RGIA INSTITUTE OF TECHNOLOGY

OFFICE OF CONTRACT ADMINISTRATION

PROJECT ADMINISTRATION DATA SHEET

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[ ] REVISION NO. ____

oject No. G-37-629

DATE: 6-22-81

oject Director: Dr W. F. Ames

School/Lab Mathematics

onsor: U.S. Army Research Office, P.O. Box 12211, Research Triangle Park, N.C. 27709

pe Agreement: Short Form Research Contract (SFRC) No DAAG29-81-K-004

ard Period: From 6-14-81 To ~~6-13-83~~ 10-31-83 (Performance) ~~8-13-83~~ 12/31/83 (Reports)

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GTRI/~~GIT~~

tle: Linear or Nonlinear Wave Problems With Input Sets

MINISTRATIVE DATA

OCA CONTACT Don Harty

Sponsor Technical Contact: Dr JAGDISH CHANDRA, U S Army Research Office PO Box 12211, Research Triangle Park, N.C. 27709

Sponsor Admin./Contractual Contact: T. A. Bryant, ONR-RR, Campus
phone 881-4213

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STRICTIONS

e Attached DOD Supplemental Information Sheet for Additional Requirements.

avel: Foreign travel must have prior approval - Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

uipment: Title vests with N/A - None included

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EORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 2/20/84

oject No. G-37-629 School/KXX Mathematics

cludes Subproject No.(s) N/A

oject Director(s) W. F. Ames GTRI / XXX

onsor U. S. Army Research Office, Research Triangle Park, N. C.

tle Linear or Nonliniar Wave Problems With Input Sets

fective Completion Date: 10/31/83 (Performance) 12/31/83 (Reports)

ant/Contract Closeout Actions Remaining:

- [ ] None
- [ ] Final Invoice or Final Fiscal Report
- [X] Closing Documents
- [X] Final Report of Inventions - Patent Questionnaire attached for Project Director.
- [X] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other

ntinues Project No. N/A Continued by Project No. N/A

PIES TO:

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F 37-621

PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: Not available (our apologies)

2. PERIOD COVERED BY REPORT: June 14, 1981 to Dec. 31, 1981

3. TITLE OF PROPOSAL: Linear or Nonlinear Wave Problems with Input Sets

4. CONTRACT OR GRANT NUMBER: DAAG-29-81-K-0042

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: William F. Ames, Regents' Professor of Mathematics

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:
   a) "Some exact solutions for wave propagation in viscoelastic, viscoplastic materials and electrical transmission lines", accepted for publication in the Journal of Nonlinear Mechanics.
   b) "On group analysis of the von Karman equations" submitted to the Journal of Nonlinear Analysis: TMA
   c) "New solutions of the Navier-Stokes equations by group methods" - to be published in the proceedings of the IMACS 10th World Congress, Montreal, Aug. 1982

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

William F. Ames	1 month + cost shared time	
(s) Robert E. Boisvert	3 summer months	1/3 time
(c) Walter Zulehner	3 months	1/4 time
(s) Dietrich Cordes	3 months	1/3 time

(s) Graduate Students
(c) Faculty Colleague

Dr. William F. Ames 17723-M
Georgia Institute of Technology
Mathematics Department
Atlanta, GA 30332

Outline of Research Findings

a) Exact solutions for wave propagation in viscoelastic, viscoplastic materials and electrical transmission lines.

Here we studied the governing equations for wave propagation $\rho v_t - \sigma_x = 0$, $\varepsilon_t - v_x = 0$ with a nonlinear constitutive law $\sigma_t - f(\varepsilon,\sigma,\varepsilon_t) = 0$ where ρ is the mass density (inductance), v is the particle velocity (electric current), σ is the stress ($-\sigma$ the voltage), ε is the strain ($-\varepsilon$ the electrical charge) and f characterizes the body.

