

PROJECT ADMINISTRATION DATA SHEET



ORIGINAL



REVISION NO. \_\_\_\_\_

Project No. G-35-689

DATE: 4-3-81

Project Director: Dr. D. M. Cunnold School/Lab Geophys. Sciences

Sponsor: Massachusetts Institute of Technology,  
Cambridge, Massachusetts, 02139

Type Agreement: Purchase Order No. SR-213943

Award Period: From 7-15-80 To 7-15-82 (Performance) 6/30/83 (Reports)

Sponsor Amount: \$ 49,209 (G-35-689) Contracted through:

Cost Sharing: \$ 2,590 (G-35-358) GTRI/GIT

Title: An Analysis of Observations of Gravity Waves  
and Turbulence at Millstone Hill

ADMINISTRATIVE DATA

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Reports: See Deliverable Schedule Security Classification: None

Defense Priority Rating: None

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Equipment: Title vests with N/A - None included

COMMENTS: Subcontract under prime NSF Grant  
No. ATM-800060

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SPONSORED PROJECT TERMINATION SHEET

Date 10/7/83

Project Title: "An Analysis of Observations of Gravity Waves and Turbulence at Millstone Hill

Project No: G-35-689

Project Director: Dr. D.M. Cunnold

Sponsor: Massachusetts Institute of Technology.

Effective Termination Date: 6/30/83

Clearance of Accounting Charges: 8/31/83

Grant/Contract Closeout Actions Remaining:

- Final Invoice ~~and Closing Documents~~
- Final Fiscal Report
- Final Report of Inventions
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CUNNOLD

GTRI

PROGRESS REPORT

for

An analysis of observations of gravity waves and  
turbulence at Millstone Hill

from

Dr. Derek M. Cunnold  
School of Geophysical Sciences  
Georgia Institute of Technology  
Atlanta, Georgia 30332

November 1982

Through discussions with Dr. Rastogi, the observations which were made at Millstone Hill on December 22-23, 1981 have been identified as being particularly interesting. The radar observations show the presence of a wave with a period of approximately one hour at altitudes below 14 km and a suggestion of higher frequency waves between 14 and 16 km.

The observations are being analyzed by a PhD candidate, Mr. Jayakumar, and myself with considerable assistance from Dr. Einaudi. We have obtained the radiosonde observations from Chatham, Mass.; Portland, Maine; and Albany, New York for the period 19-23 December, 1981. The individual radiosonde observations are being processed with a computer code which determines the Richardson number at each point on the profile. During this time period, the jet stream axis was oriented ENE-WSW and was located approximately over Millstone Hill. As a result, there are substantial vertical shears of the horizontal wind and we obtain a small Richardson number on the underside of the jet. The Richardson number is particularly sensitive, however, to the vertical gradient of temperature which is somewhat uncertain because of (standard) uncertainties in the temperature measurements and because of the vertical resolution of the measurements. We are currently investigating how these uncertainties translate into uncertainties in the Richardson number.

Having established that the windshear profile may be unstable at certain heights (where  $R_i < 0.25$ ), we plan to determine the expected vertical structure of the unstable modes using a computer code which solves the Taylor-Goldstein equation. Because the radar provides information along a single line of sight only, we are investigating the availability of surface pressure observations from a microbarograph array located in New England. If available, this infor-

mation should provide additional details on the horizontal wavelength and phase velocity of the waves.

Finally, we plan to examine whether there is evidence of an exchange of material between the troposphere and stratosphere during this event. For this purpose, we shall investigate the availability of columnar ozone observations with a horizontal resolution of approximately 50 km from the TOMS experiment on Nimbus-7.

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I—PROJECT IDENTIFICATION INFORMATION

1. Institution and Address School of Geophysical Sciences Georgia Institute of Technology Atlanta, Georgia 30332	2. NSF Program	3. NSF Award Number
	4. Award Period From 7/15/80 To 6/30/83	5. Cumulative Award Amount \$49,209

6. Project Title  
An analysis of observations of gravity waves and turbulence at Millstone Hill

PART II—SUMMARY OF COMPLETED PROJECT (FOR PUBLIC USE)

This project consisted of an analysis of observations made by the Millstone Hill stratosphere-troposphere (ST) radar for the period 2300-0300 GMT on December 22-23, 1981. The observations of Doppler shift indicate the presence of gravity waves at several heights with a period of approximately 1 hour. Analysis of this event has consisted of a spectral analysis to identify the principal periods present followed by processing with a bandpass filter to isolate the waves. A correlation analysis has also been used to identify phase relationships in height. A wave with a period of 1.7 hours is present in the vicinity of the strong shear and small Richardson number (indicated by radiosonde records) below the jet-stream maximum and a wave with a period of approximately 0.9 hours is present in the vicinity of the strong shear and small Richardson number above the jet maximum. Simultaneous observations from a network of ionospheric Doppler radars have been analyzed to determine whether these waves penetrated to the ionosphere. For this particular event, although waves are evident in the ionospheric data, the correspondence with the waves in the lower atmosphere appears to be poor. The radiosonde profile for 00 GMT December 23 at Chatham, Mass. is now being used in the model of Dr. Einaudi (J. Atmos. Sci., 33, 1730-1738, 1976) to estimate what wave periods with what vertical structure should be produced by the wind shears at this time.

PART III—TECHNICAL INFORMATION (FOR PROGRAM MANAGEMENT USES)

I. ITEM (Check appropriate blocks)	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM	
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a. Abstracts of Theses	X				
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d. Information on Inventions	X				
e. Technical Description of Project and Results		X			
f. Other (specify)					
2. Principal Investigator/Project Director Name (Typed) Derek M. Cunnold	3. Principal Investigator/Project Director Signature			4. Date 9/26/83	

Final Report

for

An analysis of observation of gravity wave  
and turbulence at Millstone Hill

by

D. M. Cunnold  
School of Geophysical Sciences  
Georgia Institute of Technology  
Atlanta, Georgia 30332

Under MIT requisition 79362 and Georgia Tech contract G-35-689

September 1983

## Introduction

The investigations proposed under this subcontract consisted of aiding in the analysis of Millstone data with the objective of examining the excitation and propagation of long period gravity waves in the atmosphere. This research was being conducted in the context of improving our understanding of the transport of trace species, particularly with a view toward large-scale modeling of this transport. The role of gravity waves in this regard has assumed increased importance in the last few years following Dr. Holton's suggestion (e.g., Holton, 1983) that gravity waves are responsible for substantial momentum transport in the mesosphere and, in fact, provide the transport necessary to close the stratospheric jet streams. If this conjecture proves to be correct, an important feedback (in both directions!) between large and small scale motions in the atmosphere will have been established. It will then be necessary to understand on what time scales the large scale circulation of the middle atmosphere may be modified by variations in the upward gravity wave flux. At the current time, for example, a universal spectrum of gravity wave energies has been proposed for the lower atmosphere (Van Zandt, 1982), but there is evidence for substantial day to day variability in the gravity wave energy in the middle atmosphere (Hirota, 1983). MST radars, such as that at Millstone Hill, are important tools for examining gravity wave propagation from the surface to the upper atmosphere.

Through discussions with Dr. Rastogi it was decided that we would analyze observations which were made in December 1981 when a strong tropospheric jet stream was located approximately over Millstone Hill. A 4 hour period on December 22, 23 was selected on the basis of evidence of wave activity at that time



noted by Dr. Rastogi. The objective of the research was then to examine whether wave coherence existed over the height range observed and whether the wave could be related to shear instabilities of the jet stream. To provide quantitative evidence of this possibility it was planned that the study should ultimately lead to calculations with the numerical model developed by Dr. Einaudi (see, for example, Mastrantonio et al., 1976). To constrain the projected calculations an attempt was made to obtain microbarograph array data for New England so as to define the horizontal propagation velocity of the waves. Unfortunately, this data was unavailable for this time period. We were, however, able to obtain Doppler shift data from a nearby array of ionospheric radars from Dr. Balanchandran. Although this data is not expected to provide as much information on tropospheric wave activity, because of the large altitude separation involved, it could be particularly useful for indicating how tropospheric gravity waves, and those generated by the jet stream in particular, propagate to the upper atmosphere.

