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GEORGIA INSTITUTE OF TECHNOLOGY
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SPONSORED PROJECT INITIATION

Date: June 8, 1979

Project Title: Problems in Elastostatics Involving Unbounded Bodies

Project No: G-37-615 Green card

Project Director: Dr. Kenneth B. Howell

Sponsor: National Science Foundation

Agreement Period: From 6/15/79 Until 11/30/81 (Grant Period)

Type Agreement: Grant No. MCS-7906636, dtd. May 21, 1979

Amount: $14,795 NSF
4,428 GIT (G-37-324)
$19,223 TOTAL

Reports Required: Annual Progress Report(s); Final Project Report

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Defense Priority Rating: n/a

Assigned to: Mathematics (School/Laboratory)

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CA-3 (3/76)

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GEORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION SHEET

Date 7/14/82

Project Title: Problems in Elastostatics Involving Unbounded Bodies

Project No: G-37-615

Project Director: Dr. Kenneth B. Howell

Sponsor: National Science Foundation

Effective Termination Date: 11/30/81

Clearance of Accounting Charges: 2/28/82

Grant/Contract Closeout Actions Remaining:

- [x] Final Invoice and Closing Documents
- [ ] Final Fiscal Report
- [x] Final Report of Inventions
- [x] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other ______

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G-37-615

Annual Progress Report

NSF Grant No. MCS-7906636

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The research supported by NSF Grant No. MCS-7906636 is concerned with the theory of elasticity as applied to problems in which the elastic body is unbounded (i.e., of infinite extent). Initially, the main thrust of the study was directed toward determining which "physically reasonable" conditions are necessary and sufficient to assure the uniqueness of solutions to the linear equations of equilibrium. This remains a central issue in this research but has led naturally to an interest in the general behavior of the solutions which arise when considering particular classes of problems - e.g., problems involving symmetry about some axis.

As of the first anniversary of the effective date of this grant, two papers based largely on the supported research had been written and accepted for publication. In addition, a third paper had been begun. Preprints of all these papers are available. The contents of the individual papers (including, for the sake of completeness, the third paper) are briefly outlined in the following paragraphs. In that these papers averaged in length approximately fifty typed pages each, it should be understood that the following summaries are not very detailed.

The first paper, "Uniqueness in Linear Elastostatics for Problems Involving Unbounded Bodies," should appear in the January, 1981 issue of the Journal of Elasticity. In it, three rather general uniqueness theorems were developed involving, respectively, the general boundary value problem, the displacement problem, and the traction problem. Considerable attention was devoted to showing that the uniqueness result for the general boundary value problem also held in those mathematically unpleasant, but physically important, problems in which the solutions could be expected to possess discontinuities and singularities. In addition, a type of mean-value theorem was derived in which bounds on the magnitude of the displacement and its gradient were determined in terms of local bounds on the magnitude of the stresses and the distance to the boundary of the elastic body. This has proved quite useful in showing that, in many cases, sufficient knowledge of the stresses in a neighborhood of infinity guarantees uniqueness. It is also the type of result one could expect if the linear theory is to be a useful model of elastic processes.

The second paper, "Periodic and 'Slightly' Periodic Boundary Value Problems In Elastostatics On Bodies Bounded In All But One Direction," has also been accepted by the Journal of Elasticity. Its publication date is yet unknown by the writer. The research communicated in this paper dealt with problems in which some or all of the functions involved were periodic in some spatial direction. As indicated by the title, attention was focused on those problems in which the body was unbounded only in the direction of periodicity. For these problems, the concept of mean stress is highly relevant. One of the major results was a theorem of work and energy involving mean stress terms. The paper begins with an analysis of the displacement corresponding to periodic strain (periodic strain does not necessarily imply periodic displacement). This, along with the aforementioned theorem of work and energy, leads to the proof of a rather general uniqueness theorem for periodic strain problems. By then developing theorems showing which problems must have periodic strain solutions, it was possible to extend the uniqueness result so as to include the fairly broad class of problems in which only the corresponding null problem (not necessarily the original problem) is described by periodic boundary data. For many of the results, counter-examples were constructed to demonstrate the necessity of the given assumptions.