First, all those functions f are characterized which remain invariant under the dilatation group and the screw (spiral) group. Then exact solutions are constructed for three special cases. Lastly, an algorithm is formulated for solving a large (nonintegrable) class of viscoplastic problems by numerical integration of the reduced ordinary differential equations.

b) On Group Analysis of the von Karman equations

The von Karman equations

$$\Delta^2 F = E\,[(w_{xy})^2 - w_{xx}w_{yy}]$$

$$\Delta^2 w = \frac{q}{D} + \frac{h^*}{D}\,[F_{yy}w_{xx} + F_{xx}w_{yy} - 2\,F_{xy}\,w_{xy}]$$

with $q = 0$ (steady state) and $q = -\rho w_{tt}$ are analyzed by group methods. These equations arise in studies of the large deflections (w) of rectangular elastic plates under the combined action of a lateral load and tensile force. Group analysis is the machinery used to construct five new exact solutions.

These solutions have potential use in checking general computer codes developed for these equations. The equations themselves are used in studying the dynamics of helicopter rotors and airplane fuselages and wings.

c) Navier-Stokes Equations

The very limited number (3?) of known non-trivial exact solutions of the Navier-Stokes equations prohibits comparative evaluations of computer codes for those equations, In this study by a doctoral student (Boisvert) the group analyses generate the full Lie group and Lie algebra of that system in Cartesian coordinates. Our research shows this was done by several Russians. Their obscure work was not completely general, however. In addition it did not exploit the full potential for constructing solutions.

This analysis is completely general, constructs eight new solutions of the two and three dimensional steady equations, demonstrates the machinery for constructing more and how to associate an infinite number (2 or 3 arbitrary function family) of time dependent solutions with the steady state ones.

This thesis by Boisvert is completed and is in the typing stage. One Summer's work was partially supported.

d. Nonlinear equations with input sets.

The algorithm has been constructed for second order equations and examples from the classes $u_{tt} = (f(u)u_x)_x$ and $u_{xy} = f(u)$ are being studied.

G-37-629

PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: Not Available

2. PERIOD COVERED BY REPORT: January 1, 1982 - June 30, 1982

3. TITLE OF PROPOSAL: Linear or Nonlinear Wave Problems with Input Sets

4. CONTRACT OR GRANT NUMBER: DAAG 29-81-K-0042

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Regents' Professor William F. Ames

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:
   a) "On group analysis of the von Karman equations", accepted for publication in the Journal of Nonlinear Analysis: TMA
   b) "Wave Propagation in Reacting Gases", work completed, in typing stage.
   c) "Implicit Separation of Semilinear Wave Equations", work completed, in typing stage.
   d) "Linear or Nonlinear Hyperbolic Wave Problems with Input Sets, II" in preparation.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

	William F. Ames	Cost shared time
(s)	Dietrich Cordes	6 months 1/3 time
(c)	Walter Zulehner	6 months 1/4 time
(c)	Ernst Adams	1 month

(s) Graduate student
(c) Faculty colleague

Dr. William F. Ames 17723-M
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BRIEF OUTLINE OF RESEARCH FINDINGS

a) Wave Propagation in Reacting Gases

Solutions of the system $u_t + (1 + a\,u^p)u_x = bu^{n+1}$, $u(x,0) = g(x)$, $u_x(0,t) = 0$, modeling wave propagation in a reacting flowing gas, have been constructed by means of a Taylor expansion in the physical parameters (a and b). An estimation of the solution for all admitted values of the parameters (sets) from the small interval expansion is obtained.
This research was just completed and will be submitted to Ingenieur Archiv.

b) Nonlinear equations with input sets

Efforts were concentrated on attempts to develop practical methods for the construction of sharp intervals, depending on the spatial variable and time, which contain the range of the solution of a linear or nonlinear mathematical model of a wave propagation problem with or without input sets. This task is most easily accomplished numerically if the problem is (a) reducible to ordinary differential equations or (b) discretized. Efficient algorithms and computer programs for the construction of sharp bounds for the solution of ordinary initial or boundary value problems have been developed. An a-priori test for the determination of suitable computational step sizes has been developed. The test automatically accounts for input sets in the problem. This work is included in paper d) given in item 7.

c) Implicit Separation and semilinear wave equations

Semilinear wave equations of the form $u_{xx}-u_{tt} = f(u)$ have been analyzed by a new method called "implicit separation". New exact solutions have been constructed in the cases $f(u) = \sin u$, $\cos u$, $\sinh u$, $\cosh u$, e^u and u^n. These solutions contain a number of free parameters. The specific cases were chosen because of the physical interest in optics, electromagnetic theory and elsewhere. The machinery for arbitrary f(u) is also constructed.