In this report a spectral analysis of the Millstone Hill radar data for a selected 4 hour period is described together with analysis of the simultaneously obtain ionospheric data. Based on the radiosonde data we then outline the expected results from the instability model calculations which should be completed shortly.

## Radiosonde observations

The general meteorological situation over the United States is shown in the 500 mb chart of 0Z on December 23 given in Figure 1. The jet stream passes directly over New England flowing from WSW to ENE with a speed of approximately 70 knots. The surface weather map shows significant frontal activity associated with a North-South front over central Pennsylvania. This front moves rapidly through New England and 24 hours later is a long way East of the New England coast.

The radiosonde stations closest to Millstone Hill are located at Chatham, Massachusetts (approximately 170 km South-East); Albany, New York (approximately 200 km West); and Portland, Maine (approximately 150 km North-East). The soundings for 2300Z on December 22 at Chatham are shown in Figure 2. To examine the possibility of generation of gravity waves by the jet stream we need to calculate Richardson numbers for the flow. This requires the creation of a smooth curve from the radiosonde data. We have utilized a cubic spline function fit to the data but have ensured (by the addition of simulated measurements into the profile) that the spline function curvatures are not introducing spurious regions of small Richardson number into the results. The regions having a small Richardson number are highlighted in Figure 3. In particular, there are two regions where the Richardson number is less than 0.5—7.0 km and 13.8 km. These two regions correspond to almost adiabatic temperature gradients together with strong shears below and above the jet stream respectively. It is particularly significant for the instability calculations that the wind speed in both of these regions is approximately 48 m/sec. Moreover, it would only take 1°K error in the temperature measurements to produce Richardson numbers less than

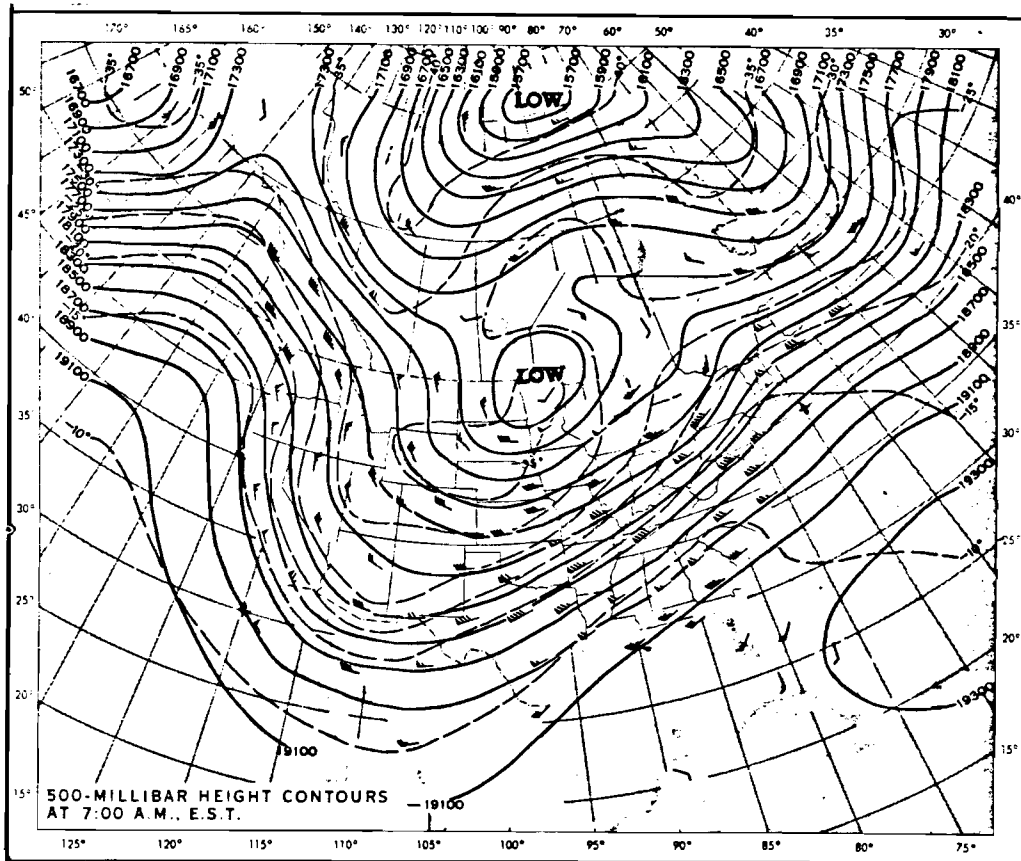
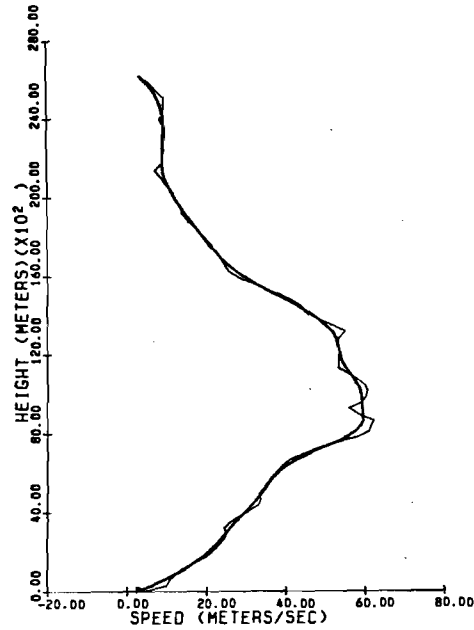
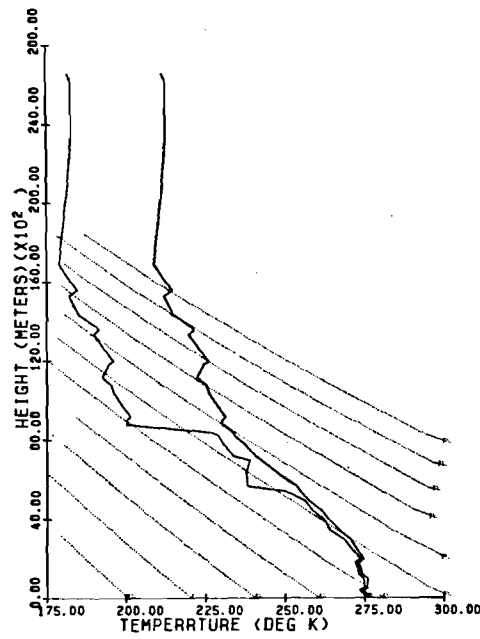


Figure 1: The 500 mb chart for the United States for 0Z December 23, 1981



(a)



(b)

Figure 2: The wind (a) and temperature (b) profile obtained by the radiosonde at Chatham, Massachusetts for 2300Z, December 22, 1981. The solid line is a curve fit to the profile which could be used in the model calculations. Also shown on the temperature profile are the dry adiabats (dashed line) and the dew point profile (lighter line).

