenieurhochschule Zwickau, (an invited paper; to appear), 1983. A copy has been sent to the ARO.

(c). J. vom Scheidt and A. T. Bharucha-Reid, "On the averaging problem for the roots of random algebraic polynomials," Ibid. (to appear), 1983.

(d). J. vom Scheidt and A. T. Bharucha-Reid, "On the distribution of the roots of random algebraic polynomials," Ibid. (to appear), 1983.

The manuscripts of 7(c) and (d) are being retyped in Germany, where they will be published.

(e). M. Sambandham and V. Thangaraj, "On the real zeros of random trigonometric polynomials," J. Indian Math. Soc. (to appear).

(f). M. J. Christensen and M. Sambandham, "An improved lower bound for the expected number of real zeros of random algebraic polynomials," submitted to Probability and Statistics Letters.

(g). G. Ladde, V. Lakshmikantham, and M. Sambandham, "Error estimates of solutions and mean solutions of random differential equations through comparison principles," to appear in Stochastic Anal. Appl.

(h). M. J. Christensen and M. Sambandham, "A limit theorem for random trigonometric polynomials," J. Math. Anal. Appl., (in press).

BRIEF OUTLINE OF RESEARCH FINDINGS

1. Publication 7(b). This is an invited survey paper, which, as its title indicates stresses the importance of systems of random algebraic equations in the numerical analysis of random integral equations.

2. Publication 7(c). This paper, using perturbation techniques, gives error estimates for the difference between the expectation of the roots of a random algebraic polynomial and the roots of the mean (deterministic) polynomial. This paper improves the estimate due to Christensen and Bharucha-Reid on the stability of the roots of random algebraic polynomials.

3. Publication 7(d). In this paper we prove a theorem which gives the limiting distribution of the roots of a random algebraic polynomial. We will use this result to study the approximation of the eigenvalues of a random differential and integral operators.

4. Publication 7(e). When the random coefficients are general random variables with finite second and third moments and mean zero, or nonzero, the expected number of real zeros of random trigonometric polynomials is estimated.

5. Publication 7(f). For the random polynomial $\sum_{k=0}^{n} a_k(\omega)x^k = 0$, $a_k(\omega) \varepsilon N(0,1)$, the authors show that the expected number of real roots is greater than or equal to $\frac{2}{\pi} \log (n+1) - .02$. This is the best lower estimate that has been obtained.

6. Publication 7(g). Using comparison principles, error estimates between the solution and mean solution of random differential equations are given.

7. Publication 7(h). An analogue of the well-known limit theorem of Šparo and Šur for random algebraic polynomials is proved for random trigonometric polynomials.

8. W. Römisch and A. T. Bharucha-Reid are preparing the second draft of two papers entitled "Projection methods for the solution of random equations." In Part I we prove the convergence of random projection methods; and in Part II we prove the weak convergence of the approximate solution measures associated with random projection methods. In Part III we will apply these results to systems of random algebraic equations and random integral equations.

G-37-634

COMPUTATIONAL SOLUTION OF RANDOM EQUATIONS

FINAL REPORT

27 NOVEMBER 1983

U.S. ARMY RESEARCH OFFICE

CONTRACT NO. DAAG-29-81-K-0174

GEORGIA INSTITUTE OF TECHNOLOGY

APPROVED FOR PUBLIC REVIEWING
DISTRIBUTION UNLIMITED

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 1	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) COMPUTATIONAL SOLUTION OF RANDOM EQUATIONS		5. TYPE OF REPORT & PERIOD COVERED Final Report 9/1/81 - 11/30/83
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) A. T. Bharucha-Reid		8. CONTRACT OR GRANT NUMBER(s) DAAG29-81-K-0174
9. PERFORMING ORGANIZATION NAME AND ADDRESS School of Mathematics Georgia Institute of Technology Atlanta, GA 30332		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE 30 November 1983
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) O.N.R., Resident Representative GIT, 206 O'Keefe Bldg. Atlanta, GA 30332		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

THE VIEW, OPINIONS, AND/OR FINDINGS CONTAINED IN THIS REPORT ARE THOSE OF THE AUTHOR(S) AND SHOULD NOT BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION, POLICY, OR DECISION, UNLESS SO DESIGNATED BY OTHER DOCUMENTATION.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Random equations, numerical methods, simulation, fixed point theorems, limit theorems.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents a review of results obtained in connection with ARO Contract No. DAAG29-81-K-0174 for the period 1 October 1981 - 30 November 1983. Results presented concern numerical results for random polynomials, random integral equations, random integrodifferential equations and fixed point theorems.

1. Statement of the Problems Studied

The research project was concerned with the systematic development of computational methods for random equations. An earlier ARO research project (DAAG29-77-G-0164) was concerned primarily with the development of computational methods for the solution of random integral equations. This project was concerned with the computational solution of random integral equations as well as other classes of random equations, with special reference to computer implementation of general methods for obtaining approximate solution of other classes of random equations. In particular, we were concerned with computer implementation of (1) approximate methods for solving random linear algebraic systems of equations, (2) projection methods for solving random operator equations, and (3) iterative methods for solving random operator equations.

2. Summary of the Most Important Results Obtained

All of the results obtained can, in a given context, be regarded as important.

(a) Paper A-1 was presented at the Montreal meeting by Bharucha-Reid as a keynote address. It gives an up-to-date account of general methods for solving random integral equations, and surveys studies on the approximate solution of random equations.

(b) In Paper A-2 the authors consider a system of random ODEs and the corresponding deterministic system obtained by averaging first. Methods due to Christensen and Bharucha-Reid on the roots of random algebraic polynomials are used. The estimates obtained enable workers to estimate how much the determinstic solution differs from the random solution.