6-37-629

PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: Not available

2. PERIOD COVERED BY REPORT: July 1, 1982 - December 31, 1982

3. TITLE OF PROPOSAL: Linear or Nonlinear Wave Problems with Input Sets

4. CONTRACT OR GRANT NUMBER: DAAG 29-81-K-0042

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Regents' Professor William F. Ames

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

a) W. F. Ames and I. Suliciu, "Some ....," Int. Jl. Nonlinear Mechanics 17, 223-230 (1982); b) K. A. Ames and W. F. Ames, "On Group Analysis of the von Karman Equations," Jl. Nonlinear Analysis: TMA, 6, 845-853 (1982) c) W. Zulehner and W. F. Ames, "Group Analysis of a Semilinear Vector Diffusion Equation", accepted for publication in the Jl. Nonlinear Analysis; d) R. E. Boisvert, W. F. Ames & U. N. Srivastava, Group Properties & New Solutio of the Navier-Stokes Equation" accepted for publication in the Jl. of Eng. Math.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

William F. Ames	1 Month plus cost shared time
*(c) E. Adams	1 Month
**(s) V. Ervin	3 Months (1/4 time)

*Faculty Colleague
**Graduate Student

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BRIEF OUTLINE OF RESEARCH FINDINGS

a) Wave propagation in reacting gases, with input sets. An estimation (upper and lower bounds) of the solution, for all admitted values of the parameters (input sets), of $u_t + (1+au^p)u_x = bu^{n+1}$, $u(x,0) = g(x)$, $u_x(0,t) = 0$ has been improved over that from the previous report. A computer program is now available, developed by D. Cordes, in Karlsruhe.

b) Nonlinear equations with input sets (General). Numerical computations on test problems have been performed. We expect to finalize the long paper "Linear or Nonlinear Hyperbolic Wave Problems with Input Sets II" during Professor Adams' visit in March-April 1983.

c) Nonlinear Waves in Fusion. The 3 nonlinear hyperbolic equations describing the wave propagation and shocks in the pellet fusion process have been studied by group analysis. Exact solutions have been constructed and the shock properties studied. The methods employed are equally applicable to the study of other shock wave properties.

d) Waves in Fluids. Our work in group analysis of the Navier-Stokes equations and other diffusion processes is completed and two papers are to appear (see c and d of item 7).

Group Analysis of a Semilinear Vector Diffusion Equation: Abstract.

Systems of equations of the form $u_t - u_{xx} = f(u)$, where $f(u)$: $\mathbb{R}^n \to \mathbb{R}^n$ are studied to determine the conditions on $f(u)$ for which Lie transformation group invariant solutions exist. Solutions for these equations, of special physical interest, are the travelling wave solutions, steady solutions and kinetic solutions.

Group Properties and New Solutions of Navier-Stokes Equations: Abstract.

Using the machinery of Lie theory (groups and algebras) applied to the Navier-Stokes equations a number of exact solutions for the steady state are derived in (two) three dimensions. It is then shown how each of these generates an infinite number of time dependent solutions via (three) four arbitrary functions of time. This algebraic structure also provides the mechanism to search for other solutions since its character is inferred from the basic equations.

037-019

PROGRESS REPORT

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3. TITLE OF PROPOSAL: Linear or Nonlinear Wave Problems with Input Sets

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5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Regents' Professor, William F. Ames

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

   K. A. Ames and W. F. Ames, "Analyses of the von Karman Equations by Group Methods", to appear in Proceedings of the First Army Conference on Applied Mathematics and Computing

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

William F. Ames	1 month plus Cost Shared Time
*E. Adams	1/2 month
**V. Ervin	6 months (1/4 time)
***M. Kvale	6 months (1/8 time)

*Faculty Colleague
**Graduate Student
***Senior Assistant

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BRIEF OUTLINE OF RESEARCH FINDINGS

Nonlinear equations with input sets (General).
All numerical computations have been successfully performed and the long paper "Linear or Nonlinear Hyperbolic Wave Problems with Input Sets II" is in rough draft. Adams hospitalization in June has delayed our completion of the manuscript.