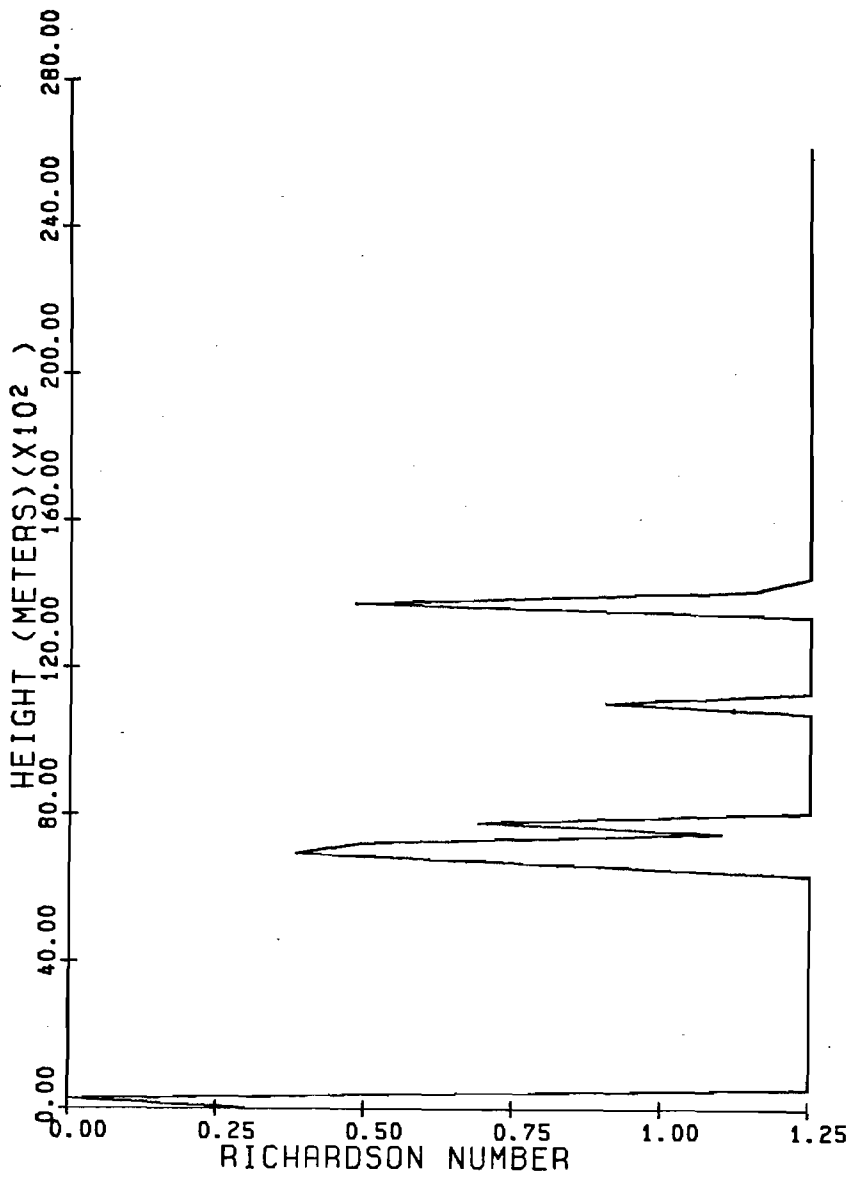


Figure 3: Richardson numbers (truncated at 1.25) for the 2300Z observations on December 22, 1981 at Chatham, Massachusetts.

0.25 in both regions (and much less than 0.25 at 13.8 km). The wind direction was  $280^\circ$  from North in the lower region and  $290^\circ$  in the upper region.

Unfortunately, the radiosonde observations at Albany and Portland at this same time both resulted in wind data below the jet stream maximum only. They do, however, indicate that over the entire New England region the Richardson number between approximately 7.2 and 7.5 km was approximately 0.5 where the wind speed was approximately 50 m/sec. Strong temperature gradients, although approximately 10% weaker than those at Chatham, were also observed near 13.8 km.

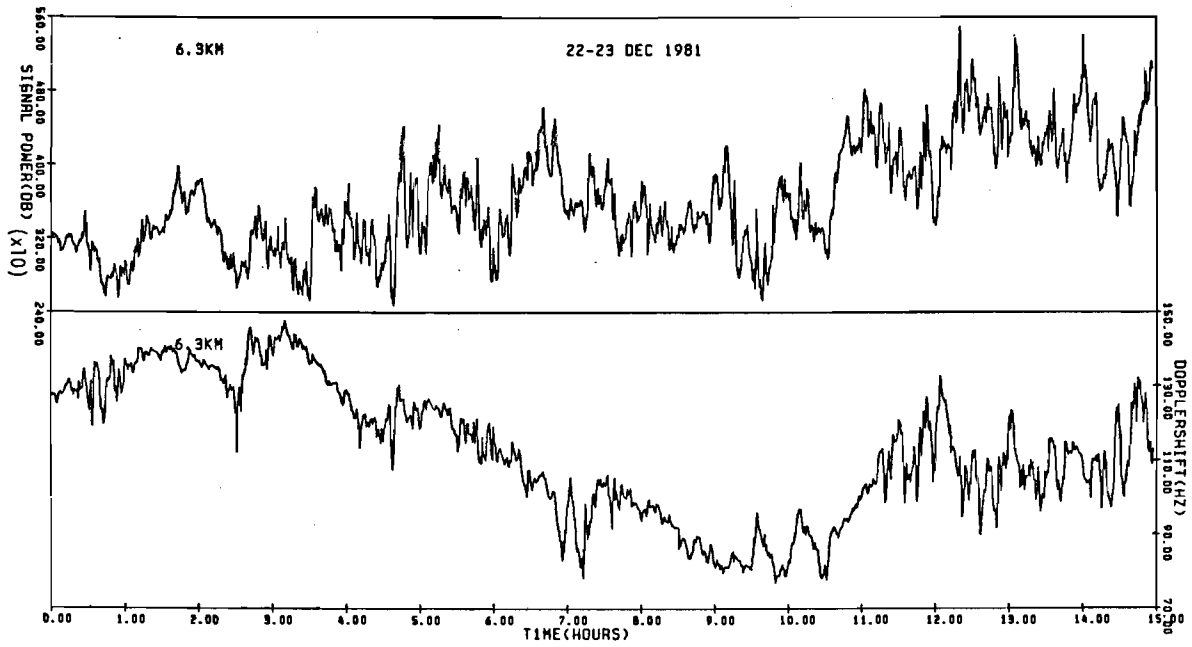
Twelve hours later the radiosonde observations suggest the possibility of instability 1 km or so higher up, near the jet stream maximum because of a strong temperature gradient there. The Albany and Portland observations also suggest the possibility of instability at approximately 15 km but this is not confirmed by the Chatham observations and in contrast to the situation 12 hours earlier the two levels of possible instability possess wind speeds which differ by almost a factor of 2.

## Millstone Hill radar observations

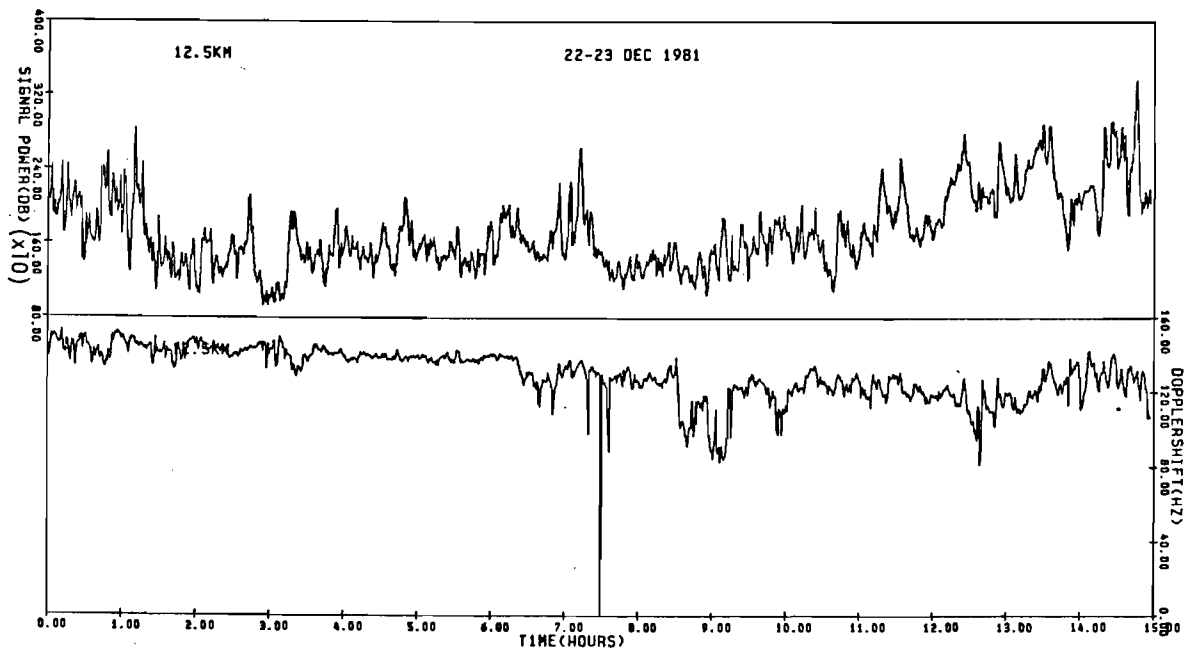
A tape of observations for continuous periods on December 19 and 20 and December 22 and 23, 1981, made with the incoherent scatter (440 MHz) radar at Millstone has been supplied by Dr. Rastogi. The observations were made at a 20° elevation angle and with the antenna pointed West (azimuth 270°). Doppler shifts and signal power are given at intervals of less than one minute and at altitude intervals of less than 1 km (see, for example, Wand et al., 1983). The vertical resolution of the experiment is approximately 1 km; we have selected the data records approximately every 2 km between 4.2 and 16.7 km to investigate for wave activity.