Both of the above papers were reviewed for the _Journal of Elasticity_ by Professor M. Gurtin of Carnegie-Mellon University. It was a pleasure to learn that, in sending the papers on to Professor D. Carlson for publication in the _Journal_, Professor Gurtin commented that "Uniqueness..." was "a very nice paper" and that "Periodic..." was "a very interesting paper".

The work discussed in the third paper, "Periodic and 'Slightly' Periodic Boundary Value Problems In Elastostatics On Bodies Unbounded In Several Directions," roughly parallels that communicated in the previous paper, but concentrates on those problems in which the body is unbounded in a direction other than the direction of periodicity. The analysis is complicated by the fact that the concept of mean stress is, in general, no longer meaningful, and no general theorem of work and energy appears valid. Nonetheless, it was still possible to prove results analogous to all those described in the previous paragraph (excluding the theorem of work and energy). In addition, results concerning the asymptotic behavior of periodic strain states "near infinity" (as measured in a direction perpendicular to the direction of periodicity) were developed. These asymptotic behavior results were, in turn, invaluable in proving the subsequent uniqueness and periodicity results. This paper included a special section in which half-plane problems were analyzed using the inherent arbitrary periodicity of the corresponding null problems. In this manner, uniqueness theorems similar to those already known for the displacement and traction problems on homogeneous isotropic half-planes were proven for the same problems without requiring that the material be isotropic.

It is likely that the above described paper will be submitted to the International Journal of Engineering Science by the end of 1980.

At present, the work just described is being continued for the special case in which the elastic body is composed of a homogeneous and isotropic material. This allows the use of (and development of additional) mean value and asymptotic behavior theorems. One major result is that it can be shown that certain components of the gradient of the displacement rapidly approach fixed values "near infinity". In fact, when the body is two dimensional, it has been found that the entire (periodic strain) solution has a particularly simple form "in the neighborhood of infinity" which depends, essentially, on four constants - two of which can be directly computed from the surface traction, while the other two measure the stress "at infinity" or, equivalently, the lack of periodicity in the displacement. Where these results hold, much stronger versions of the results mentioned in the previous paragraphs can and will be proven.

Preliminary work indicates that the study of problems involving periodicity in more than one direction may prove surprisingly fruitful. For one thing, it has been discovered that multiple periodicity imposes more restrictions on the solutions than might initially be suspected from the study of problems involving periodicity in one direction only. Also, it appears that a study of these problems will provide a means to extend the just described "two-dimensional asymptotic behavior near infinity" results to certain higher dimensional problems. Needless to say, this investigator is anxious to study these results further and to see how they may be used as tools in studying the asymptotic behavior, periodicity, and uniqueness of solutions both to multiply periodic and, especially, to "slightly" multiply periodic boundary value problems.

It, perhaps, should be mentioned that the literature contains many particular examples of each type of problem alluded to in the previous paragraphs. These examples have certainly influenced and, often, inspired the work described. As far as periodicity is concerned, however, all of the examples are solutions to or methods of solving particular problems. Outside of these attempts to solve particular problems, it would appear that the work summarized here is the first serious development and study of the "general theory" behind periodic boundary value problems in elasticity, and is apparently the first instance where questions concerning nonperiodic problems were answered using this theory.

It must be admitted that the proposer had not originally envisioned devoting so much time to the periodic boundary value problem, and had, in fact, anticipated that he would have been further along in the program which had been tentatively sketched out in the original grant proposal. On the other hand, the study of the periodic boundary value problem has proved much more rewarding and fruitful than expected - even leading to results concerning nonperiodic boundary value problems. As a result this researcher, although a little surprised, is not disappointed in the progress of his research.