Nonlinear waves in Fusion. Calculations have been performed illustrating the shocks in the group analysis solution. Further analysis of a transformed system continues. The full group is also being constructed. Portions of this research were presented at the Toronto Conference on Wave Phenomena (June 20-24, 1983) in an invited lecture.

Large deflection of plates (von Karman equations). The earlier work, reported in the previous Progress Report, has been generalized and the full group constructed. This paper was presented at the First Army Conference on Applied Mathematics and Computing. The results and generalizations have application to impact problems.

Analysis of the von Karman Equations by Group Methods.

ABSTRACT. One of the system of equations approximating the large deflection of plates consists of two coupled nonlinear fourth order partial differential equations, known as the von Karman equations. The full symmetry group for the steady equations is a finitely generated Lie group with ten parameters. For the time dependent system the full symmetry group is an infinite parameter Lie group. Several subgroups of the full group are used to generate exact solutions of the time-independent and the time-dependent system. These include the dilatation group (similar solutions), rotation group, screw group and others. Physical implications and applications are discussed.

G-37-629

LINEAR OR NONLINEAR WAVE PROBLEMS WITH INPUT SETS

FINAL REPORT

21 DECEMBER 1983

U. S. ARMY RESEARCH OFFICE

CONTRACT NO. DAAG 29-81-K-0042

SCHOOL OF MATHEMATICS
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GA 30332

REPORT DOCUMENTATION PAGE

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18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Nonlinear hyperbolic equations, group analysis, input data sets, numerical enclosure of solutions, nonlinear parabolic equations, Navier-Stokes equations, von Karman equations.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents a review of results and a list of publications developed in connection with ARO Contract No. DAAG 29-81-K-0042 for the period June 14, 1981 to November 30, 1983. Results presented concern the group analysis of hyperbolic and parabolic systems, their types of solutions and numerical enclosure of their solutions when input data sets are present.

1. General Discussion of Research

This research project is concerned with analytical and numerical studies of linear and nonlinear wave problems with input data sets. On the analytical side, we were concerned with the examination of a variety of analytic techniques and applying them to nonlinear wave problems. The most general of those methods, that of group analysis, was applied with much success as will be seen in the List of Publications (Paragraph 3).

Group theoretic methods were found useful in the study of various nonlinear equations. In particular, they permit us to (i) predict the forms of various types of solutions which the system will admit; (ii) sometimes obtain exact solutions useful in ascertaining whether numerical algorithms give required accuracy (by comparison); (iii) provide a systematic way of improving numerical algorithms; (iv) permit the conversion of difficult boundary value problems to initial value problems (which are easier to solve numerically); (v) provide a systematic way of deciding whether a system has a "similar" solution; (vi) provide a systematic way of ascertaining whether an approximate method is compatible with some underlying theory (such as Newtonian Mechanics); (vii) providing assistance in sensitivity analysis, via a reduction in dimension.

Indeed, group theoretic methods seem to be the only analytic methods applicable to both linear and nonlinear problems.

On the numerical side, our goal was the development of a constructive, easily computed enclosure ("good" upper and lower bounds) of the set of solutions (depending on input parameters varying in intevals) for nonlinear wave problems. In many physical problems, the parameters (density, modulus of elasticity viscosity) are poorly known and can only be specified to lie in some intevals. Rather than solving a large number of problems with specified parameter values, we suggest that an enclosure of the solutions is preferable for the purposes of the design engineer, for example. We have been successful in developing such an approach for the class of problems $u_{tt} + cu_t - f(u_x)\, u_{xx}=0$ as will be discussed in Paragraph 2. A paper describing the method and numerical calculations will be included in the final report.