The time series of the observations at 6.3 and 12.5 km are shown in Figure 4 for December 22 and 23 (with time being measured from 2215 GMT on December 22). A spectral analysis by Dr. Rastogi suggested the presence of waves of approximately 1 hour period early in this data record. The Doppler shift record at 12.5 km in particular suggests the presence of such a wave during the first 4 hours of operation. The largest signal powers, of the heights analyzed, are obtained at 6.3 km presumably because this is close to the altitude of maximum shear in the horizontal wind and to a location of 7.2 km where gravity waves of various scales might be generated. The signal power at 6.3 km also appears to be modulated on a 1 hour time scale, perhaps because a gravity wave is modulating the generation of the small scale irregularities which the radar observes. This possibility will be pursued no further in this report and instead we concentrate solely on the modulation of the Doppler shift (i.e., horizontal wind) at the various levels.

As a first step in analyzing the observations between 2300 GMT December 23



(a)



(b)

Figure 4: The Millstone Hill radar observations of signal power (upper curve) and Doppler shift (lower curve) at 6.3 km (a) and 12.5 km (b) starting at approximately 2215 GMT on December 22, 1981.



and 0300 GMT December 23, 1981, a spectral power analysis was made. This analysis used a standard procedure which is described, for example, in Parzen (1964) to obtain the autocorrelation function for the time series and utilized a weighting (window) function of the form

$$\begin{aligned}
 K(\tau) &= 1 - 6\tau^2 + 6\tau^3 & 0 \leq |\tau| \leq 0.5 \\
 &= 2(1 - u)^3 & 0.5 \leq |\tau| \leq 1 \\
 &= 0 & |\tau| \geq 1,
 \end{aligned}$$

where  $\tau$  is the time lag divided by  $M$ , to obtain the Fourier transform of the autocorrelation function and hence the power spectrum of the time series. We have used an  $M$  value corresponding to 40% of the available data points which implies error bars on the spectral estimates of a factor of approximately 1.6. Prior to processing, the mean and linear trend over the 4 hour period were removed. Figure 5 shows the resulting spectra at 6.3, 12.5 and 14.6 km.

At each of the heights shown in Figure 5, a wave with a period of approximately 1 hour appears to be present in the data and at the upper levels there is also evidence of a wave with a period of approximately 25 minutes. To further examine whether a coherent wave exists in this data set we have used a passband filter based on the design described by Rabiner, McGonegal and Paul (1979). This filter consists of a rectangular function in the spectral domain which is convolved with delta functions at zero frequency and at a frequency corresponding to the temporal resolution of the data. This filter has negligible sidelobes in the spectral domain and produces no phase shift bias in the passband.

The radar data was processed with a passband of 0.7 to 2.0 hours. The

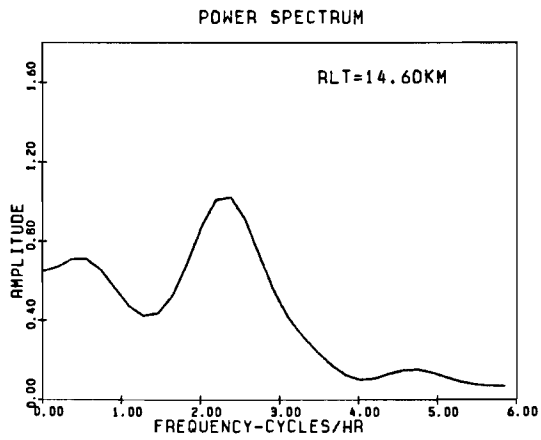
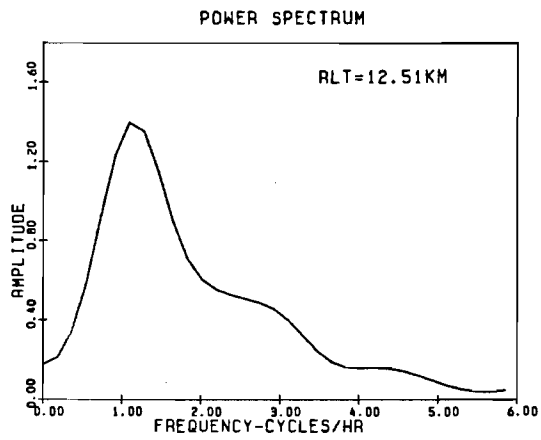
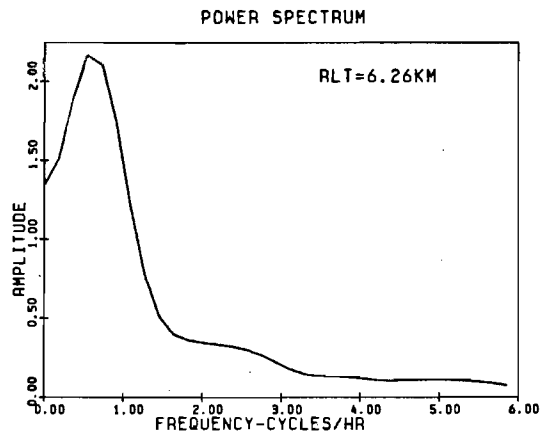


Figure 5: Power spectra obtained from the Millstone Hill observations at 6.3, 12.5, and 14.6 km for the period 2300 GMT, December 22, 1981 to 0300 GMT, December 23, 1981.

resulting signals at 6.3 and 12.5 km are shown in Figure 6. Wave activity in this range of periods is evident throughout almost the entire period at 12.5 km with the purest sine wave (with a period of less than 1 hour) being evident after the first half hour. At 6.3 km, on the other hand, the wave activity appears to have a period closer to 2 hours. To assist in the interpretation of these data and to see in particular whether there is a phase shift with height, we have cross-correlated the filtered data records between neighboring heights. Between 6.3 and 8.3 km there is a highly significant correlation for a wave of period 1.7 hours with the 8.3 km phase leading that at 6.3 km by approximately 0.4 hours (see Figure 7). This would correspond to downward phase propagation at 1.4 m/sec. This wave also appears in the 10.4 km record with the phase leading that at 8.3 km by approximately 0.3 hours. The wave does not appear to be present in the 4.2 km record or the 12.5 km record (although there is some evidence of it at 14.6 km).

The correlation between 12.5 km and 14.5 km shows evidence of a common periodicity of 0.9 hours and with the two altitudes possessing approximately opposite phases (Figure 7). The largest correlation for this wave actually occurs between the 12.5 and 10.4 km records (reaching 0.9) with the phase at 12.5 km leading that at 10.4 km by approximately 0.3 hours. Averaging these results suggests that this wave also possesses downward phase propagation at a speed of approximately 1.4 m/sec. The 16.7 km data is considerably noisier than the data at the other levels. It primarily shows evidence of this wave only in the second half of the 4 hour period and accordingly does not show a strong correlation with 14.6 km.