It is intended, after "completing" the study of the periodic boundary value problem in elasticity, that a study of the axisymmetric problem in elasticity will be started. This research will actually be a renewal of some research which had been carried out by the researcher previously. Then, the Aleksandrov transformation had been used in an attempt to determine the relationship between axisymmetric problems and corresponding two-dimensional problems. It was hoped that certain uniqueness results which held for two-dimensional problems could be carried over to axisymmetric problems. Though progress had been made, it was not apparent that the boundary data transformed appropriately. It is hoped that these studies can be continued along various distinct lines. One is to replace the Aleksandrov transform with a modified version. In this manner one difficulty which arises when the axisymmetric body is not an exterior body - the convergence of certain integrals - will be overcome. Another would be the development of a new transform which would be more directly based on axisymmetry. The main advantage of this new transform is that it is expected that the general behavior of the axisymmetric solution will be directly carried over to the image solution. A possible difficulty, however, is that the image body is not expected to be homogeneous and isotropic whenever the axisymmetric body is homogeneous and isotropic. A third approach would be to use the "angular periodicity" inherent in the axisymmetric problem along with ideas mentioned in previous paragraphs. As, perhaps, already suspected particular attention will be given to the special case of the axisymmetric periodic boundary value problem.

It should be noted that the above described research - both that completed and that planned - does not exhaust the program outlined in the original proposal. Doubtlessly, however, by the time the above research will be "completed" the original grant period will be exhausted. Further support has been requested.

G-37-615

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT

NSF FORM 98A

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I—PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
School of Mathematics Georgia Inst. of Tech. Atlanta, GA 30332	Applied Mathematics	MCS-7906636
	4. Award Period From 6/15/79 To 11/30/81	5. Cumulative Award Amount

6. Project Title

Problems in Elastostatics Involving Unbounded Bodies

PART II—SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

This project was concerned with determining the appropriate "boundary conditions" in problems involving linearly elastostatic bodies of infinite extent. Here, "boundary conditions" include assumptions made on the behavior of the solutions at great distances from some fixed region. For the most general bodies, a differential inequality was employed to show that solutions are uniquely determined when the displacement is assumed suitably bounded at great distances from some fixed point. This bound depends on whether the body is essentially one-, two-, or three-dimensional and corresponds favorably to the assumptions often used in solving particular problems. Analogous results were also obtained for a large class of traction problems after developing a integral formula especially suited to traction problems. In this case suitable bounds were assumed on the stress field. Of especial interest in this study were those problems in which certain of the functions are periodic in some spatial direction. The investiagation of these problems was extensive and produced many general results showing--among other things--which assumptions implied uniquely determined and/or periodic solutions, and, via the construction of counterexamples, which assumptions allowed non-uniqueness or non-periodicity in the solutions. These results were then applied to problems involving a minimum of periodicity, e.g.: determining the uniqueness of solutions to the general traction problem on a homogeneous anisotropic half-plane.

PART III—TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (✓)	TO BE FURNISHED SEPARATELY TO PROGRAM: Approx. Date
a. Abstracts of Theses	X				
b. Publication Citations		X			
c. Data on Scientific Collaborators	X				
d. Information on Inventions	X				
e. Technical Description of Project and Results			X		
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
Dr. Kenneth B. Howell		2/15/82

Publications:

Uniqueness in Linear Elastostatics for Problems Involving Unbounded Bodies, _J. Elasticity_ 10 (1980) 407--427 .

Periodic and "Slightly" Periodic Boundary Value Problems in Elastostatics on Bodies Bounded in All But One Direction, _J. Elasticity_ 11 (1981) 293--316 .

Periodic and Slightly Periodic Boundary Value Problems in Elastostatics on Bodies Unbounded in Several Directions, _Int. J. Engng. Sci._ 20 (1982) 455--481 .