Applications to other more general classes of nonlinear hyperbolic or parabolic problems is straightforward provided there are only input sets of constant parameters. Input sets of (continuous) functions are not permitted since sample problems from an earlier paper (Adams, E. and Ames, W.F., "Linear or nonlinear hyperbolic wave problems with input sets (Part I)", Jl. Engineering Math. 16 (1982), 23-45) revealed that this may give rise to instabilities of the resonance or parameter resonance type, unless there is sufficiently strong dissipation (damping) in the equations.

The method makes use of (i) a Fourier-polynomial whose time-dependent coefficients are such that the partial differential equation is satisfied on grid lines $x = x_i$ and, (ii) an interval extension of the first order remainder term in a Taylor expansion of the set of solutions in terms of the initial parameters and equation parameters. Using (i), the problem is approximated by a vector nonlinear ordinary ivp (initial value problem) with initial and parameter intervals. A Taylor-approximation of a set of solutions may be inaccurate for the case of sufficiently large time t. Therefore, here the Taylor-representation is computed only for an interval $[0,t^*]\ni t$ comparable with one timewise wave length of the (almost) periodic solutions of the ibvp. Then, it follows from the autonomous character of the ibvp that a "mapping into themselves" of the input intervals at time t* establishes the confinement of the set of values, of the set of solutions, to uniformly bounded and known intervals for all $t \geq 0$.

The set of solutions to be enclosed arises from perturbations in a neighborhood of the trivial* solution of the ibvp, i.e., the analysis provides a quantitative and practical stability analysis for the trivial solution in a finite

*Neighborhoods of nontrivial periodic or steady-state solutions can also be investigated.

initial and parameter neighborhood. In a "practical stability analysis", perturbations of both initial data and coefficients are admitted.

2. Summary of the Most Important Results

a) Paper A-1 was motivated by a physical problem in which the constitutive law has (natural) creep properties in one domain of the stress-strain plane and relaxation properties in another domain. General constitutive laws remaining invariant under the dilatation and spiral groups are determined. Then exact invariant solutions are constructed for a classical (linear) material, for a quadratic constitutive law and for a linear viscoelastic material. Lastly, a viscoplastic material is studied in the light of group analysis and the machinery to determine the solution (numerically) for a variety of viscoplastic problems is established.

b) Papers A-2 and B-1 form a complete picture of the possible invariant solutions for the celebrated von Karman equations for the large deflection of elastic plates. The full symmetry group for the steady equations is a finitely generated Lie group with ten parameters. For the time dependent system the full symmetry group is an infinite parameter Lie group. Several subgroups of the full group are used to generate exact solutions. This system of equations is shown to be consistent with Newtonian Mechanics. In a related paper, B-3, questions are raised about the consistency with Newtonian Mechanics of the Tadjbakhsh-Saibel generalization of these equations. Of considerable mathematical interest is the idea of raising the order to integrate certain nonlinear second, third and higher order equations. The contents of these papers was reported at the First Army Conference on Applied Mathematics and Computing held at George Washington University (1983).

c) Papers A-3 were reported at the Tenth World Congress of IMACS (International Association for Mathematics and Computers in Simulation) held in Montreal, August, 1982. A preliminary report was also given to the Chemical Engineering World Congress (Montreal 1981) in an invited paper.

Using the machinery of Lie theory (groups and algebras) applied to the Navier-Stokes equations a number of exact solutions for the steady state are derived in (two) three dimensions. It is then shown how each of these generates an infinite number of time dependent solutions via (three) four arbitrary functions of time. This algebraic structure also provides the mechanism to search for other solutions since its character is inferred from the basic equations.

d) Paper A-4 attacks a problem related to part c) above. Nonlinear models of pollutant transport in air and water leads to vector semi-linear diffusion equations of the form $u_t - u_{xx} = f(u)$, where $f(u)$ is a nonlinear (vector) function from R^n to R^n. The goal of the paper is to find conditions on $f(u)$, the so-called classification problem, for which invariant solutions exist.

e) Paper B-2 constitutes preliminary work to a PhD Dissertation by V.J. Ervin. It was presented in an invited paper to Wave Phenomena 83, a conference held at the University of Toronto in June, 1983.