The wave with a period of approximately 25 minutes noted in the spectrum

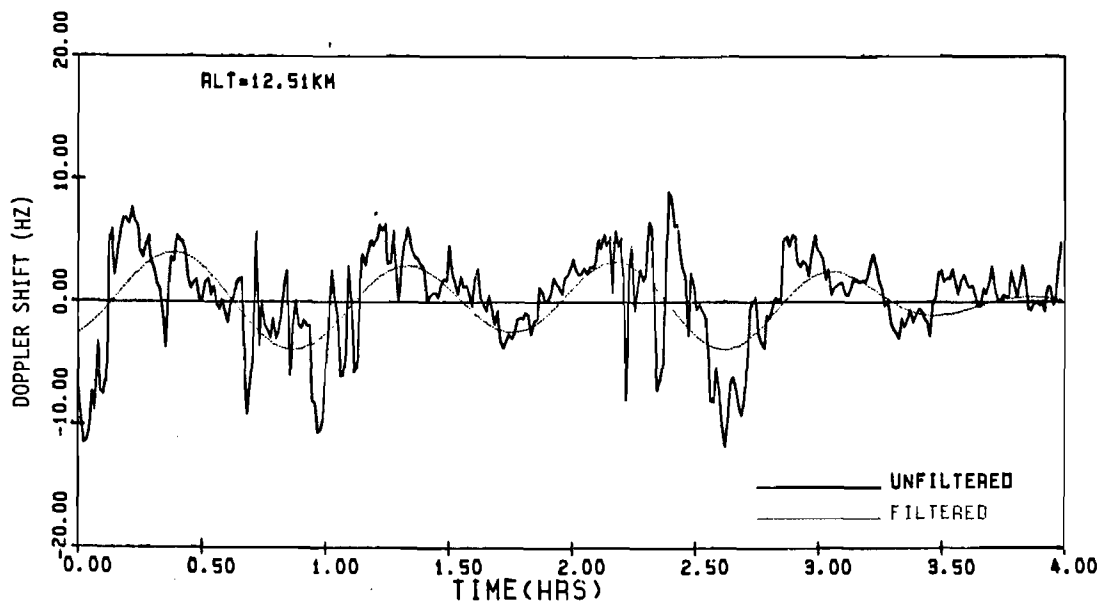
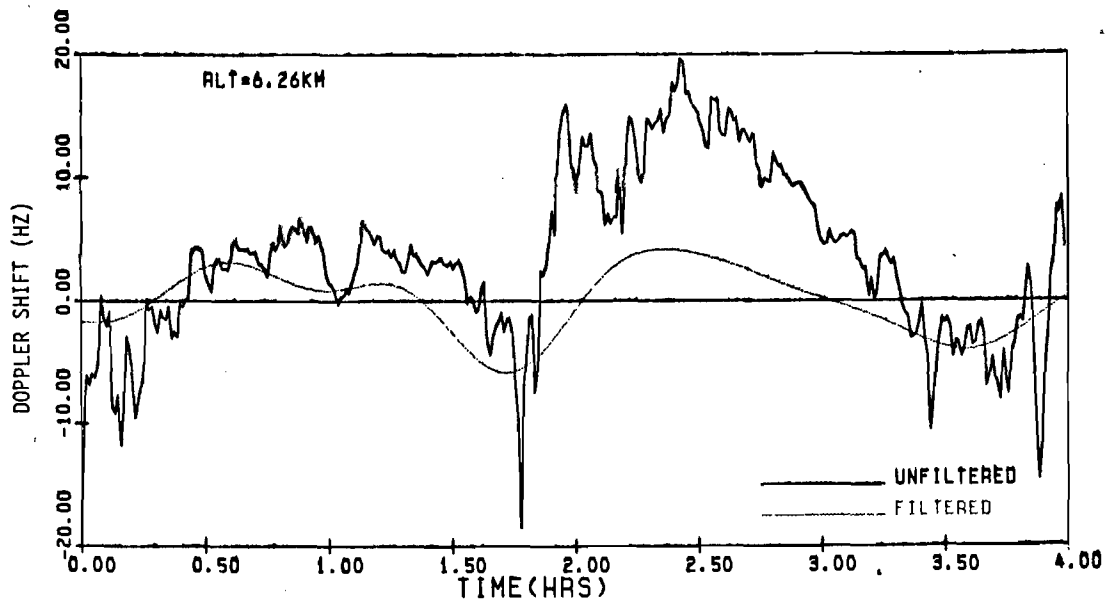


Figure 6: The Millstone Hill radar data at 6.3 and 12.5 km starting at 2300 GMT December 22, 1981 before and after a 0.7 to 2.0 hour period band-pass filter was applied to the data.

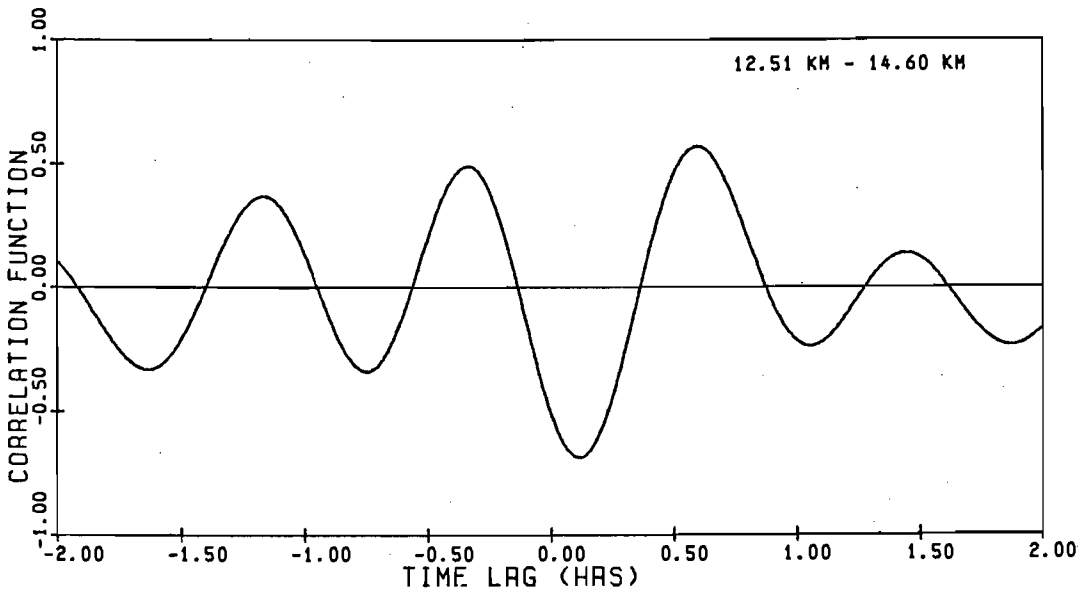
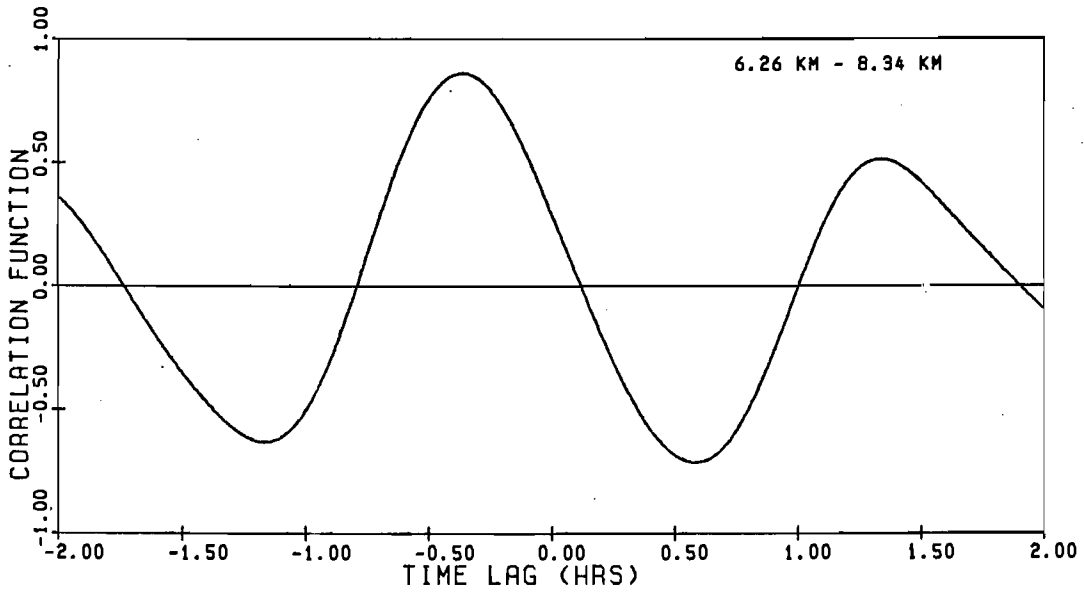