(c) In Paper A-3 we give estimates of the variance of the number of real roots in the general case of random algebraic polynomials with dependent coefficients. Using simulation methods we also determine the coefficient of variation of the number of real roots.

(d) In Paper B-1 the author discusses the importance of the study of the limiting distribution of systems of linear algebraic equations with random coefficients in the numerical analysis of random integral equations.

(e) In Paper B-2 the authors use methods developed by von Scheidt and Purkert (see their forthcoming book on random eigenvalue problems) to obtain the best estimate to-date of the difference between the roots of a random algebraic polynomial and the roots of the mean (or deterministic) polynomial. The estimates are better than those obtained earlier by Christensen and Bharucha-Reid.

(f) In Paper B-3 the authors prove a theorem on the limiting distribution of random algebraic polynomials. (We are now considering the application of this result to random integral equations.)

(g) In Paper B-4 the authors obtain estimates of the number of real roots of random trigonometric polynomials with dependent coefficients.

(h) In Paper B-5 the authors obtain the best lower bound to-date of the expected number of real roots of random algebraic polynomials.

(i) In Paper B-6 the authors prove a beautiful result on the limiting distribution of the roots of random trigonometric polynomials. They show that the roots are uniformly distributed in a symmetric region around the imaginary axis.

(k) The results of Paper B-7 are clearly indicated by the title.

(l) In Paper C-1 the authors use random Chebyshev polynomials to solve random Fredholm equations. A concrete example is presented.

(m) In Paper C-2 and 3 the authors study the random integro-differential equation.

$$x'(t,\omega) = A(t) \times (t, \omega) + \int_0^t B(t,r) \times (\tau,\omega)d\tau + f(t,\omega).$$

The titles of these papers indicate the problems considered.

(n) In Paper C-4 the authors introduce for the first time various measures of noncompactness in order to obtain fixed point theorems for random operators.

(o) In Papers C-5, 6, and 7 the authors consider in a very rigorous way the notions of random projection methods for random equations. Those papers are now in the second draft stage, and will be finished by March 1984.

3. List of Publications

A. Published Papers

1. Bharucha-Reid, A. T. and Christensen, M. J., "Approximate solution of random integral equations: General methods," Proc. 10th IMACS World Congress on System Simulation and Scientific Computation, 4 (1982), 299-304. (A revised version of this paper will appear in a special issue of the journal Mathematics and Computers in Simulation, which will be édited by Bharucha-Reid and Tsokos.

2. Ladde, G. S. and Sambandham, M., "Stochastic and deterministic," Math. Comp. Simulation 24 (1982), 507-514.

3. Sambandham, M., Thangaraj, V., and Bharucha-Reid, A.T., "On the variance of the number of real roots of random algebraic polynomials," Stochastic Anal. Appl. 1 (1983), 215-238.

B. Papers in Press

1. Bharucha-Reid, A. T., "Systems of linear algebraic equations with random coefficients and the numerical solution of random integral equations," Wissenschaftliche Beiträge Ingenieurhochschule, 1983. (This is an invited paper.)

2. vom Schedit, T. and Bharucha-Reid, A. T., "On the averaging problem for the roots of random algebraic polynomials," Beiträge zur numerische Mathematik.

3. vom Scheidt, J. and Bharucha-Reid, A. T., "On the distribution of the roots of random algebraic polynomials," Zeitschrift für angewandte Mathematik.

4. Sambandham, M. and Thangaraj, V., "On the real zeros of random trigonometric polynomials," J. Indian Math. Soc.

5. Ladde, G. S., Lakshmikantham, V. and Sambandham, M., "Error estimates of solutions and mean solutions of random differential equations through comparison principles," Stochastic Anal. Appl.

C. Papers in Preparation

1. Bharucha-Reid, A. T. and Sambandham, M., "On the solution of random Fredholm integral equations using random Chebyshev polynomials."

2. Kanna, D., Bharucha-Reid, A. T. and Martin, B., "On a class of random integrodifferential equations: I. Existence, uniqueness, and uniqueness of solution, and properties of the solution process.

3. Bharucha-Reid, A. T. and Martin, B., "On a class of random integrodifferential equations: II. Numerical methods of solution."

4. Chandrasekharan, P. S. and Bharucha-Reid, A. T., "On measure of noncompactness and fixed points of random operators."

5. Römisch, W. and Bharucha-Reid, A. T., "Projection methods for the solution of random equations: I. Convergence of projection method."

6. Römisch, W. and Bharucha-Reid, A. T., "Projection methods for the solution of random equations: II. Convergence of solution measures."

7. Römisch, W. and Bharucha-Reid, A. T., "Projection methods for the solution of random equations: III. Applications."

8. Christensen, M. J. and Sambandham, M., "A limit theorem for random trigonometric polynomials."

D. Books and Monographs in Preparation

1. Bharucha-Reid, A. T. and Sambandham, M., "Random Polynomials," Academic Press (to appear in 1984).

2. Bharucha-Reid, A. T. and Christensen, M. J., "Approximate Solution of Random Integral Equations."

4. <u>List of all Participating Scientific Personnel</u>

*A. T. Bharucha-Reid, Principal Investigator

*M. J. Christensen, Faculty Associate

K. Kannan

G. S. Ladde

V. Laushmikantham

B. Martin

W. Romisch

*M. Sambandham, Post-doctoral research associate

J. von Scheidt

V. Thangaraj

P. S. Chandrasekharan

---

*Supported by ARO funds.