A gas dynamic model of the pellet fusion process having a time-invariant source term is studied by means of group analyses. Some exact solutions of this nonlinear system are constructed for specific (physical) values of parameters. The development of multiple shock waves is demonstrated in several cases analytically.

f) Paper C-1 introduces a generalized idea of separation of variables, called implicit separation, of which simple separation is a special case. Equations of the form $u_{xt} = f(u)$ are solved.

g) Paper C-2 forms the heart of the numerical method described in the first section. A nonlinear autonomous dissipative hyperbolic initial boundary value problem with a parameter B given on an interval $B \in [\underline{B}, \bar{B}]$ is studied and initial data on intervals is also admitted. The algorithm for determining the enclosure of the set of values of classical solutions is obtained by using a Fourier polynomial with time dependent coefficients and an interval extension of the remainder term in a Taylor representation of the set of solutions in terms of the initial parameters and equation parameter. The analysis provides an enclosure and a quantitative and easily computed stability analysis for the trivial solution. It is also demonstrated that standard finite difference methods cannot be employed because of local discretization errors.

h) Paper D-1 concerns wave propagation in reacting gases with input sets. An estimation (upper and lower bounds) of the solution, for all admitted values of the parameters (input sets) of $u_t + (1+a\, u^p)\, u_x = b\, u^{n+1}$, $u(x,o)=g(x)$, $u_x(o,t)=o$ has been developed. The computations, having been delayed, are nearing completion.

3. List of Publications

A. Published Papers

1. Ames, W. F. and Suliciu, I., "Some exact solutions for wave propagation in viscoelastic, viscoplastic materials and electrical transmission lines", Int. Jl. Nonlinear Mechanics 17 (1982), 223-230.

2. Ames, K. A. and Ames, W. F., "On Group Analysis of the von Karman equations", Nonlinear Analysis: TMA, 8 (1982), 845-853.

3. Boisvert, R.E., Ames, W.F. and Srivastava, U.N., "Solutions of the Navier-Stokes equations by group methods" Proceedings IMACS 10th World Congress, Montreal, Canada (August 8-13, 1982) Vol. 3, 31-35
"Group properties and new solutions of the Navier-Stokes equations", Jl. Engineering Mathematics, 17 (1983), 203-221.

4. Zulehner, W. and Ames, W.F., "Group Analysis of a semilinear vector diffusion equation", Nonlinear Analysis: TMA, 7 (1983), 945-969.

B. Papers in Press

1. Ames, K.A. and Ames, W.F., "Analysis of the von Karman equations by group methods", Proceedings of the First Army Conference on Applied Mathematics and Computing (George Washington University 1983)

2. Ervin, V.J., Ames, W.F. and Adams, E., "Nonlinear waves in the pellet fusion process", WAVE Phenomena 83 (Proceedings of a conference at the University of Toronto, 1983) North Holland Publishing Company.

3. Ames, W. F. "Invariant solutions for large deflection of elastic plates", Saibel Anniversary Volume (1984).

C. Papers Submitted

1. Ames, W.F. and Adams, E. "Implicit Separation of semi-linear wave equations".

2. Adams, E., Lohner, R., and Ames, W.F., "Linear or nonlinear hyperbolic wave problems with input sets II".

D. Papers in Preparation

1. Cordes, D., Ames, W.F. and Adams, E., "Wave propagation in reacting gases".

4. List of Participating Scientific Personnel

Name	Status	Permanent Address
W. F. Ames*	Principal Investigator	Georgia Institute of Technology
K. A. Ames	Faculty Colleague	Iowa State University
E. Adams*	Senior Associate	Univ. of Karlsruhe, W. Germany
R. E. Boisvert*	Graduate Student	Lincoln Lab, MIT
D. Cordes*	Graduate Student	University of Karlsruhe
V. Ervin*	Graduate Student	Georgia Institute of Technology
M. Kvale*	Senior	Georgia Institute of Technology
R. Lang	Graduate Student	Georgia Institute of Technology
R. Lohner	Graduate Student	University of Karlsruhe
W. Zulehner*	Faculty Colleague	Georgia Institute of Technology

*Supported by ARO Funds