Figure 7: The correlation between the filtered signals at altitudes differing by approximately 2 km for Millstone Hill radar data for the period 2300 GMT, December 22 to 0300 GMT December 23, 1981.

at 14.6 km has also been analyzed by using a passband filter with cutoffs at 0.3 and 0.7 hours. The wave is predominantly present during the first 2.5 hours of that record (see Figure 8) but appears more in the second half of the 2 hour period at 12.6 and 16.7 km. The correlations between the levels thus only attain values of 0.5. We conclude that the 25 minute wave which shows up as a strong peak in the 14.6 km power spectrum is not well correlated with similar wave activity at the other levels analyzed.

Summarizing these results, waves possessing downward phase propagation of approximately 1.4 m/sec. have been detected. A wave with a period of 1.7 hours is present in the vicinity of the strong shear and small Richardson number below the jet maximum and a wave with a period of approximately 0.9 hours is present in the vicinity of the strong shear and small Richardson number above the jet maximum.

The amplitude of those waves corresponds to approximately 3.0 Hz in Doppler shift. If the waves are propagating in the direction of the jet axis, this implies a wave amplitude of 1 m/sec. in horizontal velocity.

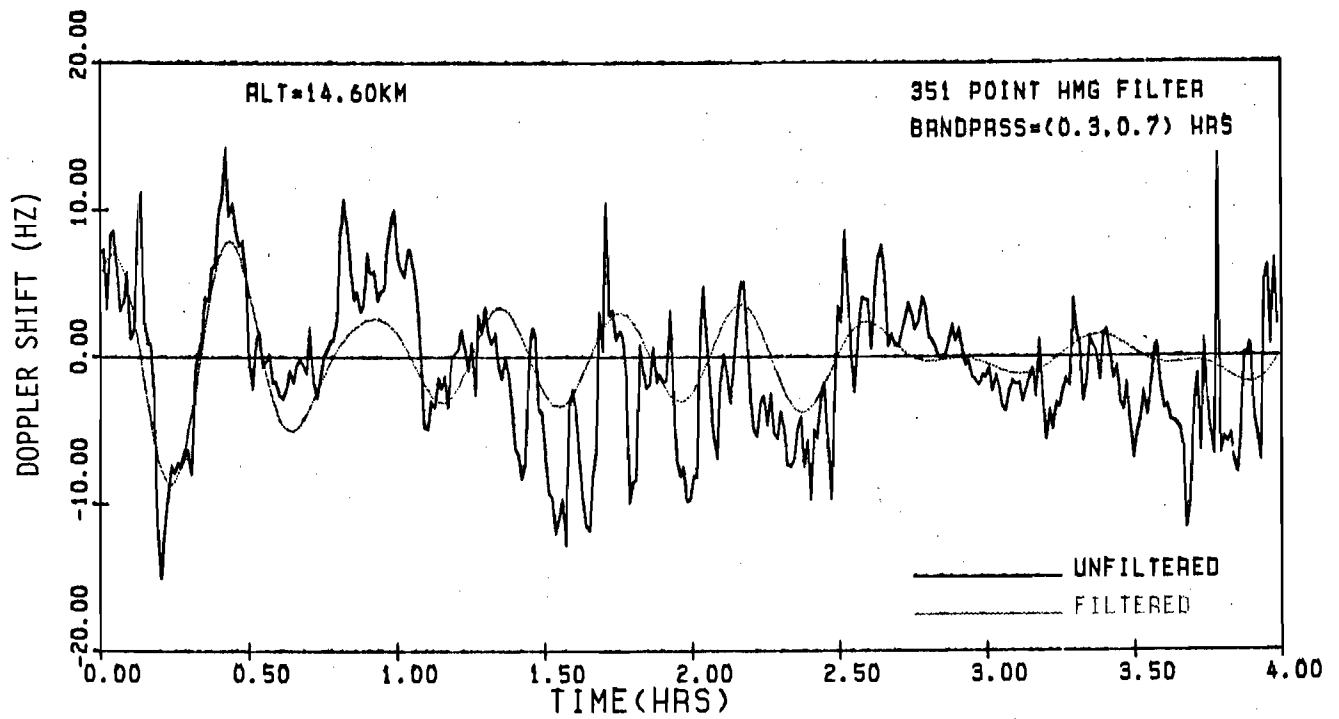


Figure 8: The Millstone Hill data at 14.6 km for the period commencing at 2300 GMT December 22, 1981 before and after a band-pass filter with cutoff periods of 0.3 and 0.7 hours was applied to the data.

## Ionospheric data

Chart records of Doppler shifts at a network of ionospheric radar stations have been obtained from Dr. Balachandran (Lamont-Doherty Geological Observatory, New York). The transmitting stations are roughly uniformly distributed in direction approximately 100 km away from the receivers at the Lamont-Doherty Geological Observatory, Palisades, New York (approximately 270 km South-East of Millstone Hill). Ionospheric reflection points occur at the corners of a square whose sides are approximately 50 km. The transmitting stations are at Farmingdale (East of Palisades), Evans (South), Newton (West) and Newburgh (North). Two additional transmitters operating on different frequencies are located at the Newton site to provide vertical information on waves. Doppler shifts of 0.1 Hz correspond to a speed of 6 m/sec. (Balachandran, personal communication). These shifts arise from variations in the ionospheric (~ 250 km) reflection level or variations in electron density close to that level and in the case of travelling waves correspond to vertical phase velocities of those disturbances. Phase differences between the records from the different transmitting sites provide information on the horizontal velocities of the disturbances.

The chart records have been digitized by hand at intervals of approximately 4 minutes (which was considered adequate to resolve waves whose period were approximately 1 hour) and analyzed by procedures similar to those used for the radar data. The power spectra (e.g., Figure 9) for the period 22:15 GMT December 22nd to 02:40 GMT December 23rd, 1981, show a wave with a period of approximately 40 minutes as well as some activity at a period of roughly 20 minutes. The spectral peak at 40 minutes is more consistent from station to station than in the peak at 20 minutes.



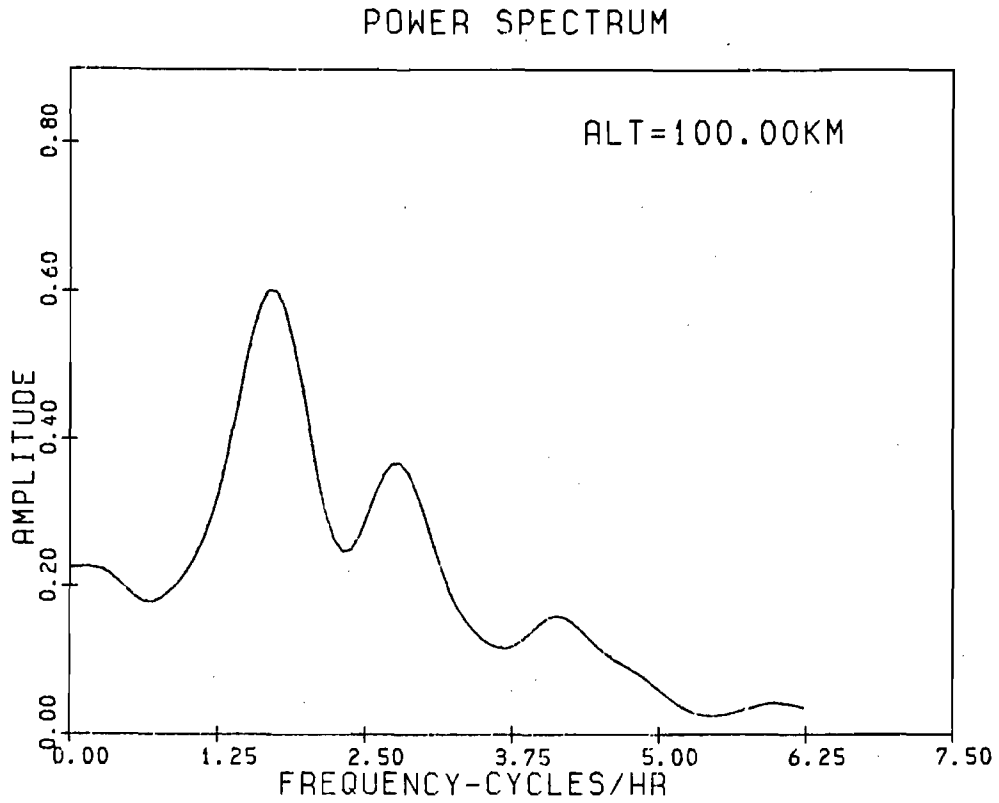


Figure 9: The power spectrum of the ionospheric observations at Newton, New York at 4.5 MHz for the period 2215 GMT December 22 to 0240 GMT December 23, 1981.

The 40 minute spectral peak has been examined using a passband filter with cutoffs at periods of 0.5 and 2.0 hours. The filters (plus the raw) signal at Newton is shown in Figure 10. Note that on this curve, zero time corresponds to a time of 0.3 hours on the radar data of Figure 6. Early in the data record a wave with a period of 0.6 hours seems to be present but after 0100 (1.7 hours) a wave with a period closer to 1 hour appears. This change from a short period wave early on to a longer period later is shown in all the radar records. Correlations near 0.9 are found for small time lags between all of the stations. However, only for the Newburgh-Farmingdale correlation (Figure 11) does the 1 hour period wave appear to control this correlation (at Newton the correlation between different altitudes seems to be controlled by the 0.6 hour period wave).

To study the 1 hour period wave we need to isolate the 0Z observations and extend the data record past 02:40Z. However, both visual inspection of the last 2 hours of data and our station to station correlation analysis indicates that the low frequency waves at Evans and Newburgh are approximately in phase and that small differences of order 0.1 hours exist with Farmingdale and Newton. During the period when the 1 hour period wave appears to be present there are intermittantly two ionospheric reflections as well as weaker than normal reflections which render the data interpretation difficult. Preliminary indications are that the wave is propagating in the direction opposite to that of the tropospheric jet stream at a speed in excess of 100 m/sec. The correspondence between the tropospheric and the ionospheric observations is thus not obvious at this time. Further analysis of the ionospheric record is needed together with an investigation of whether wind conditions in the stratosphere were such that a wave generated in the tropospheric jet stream could have reached the ionosphere

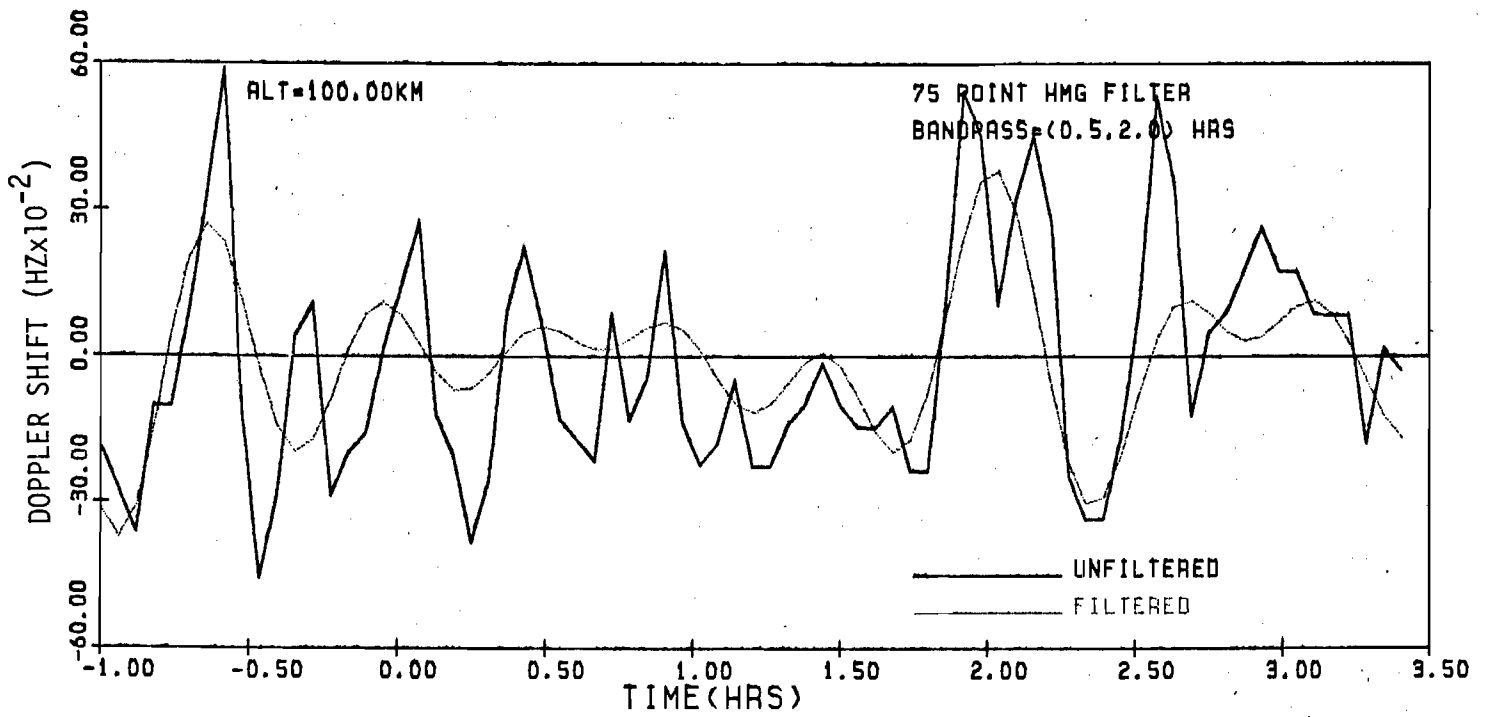


Figure 10: The ionospheric data for Newton, New York for 2215 GMT December 22 to 0240 GMT December 23, 1981 before and after a band-pass filter with cutoff periods of 0.5 and 2.0 hours was applied to the data.

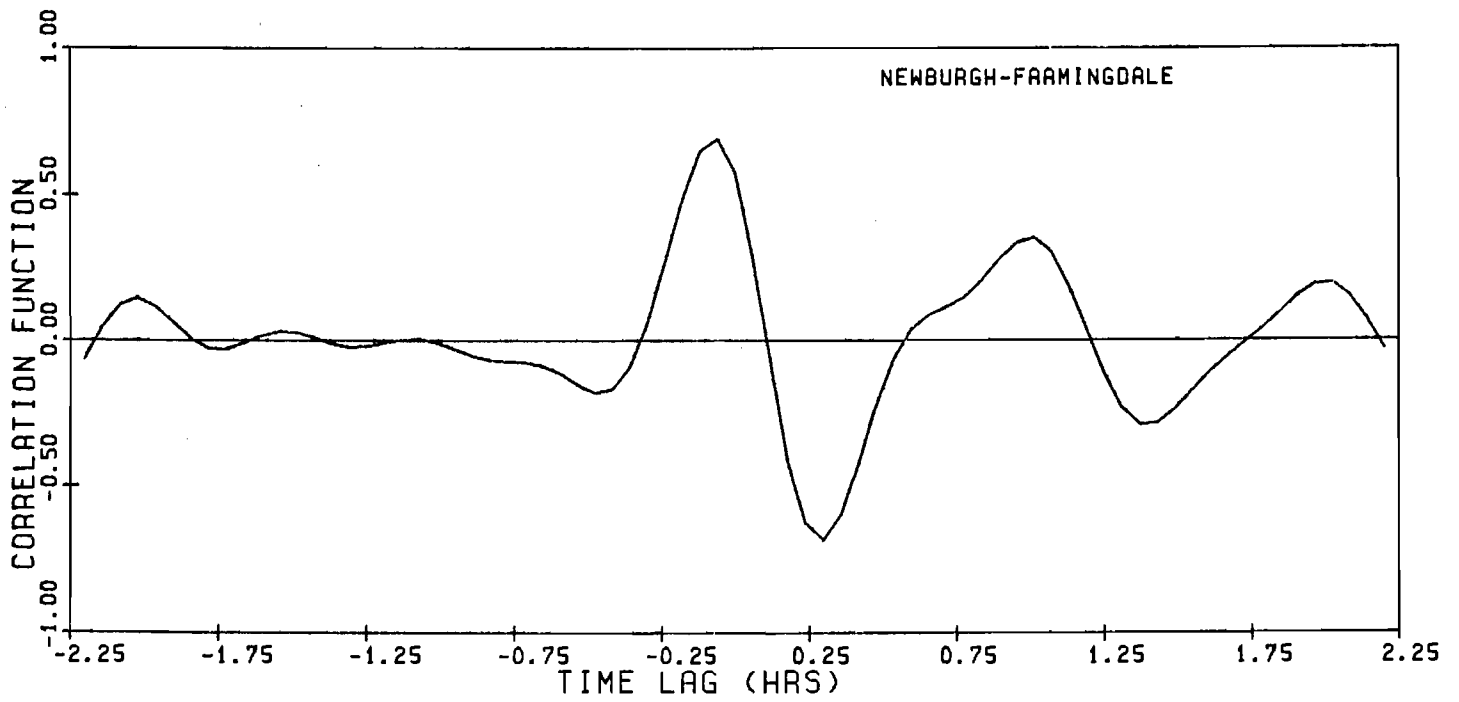


Figure 11: The correlation between the ionospheric reflections from transmitting sites at Newburgh and Farmingdale after processing the chart records through a filter with a passband of 0.5 to 2.0 hours.

(that is, that there was no critical level for the wave in the stratospheric polar night jet).

## Modeling considerations

It has been shown by Lalas and Einaudi (1976) that unstable waves having relatively large horizontal wavelengths may be generated in a region where the Richardson number is less than 0.25. The longest wavelengths are generated in their mode 3 which may exist because of resonance in the region between the surface and the critical layer where the wave is generated (Lalas and Einaudi, 1976). The radar results indicate the presence of a wave having a 1 hour period and we propose that it is generated where the wind speed is approximately 50 m/sec. This must then approximately be the critical level for this wave which implies that its wavelength (in the direction of the jet) is  $50 \times 3.6 = 180$  km. In order to preferentially produce such a wave, the results of Lalas and Einaudi suggest that the minimum Richardson number would have to be less than 0.1. Their mode 3 propagates both below and above the shear region for a critical (i.e., generation) level 10 km above the ground and possesses a vertical wavelength of approximately 20 km.

A more relevant atmospheric wind and temperature profile is used in similar model calculations by Mastrantonio et al. (1976). These calculations allow for the fact that a wave generated by the jet stream will possess two critical levels which may produce significant enhancement or destruction of an unstable wave. The calculations, inasmuch as they attempt to allow more degrees of freedom than the less complex calculations of Lalas and Einaudi (1976), are less complete than those calculations. They are, however, the basis of a computer code which is capable of accepting fairly realistic wind and temperature profiles. They also provide a framework within which the generation of gravity waves by the jet stream can be further discussed.

In the calculations of Mastrantonio et al. (1976) there exists a mode, with a small horizontal wavenumber, similar to that found by Lalas and Einaudi (1976). Again, to generate the large horizontal wavelength found in the radar observations, the Richardson number would have to be less than 0.1. They obtain a wave (their mode 1) which is responsible for vertical energy propagation above the upper critical level and which is evanescent below the lower critical level. The vertical wavelength substantially above the upper critical level is roughly 20 km but in the vicinity of the critical level it appears to be much shorter than this (the radar data implied a vertical wavelength near the critical level of approximately 5 km). The largest Doppler shifts (in the radar data) would be expected at and above the upper critical level because of the tendency for the wave to amplify as the square root of the density. In those model calculations mode 1 disappeared as the thickness of the jet increased. We suspect, however, that this is because of the absence of a Richardson number less than 0.25 at the lower critical level rather than because of the change in layer thickness per se. In fact, since the thickness of the jet on December 22nd, 1981, was comparable to that in their second calculation in which mode 1 was absent, our primary modeling emphasis is to evaluate what values of Richardson number (which is of course poorly defined by radiosonde observations where the temperature gradient is almost superadiabatic) would be needed at the upper and lower critical levels to produce mode 1.

Our model calculations are based on solving the linearized equations of motion in the Boussinesq limit with rotational effects neglected for a perturbation of the form

$$\rho_0^{-1/2} \operatorname{Re}\{\psi(z) \exp[i(k_x X - \omega t)]\}$$

where  $\rho_0$  is the background density at altitude  $z$ ,  $k_x$  is the horizontal wavenumber (in the direction of the jet axis), and  $\omega$  is the frequency of the disturbance. The code then solves the equation (Mastrantonio et al., 1976)

$$\frac{d^2 \psi}{dy^2} - \Lambda(y) \psi = 0$$

where

$$\Lambda = \alpha^2 - \frac{1}{4} \frac{1}{\rho_0^2} \left( \frac{d\rho_0}{dy} \right)^2 + \frac{1}{2\rho_0} \frac{d^2 \rho_0}{dy^2} - \frac{n^2 h^2}{V^2 \Omega^2} - \frac{1}{\Omega} \left[ \frac{1}{\rho_0} \frac{d\rho_0}{dy} \frac{du_0}{dy} + \frac{d^2 u_0}{dy^2} \right]$$

$$\alpha = h k_x ,$$

$$\Omega = \frac{w}{k_x V} - u_0 ,$$

$u_0(y)$  is the horizontal wind profile (with maximum  $V$ ),

$\rho_0(y)$  is the profile of atmospheric density,

$y = z/h$  where  $h$  is an assignable characteristic length scale, and

$n^2$  is the Brunt-Väisälä frequency. The current code is capable of

allowing for compressibility. Modifications needed to include the effect of rotation and a horizontal wavenumber component perpendicular to the jet stream have been worked out by Stobie (1983).

Using this model and inserting reasonable wind and temperature profiles for December 22/23, 1981, we hope to be able to demonstrate that a 1 hour period gravity wave could have been generated on the upper side of the jet stream pro-



vided that reasonable assumptions are made about possible errors in the temperature (and perhaps wind) profiles. We expect to need a Richardson number less than 0.1, for example. We believe, however, that the observed conditions are particularly promising for wave generation because of the small Richardson number obtained at two altitudes possessing similar wind speeds.

## Accomplishments

This study indicates that a gravity wave of approximately 1 hour period may have been produced by the horizontal wind shear on the upper side of the jet stream over Millstone Hill on December 22, 1981. Analysis of ionospheric records has not yet yielded any good evidence that the wave was able to penetrate to ionospheric altitudes. Of more importance than the analysis of this single event, spectral analysis procedures have been coded for the Data General Eclipse computer at Georgia Tech so that similar Millstone Hill radar data and ionosonde data can be analyzed for the presence of waves. Moreover, numerical model calculations concerning instability of individual wave modes are now being made for wind and temperature profiles obtained simultaneously from the nearest radiosonde stations to Millstone Hill. This model can provide support for conclusions concerning jet stream generated waves observed with the Millstone Hill radar. Moreover, the type of data sets analyzed here together with an existing microbarograph array (from which data was unavailable for the time period studied) represent a powerful resource for studying the gravity wave connection between the lower and upper atmosphere.

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